

From Quasi-Synchrony to LTTA

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Timothy Bourke
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Context

Real-Time Fidelity in Synchronous Languages

Study Quasi-Synchrony as an example

- Mix of real- and discrete-time
- Notion of tolerance (e.g., “jitter”)

Outline

1. Quasi-Synchrony
2. Discrete Abstraction
3. Loosely Time-Triggered Architecture

Quasi-Synchrony

[Caspi 2000]

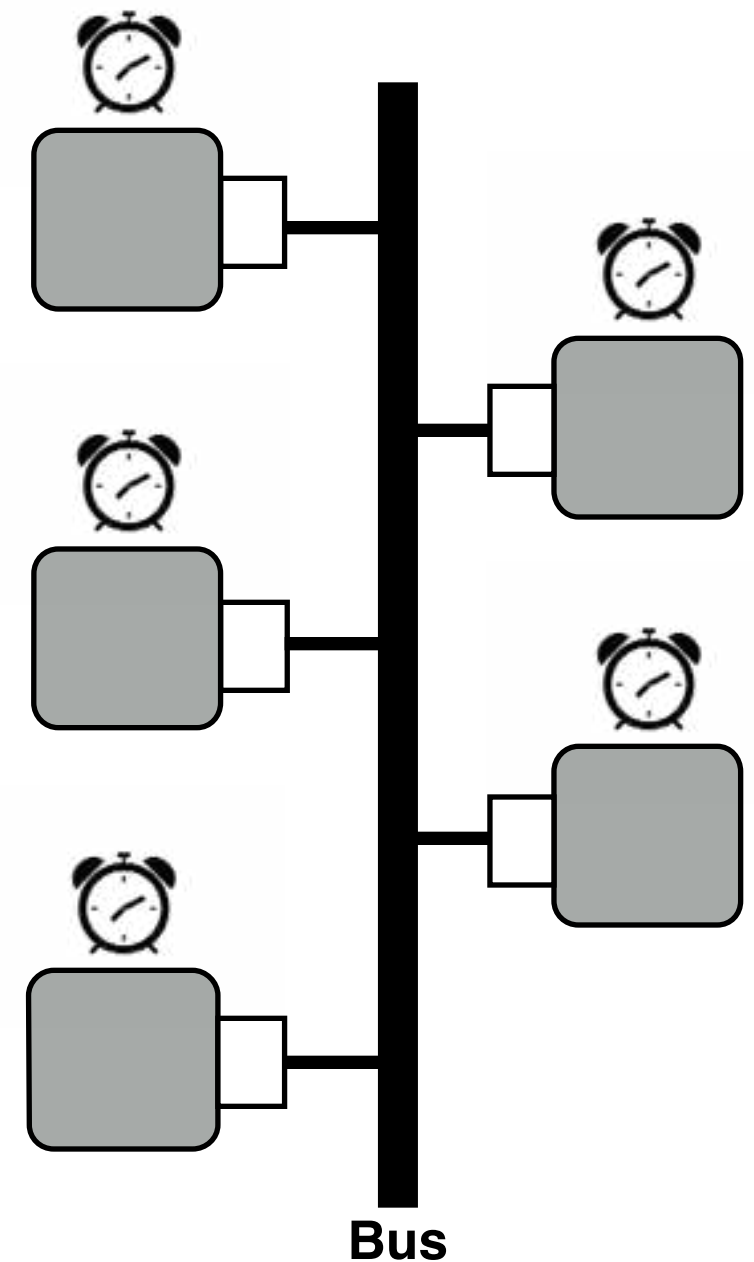
Quasi-Periodic Architecture

- A set of “quasi-periodic” processes with local clocks and nominal period T^n (jitter ε)

$$0 < T_{\min} \leq T^n \leq T_{\max} \quad \text{or} \\ T^n - \varepsilon \leq \kappa_i - \kappa_{i-1} \leq T^n + \varepsilon$$

$(\kappa_i)_{i \in \mathbb{N}}$ clock activations

- Buffered bus-based communication
- Bounded communication delay



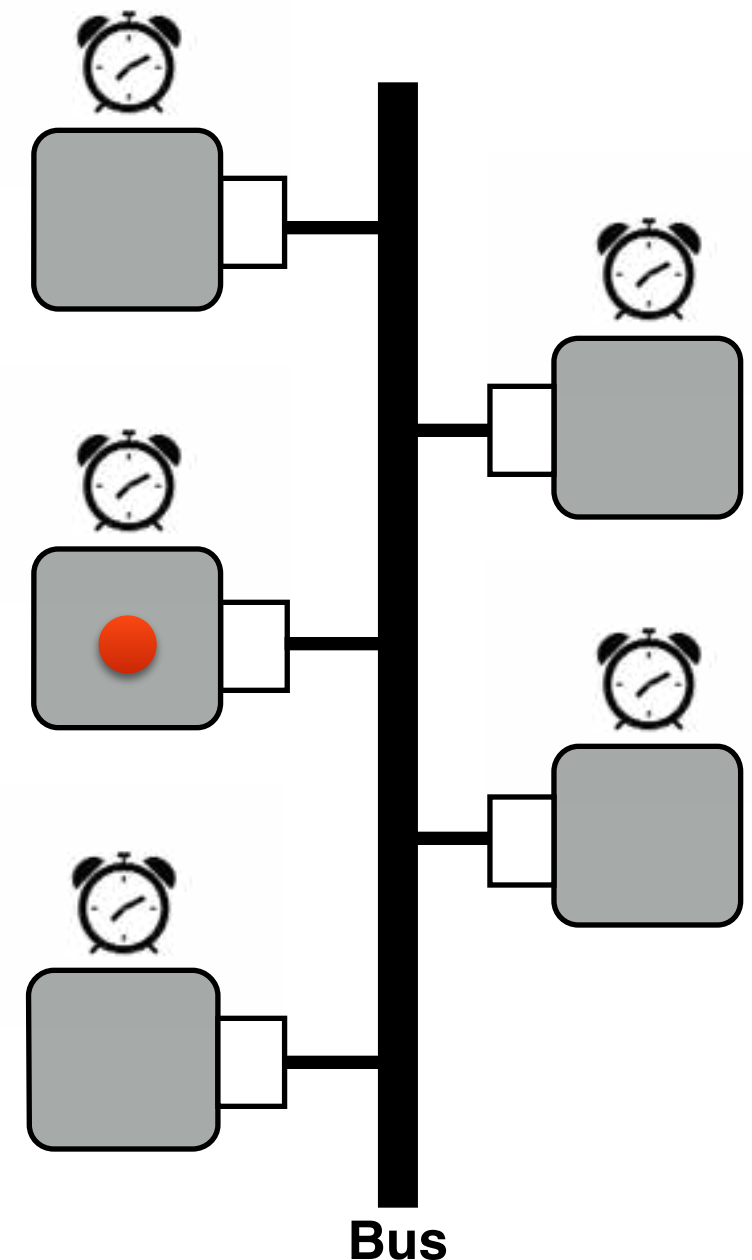
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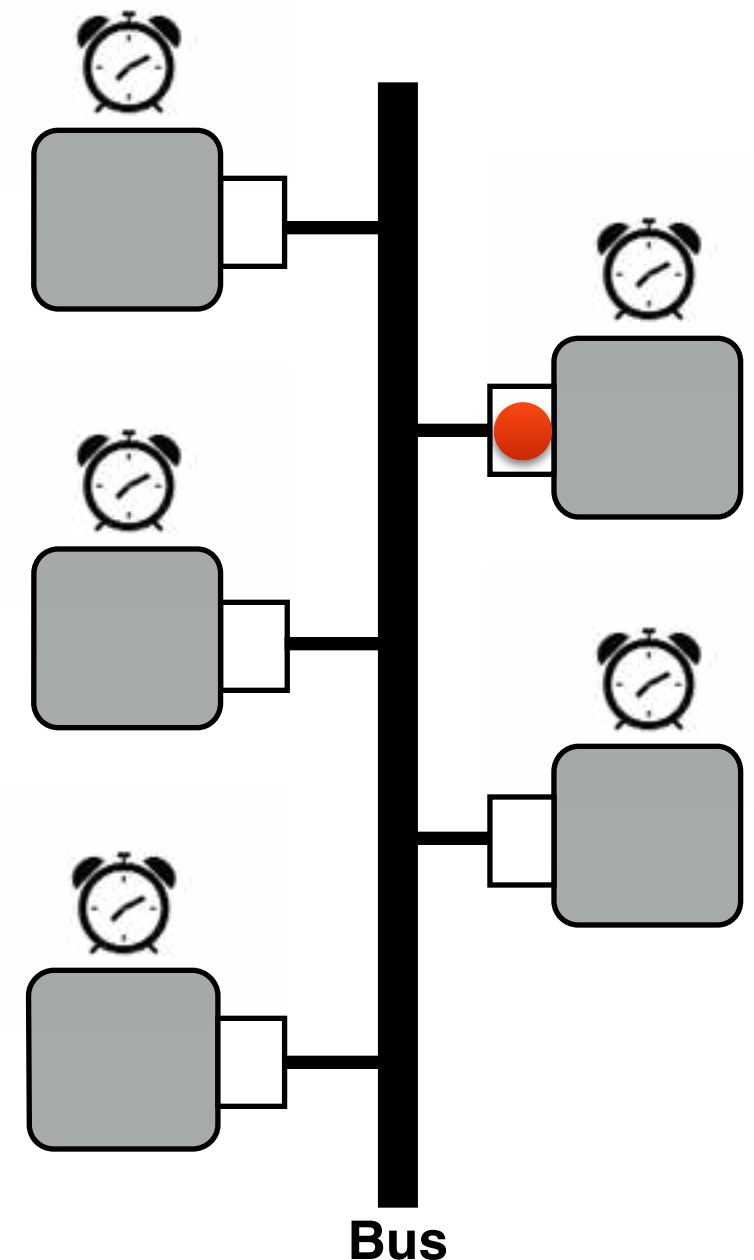
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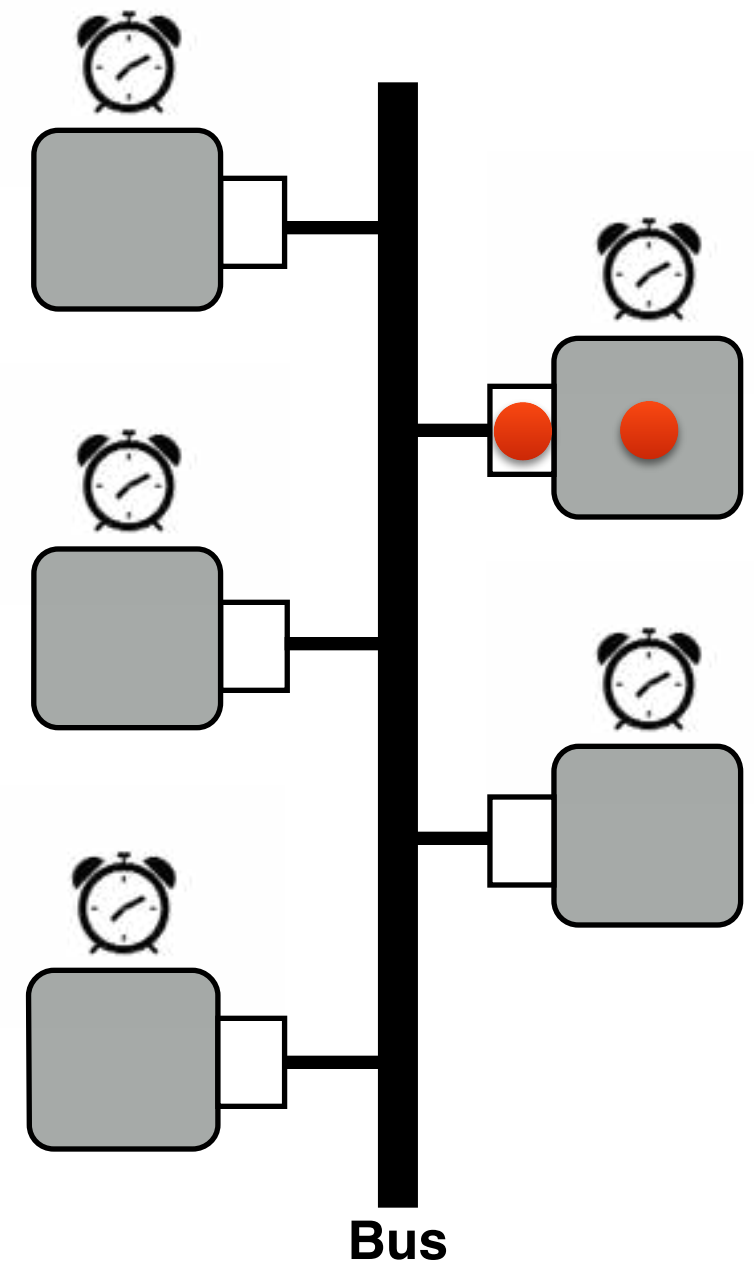
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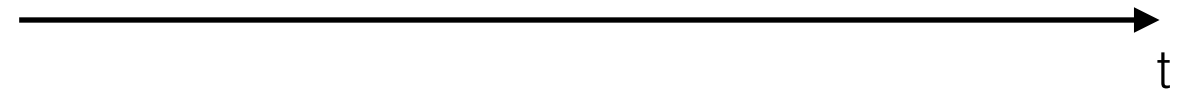
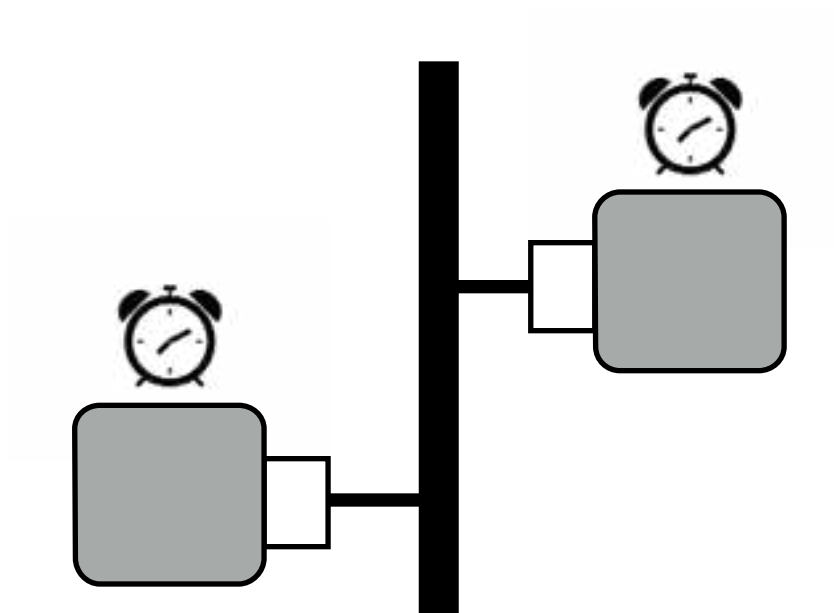
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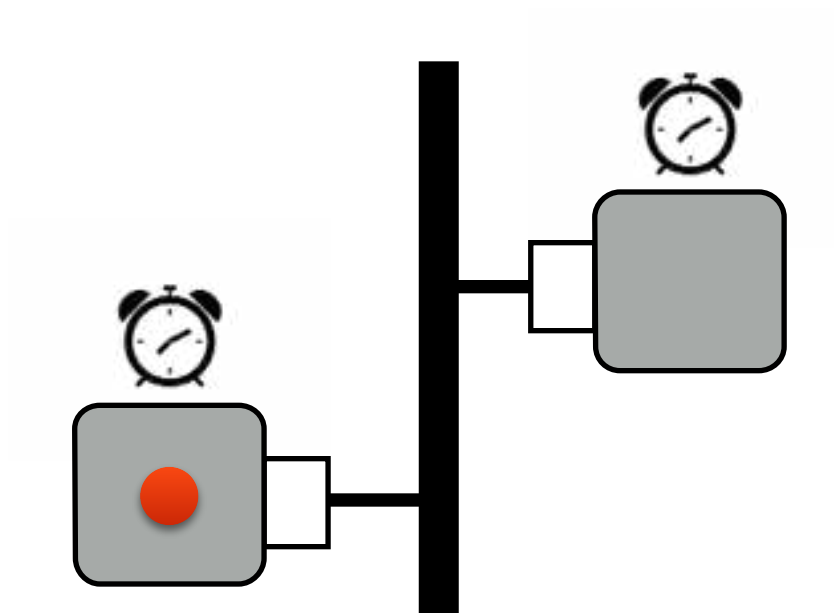
Two phenomena

Overwriting



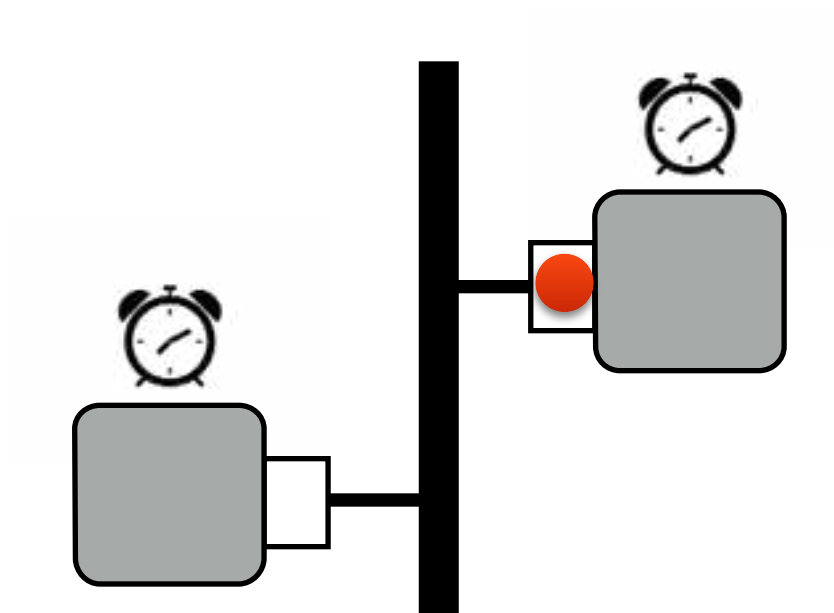
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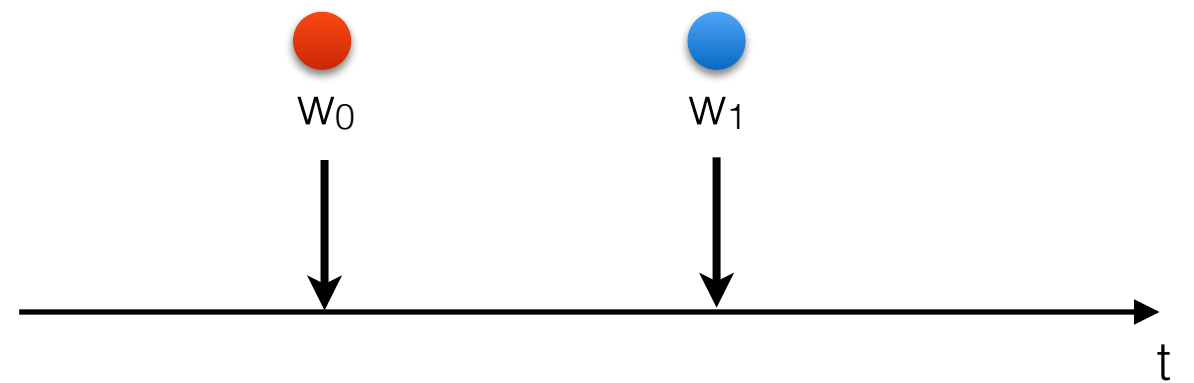
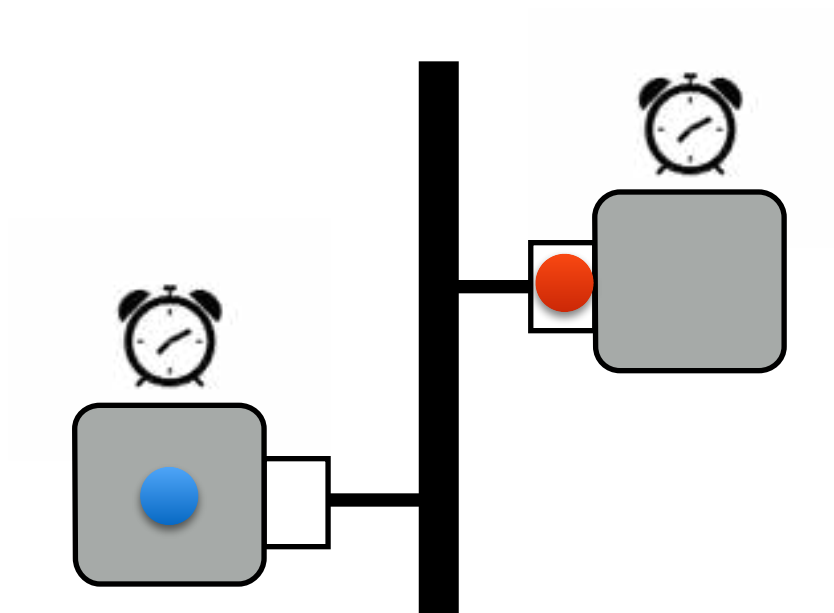
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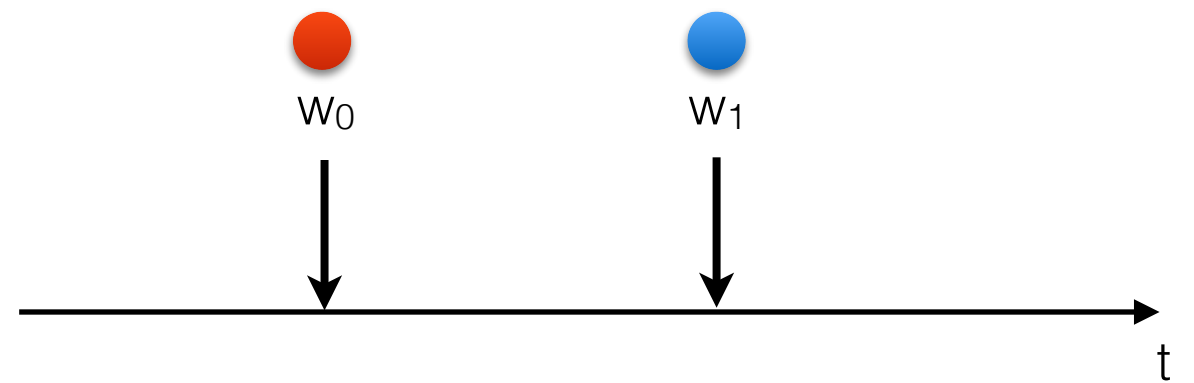
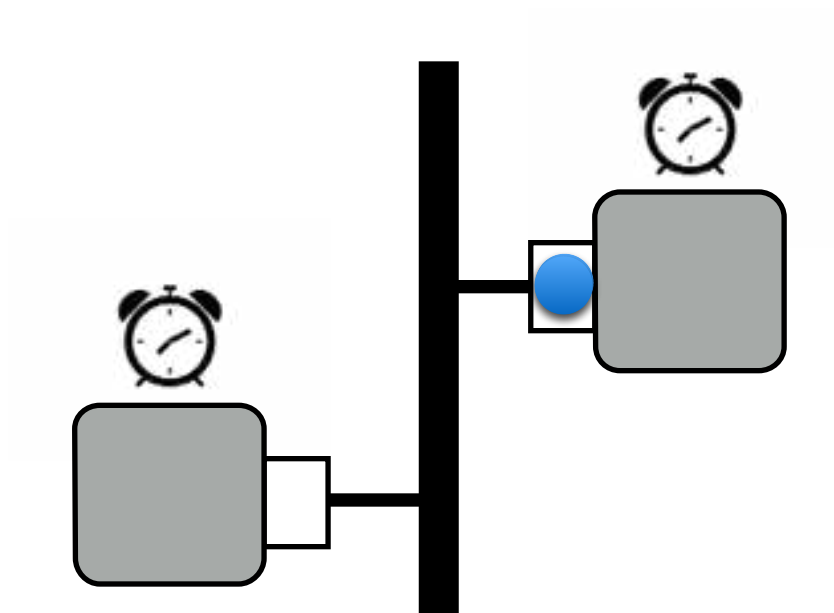
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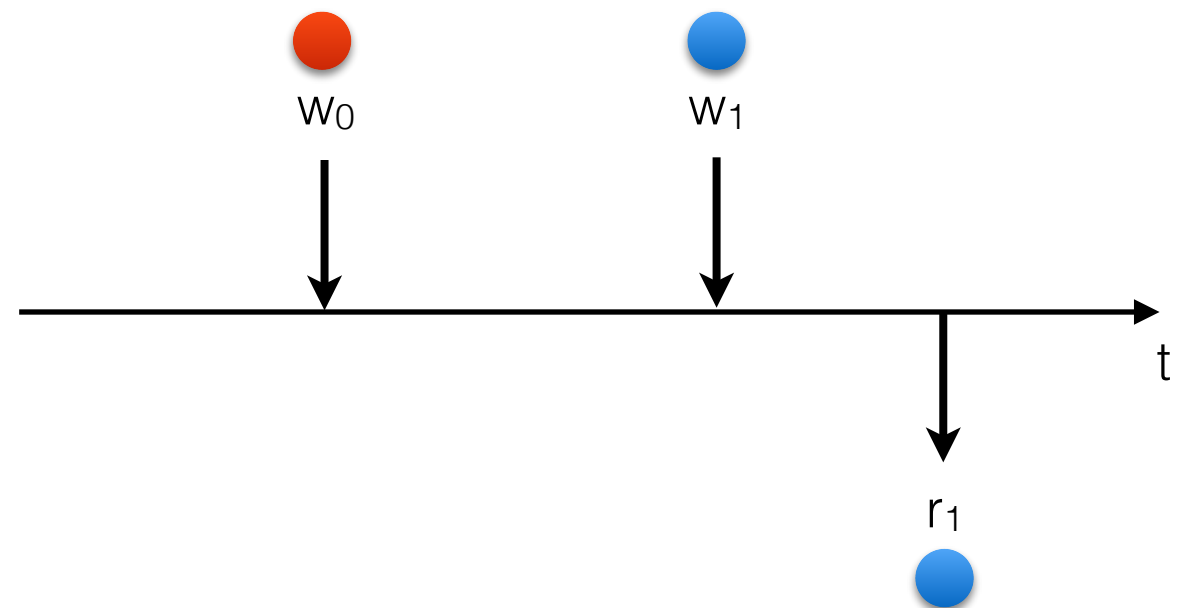
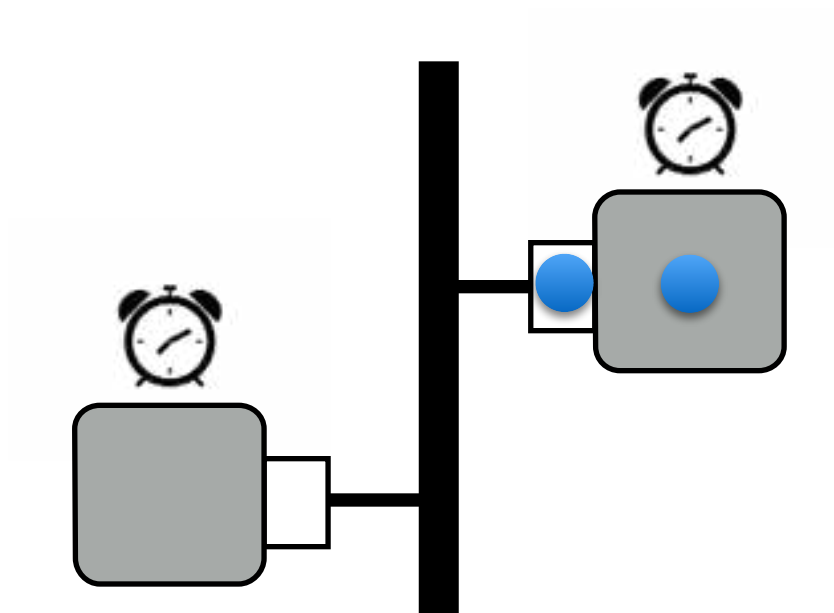
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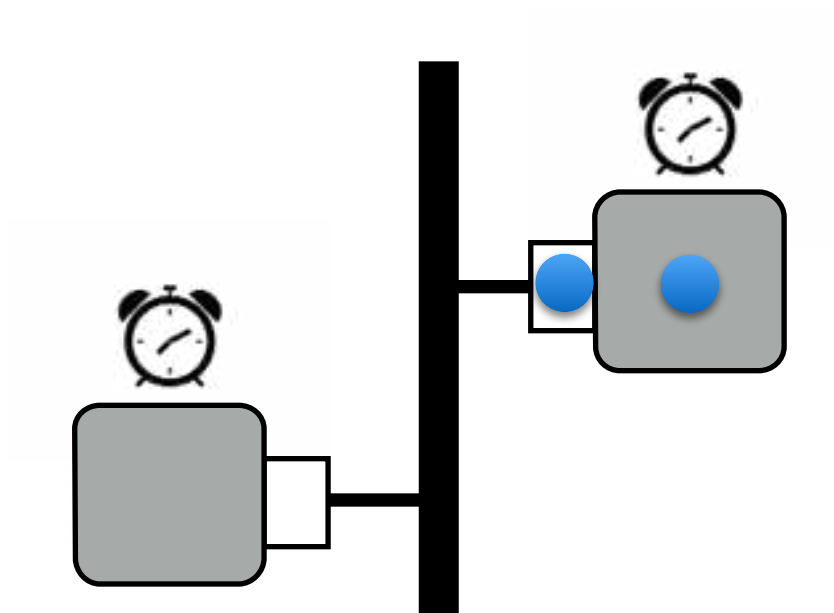
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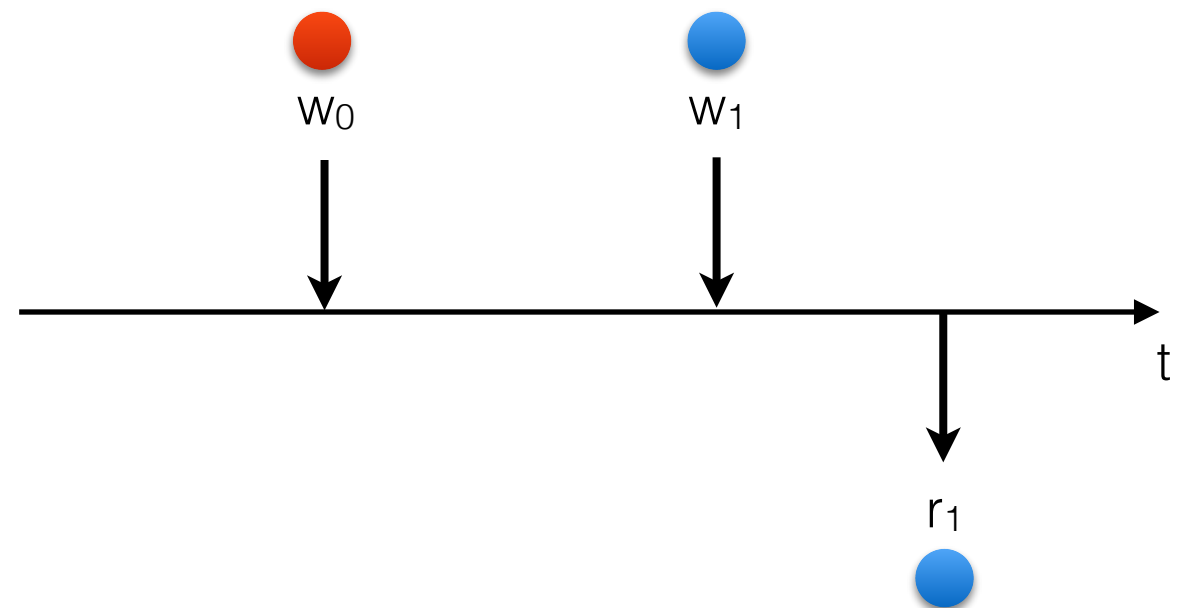


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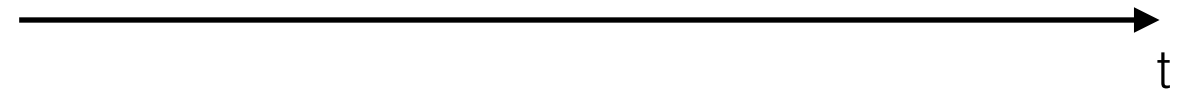
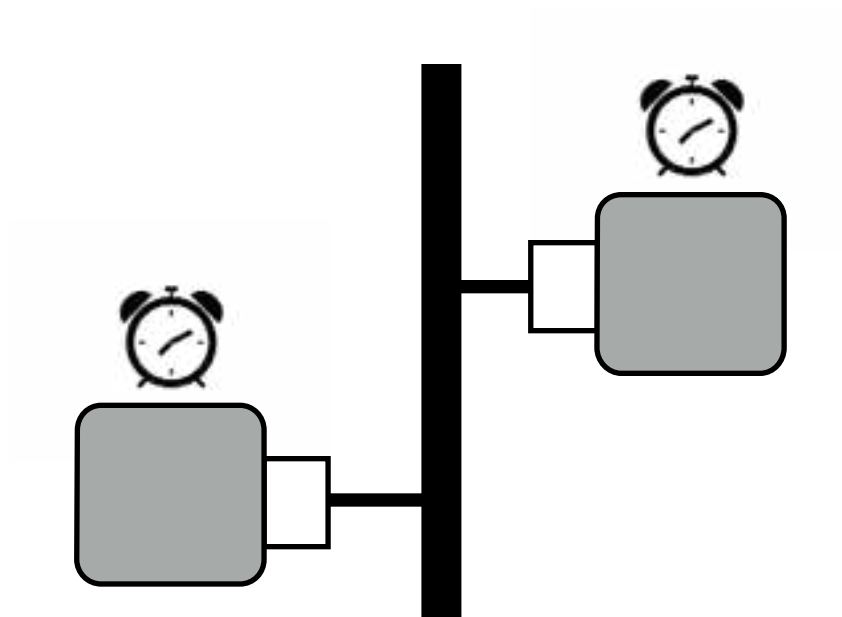


● missed



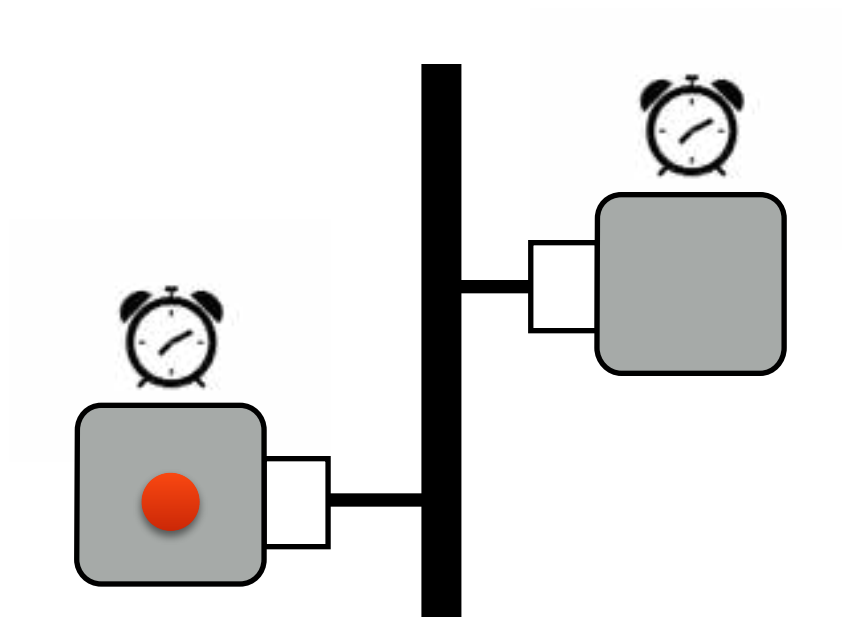
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Oversampling



