# Computer Graphics

## - Exercise Booklet -

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## Contents



**Note:** All test questions and exercises marked with "*" appeared on previous exams.

# 1 Math Fundamentals

## Exercise 1

You are given two vectors $\mathbf{x} = (\sqrt{2}, 1, 0)^T$, $\mathbf{y} = (1, 1, 1)^T$, and matrix

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ -1 & -3 & -3 \end{pmatrix}$$

- Compute the cosine of the angle between vectors $\mathbf{x}$ and $\mathbf{y}$.
- Compute vector $\mathbf{z}$ that is the vector perpendicular to vectors $\mathbf{x}$ and $\mathbf{y}$. Because there is many vectors that fulfill this requirement, report the one that additionally has magnitude equal $1$, i.e., it is normalized.
- Compute vector $\mathbf{u}$ defined as $\mathbf{u} = A\mathbf{z}$

# 2 Raytracing basics

## Exercise 1

Derived the solution for ray-sphere intersection using directly the ray equation and the implicit definition of the sphere. You can follow the steps below:

1. Assume a ray defined as $\gamma(t) = t\mathbf{d}$, where $\mathbf{d}$ is the ray direction, and a sphere defined using implicit representation given by

   $$F(x, y, z) = (x - c_x)^2 + (y - c_y)^2 + (z - c_z)^2 - r^2 = 0,$$

   where $r$ is the radius of the sphere and $\mathbf{c} = (c_x, c_y, x_z)^T$ is the center of the sphere.

2. Show that the implicit representation can also be expressed as

   $$F(\mathbf{p}) = \|\mathbf{p} - \mathbf{c}\|^2 - r^2 = 0,$$

   where $\mathbf{p} = (x, y, z)^T$.

3. Show that to find the intersections of the ray with the sphere, you need to find $t$ such that

   $$\|t\mathbf{d} - \mathbf{c}\|^2 - r^2 = 0.$$

4. Use properties of the dot product to transform the above equation to the form:

   $$t^2 - 2\langle d, c\rangle t + \|c\|^2 - r^2 = 0$$

5. Solve the above quadratic equation to find $t$, write down the formulas for the intersection points.

## Exercise 2

Consider a tasks of raytracing a sphere with center at $c = (1, 1, 1)^T$ and radius $r = \sqrt{2}/2$. Assume that the camera is located at the origin of the coordinate system. Let $l$ be the line that passes through the camera location and the center of the sphere, and $d$ is the direction of a ray that intersect the sphere exactly at one location. Compute the angle between $d$ and $l$.

### Exercise 3

Derive a formula for computing an intersection point between a ray, $\gamma(t) = t\mathbf{d} + \mathbf{o}$, and a plane defined by its normal $\mathbf{n}$, and one point $\mathbf{p}$ that is known to lie on the plane. What is the condition that indicates no intersection?

## 3 Lighting Models

### Exercise 1

Consider a shiny ground plane $y = 0$ illuminated by a directional light source. Assume that the direction towards the light is $(1, 2, 2)$ and the viewer/camera is at $(4, 6, 7)$. Note this is different from what we currently assume in our raytracer implementation where the camera is at $(0, 0, 0)$.

1. Compute a position on the ground plane at which the viewer observes the peak of the highlight. You can assume that the peak occurs where a perfect mirror reflection takes the ray from the light source and reflects it directly towards the viewer.

2. Let us model the appearance of the plane using Phone lighting model. Assume that the plane is reflecting half of the incoming light according to the diffuse reflection and half according to specular reflection. The shininess coefficient of the plane is $k = 2$. The plane does not emit any light, there is no ambient illumination, and the intensity of the directional light is $I = 1$. There is also no color in the scene, i.e., everything is grey. Compute the intensity of the light observed by a viewer at the peak of the highlight, i.e., the location of the plane computed in the previous task. Do you need to account for the distance between the light source and the plane? If so, assume for your calculations any distance you want.

### Exercise 2

Consider a ray intersecting a sphere at a certain point and $n$ being the normal vector at the intersection, $v$ direction towards camera, $l$ direction to the light source, and $r$ the reflected light direction. Proof that if all the vectors are coplanar, the angle between the half-vector $(l + v)/2$ and the normal vector $n$ is equal to half the angle between the $v$ and $r$.

### *Exercise 3

Consider an infinitely tall cylinder whose axis coincides with y-axis of the coordinate system and whose radius is $r = \sqrt{2}$.

1. Provide an equation that describes all the points lying on the surface of the cylinder.

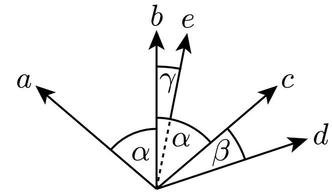
2. Consider a ray with an origin $o = (1, 4, 2)$ and a direction $d = (0, 0, -1)$. Compute all the intersection points of this ray with the cylinder.

3. What is the brightness of the cylinder observed along the ray assuming that:

   - the lighting is modeled using Phong's model,
   - the reflection constants for diffuse and specular reflections are $\rho_d = \rho_s = 1$, and the shininess constant is $k = 2$,
   - the cylinder is illuminated with a point-light source located at point $l = (4, 4, 4)$ with intensity $I = 1$,
   - the cylinder does not emit any light and there is no no ambient light in the scene,
   - we consider a gray world, no colors, only brightness.

## Exercise 4

Consider the configuration of planar vectors shown on the right. Let $\alpha$ be the angle between the unit vectors $a$ and $b$ and let $c = 2b\langle a, b\rangle - a$, so that the angle between $b$ and $c$ is also $\alpha$. We further consider the unit vector $d$ and denote the angle between $c$ and $d$ by $\beta$. We finally compute the *halfway-vector* $e = \frac{a+d}{\|a+d\|}$ and denote the angle between $b$ and $e$ by $\gamma$. Show that $\gamma = \beta/2$. Note that $d$ may also lie on the other side of $c$, i.e. between $b$ and $c$ or even between $a$ and $b$, so you need to distinguish these different cases.




## Exercise 5

Imagine it is night and you see a full moon in the sky. Ignoring the shading artifacts caused by craters, it appears as a white disk with constant brightness rather than a sphere shaded according to the Phong illumination where the color is modulated by the cosine of the angle between the normal vector and the light direction. See the image on the right for the comparison. What could be the reason for this?