

Q1  $E(\text{Score}) = 4 \cdot P(\text{score}=4) + 3 \cdot P(\text{score}=3) + 2 \cdot P(\text{score}=2) + 0$

$$4 \cdot P(\text{score}=4) = 4 \int_0^{\frac{1}{3}} f(x) dx = 4 \cdot \frac{1}{3} = \frac{4}{3}$$

$$3 \cdot P(\text{score}=3) = 3 \int_{\frac{1}{3}}^{\frac{1}{2}} f(x) dx = 3 \cdot \frac{1}{6} = \frac{1}{2}$$

$$2 \cdot P(\text{score}=2) = 2 \int_{\frac{1}{2}}^{\frac{2}{3}} f(x) dx = 2 \cdot \frac{1}{6} = \frac{1}{3}$$

therefore

Expected score of single shot  
 $= \frac{4}{3} + \frac{1}{2} + \frac{1}{3} = \frac{8+3+2}{6} = \frac{13}{6}$

Q2  $n$  observations  $x_1, x_2, x_3, \dots, x_n$  independent

$$P(x_1, x_2, \dots, x_n) = P(x_1)P(x_2) \dots P(x_n)$$

$$= \theta e^{-\theta x_1} \theta e^{-\theta x_2} \dots \theta e^{-\theta x_n}$$

Want to Maximize this value  $\theta^n e^{-\theta(\sum_{i=1}^n x_i)}$

$$\left[ \theta^n e^{-\theta(\sum_{i=1}^n x_i)} \right]' = n \theta^{n-1} e^{-\theta(\sum_{i=1}^n x_i)} + \theta^n e^{-\theta(\sum_{i=1}^n x_i)} \cdot \left( -\sum_{i=1}^n x_i \right) = 0$$

$$\text{RR} \quad n = \theta \sum_{i=1}^n x_i$$

$$\theta = \frac{n}{\sum_{i=1}^n x_i}$$

Q3 a7.  $X^T A X = [x_1 \dots x_n] \begin{bmatrix} \sum_{i=1}^n a_{1i} x_i \\ \sum_{i=1}^n a_{2i} x_i \\ \vdots \\ \sum_{i=1}^n a_{ni} x_i \end{bmatrix}$

$$= x_1 \sum_{i=1}^n a_{1i} x_i + x_2 \sum_{i=1}^n a_{2i} x_i \dots + x_n \sum_{i=1}^n a_{ni} x_i$$

$$= \sum_{i=1}^n \sum_{j=1}^n a_{ij} x_i x_j$$

b) Symmetric then ~~if~~  $a_{ij} = a_{ji}$

plug in  $x = e_1, e_2, e_3, \dots$

$$x_i = e_i = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \rightarrow i\text{th}$$

$$x_i^T A x_i = a_{ii} > 0 \quad \text{according to the definition of positive matrix}$$

Q4: a7.  $A$  positive  $X^T A X > 0$ .

~~$A$~~   $X^T (A + rI) X = X^T A X + X^T rI X = X^T A X + r X^T X$   
 from definition.  $X^T A X > 0$  and  $r X^T X$  also  $> 0$ .  
 then  $A + rI$  also positive.

b7.  $V$  eigenvector and  $\lambda$  the corresponding eigenvalue.  
 then  $AV = \lambda V$ .

$$V^T A V = V^T \lambda V = \lambda V^T V > 0 \Leftrightarrow V^T A V > 0$$

and  $V^T V > 0$  then  $\lambda > 0$

$\therefore$  all eigenvalue of  $A$  greater than zero

c7.  $A$  positive  $\Rightarrow \forall x \in \mathbb{R}^n \quad X^T A X > 0$ .

then Nullspace of  $A$  is  $\emptyset$ .

Because if  $\exists x \quad Ax = 0$  then  $X^T A X = 0$   $\otimes$

Null space  $= 0$   $A$  has  $n$  independent column span  $\mathbb{R}^n$   
 thus invertible.

d7.  $A$  Symmetric then  $A$  can be written as  $U^T \Lambda U$ .