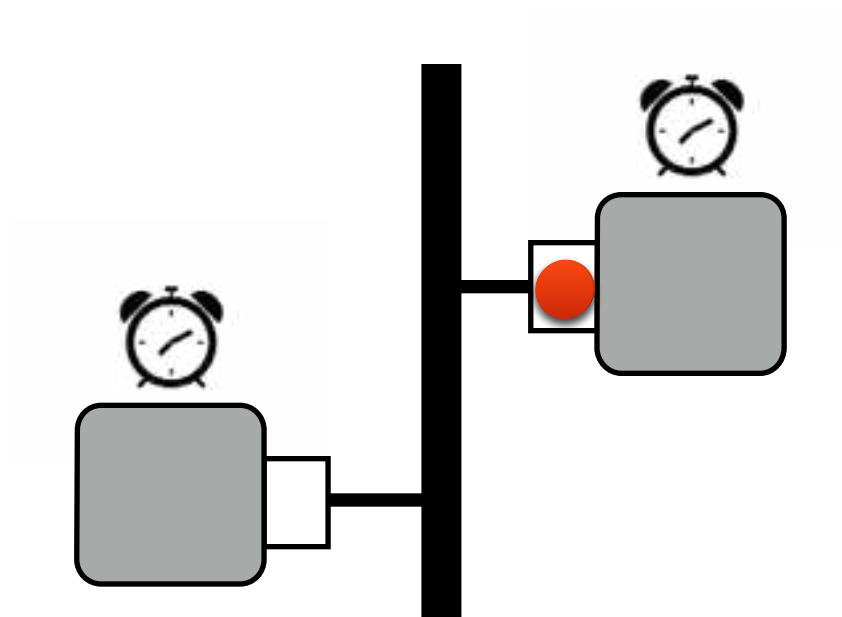
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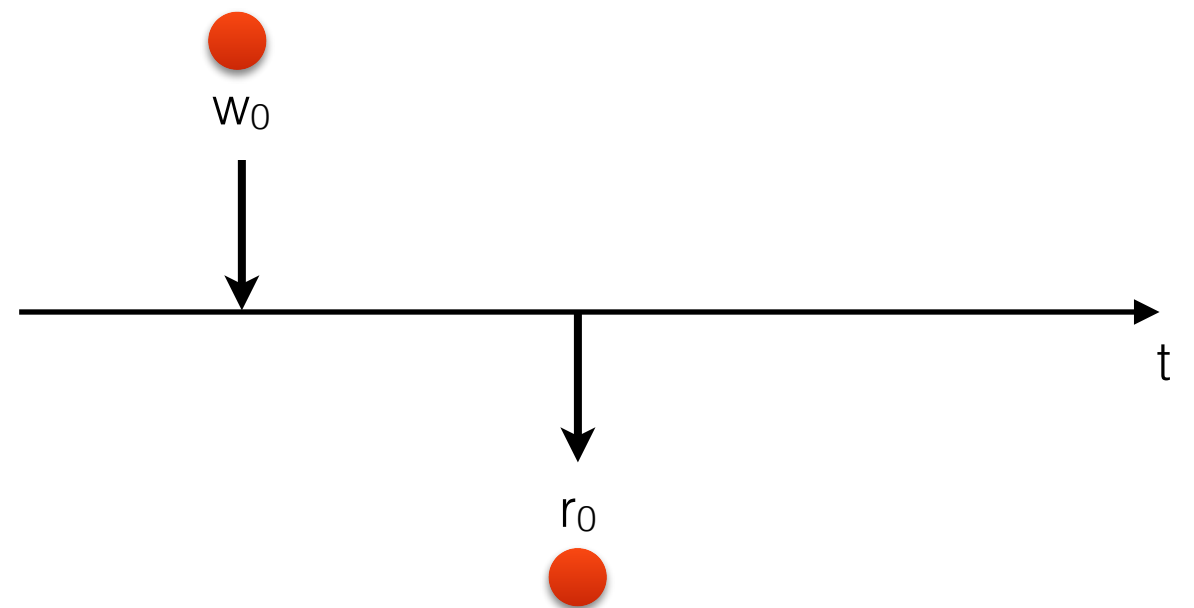
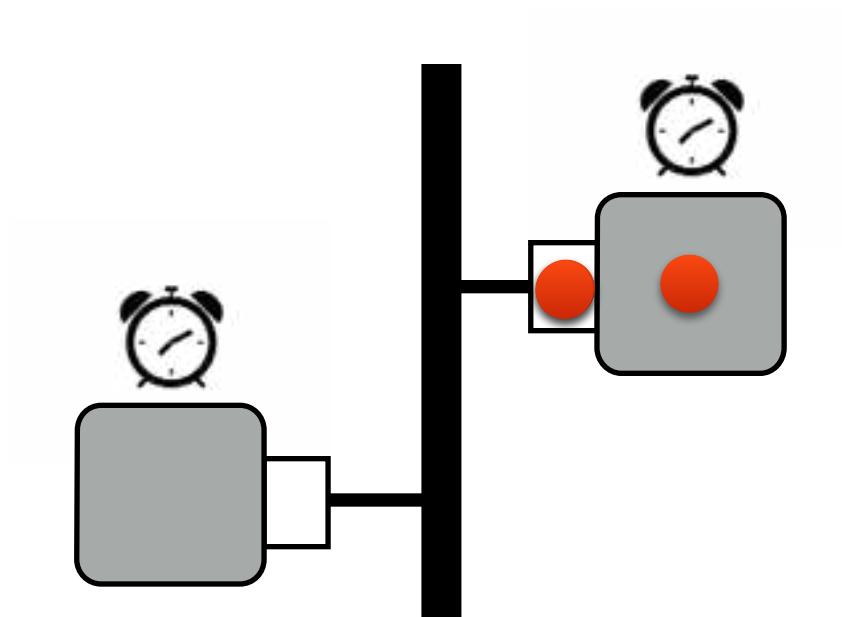
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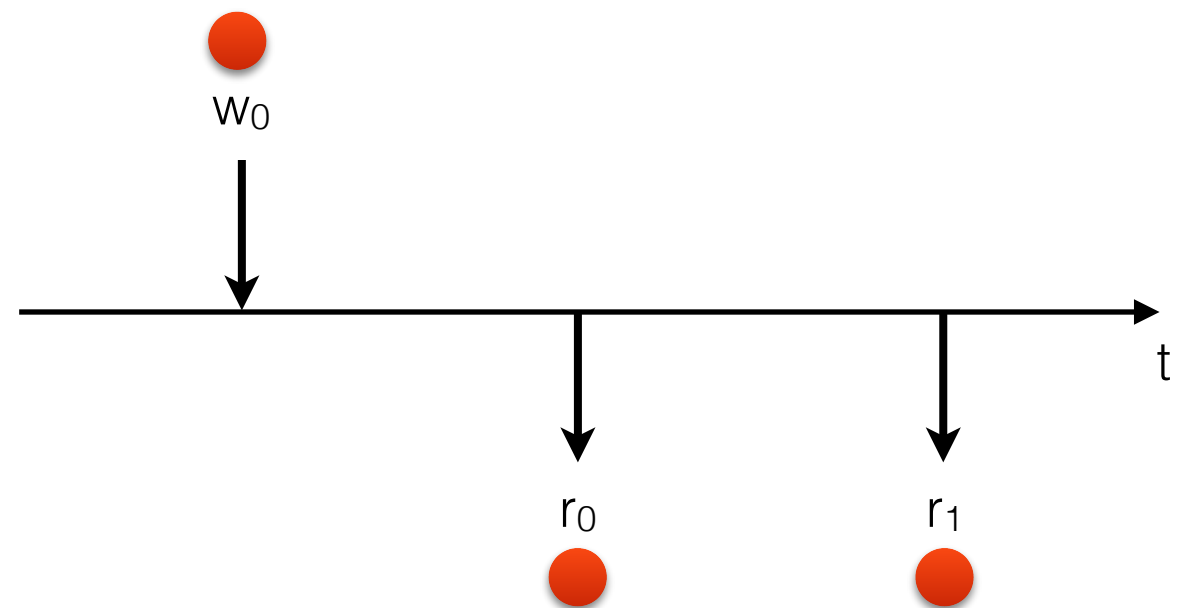
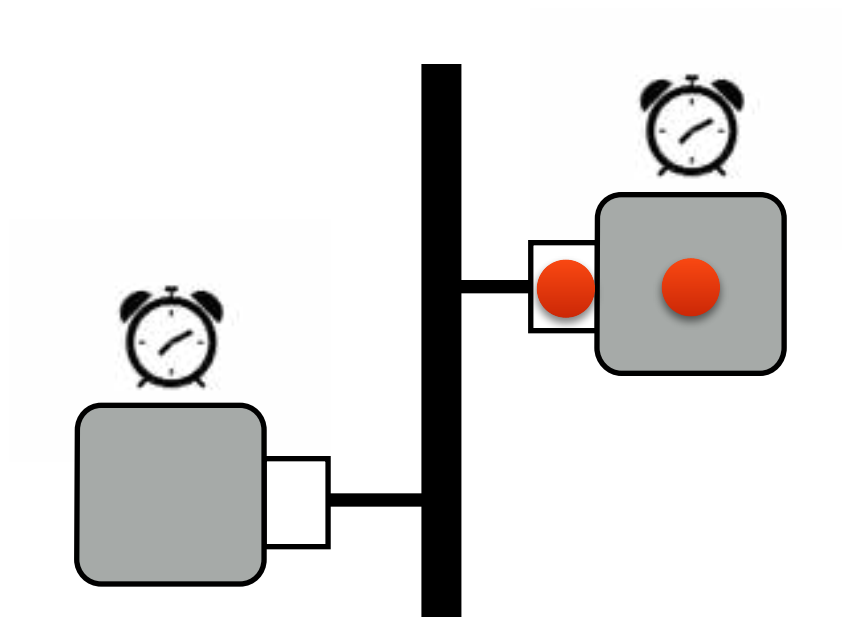
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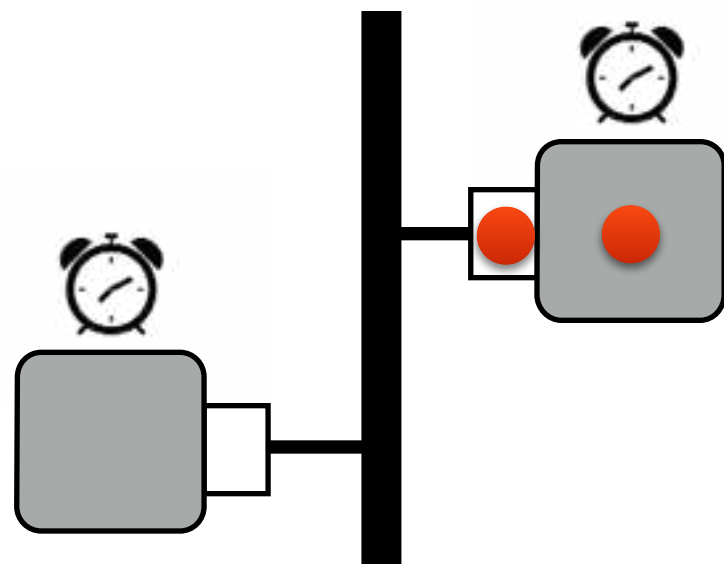
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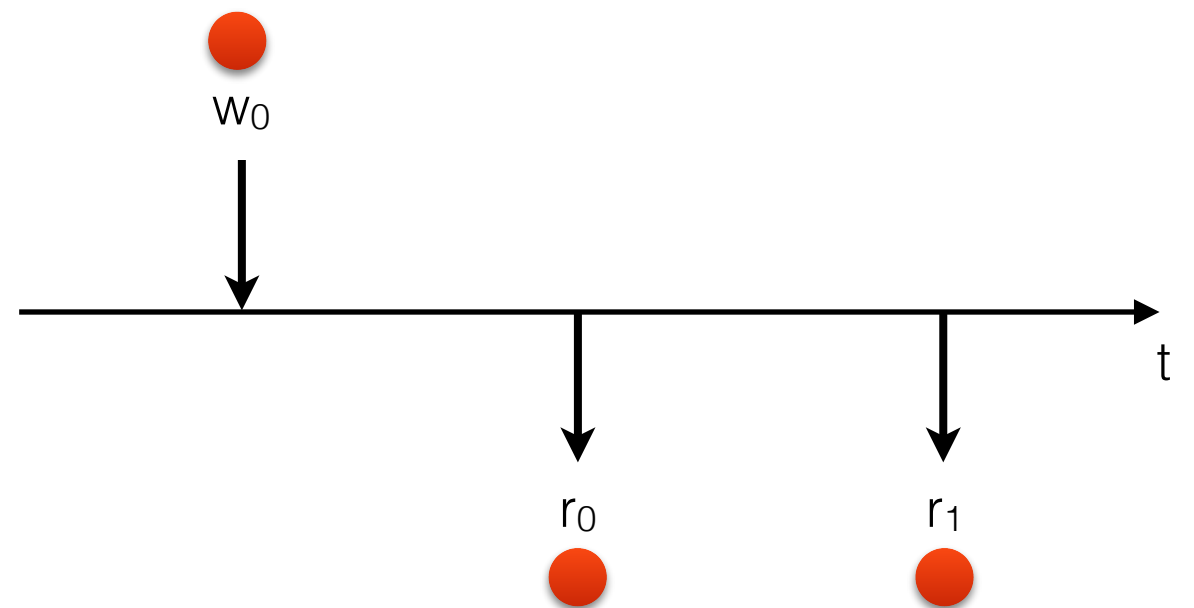


Two phenomena

Oversampling



● read twice



Bounding Overwritten Values

Proposition: Given a writer and a reader, the maximum number of consecutive overwrites is

$$n_o = \left\lceil \frac{T_R^{max}}{T_W^{min}} \right\rceil - 1$$

Proof: a Fencepost Problem



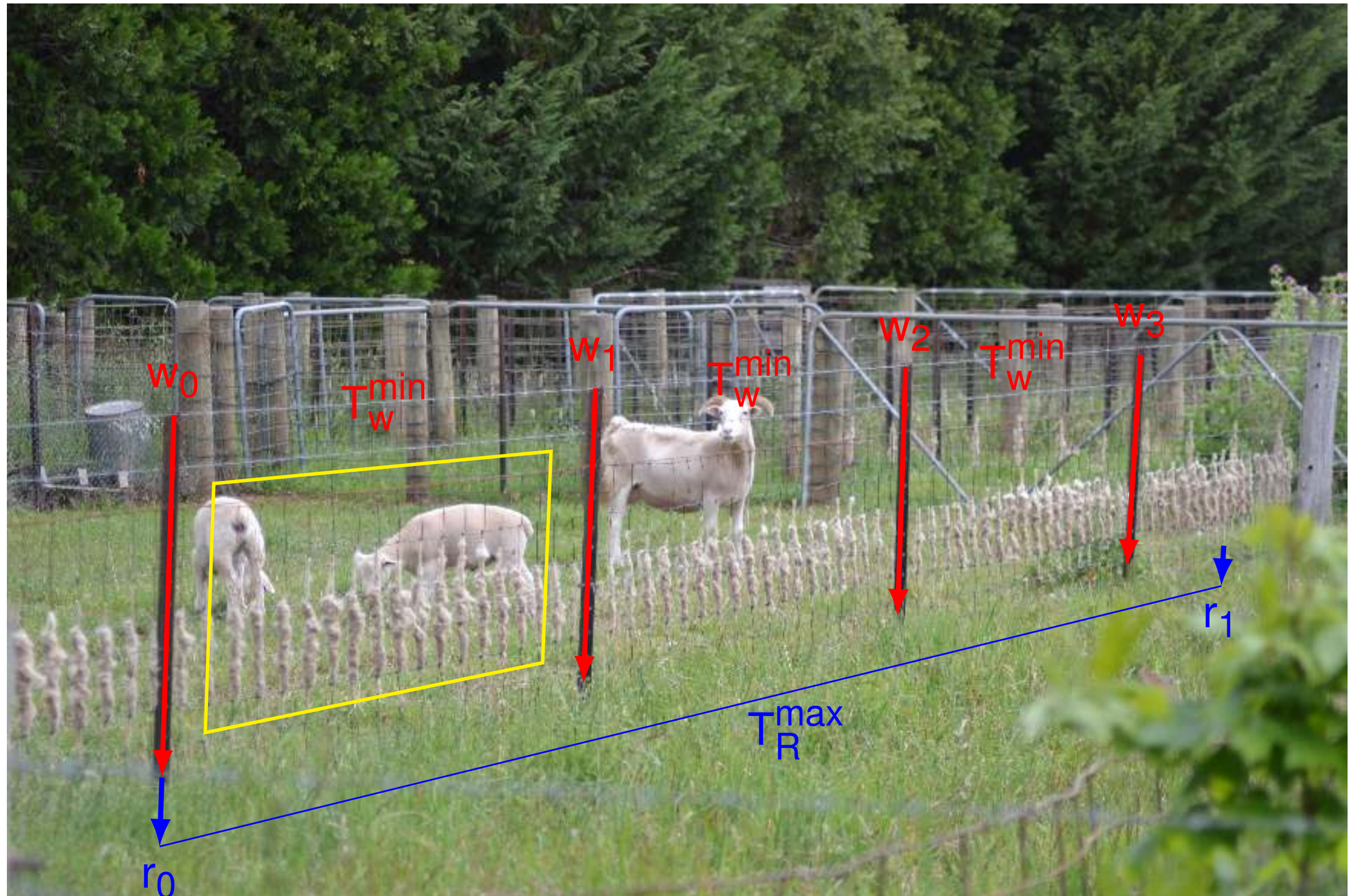
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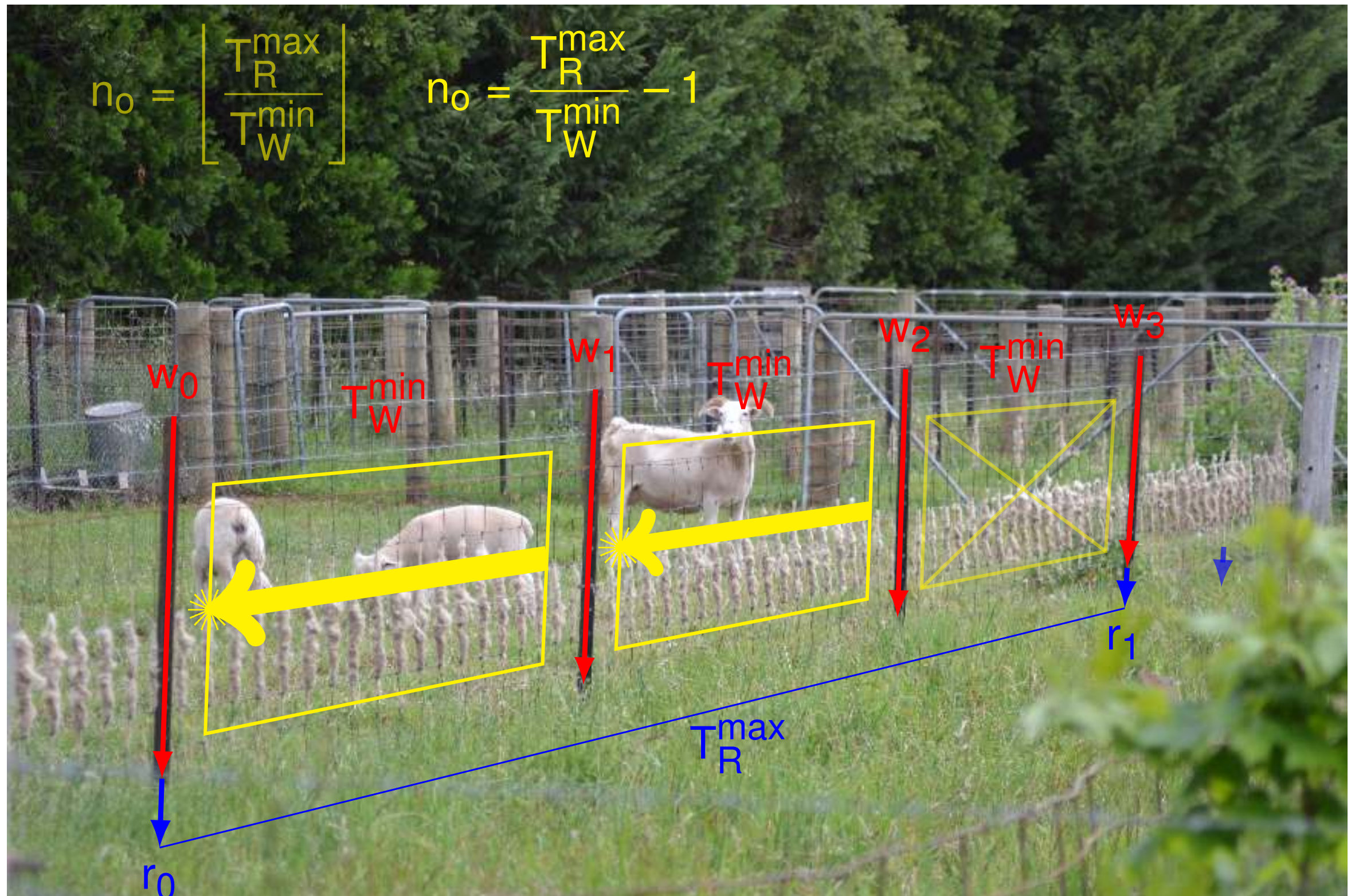
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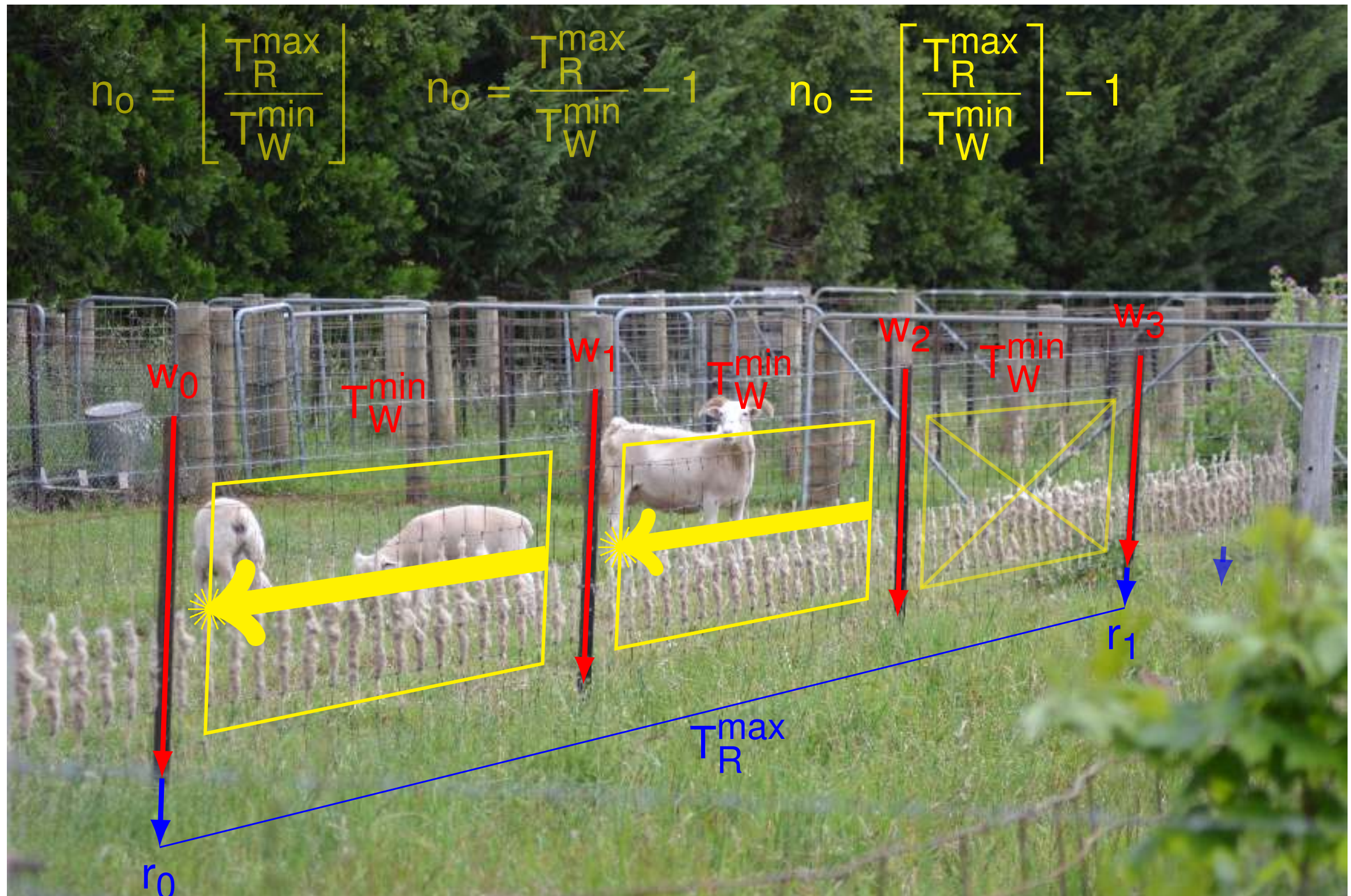
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Bounding Overwritten Values

Corollary: Given a writer and a reader with the same nominal period and a small jitter of $0 < \varepsilon \leq \frac{1}{3}T^n$ it follows that $n_o = 1$

$$\begin{aligned} \text{Proof. } n_o &= \left\lceil \frac{T^n(1 + 1/3)}{T^n(1 - 1/3)} \right\rceil - 1 \\ &= \left\lceil \frac{4/3}{2/3} \right\rceil - 1 \\ &= 1 \end{aligned}$$

Conclusion:

In a Quasi-periodic System where all process have the same period and a small jitter, **only one** value can ever be lost between two reads

Bounding Oversampled Values

Proposition: Given a writer and a reader, the maximum number of consecutive oversampling is

$$n_s = \left\lceil \frac{T_W^{max}}{T_R^{min}} \right\rceil - 1$$

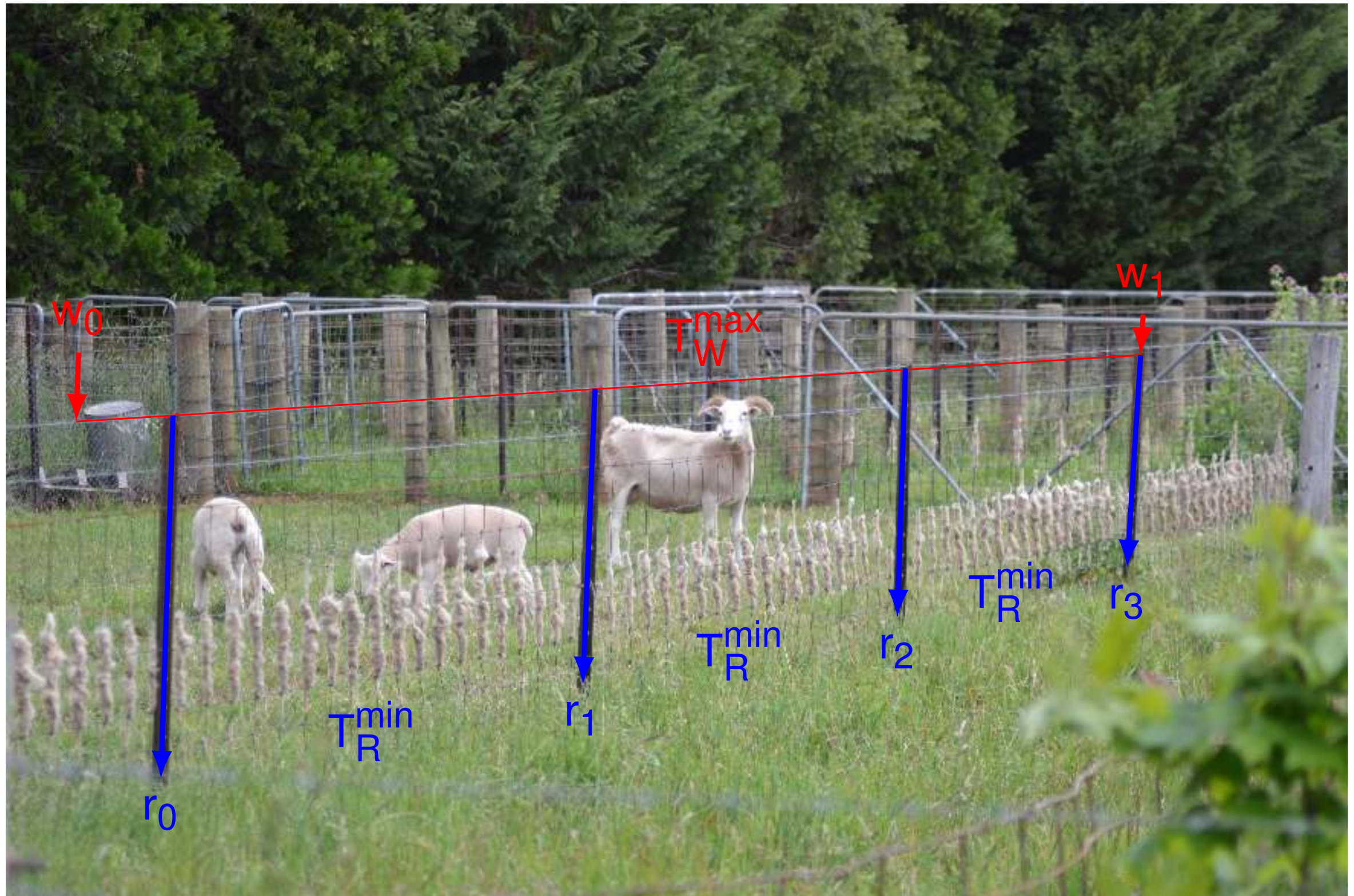
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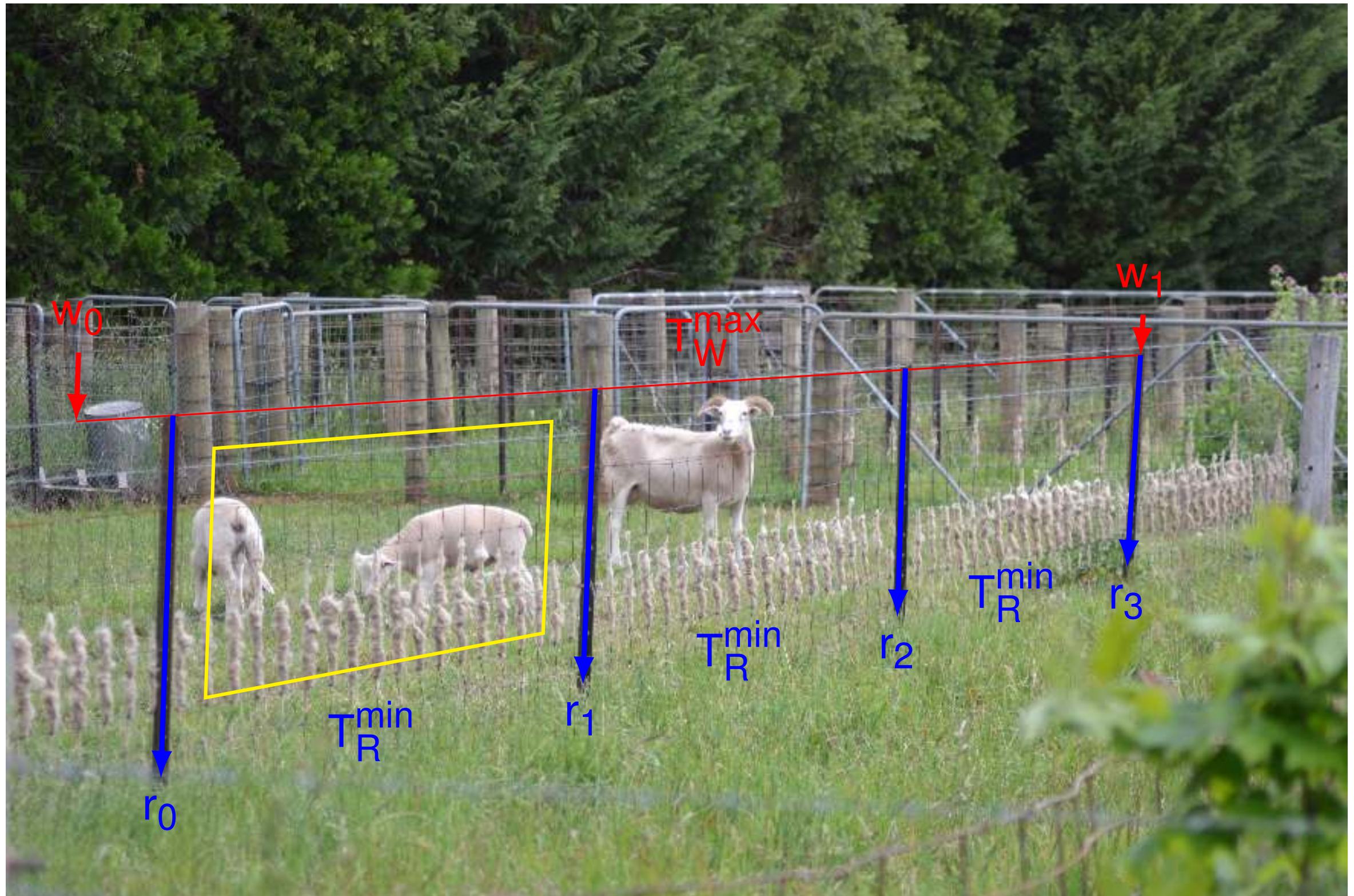
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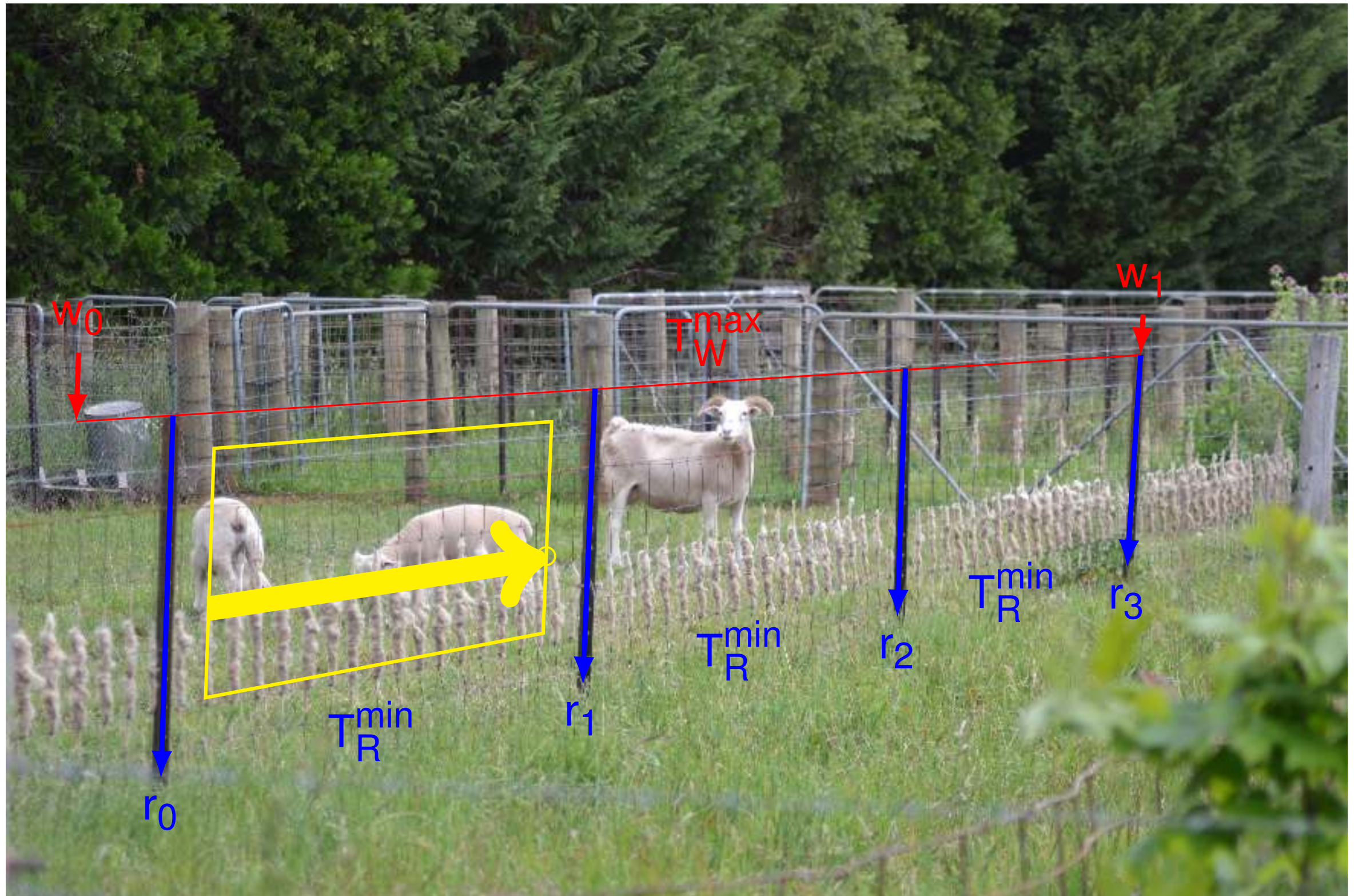
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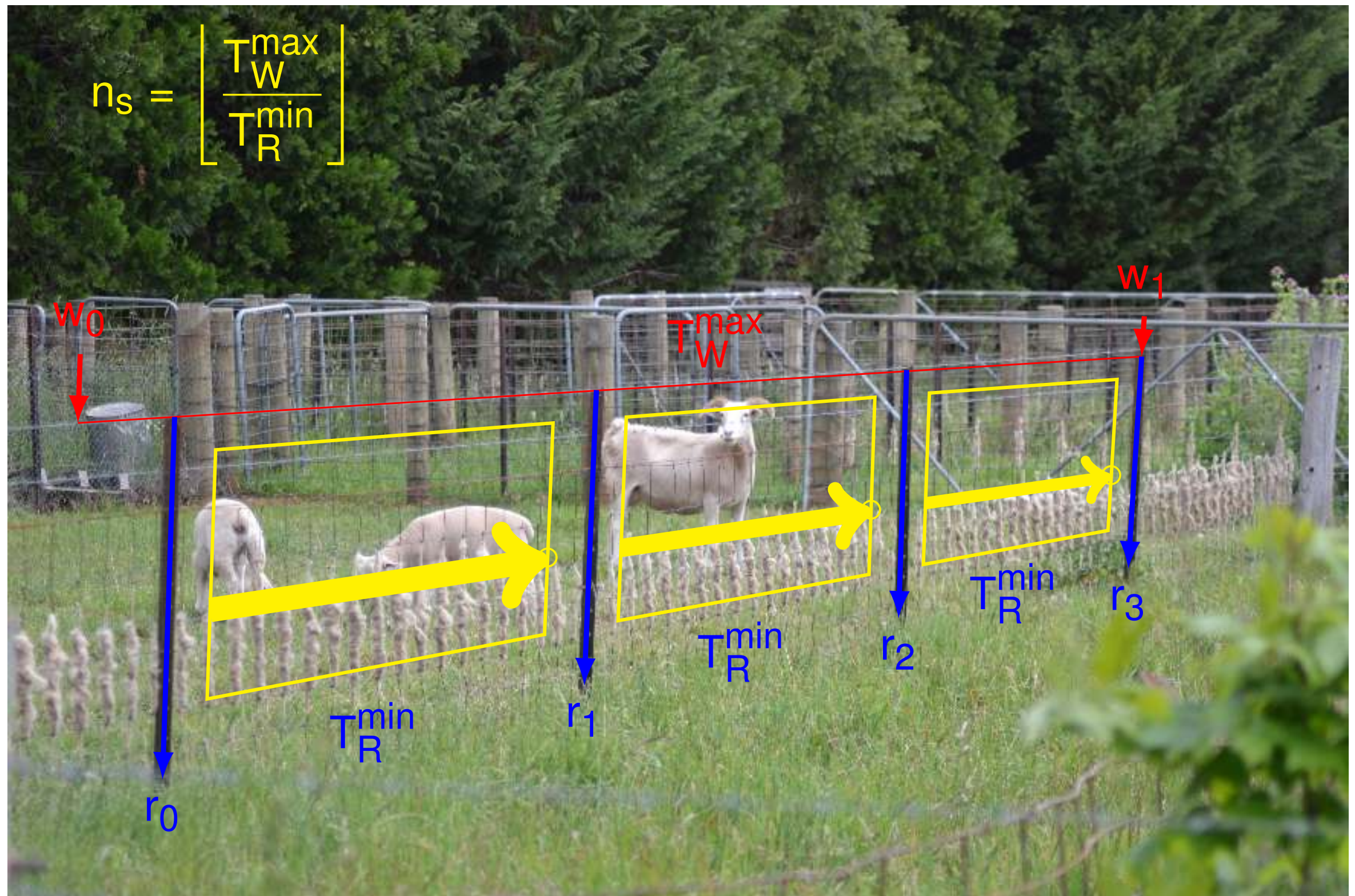
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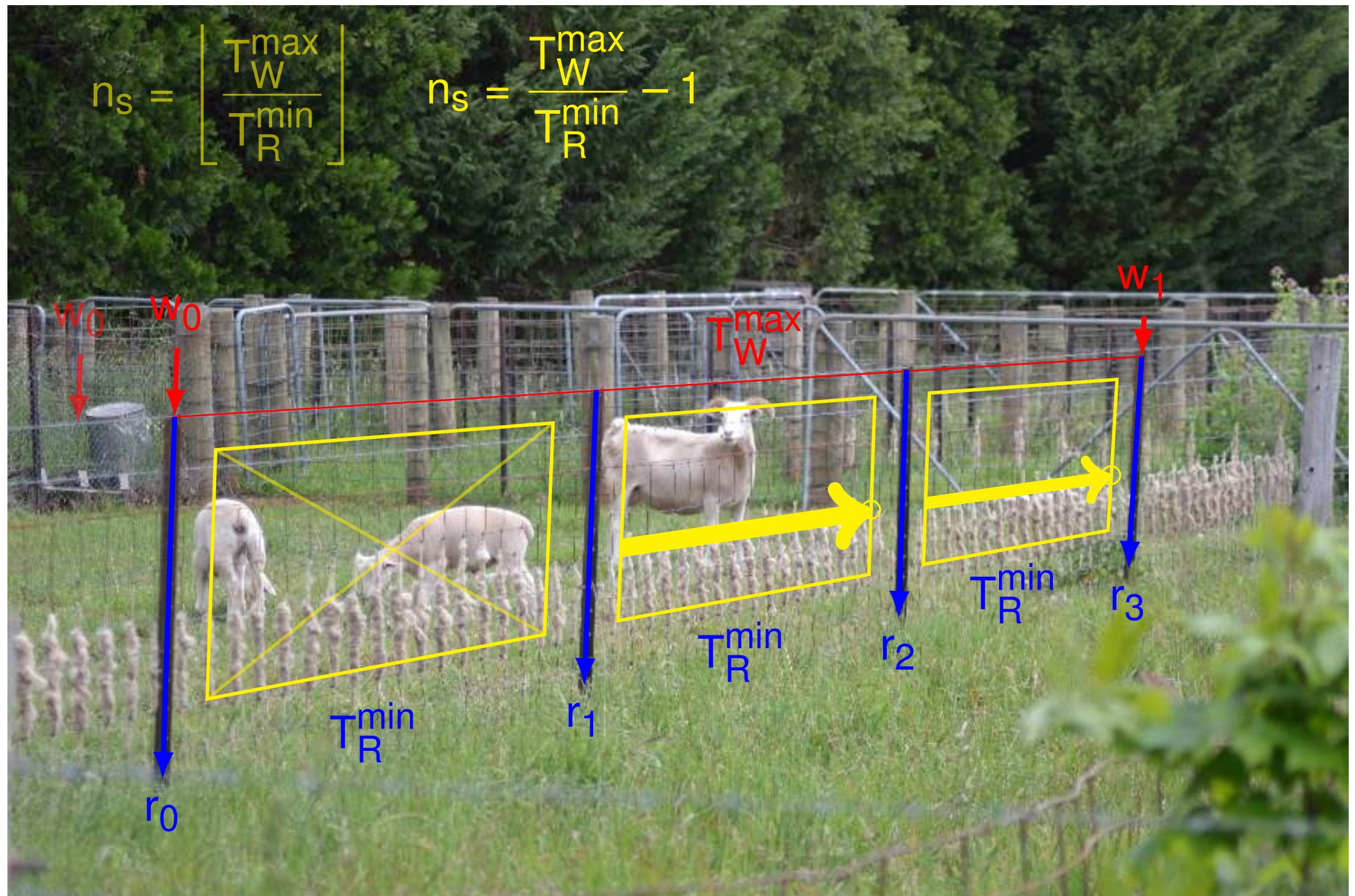
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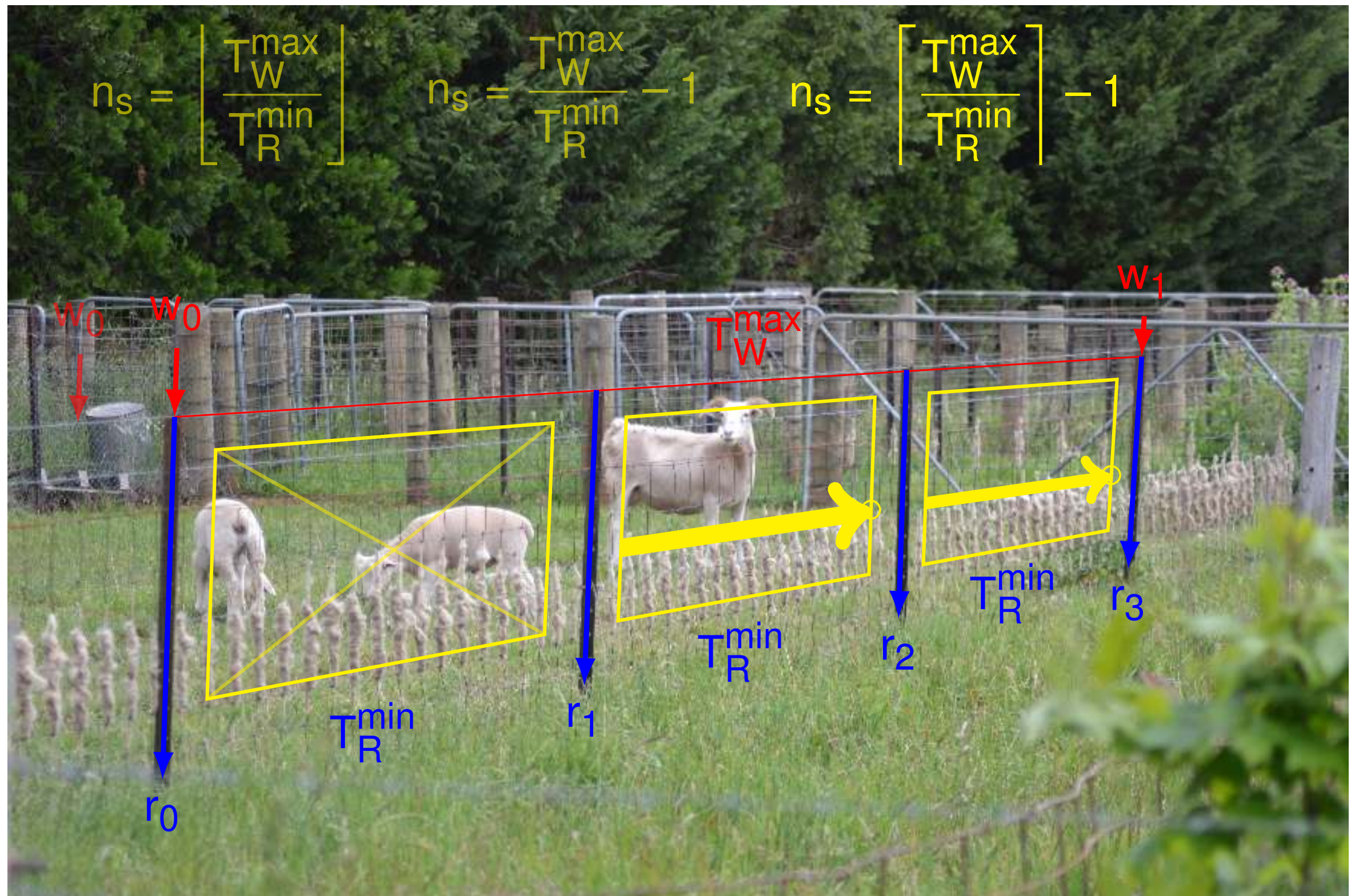
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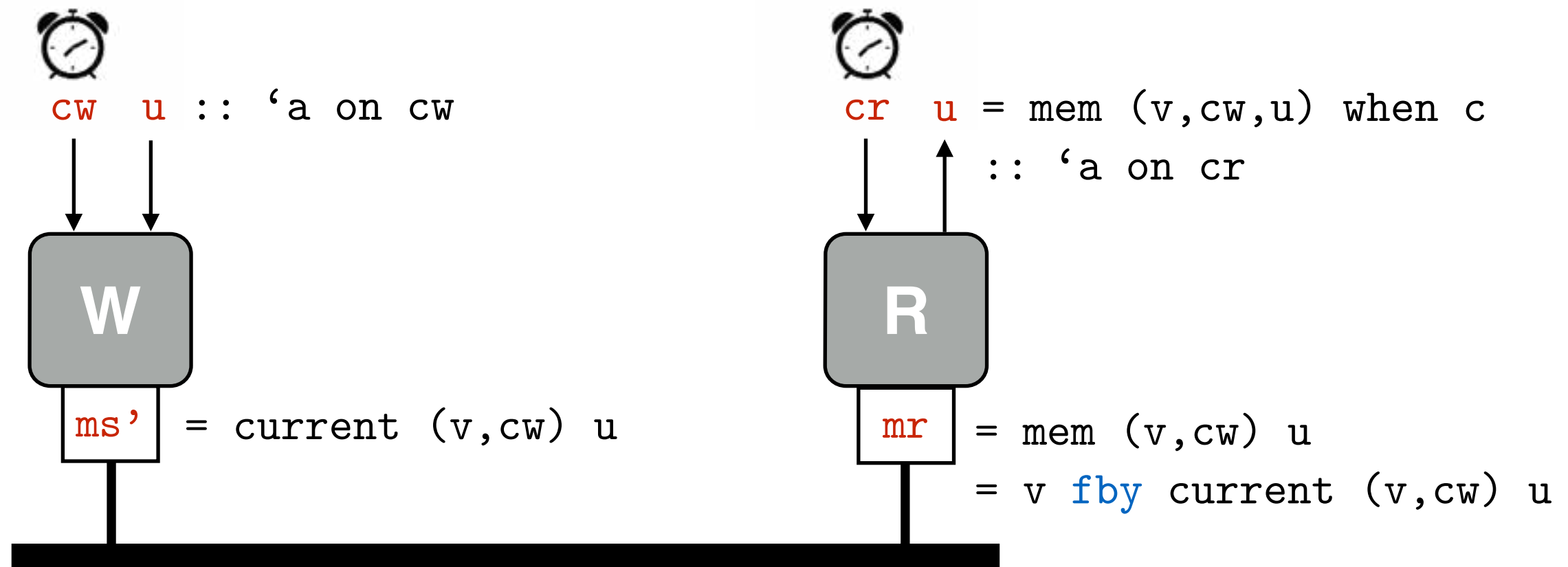


