**UNIVERSITY OF BRISTOL**

**May/June 2023 Examination Period**

**FACULTY OF ENGINEERING**

**Second Year Examination for the Degree of**
**Bachelor of Science and Master of Engineering**

**COMS20011**
**Data-Driven Computer Science**

**TIME ALLOWED:**
**2 Hours**

# Answers to COMS20011: Data-Driven Computer Science

**Intended Learning Outcomes:**

**Help Formulas:**

Minkowski distance:

$$D(x,y) = (\sum_{i=1}^{n} |x_i - y_i|^p)^{\frac{1}{p}}$$

One-dimensional Gaussian/Normal probability density function:

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Multi-dimensional Gaussian/Normal probability density function:

$$p(\mathbf{x}) = \frac{1}{\sqrt{(2\pi)^M |\Sigma|}} e^{-\frac{1}{2}(\mathrm{x}-\boldsymbol{\mu})^T \Sigma^{-1} (\mathrm{x}-\boldsymbol{\mu})}$$

2D Convolution:

$$g(x,y) = \sum_{m=-1}^{1} \sum_{n=-1}^{1} h(m,n) f(x-m, y-n)$$

Least Squares Matrix Form:

$$\mathbf{w}^* = (\mathbf{X}^T \mathbf{X})^{-1} \ \mathbf{X}^T \ \mathbf{y}$$

Matrix inversion:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

Matrix Determinant:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

**Q1**. Consider the pixel values of a small image below:

| | | | | | |
|---|---|---|---|---|---|
| 10 | 2 | 2 | 2 | 2 | 2 |
| 10 | 2 | 1 | 14 | 2 | 2 |
| 10 | 2 | 2 | 13 | 0 | 2 |
| 10 | 2 | 1 | 15 | 2 | 2 |
| 10 | 2 | 2 | 15 | 1 | 2 |
| 10 | 2 | 2 | 2 | 2 | 2 |

Using the position of the pixel with value 13 as the centre pixel to be convolved, apply the following convolution filter once using 8-connectivity and once using 4-connectivity:

$$\begin{pmatrix} -1 & 1 & -1 \\ -1 & 3 & -1 \\ -1 & 1 & -1 \end{pmatrix}$$

Which of the options below is the correct answer for the new pixel value in each connectivity case?

A. For 8-connectivity it is 76, and for 4-connectivity it is 70

**B. For 8-connectivity it is 60, and for 4-connectivity it is 66**

C. For 8-connectivity it is 60, and for 4-connectivity it is 76

D. For 8-connectivity it is 66, and for 4-connectivity it is 60

E. For 8-connectivity it is 70, and for 4-connectivity it is 76

*[6 marks]*

**Solution:** B - convolve or in fact correlate (since the filter is symmetric) with 8 neighbours or 4 neighbours.

**Q2**. Which of the following statements is TRUE:

A. The outer regions of the Fourier space represent the detail in the image and are used for smoothing.

**B. The outer regions of the Fourier space represent the detail in the image and are used for sharpening.**

C. The central regions of the Fourier space represent the detail in the image and are used for smoothing.

D. Options A, B, and C are all true.

(cont.)

E. Options A, B, and C are all false.

*[3 marks]*

**Solution:** B

**Q3**. Naive Bayes Classifier - The table below shows the probability of certain words from amongst a large selection of spam and not spam emails received at a university. The occurrence of the words and their probabilities are independent of each other.

| Word | $p(word\|spam)$ | $p(word\|\neg spam)$ |
|---|---|---|
| Ink | 0.80 | 0.30 |
| Term | 0.02 | 0.93 |
| Summer | 0.40 | 0.65 |
| Printer | 0.18 | 0.75 |
| Bulk | 0.70 | 0.10 |

Making a Naive Bayes assumption, compute the probability of sentence $S1$ below being spam and the probability of sentence $S2$ below not being spam:

$S1$- *Buy printer ink in bulk at prices seen last Summer.*
$S2$- *The Summer term notes are by the printer.*

Choose the correct option for $P(S1|spam)$ and $P(S2|not\ spam)$:

A. $P(S1|spam) = 0.0403$ and $P(S2|not\ spam) = 0.0146$

B. $P(S1|spam) = 0.0146$ and $P(S2|not\ spam) = 0.4534$

C. $P(S1|spam) = 0.0403$ and $P(S2|not\ spam) = 0.0014$

D. $P(S1|spam) = 0.0146$ and $P(S2|not\ spam) = 0.4095$

**E.** $P(S1|spam) = 0.0403$ **and** $P(S2|not\ spam) = 0.4534$

*[6 marks]*

**Solution:** E - $P(S1|spam) = 0.18\times0.80\times0.70\times0.40 = 0.0403$ and $P(S2|not\ spam) = 0.65 \times 0.93 \times 0.75 = 0.4095$

**Q4**. For a digitised sample acquired using 4Hz sampling and 4 quantisation levels, the following file has been provided:

0010101101010010011010111010 10

The first sample collected after the first second of recording has passed is equal to:

A. 0110

B. 1011

(cont.)

