c). ~~### ### ###~~ sample from

convert X Back to ~~Linearly Independent~~ $Z = \begin{vmatrix} N(0,1) \\ N(0,1) \\ N(0,1) \\ \vdots \end{vmatrix}$

$\Sigma = VV^T$ $\qquad X = VZ + b \rightarrow b=0$

$\Sigma^{-1} = V^{T-1}V^{-1}$ $\qquad = VZ$

$x^T \Sigma^{-1} x = x^T V^{T-1} V^{-1} x$ $\qquad x = Vz + \cancel{b}$. $\quad x, z$ are samples from X, Z

$= \underbrace{z^T V^T V^{T-1}}_{I} \underbrace{V^{-1} V}_{I} z = z^T z = \|Ax\|_2^2$

change sample space.

d). $\|Ax\|_2^2 = x^T \Sigma^{-1} x = x^T U \Lambda U^T x$. $\quad U^T$ change ~~base~~ base

therefore $\qquad \|U^T x\|_2 = 1$

Max = Maximum entry of Λ

Min = ~~Min entry~~ Minimum entry of Λ

entries of Λ are eigenvalues of Σ^{-1}

if $X_i \perp\!\!\!\perp X_j$ then Σ diagonal Matrix entries correspond to variance of X_i

Maximum of $\|Ax\|_2^2 = \frac{1}{a}$ $\quad a$ = Minimum variance among X_i

Minimum of $\|Ax\|_2^2 = \frac{1}{b}$ $\quad b$ = Maximum variance among X_i

choose X_i makes $\|Ax\|_2^2 = \frac{1}{b}$ $\quad b$ Max Variance

X_i is the ~~normalized eigenvector~~ variable with Largest ~~###~~variance