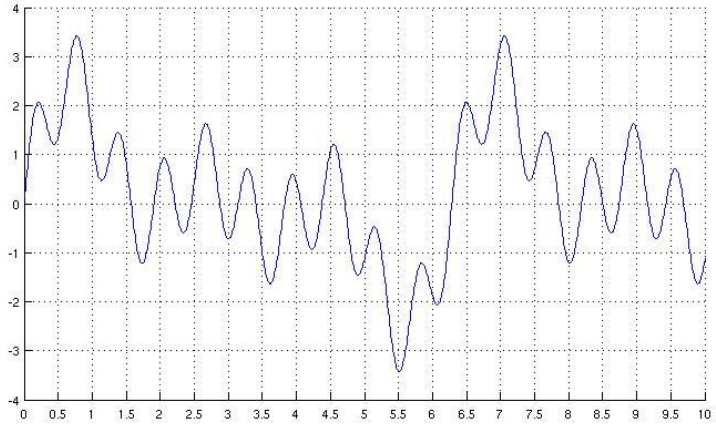
COMS20011 – Data-Driven Computer Science

Problem Sheet MM03

1 – Using *sin(2πnx)*, demonstrate the concept of superposition as follows (in Matlab or Python):

(a) first plot three sine functions over the range ±3 in steps of 0.1 using *n={1/4,1,2}*. Note, plots should appear in the same graph to give a better sense of what is happening.
(b) Now plot in a different colour the sum of all the sines above.
(c) Add more sine functions over the same range and repeat step (b).

2 – Based on your understanding of the Nyquist Sampling Rate theorem, what is a sufficient sampling rate for the signal below? Hint: the signal is composed of the summation of sin(x), sin(2x), sin(3x) and sin(10x).



3 – Determine which is an even and which is an odd function:

(i) $f(x) = 7x^3 - x$

(ii) $f(x) = 3x^2 + 1$

(iii) $f(x) = 3x^2 \sin(x)$

(iv) $f(x) = \dfrac{3}{(-x)^4 - 4}$

(v) $f(x) = \cos(x) + 5x \quad - 3$

4 – The period of the signal $x(t) = 10 \sin 12\pi t + 4 \cos 18\pi t$ is:

a) π/4
b) 1/6
c) 1/9
d) 1/3
e) 1/30

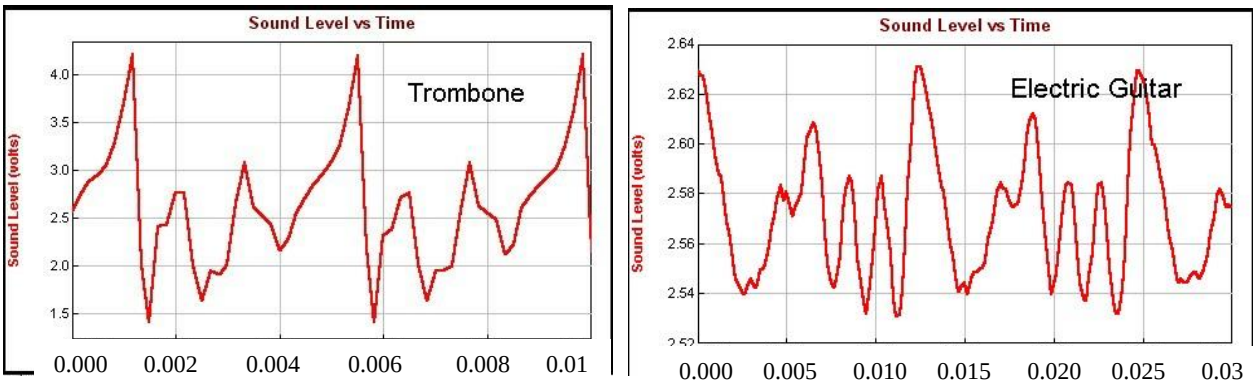
5 – The following gene sequence contains significant frequencies. Design two different symbolic encodings and in each case apply your encoding to extract some of these frequencies.

ACAGAGATACAGAGATACAG . . . . . .

6 – If the fastest oscillations that we want to measure are at 120 Hz, which of the following is the most reasonable sampling rate?

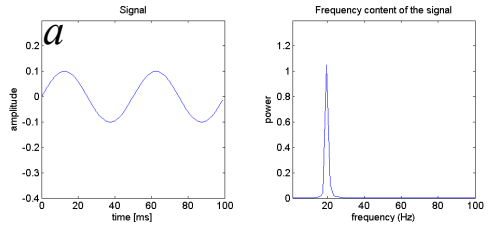
a. 60 Hz
b. 60 kHz
c. anything over 0.00833 Hz
d. 250 Hz
e. 120 Hz

7 – The graphs below display the amplitude of the sound wave for a Trombone and an Electric Guitar as a function of time. The y-axis is the amplitude axis and the x-axis is the time axis. Notice that each one is plotted over a different length of time.



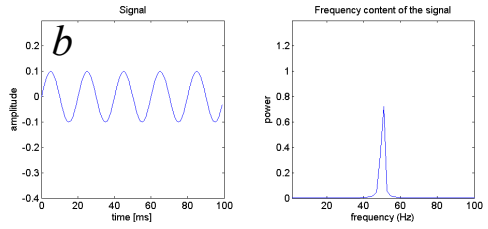
(a) Mark the period of the signal for each instrument.
(b) Approximately, how many periods are shown in these graphs for each instrument?
(c) Approximately, what is the peak amplitude in each case?
(d) Approximately, what is the frequency given the signal period in each case?
(e) Which signal contains higher frequency information? Why?

8 – Consider the three signals a, b, and c below, and their addition d.

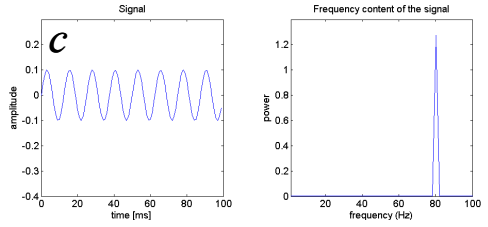


time domain frequency domain

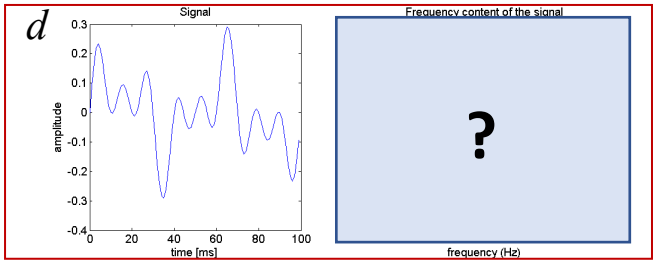
$d = a + b + c$



time domain frequency domain



time domain frequency domain



time domain frequency domain

(a) What would the frequency of the signal d look like?

(b) How many oscillations per second does signal a have?

(c) How can you determine the frequency of signal c if you did not have the frequency domain plot of that signal?

9 – What are the two 1D filters that can replace the 2D filter (in each example for W and X) if they were applied consecutively?

$$W = \frac{1}{9}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \qquad X = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{pmatrix}$$