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Study of Chemical Potential Effects on Hadron by Lattice QCD

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Contents

一。 introduction

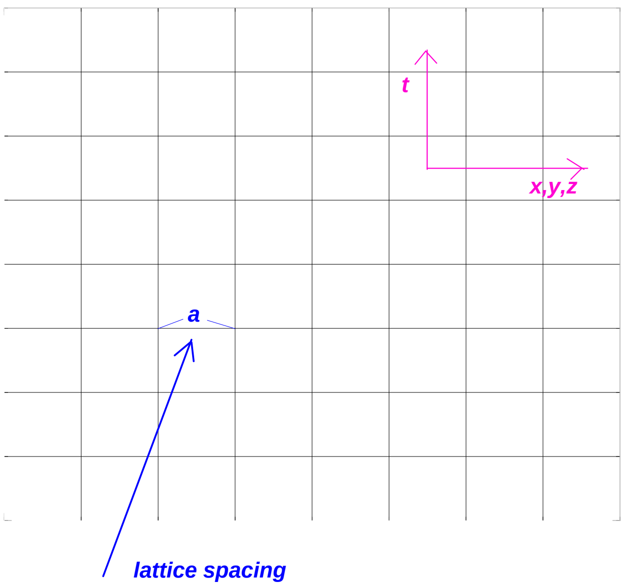
1. about Lattice QCD

(1). Minkowski space⟶Euclidean space

$$(\quad x^0 \longrightarrow \textbf{-i}x^4 \quad x^4 \in \mathbf{R} \quad)$$

$$\begin{aligned} S^{(eucl.)} &= S_G^{(eucl.)} + S_F^{(eucl.)} \\ S_G^{(eucl.)} &= \frac{1}{4}\int d^4x \mathrm{Tr}\left[F_{\mu\nu}F_{\mu\nu}\right] \\ S_F^{(eucl.)} &= \int d^4x \bar{\psi}(\gamma_\mu D_\mu + M)\psi \end{aligned}$$

$$\langle O \rangle = \frac{\int dA_\mu d\bar{\psi} d\psi O e^{-S^{(eucl.)}}}{\int dA_\mu \bar{\psi} d\psi e^{-S^{(eucl.)}}}$$



$$x_\mu \longrightarrow n_\mu a$$

$$M \longrightarrow \frac{1}{a}\hat{M}$$

$$\psi(x) \longrightarrow \frac{1}{a^{\frac{3}{2}}}\hat{\psi}(n)$$

$$\int d^4x \longrightarrow a^4 \sum_n$$

$$\partial_\mu \psi(x) \longrightarrow \frac{1}{a^{\frac{5}{2}}}\hat{\partial}_\mu \hat{\psi}(n)$$

$$\hat{\partial}_\mu \hat{\psi}(n) = \frac{1}{2}\left[\hat{\psi}(n+\hat{\mu}) - \hat{\psi}(n-\hat{\mu})\right]$$

(2). gauge action and link variables

$$
\begin{aligned}
S_G^{Latt.} &= \beta \sum_P \left[1 - \frac{Tr}{2}(U_P + U_P^\dagger) \right] \qquad \beta = \frac{1}{g^2} \\
U_{\mu\nu} &= e^{iga^2 F_{\mu\nu}(n)} \\
F_{\mu\nu}(n) &= \frac{1}{a} \Big[(A_\nu(n+\hat{\mu}) - A_\nu(n)) \\
&- \ (A_\mu(n+\hat{\nu}) - A_\mu(n)) \ \Big]
\end{aligned}
$$

$$\Downarrow a = 0$$

$$S_G^{Latt.} \Longrightarrow S_G^{Cont.} = \frac{1}{4} \int d^4x F_{\mu\nu}(x) F_{\mu\nu}(x)$$

(3). fermion action

$$S_F^{Latt.}[\bar{\psi},\psi,U] = (\hat{M}+4r)\sum_n \bar{\psi}(n)\psi(n) \quad - \quad \frac{1}{2}\sum_{n,\mu}\Big[\,\bar{\psi}(n)(r-\gamma_\mu)U_\mu(n)\psi(n+\hat{\mu})$$
$$+ \quad \bar{\psi}(n+\hat{\mu})(r+\gamma_\mu)U_\mu^\dagger(n)\psi(n)\,\Big]$$

(Wilson fermion action *K. G. Wilson 1975***)**

$$\Downarrow a = 0$$

$$S_F^{Latt.} \Longrightarrow S_F^{Cont.} = \int d^4x\bar{\psi}(x)(\gamma_\mu D_\mu + M)\psi x$$

$$\Uparrow a = 0,\ r = 0, \psi(n) = T(n)\chi(n)$$
$$\Big\| \qquad (T^\dagger(n)\gamma_\mu T(n+\hat{\mu}) = \eta_\mu(n)\mathbf{1})$$

$$S_F^{Latt.} = \hat{M}\sum_n \bar{\chi}(n)\chi(n) + \frac{1}{2}\sum_{n,\mu}\eta_\mu(n)\left[\bar{\chi}_\mu(n)U_\mu(n)\chi(n+\hat{\mu}) - \bar{\chi}_\mu(n)U_\mu^\dagger(n-\hat{\mu})\chi(n-\hat{\mu})\right]$$

(staggered fermion action *H. Kluberg-Stern et. al. 1983***)**

(4). what can be studied on Lattice

$\boxed{\text{zero temperature}}$ and $\boxed{\text{finite temperature}}$

$(^{\odot}\smile^{\odot})$ **hadron mass (π, ρ)**

$(^{\odot}\smile^{\odot})$ **energy, string tension and potential**

$(^{\odot}\smile^{\odot})$ **chiral condensate**

(5). what can not be studied

$(^{\odot}\frown^{\odot})$ **$\boxed{\text{odd flavors}}$**

$(^{\odot}\frown^{\odot})$ **$\boxed{\text{finite density}}$**

$$
\begin{aligned}
S_F^{Latt.} &= \hat{M}\sum_n \bar{\chi}(n)\chi(n) \\
&+ \frac{1}{2}\sum_{n,\sigma=1}^{\sigma=3}\eta_\sigma(n)\left[\bar{\chi}_\sigma(n)U_\sigma(n)\chi(n+\hat{\sigma}) - \bar{\chi}_\sigma(n)U_\sigma^\dagger(n-\hat{\sigma})\chi(n-\hat{\sigma})\right] \\
&+ \frac{1}{2}\sum_n \eta_t(n)\left[\bar{\chi}_t(n)U_t(n)e^{a\mu}\chi(n+\hat{t}) - \bar{\chi}_t(n)U_t^\dagger(n-\hat{t})e^{-a\mu}\chi(n-\hat{t})\right]
\end{aligned}
$$

$$
\begin{aligned}
D(U,\hat{\mu})_{n,m} &= ma\delta_{n,m} + \frac{1}{2}\sum_{\sigma=x,y,z}\eta_\sigma(n)\left[U_{\hat{\sigma}}(n)\delta_{n+\hat{\sigma},m} - U_{\hat{\sigma}}^\dagger(n-\hat{\sigma})\delta_{n-\hat{\sigma},m}\right] \\
&+ \frac{1}{2}\eta_t(n)\left[U_{\hat{t}}(n)e^{\hat{\mu}}\delta_{n+\hat{t},m} - U_{\hat{t}}^\dagger(n-\hat{t})e^{-\hat{\mu}}\delta_{n-\hat{t},m}\right]
\end{aligned}
$$

$$
\langle O\rangle = \frac{\int [dU]\, O\Delta e^{-S_G}}{\int [dU]\, \Delta e^{-S_G}}
$$

$$
\Delta = \prod_{i=1}^{N_f} det\left[D(\hat{\mu}_i)\right] \quad (\ D = D^\dagger(\mu_i = 0)\ ;\ \ D \neq D^\dagger(\mu_i \neq 0)\ )
$$

$$
\langle O(U)\rangle = \frac{\langle O(U)e^{i\varphi}\rangle_{S_{eff}}}{\langle e^{i\varphi}\rangle_{S_{eff}}}
$$