Discrete Abstraction

Discrete Model

[From the Cooking Book]

- Model in Lustre or Lucid Synchronic
- Clocks represent activations of a node
- Transmission delay: one tick of the base clock ('a)



Clock Condition

“

Neither of the clocks can take the value 1 more than twice between two successive 1 of the other

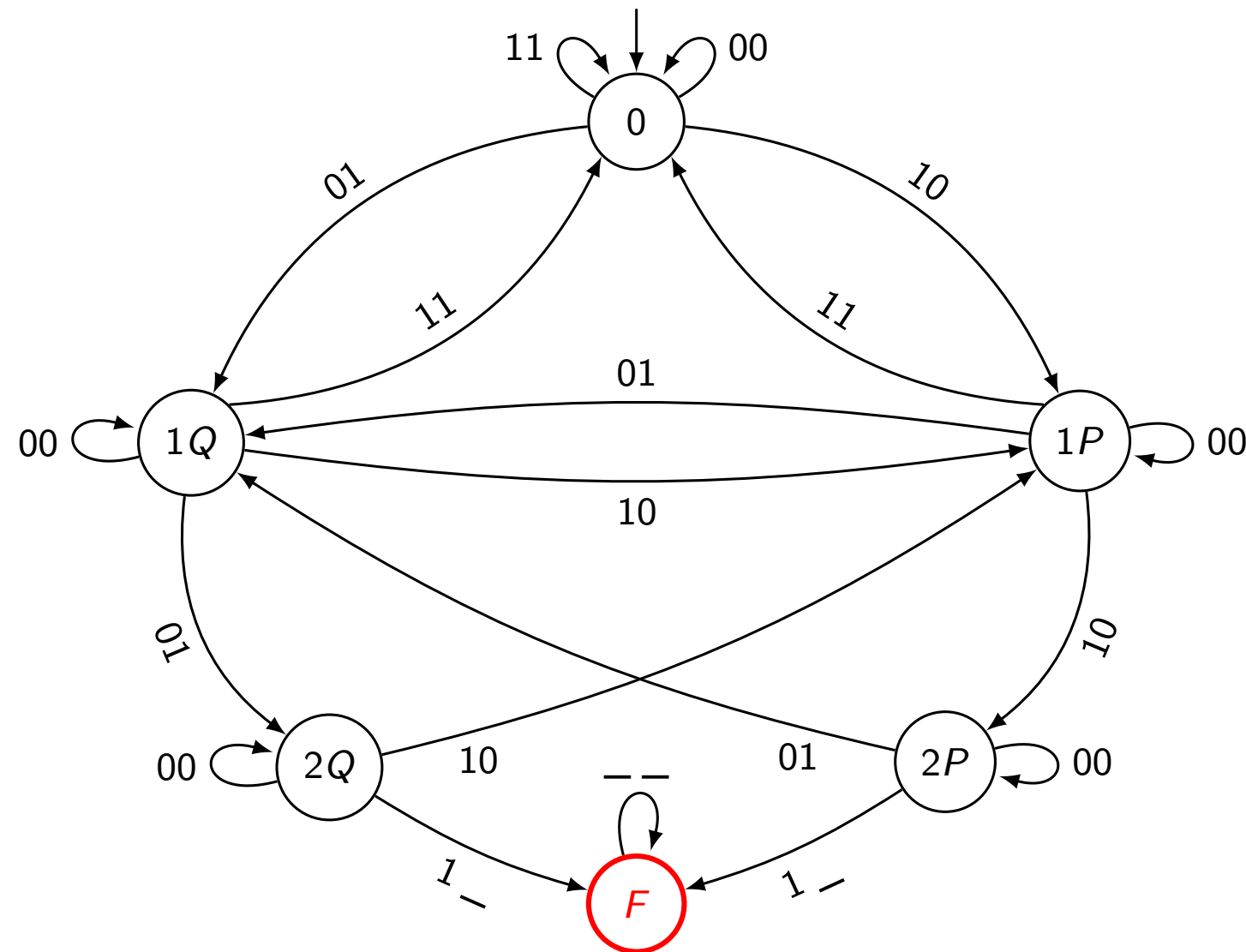
P. Caspi

Idea: forbid the following sequence and the symmetrical one

$$\begin{array}{lcl} C_1 & \begin{bmatrix} 1 \\ - \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \end{bmatrix}^* \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \end{bmatrix}^* \cdot \begin{bmatrix} 1 \\ - \end{bmatrix} & n_o = 1 \\ C_2 & & n_s = 1 \end{array}$$

Clock Condition

Directly leads to a finite automaton



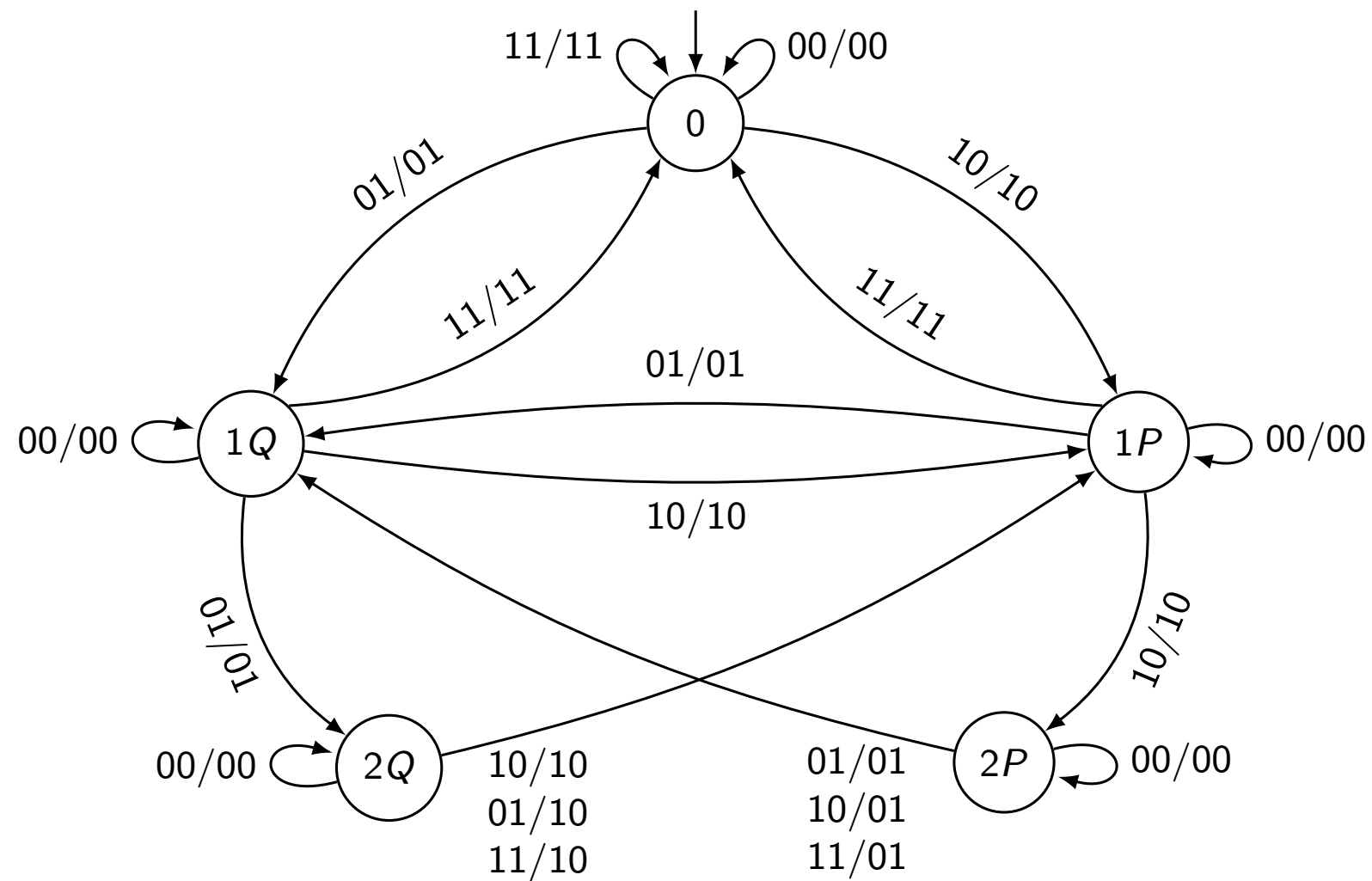
Transitions are labelled by (c_w, c_r)

If state **F** is reached, the trace is not quasi-synchronous

Clock Condition

Or a scheduler for simulation

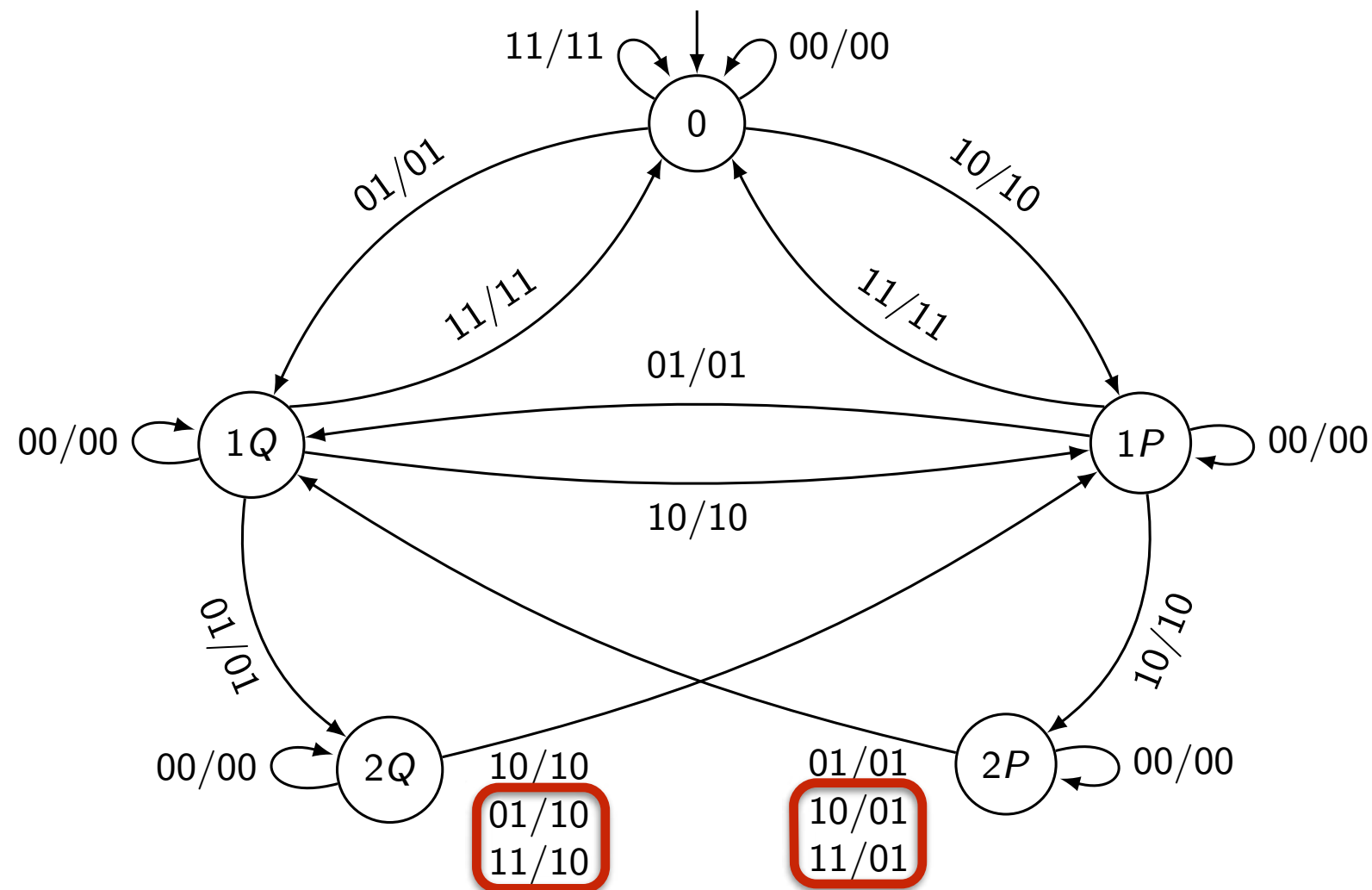
[Halbwachs Mandel 2006]



Clock Condition

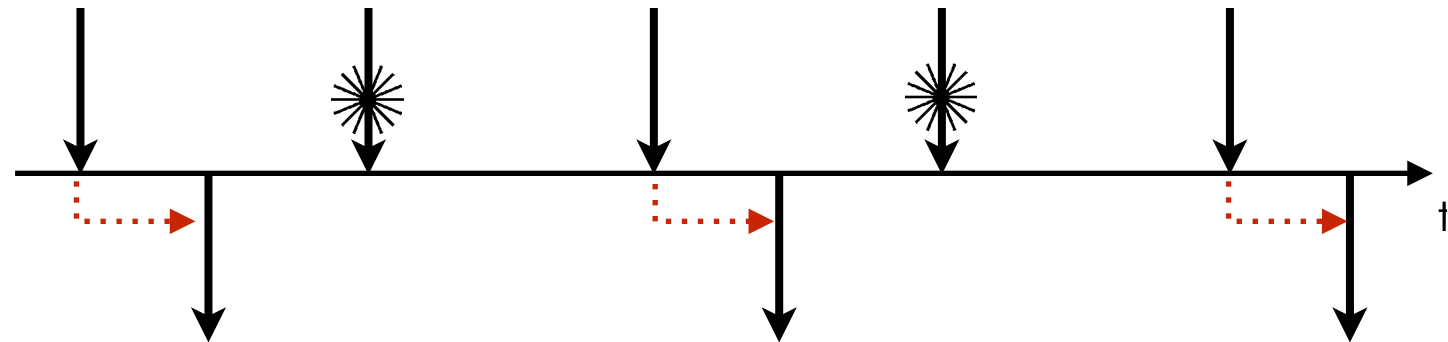
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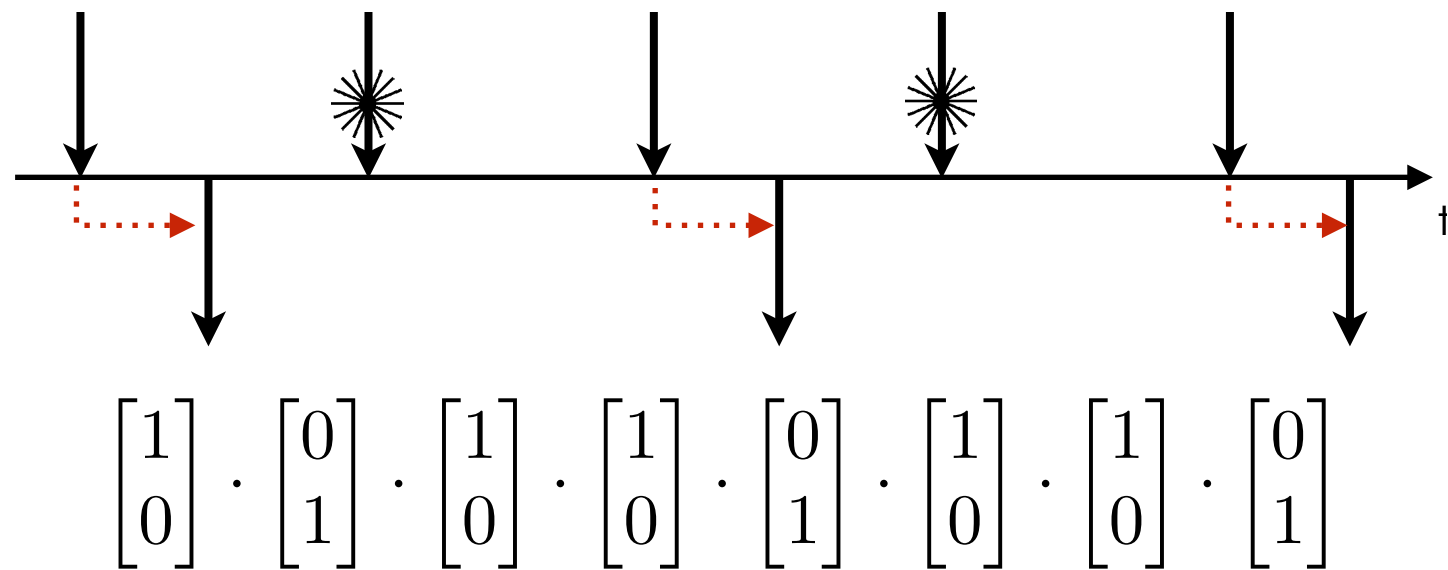


Non quasi-synchronous traces are corrected

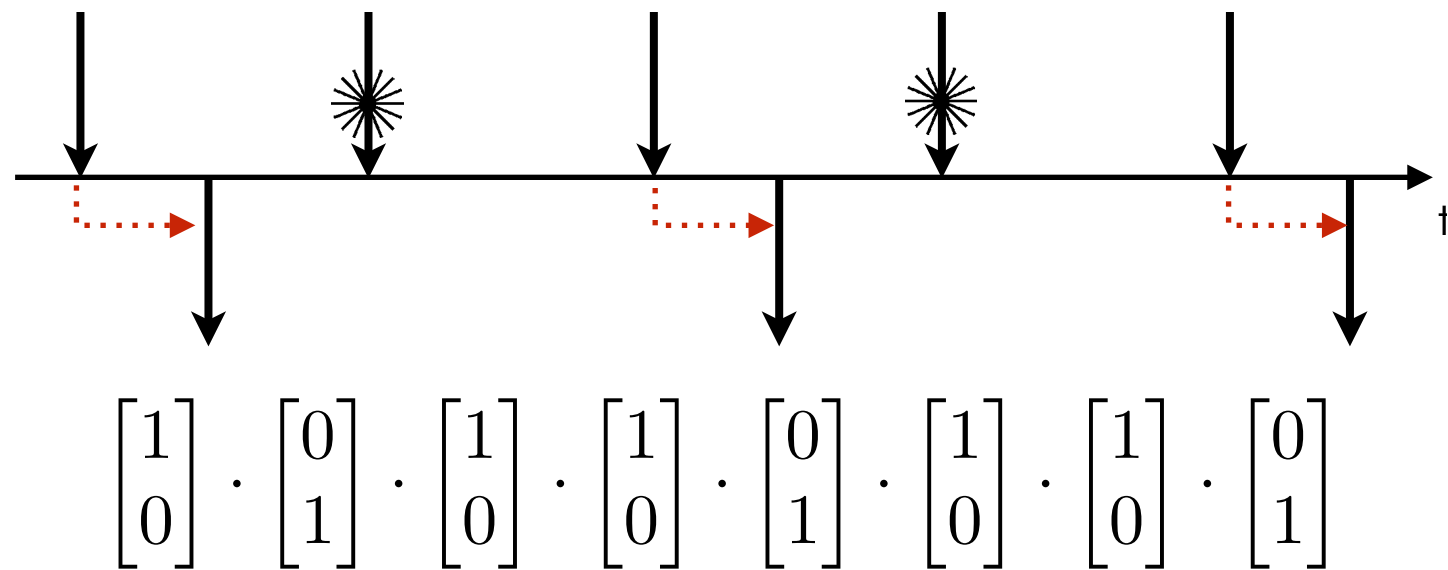
Too Restrictive?



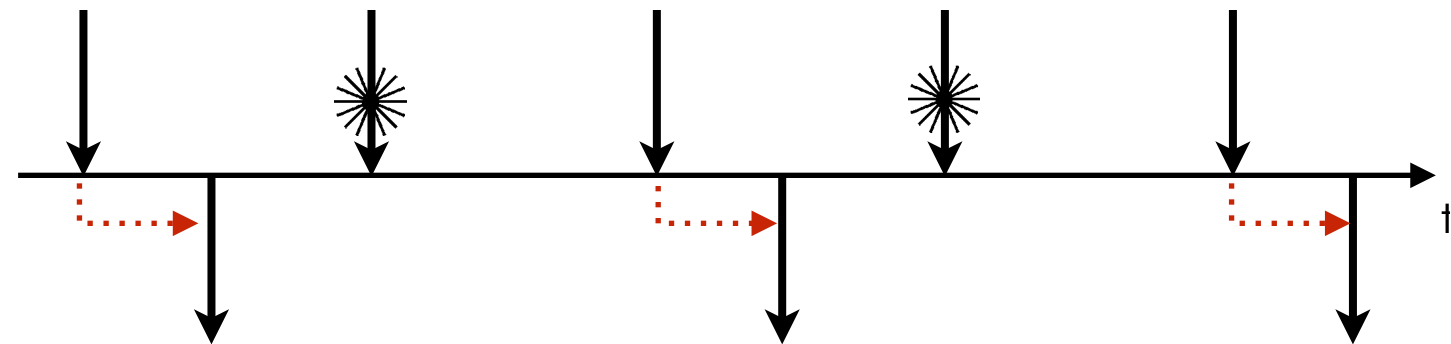
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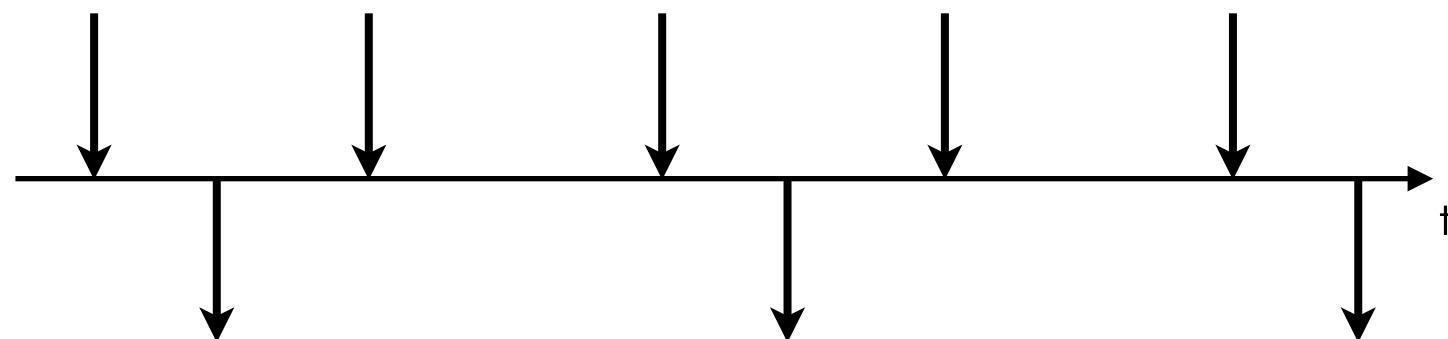
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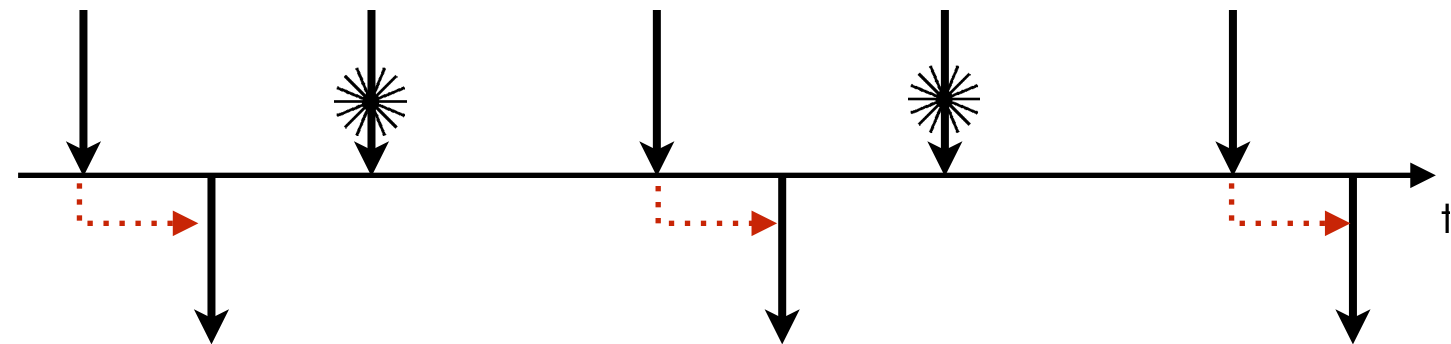
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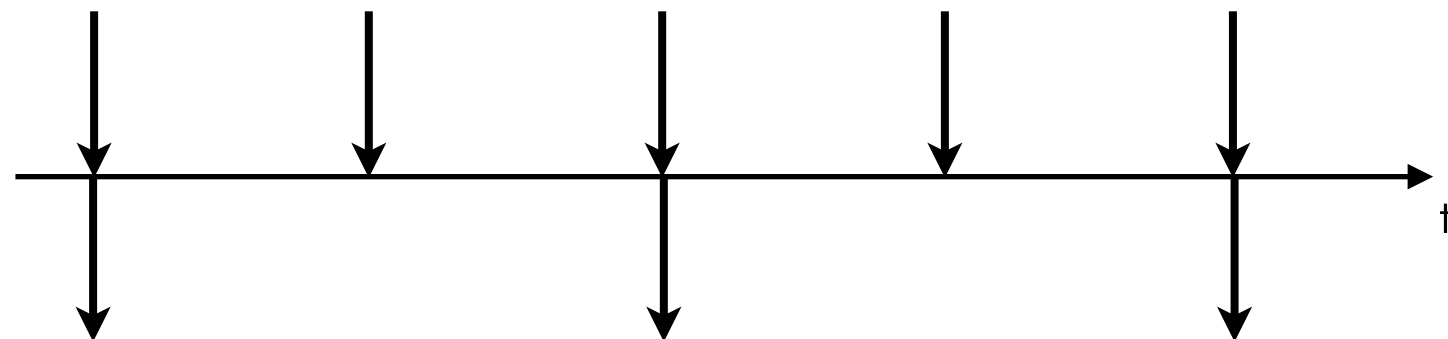
$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$



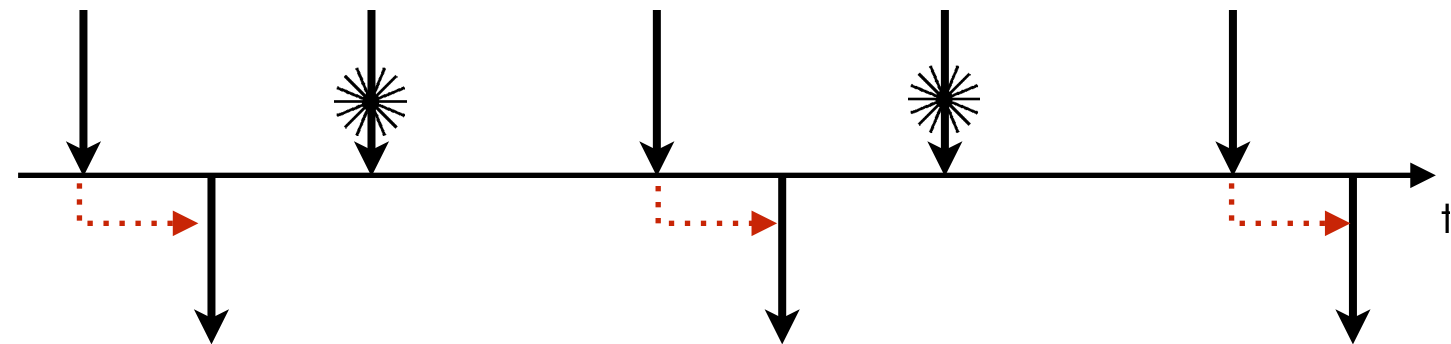
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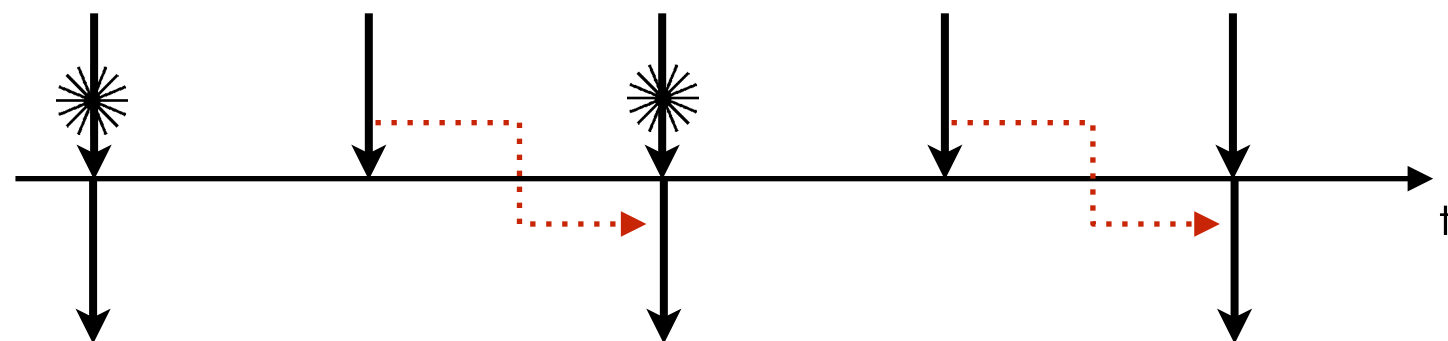
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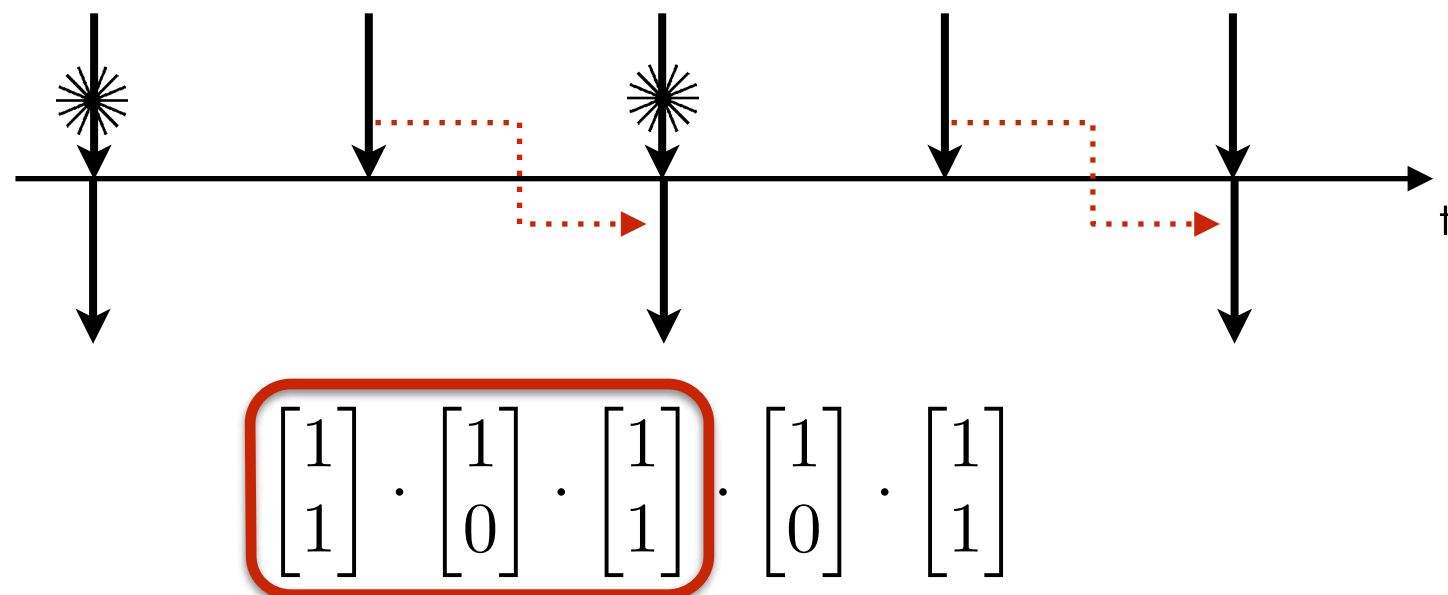
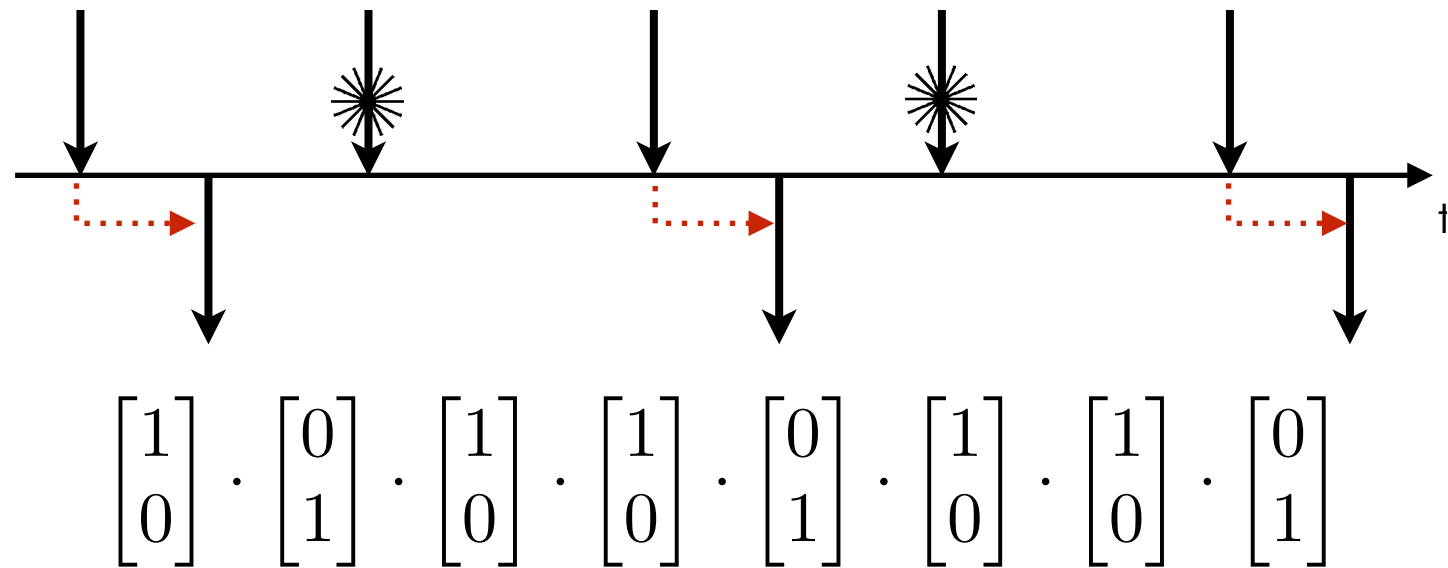


