COMS20011 – Data-Driven Computer Science
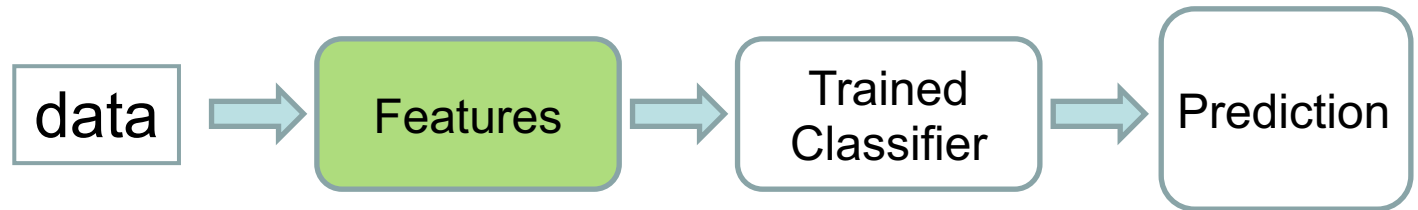

March 2024
Features

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Lecture MM-06

Next in DDCS

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data  →  Features  →  Trained      →  Prediction
                       Classifier
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Feature Selection and Extraction

➢ Signal basics and Fourier Series

➢ 1D and 2D Fourier Transform

➢ **Another look at features**

➢ PCA for dimensionality reduction

➢ Convolutions

Features for recognizing a chair?

Features for recognizing good/bad apples?

Features for recognizing masculine/feminine words in French?

- le fromage (cheese)
- le monument (monument)
- le sentiment (feeling)
- le couteau (knife)
- le téléphone (telephone)
- le microscope (microscope)
- le romantisme (romanticism)

la salade (salad, lettuce)
la fourchette (fork)
la télévision (television)
la culture (culture)
la situation (situation)
la société (society)
la différence (difference)

Image Features for Recognising Buildings?

Matching features (while also exploiting how they are arranged in the scene) would be much more efficient than matching all pixels

Common features between images allows us to perform tasks such as scene matching, face recognition, 3D model generation, and much more!

Examples of Features

- Primitive features, e.g.: (or Discrete/Continuous or Quantitative)
  - weight, length, width, height, volume …
  - amplitude, frequency, phase, duration, roll-off, flux …
  - beats per minute, temperature, pressure,...
  - edges, corners, lines, curvature, …
  - mean RGB colour, colour histogram, …
- Semantic features, e.g.: (or Nominal or Qualitative)
  - colour layout (red, cyan, magenta,…)
  - texture descriptors (coarse, fine, rough, smooth,…)
  - shape descriptors (rectangular, circular, elliptical,…)
  - kind of day (warm, cold, sunny, rainy, …)
- Ordinal features, e.g.: (or Ordered)
  - Education Level (Higher, Secondary, Primary)
  - Body Mass Index (BMI): Underweight, normal weight, overweight, obese
  - Letter grades in the exam (A, B, C, D, etc.)
- Statistical features, e.g.:
 - mean, median, variance, percentiles, moments, ...
- ...

Quick Review: Features

$$\mathbf{p}_i = \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_d \end{bmatrix}$$

- Features describe characteristics of our data.

- The combination of d features is represented as a d-dimensional column vector called a *feature vector*.

- The d-dimensional space defined by the feature vector is called the *feature space*.
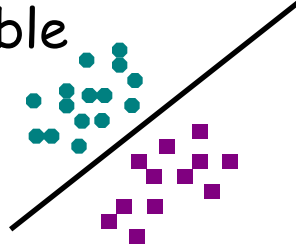
$\mathbf{p}_i \in X$ is a point in feature space X

