3.13

解：设 m_1 的初速度为 V_{01}

$$\begin{cases} V_{01}' = \dfrac{\sqrt{m_1^2+m_2^2+2m_1m_2\cos\theta}}{m_1+m_2} V_{01} \\ V_{02}' = \dfrac{2m_1V_{01}}{m_1+m_2}\sin\dfrac{\theta}{2} \end{cases}$$

$$\tan\theta_1 = \frac{m_2\sin\theta}{m_1+m_2\cos\theta} = \frac{m_2\sqrt{1-\cos^2\theta}}{m_1+m_2\cos\theta}$$

~~θ_2~~ 将 $\theta = \pi - 2\theta_2$ 代入

$$\begin{cases} V_{01}' = \left(\dfrac{m_1}{m_1+m_2}\cos\theta_1 \pm \dfrac{\sqrt{m_2^2-m_1^2\sin^2\theta_1}}{m_1+m_2}\right) V \\ V_{02}' = \dfrac{2m_1V_{01}}{m_1+m_2}\cos\theta_2 \, V \end{cases}$$

4.1 (1) $w_p = 0$　　$V_S = (r_1+r_2)\dot\theta = r_2 w_S$

$$w_S = \frac{r_1+r_2}{r_2}\dot\theta$$

(2) $w_1 = \dot\varphi$　　$V_S = (r_1+r_2)\dot\theta = -r_1\dot\varphi + r_2 w_S$

$$w_S = \frac{r_1+r_2}{r_2}\dot\theta + \frac{r_1}{r_2}\dot\varphi$$

(3) $w_S = w_p = 0$ 或为常数

4.2 (1) $w_L = \dot\varphi$

相对于P点　　$V_S = (r_1-r_2)\dot\theta = r_2 w_S - r_1\dot\varphi$

$$w_S = \frac{r_1-r_2}{r_2}\dot\theta + \frac{r_1}{r_2}\dot\varphi$$

(2) $w_L = \dot\varphi$ $\dot w$

$w_S = 0$ 或常数

$$d\vec r = dr\,\vec e_r + r\,d\theta\,\vec e_\theta + r\sin\theta\,d\varphi\,\vec e_\varphi$$

$$\vec v = \frac{d\vec r}{dt} = \dot r\vec e_r + r\dot\theta\vec e_\theta + r\sin\theta\,\dot\varphi\vec e_\varphi$$

~~$\vec e_r = \sin\theta\cos\varphi$~~ $\dot{\vec e}_r = \dot\theta\vec e_\theta + \sin\theta\,\dot\varphi\vec e_\varphi$