

2.20 (1) 以半平面界线为x轴, 则 P_x 守恒 (空间均匀性守恒)

(2) 以两点源连线为x轴, 则 L_x

坐标轴的厚点方向任意选取不会改变力学性质,

任意微小平移或转动不会引起 L_x 的改变

(3) ~~2.20~~ 在圆柱体轴小微小转动引起改变 L_z

其中z轴为圆柱对称轴

(4) 在柱对称: $V(r, \theta, z) = V(r, z)$

势能 $\frac{\partial V}{\partial \theta} + \frac{\partial V}{\partial z} \frac{h}{2\pi} = 0$

$L = T - V$ 中动能项无坐标

$\therefore \frac{\partial L}{\partial \theta} + \frac{\partial L}{\partial z} \frac{h}{2\pi} = 0$

关于 θ 的拉格朗日方程 $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = 0$ ①

关于 z $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{z}} \right) - \frac{\partial L}{\partial z} = 0$ ②

② $\times \frac{h}{2\pi}$ ① 得 $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} + \frac{h}{2\pi} \frac{\partial L}{\partial \dot{z}} \right) = 0$

即 $L_z + \frac{h}{2\pi} P_x$ 为守恒量

8.1 拉格朗日在 $H(p, q, t)$ 中利用 $L(p, q, t)$ 和 $\dot{q} = q(p, q, t)$ 成

$H(p, q, t) = p_i \dot{q}_i - L$, 其中 i 无坐标

柱坐标: $H = \frac{1}{2m} \left[p_r^2 + \frac{p_\theta^2}{r^2} + p_z^2 \right] + V(r, \theta, z)$

球坐标: $H = \frac{1}{2m} \left[p_r^2 + \frac{p_\theta^2}{r^2} + \frac{p_\phi^2}{r^2 \sin^2 \theta} \right] + V(r, \theta, \phi)$

$$+ \frac{R^2}{2b^2 \sin^2 \theta} (V \sin \theta)^2$$

$$r'(\theta)$$

$$+ \frac{1}{2} m r'^2 - m g r \sin \theta$$

以 θ 为广义坐标, $r = r_0 - \frac{m g \sin \theta}{k}$

$$\text{补充题: } I \alpha = \int_0^{\Delta t} Q \alpha dt = \int_0^{\Delta t} \frac{\partial}{\partial t} \frac{\partial T}{\partial \alpha} dt$$

$$= \int_0^{\Delta t} \left(\frac{1}{2} m \dot{r}^2 + \frac{1}{2} m r^2 \dot{\theta}^2 \right) d\theta$$

$$\dot{r} = \left(r_0 - \frac{m g \sin \theta}{k} \right) = - \frac{m g \cos \theta}{k} \quad \dot{\theta} = \omega$$

$$= \int_0^{\frac{\pi}{2}} \left[\frac{m^2 g^2 \cos^2 \theta}{2k^2} + \frac{\partial}{\partial \theta} \frac{m \left(r_0 - \frac{m g \sin \theta}{k} \right)^2 \omega^2}{2} \right] d\theta$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{m^2 g^2 \sin \theta \cos \theta}{2k^2} + \frac{m^2 \omega^2 g \sin \theta r_0}{k^2} + \frac{\omega^2 \sin^3 \theta g \cos \theta}{k^2} \right) d\theta$$

$$= \frac{(m^2 - 1) m g^2}{2k^2} \sin \theta \cos \theta \Big|_0^{\frac{\pi}{2}} + \frac{m^2 \omega^2 g \cos \theta r_0}{k^2} \Big|_0^{\frac{\pi}{2}}$$

$$= \frac{(m^2 - 1) m g^2}{2k^2} - \frac{m^2 \omega^2 g r_0}{k^2}$$