Example: 3D feature space X
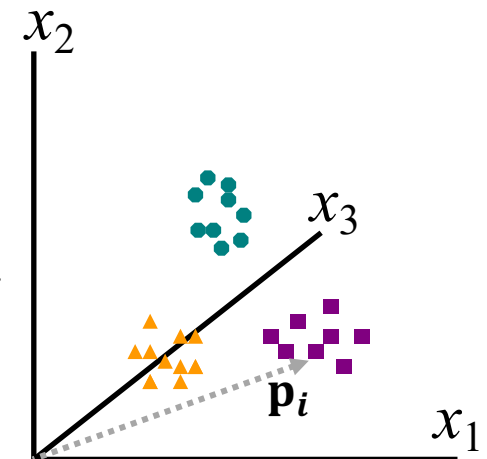
Feature Properties – *what makes a good feature vector?*
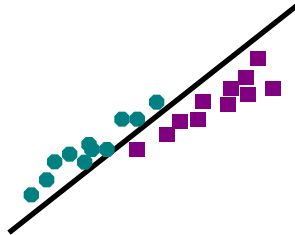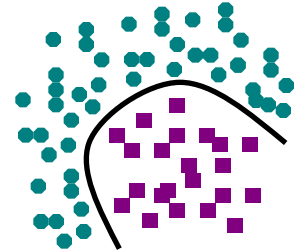
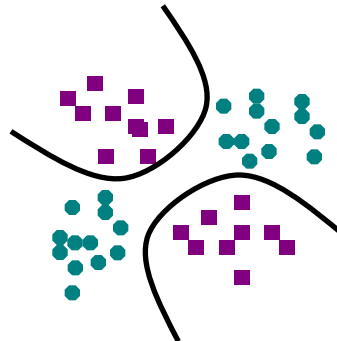good, linearly-separable features

bad features

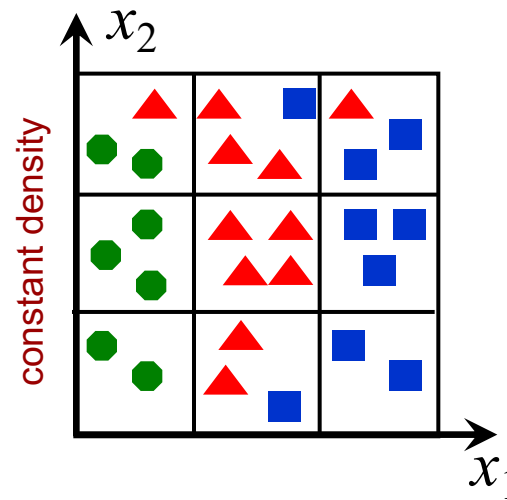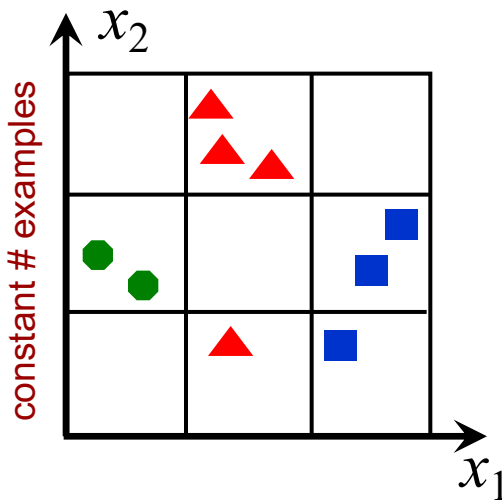highly correlated features

nonlinearly-separable features

multimodal features

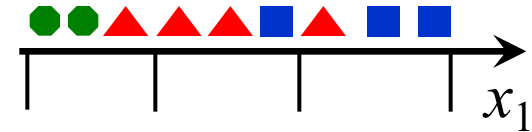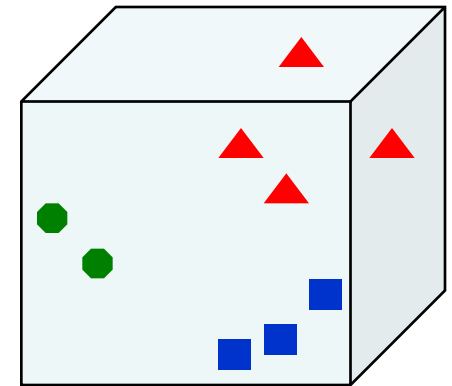# The Curse of Dimensionality