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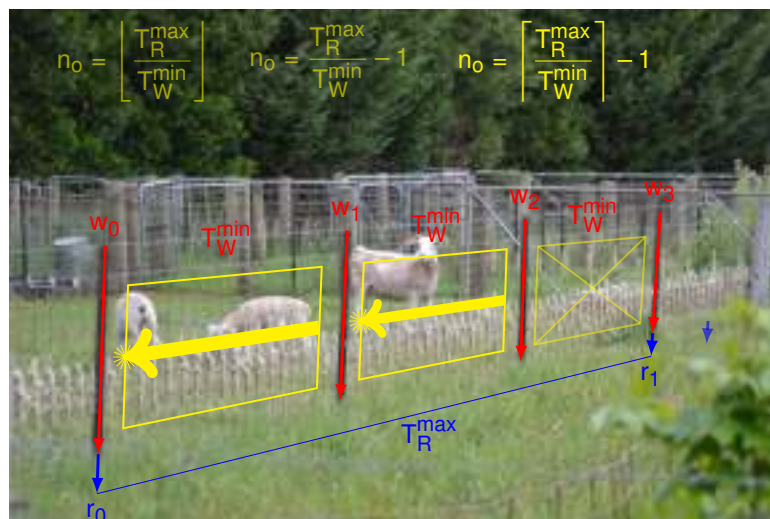


Generalization

Idea: decoupling overwriting and oversampling

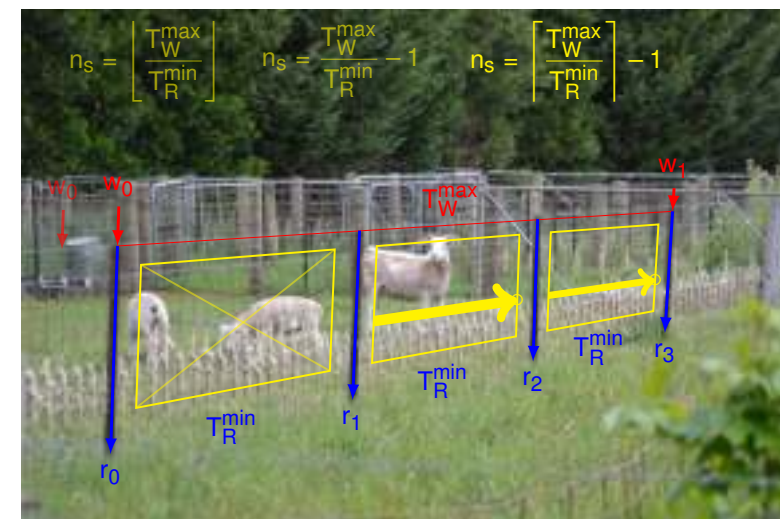
Overwriting

$$\begin{matrix} C_W \\ C_R \end{matrix} \begin{bmatrix} 1 \\ - \end{bmatrix} \cdot \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}^* \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right)^{n_o+1}$$



Oversampling

$$\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \end{bmatrix}^* \right)^{n_s+1} \cdot \begin{bmatrix} - \\ 1 \end{bmatrix}$$

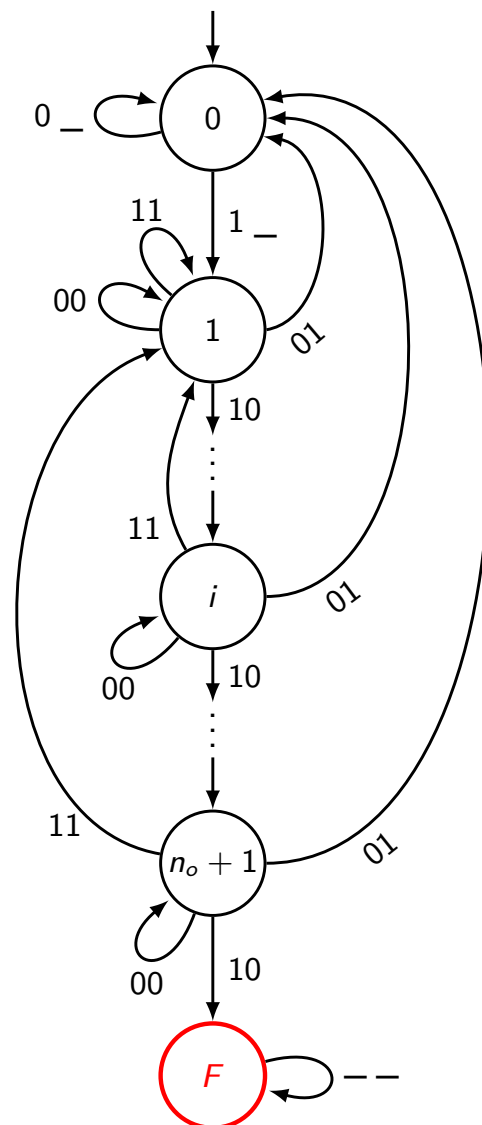


Counters of consecutive activations
Symmetrical formula

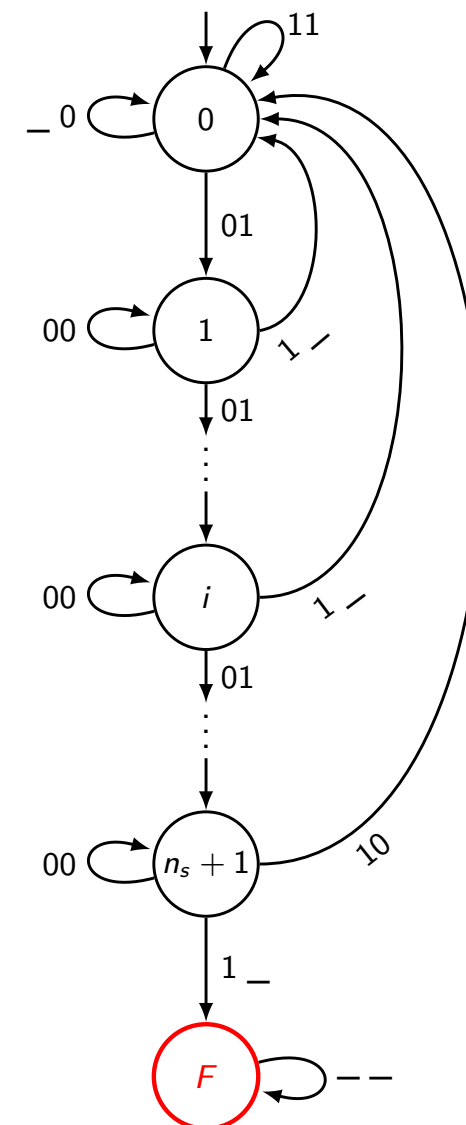
Generalization

Idea: decoupling overwriting and oversampling

Overwriting



Oversampling



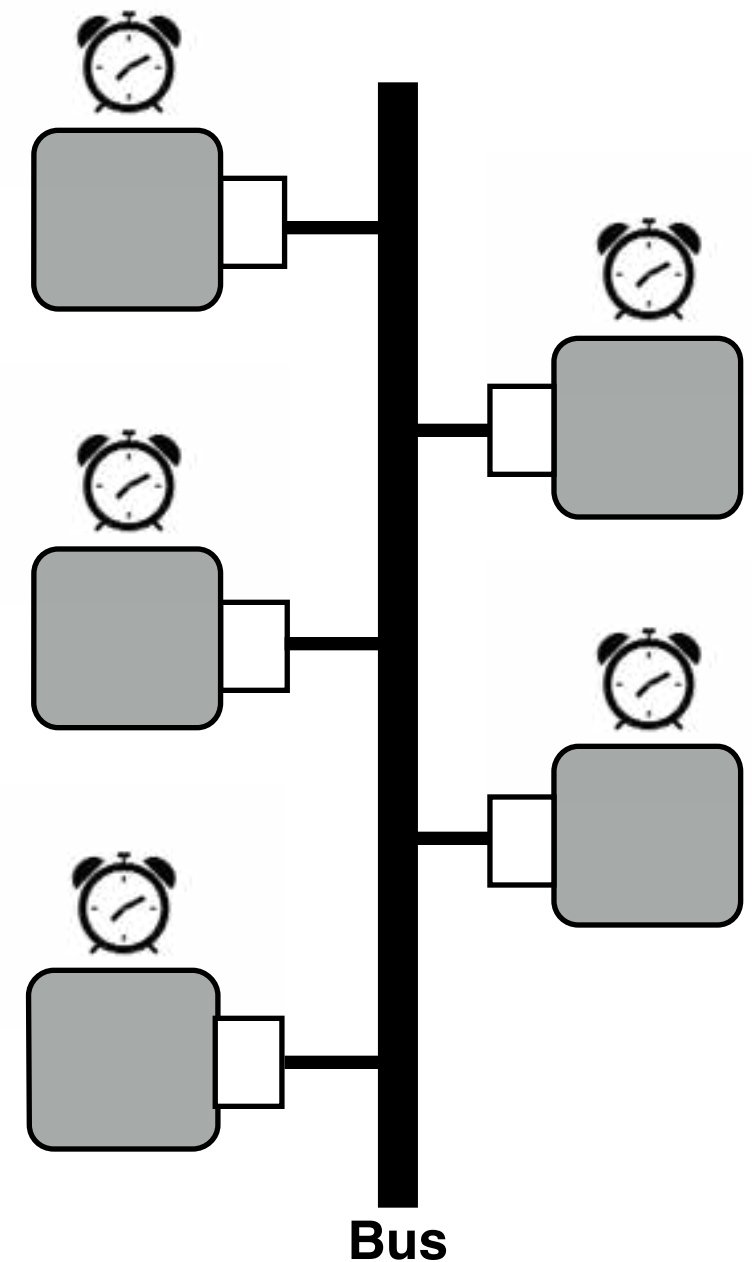
Loosely Time-Triggered Architectures

[Tripakis et al. 2008]

[Caspi, Benveniste 2008]

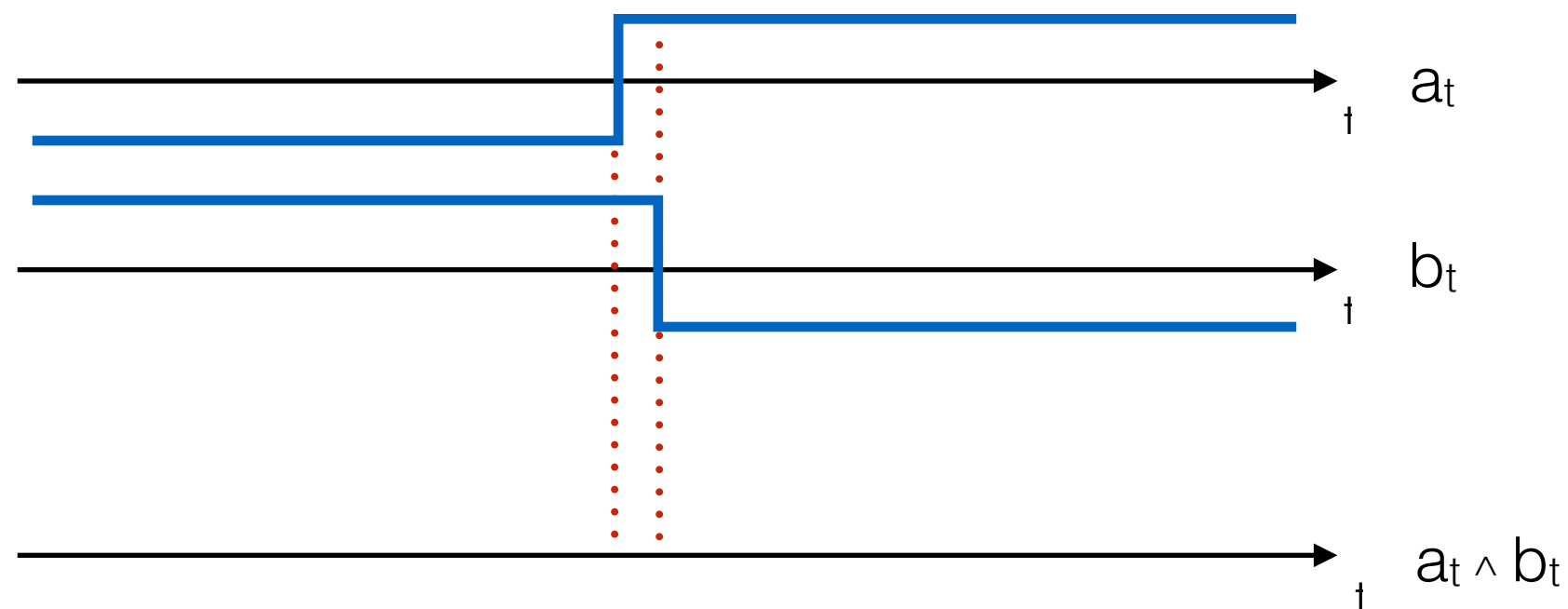
What are LTTA?

- **Base:** A quasi-periodic architecture
- **Goal:** Safely deploy a synchronous model
- **Idea:** Add a layer of middleware



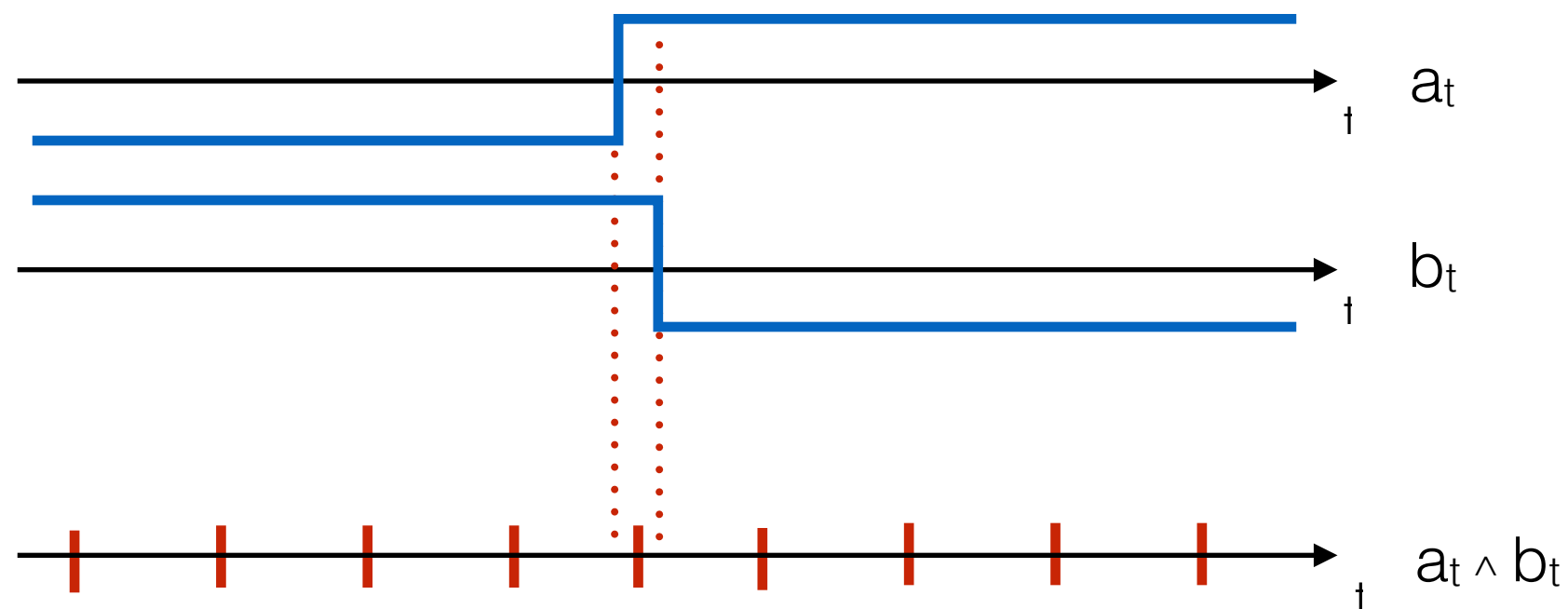
Problems

- **Overwriting:** Lost of values
- **Oversampling:** Duplication of values
- **Combination of signals**



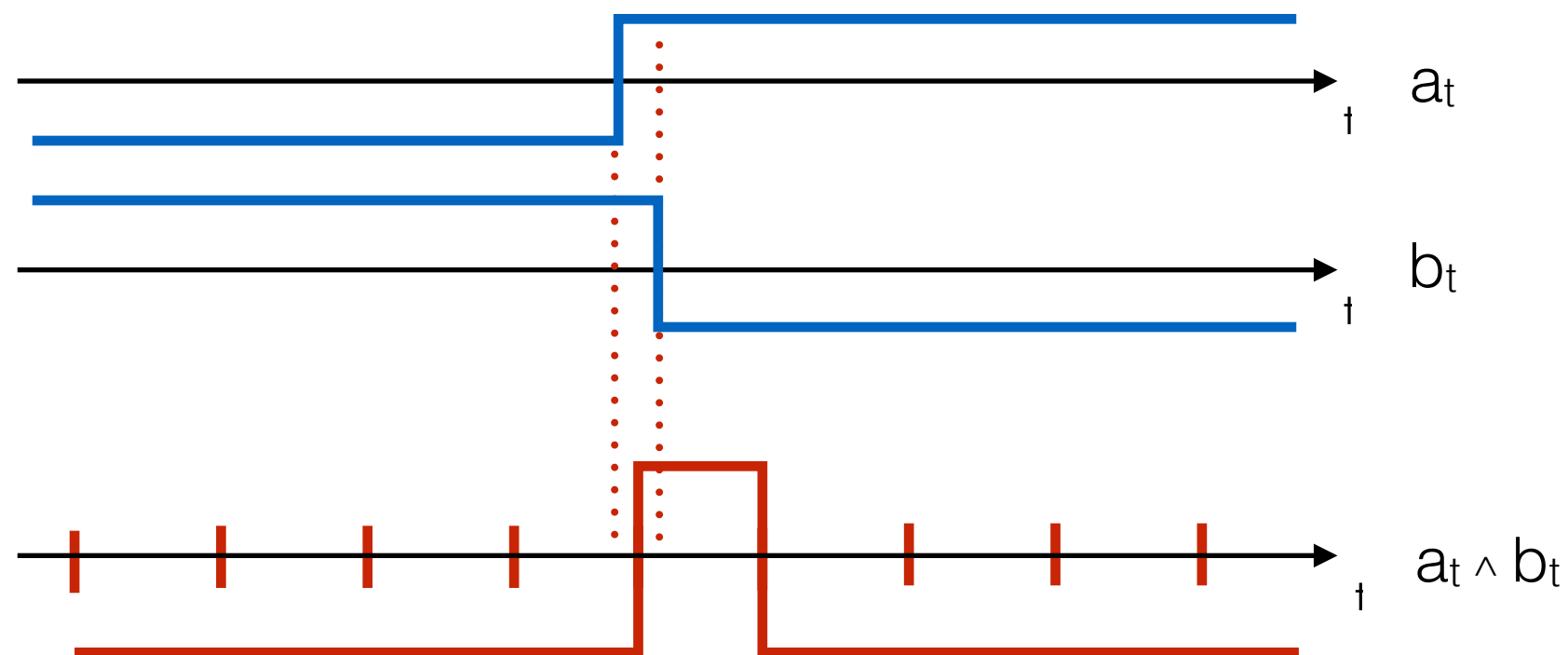
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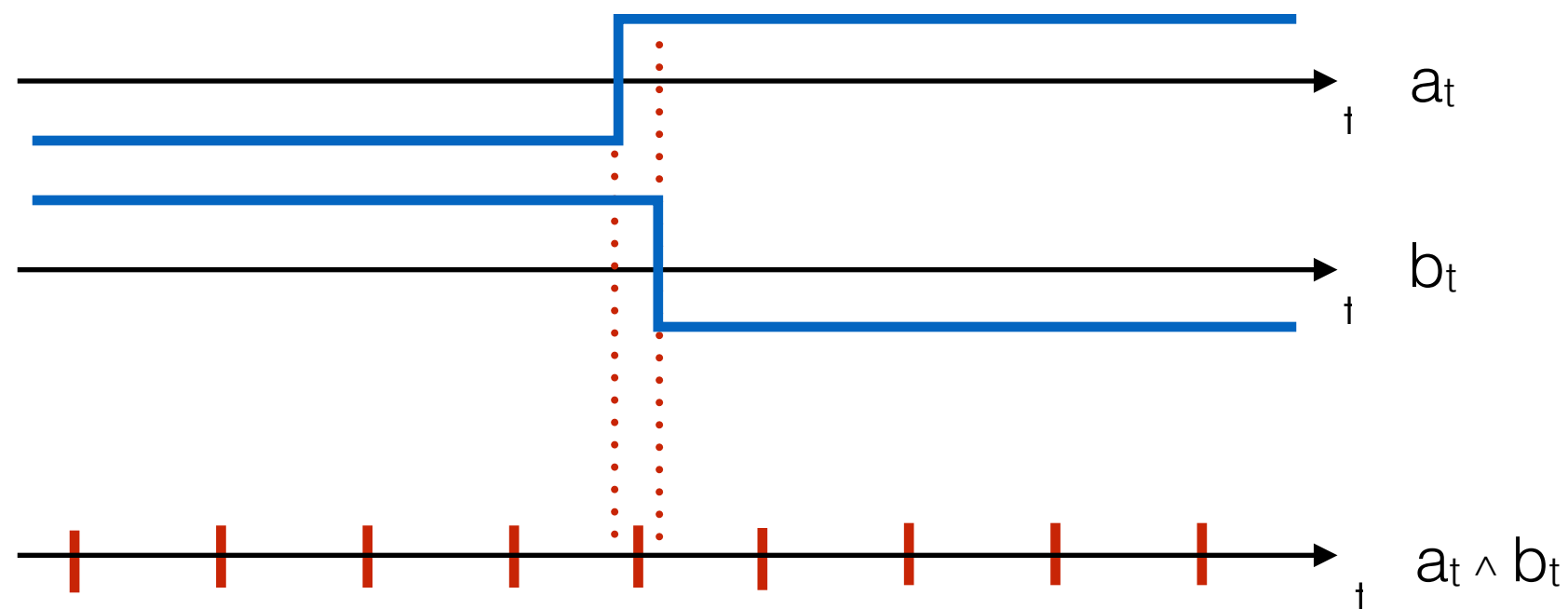
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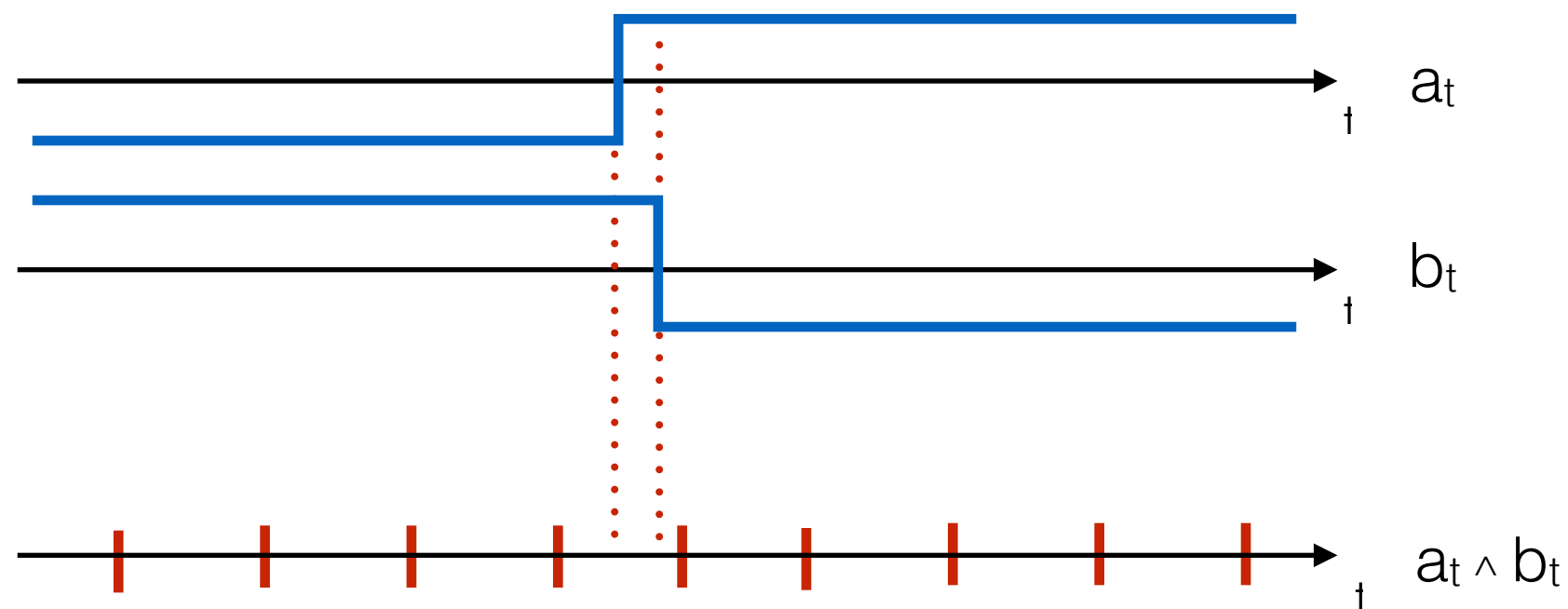
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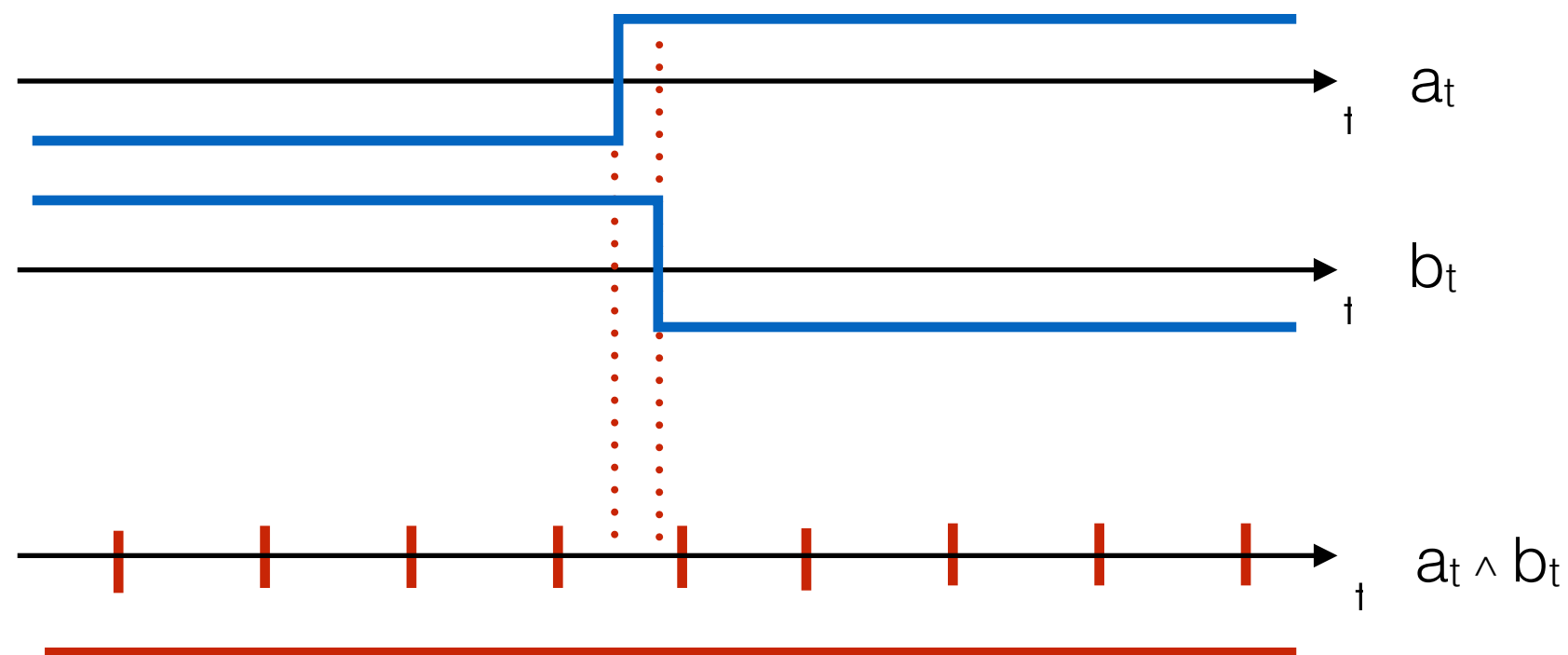
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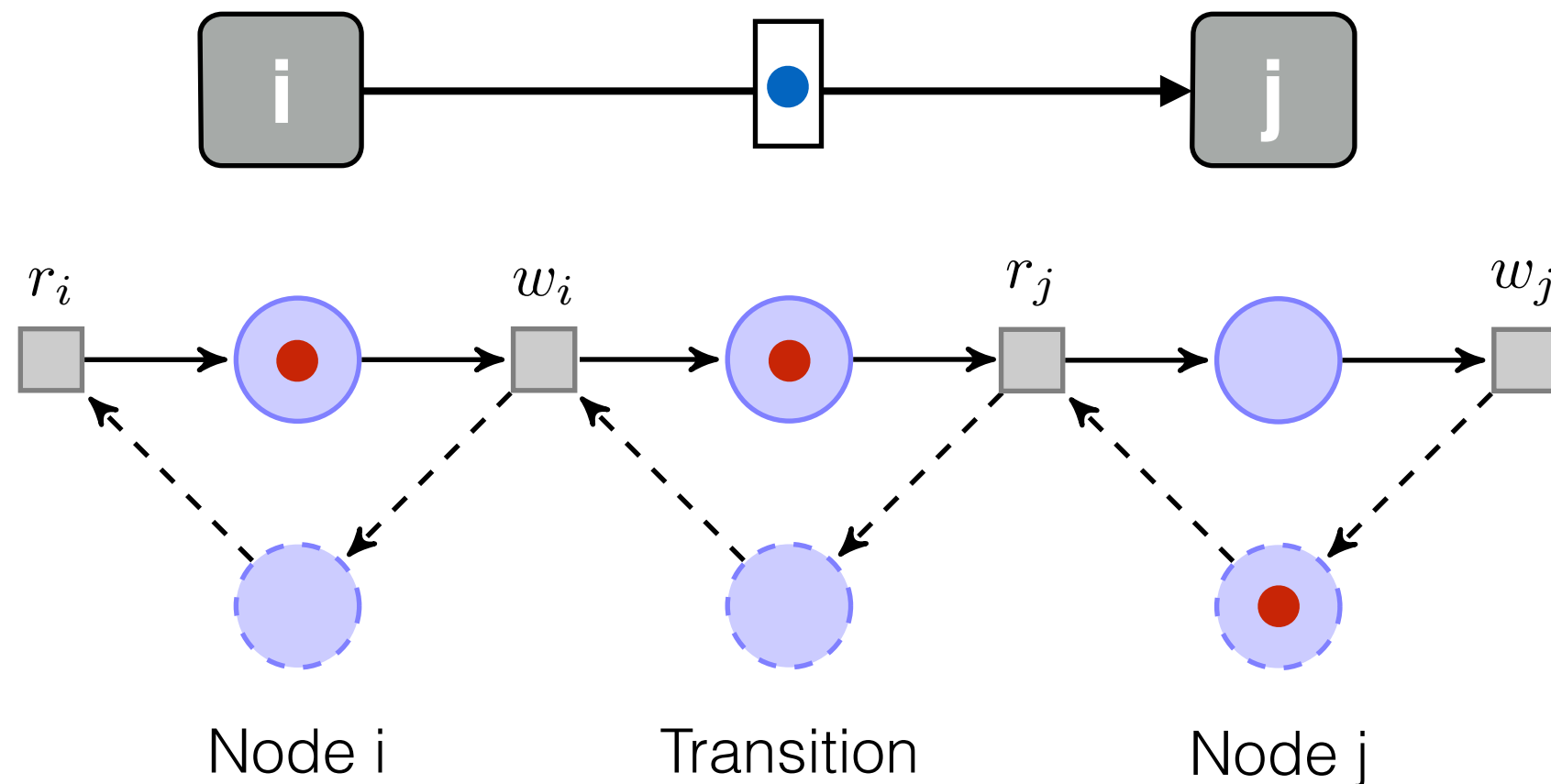


Two solutions

- **Back-Pressure LTTA** [Tripakis et al. 2008]
- **Time-Based LTTA** [Caspi, Benveniste 2008]

Back-Pressure Kahn Network

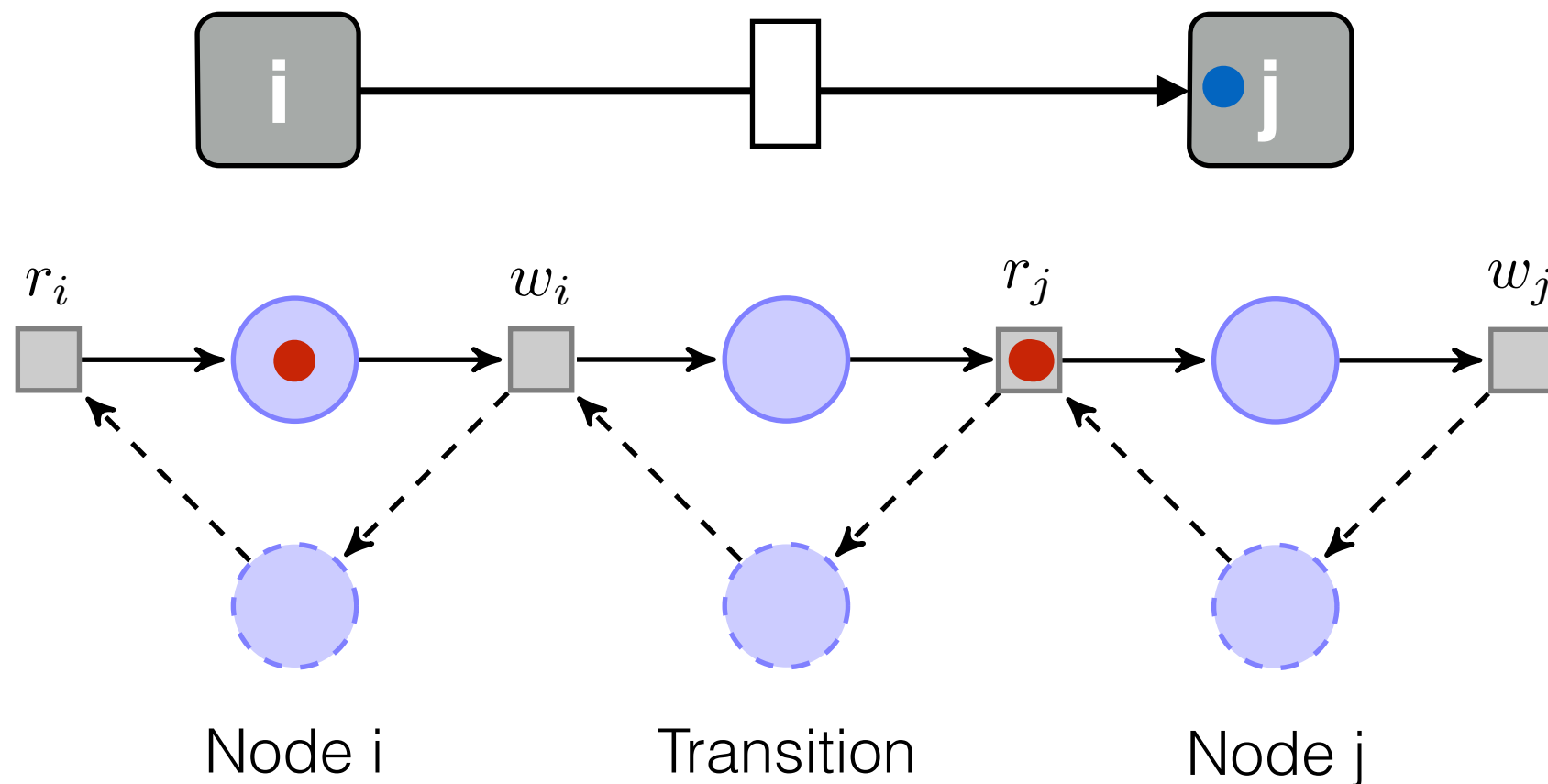
buffer of size 1



- Reading from a buffer is acknowledged to the writer
- Nodes alternate between reads and writes