2. about our study

$(^{\odot}\frown^{\odot})$ **Lattice approach to $\mu \neq 0$ is**

very difficult SU(3) Lattice QCD

(action become complex)

$(^{\odot}\smile^{\odot})$ **SU(2) accessible but still difficult**

- *N. Nakamura, Phys. Lett.* **B149**, *391 (1984)*
- *S. Hands et al., Nucl. Phys.***B557** *327 (1999)*
- *Y. Liu, et al., Non-Per. Meth. and Latt. QCD* **132** *2001*
- *S. Muroya, et al., Nucl. Phys. B(Proc. Suppl)***94**, *469 (2001)*

♡ **quenched calculation corresponds to**

different world

- *C. T. H.Davies et al., Phys. Lett.* **B256**, *68 (1991)*
- *J. B. Kogut et al., Phys. Rev.* **D51**, *1282 (1995)*
- *M.-P. Lombardo et al., Phys. Rev.* **D54**, *2303 (1996)*

♡ behavior of SU(3) Lattice QCD simulation at zero chemical potential

$$
\begin{aligned}
\chi_S &= \left(\frac{\partial}{\partial\mu_u} + \frac{\partial}{\partial\mu_d}\right)\Big(\langle n_u\rangle + \langle n_d\rangle\Big) \\
\chi_V &= \left(\frac{\partial}{\partial\mu_u} - \frac{\partial}{\partial\mu_d}\right)\Big(\langle n_u\rangle - \langle n_d\rangle\Big) \\
\chi_q &= \frac{1}{V}\langle Q^2\rangle
\end{aligned}
$$

($\langle n_{u(d)}\rangle = \frac{1}{V_s\beta}\frac{\partial}{\partial\mu_{u(d)}} lnZ$; Q : topological charge)

- *S. Gottieb et al., Phys. Rev.* **D47**,*3619 (1993);* **D55**,*6852 (1997)*
- *Y. Liu et al., Soryushiron Kenkyu (Kyoto) 103/1 A73 (2001)*
- *MENG Xiang-Fei, Y. Liu HEP and NP 1222 A30 (2006)*

idea and methods for our study

$$\left.\frac{M(\mu)}{T}\right|_{\mu} = \left.\frac{M}{T}\right|_{\mu=0} + \left(\frac{\mu}{T}\right) \left.\frac{\partial M}{\partial \mu}\right|_{\mu=0} + \left(\frac{\mu}{T}\right)^2 \frac{T}{2} \left.\frac{\partial^2 M}{\partial \mu^2}\right|_{\mu=0} + O\left[\left(\frac{\mu}{T}\right)^3\right]$$

$$\left.\frac{\langle\bar{\psi}\psi\rangle(\mu)}{T^3}\right|_{\mu} = \left.\frac{\langle\bar{\psi}\psi\rangle}{T^3}\right|_{\mu=0} + \left(\frac{\mu}{T}\right) \frac{1}{T^2} \left.\frac{\partial\langle\bar{\psi}\psi\rangle}{\partial \mu}\right|_{\mu=0} + \left(\frac{\mu}{T}\right)^2 \frac{1}{2T} \left.\frac{\partial^2\langle\bar{\psi}\psi\rangle}{\partial \mu^2}\right|_{\mu=0} + O\left[\left(\frac{\mu}{T}\right)^3\right]$$

$$\boxed{\left.\frac{\partial M}{\partial \mu}\right|_{\mu=0}, \left.\frac{\partial^2 M}{\partial \mu^2}\right|_{\mu=0}; \left.\frac{\partial \gamma}{\partial \mu}\right|_{\mu=0}, \left.\frac{\partial^2 \gamma}{\partial \mu^2}\right|_{\mu=0}; \left.\frac{\partial\langle\bar{\psi}\psi\rangle}{\partial \mu}\right|_{\mu=0}, \left.\frac{\partial^2\langle\bar{\psi}\psi\rangle}{\partial \mu^2}\right|_{\mu=0}}$$

- ∗ *QCD-TAQRO collab. Nucl. Phys. B* **(Proc.Suppl.) 63**, *460 (1998)*
- ∗ *QCD-TAQRO collab. Nucl. Phys. B* **(Proc.Suppl.) 73**, *477 (1999)*
- ∗ *QCD-TAQRO collab. hep-lat/0110223 (Lattice 2001)*
- ∗ *QCD-TAQRO collab. Phys. Rev. D* **65**, *0145XX. (2002)*
- ∗ *QCD-TAQRO collab. Nucl. Phys. A* **698**, *395 (2002)*

二。 formulae

suppose that a hadronic correlator dominated

by hadronic pole :

$$C(x) = \sum_{y,z,t} \langle H(x,y,z,t) H(0,0,0,0)^{\dagger} \rangle = A\left(e^{-\hat{M}\hat{x}} + e^{-\hat{M}(L_x - \hat{x})}\right)$$

where $\hat{M} = aM$**,** $\hat{x} = \frac{x}{a}$**,** $A = \frac{\hat{\gamma}}{2\hat{M}}$ **(**γ **is hadron coupling)**

the first derivative with respect to chemical potential

$$C(x)^{-1}\frac{\partial C(x)}{\partial \hat{\mu}} = A^{-1}\frac{\partial A}{\partial \hat{\mu}} + \frac{\partial \hat{M}}{\partial \hat{\mu}}\left[\left(\hat{x} - \frac{L_x}{2}\right)\tanh\left\{\hat{M}\left(\hat{x} - \frac{L_x}{2}\right)\right\} - \frac{L_x}{2}\right]$$

and second derivative

$$\begin{aligned} C(x)^{-1} \quad \frac{\partial^2 C(x)}{\partial \hat{\mu}^2} &= A^{-1}\frac{\partial^2 A}{\partial \hat{\mu}^2} \\ + \quad & \left(2A^{-1}\frac{\partial A}{\partial \hat{\mu}}\frac{\partial \hat{M}}{\partial \hat{\mu}} + \frac{\partial^2 \hat{M}}{\partial \hat{\mu}^2}\right)\left[\left(\hat{x} - \frac{L_x}{2}\right)\tanh\left\{\hat{M}\left(\hat{x} - \frac{L_x}{2}\right)\right\} - \frac{L_x}{2}\right] \\ + \quad & \left(\frac{\partial \hat{M}}{\partial \hat{\mu}}\right)^2\left[\left(\hat{x} - \frac{L_x}{2}\right)^2 + \frac{L_x^2}{4} - L_x\left(\hat{x} - \frac{L_x}{2}\right)\tanh\left\{\hat{M}\left(\hat{x} - \frac{L_x}{2}\right)\right\}\right] \end{aligned}$$

on Lattice, the hadron correlator :

$$\langle H(n)H(0)^{\dagger}\rangle = \langle G\rangle$$

where G **is the meson propagator part**

$$G = \mathrm{Tr}\left[g\left(\hat{\mu}_u\right)_{n:0}\Gamma g\left(\hat{\mu}_d\right)_{0:n}\Gamma^{\dagger}\right]$$