$U^T$  constructs by ~~eigenvectors~~ and  $\Lambda$  is diagonal, eigenvalues.  
 normalized eigenvectors

~~then assume~~  $v_1, v_2, v_3, \dots, v_n$  are  $n$  normalized eigenvector  
 then  ~~$A_{ij} = \lambda_i \delta_{ij}$~~

$\Lambda$  diagonal and all entries  $> 0$

then

$$\begin{vmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \lambda_n \end{vmatrix} = \begin{vmatrix} \sqrt{\lambda_1} & & 0 \\ & \sqrt{\lambda_2} & \\ 0 & & \sqrt{\lambda_n} \end{vmatrix}^2 = (\sqrt{\Lambda})^2$$

$\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n$

are  $n$  eigenvalues

all of them are positive

$$\text{therefore } A = U^T \Lambda U = (\sqrt{\Lambda})^T (\sqrt{\Lambda} U)$$

Q5.

a).  $x, a \in \mathbb{R}^n$  get  $\frac{\partial (x^T a)}{\partial x}$

$$f(x) = x^T a = x_1 a_1 + x_2 a_2 + x_3 a_3 + x_4 a_4 \dots$$

$$\frac{\partial (x^T a)}{\partial x} = \begin{bmatrix} \frac{\partial f(x)}{\partial x_1} \\ \frac{\partial f(x)}{\partial x_2} \\ \vdots \end{bmatrix}^T = a^T$$

$$(a^T)^T = a$$

b).  $A \in \mathbb{R}^{n \times n}$   $x \in \mathbb{R}^n$   $\frac{\partial (x^T A x)}{\partial x}$

$$Ax = \begin{bmatrix} \sum A_{1i} x_i \\ \sum A_{2i} x_i \\ \vdots \end{bmatrix}$$

$$f(x) = x^T A x = x_1 \sum A_{1i} x_i + x_2 \sum A_{2i} x_i \dots + x_n \sum A_{ni} x_i$$

$$\frac{\partial f(x)}{\partial x_k} = x_1 A_{1k} + x_2 A_{2k} + \dots + x_n A_{nk}$$

$$\sum_{i \neq k} A_{ki} x_i + 2A_{kk} x_k$$

$$= \sum_{i=1}^n A_{ki} x_i + \sum_{i=1}^n A_{ik} x_i$$

$$\frac{\partial f(x)}{\partial x} = \left( \sum_{i=1}^n A_{1i} x_i + \sum_{i=1}^n A_{i1} x_i \right), \left( \sum_{i=1}^n A_{2i} x_i + \sum_{i=1}^n A_{i2} x_i \right) \dots$$

$$= x^T A^T + x^T A$$

$$(x^T A^T + x^T A)^T = A^T x + A^T x$$

$$c) (XA)_{ij} = \sum_{k=1}^n X_{ik} A_{kj} \quad \text{Trace}(XA) = \sum_{i=1}^n (XA)_{ii}$$

$$= \sum_{i=1}^n \sum_{k=1}^n X_{ik} A_{ki}$$

$$\frac{\partial (XA)}{\partial X_{ab}} = A_{ba} \quad \left( \frac{\partial (XA)}{\partial X} = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1n} \\ A_{21} & A_{22} & \dots & A_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1} & A_{n2} & \dots & A_{nn} \end{bmatrix} = A^T \right)$$

d) want to proof

$$\sqrt{\sum X_i^2} \leq \sum |X_i| \leq \sqrt{n} \sqrt{\sum X_i^2}$$

$$(\sqrt{\sum X_i^2})^2 \leq (\sum |X_i|)^2$$

$$\sum X_i^2 \leq \sum |X_i|^2 + 2 \sum_{i=1}^n \sum_{j=1}^n |X_i| \cdot |X_j| \quad i \neq j$$

greater or equal to zero

therefore

$$\|X\|_2 = \|X\|_1$$

$$\text{want to show } \sum |X_i| \leq \sqrt{n} \sqrt{\sum |X_i|^2}$$

$$(\sum |X_i|)^2 \leq \sum |X_i|^2 \sum 1^2$$

Apply Cauchy inequality

$$\sum a_i^2 \sum b_i^2 \geq (\sum a_i b_i)^2$$

$$\sum |X_i|^2 \sum 1^2 \geq (\sum |X_i| \cdot 1)^2$$

Done

$$e) \text{ Maximize } pX = X^T Z \quad | \|X\|_1 \leq 1$$

$$X^T Z = X_1 Z_1 + X_2 Z_2 + \dots + X_n Z_n \leq \max(|Z_i|) (Z_1 + Z_2 + \dots + Z_n) \leq \max(|Z_i|) \cdot 1$$

