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In[30]:= SetOptions[SelectedNotebook[],
  PrintingStyleEnvironment -> "Printout", ShowSyntaxStyles -> True]

In[31]:= << Notation`

In[32]:= (* Simplify the notation for the
  formulas: index vectors based on a subscript notation *)
Notation[ $x_{n_}$   $\Leftrightarrow$   $x_{[n]}$ ]

In[33]:=  $\alpha$  = Reverse@{13.00773, 1.962079, 0.444529, 0.1219492} & /@ Range[1, 2] // Flatten
Out[33]= {0.121949, 0.444529, 1.96208, 13.0077, 0.121949, 0.444529, 1.96208, 13.0077}

In[34]:= MatrixNormalize[v_, M_] :=  $\frac{v}{\text{Sqrt}[v.M.v]}$ ;

In[35]:= SCFLoop[d_,  $\epsilon$ _,  $\alpha$ _] := Module[{Z, K, R, S, T, V, H, Q, P, G, F,  $\lambda$ ,  $\psi$ ,  $\xi$ },
  P = {{0, 0, 0}, {d, 0, 0}}; (* Atomic positions *)
  Z = Partition[Table[#, {n, 1, 4}] & /@ P // Flatten, 3];

  (* Array of positions for the coefficients  $\alpha$  *)
  K = Array[Exp[- $\frac{\alpha_{\#1} \alpha_{\#2}}{\alpha_{\#1} + \alpha_{\#2}}$  Norm[Z#1 - Z#2]2] &, {8, 8}];
  (* Coefficients matrix Kp,q *)
  R = Array[ $\frac{\alpha_{\#1} Z_{\#1} + \alpha_{\#2} Z_{\#2}}{\alpha_{\#1} + \alpha_{\#2}}$  &, {8, 8}]; (* Distances matrix Rp,q *)
  S = Array[( $\frac{\pi}{\alpha_{\#1} + \alpha_{\#2}}$ )3/2 K#1,#2 &, {8, 8}];
  (* Overlap matrix Sp,q *)

  T = Array[ $\frac{\alpha_{\#1} \alpha_{\#2}}{\alpha_{\#1} + \alpha_{\#2}}$  (3 - 2  $\frac{\alpha_{\#1} \alpha_{\#2}}{\alpha_{\#1} + \alpha_{\#2}}$  Norm[Z#1 - Z#2]2) S#1,#2 &, {8, 8}];
  (* Kinetic energy matrix Tp,q *)
  V = Total@Table[Array[If[Norm[R#1,#2 - U] == 0, - $\frac{2 \pi}{\alpha_{\#1} + \alpha_{\#2}}$  K#1,#2,
    -S#1,#2  $\frac{1}{\text{Norm}[R_{\#1,\#2} - U]}$  Erf[ $\sqrt{\alpha_{\#1} + \alpha_{\#2}}$  Norm[R#1,#2 - U]]] &, {8, 8}], {U, P}];
  (* Potential energy matrix Vp,q *)
  H = T + V; (* Hamiltonian matrix Hp,q *)

  Q = Array[S#1,#3 S#2,#4 If[Norm[R#1,#3 - R#2,#4] == 0,  $\frac{2}{\sqrt{\pi}}$   $\sqrt{\frac{(\alpha_{\#1} + \alpha_{\#3})(\alpha_{\#2} + \alpha_{\#4})}{\alpha_{\#1} + \alpha_{\#3} + \alpha_{\#2} + \alpha_{\#4}}}$ ,
     $\frac{1}{\text{Norm}[R_{\#1,\#3} - R_{\#2,\#4}]}$  Erf[ $\sqrt{\frac{(\alpha_{\#1} + \alpha_{\#3})(\alpha_{\#2} + \alpha_{\#4})}{\alpha_{\#1} + \alpha_{\#3} + \alpha_{\#2} + \alpha_{\#4}}}$  Norm[R#1,#3 - R#2,#4]]] &,
    {8, 8, 8, 8}]; (* Qp,r,q,s matrix, it's an 8 by 8
  matrix containing 8 by 8 matrices *)

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P = ConstantArray[0., {8, 8}];
(* Initial guess for the density matrix *)
(* Perform a reset before the while-loop to assert correctness *)
λ = ConstantArray[0., {8}];
ξ = λ + ε;