**C. 01**

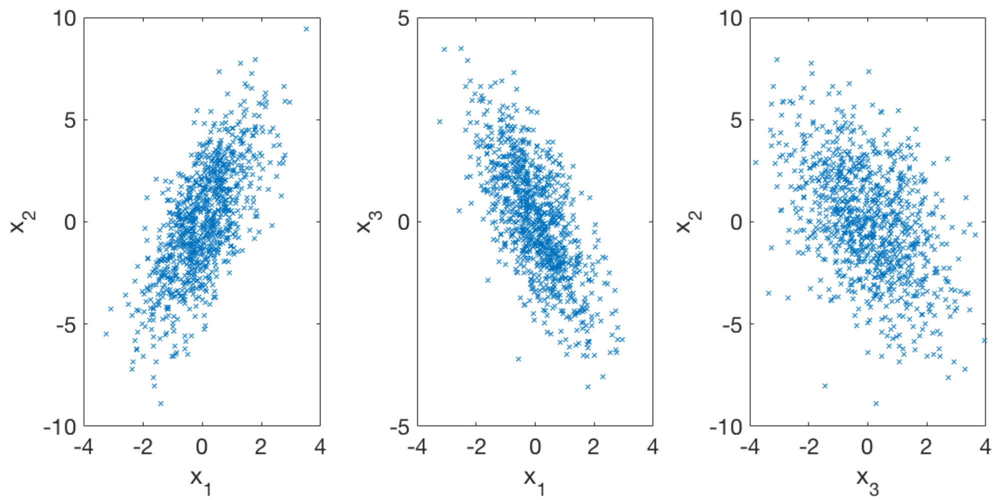
D. 10

E. 0010

*[3 marks]*

**Solution:** C - 4 quantisation levels requires 2 binary digits. 4Hz is 1/4 seconds, then at 1 second we have 00101011, and the first sample collected next is 01.

**Q5**. For three-dimensional data $X = (x_1, x_2, x_3)$, we plot each variable against the other as shown below:




Given these plots, determine which of the following is a reasonable estimate of the co-variance matrix $\Sigma$ of dataset $X$?

**A.** $\Sigma = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 7 & -2 \\ -1 & -2 & 2 \end{bmatrix}$

B. $\Sigma = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ -1 & -2 & 7 \end{bmatrix}$

C. $\Sigma = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 1 & 2 \\ 1 & 2 & 7 \end{bmatrix}$

D. $\Sigma = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 7 & -2 \\ -1 & -2 & 2 \end{bmatrix}$

E. $\Sigma = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 7 & -2 \\ -1 & -2 & 0 \end{bmatrix}$

*[5 marks]*

**Solution:** A

**Q6**. Which of the following 2D matrices are NOT separable? Ignore normalisation factors which are not stated here.

$$M_1 = \begin{pmatrix} -1 & 3 & -1 \\ -3 & 9 & -3 \\ 1 & -3 & 1 \end{pmatrix} \qquad M_2 = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ -1 & 0 & 1 \end{pmatrix} \qquad M_3 = \begin{pmatrix} -1 & 4 & -1 \\ -1 & 8 & -1 \\ -1 & 4 & -1 \end{pmatrix}$$

$$M_4 = \begin{pmatrix} 1 & 2 & -1 & 2 & 4 \\ 2 & 4 & -2 & 4 & 8 \\ -1 & -2 & 1 & -2 & -4 \\ 2 & 4 & -2 & 4 & 8 \\ 4 & 8 & -4 & 8 & 16 \end{pmatrix} \qquad M_5 = \begin{pmatrix} 1 & 0 & 0 & -1 \\ 2 & 1 & 1 & -2 \\ 1 & 0 & 0 & -1 \\ 2 & 1 & 1 & -2 \end{pmatrix}$$

Choose the correct option:

A. $M_1$ and $M_5$

B. $M_2$ and $M_3$ and $M_5$

C. $M_1$ and $M_4$

D. $M_2$ and $M_3$ and $M_4$

**E. $M_3$ and $M_5$**

*[5 marks]*

**Solution:** E - Only $M_3$ and $M_5$ are not separable - the others can be arrived at with one column and one row vector. Verify using Python or Matlab.

