年 月 日 第 页

2-2. 以碗球心为原点建立球坐标系

$r = Re_r + zk$

$\delta \vec{r} = \delta R e_r + R\delta\phi e_\phi + \delta z k$

$\dot{r} = \dot{R} e_r + R\dot{\phi} e_\phi + \dot{z} k$

$\ddot{r} = \ddot{R} e_r + 2\dot{R}\dot{\phi} e_\phi + R\ddot{\phi} e_\phi - R\dot{\phi}^2 e_r + \ddot{z} k$

$= (\ddot{R} - R\dot{\phi}^2) e_r + (2\dot{R}\dot{\phi} + R\ddot{\phi}) e_\phi + \ddot{z} k$

$F = -mgk$

代入达朗贝尔方程

$m[-gk - (\ddot{R} - R\dot{\phi}^2) e_r + (2\dot{R}\dot{\phi} + R\ddot{\phi}) e_\phi + \ddot{z} k] \cdot (\delta R e_r + R\delta\phi e_\phi + \delta z k) = 0$

解得 $\begin{cases} -\frac{z}{R}\ddot{R} + z\dot{\phi} + \ddot{z} + g = 0 \\ R\ddot{\phi} + 2\dot{R}\dot{\phi} = 0 \end{cases}$

2-3 取 m 径矢 OM 与 OO′ 连线夹角 θ，取极坐标系 $\vec{r} = 2R\cos\theta\, \vec{e}_r$

则 $\dot{\vec{r}} = 2R[-\sin\theta\dot{\theta}\vec{e}_r + \cos\theta(\dot{\theta} + w)\vec{e}_\theta]$

$\ddot{\vec{r}} = 2R\{[-\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta} - \cos\theta(\dot{\theta} + w)]\vec{e}_r + [-2\sin\theta\dot{\theta}(\dot{\theta} + w) + \cos\theta\ddot{\theta}] \cdot \vec{e}_\theta\}$

$\delta\vec{r} = 2R[-\sin\theta\,\delta\theta\,\vec{e}_r + \cos\theta\,\delta(\theta + wt)\vec{e}_\theta] = 2R\,\delta\theta(-\sin\theta\vec{e}_r + \cos\theta\vec{e}_\theta)$

代入达朗贝尔方程，化简得

$\begin{cases} (\ddot{\theta} + \sin\theta\cos\theta \cdot w^2)\delta r = 0 \\ \ddot{\theta} + \sin\theta\cos\theta\, w^2 = 0 \end{cases}$

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年 月 日 第

例 1. 取广义坐标 R、φ

$T = \frac{1}{2}m(\dot{R}^2 + R^2\dot{\varphi}^2) + \frac{1}{2}m'\dot{z}^2$

$V = m'gz$

由于约束关系 $R - z = l$　$z = R - l$ 代入

$L = T - V = \frac{1}{2}(m + m')\dot{R}^2 + \frac{1}{2}mR^2\dot{\varphi}^2 - m'g(R - l)$

得到 $\begin{cases} (m + m')\ddot{R} - mR\dot{\varphi}^2 + m'g = 0 \\ \frac{d}{dt}(mR^2\dot{\varphi}) = 0 \end{cases}$