$\sum_i (F_i - m_i \ddot{r}_i)\delta r_i = 0$

对于移动 $\delta r = \left(\frac{\partial y}{\partial \alpha}\right)_{\alpha=0} d\alpha$

$\delta S = \left(\frac{dS}{d\alpha}\right)_{\alpha=0} d\alpha$

$\therefore \delta S = \int_{x_1}^{x_2} \left(\frac{\partial f}{\partial y} - \frac{d}{dx}\frac{\partial f}{\partial y'}\right)\delta y\, dx = 0$ 换成 $\delta \int_{t_1}^{t_2} L\, dt = 0$

而加上非保守力 $\delta S = \int_{t_1}^{t_2} [\delta T(q_\alpha, \dot{q}_\alpha, t) - \sum Q_\alpha \delta q_\alpha]\, dt = 0$

例1： $L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\,\dot{\varphi}^2) + \frac{\alpha}{r}$

$$\begin{cases} P_r = m\dot{r} \\ P_\theta = mr^2\dot{\theta} \\ P_\varphi = mr^2\sin^2\theta\,\dot{\varphi} \end{cases}$$

$\therefore \dot{r} = \frac{P_r}{m} \quad \dot{\theta} = \frac{P_\theta}{mr^2} \quad \dot{\varphi} = \frac{P_\varphi}{mr^2\sin^2\theta}$

哈密顿函数 $H = \frac{1}{2m}\left(P_r^2 + \frac{P_\theta^2}{r^2} + \frac{P_\varphi^2}{r^2\sin^2\theta}\right) - \frac{\alpha}{r}$

正则方程

$$\begin{cases} \dot{r} = \frac{P_r}{m} \\ \dot{P}_r = \frac{P_\theta^2}{mr^3} + \frac{P_\varphi^2}{mr^3\sin^2\theta} - \frac{\alpha}{r^2} \end{cases}$$

$$\begin{cases} \dot{\theta} = \frac{P_\theta}{mr^2} \\ \dot{P}_\theta = \frac{-2P_\varphi^2\cos\theta}{r^2\sin^3\theta} \end{cases}$$

$$\begin{cases} \dot{\varphi} = \frac{P_\varphi}{mr^2\sin^2\theta} \\ \dot{P}_\varphi = 0 \end{cases}$$

$\frac{dv}{dt} = g - \frac{kv^2}{m}$

$\frac{dv}{g - \frac{kv^2}{m}} = dt$

3 $H = -mgL + \frac{1}{2}m\dot{L}^2$

$\ddot{L} = g - \frac{k\dot{L}^2}{m}$ ~~$\dot{L} = \sqrt{\ }$~~ ~~$L = \frac{m}{2k}\ln\frac{mg}{mg-kv^2}$~~

$P = m\dot{L} \qquad \dot{P} = k\frac{P^2}{m^2} - mg$

~~$H = \frac{P^2}{2m} - mg\frac{m^2 g}{2k}\ln\frac{mg}{mg-kv^2}$~~

$$\begin{cases} \dot{L} = \frac{P}{m} \\ \dot{P} = \frac{kP^2}{m^2} - mg \end{cases}$$

3.5 设短程线 $y = y(x)$ $H = H(x, y)$

$H_y - \frac{d}{dx}H_{y'} = 0$

~~$H = \sqrt{1+y'^2}$~~ $\frac{\partial H}{\partial y} = \frac{\partial H}{\partial y'} \cdot \frac{d}{dx}$

$\therefore \partial y' = \frac{\partial y}{\partial x}$

$\therefore \partial y = k\partial x$

$\therefore y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + y_1$

.6 $\int_{t_1}^{t_2} \left[\sum_{\alpha=1}^{s} \frac{\partial L}{\partial q_\alpha}\delta q_\alpha + \frac{\partial L}{\partial \dot{q}_\alpha}\delta \dot{q}_\alpha\right] dt = 0 \quad \left(\frac{\partial L}{\partial t}\delta t = 0\right)$

$\sum_{\alpha=1}^{s} \frac{\partial L}{\partial \dot{q}_\alpha}\delta \dot{q}_\alpha = \sum_{\alpha=1}^{s} \frac{\partial L}{\partial \dot{q}_\alpha}\frac{d}{dt}\delta q_\alpha = \frac{d}{dt}\left[\sum_{\alpha=1}^{s}\frac{\partial L}{\partial \dot{q}_\alpha}\delta q_\alpha\right] - \sum_{\alpha=1}^{s}\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_\alpha}\right)\delta q_\alpha]dt = 0$

代入，得 $\int_{t_1}^{t_2}\left[\sum_{\alpha=1}^{s}\frac{\partial L}{\partial q_\alpha}\delta q_\alpha + \frac{d}{dt}\left[\sum_{\alpha=1}^{s}\frac{\partial L}{\partial \dot{q}_\alpha}\delta q_\alpha\right] - \sum_{\alpha=1}^{s}\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_\alpha}\right)\delta q_\alpha\right] dt = 0$

$\therefore \delta q_\alpha \Big|_{t_1}^{t_2} = 0 \qquad \therefore \int_{t_1}^{t_2}\left[\sum_{\alpha=1}^{s}\frac{\partial L}{\partial q_\alpha} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_\alpha}\right)\right]\delta q_\alpha\, dt = 0$

由于 δq_α 任意

$\therefore \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_\alpha} - \frac{\partial L}{\partial q_\alpha} = 0$

vivo X60 · ZEISS
2022/10/09 16:31

vivo X60 · ZEISS
2022/10/09 16:31