Q6 a7.  $P(X|W_i) = N(\mu_i, \sigma^2)(x)$ .  $P(W_i|x) = \frac{P(X|W_i)P(W_i)}{P(X)}$

$$P(X) = P(X|W_1)P(W_1) + P(X|W_2)P(W_2) = \frac{1}{2}N(\mu_1, \sigma^2)(x) + \frac{1}{2}N(\mu_2, \sigma^2)(x)$$

$$= \frac{\frac{1}{2} \cdot N(\mu_1, \sigma^2)(x)}{\frac{1}{2} [N(\mu_1, \sigma^2)(x) + N(\mu_2, \sigma^2)(x)]}$$

$$P(W_1|x) = \frac{\frac{1}{2} N(\mu_1, \sigma^2)(x)}{\frac{1}{2} [N(\mu_1, \sigma^2) + N(\mu_2, \sigma^2)](x)} = P(W_2|x) = \frac{\frac{1}{2} N(\mu_2, \sigma^2)(x)}{\frac{1}{2} [N(\mu_1, \sigma^2) + N(\mu_2, \sigma^2)](x)}$$

$$\mu_1 \leq \mu_2$$

$$N(\mu_1, \sigma^2)(x) = N(\mu_2, \sigma^2)(x)$$

$$\frac{1}{\sqrt{2\sigma^2\pi}} e^{-\frac{(x-\mu_1)^2}{2\sigma^2}} = \frac{1}{\sqrt{2\sigma^2\pi}} e^{-\frac{(x-\mu_2)^2}{2\sigma^2}}$$

$$(x-\mu_1)^2 = (x-\mu_2)^2 \quad x_0 = \frac{\mu_1 + \mu_2}{2}$$

Boundary  $x_0 = \frac{\mu_1 + \mu_2}{2}$  when  $x > x_0$   $W_2$   
~~or~~  $W_1$



$$b) P(\text{error}) = P(\text{misclassified } w_2 | w_1) P(w_1) + P(\text{misclassified as } w_1 | w_2) P(w_2)$$

$$X_c = \frac{u_1 + u_2}{2} \quad A = \frac{1}{2} \int_{X_c}^{\infty} \frac{1}{\sqrt{2\pi}b} e^{-\frac{(x-u_1)^2}{2b^2}} dx$$

$$B = \frac{1}{2} \int_{-\infty}^{X_c} \frac{1}{\sqrt{2\pi}b} e^{-\frac{(x-u_2)^2}{2b^2}} dx$$

$$Z_1 = \left( \frac{x - u_1}{b} \right) \quad \text{replace } x \text{ in } A \text{ as } Z_1$$

$$A \Rightarrow \frac{1}{2} \int_{\frac{u_2 - u_1}{b}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$$

$$Z_2 = \left( \frac{x - u_2}{b} \right) \quad \text{replace } x \text{ in } B \text{ as } Z_2$$

$$B \Rightarrow \frac{1}{2} \int_{-\infty}^{\frac{u_2 - u_1}{b}} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$$

$$A+B = P(\text{error})$$

~~$$A = \frac{1}{2} \int_{\frac{u_2 - u_1}{b}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$$~~

$$B = \frac{1}{2} \int_{-\frac{(u_2 - u_1)}{b}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$$

$$= A$$

since  $e^{-\frac{z^2}{2}}$  is even function.

therefore

$$P(\text{error}) = \int_{\frac{u_2 - u_1}{2b}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$$

Q7. a)  $f(w) = \frac{1}{2} \|Xw - y\|_2^2 + \frac{\lambda}{2} \|w\|_2^2$

$$= \frac{1}{2} (Xw - y)^T (Xw - y) + \frac{\lambda}{2} w^T w$$

$$\frac{\partial f(w)}{\partial w} = (Xw - y) = k(w) \quad \frac{\partial \|k\|_2^2}{\partial k} \frac{\partial k}{\partial w} = 2k^T \cdot X$$

therefore  $\frac{\partial f(w)}{\partial w} = \frac{1}{2} \cdot 2 (Xw - y)^T \cdot X + \frac{\lambda}{2} \cdot 2w^T$

