# Math 63: Real Analysis

Prishita Dharampal

**Credit Statement:** Talked to Sair Shaikh'26, and Math Stack Exchange. **TODO:**

**Problem 1**. If $A, B, C$ are sets show that:

$$A - (B - C) = (A - B) \cup (A \cap B \cap C)$$

**Problem 2**. Let $I$ be a set and for each $i \in I$ let $X_i$ , be a set. Prove that for any set $B$ we have:

$$B \cap \bigcup_{i \in I} X_i = \bigcup_{i \in I} (B \cap X_i)$$

**Problem 3**. Let $f : X \to Y$ be a function, let $A$ and $B$ be subsets of $X$ , and let $C$ and $D$ be subsets of $Y$. Prove that:

1. $f(A \cap B) \subset f(A) \cap f(B)$
2. $f^{-1}(C \cap D) \subset f^{-1}(C) \cap f^{-1}(D)$

**Problem 4**.

1. How many functions are there from a nonempty set $S$ into the $\emptyset$?
2. Show that the notation $\{X_i\}_{i \in I}$ implicitly involves the notion of function.

**Problem 5**. Prove in detail that for any $a, b \in \mathbb{R}$:

$$-(a - b) = b - a$$

**Problem 6**. Show that if $a, b, x, y \in \mathbb{R}$ and $a < x < b, a < y < b$, then $\mid y - x \mid < b - a$.

**Problem 7**. Find the g.l.b. and l.u.b. of $\left\{1, \frac{1}{2}, \frac{1}{4}, \cdots\right\}$, giving reasons if you can.

**Problem 8**. Prove that if $a \in \mathbb{R}, a > 1$, then the set $\{a, a^2, a^3, ...\}$ is not bounded from above. (Hint: First find a positive integer $n$ such that $a > 1 + \frac{1}{n}$ and prove that $a^n > \left(1 + \frac{1}{n}\right)^n \geq 2$).

**Problem 8**. If $S_1, S_2$ are nonempty subsets of $\mathbb{R}$ that are bounded from above, prove that

$$\text{l.u.b}\ \{x + y : x \in S_1, y \in S_2\} = \text{l.u.b.}\ S_1 + \text{l.u.b.}\ S_2$$