Back-Pressure Kahn Network

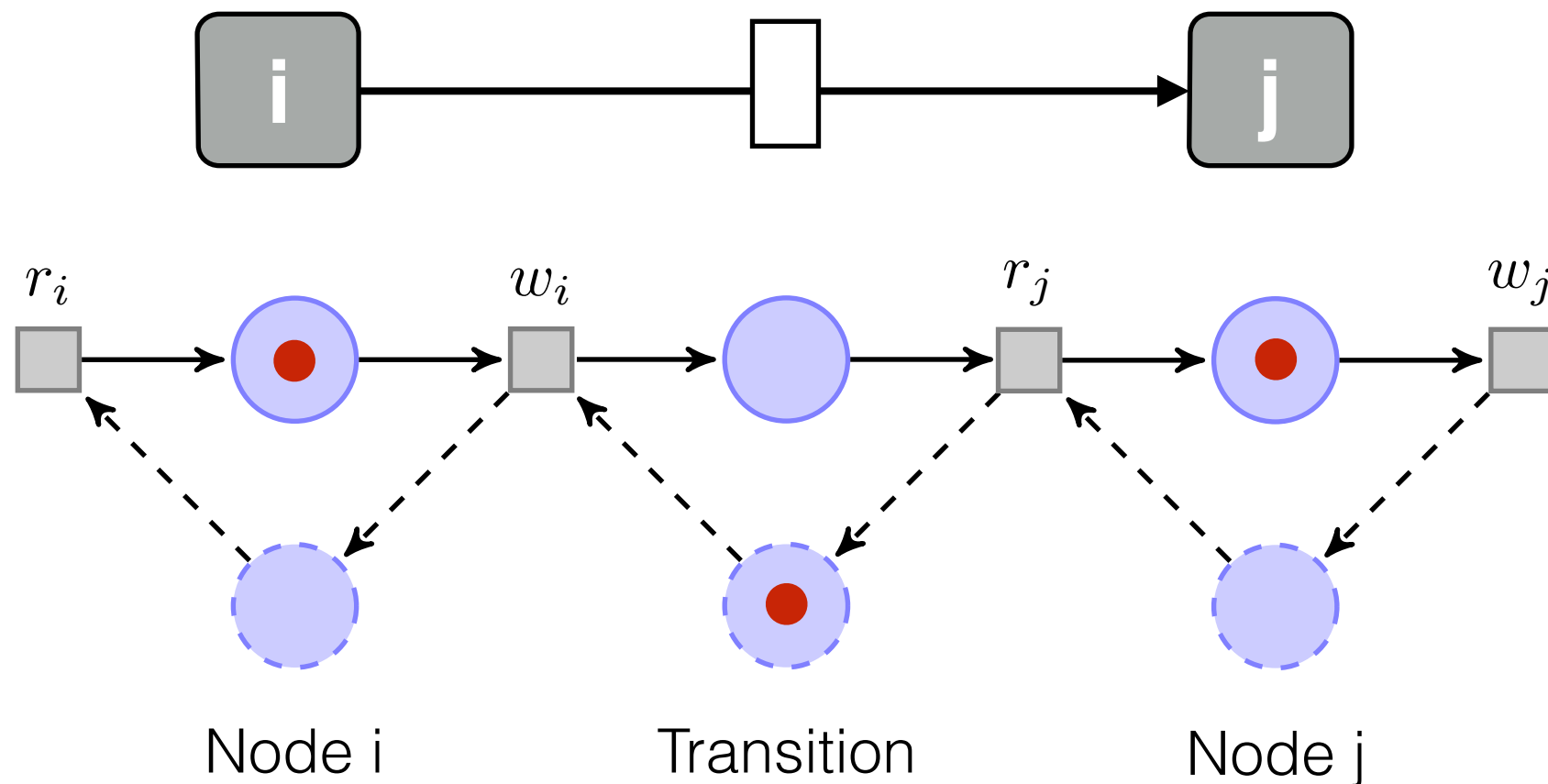
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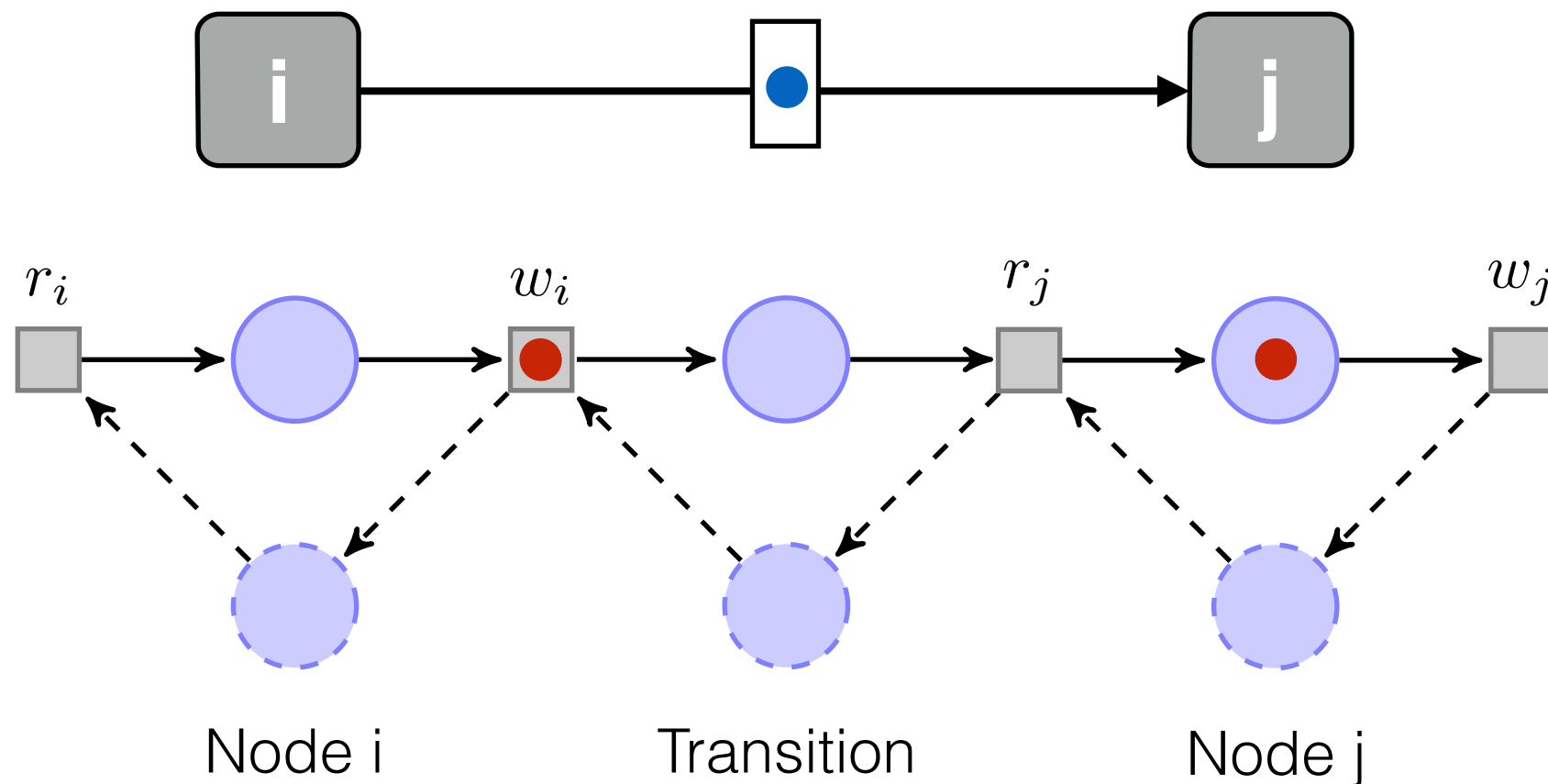
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- Nodes alternate between reads and writes

Back-Pressure Kahn Network

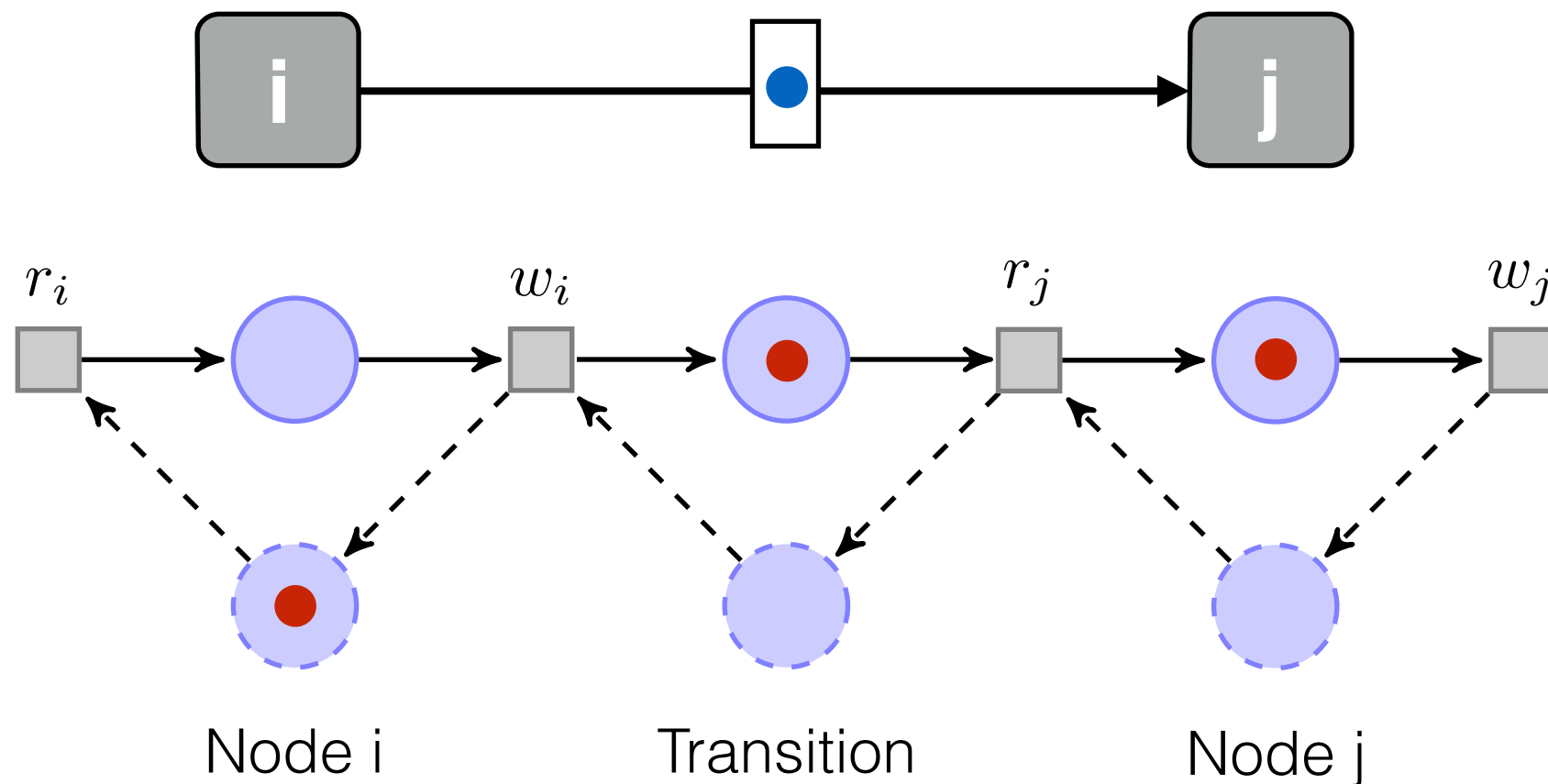
buffer of size 1



- Reading from a buffer is acknowledged to the writer
- Nodes alternate between reads and writes

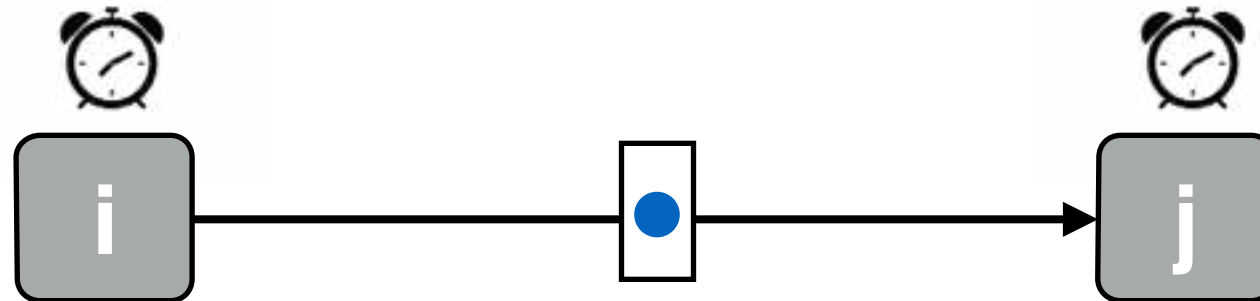
Back-Pressure Kahn Network

buffer of size 1



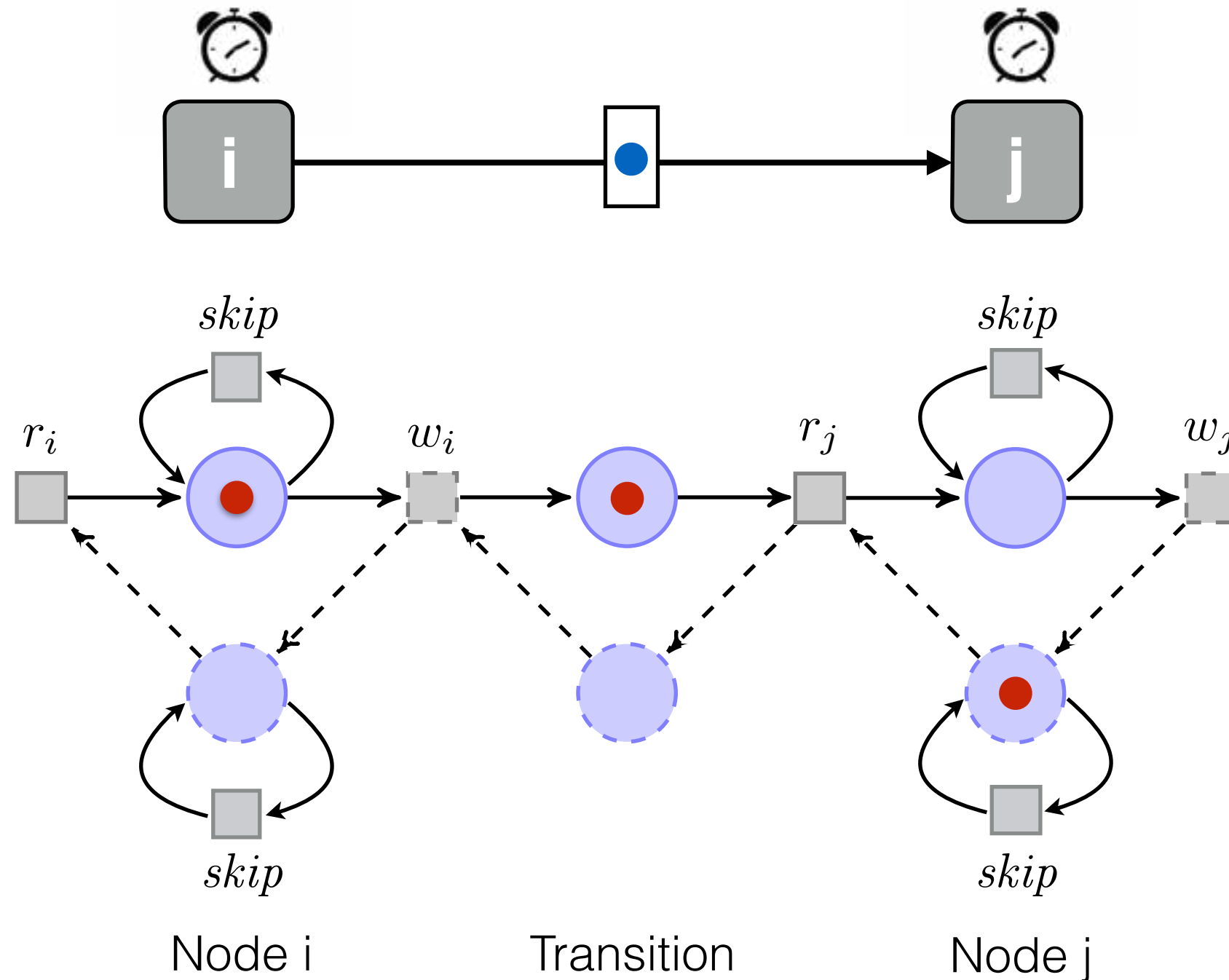
- Reading from a buffer is acknowledged to the writer
- Nodes alternate between reads and writes

Back-Pressure LTTA

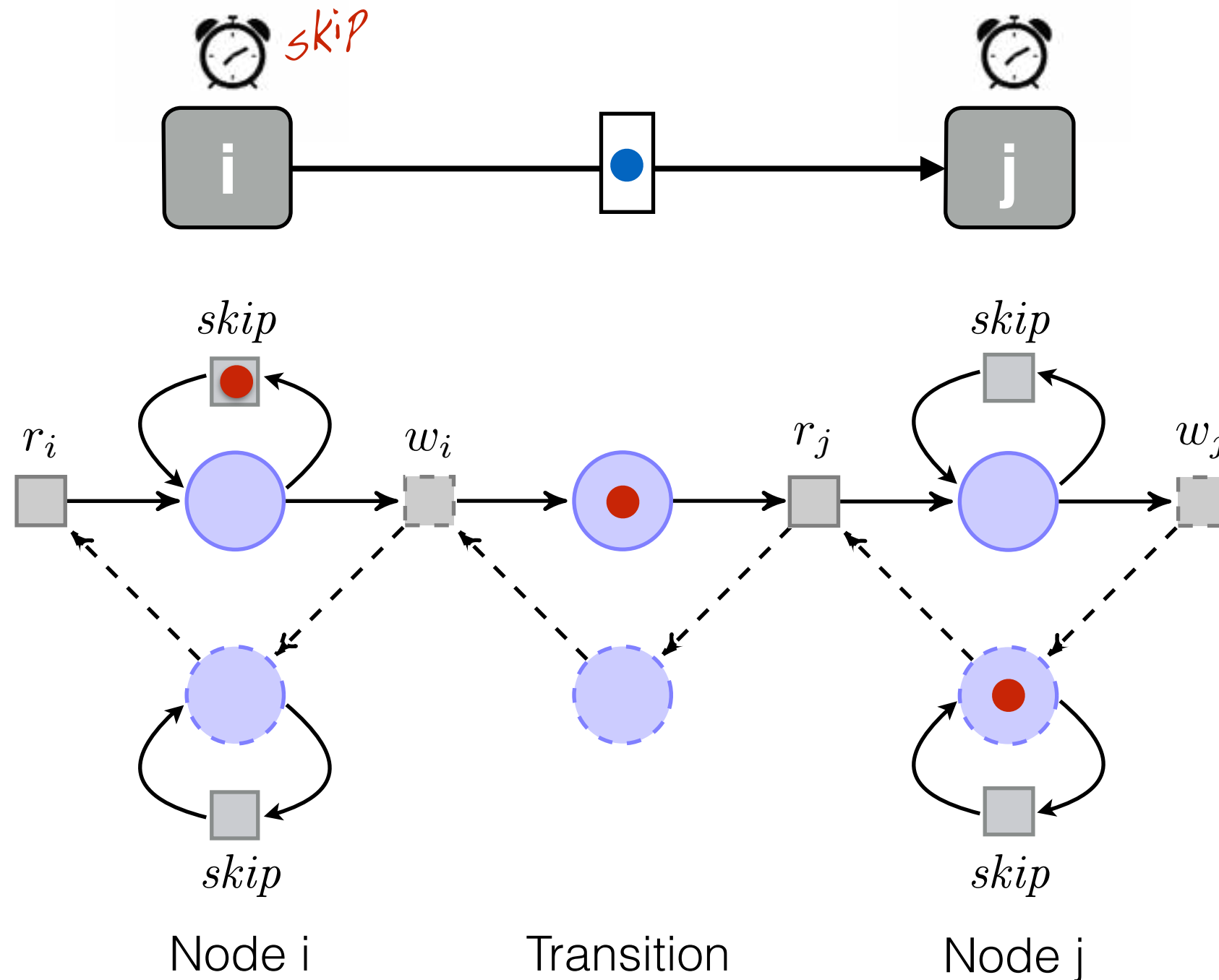


- **Difference:** nodes are triggered by their local clock
- **Idea:** adding skipping mechanism