**Q7**. Two eigenvalues of the matrix below are 1 and 8:

$$\begin{bmatrix} 1 & -2 & 0 & 5 \\ 0 & 7 & 1 & 5 \\ 0 & 4 & 4 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

What are the other two eigenvalues?

A. 3 and 4

B. 4 and 2

**C. 2 and 3**

D. 1 and 3

E. 0 and 4

*[4 marks]*

(cont.)

**Solution:** C - Sum of the variances (main diagonal elements = 14) = sum of the eigenvalues, so answer has to be 2 and 3 for all the eigenvalues to sum up to 14 too

**Q8**. Fig. 1 shows an image of 'lines & numbers' and its Fourier Transform output.

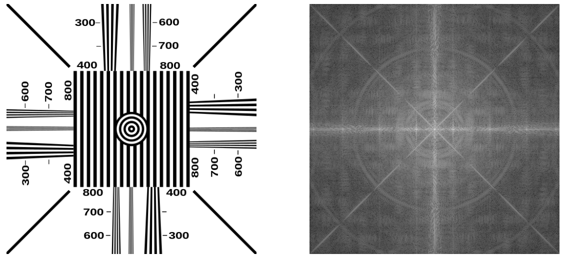


Figure 1: An image and its FFT space.

The top row in Fig. 2 shows 4 filtered versions of the 'lines & numbers' FFT space: ($F1, F2, F3, F4$). The 4 images in the bottom row, ($W, X, Y, Z$), show *in a random order*, the inverse FFT results of those filtered FFT outputs. Select the choice that correctly states which filtered FFT image corresponds to which inverse filtered FFT image.

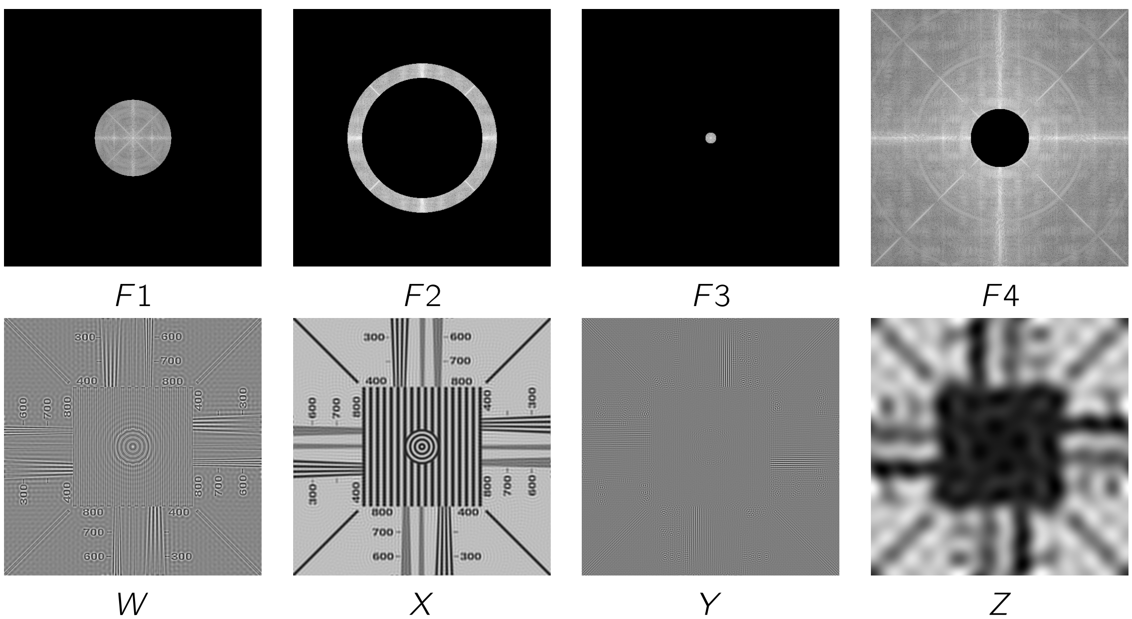



Figure 2: (top row) Filtered versions of the FFT result of the 'lines & numbers' image, and (bottom row) Inverse FFT results of tose in the top row but in a random order.

