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In[*]:= SetOptions[SelectedNotebook[],
  PrintingStyleEnvironment -> "Printout", ShowSyntaxStyles -> True]

In[*]:= << Notation`

In[*]:= (* Simplify the notation for the
  formulas: index vectors based on a subscript notation *)
Notation[ $x_{n_}$   $\Leftrightarrow$   $x_{[n]}$ ]

In[*]:=  $\alpha$  = Reverse@{13.00773, 1.962079, 0.444529, 0.1219492} & /@ Range[1, 2] // Flatten
Out[*]= {0.121949, 0.444529, 1.96208, 13.0077, 0.121949, 0.444529, 1.96208, 13.0077}

matNorm[v_, M_] :=  $\frac{v}{\text{Sqrt}[v.M.v]}$ ;

scfLoop[d_,  $\epsilon$ _,  $\alpha$ _] :=
Module[{Z, K, R, S, T, V, H, Q, atomPts, G, F, evals, vecs, newEvals},
  atomPts = {{0, 0, 0}, {d, 0, 0}}; (* Atomic positions *)
  Z = Partition[Table[#, {n, 1, 4}] & /@ atomPts // Flatten, 3];

  (* Array of positions for the coefficients  $\alpha$  *)
  K = Array[Exp[- $\frac{\alpha_{\#1} \alpha_{\#2}}{\alpha_{\#1} + \alpha_{\#2}}$  Norm[Z#1 - Z#2]2] &, {8, 8}];
  (* Coefficients matrix Kp,q *)
  R = Array[ $\frac{\alpha_{\#1} Z_{\#1} + \alpha_{\#2} Z_{\#2}}{\alpha_{\#1} + \alpha_{\#2}}$  &, {8, 8}]; (* Distances matrix Rp,q *)
  S = Array[( $\frac{\pi}{\alpha_{\#1} + \alpha_{\#2}}$ )3/2 K#1,#2 &, {8, 8}];
  (* Overlap matrix Sp,q *)

  T = Array[ $\frac{\alpha_{\#1} \alpha_{\#2}}{\alpha_{\#1} + \alpha_{\#2}}$  (3 - 2  $\frac{\alpha_{\#1} \alpha_{\#2}}{\alpha_{\#1} + \alpha_{\#2}}$  Norm[Z#1 - Z#2]2) S#1,#2 &, {8, 8}];
  (* Kinetic energy matrix Tp,q *)
  V = Total@Table[Array[If[Norm[R#1,#2 - U] == 0, - $\frac{2 \pi}{\alpha_{\#1} + \alpha_{\#2}}$  K#1,#2, -S#1,#2
     $\frac{1}{\text{Norm}[R_{\#1,\#2} - U]}$  Erf[ $\sqrt{\alpha_{\#1} + \alpha_{\#2}}$  Norm[R#1,#2 - U]]] &, {8, 8}], {U, atomPts}];
  (* Potential energy matrix Vp,q *)
  H = T + V; (* Hamiltonian matrix Hp,q *)

  Q = Array[S#1,#3 S#2,#4 If[Norm[R#1,#3 - R#2,#4] == 0,  $\frac{2}{\sqrt{\pi}}$   $\sqrt{\frac{(\alpha_{\#1} + \alpha_{\#3})(\alpha_{\#2} + \alpha_{\#4})}{\alpha_{\#1} + \alpha_{\#3} + \alpha_{\#2} + \alpha_{\#4}}}$ ,
     $\frac{1}{\text{Norm}[R_{\#1,\#3} - R_{\#2,\#4}]}$  Erf[ $\sqrt{\frac{(\alpha_{\#1} + \alpha_{\#3})(\alpha_{\#2} + \alpha_{\#4})}{\alpha_{\#1} + \alpha_{\#3} + \alpha_{\#2} + \alpha_{\#4}}}$  Norm[R#1,#3 - R#2,#4]]] &,
    {8, 8, 8, 8}]; (* Qp,r,q,s matrix, it's an 8 by 8

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matrix containing 8 by 8 matrices *)

P = ConstantArray[0., {8, 8}];
(* Initial guess for the density matrix *)
(* Perform a reset before the while-loop to assert correctness *)
evals = ConstantArray[0., {8}];
newEvals = evals +  $\epsilon$ ;

(* While the new and previous eigenvalues are still different enough... *)
While[Norm[newEvals - evals] >  $\epsilon$ ,
  newEvals = evals; (* Overwrite the previous eigenvalues *)

  G = Array[ $\sum_{r=1}^8 \sum_{s=1}^8 P_{r,s} (2 Q_{\#1,r,\#2,s} - Q_{\#1,r,s,\#2})$  &, {8, 8}] (*  $G_{p,q}$  matrix *);