$g(\hat{\mu})$ **is the quark propagator at finite chemical potential, and** Γ **is the Dirac matrix which specifies the spin of the meson. The quark propagator is related to the Dirac operator** $D[U;\hat{\mu}]$ **in the background gauge field configuration** U **as**

$$g(\hat{\mu}) = D(\hat{\mu})^{-1}$$

$$\langle O \rangle = \frac{\int [dU]\, O \Delta e^{-S_G}}{\int [dU]\, \Delta e^{-S_G}}$$

where S_G is the gluonic action and Δ is the fermion determinant,

$$\Delta = \prod_{i=1}^{N_f} \det\Big(D\left(\hat{\mu}_i\right)\Big)$$

for two light flavors fermion, i.e., $N_f = 2$. Δ can be written as

$$\Delta = \det\Big(D(\hat{\mu}_u)\Big) \det\Big(D(\hat{\mu}_d)\Big)$$

the first and the second derivatives are

$$\frac{\partial}{\partial\hat{\mu}} \Big\langle H(n)H(0)^{\dagger} \Big\rangle = \Big\langle \dot{G} + G\frac{\dot{\Delta}}{\Delta} \Big\rangle - \Big\langle G \Big\rangle \Big\langle \frac{\dot{\Delta}}{\Delta} \Big\rangle$$

and

$$\begin{aligned} \frac{\partial^2}{\partial\hat{\mu}^2} \Big\langle H(n)H(0)^{\dagger} \Big\rangle &= \Big\langle \ddot{G} + 2\dot{G}\frac{\dot{\Delta}}{\Delta} + G\frac{\ddot{\Delta}}{\Delta} \Big\rangle - 2\Big\langle \dot{G} + G\frac{\dot{\Delta}}{\Delta} \Big\rangle \Big\langle \frac{\dot{\Delta}}{\Delta} \Big\rangle \\ &- \langle G \rangle \left[\Big\langle \frac{\ddot{\Delta}}{\Delta} \Big\rangle - 2\left\{ \Big\langle \frac{\dot{\Delta}}{\Delta} \Big\rangle \right\}^2 \right] \end{aligned}$$

for *Isoscalar chemical potential channel* **channel**

$$\boxed{\hat{\mu}_S = \hat{\mu}_u = \hat{\mu}_d}$$

$$\frac{\partial}{\partial \hat{\mu}} \mathrm{Re} \left\langle H(n) H(0)^\dagger \right\rangle = 0$$

$$\begin{aligned}
\frac{\partial^2}{\partial \hat{\mu}^2} \quad \mathrm{Re} \quad & \left\langle H(n) H(0)^\dagger \right\rangle = 4\mathrm{Re} \left\langle \mathrm{Tr} \left[\left(g \dot{D} g \dot{D} g \right)_{n:0} \Gamma \gamma_5 g^\dagger_{n:0} \gamma_5 \Gamma^\dagger \right] \right\rangle \\
- \quad & 2\mathrm{Re} \left\langle \mathrm{Tr} \left[\left(g \ddot{D} g \right)_{n:0} \Gamma \gamma_5 g^\dagger_{n:0} \gamma_5 \Gamma^\dagger \right] \right\rangle \\
- \quad & 2\mathrm{Re} \left\langle \mathrm{Tr} \left[\left(g \dot{D} g \right)_{n:0} \Gamma \gamma_5 \left(g \dot{D} g \right)^\dagger_{n:0} \gamma_5 \Gamma^\dagger \right] \right\rangle \\
+ \quad & 8 \left\langle \mathrm{Im}\mathrm{Tr} \left[\left(g \dot{D} g \right)_{n:0} \Gamma \gamma_5 g^\dagger_{n:0} \gamma_5 \Gamma^\dagger \right] \mathrm{Im}\mathrm{Tr} \left[\dot{D} g \right] \right\rangle \\
+ \quad & 2\mathrm{Re} \left\{ \left\langle \mathrm{Tr} \left[g_{n:0} \Gamma \gamma_5 g^\dagger_{n:0} \gamma_5 \Gamma^\dagger \right] \left(\mathrm{Tr} \left[\ddot{D} g \right] - \mathrm{Tr} \left[\dot{D} g \dot{D} g \right] + 2\mathrm{Tr} \left[\dot{D} g \right]^2 \right) \right\rangle \right. \\
- \quad & \left. \left\langle \mathrm{Tr} \left[g_{n:0} \Gamma \gamma_5 g^\dagger_{n:0} \gamma_5 \Gamma^\dagger \right] \right\rangle \left\langle \mathrm{Tr} \left[\ddot{D} g \right] - \mathrm{Tr} \left[\dot{D} g \dot{D} g \right] + 2\mathrm{Tr} \left[\dot{D} g \right]^2 \right\rangle \right\}
\end{aligned}$$

for *Isovector chemical potential* **channel**

$$\boxed{\hat{\mu}_V = \hat{\mu}_u = -\hat{\mu}_d}$$

$$\frac{\partial}{\partial\hat{\mu}}\mathrm{Re}\left\langle\, H(n)H(0)^{\dagger}\right\rangle = -2\mathrm{Re}\mathrm{Tr}\left[\left(g\dot{D}g\right)_{n:0}\Gamma\gamma_5 g^{\dagger}_{n:0}\gamma_5\Gamma^{\dagger}\right]$$

$$\begin{aligned}
\frac{\partial^2}{\partial\hat{\mu}^2}\mathrm{Re}\left\langle\, H(n)H(0)^{\dagger}\right\rangle \;&=\; 4\mathrm{Re}\left\langle\mathrm{Tr}\left[\left(g\dot{D}g\dot{D}g\right)_{n:0}\Gamma\gamma_5 g^{\dagger}_{n:0}\gamma_5\Gamma^{\dagger}\right]\right\rangle \\
&-\; 2\mathrm{Re}\left\langle\mathrm{Tr}\left[\left(g\ddot{D}g\right)_{n:0}\Gamma\gamma_5 g^{\dagger}_{n:0}\gamma_5\Gamma^{\dagger}\right]\right\rangle \\
&+\; 2\mathrm{Re}\left\langle\mathrm{Tr}\left[\left(g\dot{D}g\right)_{n:0}\Gamma\gamma_5\left(g\dot{D}g\right)^{\dagger}_{n:0}\gamma_5\Gamma^{\dagger}\right]\right\rangle \\
&+\; 2\mathrm{Re}\left\{\left\langle\mathrm{Tr}\left[g_{n:0}\Gamma\gamma_5 g^{\dagger}_{n:0}\gamma_5\Gamma^{\dagger}\right]\left(\mathrm{Tr}\left[\ddot{D}g\right]-\mathrm{Tr}\left[\dot{D}g\dot{D}g\right]\right)\right\rangle\right. \\
&-\; \left.\left\langle\,\mathrm{Tr}\left[g_{n:0}\Gamma\gamma_5 g^{\dagger}_{n:0}\gamma_5\Gamma^{\dagger}\right]\right\rangle\left\langle\,\mathrm{Tr}\left[\ddot{D}g\right]-\mathrm{Tr}[\dot{D}g\dot{D}g]\right\rangle\right\}
\end{aligned}$$

chiral condensate on lattice

$$\left\langle \bar{\psi}\psi \right\rangle = \mathrm{Re}\left\langle G \right\rangle$$

for *Isoscalar chemical potential* **channel**

$$G = \frac{1}{2}\left\{ \mathrm{Tr}\left[g(\hat{\mu}_u)\right] + \mathrm{Tr}[g(\hat{\mu}_d)]\right\} = \mathrm{Tr}\left[g(\hat{\mu}_S)\right]$$