A. ($F1, F2, F3, F4$) correspond to ($Y, X, Z, W$)

B. ($F1, F2, F3, F4$) correspond to ($Y, X, W, Y$)

C. ($F1, F2, F3, F4$) correspond to ($Z, W, X, Y$)

**D. ($F1, F2, F3, F4$) correspond to ($X, Y, Z, W$)**

E. ($F1, F2, F3, F4$) correspond to ($X, Y, W, Z$)

*[10 marks]*

**Solution:** D - the filters are high pass ($F4 \Rightarrow W$), low pass ($F1 \Rightarrow X$), band pass ($F2 \Rightarrow Y$), and very low pass ($F2 \Rightarrow Z$).

**Q9**. The eigenvalues of a dataset are: [26.0, 16.0, 13.0, 5.0, 4.0, 3.0, 1.95, 0.85, 0.60]. Approximately what variance in the dataset do the first 4 eigenvalues represent?

A. 91.2%

B. 88.6%

C. 93.3%

D. 77.3%

**E. 85.2%**

*[5 marks]*

**Solution:** E - Sum of the first 4 eigenvalues divided by the sum of all the eigenvalues, multiplied by 100 and rounded to 1 decimal point.

**Q10**. What is the minimum Edit Distance between the words "Sunday" and "Saturday"?

A. 4

B. 6

C. 7

**D. 3**

E. 2

*[3 marks]*

**Solution:** D - We would need to convert "un" to "atur" using 3 operations: substitute 'n' with 'r', insert 'a', insert 't'

**Q11**. Which of these is NOT a potential cause of overfitting?

A. Choosing a function class that is too complex

**B. Choosing a function class that is too simple**

C. No regularisation.

D. Too few datapoints.

E. Datapoints only cover a small region in the input space.

*[5 marks]*

**Q12**. Which of these statements about cross-validation is FALSE:

A. Cross-validation can be used to assess overfitting.

**B. Cross-validation reports performance on the training data used to fit the function.**

C. Cross-validation can be used to choose the function class.

D. Cross-validation can be used to choose the amount of regularisation.

E. Cross-validation can be computationally expensive if we have more than one or two hyperparameters.

*[5 marks]*

**Q13**. Which of these statements about the logarithm and its use in data-science is FALSE:

A. The logarithm converts products into sums, i.e. $\log ab = \log a + \log b$.

B. The logarithm converts powers into products, i.e. $\log a^b = b \log a$.

**C. The gradient of the logarithm is $\frac{\partial \log p}{\partial p} = 2p^{-1}$.**

D. Using log-probabilities rather than "raw" probabilities helps us avoid numerical under/overflow.

E. When doing maximum-likelihood fitting, the parameters with the highest log-likelihood are the same as the parameters with the highest "raw" likelihood.

*[5 marks]*

**Q14**. Find the value of:

$$\sum_{i=1}^{5} (\delta_{i2} i^3 + \delta_{i5} i^2)$$

where $\delta$ is the Kronecker-delta.

A. 30

**B. 33**

C. 34

(cont.)

D. 36

E. 40

*[5 marks]*

**Solution:**

$$\sum_{i=1}^{5}(\delta_{i2}i^3 + \delta_{i5}i^2) = 2^3 + 5^2 = 8 + 25 = 33. \tag{1}$$

**Q15**. For the data in the table, fit a model of the form $\hat{y} = w_1 + w_2 x$

| x | y |
|---|---|
| 0 | -4.2 |
| 1 | -2.3 |
| 2 | -0.1 |
| 3 | 2.1 |
| 4 | 3.9 |

*[5 marks]*

A. $w_1 = -4.20$, $w_2 = 2.12$.

**B.** $w_1 = -4.24$, $w_2 = 2.06$**.**

C. $w_1 = -4.30$, $w_2 = 2.12$.

D. $w_1 = -4.32$, $w_2 = 2.06$.

E. $w_1 = -4.35$, $w_2 = 2.12$.

**Q16**. For the data in the table, fit a model of the form $\hat{y} = w_1 x + w_2 x^2$

| x | y |
|---|---|
| 0 | -4.2 |
| 1 | -2.3 |
| 2 | -0.1 |
| 3 | 2.1 |
| 4 | 3.9 |