- Example: one feature, no. of bins = 3, but results not good enough, so incorporate another feature.
- But, the no. of bins → $3^2 = 9$, so to maintain the density, increase the no. of examples from 9 to 27

Moving to 3 features makes it even worse…

The amount of data to sustain a given spatial density increases exponentially with the dimensionality of the input space

Dimensionality Reduction

➢ Strive for compact representation of the *properties* of data, such that redundancy/irrelevancy is removed.

➢ The choice of features is very important, as it influences:

- accuracy of classification

- time needed for classification

- difficulty in performing classification

- no. of learning examples
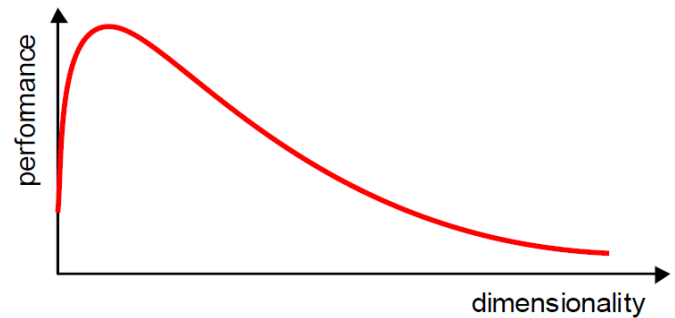
Feature Selection/Extraction

- Feature selection: Choosing $d<N$ important features, ignoring the remaining $N-d$ features

  Subset selection algorithms

- Feature extraction: Project the original m_i, $i=1,...,N$ dimensions to new $d<N$ dimensions, x_j, $j=1,...,d$

  Principal Components Analysis (PCA),

  Linear Discriminant Analysis (LDA), & many more…

- Feature construction: create new features based on old features: $f=(…)$ where f is usually a non-linear function, e.g Support Vector Machines,…. (not within scope)

Selection or Extraction?

Two general approaches to dimensionality reduction:

➢ Feature Selection: Selecting a subset of the existing features without a transformation

➢ Feature Extraction: Transforming the existing features into a lower dimensional space
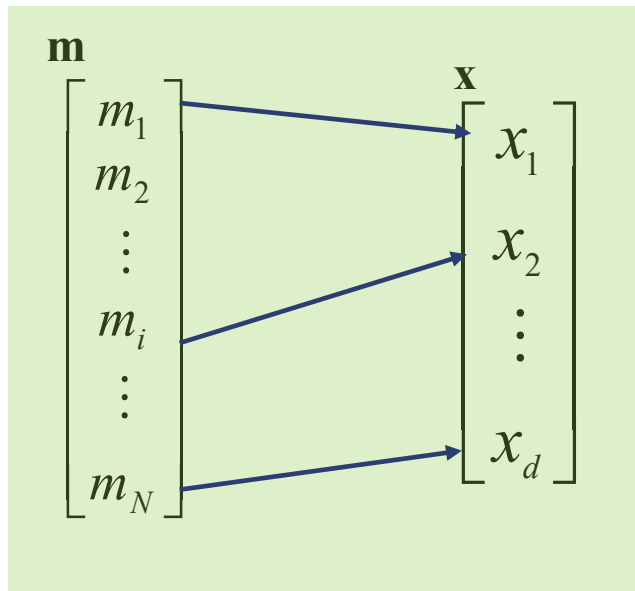
$$d \ll N$$

Exhaustive Feature Selection

- Feature Selection is necessary in a number of situations, e.g. there may be too many features or may be too expensive to obtain.