$$\frac{\partial}{\partial \hat{\mu}}\mathrm{Re}\left\langle G \right\rangle = 0$$

$$\begin{aligned} & \frac{\partial^2}{\partial \hat{\mu}^2}\mathrm{Re}\left\langle G \right\rangle = \\ & 2\mathrm{Re}\left\langle \mathrm{Tr}\left[g\dot{D}g\dot{D}g)\right]\right\rangle - \mathrm{Re}\left\langle \mathrm{Tr}[g\ddot{D}g]\right\rangle \\ + \quad & 4\left\langle \mathrm{ImTr}\left[g\dot{D}g\right]\mathrm{ImTr}\left[\dot{D}g\right]\right\rangle \\ + \quad & \left\langle \mathrm{ReTr}\left[g\right]\circ\left(2\mathrm{ReTr}\left[\ddot{D}g\right] - 2\mathrm{ReTr}\left[\dot{D}g\dot{D}g)\right] - 4\left\{\mathrm{ImTr}\left[\dot{D}g\right]\right\}^2\right)\right\rangle_{cc} \end{aligned}$$

for *Isovector chemical potential* **channel**

$$G = \frac{1}{2}\left\{ \mathrm{Tr}\left[g(\hat{\mu}_u)\right] + \mathrm{Tr}\left[g(\hat{\mu}_d)\right]\right\} = \frac{1}{2}\left\{ \mathrm{Tr}\left[g\left(\hat{\mu}\right)\right] + \mathrm{Tr}\left[g\left(-\hat{\mu}\right)\right]\right\}$$

$$\frac{\partial}{\partial \hat{\mu}}\mathrm{Re}\left\langle G\right\rangle = 0$$

$$\begin{aligned}\frac{\partial^2}{\partial \hat{\mu}^2}\mathrm{Re}\left\langle G\right\rangle &= 2\mathrm{Re}\left\langle \mathrm{Tr}\left[g\dot{D}g\dot{D}g\right]\right\rangle - \mathrm{Re}\left\langle \mathrm{Tr}\left[g\ddot{D}g\right]\right\rangle \\ &+ \left\langle \mathrm{ReTr}\left[g\right] \circ \left\{ 2\mathrm{ReTr}\left[\ddot{D}g\right] - 2\mathrm{ReTr}\left[\dot{D}g\dot{D}g\right]\right\}\right\rangle_{cc}\end{aligned}$$

$$\langle A \circ B\rangle_{cc} = \langle AB\rangle - \langle A\rangle\langle B\rangle$$

四。simulation results

Simulation parameters (Lattice size=$16 \times 8^2 \times 4$)

ma	β	$\#conf.$	T/T_C
0.0125	**5.26**	**300**	**0.985**
	5.34	**300**	**1.098**
0.0170	**5.26**	**300**	**0.977**
	5.34	**300**	**1.089**
0.0250	**5.20**	**300**	**0.888**
	5.26	**300**	**0.963**
	5.28	**300**	**0.990**
	5.29	**300**	**1.003**
	5.30	**300**	**1.017**
	5.32	**300**	**1.045**
	5.34	**300**	**1.074**
	5.36	**300**	**1.104**
	5.40	**300**	**1.165**

For two light flavors of staggered fermion, the critical coupling β_c is carried out at $N_t = 4$, i.e.,

$$\beta_c = 5.271 \text{ for } ma = 0.0125,$$

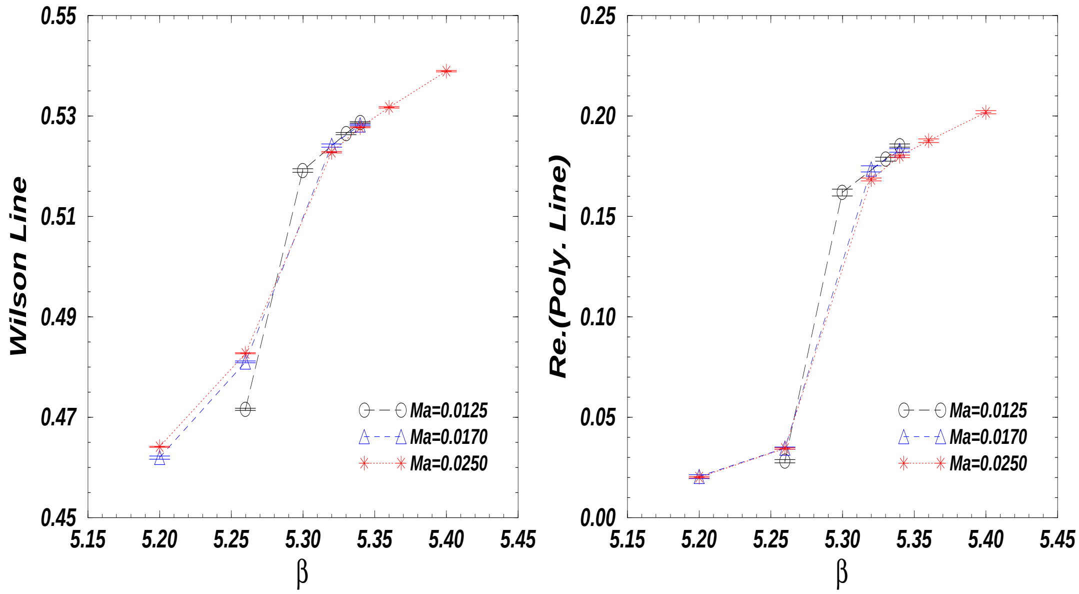
$$\text{and}$$

$$\beta_c = 5.288 \text{ for } ma = 0.025$$

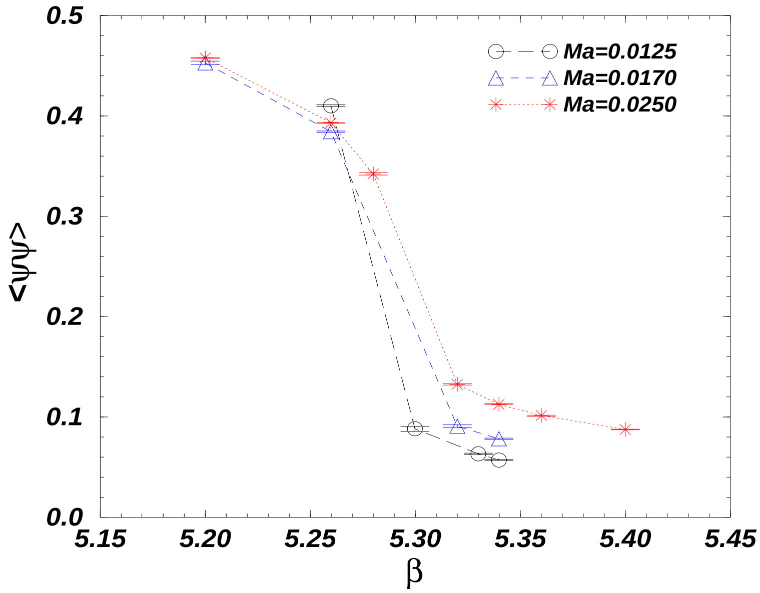
Our simulations are carried out with the R-algorithm. The time step of molecular dynamics is taken as $\delta = 0.01$.

To evaluate the trace, 'Tr', the Z_2 noise method is used, and the number of noise vectors is two hundred.