$$= (Xw - y)^T X + \lambda w^T$$

b) want derivative  $\Rightarrow \lambda w^T + (Xw - y)^T X$

$$= \lambda w^T + w^T X^T X - y^T X$$

~~@  $w^T (X^T X + \lambda I)$~~   $y^T X = w^T (\lambda I + X^T X)$

~~$X^T y = (\lambda I + X^T X)^T w$~~

$X^T y = (\lambda I + X^T X) \cdot w$

Since  $\lambda > 0$  therefore  $(\lambda I + X^T X)$  is ~~invertible~~ invertible

c)  $\Rightarrow (\lambda I + X^T X)^{-1} X^T y = w$

the first term

$$\frac{1}{2} \|Xw - y\|_2^2 = \frac{1}{2} (Xw - y)^T (Xw - y) \quad w = w_n + X^T a$$

$Xw = Xw_n + XX^T a$  since  $w_n \in \text{Null}(X)$

$Xw = XX^T a$

therefore  $\frac{1}{2} \|Xw - y\|_2^2 = \frac{1}{2} (XX^T a - y)^T (XX^T a - y)$

only depends on  $a$ .

d)  $\frac{\lambda}{2} w^T w \quad w = w_n + X^T a$

$$\frac{\lambda}{2} (w_n^T + a^T X^T) (w_n + X^T a)$$

$$= \frac{\lambda}{2} (w_n^T w_n + w_n^T X^T a + a^T X w_n + a^T X^T X a)$$

$$w_n^T X^T a = w_n \cdot X^T a$$

$$a^T X w_n = (X a)^T \cdot w_n$$

$$= X^T a \cdot w_n = 0$$



therefore  $\frac{1}{2} \|W\|_2^2 = \frac{\lambda}{2} [ \underbrace{W^T W}_{\geq 0} + \underbrace{(\pi^T a)^T (\pi^T a)}_{\geq 0} ]$

if  $a$  fixed  $\frac{\lambda}{2} \|W\|_2^2 \geq \frac{\lambda}{2} (\pi^T a)^T (\pi^T a)$   
 equal when  $W_n = 0$

e7. minimize  $\frac{1}{2} \|X^T a - y\|_2^2 + \frac{\lambda}{2} \|a\|_2^2 \Rightarrow$  depend only on  $a$

Since from c and d. we find that

the first term of (1) not depend on  $W_n$  and the second term of (1) is minimized (over  $W_n$ ) when  $W_n = 0$ .

f7. then take derivative of  $\frac{1}{2} \|X^T a - y\|_2^2 + \frac{\lambda}{2} \|a\|_2^2$  (over  $a$ )

$$\frac{d}{da} \left[ \frac{1}{2} (a^T X X^T - y^T) (\pi^T a - y) + \frac{\lambda}{2} (a^T a) (\pi^T a) \right]$$

$$= \frac{d}{da} \left[ \frac{1}{2} (a^T X X^T X X^T a - a^T X X^T y - y^T X X^T a + y^T y) + \frac{\lambda}{2} a^T X X^T a \right]$$

$$= \frac{1}{2} (a^T X X^T X X^T + a^T X X^T X X^T - y^T X^T X - y^T X X^T) + \lambda a^T X X^T$$

$X X^T = A$       Asymmetric

$$\Rightarrow \text{then } a^T A^2 - y^T A + \lambda a^T A = 0$$

$$a^T (A^2 + \lambda A) = y^T A \quad A \text{ invertible}$$

$$(A^T A + \lambda A) a = A y$$

$$A^T (A^T A + \lambda A) a = A^T A y$$

$$\Rightarrow (A^T + \lambda I) a = y$$

$$A^T = X^T X^T \text{ semipositive.}$$

By Q4. part a.

$$(A^T + \lambda I) \quad \lambda > 0 \text{ invertible}$$

Since  $A^T = X^T X^T$  semipositive

therefore

$$a = (A^T + \lambda I)^{-1} y = (X^T X^T + \lambda I)^{-1} y$$

$$v^T X^T X^T v = \|X v\|_2^2 \geq 0.$$

#7

q7. find  $a_*$ :  $(XX^T + \lambda I)^{-1} y$

$X = n \times d$ .

get  $XX^T$  takes  $n \times n \times d$ .

inverse takes  $n^3$ .

$y = n \times 1$

then takes  $n^2$ .

$O(n^3 + n^2 + n^2 d)$ .

$= O(n^3 + n^2 d)$  if  $n, d$  huge.