- Feature Selection involves a search strategy that may explore the space of all possible combinations of features.
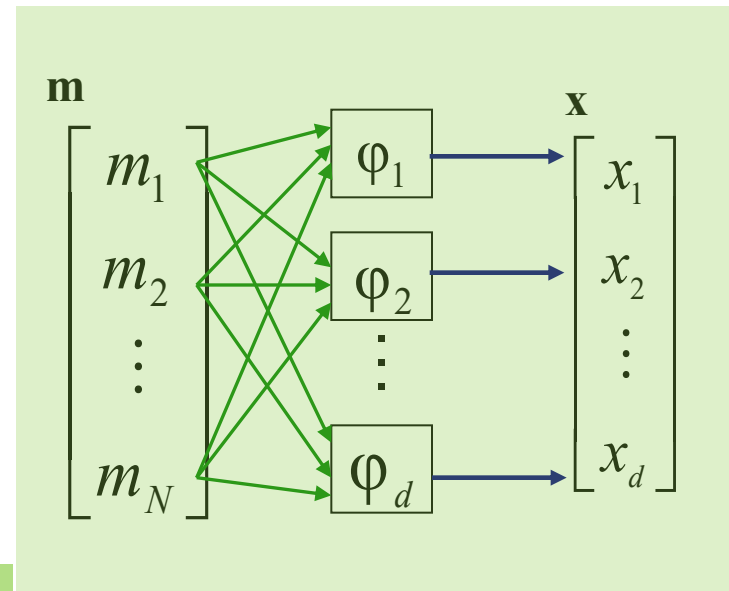
Given a feature set $\{x_i\}, i = 1, ..., N$, find a subset $\mathbf{X}$ of size d with $d < N$, that optimizes an objective function $J(\mathbf{X})$, e.g. P(correct classification). This function would have to be evaluated many times:

e.g. for 10 features out of 25 one would still have to consider 3,268,760 feature sets. 🤔

$$\frac{N!}{(N-d)!\,d!}$$

Exhaustive Feature Selection

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Given a feature set $\{x_i\}, i = 1, \ldots, N$, find a subset $\mathbf{X}$ of size d with $d < N$, that optimizes an objective function $J(\mathbf{X})$, e.g. P(correct classification). This function would have to be evaluated many times:

e.g. for 100 features out of 10000 one would have to consider K feature sets. What is K?

$$\frac{N!}{(N-d)!d!}$$

Exhaustive Feature Selection

K = 5,000,000,000,000,000,000,000,000,000,
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And continue with as many more 0s as above for 322 more slides....

