Math 63: Real Analysis

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Problem 1. Prove that if the points of a convergent sequence of points in a metric space are reordered, then the new sequence converges to the same limit.

Problem 2. Show that if $a_1, a_2, a_3, \cdots$ is a sequence of real numbers that converges to a, then,

$$\lim_{n\to\infty}\left(\frac{\sum_{n}^{i=1} a_i}{n}\right) = a.$$

Problem 3. Prove that any sequence in $\mathbb{R}$ has a monotonic subsequence.
(Hint: This is easy if there exists a subsequence with no least term, hence we may suppose that each subsequence has a least term.) (Note that this result and the theorem on the convergence of bounded monotonic sequences gives another proof that $\mathbb{R}$ is complete.)

Problem 4. Let S be a subset of the metric space E . Define the closure of S, denoted $\bar{S}$, to be the intersection of all closed subsets of E that contain S. Show that

1. $\bar{S} \supset S$, and S is closed if and only if $S = S$.
2. $\bar{S}$ is the set of all limits of sequences of points of S that converge in E.
3. a point $p \in E$ is in S if and only if any ball in E of center p contains points of S, which is true if and only if p is not an interior point of S' (cf. Prob. 15).

Problem 5. Show that a complete subspace of a metric space is a closed subset.