

$$H(\text{class}) = H\left(\frac{3}{6}, \frac{3}{6}\right) = 1$$

$$H(\overset{y}{\text{class}} | \overset{x}{\text{color}}) = P(X=\text{red}) \cdot H(Y | X=\text{red}) \\ + P(X=\text{blue}) \cdot H(Y | X=\text{blue}) \\ + P(X=\text{green}) \cdot H(Y | X=\text{green})$$

$$\begin{aligned} P(X=\text{red}) &= \frac{3}{6} & P(X=\text{blue}) &= \frac{1}{6} & P(X=\text{green}) &= \frac{2}{6} \\ H(Y|X=\text{red}) &= H\left(\frac{2}{3}, \frac{1}{3}\right) = \sum_{i=1}^2 -p_i \log(p_i) \\ &\quad \begin{array}{c} \nearrow \oplus \quad \nearrow \ominus \\ 3 \text{ reds} \quad \searrow \begin{array}{c} 2 \oplus \\ 1 \ominus \end{array} \end{array} \\ H(Y|X=\text{blue}) &= H(1,0) = 0 \\ H(Y|X=\text{green}) &= H(0,1) = 0 \\ &= -\frac{2}{3} \log_2\left(\frac{2}{3}\right) - \frac{1}{3} \log_2\left(\frac{1}{3}\right) \\ &= +0.98 \end{aligned}$$

$$H(\text{class}|\text{color}) = \underbrace{P(x=\text{red})}_{3/6} \times 0.98 \\ + 0 \\ + 0 \\ = 0.459$$

$$I(\text{class}, \text{color}) = H(\text{class}) - H(\text{class} | \text{color}) = 1 - 0.459 = 0.541$$