Exercise 12, Discrete Mathematics for Bioinformatics

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12.1 Inverse Queens Problem

a) Variables

$$x_i \in \{1, ..., n\} \quad \text{for } 1 \leq i \leq n$$

Constraints

$$x_i = x_j \vee |x_i - x_j| = |i - j| \quad \forall i \neq j$$

b) Solve for $n = 4$ and $D_1 = \{2\}$.

Forward checking: $D_1 = D_2 = D_3 = D_4 = \{1, 2, 3, 4\}$

- $x_1 = 2 \Rightarrow D_2 = \{1, 2, 3\}, D_3 = \{2, 4\}, D_4 = \{2\}$
  - $x_2 = 1 \Rightarrow D_3 = \{2\}, D_4 = \{\}$ ... dead end.
  - $x_2 = 2 \Rightarrow D_3 = \{2\}, D_4 = \{2\}$
    - Solution found

Patial lookahead: $D_1 = D_2 = D_3 = D_4 = \{1, 2, 3, 4\}$

- $x_1 = 2 \Rightarrow D_2 = \{2\}$ because values 1 or 3 are not arc consistent with x_4. $D_3 = \{2\}$ because value 4 is not arc consistent with x_4. $D_4 = \{2\}$.
  - Solution found

12.2 Task Scheduling

12.3 Bin Packing

12.4 IP

Each constraint of the form $|x_i - x_j| \geq 2$ can be rewritten as

$$x_i - x_j \geq 2 \vee x_j - x_i \geq 2$$

We can express the logical *or* by adding a new variable d_{ij} (decision variable):

$$\begin{array}{rcl} x_i - x_j \geq 2 & \vee & x_j - x_i \geq 2 \\ \Leftrightarrow x_i - x_j \geq d_{ij}(-2 - m) + 2 & \wedge & x_j - x_i \geq (1 - d_{ij})(-2 - m) + 2 \\ \Leftrightarrow x_i - x_j + (2 + m)d_{ij} \geq 2 & \wedge & x_j - x_i - (2 + m)d_{ij} \geq -m \end{array}$$

The decision variable decides which constraint must be satisfied:

$$\begin{array}{lrcl} \text{if } d_{ij} = 0 & x_i - x_j \geq 2 & \wedge & x_j - x_i \geq -m \\ \text{if } d_{ij} = 1 & x_i - x_j \geq -m & \wedge & x_j - x_i + \geq 2 \end{array}$$

Note that these equivalencies only hold since $x_i - x_j \geq 2$ and $x_j - x_i \geq 2$ cannot be true at the same time.

In order to model these inequalities $\forall i \neq j$, we have to apply this technique to $n^2 - n$ constraints, thus introducing $(n^2 - n)$ new binary variables.