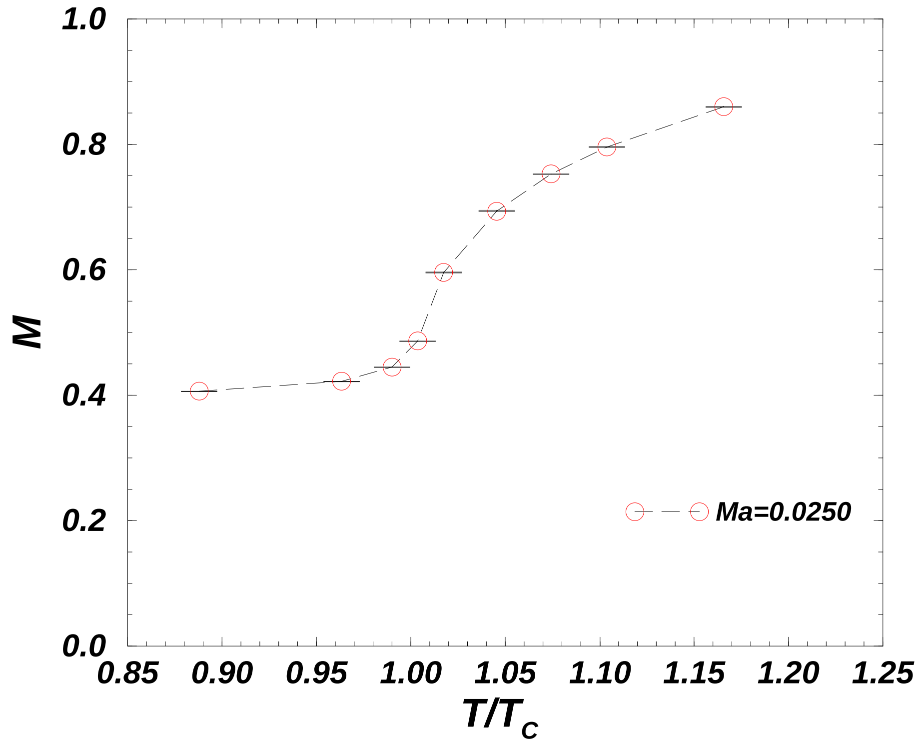
Hadronic correlators are measured by using the corner-type wall source after**Coulomb gauge fixing** in each x-plane, and we choose three quark masses, $ma = 0.0125$, 0.017, 0.025.



Wilson Line
β
Ma=0.0125
Ma=0.0170
Ma=0.0250
Re.(Poly. Line)

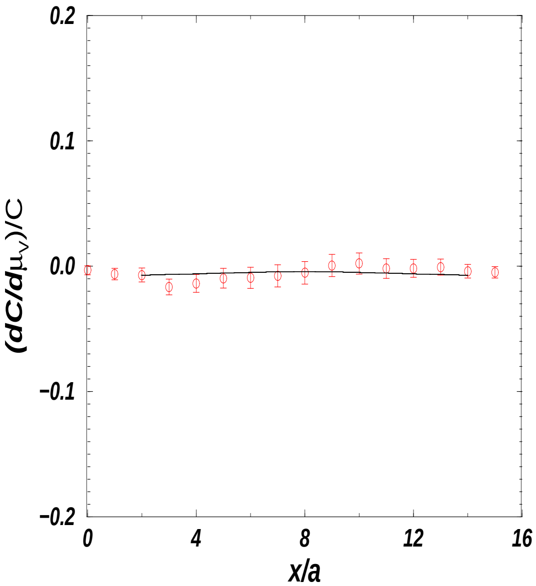


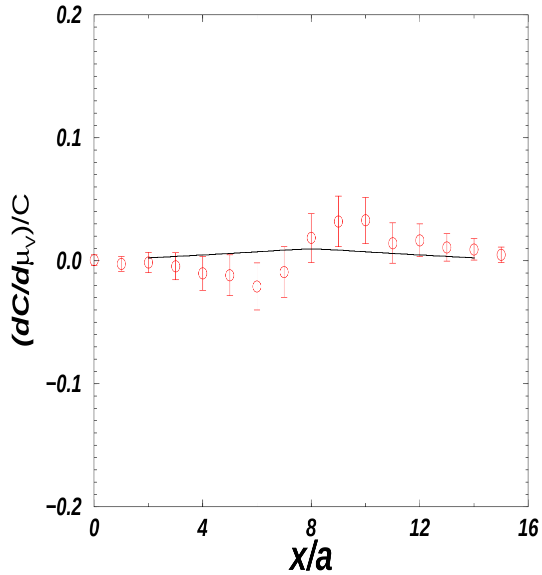
Ma=0.0125
Ma=0.0170
Ma=0.0250
<ψψ>
β
0.5
0.4
0.3
0.2
0.1
0.0
5.15
5.20
5.25
5.30
5.35
5.40
5.45



screening mass of the pseudoscalar meson $(ma = 0.025)$

$$\left.\frac{M(\mu)}{T}\right|_{\mu} = \left.\frac{M}{T}\right|_{\mu=0} + \left(\frac{\mu}{T}\right) \left.\frac{\partial M}{\partial \mu}\right|_{\mu=0} + \left(\frac{\mu}{T}\right)^2 \frac{T}{2} \left.\frac{\partial^2 M}{\partial \mu^2}\right|_{\mu=0} + O\left[\left(\frac{\mu}{T}\right)^3\right]$$





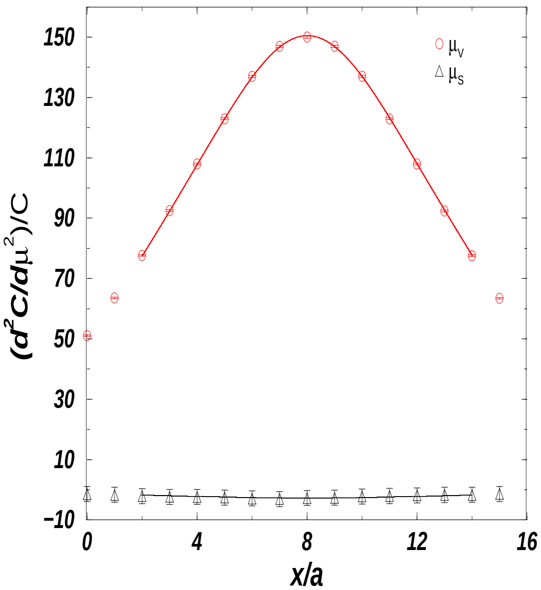
the first order response of the pseudoscalar meson correlator at $\beta = 5.26$ **and** $\beta = 5.34$ $(ma = 0.0170)$

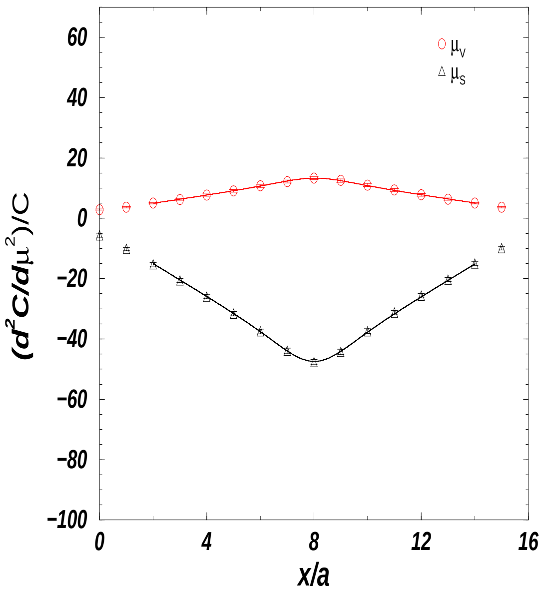
$$C(x)^{-1}\frac{\partial C(x)}{\partial\hat{\mu}} = A^{-1}\frac{\partial A}{\partial\hat{\mu}} + \frac{\partial\hat{M}}{\partial\hat{\mu}}\left[\left(\hat{x}-\frac{L_x}{2}\right)\tanh\left\{\hat{M}\left(\hat{x}-\frac{L_x}{2}\right)\right\}-\frac{L_x}{2}\right]$$

$$\frac{\partial}{\partial\hat{\mu}_S}\mathrm{Re}\left\langle\, H(n)H(0)^\dagger\right\rangle = 0$$

$$\frac{\partial}{\partial\hat{\mu}_V}\mathrm{Re}\left\langle\, H(n)H(0)^\dagger\right\rangle = -2\mathrm{ReTr}\left[\left(g\dot{D}g\right)_{n:0}\Gamma\gamma_5 g^\dagger_{n:0}\gamma_5\Gamma^\dagger\right]$$