*[5 marks]*

A. $w_1 = -1.42$, $w_2 = 0.603$.

B. $w_1 = -1.55$, $w_2 = 0.654$.

**C.** $w_1 = -1.60$, $w_2 = 0.674$**.**

D. $w_1 = -1.78$, $w_2 = 0.687$.

E. $w_1 = -1.82$, $w_2 = 0.742$.

**Q17**. For the data in the table, fit a model of the form $\hat{y}_i = w_1 X_{i1} + w_2 X_{i2}$

| $X_{i1}$ | $X_{i2}$ | $y_i$ |
|---|---|---|
| -1 | -1 | -2.6 |
| -1 | 1 | -0.2 |
| 1 | -1 | 0.5 |
| 1 | 1 | 2.4 |

*[5 marks]*

A. $w_1 = 1.275$, $w_2 = 0.925$

B. $w_1 = 1.305$, $w_2 = 0.970$

C. $w_1 = 1.350$, $w_2 = 1.025$

D. $w_1 = 1.400$, $w_2 = 1.505$

**E.** $w_1 = 1.425$, $w_2 = 1.075$

**Q18**. Compute $\sum_i \log P(y_i|x_i)$ for binary classification, where

$$P(y_i = 1|x_i) = \sigma(1 + x_i - 2x_i^2)$$

with data,

| x | y |
|---|---|
| -2.1 | 0 |
| -0.9 | 0 |
| 0.2 | 1 |
| 1.2 | 1 |
| 2.4 | 1 |

*[5 marks]*

A. -9.32

B. -9.56

C. -9.61

**D. -9.69**

E. -9.83

**Q19**. We have $N$ datapoints, $x_1, \ldots, x_N$, distributed according to,

$$P(x_i|\mu) \propto \frac{1}{x_i} e^{-(\log x_i - \mu)/2}$$

What is the maximum-likelihood solution for $\mu$?

A. $\mu = \frac{1}{N} \sum_{i=1}^N x_i$.

B. $\mu = \frac{1}{2N} \sum_{i=1}^N \log x_i$

**C.** $\mu = \frac{1}{N} \sum_{i=1}^N \log x_i$

D. $\mu = \frac{1}{2N} \sum_{i=1}^N e^{x_i}$

E. $\mu = \frac{1}{N} \sum_{i=1}^N e^{x_i}$

*[5 marks]*

**Solution:**

$$\log P(x|\mu) = \sum_i (-\log x_i - (\log x_i - \mu)^2/2) \tag{2}$$

$$\log P(x|\mu) = \sum_i (-\log x_i - (\log x_i)^2/2 + \mu \log x_i - \mu^2/2) \tag{3}$$

$$0 = \frac{\partial}{\partial \mu} \log P(x|\mu) = \sum_i (\log x_i - \mu) \tag{4}$$

$$0 = \sum_i (\log x_i) - N\mu \tag{5}$$

$$N\mu = \sum_i (\log x_i) \tag{6}$$

$$\mu = \tfrac{1}{N} \sum_i (\log x_i) \tag{7}$$

**Q20**. We have $N$ datapoints, $x_1, \ldots, x_N$, distributed according to,

$$P(x_i|\beta) \propto \beta^2 \tfrac{1}{x_i^3} e^{-\beta/x_i}$$

What is the maximum-likelihood solution for $\beta$?

A. $\frac{1}{2}(\frac{1}{N}\sum_{i=1}^N x_i)$

B. $\frac{1/2}{\frac{1}{N}\sum_{i=1}^N x_i}$

C. $\frac{1/2}{\frac{1}{N}\sum_{i=1}^N (1/x_i)}$

D. $\frac{2}{\frac{1}{N}\sum_{i=1}^N x_i}$

**E.** $\frac{2}{\frac{1}{N}\sum_{i=1}^N (1/x_i)}$

*[5 marks]*

**Solution:**

$$\log P(x|\beta) = \sum_{i=1}^{N}(2\log\beta - 3\log x - \beta/x_i) \tag{8}$$

$$= N(2\log\beta - 3\log x) - \beta\sum_{i=1}^{N}(1/x_i) \tag{9}$$

$$0 = \frac{\partial}{\partial\beta}\log P(x|\beta) = 2N\tfrac{1}{\beta} - \sum_{i=1}^{N}(1/x_i) \tag{10}$$

$$\sum_{i=1}^{N}(1/x_i) = 2N\tfrac{1}{\beta} \tag{11}$$

$$\beta = \frac{2N}{\sum_{i=1}^{N}(1/x_i)} \tag{12}$$

$$\beta = \frac{2}{\frac{1}{N}\sum_{i=1}^{N}(1/x_i)} \tag{13}$$



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