(* While the new and previous eigenvalues are still different enough... *)
While[Norm[ξ - λ] > ε,
  ξ = λ; (* Overwrite the previous eigenvalues *)
  G = Array[Sum[Sum[Pr,s (2 Q#1,r,#2,s - Q#1,r,s,#2) &, {8, 8}]] (* Gp,q matrix *)];
  F = H +  $\frac{1}{2}$  G; (* Compute the Fock operator from the G matrix *)
  {λ, ψ} = Eigensystem[{F, S}];
  (* Calculate the eigenvalues and eigenvectors *)
  {λ, ψ} = {λ[[#]], ψ[[#]]} &@Ordering[λ]; (* Sort them in ascending order *)
  ψ = MatrixNormalize[#, S] &/@ψ;
  (* Normalize them w.r.t. the overlap matrix S *)
  (* Perform a numerical divergence check *)
  If[AllTrue[Thread[#.S.# ≠ 1] &/@ψ, TrueQ], Abort[], Nothing];
  (* Calculate the density matrix from the
    eigenvector ψ1 associated with the minimal eigenvalue *)
  P = 2 TensorProduct[ψ1, ψ1];
];

(* Return the first eigenvalue and the equilibrium bonding energy,
where  $\frac{1}{d}$  is the Enuc1 term *)
(* The `Total[#, 2]` function
gives the sum of all elements of the matrix # *)
{λ1, Total[P (H +  $\frac{1}{4}$  G), 2] +  $\frac{1}{d}$ }
];

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In[36]:= δ = Range[0.3, 10, 0.01] // N;
ε = 10-4 // N; (* Accuracy level *)

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{τ, ρ} = ParallelMap[Quiet@SCFLoop[#, ε, α] &, δ] // Transpose // AbsoluteTiming;
```

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In[39]:= Length@δ (* Number of distances used *)
Quantity[τ, "Seconds"]
(* Time it took the loop to finish, measured in seconds *)

```

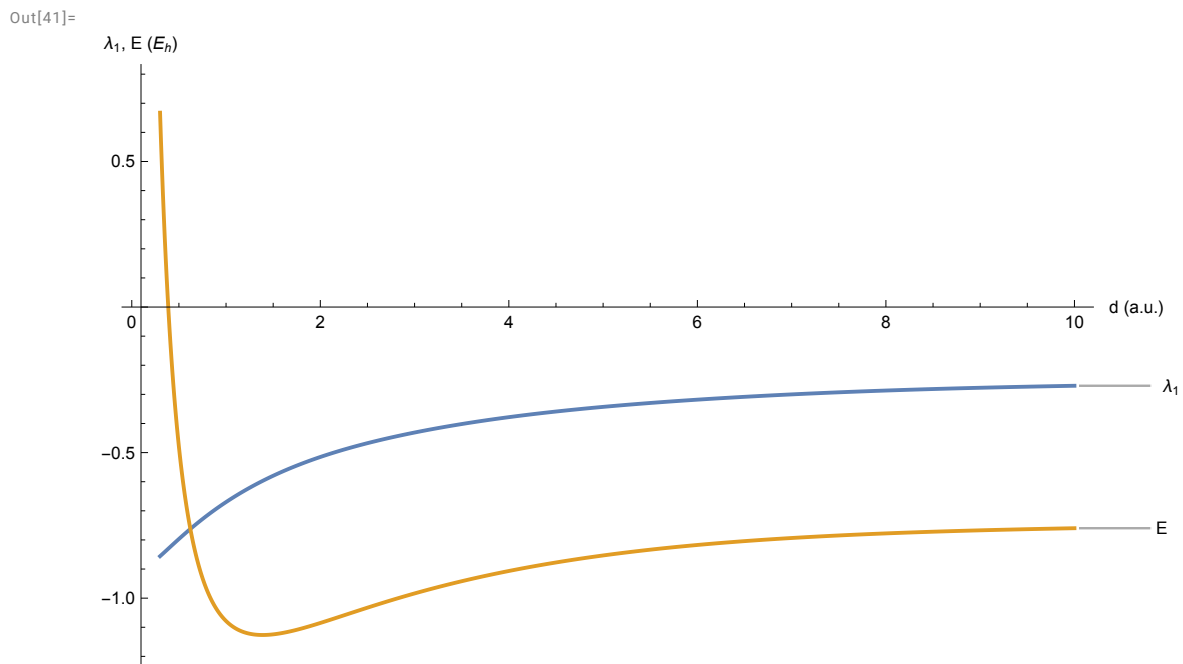
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Out[39]=
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```
971
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```
Out[40]=
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```
8.97029 s
```

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In[41]:= (* Smallest eigenvalue  $\lambda_1$  and equilibrium bonding energy E *)
ListLinePlot[Thread[{ $\delta$ , #}] & /@  $\rho$ , AxesLabel → {"d (a.u.)", " $\lambda_1$ , E ( $E_h$ )"},
PlotRange → Full, ImageSize → Large, PlotLabels → {" $\lambda_1$ ", "E"}]
```



```
In[42]:= Quantity[Min[Last@ $\rho$ ], "HartreeEnergy"] (* Smallest energy value*)
UnitConvert[%, "Electronvolts"] (* ...converted in electronvolts *)
```

Out[42]=
-1.12654 E_h

Out[43]=
-30.6548 eV