6.1 $V=-mgr\cos\theta+\frac{1}{2}k(r-l)^2$

$$\frac{\partial V}{\partial\theta}=mgr\sin\theta=0 \quad \theta_0=0$$

$$\left.\frac{\partial V}{\partial r}\right|_{\theta_0=0}=-mg+k(r_0-l)=0$$

$$r_0=l+\frac{mg}{k}$$

$$x=r-r_0$$

$$L=\frac{1}{2}m(\dot r^2+r^2\dot\theta^2)+mgr\cos\theta-mg\left(l+\frac{mg}{k}\right)-\frac{1}{2}mk(r-l)^2$$

$$=\frac{1}{2}m\left(\dot x^2+\left(l+\frac{mg}{k}\right)^2\dot\theta^2\right)-\frac{1}{2}mg\left(l+\frac{mg}{k}\right)\theta^2-\frac{1}{2}kx^2$$

即 $a_{11}=m,\ a_{22}=ml^2 \quad b_{11}=-k \quad b_{22}=mg\left(l+\frac{mg}{k}\right)$

本征方程 $\begin{vmatrix} k-m\omega^2 & 0 \\ 0 & mg\left(l+\frac{mg}{k}\right)-m\left(l+\frac{mg}{k}\right)^2\omega^2 \end{vmatrix}=0$

解得 $w_1=\sqrt{\frac{k}{m}} \qquad w_2=\sqrt{\frac{g}{l+\frac{mg}{k}}}$

~~因此~~ $\cancel{r=l+\frac{mg}{k}+A\cos(\omega_1t+\ldots}$

.4. $V=\frac{1}{2}k\Delta x_1^2+\frac{1}{2}k\Delta x_2^2+\frac{1}{2}k\Delta x_3^2$

$T=\frac{1}{2}m_1\Delta\dot x_1^2+\frac{1}{2}m_2\Delta\dot x_2^2+\frac{1}{2}m_3\Delta\dot x_3^2$

代入拉格朗日方程

$$\begin{cases} m_1\Delta\ddot x_1+k_1\Delta x_1=0 \\ m_2\Delta\ddot x_2+k_2\Delta x_2=0 \\ m_3\Delta\ddot x_3+k_3\Delta x_3=0 \end{cases}$$

$$\begin{vmatrix} k_1-m_1w^2 & & \\ & k_2-m_2w^2 & \\ & & k_3-m_3w^2 \end{vmatrix}$$

$W=\pm\sqrt{\frac{k_1}{m_1}} \quad \pm\sqrt{\frac{k_2}{m_2}} \quad \pm\sqrt{\frac{k_3}{m_3}}$

5 $T=\frac{1}{2}ml^2\dot{\theta}_1^2+\frac{1}{2}ml^2\dot{\theta}_2^2$

$V=-mgl\cos\theta_1-mgl\cos\theta_2+\frac{1}{2}k\left(\sqrt{(l\sin\theta_1-l\sin\theta_2+d)^2+(l\cos\theta_1-l\cos\theta_2)^2}-d\right)$

$\approx\frac{1}{2}mgl(\theta_1^2+\theta_2^2)+\frac{1}{2}kl^2(\theta_1+\theta_2)^2$

$=\frac{1}{2}(mgl+kl^2)(\theta_1^2+\theta_2^2)+kl^2\theta_1\theta_2$

$$\begin{vmatrix} mgl+kl^2-ml^2w^2 & kl^2 \\ kl^2 & mgl+kl^2-ml^2w^2 \end{vmatrix}=0$$

$w=\sqrt{\frac{g}{l}+\frac{k}{m}\pm\frac{k}{m}}$ $w_1=\sqrt{\frac{g}{l}}$ $w_2=\sqrt{\frac{g}{l}+\frac{2k}{m}}$

$T=\frac{1}{2}m_1\dot{x}_1^2+$ ~~…~~ $\frac{1}{2}m_2[(l\dot{\theta}\cos\theta+\dot{x}_1)^2+l^2\dot{\theta}^2\sin^2\theta]$

$V=\frac{1}{2}kx_1^2-m_2gl\cos\theta$

代入拉格朗日方程

$\begin{cases} m_1\ddot{x}_1+m_2\ddot{x}_1+kx_1+m_2l\ddot{\theta}=0 \\ m_2l^2\ddot{\theta}+m_2gl\sin\theta+m_2l\cos\theta\,\ddot{x}_1 \end{cases}$

$\cos\theta\approx1$
$\sin\theta\approx\theta$

$[k-(m_1+m_2)w^2][m_2gl-m_2l^2w^2]-m_2l\,m_2\ldots=0$

$$\begin{vmatrix} k-(m_1+m_2)w^2 & m_2l \\ m_2l & m_2gl-m_2l^2w^2 \end{vmatrix}=0$$

$$w=\sqrt{\frac{kl+m_1g+m_2g\pm\sqrt{(m_1+m_2)^2g^2k^2l^2-4(m_1+m_2)(kg-m_2l)}}{2(m_1+m_2)}}$$

6.9 $T=\frac{1}{2}m\dot{x}^2+\frac{1}{2}m\dot{y}^2$

$V=2k(x^2+y^2)$

代入拉格朗日方程，

$\begin{cases} m\ddot{x}+4kx=0 \\ m\ddot{y}+4ky=0 \end{cases}$

$a_{11}=m$ $b_{11}=4k$
$a_{22}=m$ $b_{22}=4k$

$$\begin{vmatrix} 4k-mw^2 & 0 \\ 0 & 4k-mw^2 \end{vmatrix}=0$$

$w=2\sqrt{\frac{k}{m}}$