Math 63: Real Analysis

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Problem 1. Verify that the following are metric spaces:

1. all n-tuples of real numbers, with

$$d((x_1, \cdots, x_n)(y_1, \cdots, y_n)) = \sum_{i=1}^{n} |x_i - y_i|$$

2. all bounded infinite sequences $x = (x_1, x_2, x_3, \cdots)$ of elements of $\mathbb{R}$ with $d(x, y) = l.u.b. \{|x_1 - y_1|, |x_2 - y_2|, |x_3 - y_3|, \cdots\}$

Problem 2. What are the open and closed balls in the metric space of example (4), § 1? Show that two balls of different centers and radii may be equal. What are the open sets in this metric space?

Problem 3. Show that the subset of E^2? given by $\{(x_1, x_2) \in E^2 : x_1 > x_2\}$ is open.

Problem 4. Prove that any bounded open subset of R is the union of disjoint open intervals.