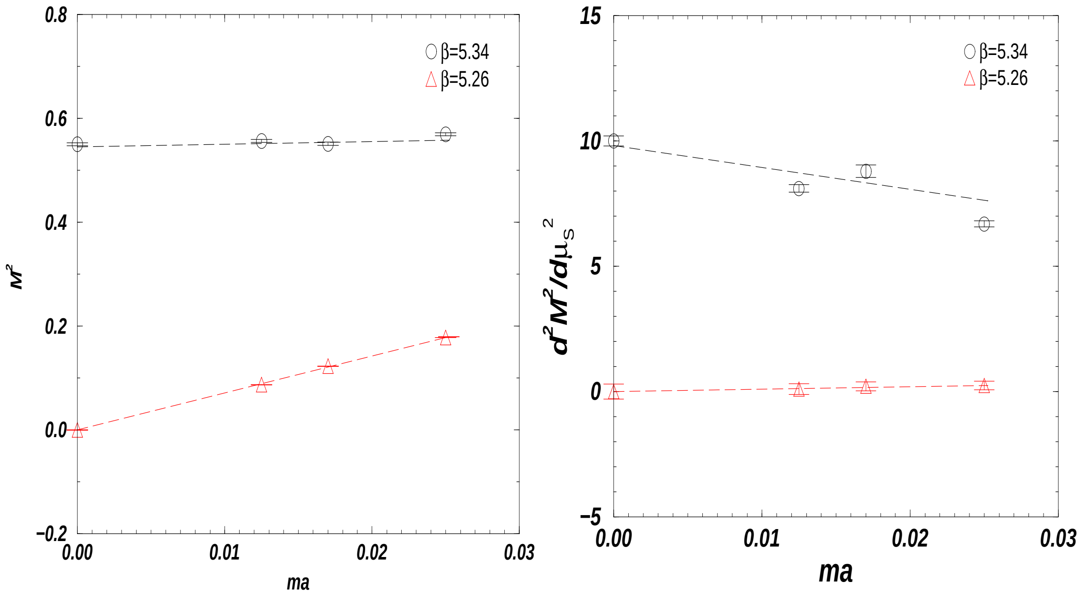
$$\boxed{\mu_S = \mu_u = \mu_d\ ,\ \mu_V = \mu_u = -\mu_d}$$





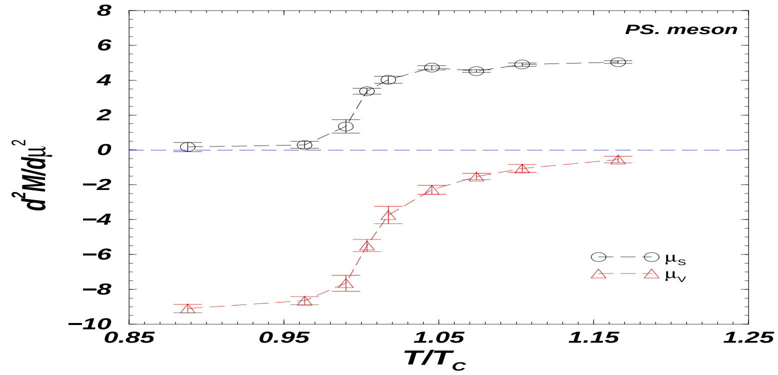
the second order response of the pseudoscalar meson correlator at $\beta = 5.26$ **and** $\beta = 5.34$ $(ma = 0.0125)$

$$
\begin{aligned}
& C(x)^{-1}\frac{\partial^2 C(x)}{\partial\hat{\mu}^2} = A^{-1}\frac{\partial^2 A}{\partial\hat{\mu}^2} \\
+ \quad & \left(2A^{-1}\frac{\partial A}{\partial\hat{\mu}}\frac{\partial\hat{M}}{\partial\hat{\mu}} + \frac{\partial^2\hat{M}}{\partial\hat{\mu}^2}\right)\left[\left(\hat{x}-\frac{L_x}{2}\right)\tanh\left\{\hat{M}\left(\hat{x}-\frac{L_x}{2}\right)\right\} - \frac{L_x}{2}\right] \\
+ \quad & \left(\frac{\partial\hat{M}}{\partial\hat{\mu}}\right)^2\left[\left(\hat{x}-\frac{L_x}{2}\right)^2 + \frac{L_x^2}{4} - L_x\left(\hat{x}-\frac{L_x}{2}\right)\tanh\left\{\hat{M}\left(\hat{x}-\frac{L_x}{2}\right)\right\}\right]
\end{aligned}
$$



$\hat{M}^2$ **and** $\frac{\partial^2 \hat{M}^2}{\partial \hat{\mu}_S^2}$ **of the pseudoscalar meson versus quark mass** ma $\beta = 5.26$ **and** $\beta = 5.34$**, the symbols plotted at** $ma = 0.0$ **are the values extrapolated to** $m = 0.0$

$$C(x) = \sum_{y,z,t} \langle H(x,y,z,t) H(0,0,0,0)^\dagger \rangle = A \left(e^{-\hat{M}\hat{x}} + e^{-\hat{M}(L_x - \hat{x})} \right)$$