i.e. $K = 5 \times 10^{35101}$

Feature Selection?
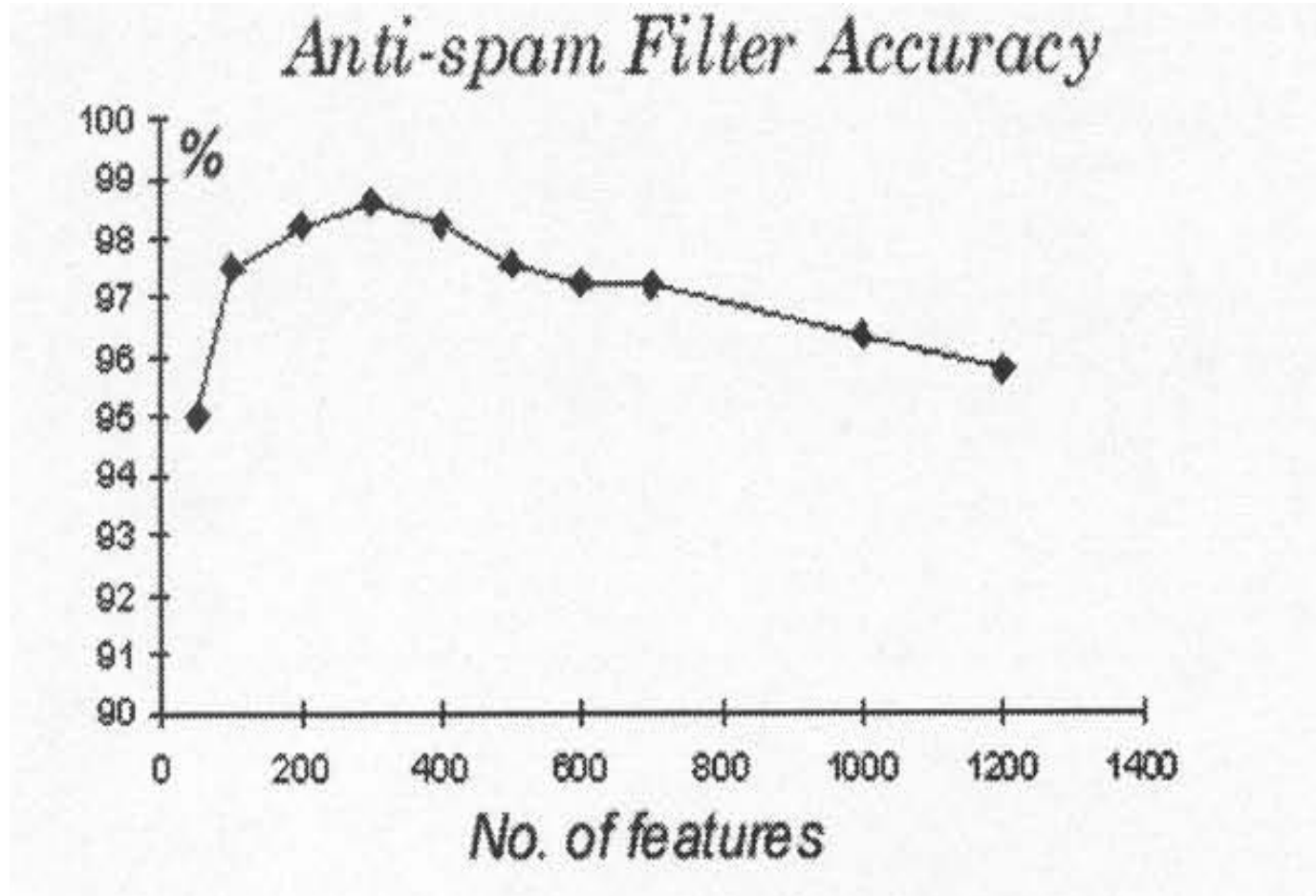
From http://elpub.scix.net/data/works/att/02-28.content.pdf

Sequential Feature Selection Methods

- Assume features are independent.

- Best single features can be chosen by significance tests.

Forward stepwise feature selection:
(bottom-up)

Build up d features incrementally,
starting with an empty set

The best single feature is picked first.

Then next best feature conditioned to the first, ...

Forward stepwise selection example with 5 variables:

Start with a model with no variables
Null Model

Add the most significant variable

Model with 1 variable

Keep adding the most significant variable until reaching the stopping rule or running out of variables

Model with 2 variables

https://quantifyinghealth.com/stepwise-selection/

Sequential Feature Selection Methods

- Assume features are independent.

