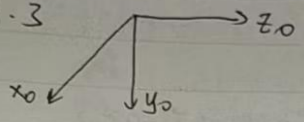
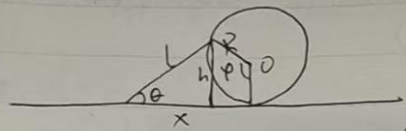
4.3



$\vec{\omega}=\vec{\omega_1}+\vec{\omega_2}$

$\frac{\omega_1}{\omega_2}=\frac{h}{R}$

4-4



$v_0=R\dot{\varphi}$ $\quad \dot\varphi=\frac{v_0}{R}$

$h=R(1-\cos\varphi)$

$\theta=\arcsin\frac{R(1-\cos\varphi)}{l}$

$=\arcsin\frac{2R}{l}\sin^2\frac{\varphi}{2}$

$$\dot\theta=\frac{\frac{2R}{l}2\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}\cdot\frac{1}{2}}{\sqrt{1-\frac{4R^2}{l^2}\sin^4\frac{\varphi}{2}}}\dot\varphi=\frac{R\sin\varphi}{l\sqrt{1-\frac{4R^2}{l^2}\sin^4\frac{\varphi}{2}}}\dot\varphi=\frac{v_0\sin\varphi}{\sqrt{l^2-4R^2\sin^4\frac{\varphi}{2}}}$$

$x=l\cos\theta$, $\quad \dot{x}=l\sin\theta\cdot\dot\theta=h\dot\theta=2R\sin^2\frac{\varphi}{2}\frac{v_0\sin\varphi}{\sqrt{l^2-4R^2\sin^4\frac{\varphi}{2}}}$

$v_A=\dot{x}+v_0(1-\cos\varphi)=2R\sin^2\frac{\varphi}{2}\frac{v_0\sin\varphi}{\sqrt{l^2-4R^2\sin^4\frac{\varphi}{2}}}+2v_0\sin^2\frac{\varphi}{2}$

4-5. $\vec{v_Q}=\vec{\omega}\times(\vec{r_a}-\vec{r_P})$ $\quad \vec{r_a}=(2R\cos\theta-a)\vec{e_r}$

$\vec{v_a}=\dot\rho\vec{e_r}+\rho\dot\theta\vec{e_\theta}$

$=\dot\theta(-2R\sin\theta\vec{e_r}+2R\cos\theta\vec{e_\theta}-a\vec{e_\theta})$

$=\dot\theta\vec{k}(\vec{r_a}-\vec{r_s})$

$\because \vec{k}\times\vec{e_r}=\vec{e_\theta}$ $\quad \vec{k}\times\vec{e_\theta}=-\vec{e_r}$

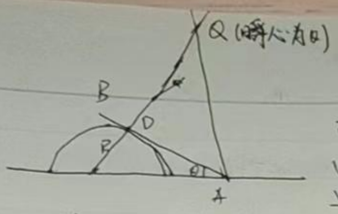
$\vec{r_Q}-\vec{r_P}=2R\sin\theta\vec{e_\theta}+2R\cos\theta\vec{e_r}-a\vec{e_r}$

$\therefore \vec{r_P}=-2R\sin\theta\vec{e_\theta}$

本体在 $x^2+y^2=4R^2$ 上

空间极迹在 $x^2+y^2=R^2$ 上

4-6



$$\vec{v_A} = \vec{v_0} = \vec{w} \times \vec{r_{QA}}$$

$$v_0 = w|QA|$$

$$\frac{v_D}{v_0} = \cos\theta$$

$$|QA| = \frac{|AD|}{\cancel{\cos}\sin\theta} = \frac{R}{\sin\theta\tan\theta}$$

$$w = \frac{v_0}{|QA|} = \frac{v_0}{R}\sin\theta\tan\theta$$

4-7 $\quad w' = \frac{w}{\sin\alpha}\cos\alpha$

$$V_A = w'|QA| = \frac{w}{\sin\alpha}\cos\alpha \cdot 2h\tan\alpha\cos\alpha = 2wh\cos\alpha$$

$$\vec{w'} = -w\cot\alpha\,\vec{i}$$

$$\vec{r} = \frac{h}{\cos\alpha}\cos 2\alpha\,\vec{i} + \frac{h}{\cos\alpha}\sin 2\alpha\,\vec{k}$$

$$\vec{a_{转}} = \frac{d\vec{w'}}{dt} \times \vec{r} = (-w\cot\alpha\, w\vec{j}) \times \vec{r} = (-w^2\cot\alpha\,\vec{j}) \times \left(\frac{h}{\cos\alpha}\cos 2\alpha\,\vec{i} + \frac{h}{\cos\alpha}\sin 2\alpha\,\vec{k}\right)$$

$$= w^2\cot\alpha\frac{h}{\cos\alpha}\cos 2\alpha\,\vec{k} - w^2\cot\alpha\frac{h}{\cos\alpha}\sin 2\alpha\,\vec{i}$$