  F = H +  $\frac{1}{2}$  G; (* Compute the Fock operator from the G matrix *)

  {evals, evecs} = Eigensystem[{F, S}];
  (* Calculate the eigenvalues and eigenvectors *)
  {evals, evecs} = {evals[[#]], evecs[[#]]} &@Ordering[evals];
  (* Sort them in ascending order *)
  evecs = matNorm[#, S] &/@evecs;
  (* Normalize them w.r.t. the overlap matrix S *)
  (* Perform a numerical divergence check *)
  If[AllTrue[Thread[#.S.# != 1] &/@evecs, TrueQ], Abort[], Nothing];
  (* Calculate the density matrix from the
    eigenvector evecs1 associated with the minimal eigenvalue *)
  P = 2 TensorProduct[First@evecs, evecs1];
];

(* Return the first eigenvalue and the equilibrium bonding energy,
where  $\frac{1}{d}$  is the  $E_{\text{nuc1}}$  term *)
(* The `Total[#, 2]` function
gives the sum of all elements of the matrix # *)
{evals1, Total[atomPts  $\left(H + \frac{1}{4} G\right)$ , 2] +  $\frac{1}{d}$ }
];

interv = Range[0.3, 10, 0.01] // N;
 $\epsilon$  = 10-4 // N; (* Accuracy level *)

{totalTime, res} =
  ParallelMap[Quiet@scfLoop[#,  $\epsilon$ ,  $\alpha$ ] &, interv] // Transpose // AbsoluteTiming;

Length@interv (* Number of distances used *)
Quantity[totalTime, "Seconds"]
(* Time it took the loop to finish, measured in seconds *)

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Out[ ]=

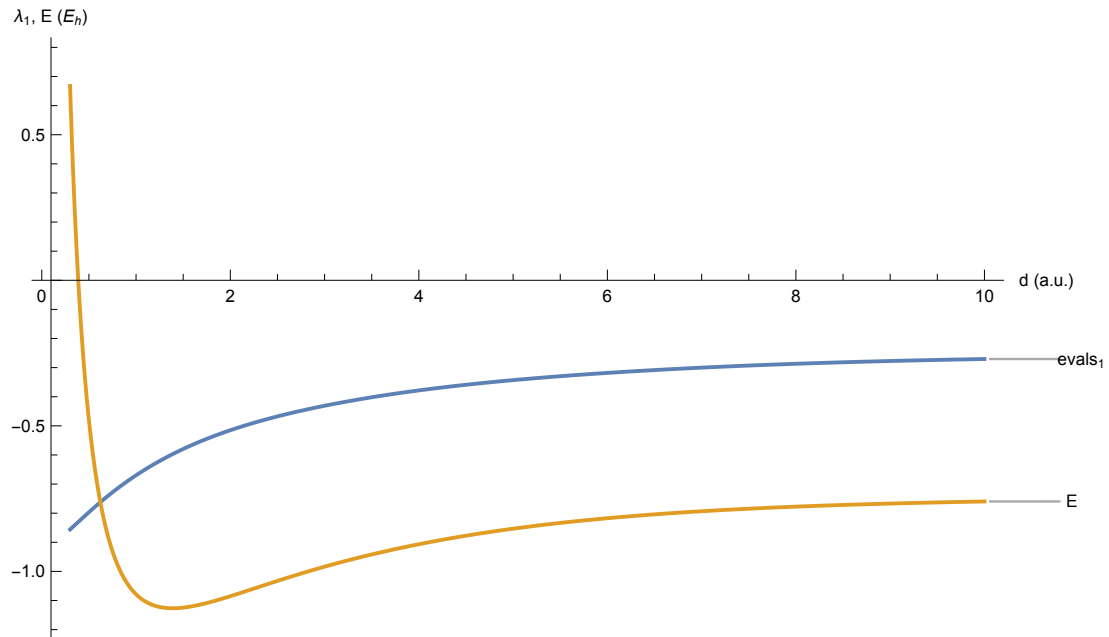
971

Out[ ]=

8.97029 s

(\* Smallest eigenvalue evals<sub>1</sub> and equilibrium bonding energy E \*)

```
ListLinePlot[Thread[{interv, #}] & /@ res, AxesLabel → {"d (a.u.)", "λ1, E (Eh)"},
  PlotRange → Full, ImageSize → Large, PlotLabels → {"λ1", "E"}]
```



Quantity[Min[Last@res], "HartreeEnergy"] (\* Smallest energy value\*)

UnitConvert[%, "Electronvolts"] (\* ...converted in electronvolts \*)

Out[ ]=

-1.12654  $E_h$

Out[ ]=

-30.6548 eV