And  $W_*$   $(\lambda I - X^T X)^{-1} X^T y$

$X^T$   $d \times n$

$X^T X$  takes  $d^2 n$

inverse takes  $d^3$ .

$X^T y$  takes  $d \times n$

multiply  $(\lambda I - X^T X)^{-1}$  and  $X^T y$   
 $d \times d$   $d \times 1$

takes  $d^2$

therefore  $O(d^2 + nd + d^3 + d^2 n)$  if  $d, n$  large

$= O(d^3 + d^2 n)$

Compare

$a_*$ :  $O(n^3 + n^2 d)$  and  $W_*$ :  $O(d^3 + d^2 n)$

if  $n \gg d$  Should choose  $W_*$

if  $n \ll d$   $a_*$

Q8 a7.  $W^*$

$$\sum_{i=1}^n (\theta_i^T W - y_i^T) (W^T \theta_i - y_i) + \lambda \sum_{j=1}^d W_{ij}^2$$

$$= \text{tr}(W^T W)$$

$$= \sum_{i=1}^n [\theta_i^T W W^T \theta_i + \theta_i^T W y_i - y_i^T W^T \theta_i + y_i^T y_i] + \text{tr}(W^T W)$$

take derivative (over  $W$ ).  $\Rightarrow$

$$= \left\{ \sum_{i=1}^n [2W^T \theta_i \theta_i^T - 2y_i \theta_i^T] + 2\lambda W^T \right\}^T$$

$$= \left\{ 2W^T \sum_{i=1}^n \theta_i \theta_i^T - 2 \sum_{i=1}^n y_i \theta_i^T + 2\lambda W^T \right\}^T$$

der. wrt  $\theta_i$

$$\sum \theta_i \theta_i^T = \frac{\sum \theta_i \theta_i^T}{\sum \theta_i \theta_i^T} = \dots$$

der. wrt  $\theta_i$

$$\sum y_i \theta_i^T = \frac{\sum y_i \theta_i^T}{\sum \theta_i \theta_i^T} = \dots$$

$$= X X^T$$

$X = [\theta_1, \theta_2, \theta_3, \dots]$

$$\sum y_i \theta_i^T = Y X$$

$Y = [y_1, y_2, y_3, \dots, y_n]$

$K \times n$

$$\text{therefore gradient} = \left\{ 2W^T X X^T - 2 Y X^T + 2\lambda W^T \right\}^T$$

Want gradient  $= 0$  then  $2W^T X X^T + 2\lambda W^T = 2 Y X^T$

$$W^T (2 X X^T + 2\lambda I) = 2 Y X^T$$

if  $\lambda > 0$   $(2 X X^T + 2\lambda I)$  positive

thus invertible.

$$(2 X X^T + 2\lambda I) W = 2 Y X^T$$

$$W = (2 X X^T + 2\lambda I)^{-1} \cdot 2 Y X^T$$

$$= (X X^T + \lambda I)^{-1} \cdot Y X^T$$

b) → python code

c) → to compute  $(XX^T + \lambda I)^{-1} \cdot X^T T$

$X: d \times n$   $\rightarrow n \times k$

$XX^T$  takes  $d^2 n$

inverse takes  $d^3$

$X^T T$  takes  $dnk$

~~therefore  $O(d^2 n + d^3 + dnk)$~~

and  $(XX^T + \lambda I)^{-1} \cdot X^T T$

takes  $d^2 k$

therefore  $O(d^2 k + dnk + d^3 + d^2 n)$

e)  $W = [w_1, w_2, \dots, w_k]$   $X: d \times n$

original classifier  $\sum_{i=0}^n \|W^T x_i - y_i\|^2 + \lambda \|W\|_F^2$

$$W^T x_i = \begin{bmatrix} w_1 \cdot x_i \\ w_2 \cdot x_i \\ \vdots \\ w_k \cdot x_i \end{bmatrix}$$

$$\|W\|_F^2 = \|w_1\|_2^2 + \|w_2\|_2^2 + \|w_3\|_2^2 + \dots + \|w_k\|_2^2$$

$$X = [x_1, x_2, \dots, x_n]$$

so we can choose  $w_i$  orthogonal to all  $x_i$   
then  $X^T w_i = 0$ .