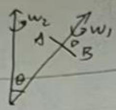
$$= w^2\cot\alpha\frac{h}{\cos\alpha}\cos 2\alpha\,\vec{k} - w^2\cot\alpha\frac{h}{\cos\alpha}\sin 2\alpha\,\vec{i}$$

$$\vec{a_{轴}} = \vec{w}' \times (\vec{w}' \times \vec{r}) = \vec{w}' \times w\cot\alpha\frac{h}{\cos\alpha}\sin 2\alpha\,\vec{j}$$

$$= \vec{w'} \times 2wh\cos\alpha\,\vec{j} = (-w\cot\alpha\,\vec{i}) \times (2wh\cos\alpha\,\vec{j})$$

$$= -2w^2h\frac{\cos^2\alpha}{\sin\alpha}\vec{k}$$

$$|\vec{a_{轴}}| = 2w^2h\frac{\cos^2\alpha}{\sin\alpha}$$



4-8

$$\vec{w}=\vec{w}_1+\vec{w}_2=\cancel{0}\cdot -w_2\sin\theta\,\vec{i}+(w_1+w_2\cos\theta\,\vec{k})$$

$$\vec{r_B}=a\vec{i}+b\vec{k}$$

$$\vec{v_B}=\vec{w}\times\vec{r_B}=(w_1 a+w_2 a\cos\theta+w_2 b\sin\theta)\vec{j}$$

4-11 $\sigma=\frac{m}{4\pi r^2}$

$$dm=\sigma\,2\pi\,r\sin\theta\,r\,d\theta$$

$$dI=dm\cdot(r\sin\theta)^2=\sigma\,2\pi(r\sin\theta)^3\,r\,d\theta$$

$$I=\int dI=\int_0^{\pi}\sigma\,2\pi(r\sin\theta)^3\,r\,d\theta$$

$$=\int_0^{\pi}\frac{m}{2}r^2\sin^3\theta\,d\theta=\frac{m}{2}r^2\cdot\frac{4}{3}=\frac{2}{3}mr^2$$

4-12 $I=\frac{1}{2}mR^2$ $\tan\theta=\frac{r}{h}$

$$\sigma=\frac{m}{\frac{1}{3}\pi r^2 h}=\frac{3m}{\pi r^2 h}$$

$$dm=\sigma\,\pi(x\tan\theta)^2\,dx$$

$$dI=\frac{3m}{2\pi r^2 h}\pi(x\tan\theta)^2dx\,(x\tan\theta)^2$$

$$=\frac{3m}{2r^2h}x^4\tan^4\theta\,dx=\frac{3m}{2r^2h}\cdot\frac{r^4}{h^4}x^4dx$$

$$=\frac{3mr^2}{2h^5}x^4dx$$

$$I=\int_0^h\frac{3mr^2}{2h^5}x^4dx=\frac{3mr^2}{2h^5}\frac{h^5}{5}=\frac{3}{10}mr^2$$

由~~垂~~直轴定理,圆盘绕直径的I: $I+I=\frac{1}{2}mR^2$, $I=\frac{1}{4}mR^2$

由平行轴定理,圆盘绕平行于直径轴的I: $I=\frac{1}{4}mR^2+mx^2$

$$I=\int_0^h m\frac{\pi(x\tan\theta)^2}{\frac{1}{3}\pi R^2 h}\left[\frac{1}{4}(x\tan\theta)^2+(h-x)^2\right]dx=\frac{1}{20}m(3r^2+2h^2)$$

4-13 $L=\frac{1}{2}mr^2w$

$$dM=df\cdot x=\mu\,dN\cdot x=\mu\frac{m}{\pi r^2}2\pi x^2dx$$

$$M=\int_0^r\frac{2\mu m x^2}{r^2}dx=\frac{2}{3}mg\mu r$$

$$\frac{dL}{dt}=\frac{1}{2}mr^2\dot{w}=-\frac{2}{3}\mu mgr$$

$$w=-\frac{4\mu gt}{3r}+w_0$$

$$t=\frac{3w_0r}{4\mu g}$$