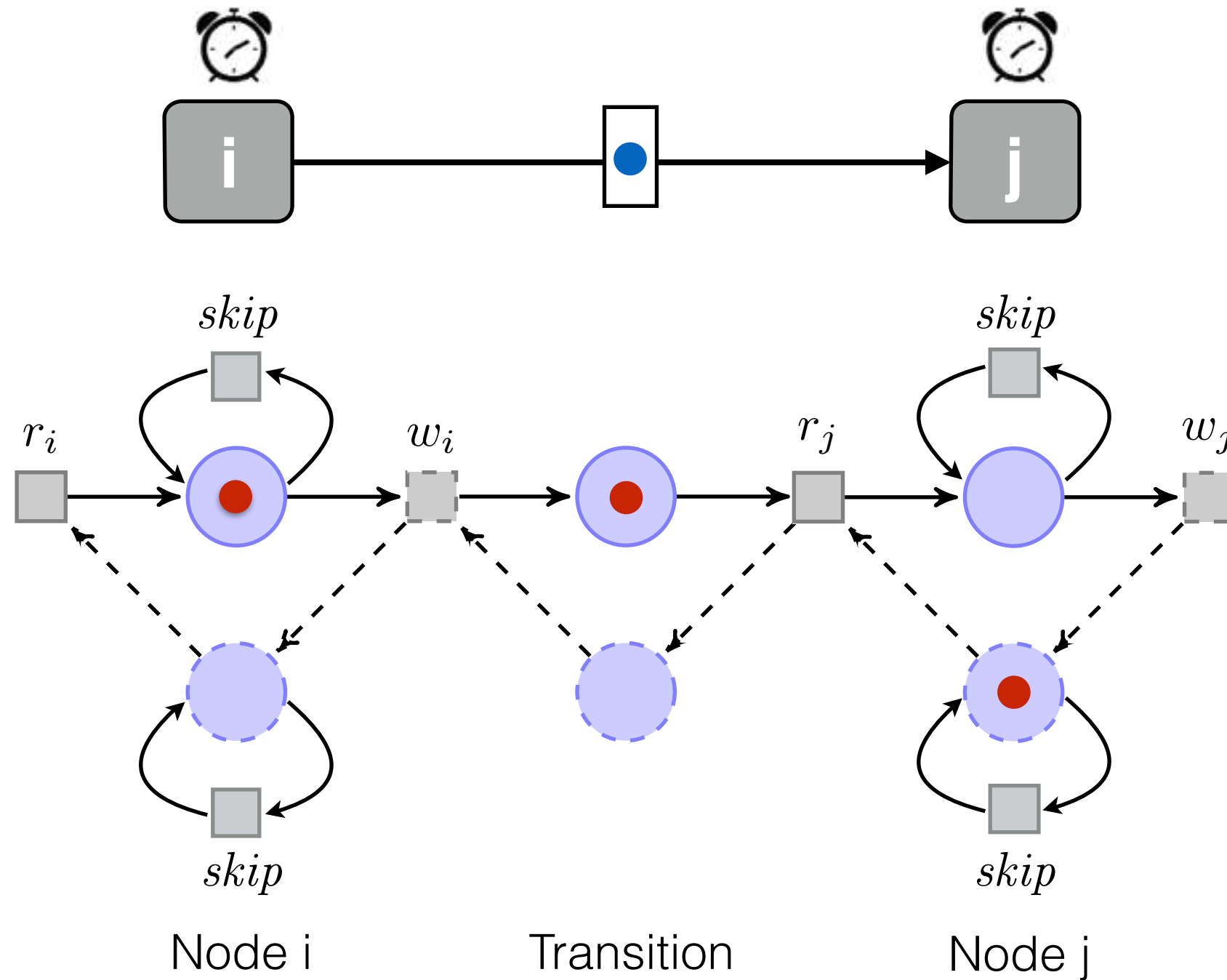
Back-Pressure LTTA



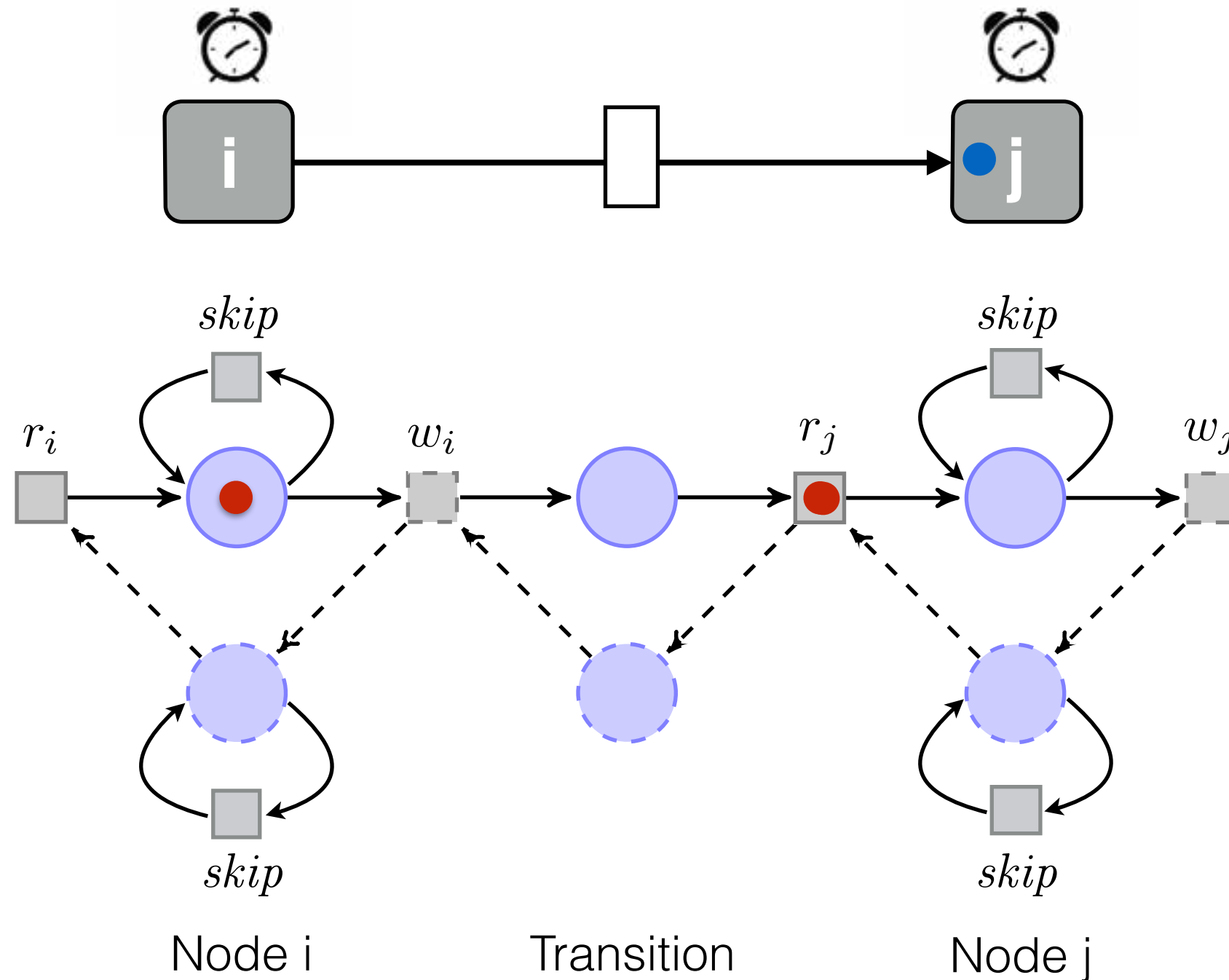
Back-Pressure LTTA



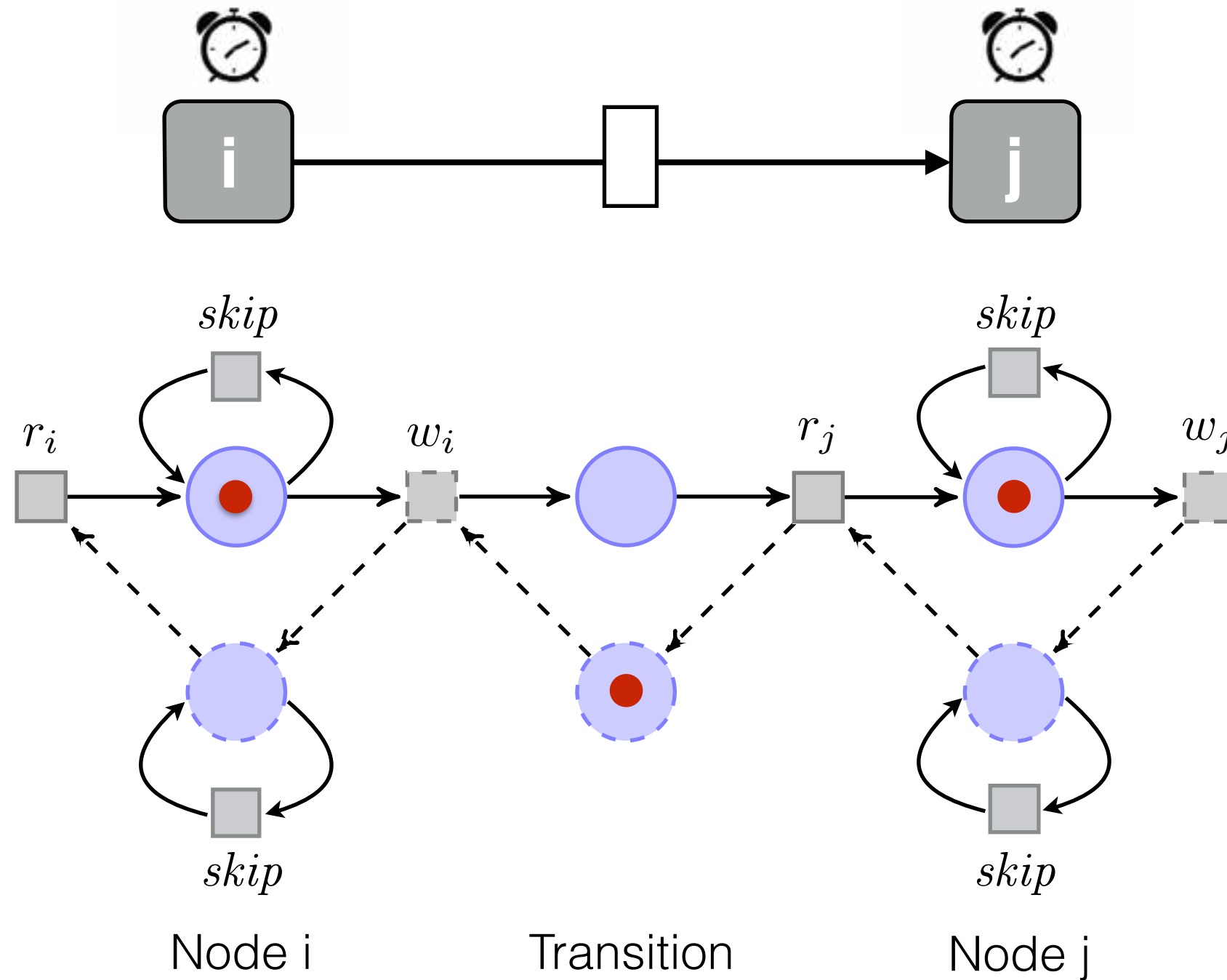
Back-Pressure LTTA



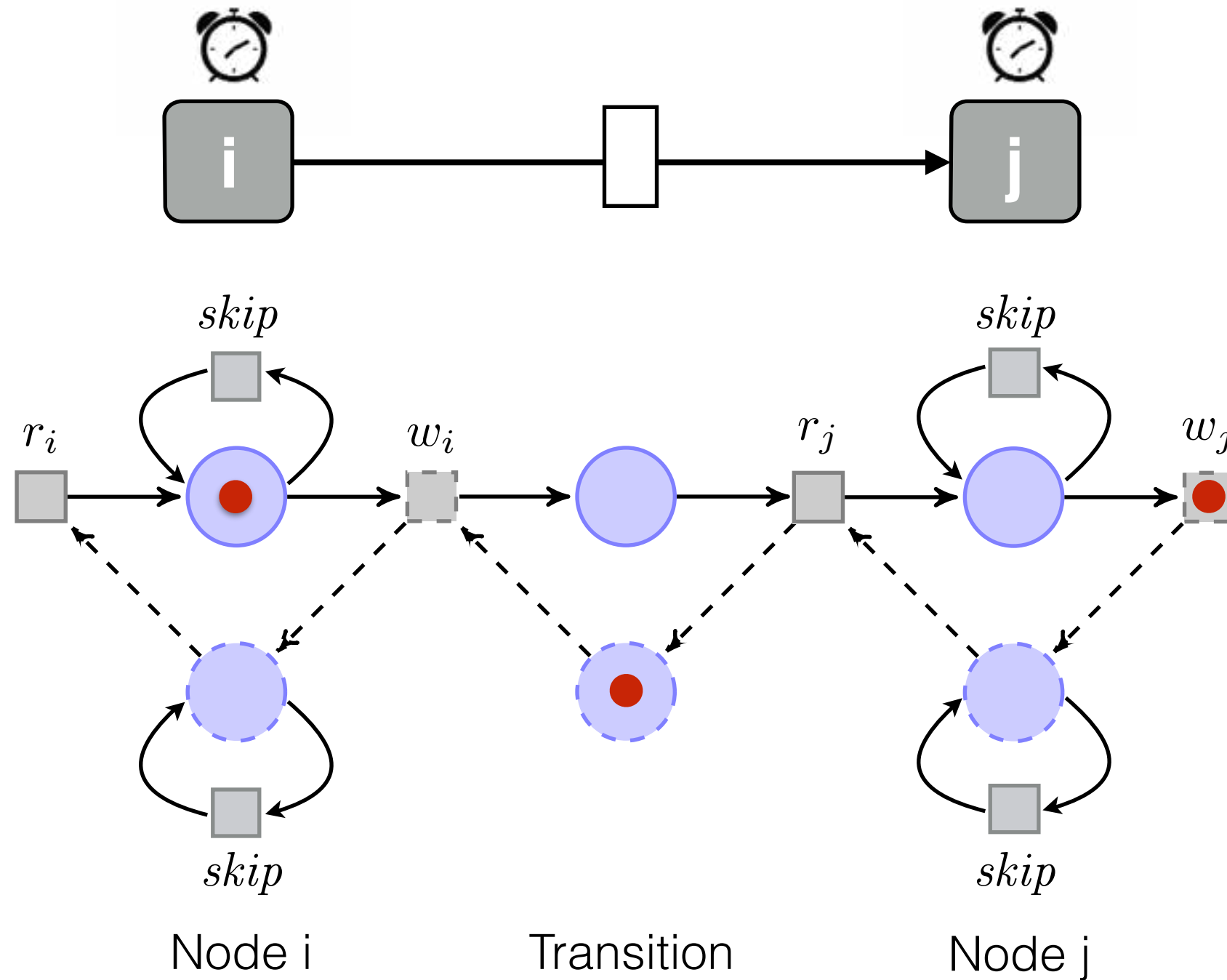
Back-Pressure LTTA



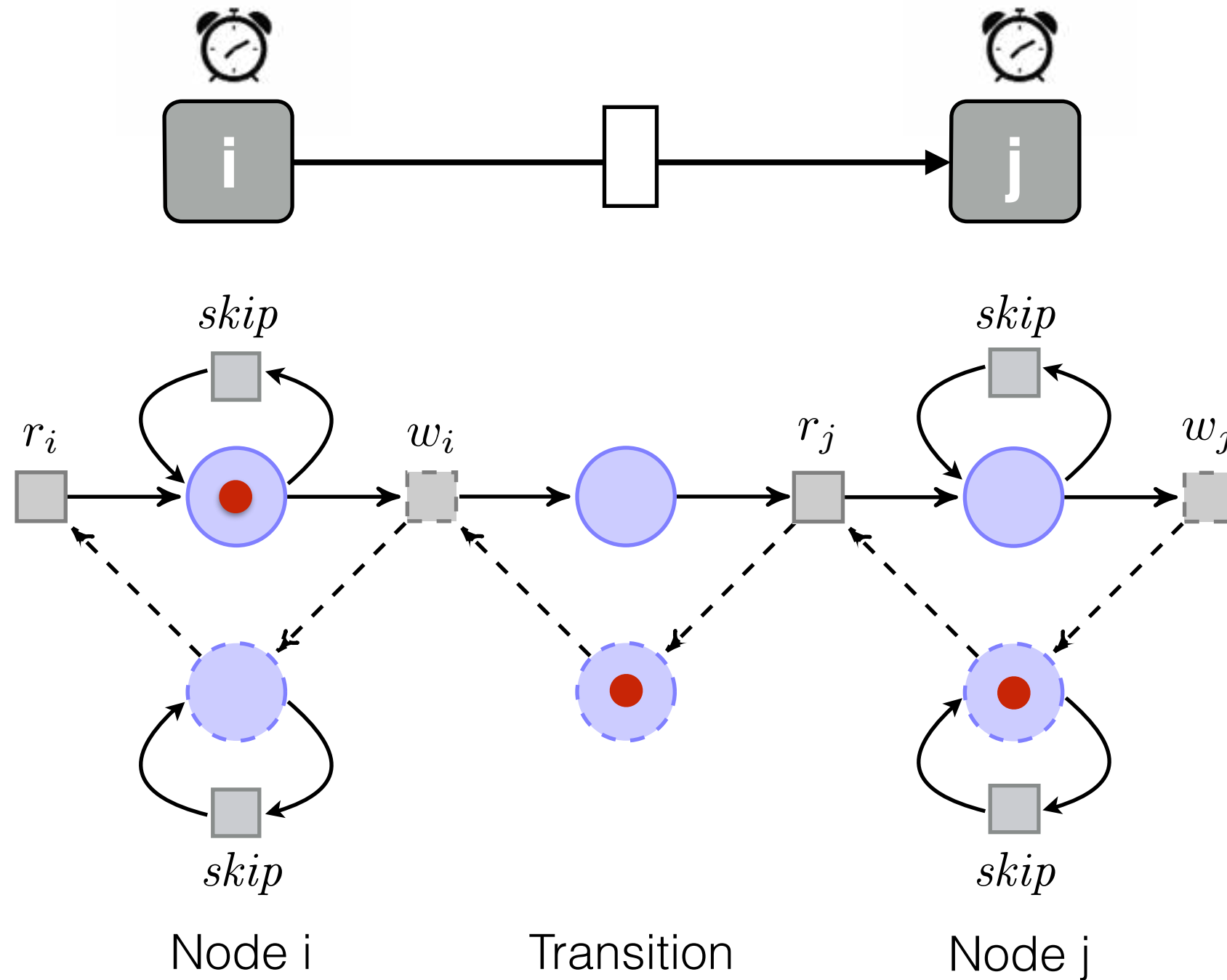
Back-Pressure LTTA



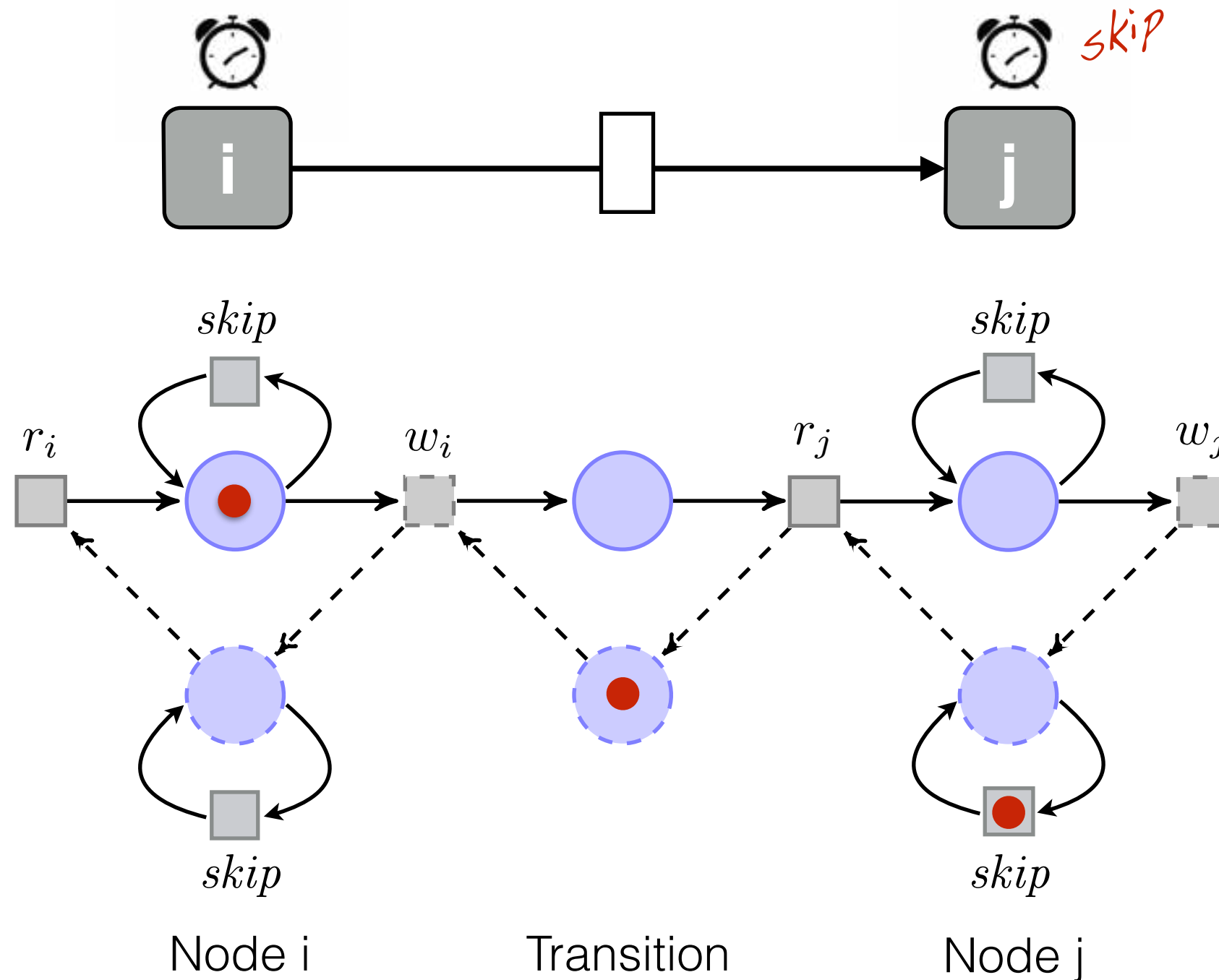
Back-Pressure LTTA



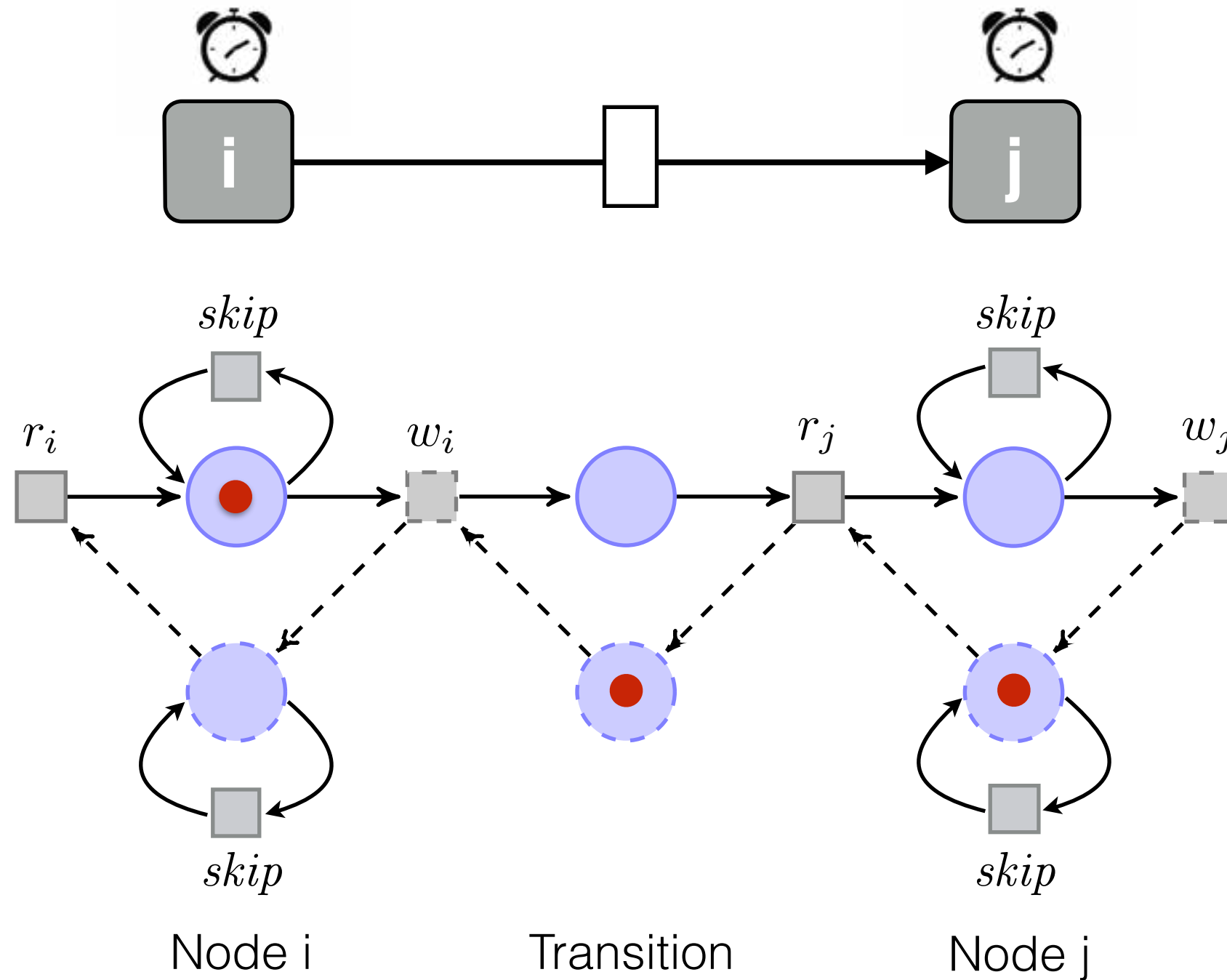
Back-Pressure LTTA



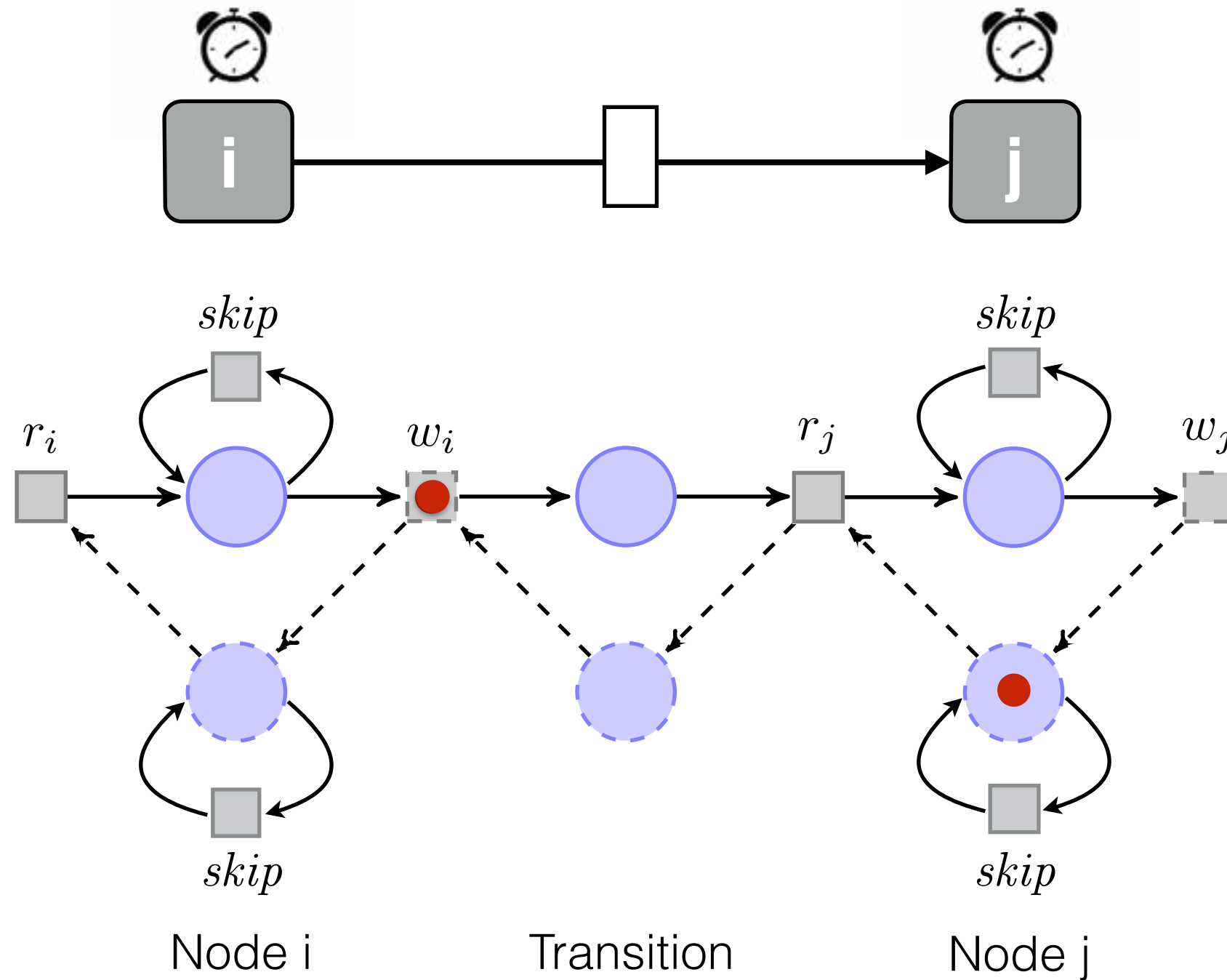
Back-Pressure LTTA



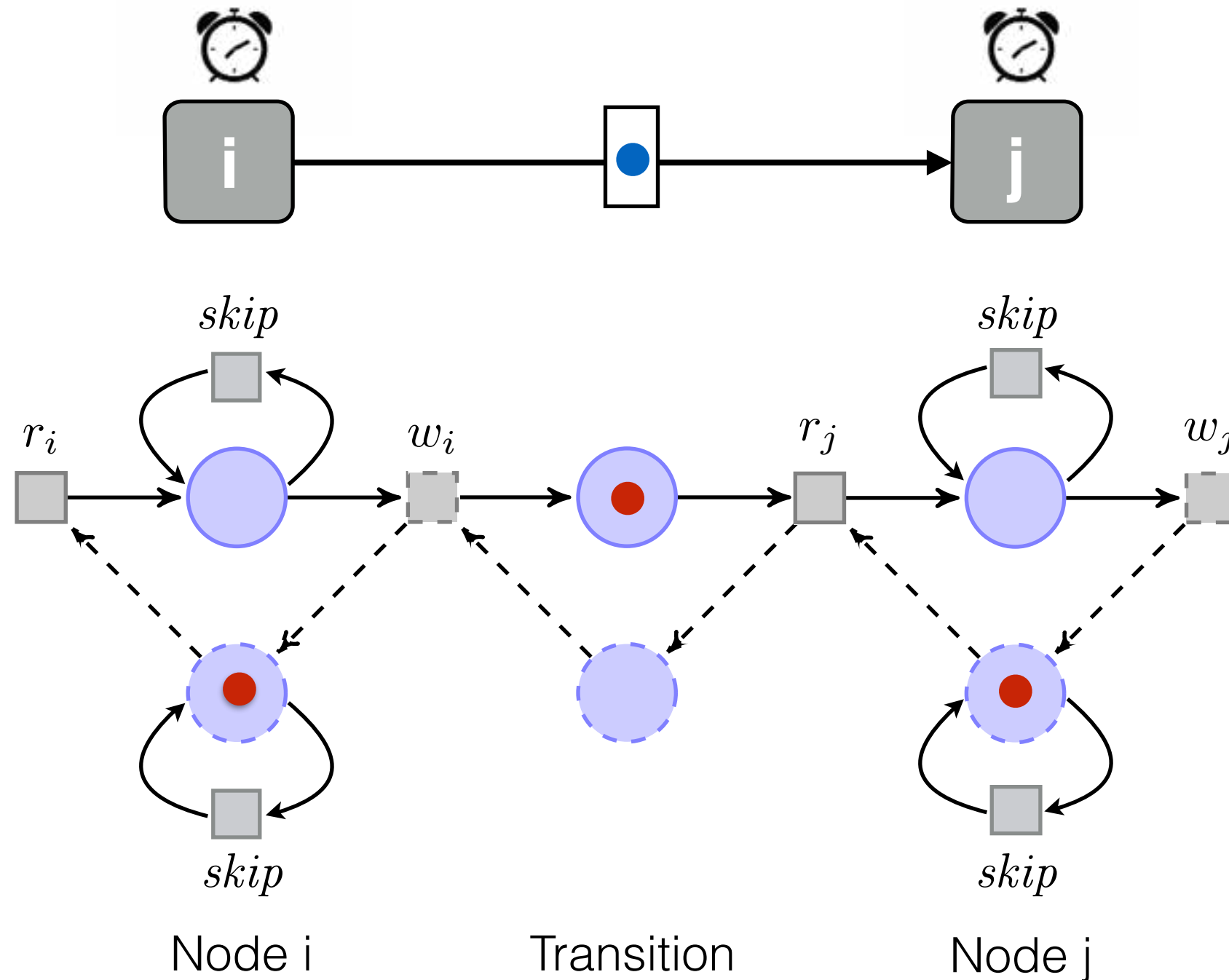
Back-Pressure LTTA



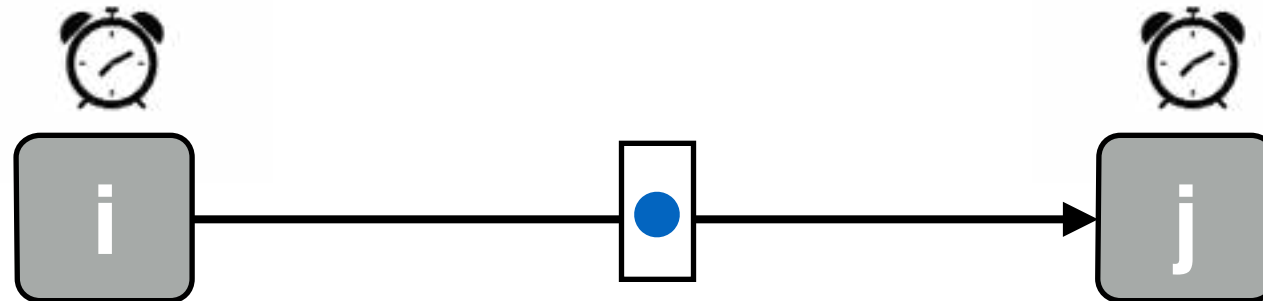
Back-Pressure LTTA



Back-Pressure LTTA



Back-Pressure LTTA

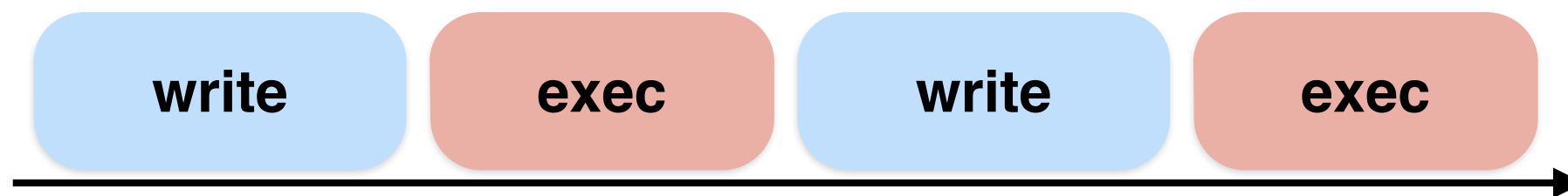
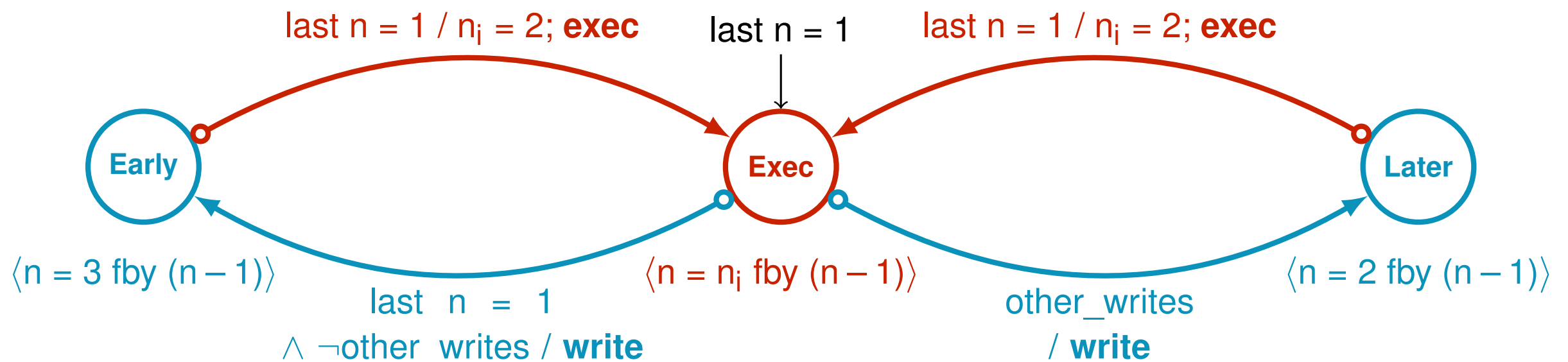


- **Difference:** nodes are triggered by their local clock
- **Idea:** adding skipping mechanism

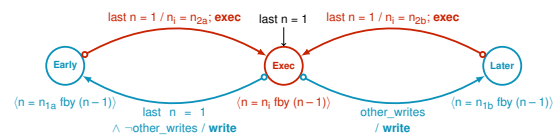
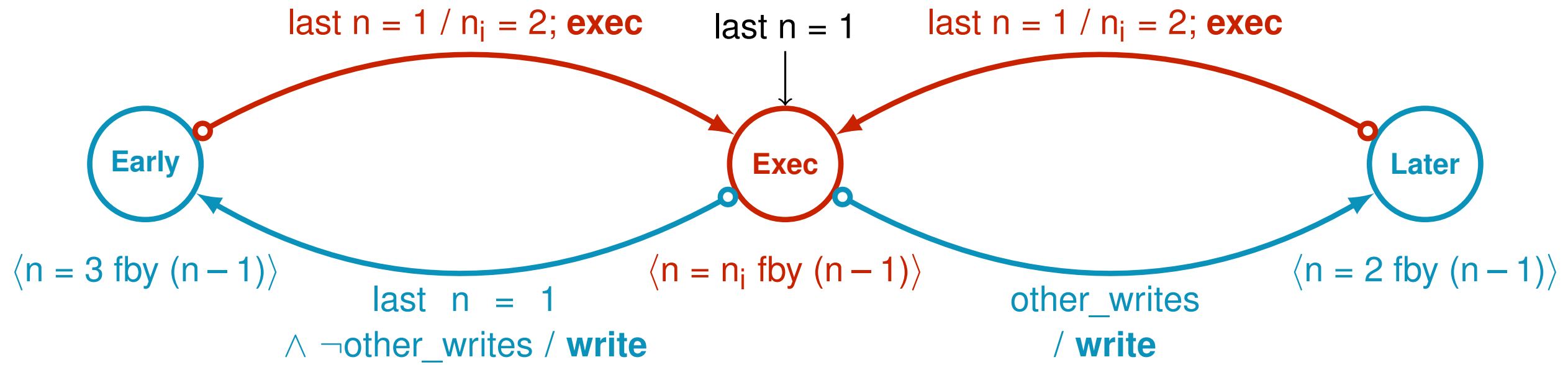
Preservation of the **discrete** synchronous semantics
(forget the skips)

Time-Based LTTA

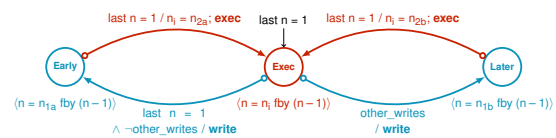
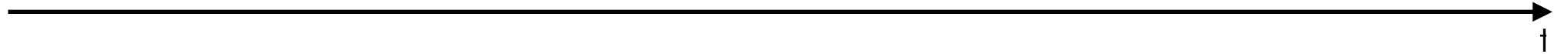
- Nodes alternate between **execution** and **broadcast**
- Nodes can spend **several ticks** in a mode



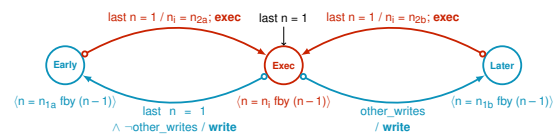
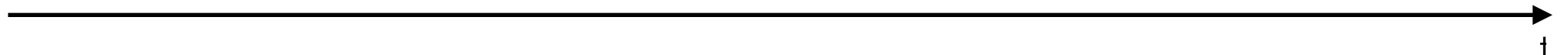
Initial counter values depend on period and communication delay bounds



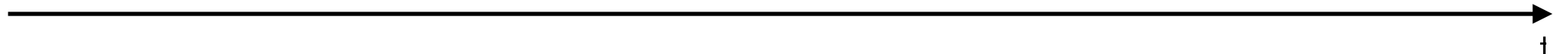
N_1

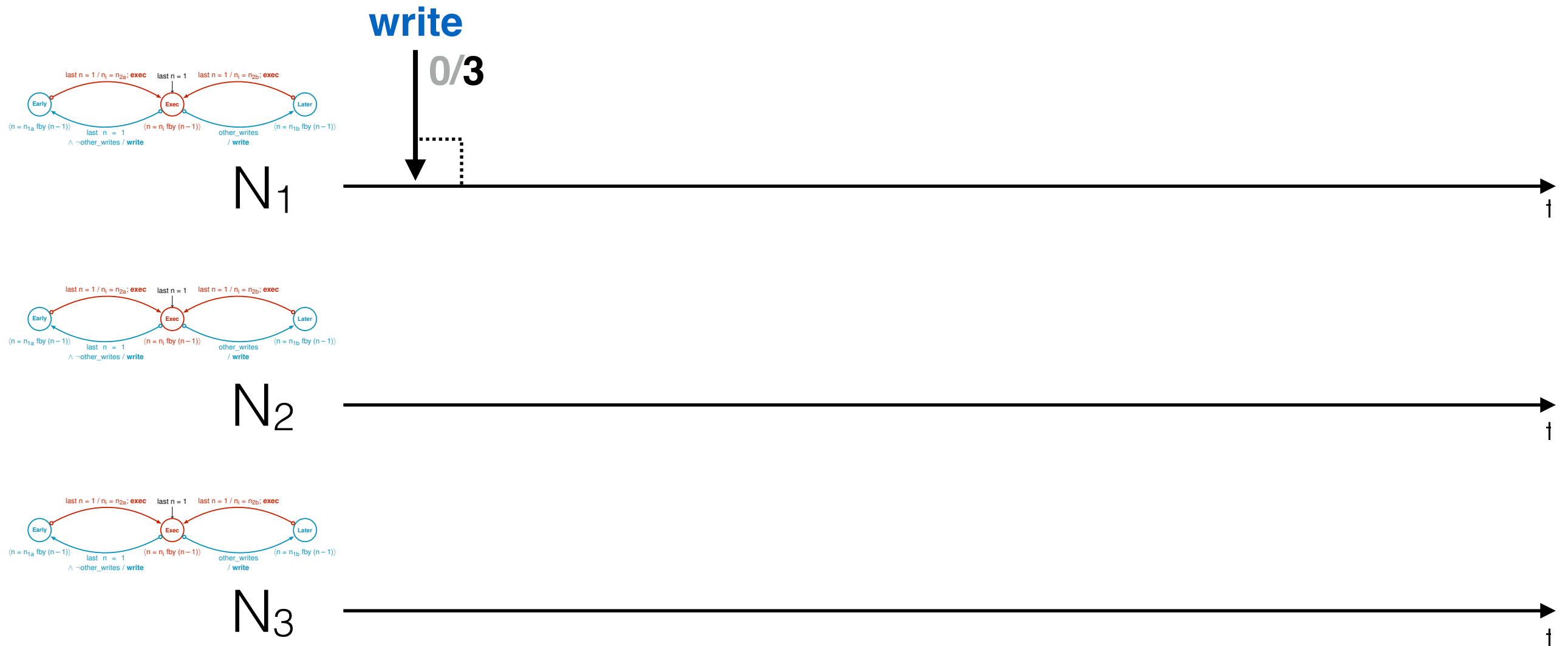
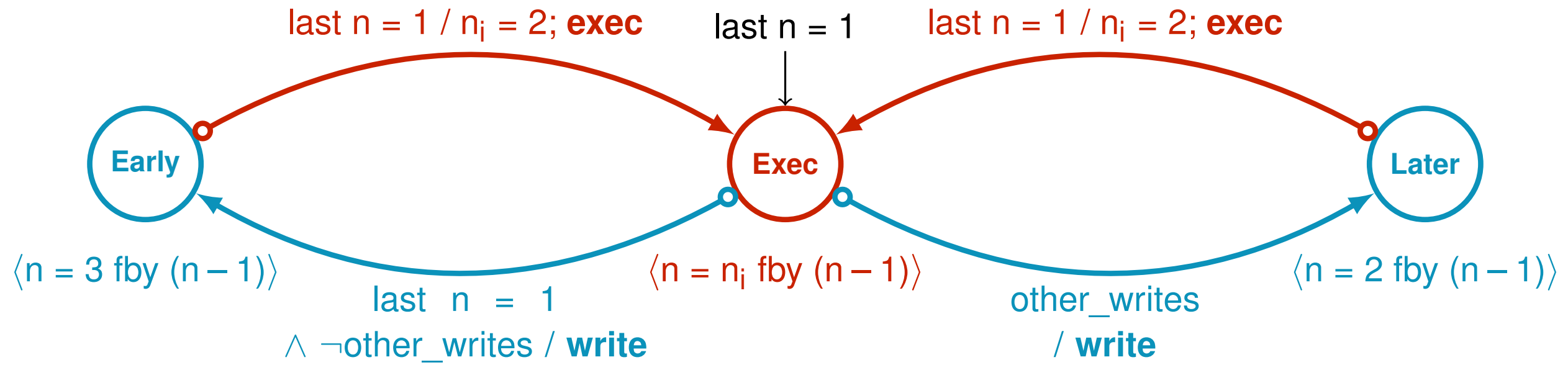


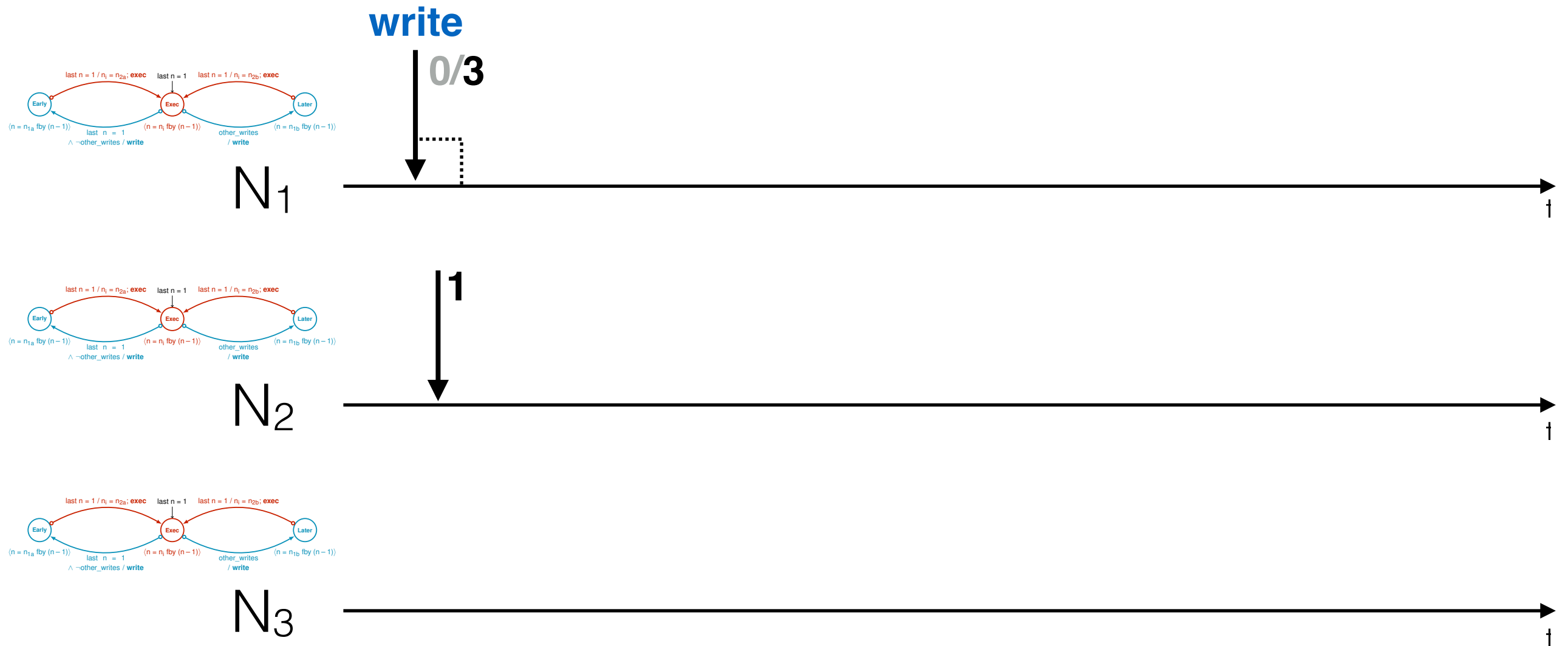
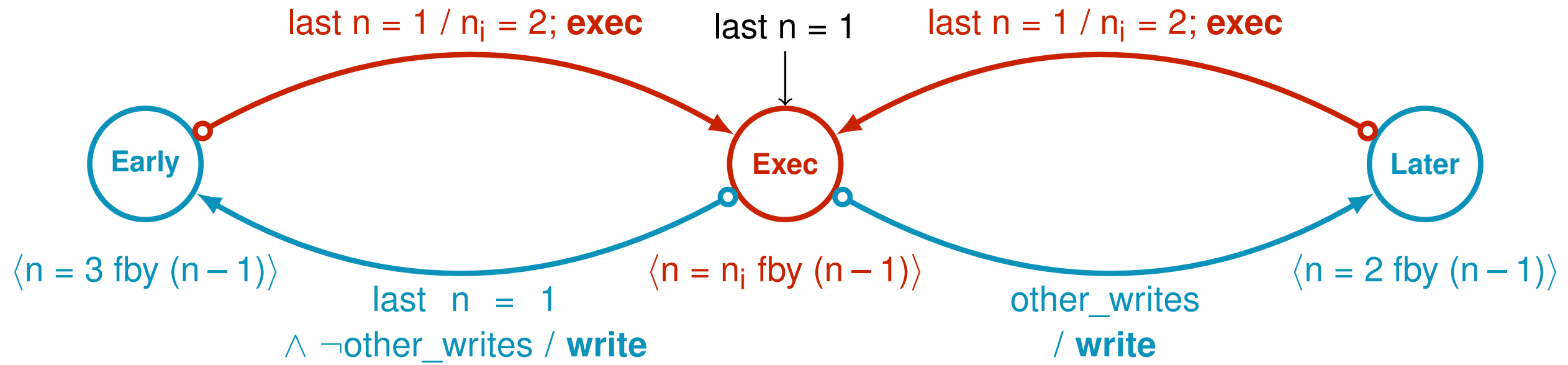
N_2

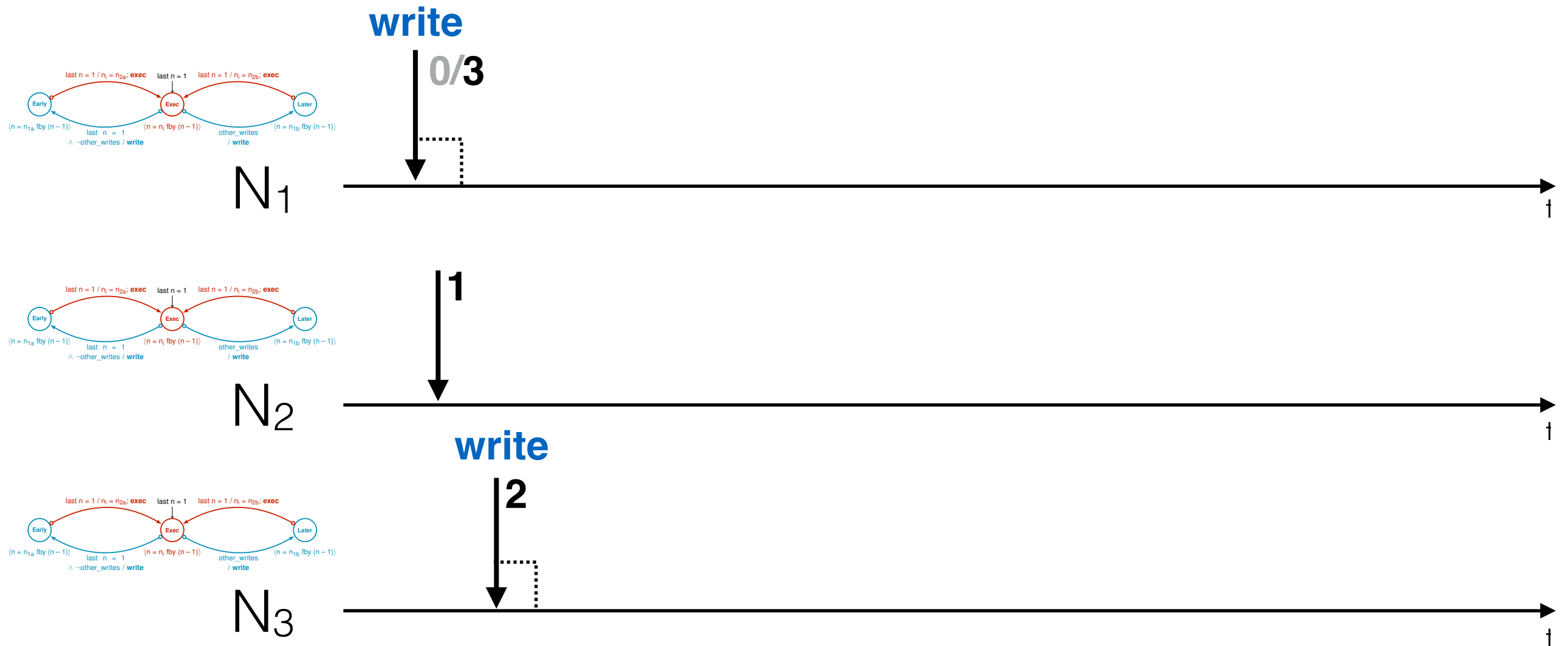
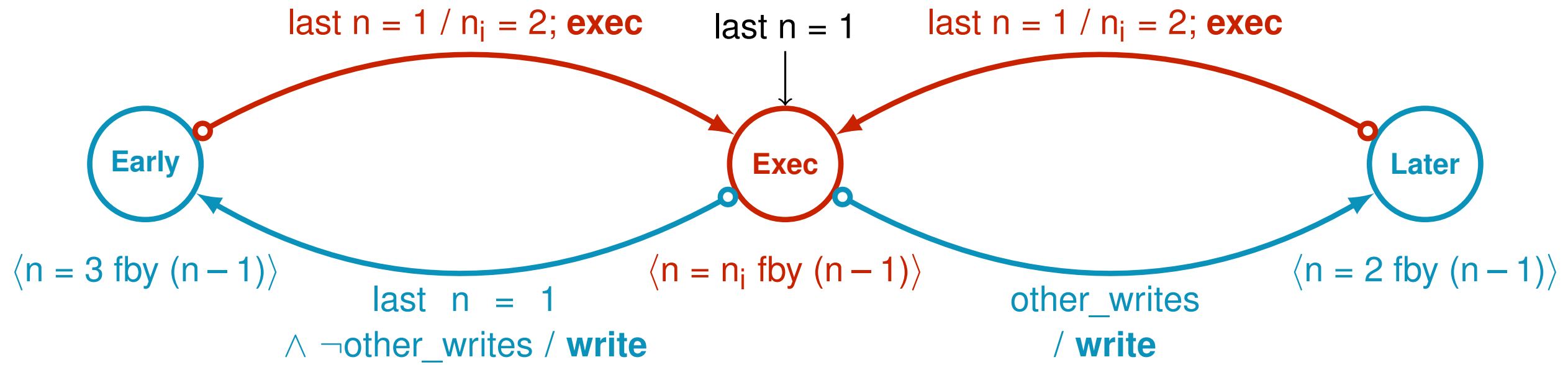


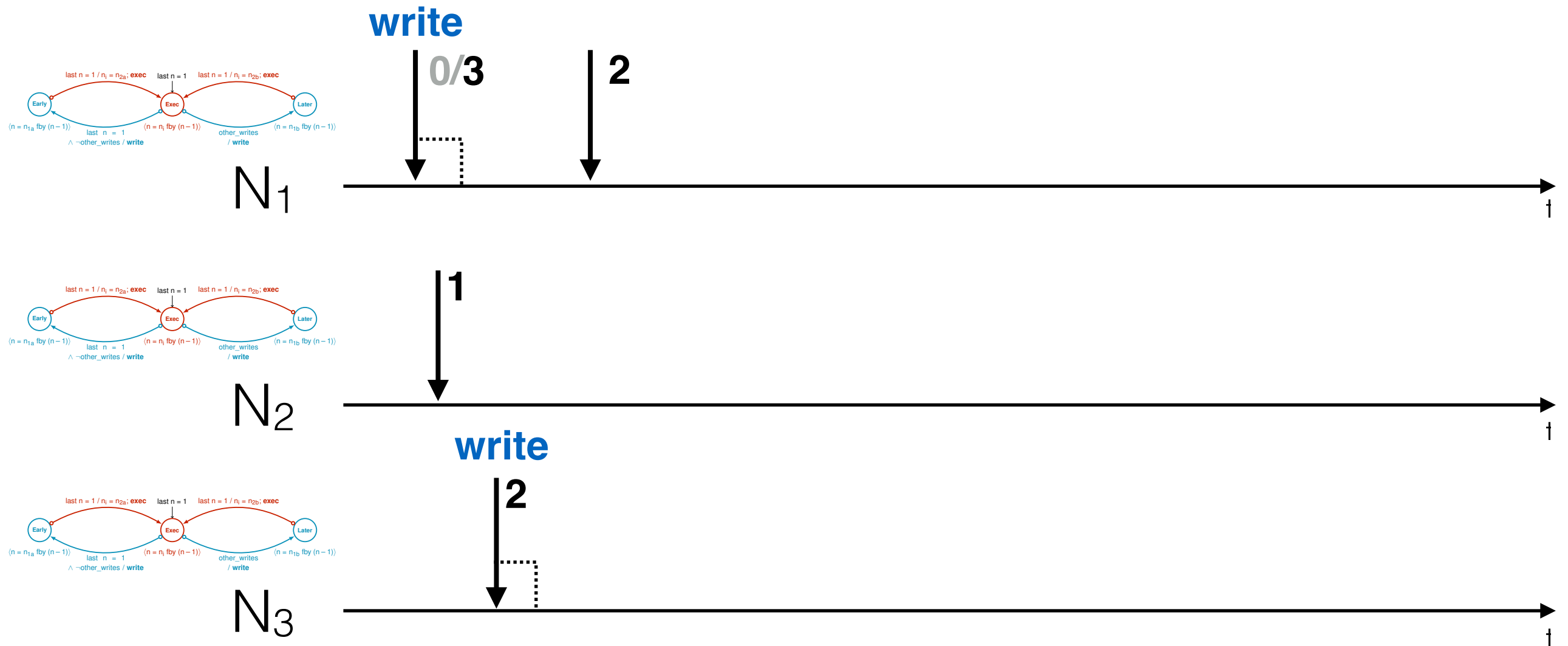
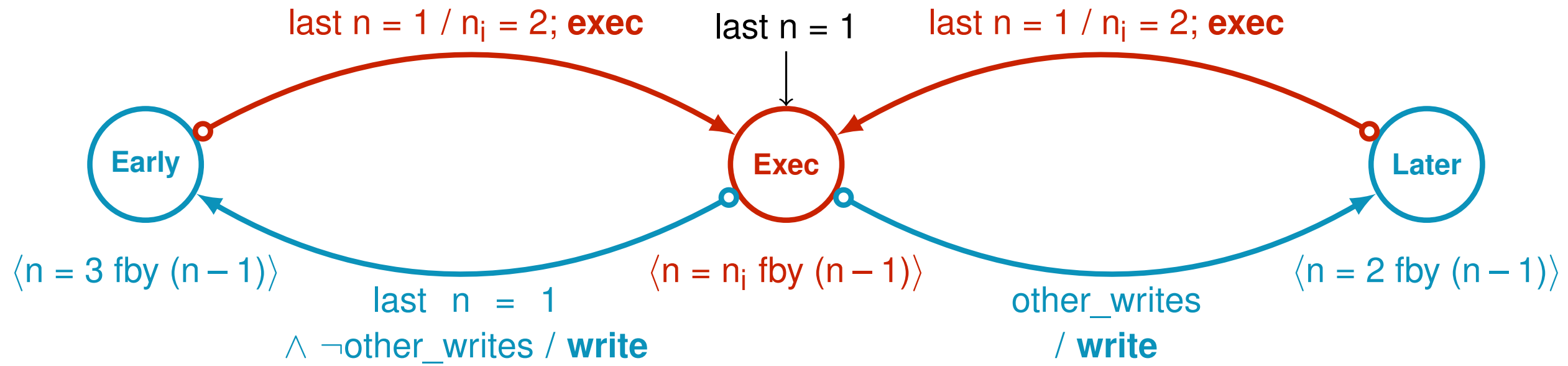
N_3

