

Lecturer

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Pattern Recognition (CS254) - Sheet 1**[10 Points]**

Preliminary Discussion 26.09.2013

Deadline 02.10.2013

This theoretical exercise does not have to be presented orally.

Exercise 1 - Multivariate Normal Distribution**[4 Points]**

Consider a bivariate normal population with $\mu_1 = 0, \mu_2 = -3, \sigma_{11} = 4, \sigma_{22} = 1$, and with cross correlation coefficient, $\rho_{12} = \frac{1}{2}$.

- Write out the bivariate normal density.
- Write out the squared generalized distance expression $(\mathbf{x} - \boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1} (\mathbf{x} - \boldsymbol{\mu})$ as a function of x_1 and x_2 .
- Determine the main axes and sketch the constant-density contour of one standard deviation.

Exercise 2 - Independence**[3 Points]**

Consider $\mathbf{X} = [X_1, X_2, X_3]^T$ distributed according to $\mathcal{N}(\mathbf{X} \mid \boldsymbol{\mu}, \boldsymbol{\Sigma})$ with

$$\boldsymbol{\mu} = \begin{bmatrix} -3 \\ 1 \\ 4 \end{bmatrix}, \quad \boldsymbol{\Sigma} = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

Which of the following random variables are independent? Explain.

- X_1 and X_2
- X_1 and X_3
- X_2 and X_3
- (X_1, X_2) and X_3
- $X_1 - X_2$ and $X_1 + X_2 - X_3$

Exercise 3 - Conditional Distribution**[3 Points]**

Specify the conditional distribution of X_1 , given that $X_2 = x_2$ for the joint distribution $\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Compare the conditional distribution $P(X_1 \mid X_2 = 1)$ to the marginal distribution $P(X_1)$ in a plot.

$$\boldsymbol{\mu} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}, \quad \boldsymbol{\Sigma} = \begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix}$$