

Exercise 5, Discrete Mathematics for Bioinformatics

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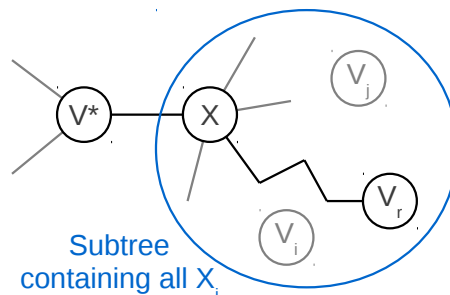
3.1 Tree decomposition

Let $G = (V, E)$ be a graph with $V = v_1, \dots, v_n$ and $E = \binom{V}{2}$. We'll prove that the graph's tree width is $n - 1$, meaning that any tree decomposition of G contains at least one piece with n elements.

Proof: Given a tree decomposition T , let V^* be the largest piece and assume that $v_n \notin V^*$ without loss of generality. We know by edge coverage property that there must be pieces containing v_n and v_i at the same time, for $1 \leq i \leq n - 1$. Let these edges be covered by the k pieces V_1, \dots, V_k with $2 \leq k \leq n - 1$ (but there could also be other pieces). k cannot be one since then V_k would be larger than V^* .

Now we analyze the structure of the tree T and regard two cases:

1. The piece V^* is somewhere "between" the pieces V_i . This means, there is at least one pair (i, j) such that V^* lies on a path from V_i to V_j . V_i and V_j both contain v_n , but V^* does not. This hurts the coherence property \Rightarrow contradiction
2. The piece V^* is not between the pieces V_i . It could be a leaf of the tree, but there could also be further pieces connected to it. However, V^* is connected to the subtree that contains all V_i by a single edge. Let X be the next piece on the path from V^* to any V_i . Usually¹ the piece X is missing at least one node $v_l \neq v_n$. We know that there is at least one piece V_r in the subtree that contains v_l , and we also know $v_l \in V^*$. By coherence property, X would also have to contain $v_l \Rightarrow$ contradiction.



¹ X can contain at most $n - 1$ nodes, so at least one node is missing, as stated. Theoretically, the missing node could be v_n . But in this case, we have $X = V^*$ and (if this is allowed at all) the argumentation (case 1 or 2) can be applied on X itself as the largest set.