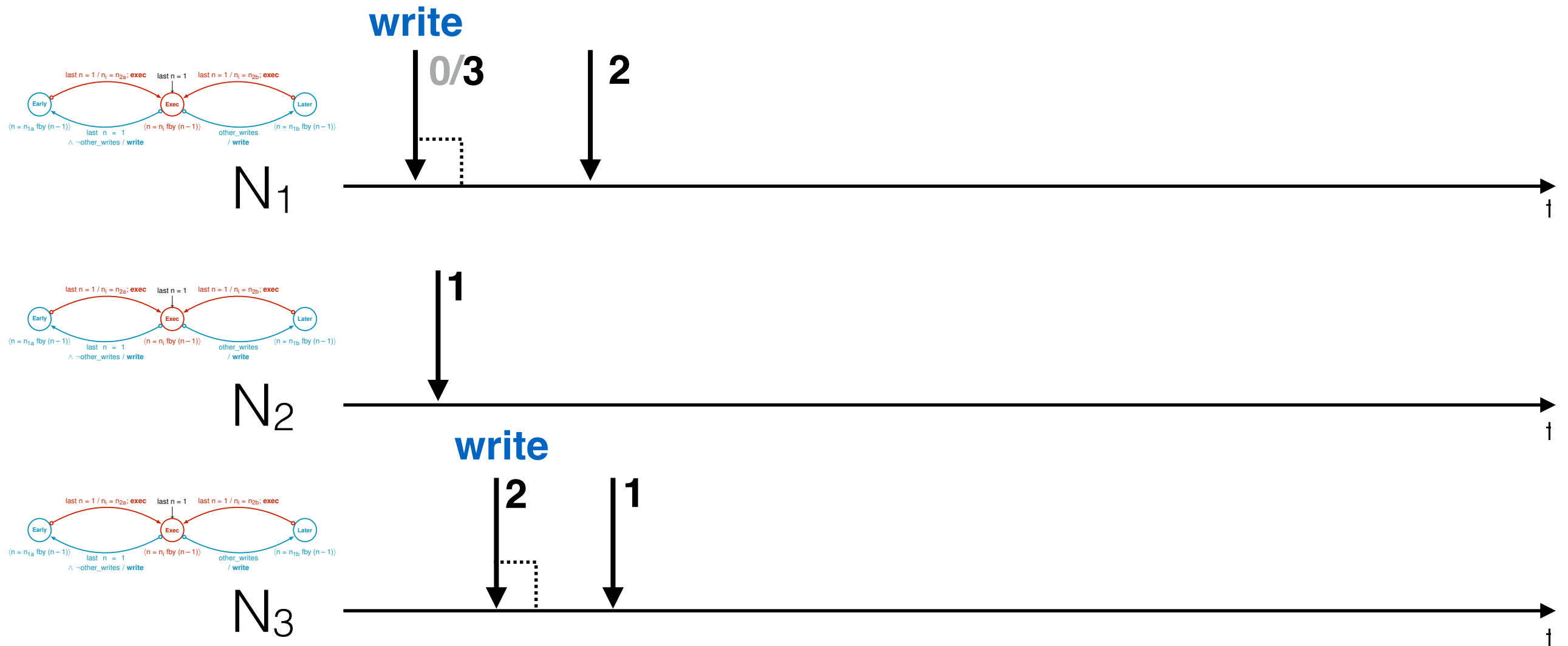
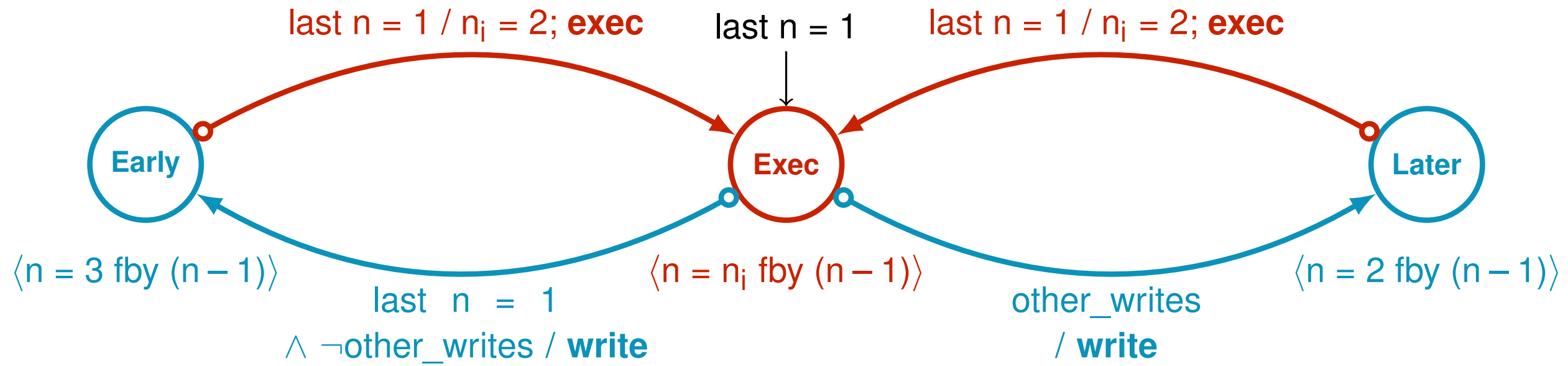


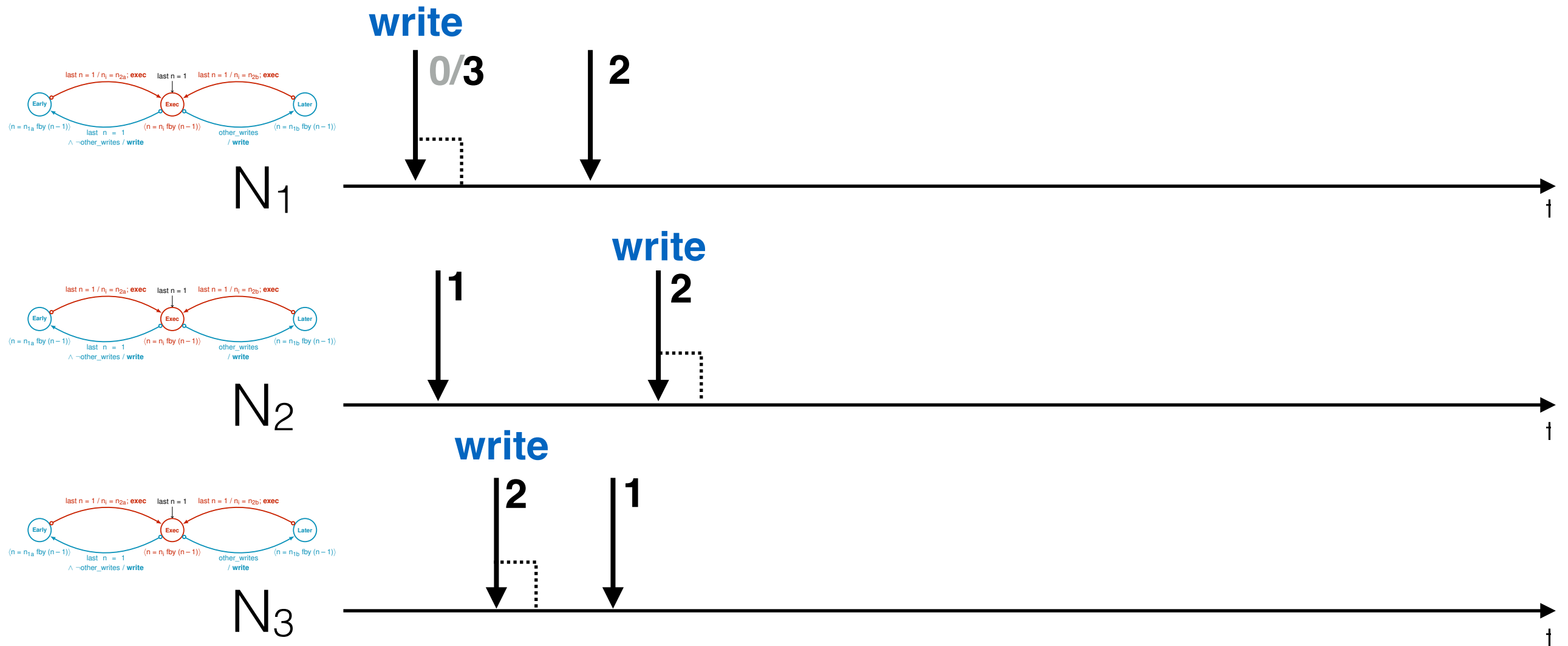
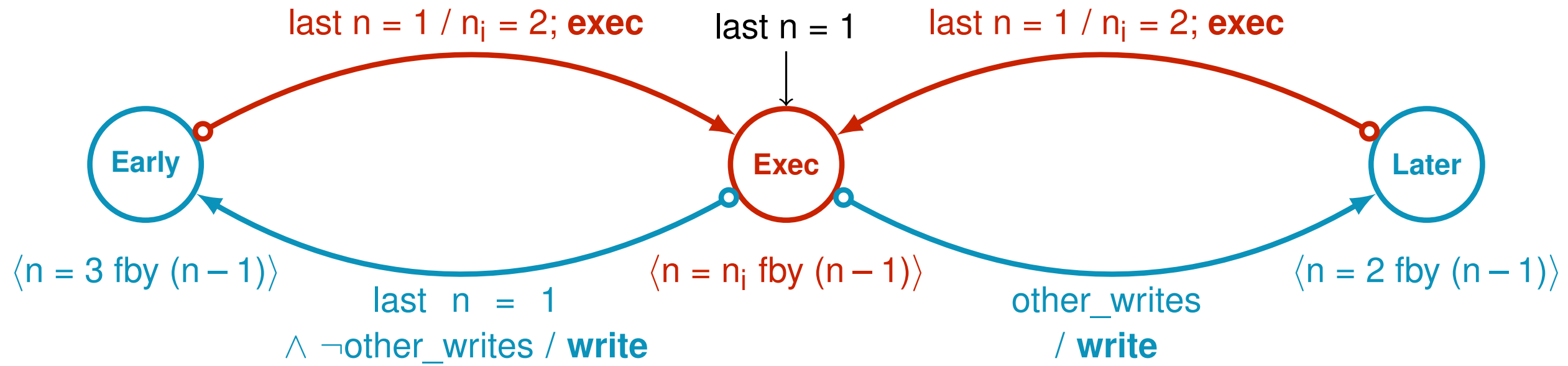


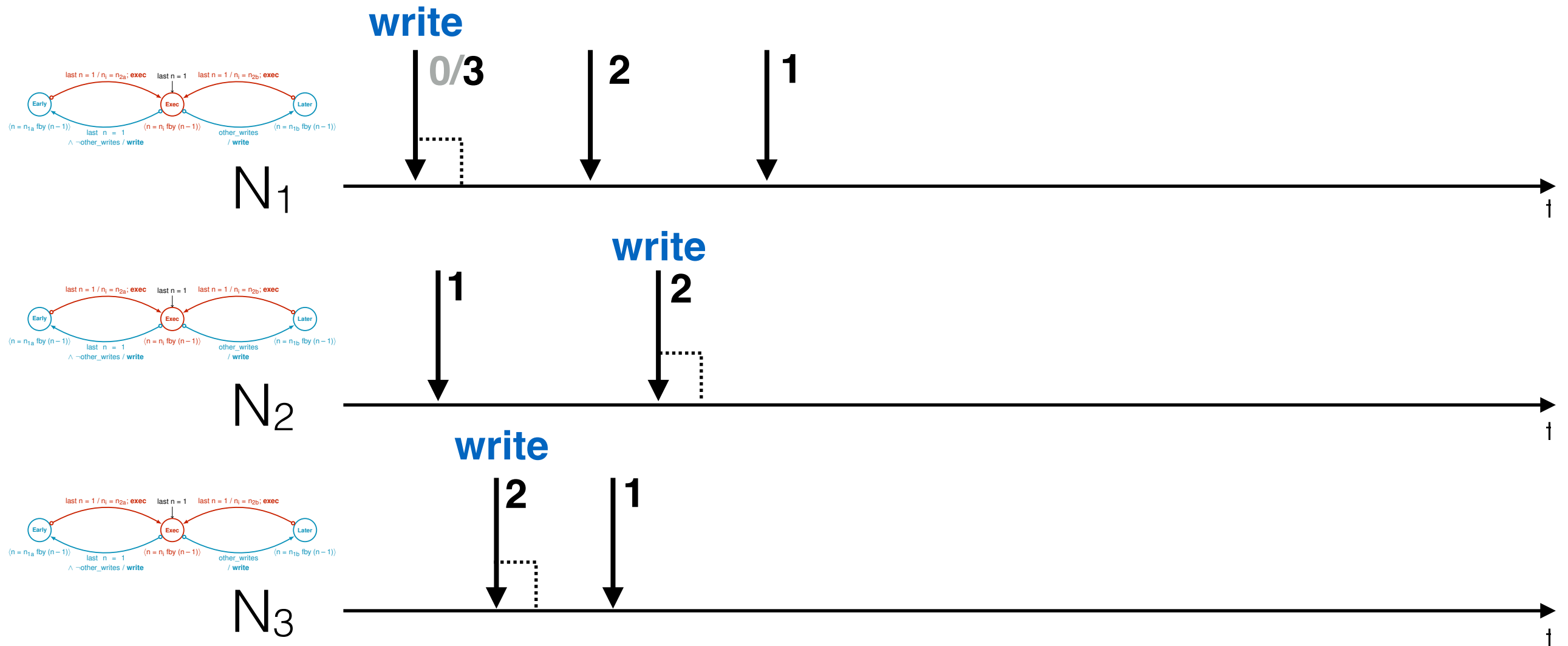
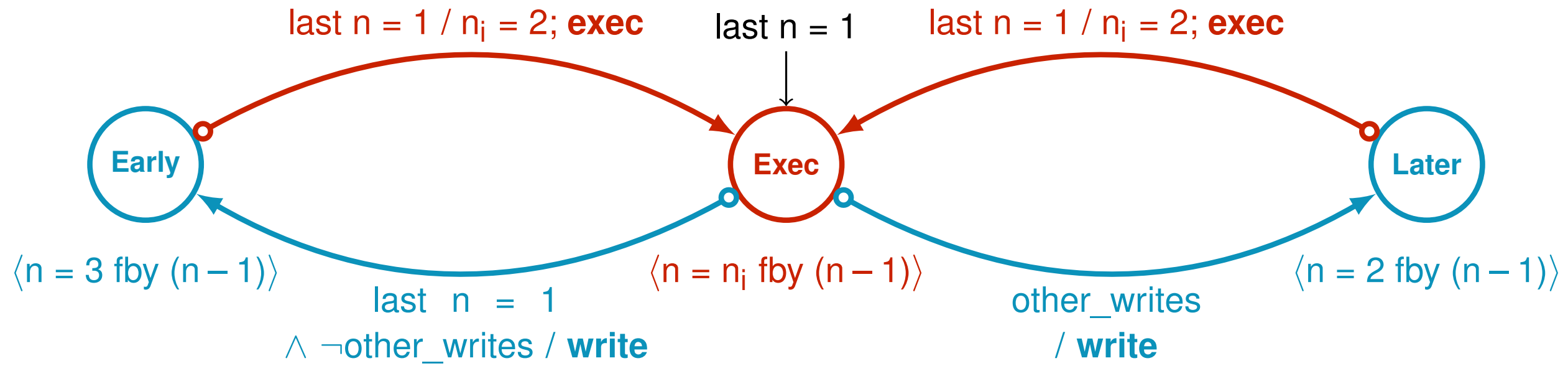


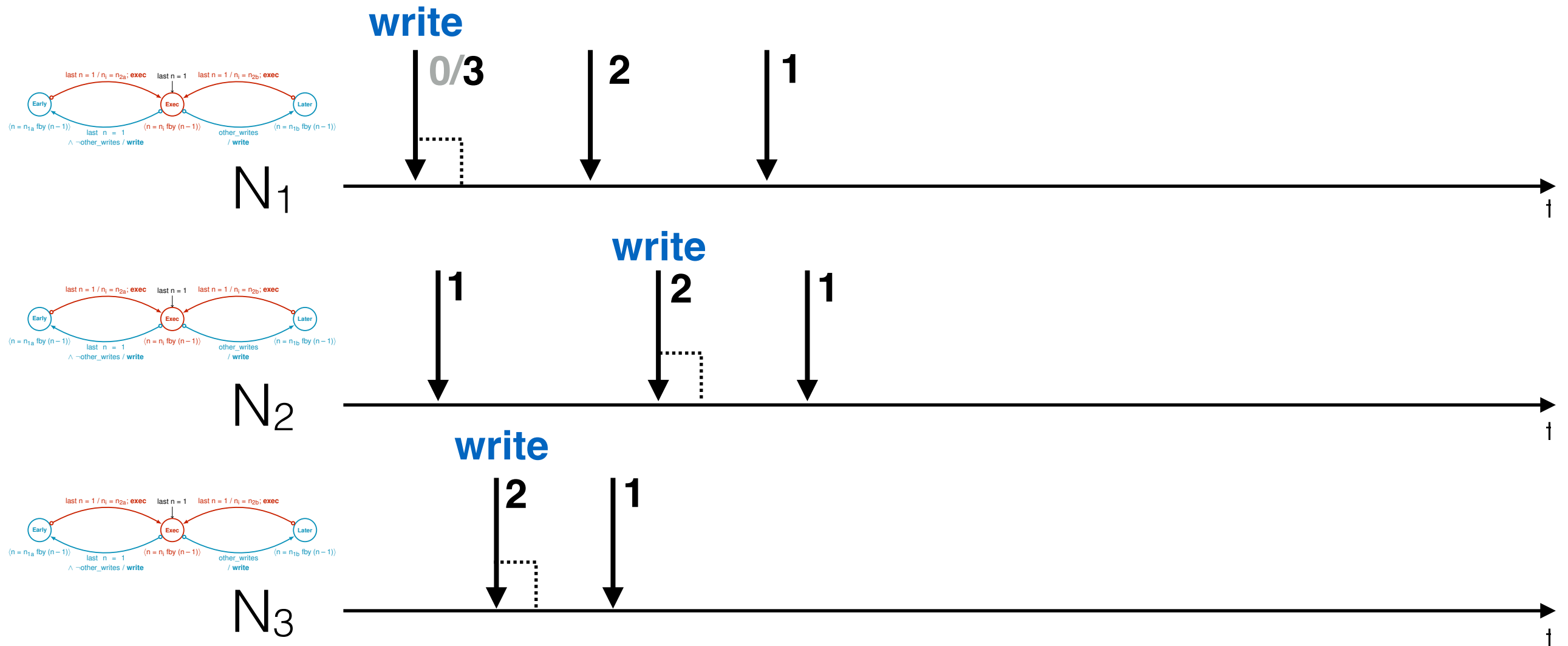
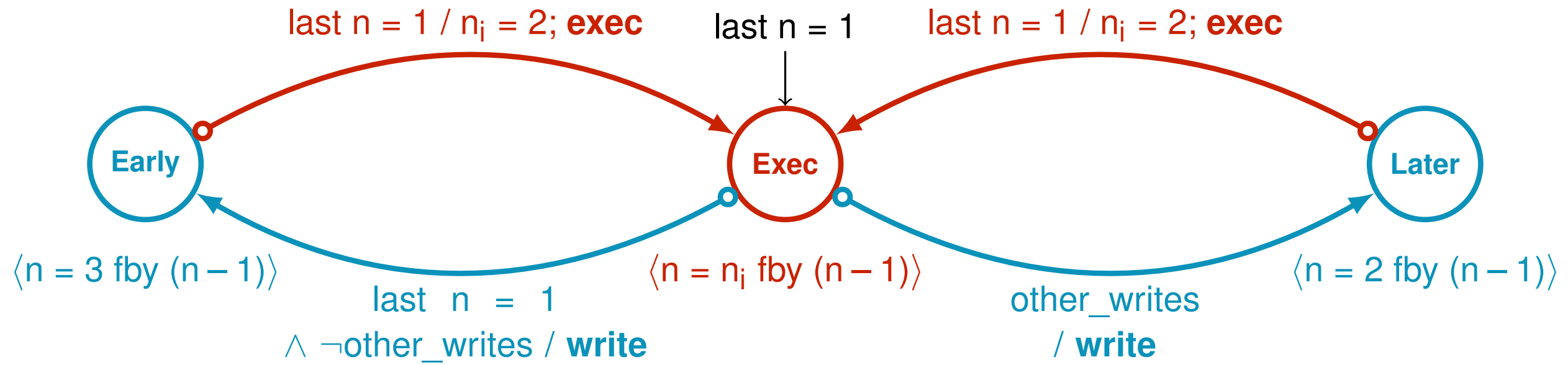


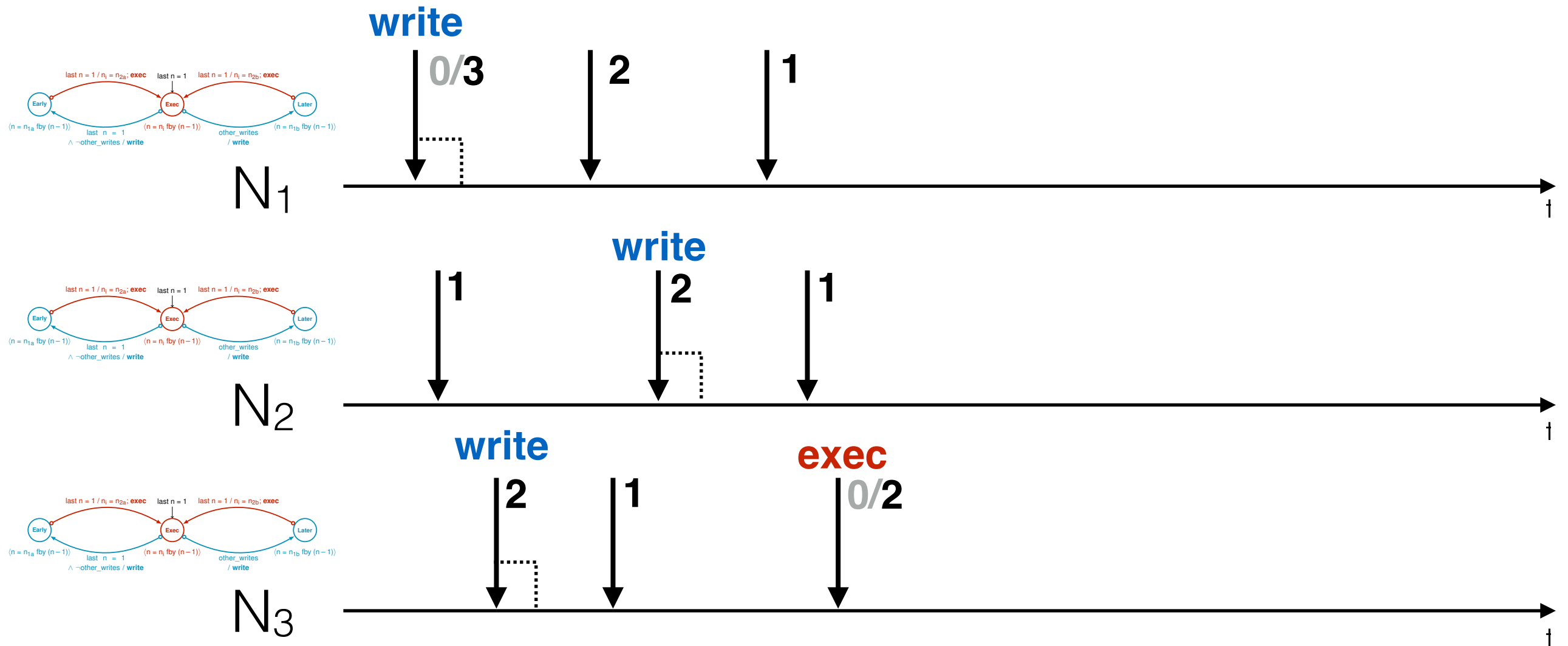
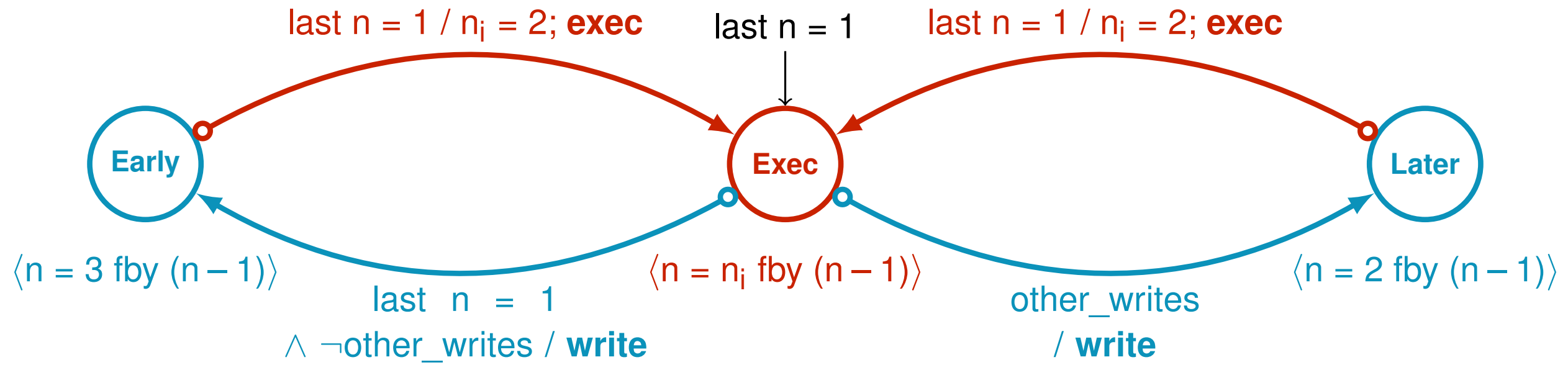


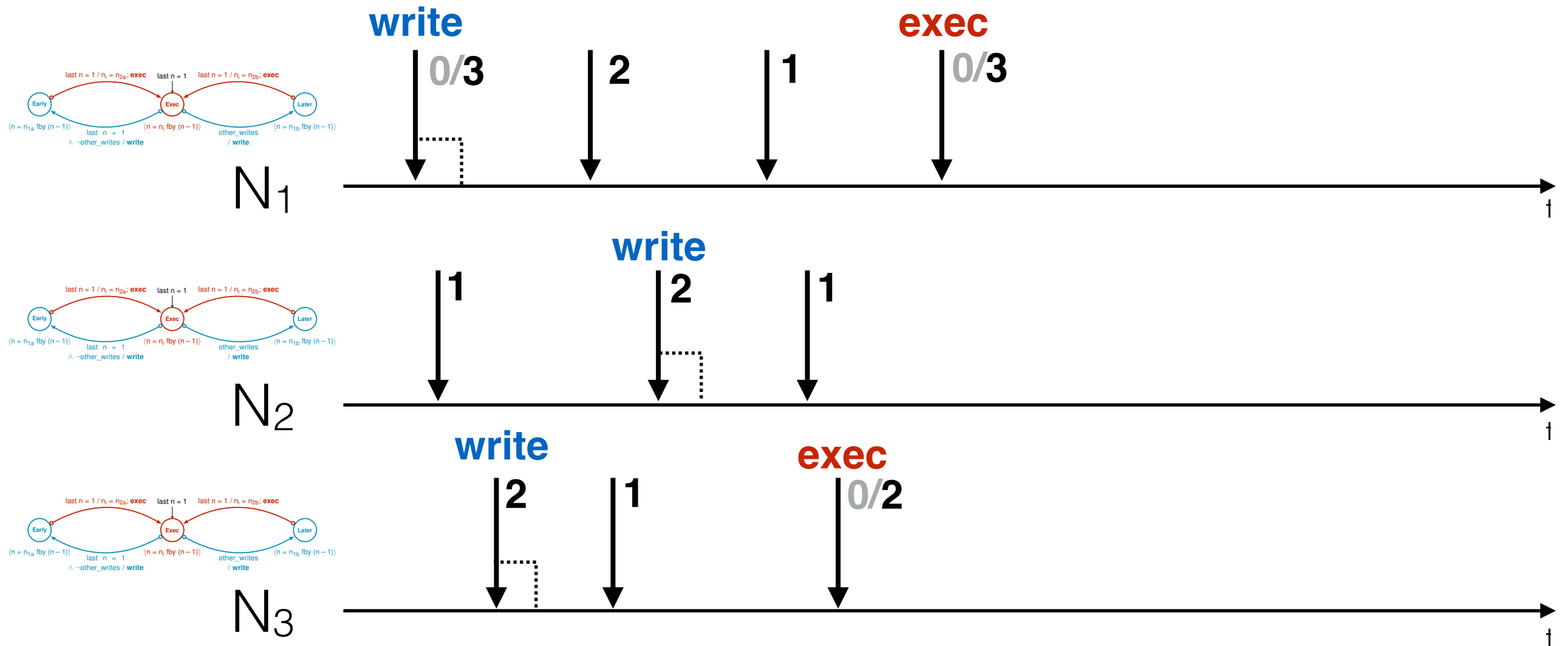
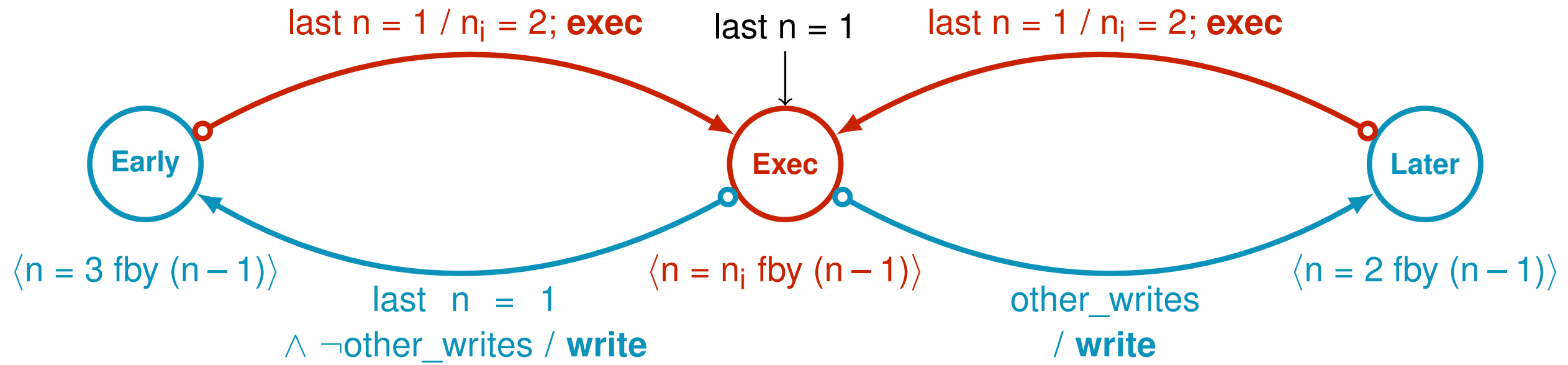


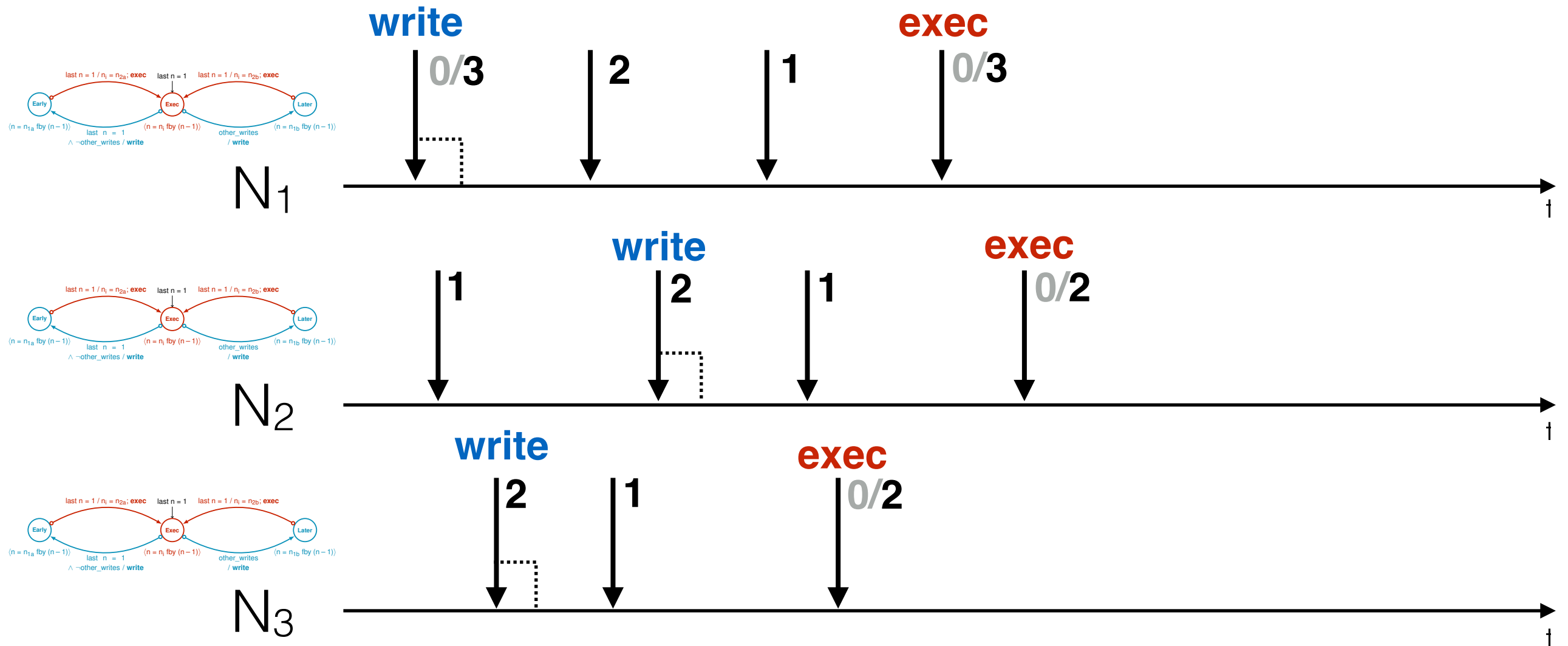
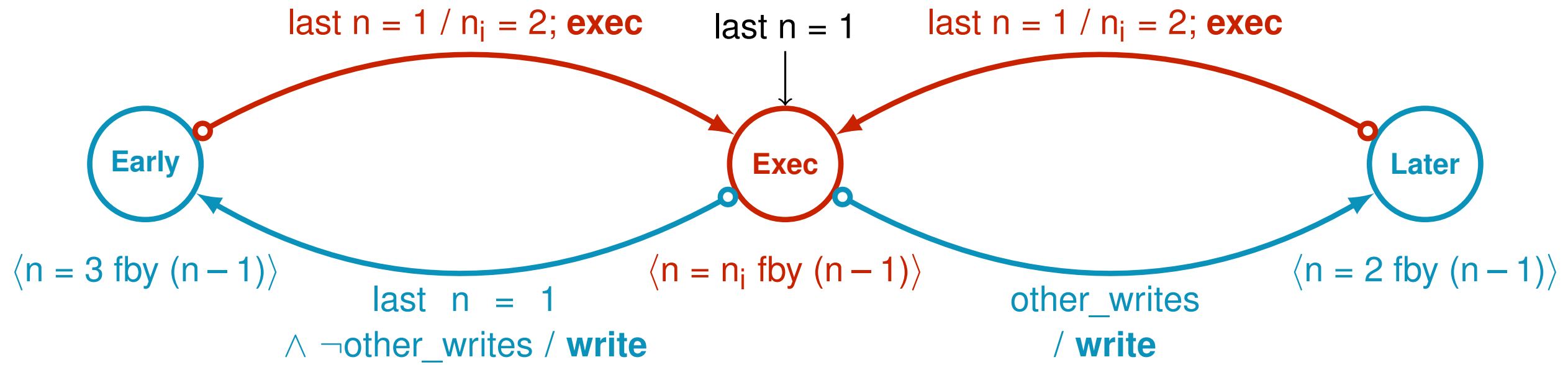