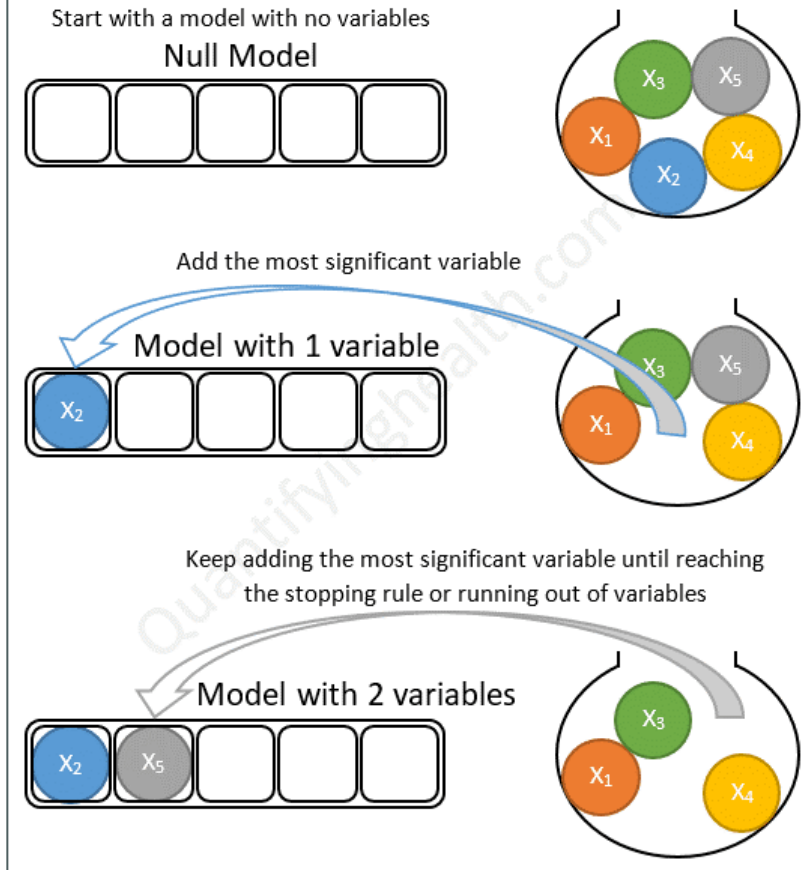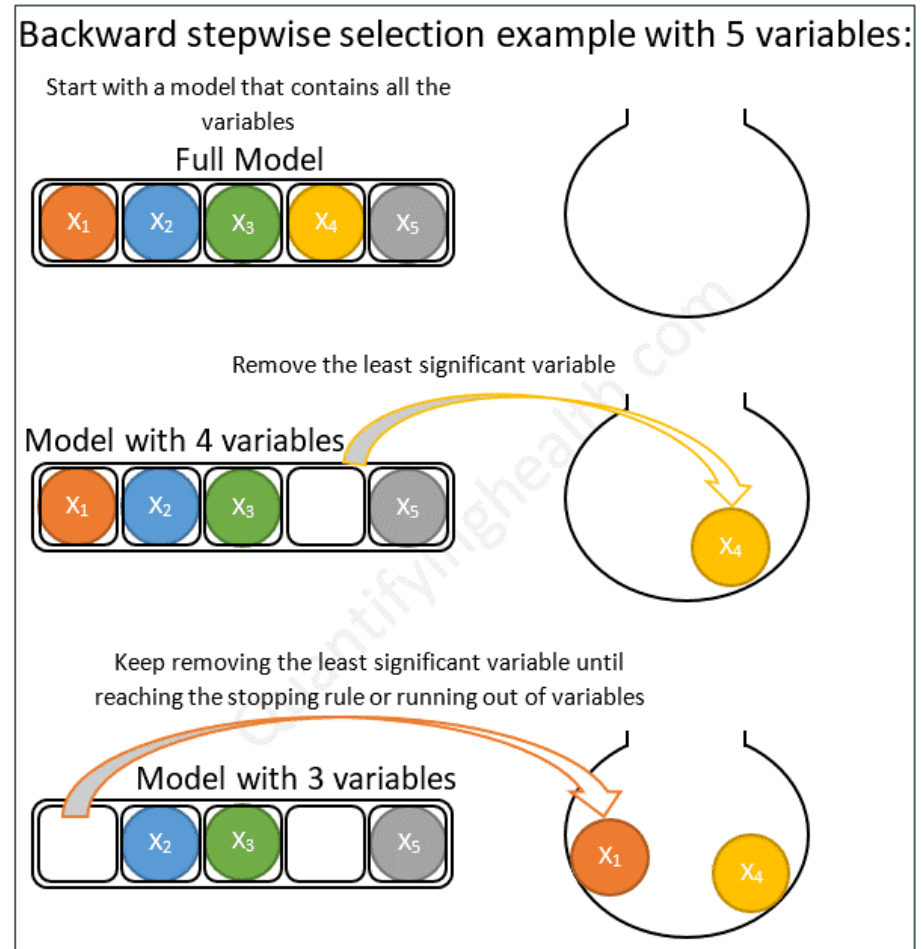- Best single features can be chosen by significance tests.

