Exercise 8, Discrete Mathematics for Bioinformatics

Sascha Meiers, Martin Seeger

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8.1 Linear Optimization

$$2x_1 + 3x_2 = \min \Leftrightarrow -2x_1 - 3x_2 = \max\,. \tag{1}$$

$$3x_1 + 6x_2 \leq 7 \Leftrightarrow 3x_1 + 6x_2 + x_3 = 7,\ x_3 \geq 0. \tag{2}$$

$$x_1 \text{ free} \Leftrightarrow x_1 = x_4 - x_5,\ x_4 \geq 0,\ x_5 \geq 0.$$

Insert the previous equation into (2):

$$6x_2 + x_3 + 3x_4 - 3x_5 = 7,\ x_3 \geq 0,\ x_4 \geq 0,\ x_5 \geq 0.$$

Finally,

$$2x_1 + 2x_2 = 5 \Leftrightarrow 2x_2 + 2x_4 - 2x_5 = 5,$$

$$-2x_1 - 3x_2 = -3x_2 - 2x_4 + 2x_5 = \max\,.$$

Summarizing,

$$\max \left\{ (-3, 0, -2, 2) \begin{pmatrix} x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \middle| \begin{pmatrix} 6 & 1 & 3 & -3 \\ 2 & 0 & 2 & -2 \end{pmatrix} \begin{pmatrix} x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 7 \\ 5 \end{pmatrix}, \begin{pmatrix} x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \geq 0 \right\}.$$

8.2 Linear Optimization

x

8.3 Linear Optimization

a) For $c^T = (-1, -1)$ we have to maximize $-x_1 - x_2$, i.e. minimize $x_1 + x_2$.
In the feasible region, this is the case for $(x_1, x_2) = (0, 0)$.

b) For $c^T = (0, -1)$ we have to maximize $-x_2$, i.e. minimize x_2.
In the feasible region, this is the case for $(x_1, x_2) = (\lambda, 0)$, $\lambda \in [0, 1]$.

c) For $c^T = (-1, 0)$ we have to maximize $-x_1$, i.e. minimize x_1.
In the feasible region, this is the case for $(x_1, x_2) = (0, \lambda)$, $\lambda \in [0, \infty[$.

d) For $c^T = (1, 1)$ we have to maximize $x_1 + x_2$.
Since e.g. the half-line $x_1 - x_2 = 0$, $x_1, x_2 \geq 0$ lies in the feasible region, and as the target function increases strictly along this half-line, there is no optimal solution.

If the following constraint is added, the problem becomes infeasible:

$$x_1 - x_2 \geq 2.$$

8.4 Profit Optimization

x