the second order response of pseudoscalar meson at $ma = 0.025$

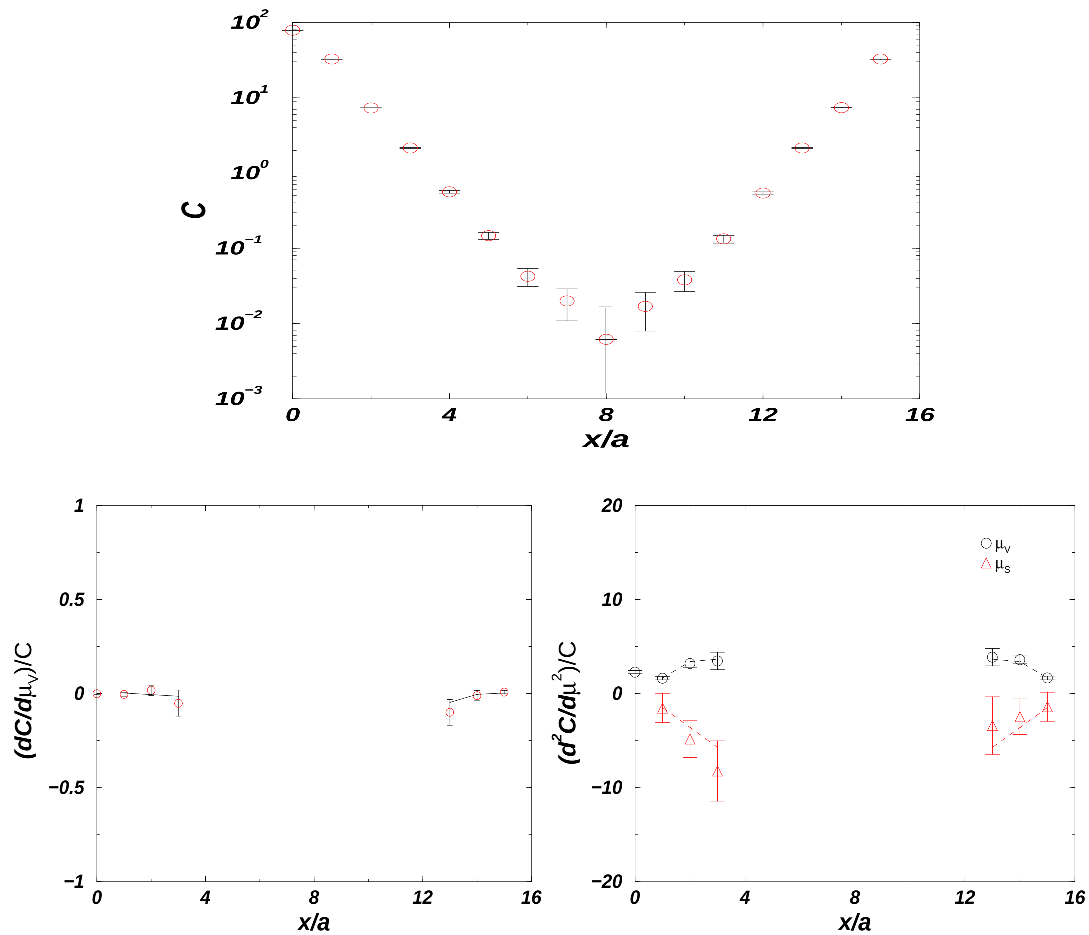
$$
\begin{aligned}
C(x)^{-1} \quad & \frac{\partial^2 C(x)}{\partial \hat{\mu}^2} = A^{-1}\frac{\partial^2 A}{\partial \hat{\mu}^2} \\
+ \quad & \left(2A^{-1}\frac{\partial A}{\partial \hat{\mu}}\frac{\partial \hat{M}}{\partial \hat{\mu}} + \frac{\partial^2 \hat{M}}{\partial \hat{\mu}^2}\right)\left[\left(\hat{x} - \frac{L_x}{2}\right)\tanh\left\{\hat{M}\left(\hat{x} - \frac{L_x}{2}\right)\right\} - \frac{L_x}{2}\right] \\
+ \quad & \left(\frac{\partial \hat{M}}{\partial \hat{\mu}}\right)^2\left[\left(\hat{x} - \frac{L_x}{2}\right)^2 + \frac{L_x^2}{4} - L_x\left(\hat{x} - \frac{L_x}{2}\right)\tanh\left\{\hat{M}\left(\hat{x} - \frac{L_x}{2}\right)\right\}\right]
\end{aligned}
$$

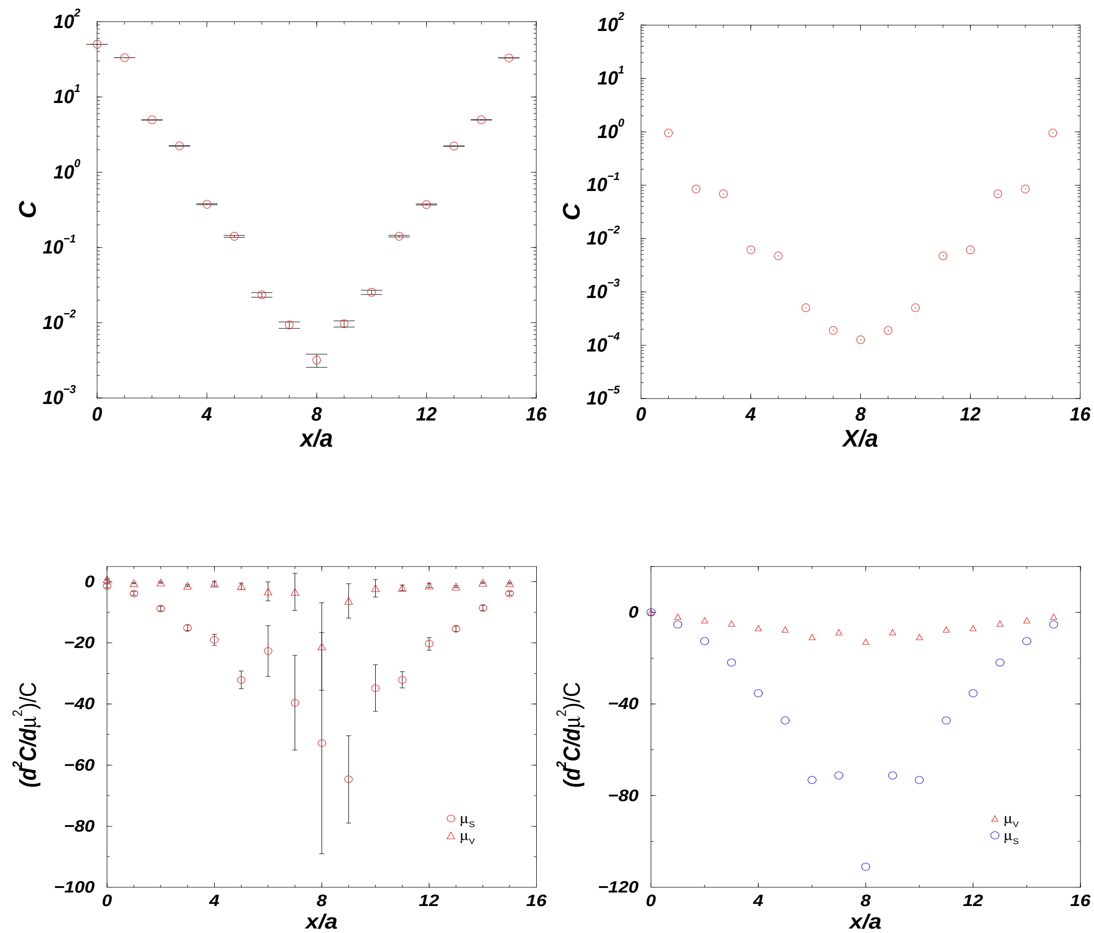
at $\beta = 5.26$, $ma = 0.0170$ **in isovector chemical potential channel**

$$
\begin{aligned}
\left.\frac{M(\mu_V)}{T}\right|_{\mu_V} &= (1.4024 \pm 0.0008) + (-0.0005 \pm 0.0010)\left(\frac{\mu_V}{T}\right) \\
&- (1.31 \pm 0.04)\left(\frac{\mu_V}{T}\right)^2 + O\left[\left(\frac{\mu_V}{T}\right)^3\right]
\end{aligned}
$$