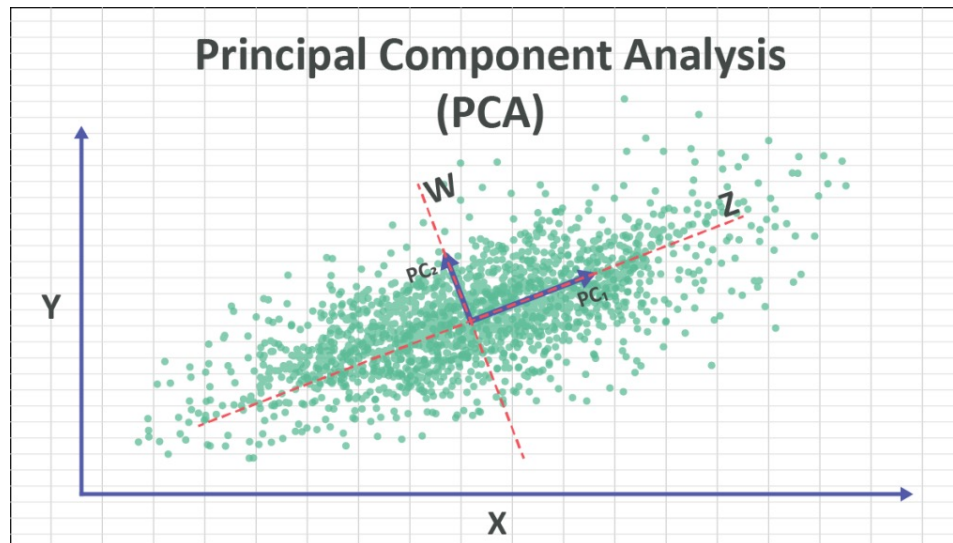*Backward stepwise feature selection*:
(top-down)

start with full set of features and remove redundant or least significant ones one after another

Until optimal d features reached

Backward stepwise selection example with 5 variables:

Start with a model that contains all the variables

Full Model

X_1 X_2 X_3 X_4 X_5

Remove the least significant variable

Model with 4 variables

X_1 X_2 X_3 X_5

X_4

Keep removing the least significant variable until reaching the stopping rule or running out of variables

