8.7 (1) 由于变换不显含时间，而

$p\,dq - P\,dQ = p\,dq - q\cot p\, d[\ln(\frac{1}{q}\sin p)] = d(p\,q + q\cot p)$ 是全微分

所以是正则变换

8.10 $p\,\delta q - P\,\delta Q = \sqrt{2kQ}\sin P\,\delta(\sqrt{\frac{2Q}{k}}\cos P) - P\,\delta Q$

$= \sqrt{2kQ}\sin P\,[\sqrt{\frac{2}{k}}\cos P\frac{1}{2\sqrt{Q}}\delta Q + \sqrt{\frac{2Q}{k}}(-\sin P)\delta P] - P\,\delta Q$

$= (\sin P\cos P - P)\delta Q - 2Q\sin^2 P\,\delta P = \delta[Q(\frac{1}{2}\sin 2P - P)]$

可写成全变分，则是正则变换

$H' = \frac{1}{2}(p^2 + k^2 q^2) = \frac{1}{2}(2kQ\sin^2 P + k^2\frac{2Q}{k}\cos^2 P) = kQ$

哈密顿正则方程为 $\dot{Q} = \frac{\partial H'}{\partial P} = 0$ $\dot{P} = -\frac{\partial H'}{\partial Q} = -k$

8.~~8~~ ~~$H = P$ $\dot{Q} = 1$, $\dot{P} = 0$~~

~~由此正则方程的解 $Q = t + \alpha$ 和 $P = \beta$ 可得~~

~~$y = \frac{1}{mg}[\beta - \frac{1}{2}mg(t+\alpha)]$ 其中 α 和 β 为积分常数~~

8.8 $H = T + V = \frac{p^2}{2m} + mgy$

$P = \frac{\partial H}{\partial y} = mg\theta$

$H = \frac{1}{2}mg^2\theta^2 + mgy$

$P = -\frac{\partial U}{\partial Q} = -\frac{1}{2}mg^2Q^2 - mgy$ 定义 $H' = -P$

$\dot{P} = -\frac{\partial H'}{\partial Q} = 0$ $\dot{Q} = \frac{\partial H'}{\partial P} = -1$ 积分得 $P = A$ $Q = B - t$

代入 $-A = \frac{1}{2}mg^2(B-t)^2 + mgy$ 将 $y = \dot{y} = 0$ 代入

$\therefore y = -\frac{1}{2}gt^2$

8.14 取 $\varphi = ar^2 + b\,r\cdot p + cp^2$

取广义坐标 $q_{1i} = x_i,\ q_{2i} = y_i,\ q_{3i} = z_i \quad i = 1, 2 \cdots$

因为 $J_{iz} = x_i p_{iy} - y_i p_{ix}$

$\varphi_i = a(x_i^2 + y_i^2 + z_i^2) + b(x_i p_{ix} + y_i p_{iy} + z_i p_{iz}) + c(p_{ix}^2 + p_{iy}^2 + p_{iz}^2)$

又 $\because \dfrac{\partial J_{iz}}{\partial x_i} = -y_i \quad \dfrac{\partial J_{iz}}{\partial y_i} = -p_{ix} \quad \dfrac{\partial J_{iz}}{\partial z_i} = 0$

$\dfrac{\partial J_{iz}}{\partial p_{ix}} = -y_i, \quad \dfrac{\partial J_{iz}}{\partial p_{iy}} = x_i \quad \dfrac{\partial J_{iz}}{\partial p_{iz}} = 0$

$\dfrac{\partial \varphi_i}{\partial x_i} = 2ax_i + bp_{ix}, \quad \dfrac{\partial \varphi_i}{\partial y_i} = 2ay_i + bp_{iy}$

$\dfrac{\partial \varphi_i}{\partial p_{ix}} = 2cp_{ix} + bx_i, \quad \dfrac{\partial \varphi_i}{\partial p_{iy}} = 2cp_{iy} + by_i$

$$\therefore [\varphi, J_z] = \sum_{i=1}^{n} [\varphi_i, J_{iz}]$$

$$= \sum_{i=1}^{n} \left[\sum_{\alpha=1}^{3} \left(\frac{\partial \varphi_i}{\partial q_{i\alpha}} \frac{\partial J_{iz}}{\partial p_{i\alpha}} - \frac{\partial J_{iz}}{\partial q_{i\alpha}} \cdot \frac{\partial \varphi}{\partial p_{i\alpha}} \right) \right]$$

$$= \sum_{i=1}^{n} [(2ax_i + bp_{ix})(-y_i) - p_{iy}(2cp_{ix} + bx_i) + (2ay_i + bp_{iy}) \cdot x_i$$

$$- (2cp_{iy} + by_i)(-p_{ix})]$$

$$= 0$$

ploission 定理：$\dfrac{d}{dt}[f, g] = \dfrac{\partial}{\partial t}[f, g] + [H, [f, g]]$

$$\frac{d}{dt}[f, g] = \left[\frac{\partial f}{\partial t}, g\right] + \left[f, \frac{\partial g}{\partial t}\right] - [f, [g, H]] - [g, [H, f]]$$

$$= \left[\left(\frac{\partial f}{\partial t} + [H, f]\right), g\right] + \left[f, \left[\frac{\partial g}{\partial t} + [H, g]\right]\right]$$

$$= \left[\frac{df}{dt}, g\right] + \left[f, \frac{dg}{dt}\right]$$

$\because f, g$ 都是运动积分

$\therefore \dfrac{df}{dt} = \dfrac{dg}{dt} = 0$

$\therefore \dfrac{d}{dt}[f, g] = 0$

$\therefore [f, g]$ 也是运动积分