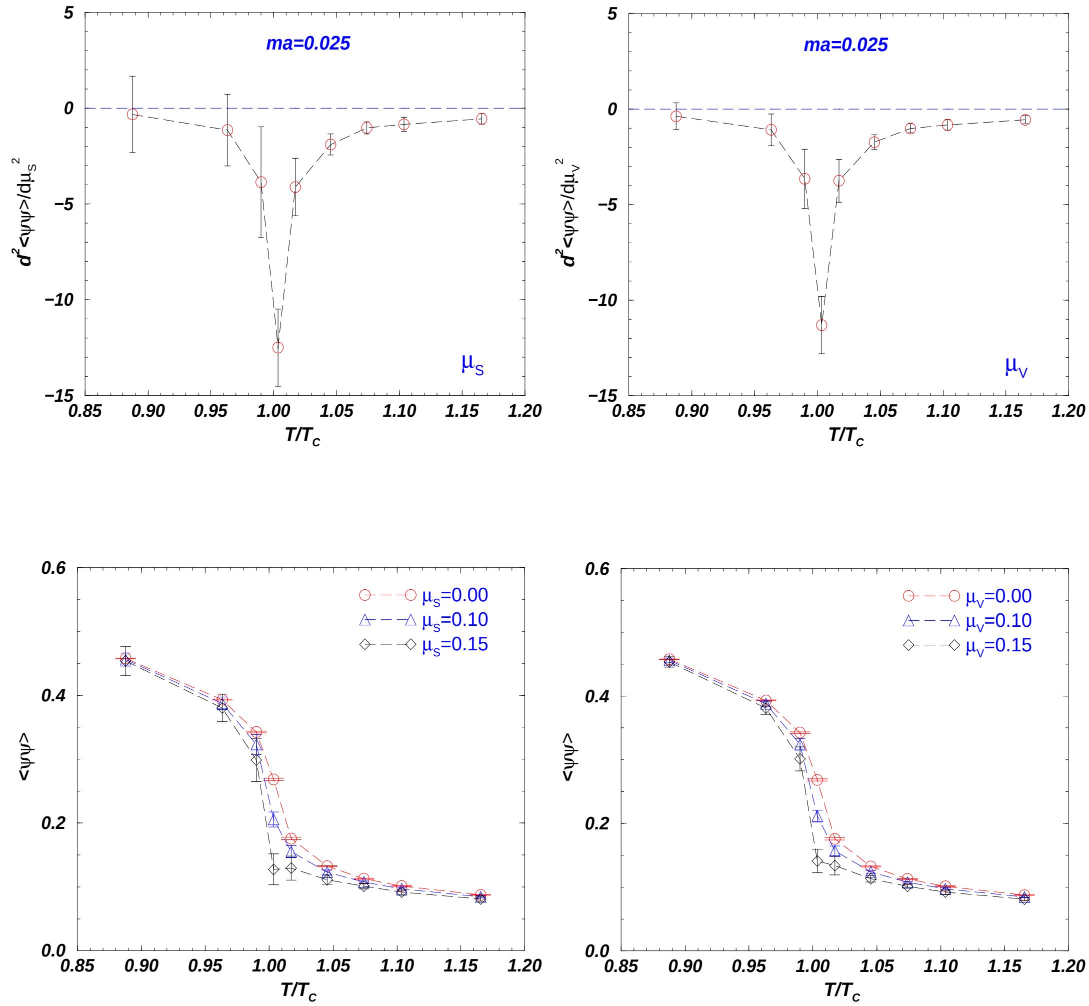
0.0125	1.287(54)	1.295(20)
0.0170	1.287(54)	1.312(17)

$ma \setminus \beta$	5.20	5.26	5.28	5.29	5.30	5.32	5.34	5.36	5.40
0.0250	1.450 (56)	1.320 (29)	1.365 (34)	1.383 (28)	1.51 (36)	1.299 (13)	1.352 (52)	1.311 (27)	1.387 (27)





the second order response of vector meson at $\beta = 5.34$ and $ma = 0.025$, and free quark response at $ma = 0.025$



the behavior of chiral condensate

$$\left.\frac{\langle\bar\psi\psi\rangle(\mu_S)}{T^3}\right|_{\mu_S} = (17.16\pm0.11)-(25.0\pm4.1)\left(\frac{\mu_S}{T}\right)^2+O\left[\left(\frac{\mu_S}{T}\right)^3\right]$$

$$\left.\frac{\langle\bar\psi\psi\rangle(\mu_V)}{T^3}\right|_{\mu_V} = (17.16\pm0.11)-(22.6\pm3.0)\left(\frac{\mu_V}{T}\right)^2+O\left[\left(\frac{\mu_V}{T}\right)^3\right]$$

五。 discussions and summary

[I]. Method to extract 1st and 2nd order responses of hadron to chemical potential

- $N_f = 2$ **staggered fermion action**
- **R-algorithm** $L = 16 \times 8 \times 8 \times 4$

[II]. behavior of pseudoscalar meson to μ_S

$\odot$ $T < T_C$

♣ **small 2nd order response of mass** ($positive$)

- **chiral limit** $\rightarrow$ **consistent with 0**
- **persistence of N-G boson nature at small** μ_S

$\odot$ $T > T_C$

presence of hadronic state in pseudoscalar meson channel

♣ **growth of** $\frac{\partial^2 \hat{M}}{\partial \hat{\mu}_S^2}$

- **liberation from N-G boson nature**
- μ_S **influence enhances mass**

[III]. behavior of pseudoscalar meson to μ_V

⊙ **1st order responses are very weak**

at both $T > T_C$ and $T < T_C$

⊙ $T < T_C$

♣ **large 2nd order response of mass ($negative$)**

$$\left.\frac{M(\mu_V)}{T}\right|_{\mu_V} = (1.4024 \pm 0.0008) + (-0.0005 \pm 0.0010)\left(\frac{\mu_V}{T}\right) - (1.31 \pm 0.04)\left(\frac{\mu_V}{T}\right)^2 + O\left[\left(\frac{\mu_V}{T}\right)^3\right]$$

$$(\ ma = 0.017, \beta = 5.26\ )$$

⊙ $T > T_C$

♣ **2nd order responses become weak**

[IV]. behavior of vector meson

⊙ **there are no large changes for vector meson mass at both $T > T_C$ and $T < T_C$**

⊙ **there are large noise, High statistical simulations shall be required to clarify the behavior for vector meson.**

[V]. behavior of chiral condensate to μ_S and μ_V

$\odot$ **1st order responses are 0 for both μ_S and μ_V**

$\odot$ **small 2nd order responses (*negative*) of $\langle\bar{\psi}\psi\rangle$ in low temperature and high temperature, and have same behavior of 2nd order responses for μ_S and μ_V channel**

$\odot$ **large 2nd order responses of $\langle\bar{\psi}\psi\rangle$ (*negative*) near T_C**

$$\left.\frac{\langle\bar{\psi}\psi\rangle(\mu_S)}{T^3}\right|_{\mu_S} = (17.16 \pm 0.11) - (25.0 \pm 4.1)\left(\frac{\mu_S}{T}\right)^2 + O\left[\left(\frac{\mu_S}{T}\right)^3\right]$$

$$\left.\frac{\langle\bar{\psi}\psi\rangle(\mu_V)}{T^3}\right|_{\mu_V} = (17.16 \pm 0.11) - (22.6 \pm 3.0)\left(\frac{\mu_V}{T}\right)^2 + O\left[\left(\frac{\mu_V}{T}\right)^3\right]$$

$(\ ma = 0.025, \beta = 5.29 \ )$

outlook

$(^{\odot}\asymp^{\odot})$ **vector meson**

$(^{\odot}\asymp^{\odot})$ **other physical quantities**

$$\left.\frac{\partial Q(\mu)}{\partial \mu}\right|_{\mu} \; ; \; \left.\frac{\partial^2 Q(\mu)}{\partial \mu^2}\right|_{\mu}$$

$(^{\odot}\asymp^{\odot})$ **for nucleon**

$(^{\odot}\asymp^{\odot})$ **large lattice size (for example $N_t > 6$)**

新 春 愉 快！

謝　謝 ！