Model with 3 variables

X_2 X_3 X_5
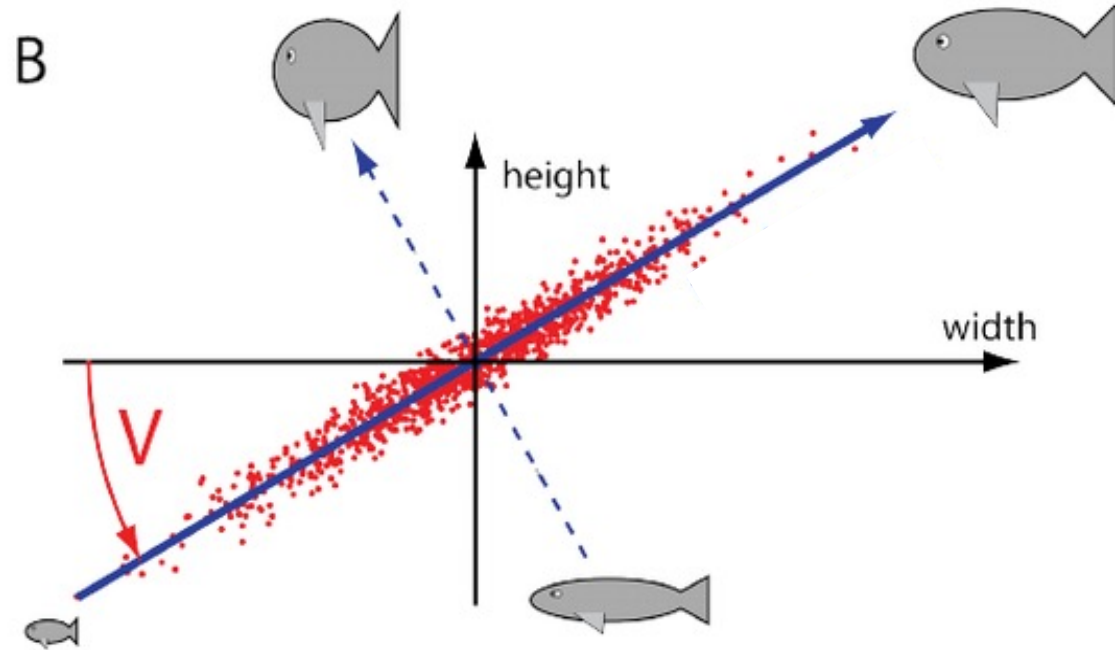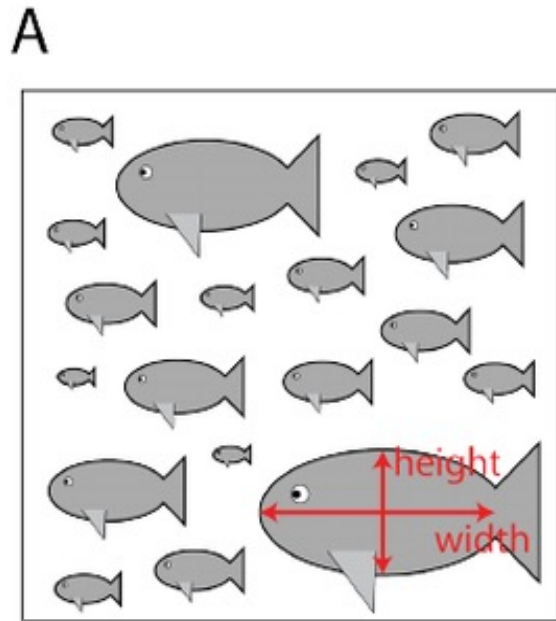
X_1 X_4

https://quantifyinghealth.com/stepwise-selection/

Feature Extraction
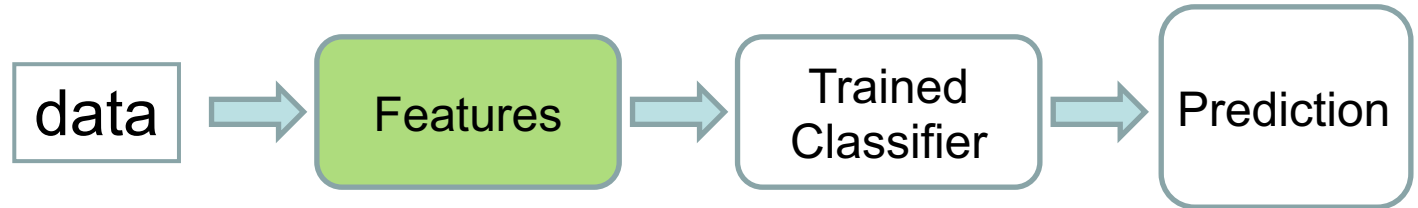
- Linear or non-linear transformation of the original variables to a lower dimensional feature space → also known as *feature selection in the transformed space*.

- Given a feature space R^N with feature vectors $\mathbf{m}$, find a mapping
  $\mathbf{x} = \varphi(\mathbf{m}): R^N \Rightarrow R^d, d < N,$ such that the transformed feature
  vector $\mathbf{x} = \{x_i\} \epsilon\, R^d$ preserves (most of) the information or structure in R^N.

- Principal Components Analysis (PCA) is an example of a transformed space for dimensionality reduction from which we can extract the most efficient features.

Principal Component Analysis (PCA)

PCA

Image from Werner et al, PLoS ONE 9(11), 2014

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data  →  Features  →  Trained Classifier  →  Prediction
```

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- ➤ **PCA for dimensionality reduction**
- ➤ Convolutions