2.6 以底面圆心为坐标原点，建立柱坐标系，设质点到z轴距离为R

$\vec{v} = \dot{R}\vec{e_r} + R\dot{\varphi}\vec{e_\varphi} + \dot{z}\vec{e_z}$

$\because R = R_2 + z\tan\alpha$，$\therefore \dot{R} = \dot{z}\tan\alpha$

$T = \frac{1}{2}m[\dot{z}^2\tan^2\alpha + (R_2+z\tan\alpha)^2\dot{\varphi}^2+\dot{z}^2] = \frac{1}{2}m[\dot{z}^2(1+\tan^2\alpha) + (R_2+z\tan\alpha)^2\cdot\dot{\varphi}^2]$

$L = T - V = T - mgz$

代入拉格朗日方程 $\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_a} - \frac{\partial L}{\partial q_a} = Q_a$

解得 $\begin{cases} \ddot{\varphi}(z\tan\alpha+R_2) + 2\dot{\varphi}\tan\alpha\cdot\dot{z} = 0 \\ \ddot{z}^2(1+\tan^2\alpha) - \dot{\varphi}^2\tan\alpha(z\tan\alpha+R_2)+g=0 \end{cases}$

2.7 取广义坐标R和φ

$\because R^2+z^2=r^2$

$V_R=\dot{R}$ $V_\varphi=R\dot{\varphi}$ $V_k=\dot{z}=\frac{-R^3}{\sqrt{r^2-R^2}}$

$\therefore L=\frac{m}{2}(R^2\dot{\varphi}^2+\frac{r^2\dot{R}^2}{r^2-R^2}\dot{R}^2)-mg\sqrt{r^2-R^2}$

代入 $\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_a}-\frac{\partial L}{\partial q_a}=Q_a$

解得 $\begin{cases} \ddot{\varphi}R+2\dot{\varphi}\dot{R}=0 \\ \frac{r^2\ddot{R}}{r^2-R^2}+\frac{Rr^2}{(r^2-R^2)^2}\dot{R}^2-R\dot{\varphi}^2-\frac{2gR}{\sqrt{r^2-R^2}}=0 \end{cases}$

2.8 以θ为广义坐标，取极坐标系

$T=\frac{m}{2}(\dot{r}^2+r^2\dot{\varphi}^2)$ $V=0$

又 $\because r=2R\cos\theta$ $\varphi=\theta+\omega t$

$L=T-V=\frac{m}{2}[(-2R\sin\theta\cdot\dot{\theta})^2+(\dot{\theta}+\omega)^2\cdot(2R\cos\theta)^2]$

$=2mR^2(\dot{\theta}^2+2\omega\dot{\theta}\cos^2\theta+\omega^2\cos^2\theta)$

代入 $\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_a}-\frac{\partial L}{\partial q_a}=Q_a$ 得 $\ddot{\theta}+\sin\theta\cos\theta\,\omega^2=0$

2.9 取x, y为广义坐标

则 $T=\frac{m}{2}(\dot{x}^2+\dot{y}^2)$ $V=-\frac{e^2}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{x^2+y^2}}$

$L=T-V=\frac{m}{2}(\dot{x}^2+\dot{y}^2)+\frac{e^2}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{x^2+y^2}}$

$\begin{cases} m\ddot{x}+\frac{e^2x}{4\pi\varepsilon_0(x^2+y^2)^{\frac{3}{2}}}=0 \\ m\ddot{y}+\frac{e^2y}{4\pi\varepsilon_0(x^2+y^2)^{\frac{3}{2}}}=0 \end{cases}$

2.11 建立以R为广义坐标的柱坐标系

$T=\frac{m}{2}(\dot{R}^2+R^2\dot{\theta}^2+\dot{z}^2)$ $\dot{z}=\frac{R}{p}\cdot\dot{R}$

$V=mgz=mg\frac{R}{2p}$

$L=T-V=\frac{m}{2}(\dot{R}^2+R^2\omega^2+\frac{R^2\dot{R}^2}{p^2})-\frac{mgR^2}{2p}=0$

$\frac{d}{dt}(\dot{R}+\frac{R^2}{p^2}\dot{R})+\frac{g}{p}R-\omega^2R-\frac{R}{p^2}\dot{R}^2=0$

得 $(p^2+R^2)\cdot\ddot{R}+R\cdot\dot{R}^2-p^2\omega^2R+pgR=0$

小环稳定时，$\dot{R}=\ddot{R}=0$

$\therefore p^2\omega^2R=pgR$ $\omega=\sqrt{\frac{g}{p}}$

2.12 以R, θ为广义坐标，建立柱坐标系

$T=\frac{m}{2}(\dot{R}^2+R^2\dot{\theta}^2+\dot{z}^2)$，$V=mgz$，$R^2=az$

$L=\frac{m}{2}(\dot{R}^2+R^2\dot{\theta}^2+\frac{4R^2\dot{R}^2}{a^2})-\frac{mg}{a}R^2$

代入，得 $\begin{cases} (1+\frac{4R^2}{a^2})\ddot{R}+\frac{8R\dot{R}^2}{a^2}-R\dot{\theta}^2+\frac{2gR}{a}=0 \\ R\ddot{\theta}+2\dot{\theta}\dot{R}=0 \end{cases}$

$\dot{R}=\ddot{R}=0$ 可得 $\dot{\theta}^2=\frac{2g}{a}$ $(R\dot{\theta})^2=2gz$

$V^2=2gz$

$t=0$时，$V_0^2=2gh$

2.13. 由杆 AC, DG 力矩平衡
$$\begin{cases} P\cdot FG = F_1\cdot EF + F_2\cdot DF \\ (P'-F_1')\cdot AB = F_2'\cdot AC \end{cases}$$

又 ∵ $F_1 = F_1'$ 且 $F_2 = F_2'$

∴ $P\cdot FG = F_1 EF + (P'-F_1)\frac{AB}{AC}DF$

∵ $\frac{DF}{EF} = \frac{AC}{AB}$ ∴ $P\cdot FG = P'EF$

∴ $P' = P\frac{FG}{EF}$

2.15 以m为原点建立直角坐标系

(1) $V = mg\left(\frac{l}{2}\cos\varphi - d\cot\varphi\right)$

体系为完整保守系统 $\frac{\partial V}{\partial \varphi} = 0$ 得 $-mg\frac{l}{2}\sin\varphi + \frac{mgd}{\sin^2\varphi} = 0$

∴ $\varphi = \arcsin\sqrt[3]{\frac{2d}{l}}$

(2) ∵ $x = \frac{l}{2}\sin\varphi - d$ $y = \frac{l}{2}\cos\varphi - d\cot\varphi$

由虚功 $mg\,\delta y = 0$

即 $mg\left(-\frac{l}{2}\sin\varphi + \frac{d}{\sin^2\varphi}\right)\delta\varphi = 0$

∴ $\varphi = \arcsin\sqrt[3]{\frac{2d}{l}}$

2.18 $V = -mg\cos\varphi_1 R - m_2 g\cos\varphi_2 R + \frac{1}{2}k\left(l - 2R\sin\frac{\varphi_1+\varphi_2}{2}\right)^2$

由 $\frac{\partial V}{\partial \varphi} = 0$

得 $\begin{cases} m_1 g\sin\varphi_1 - k\cos\frac{\varphi_1+\varphi_2}{2}(l-2R)\sin\frac{\varphi_1+\varphi_2}{2} = 0 \\ m_2 g\sin\varphi_2 - k\cos\frac{\varphi_1+\varphi_2}{2}(l-2R)\sin\frac{\varphi_1+\varphi_2}{2} = 0 \end{cases}$