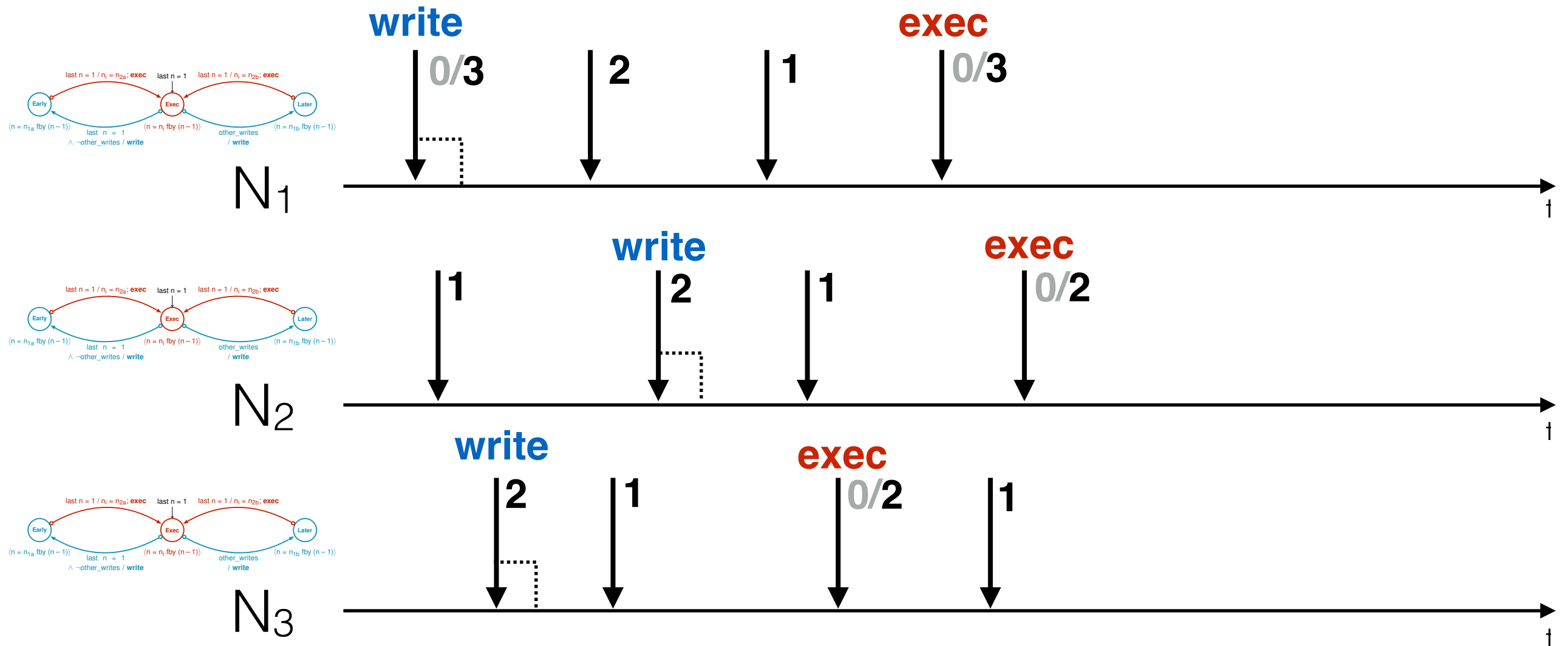
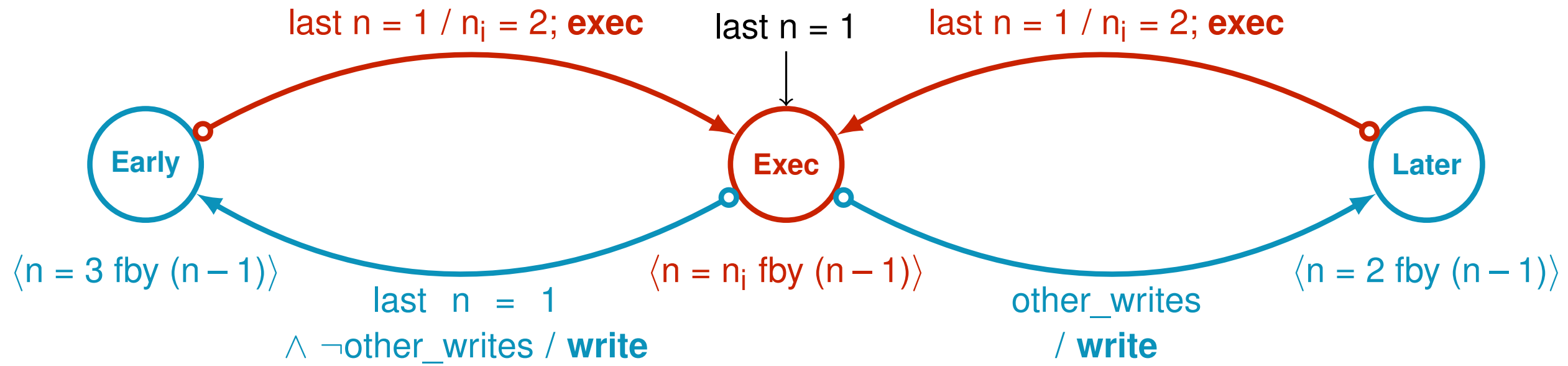


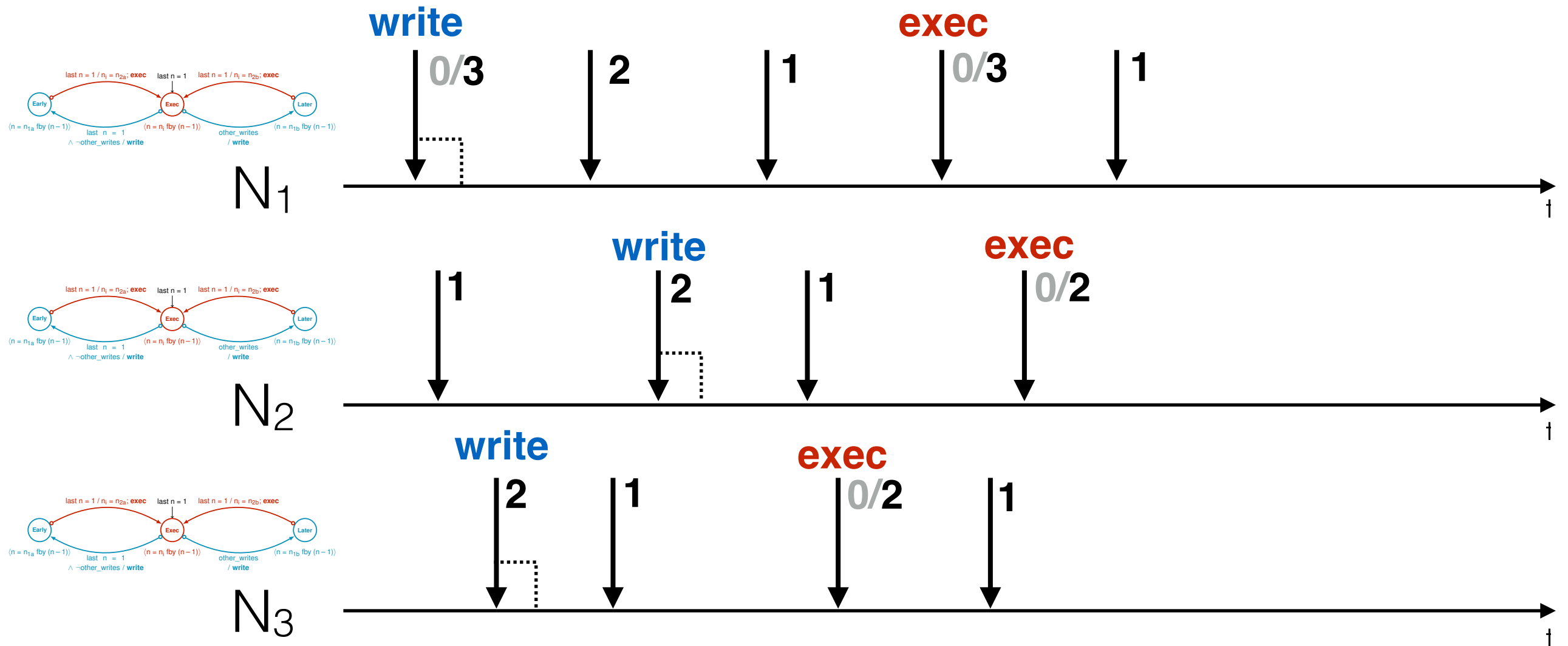
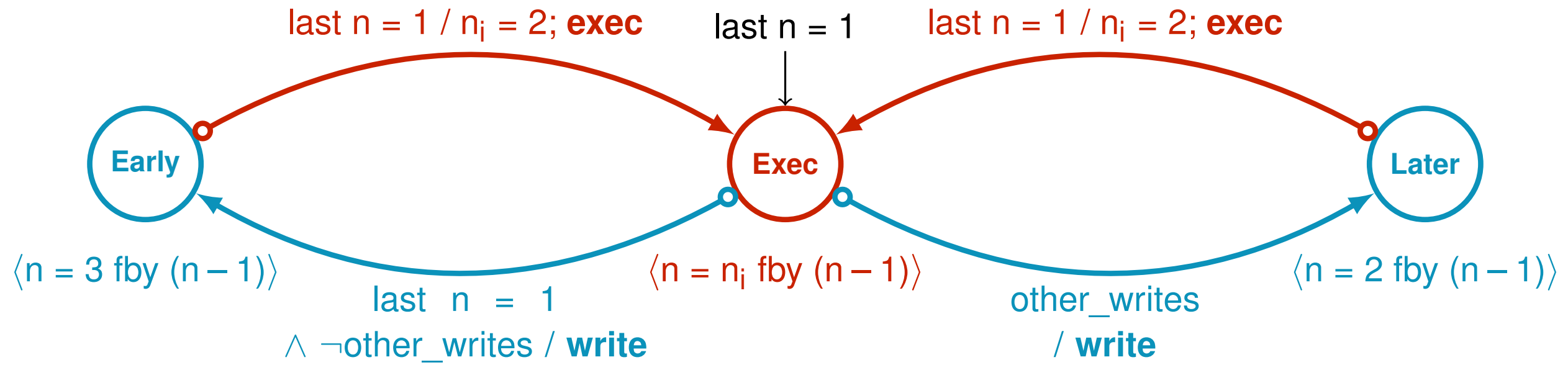


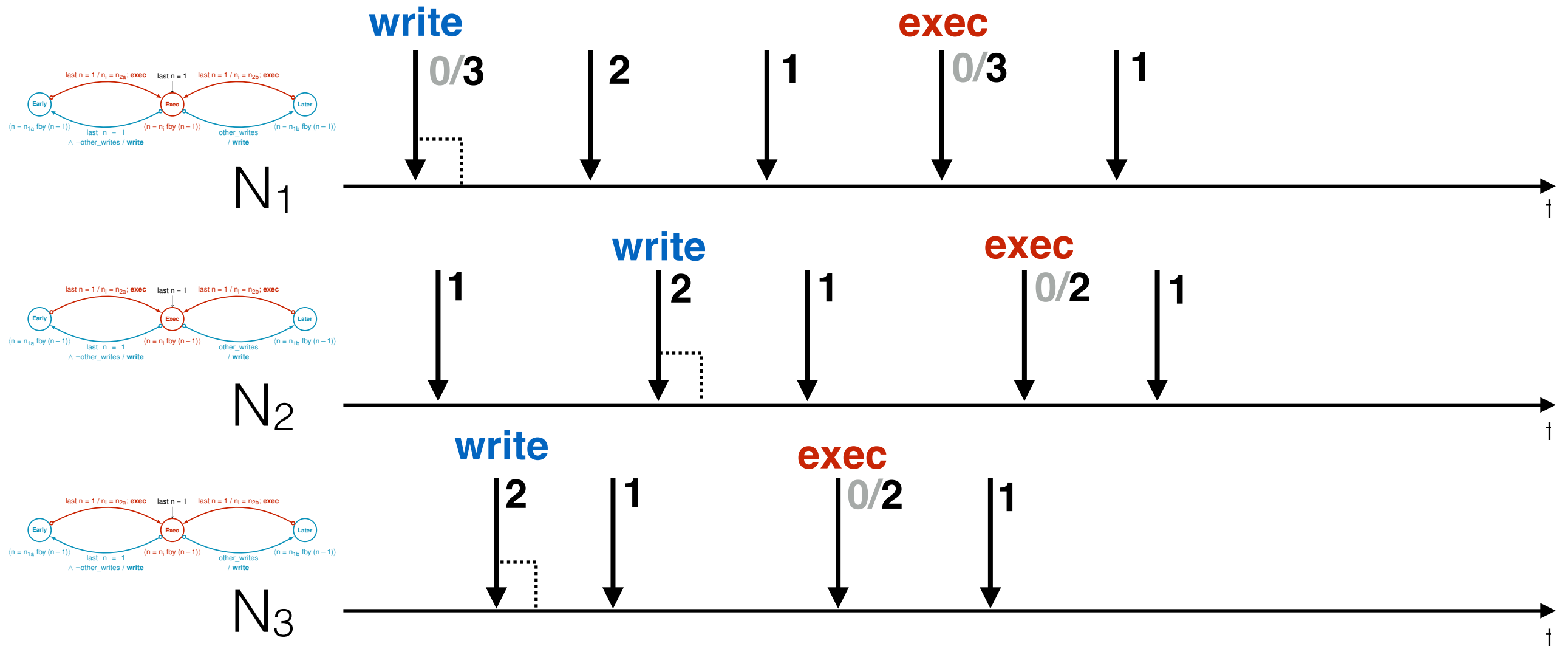
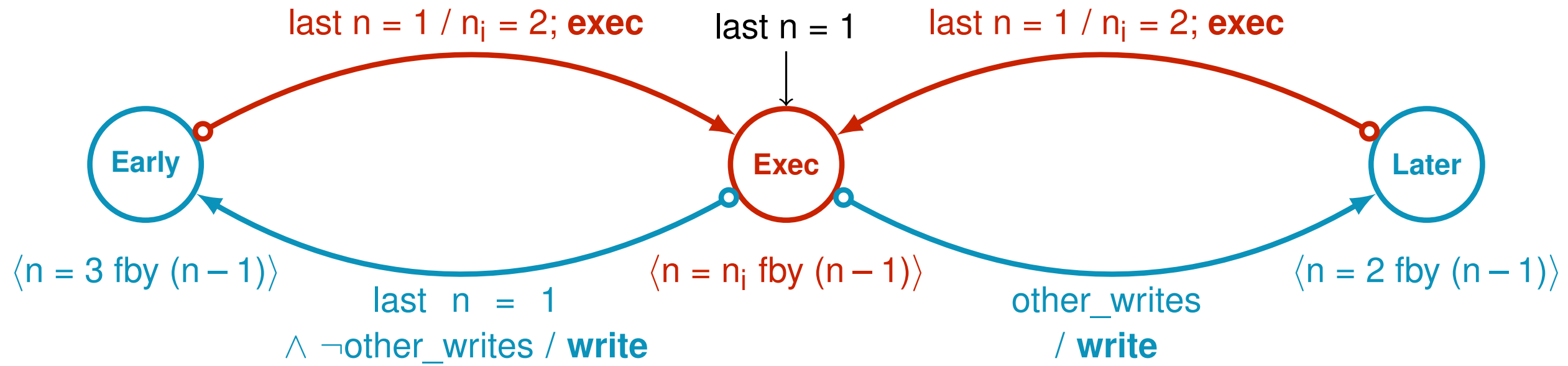


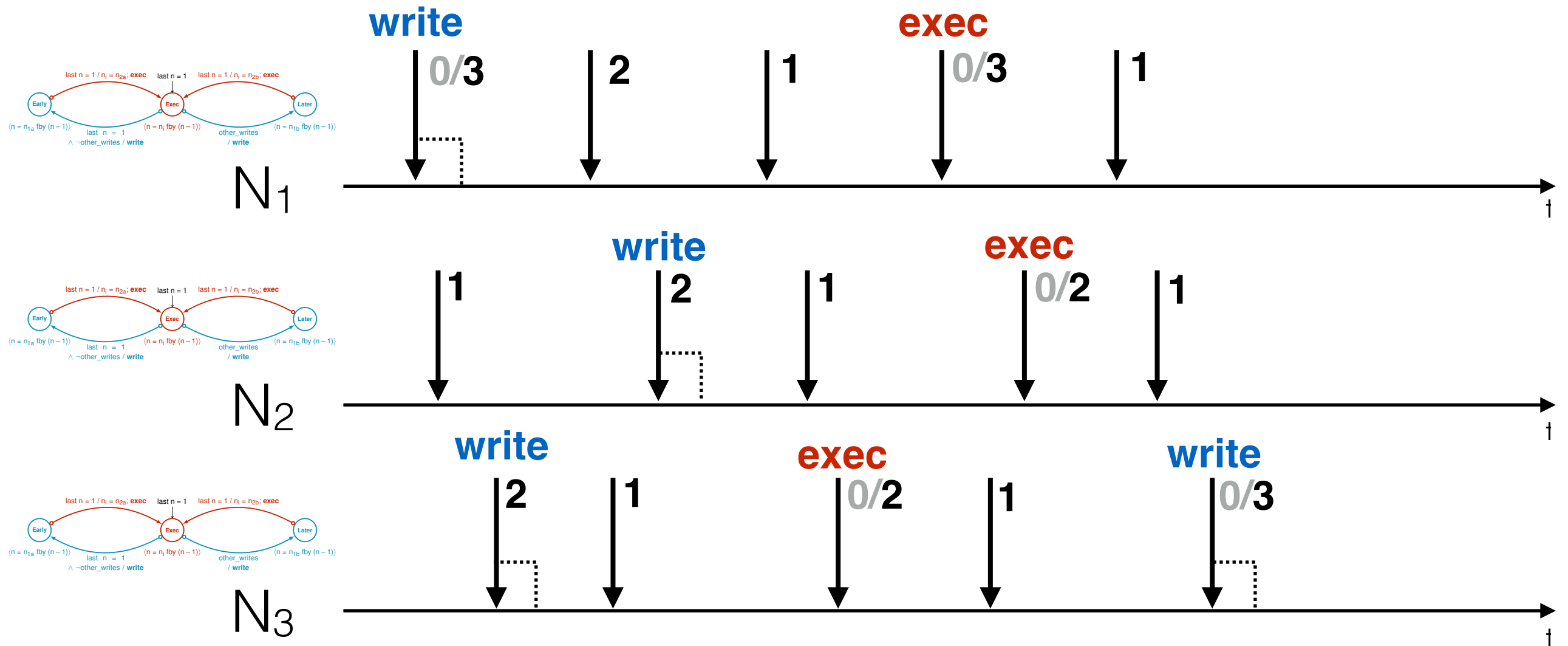
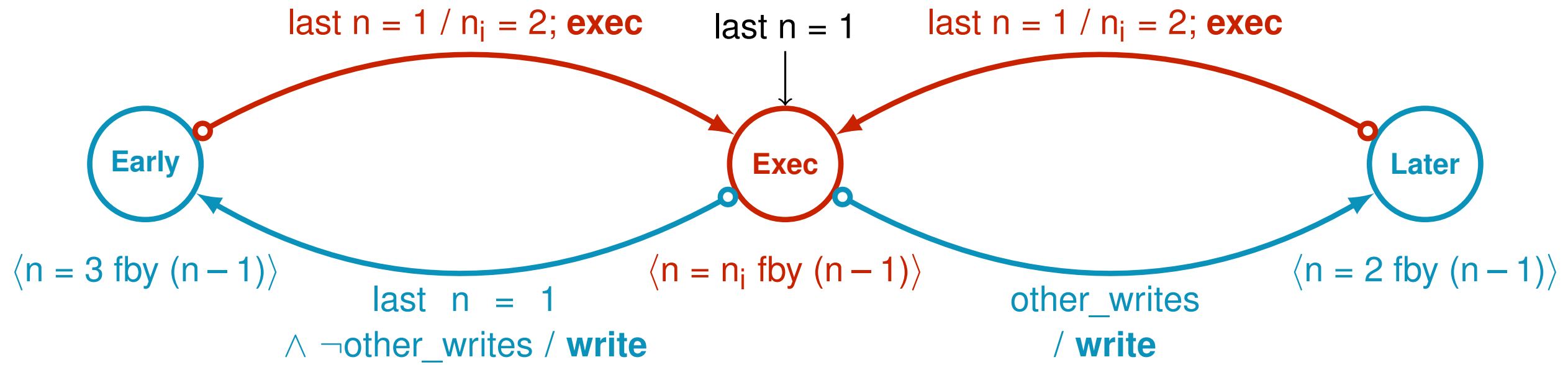


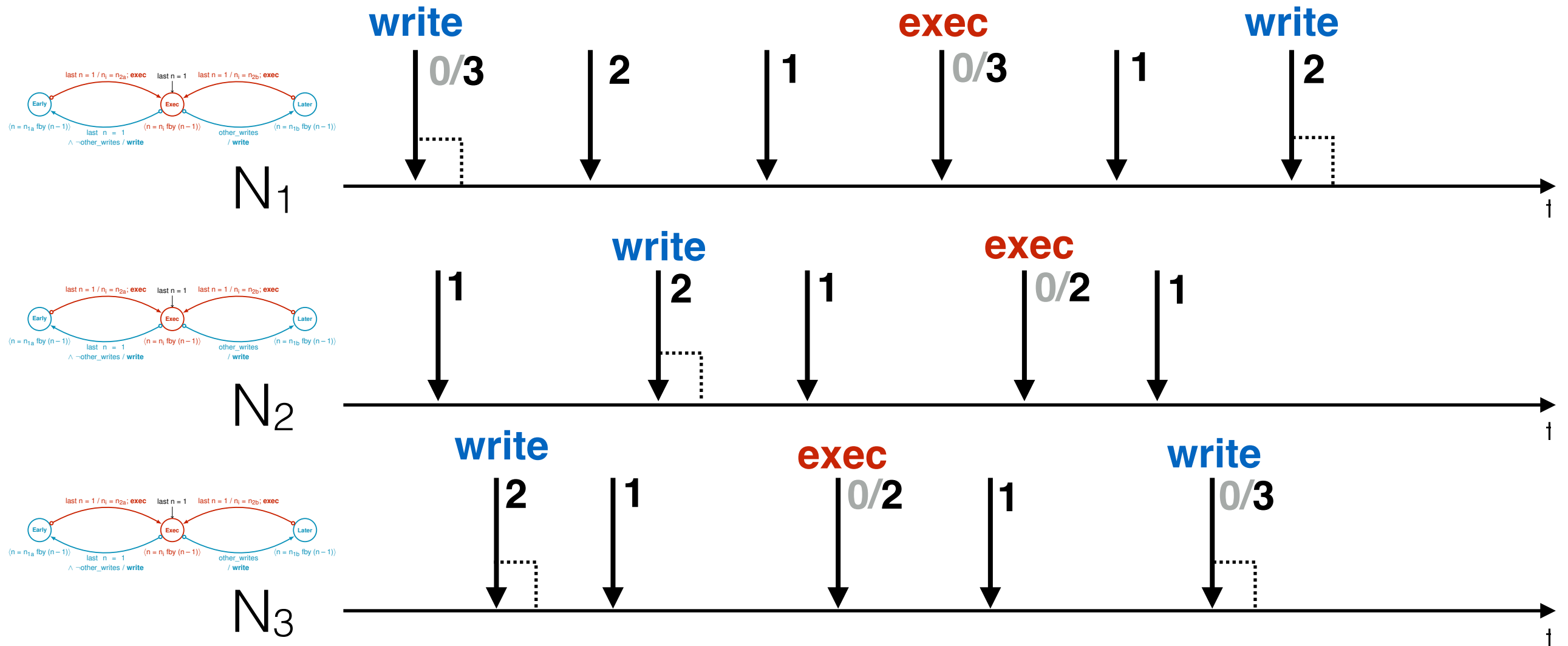
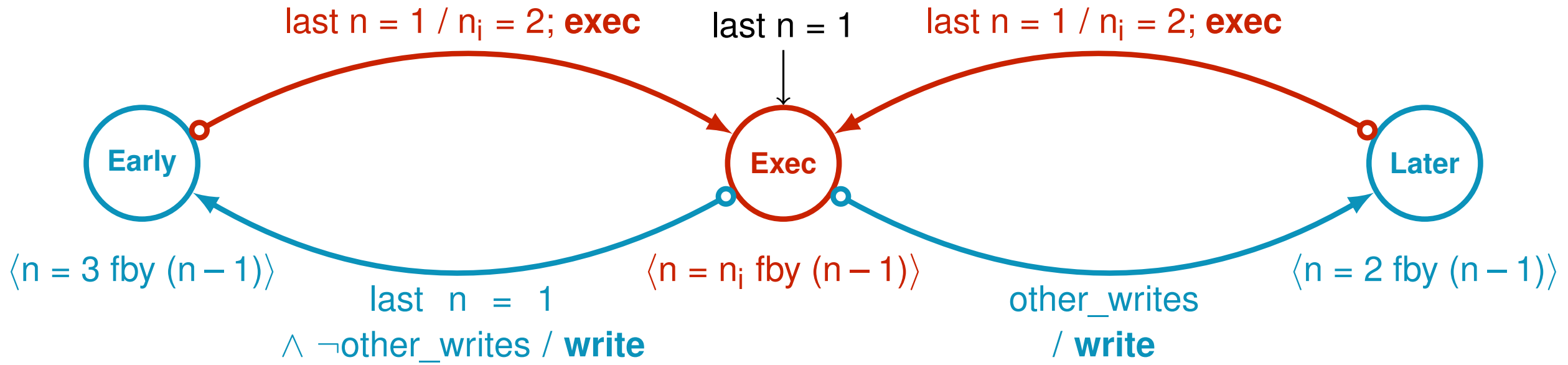


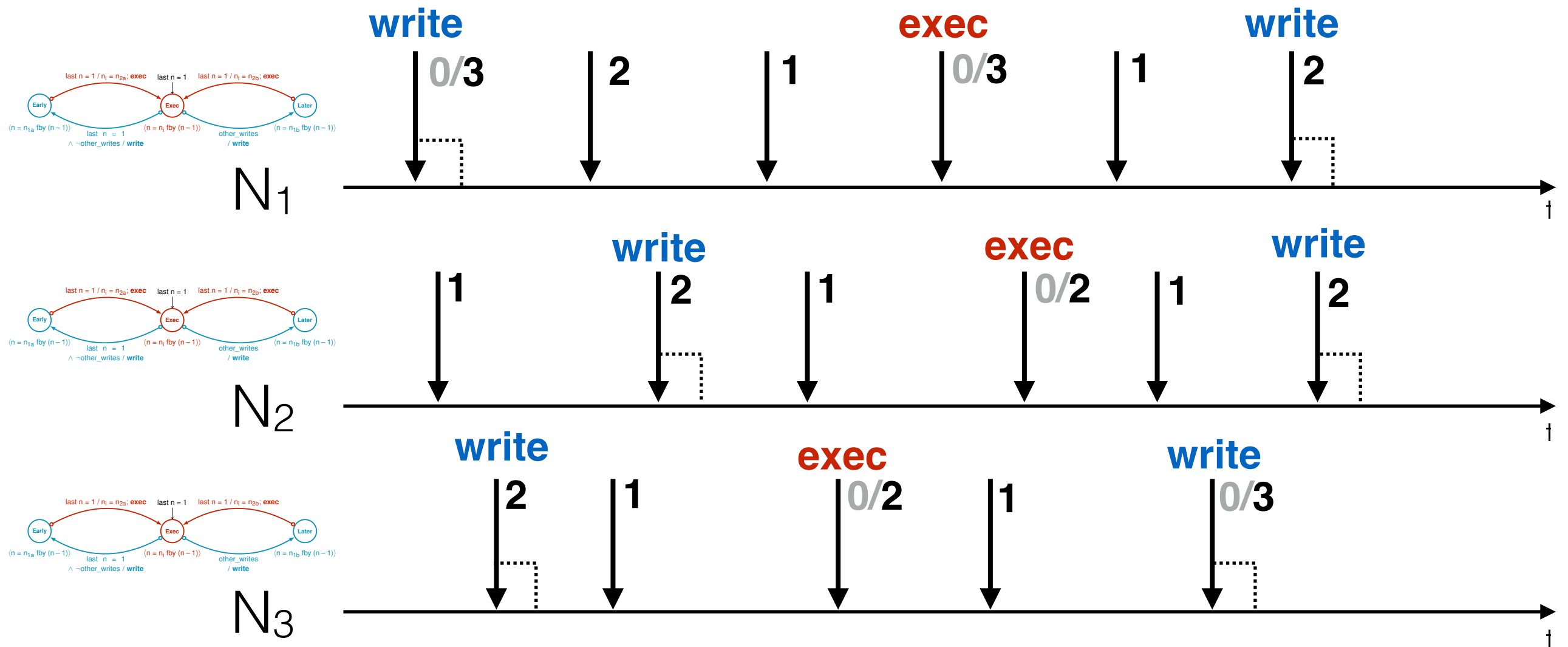
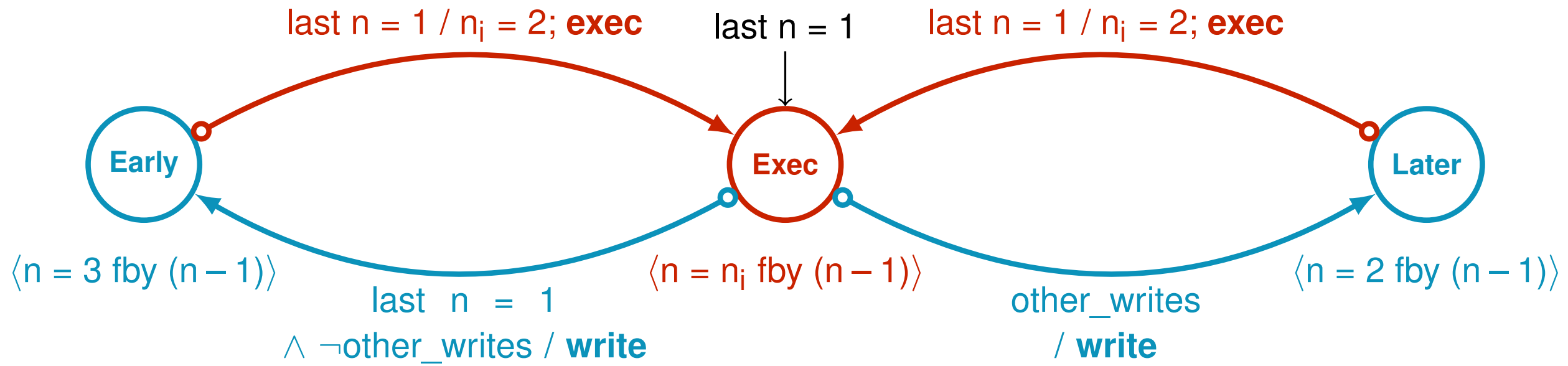


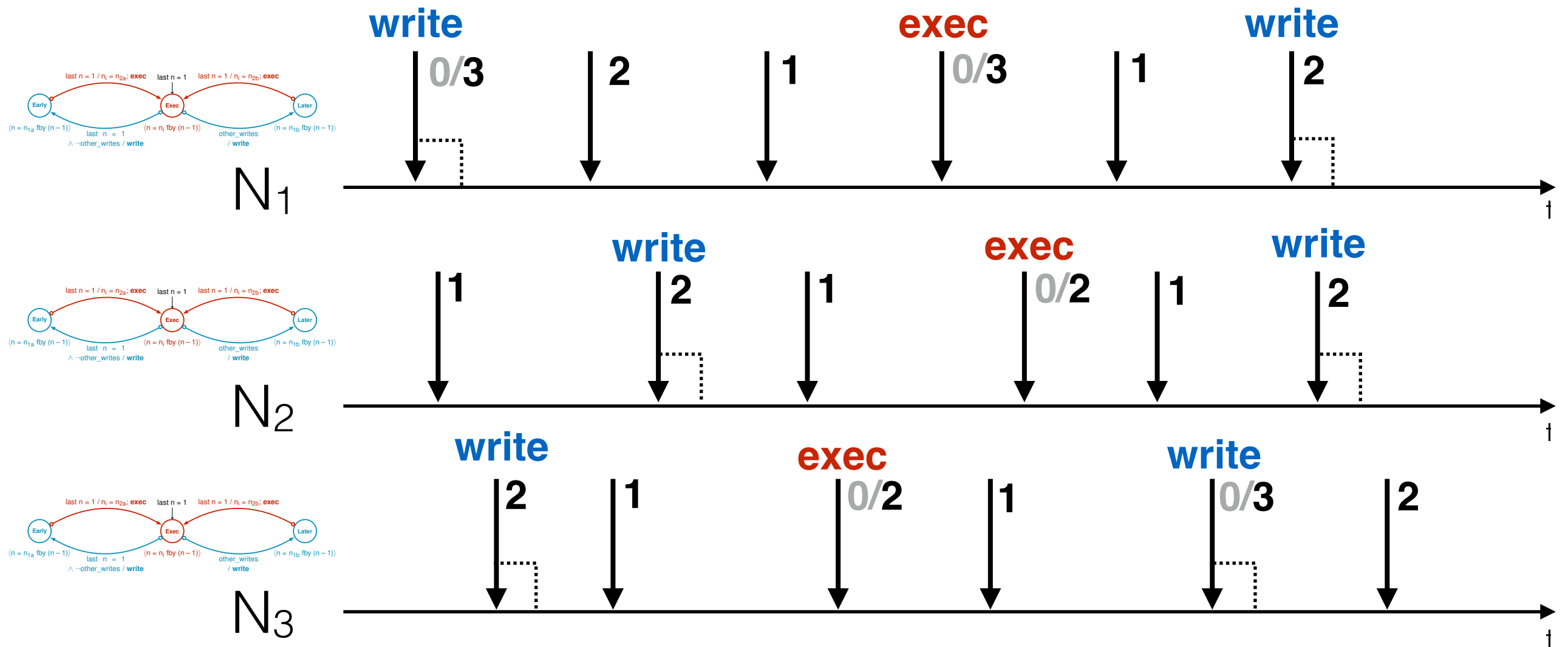
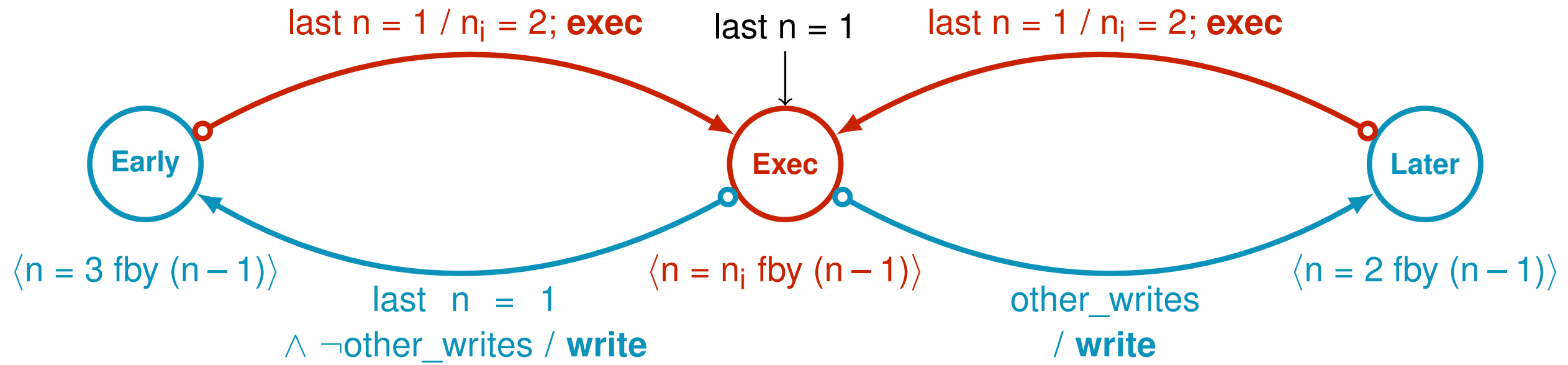


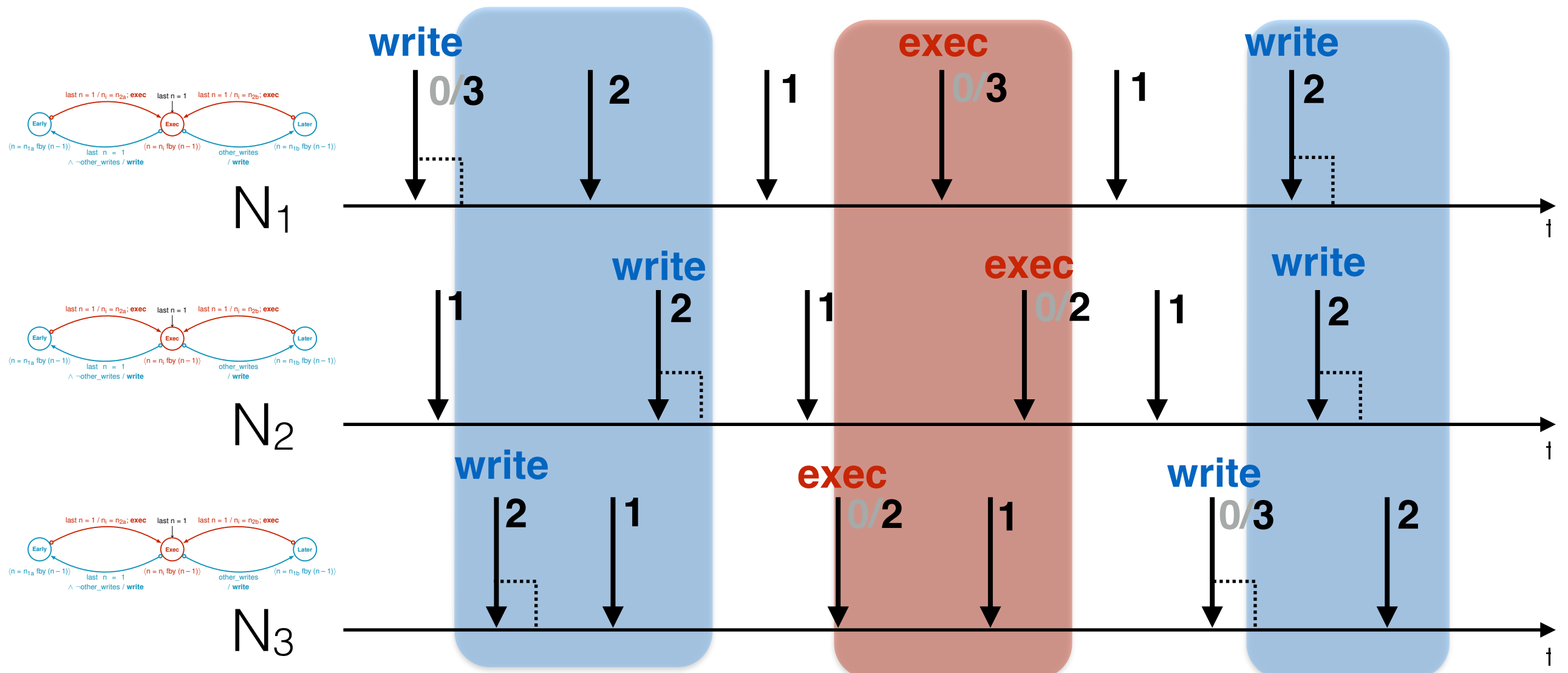
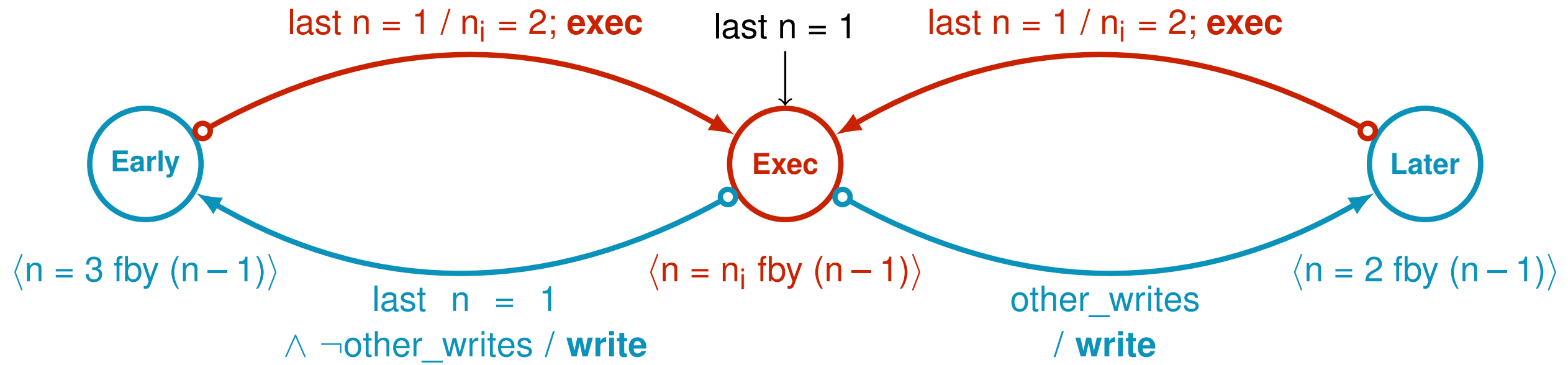
















Demo in Zélus

Comparison

	Time-Based	Back-Pressure
Flexibility		
Robustness		

Blend the two approaches [Benveniste et al. 2010]

Future Work

(i.e., my thesis)

- Zélus: language for mixing real- and discrete-time behaviors; LTTA is but one example
- We think the idea of tolerance (e.g., jitter) is important in such languages
- Continue to investigate the link between discrete- and real-time semantics (e.g., effect of skips in LTTA)

Queêêêestions?

