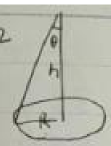
4.22 

$\tan\theta = \frac{R}{h}$, $\rho = \frac{3m}{\pi R^2 h}$

薄圆盘转动惯量为 $\frac{1}{2}mr^2$

选取高为dz, 半径为r的薄圆盘

$dI = \frac{1}{2}r^2 dm = \frac{1}{2}\rho\pi r^4 dz$

$I = \frac{1}{2}\int_0^h \rho\cdot\pi r^4 dz = \frac{3m}{2R^2 h}\cdot\int_0^h \left(\frac{Rz}{h}\right)^4 dz$

$= \frac{3m}{2R^2 h}\cdot\frac{R^4}{h^4}\int_0^h z^4 dz = \frac{3mR^2}{2h^5}\cdot\frac{h^5}{5} = \frac{3mR^2}{10}$

椭球三个轴转动惯量

4.27 $I_1 = \frac{m}{5}(b^2+c^2)$ $I_2 = \frac{m}{5}(a^2+c^2)$ $I_3 = \frac{m}{5}(a^2+b^2)$

计算为 $\rho = \frac{m}{\frac{4}{3}\pi abc}$ $2\int_0^h \rho\cdot\pi ab^2\cdot dh = \frac{m(a^2+b^2)}{5}$

~~平~~行角速度为 $\dot\psi = 0$, $\vec{\omega} = \dot\theta\cos\varphi\,\vec{i} + \dot\theta\sin\varphi\,\vec{j} + \dot\varphi\,\vec{k}$

角动量为 $\vec{L} = \frac{m}{5}(b^2+c^2)\dot\theta\cos\varphi\,\vec{i} + \frac{m}{5}(a^2+c^2)\dot\theta\sin\varphi\,\vec{j} + (b^2+c^2)\dot\varphi\,\vec{k}$

动能为 $T = \frac{1}{2}\vec{L}\cdot\vec{\omega}$

4.29 θ对应角动量改变为 $L_\theta = pd$

又 $\because L_{e_3} = I_3\omega$

z轴偏移量 $\tan\theta = \frac{pd}{I_3\omega}$

当冲量不大, 陀螺在此位置做大致对称的摆动,

则最大章动 $\theta_{max} \approx 2\theta = 2\arctan\frac{pd}{I_3\omega}$

4.36 体系的拉格朗日函数为

$L = \frac{I_1}{2}(\dot\theta^2 + \dot\varphi^2\sin^2\theta) + \frac{I_3}{2}(\dot\psi + \dot\varphi\sin\theta)^2 - \frac{I_3}{2}mgl\cos\theta$

公式郑的 $\dot\varphi_0 = \omega_2$, $\dot\psi_0 = \omega_1$

$\therefore I_1\ddot\theta - I_1\dot\varphi^2\sin\theta\cos\theta + I_3(\dot\psi + \dot\varphi\cos\theta)\dot\varphi\sin\theta - mgl\sin\theta = 0$

其中 $\dot\psi + \dot\varphi\cos\theta = \omega_3$

无章动则 $\ddot\theta = 0$

$\therefore (I_1\dot\varphi^2\cos\theta - I_3\omega_3\dot\varphi + mgl)\sin\theta = 0$

即 $I_1\dot\varphi^2\cos\theta - I_3\omega_3\dot\varphi + mgl = 0$

因此得到最终的角加速度 $\dot\varphi = \frac{I_3\omega_3 \pm \sqrt{I_3^2\omega_3^2 - 4I_1 mgl\cos\theta}}{2I_1\cos\theta}$

为使方程最终有实数根, 需要

$I_3^2\omega_3^2 \geq 4I_1 mgl\cos\theta$

$I_3^2(\omega_1 + \omega_2\cos\theta)^2 \geq 16mgl\cos\theta$