and we can take  $w$  apart  $w_{\perp}$  and  $w_{\parallel}$

$w_{\perp}$  is consisted by  $w_i$   $X^T w_i = 0$

$w_{\parallel}$  is consisted by  $w_i$  column space of  $X$ .

then  $W = w_{\perp} + XA$

$A: n \times k$  matrix

We then plug this into the equation.

$$W = W_1 + XA$$

the first term

$$\sum_{i=1}^n \|(W_1^T + A^T X^T) x_i - y_i\|_2^2 = \sum_{i=1}^n \|A^T X^T x_i - y_i\|_2^2$$

$$= \sum_{i=1}^n (x_i^T X A A^T X^T x_i - 2 x_i^T X A y_i + y_i^T y_i)$$

$$= \text{trace}(X^T X A A^T X^T X) - 2 \text{trace}[X^T X A Y] + \text{trace}(Y^T Y)$$

not about  $\lambda$

$$\text{total: } \|W\|_F^2 = \|W_1 + XA\|_F^2 = \text{trace}(W_1 + XA)^T (W_1 + XA)$$

$$= \text{trace}(A^T X^T X A)$$

$$(W_1^T + A^T X^T)(W_1 + XA) = W_1^T W_1 + A^T X^T W_1 + W_1^T X A + A^T X^T X A$$

the first term does not depend on  $W_1$  ...  
and for the second term: when  $W_1 = 0$  has minimum value

We take derivative

$$\frac{\partial}{\partial A} \left( \text{trace}(X^T X A A^T X^T X) - 2 \text{trace}[X^T X A Y] + \text{trace}(A^T X^T X A) \right) = 0$$

$$\text{then } \lambda \cdot 2 A^T X^T X + 2 A^T X^T X X^T X = 2 Y^T X^T X \quad \text{Assume } X^T X \text{ invertible}$$

$$\text{then } \lambda \cdot 2 A^T + 2 A^T X^T X = 2 Y^T$$

$$A^T (\lambda I + X^T X) = Y^T \quad \text{Since } (\lambda I + X^T X) \text{ invertible}$$

$$A = (X^T X + \lambda I)^{-1} Y^T$$

Complexity:  $X^T X$  takes  $n^2 d$  inverse  $n^3$  times  $Y^T$   $n^2 k$ .

$O(n^2 d + n^3 + n^2 k)$  if  $n \ll d$  compute  $A$  instead of  $W$ .



```
hw1.py
1  from mnist import MNIST
2  import sklearn.metrics as metrics
3  import numpy as np
4  from numpy.linalg import inv
5
6  NUM_CLASSES = 10
7
8  def load_dataset():
9      mndata = MNIST('./data/')
10     X_train, labels_train = map(np.array, mndata.load_training())
11     X_test, labels_test = map(np.array, mndata.load_testing())
12     X_train = X_train/255.0
13     X_test = X_test/255.0
14     return (X_train, labels_train), (X_test, labels_test)
15
16
17 def train(X_train, y_train, reg=0):
18     ''' Build a model from X_train -> y_train '''
19     #here involve a hyper parameter
20     inverse = inv(np.dot(np.matrix.transpose(X_train), X_train) + 0.5*np.identity(784))
21     return np.dot(inverse, np.dot(np.matrix.transpose(X_train), one_hot(y_train)))
22
23 def one_hot(labels_train):
24     '''Convert categorical labels 0,1,2,...,9 to standard basis vectors in  $\mathbb{R}^{10}$ '''
25     return np.array([[1 if i == labels_train[k] else 0 for i in range(10)] for k in range(len(labels_train))])
26
27 def predict(model, X):
28     ''' From model and data points, output prediction vectors '''
29     result = np.dot(np.matrix.transpose(model), np.matrix.transpose(X)) #get a vector
30     return [np.argmax(i) for i in np.matrix.transpose(result)] #single array with dim = 1*60000
31
32 if __name__ == "__main__":
33     (X_train, labels_train), (X_test, labels_test) = load_dataset()
34     model = train(X_train, labels_train)
35     y_train = one_hot(labels_train)
36     y_test = one_hot(labels_test)
37
38     pred_labels_train = predict(model, X_train)
39     pred_labels_test = predict(model, X_test)
40
41     print("Train accuracy: {0}".format(metrics.accuracy_score(labels_train, pred_labels_train)))
42     print("Test accuracy: {0}".format(metrics.accuracy_score(labels_test, pred_labels_test)))
43
```