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6.2 $T=\frac{1}{2}m'r^2\dot\theta^2+\frac{1}{2}mr^2(\dot\varphi+\dot\theta)^2 \quad V=\frac{mgr}{2}(\theta^2+\varphi^2)+\frac{m'gr}{2}\theta^2$

$L=T-V=\frac{1}{2}m'r^2\dot\theta^2+\frac{1}{2}mr^2(\dot\varphi+\dot\theta)^2-\frac{mgr}{2}(\theta^2+\varphi^2)-\frac{m'gr}{2}\theta^2$

代入拉格朗日方程 $\begin{cases} m'r^2\ddot\theta+mr^2\ddot\varphi+mr^2\ddot\theta+mgr\theta+m'gr\theta=0 \\ mr^2\ddot\varphi+mr^2\ddot\theta+mgr\varphi=0 \end{cases}$

$|A-B\lambda|=\begin{vmatrix} (m'+m)r^2-\lambda(mgr+m'gr) & mr^2 \\ mr^2 & mr^2-mgr\lambda \end{vmatrix} \quad \lambda_{1}=\frac{r}{g}$

$\begin{cases} (m'+m)r^2C_{11}+mr^2C_{21}=\lambda mgrC_{11} \\ mr^2C_{11}+mr^2C_{21}=\lambda mr^2C_{21} \end{cases}$ 由 $\begin{pmatrix}\theta\\ \varphi\end{pmatrix}=\begin{pmatrix}C_{11}&C_{12}\\C_{21}&C_{22}\end{pmatrix}\begin{pmatrix}q_1\\q_2\end{pmatrix}$

$q_1=\theta+\frac{m}{m'+m}\varphi \quad q_2=\theta-\varphi.$

$T=\frac{(m+m')}{2}\dot q_1^2+m'\dot q_2^2$

$V=\frac{(m'+m)}{2}q_1^2+\frac{m'+m}{2}q_2^2$

代入求本征方程，得 $w_1=\sqrt{\frac{g}{2r}}$

$w_2=\sqrt{\frac{g(m+m')}{rm'}}$

6.3 $T=J\beta=\left[\frac{1}{3}ml^2+m\left(\frac{l}{2}\right)^2\right](\dot\theta_1+\dot\theta_2)^2$

$V=\frac{mgl}{2}(\theta_1^2+\theta_2^2)$

$|A-B\lambda|=\begin{vmatrix} \frac{4ml^2}{7}-\frac{mgl}{2}\lambda & \frac{4}{7}ml^2 \\ \frac{4ml^2}{7} & \frac{4ml^2}{7}-\frac{mgl}{2}\lambda \end{vmatrix}=0 \quad \lambda=\sqrt{\frac{16l}{7g}}$

$q_1=\theta_1+\frac{4}{7}\theta_2 \quad q_2=\theta_1-\frac{4}{7}\theta_2$

$T=\frac{4m}{7}l^2\dot q_1^2+ml^2\dot q_2^2$

$V=\frac{4m}{7}l^2q_1^2+ml^2q_2^2$

代入本征方程，得 $w_1=\sqrt{(7+\sqrt{37})\frac{g}{l}}$ 简正振动模式 $\frac{\theta_1}{\theta_2}=-1.180$

$w_2=\sqrt{(7-\sqrt{37})\frac{g}{l}} \quad \frac{\theta_1}{\theta_2}=0.847$

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6.7 $T=\frac{1}{2}m(\dot x^2+\dot y^2)$ ~~$V=\frac{3}{4}k(x^2+y^2)+\frac{1}{4}k|x^2-y^2|$~~ $V=\frac{1}{2}k\cdot(\frac{1}{4}x^2+y^2)+\frac{1}{2}|\frac{1}{4}x^2-y^2|$

若$2x>y$ 若$2x<y$ $\begin{cases} m\ddot x-2kx=0 \\ m\ddot y-\frac{2k}{3}y=0 \end{cases}$

$\therefore \begin{vmatrix} 2k-mw^2 & 0 \\ 0 & 2k-3mw^2 \end{vmatrix}=0 \quad w_1=\sqrt{\frac{k}{2m}} \quad w_2=\sqrt{\frac{3k}{2m}}$

6.14 $L=\frac{1}{2}(\dot x^2+\dot y^2)-\frac{w_0^2}{2}(x^2+y^2)+\alpha xy$

代入拉格朗日方程

$\begin{cases} \ddot x-w_0^2x+\alpha y=0 \\ \ddot y-w_0^2y+\alpha x=0 \end{cases}$

$\begin{vmatrix} w_0^2-w^2 & \alpha \\ \alpha & w_0^2-w^2 \end{vmatrix}=0 \quad w=\sqrt{1\pm\frac{\alpha}{w_0^2}}$