Exercise 9, Discrete Mathematics for Bioinformatics

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9.1 PORTA — Polyhedron Representation Transformation Algorithm

a)
```
$ soplex -x problem.lp

Solution value is: 2.5000000e+00

Primal solution (name, id, value):
x2 1 1.5000000e+00
x3 2 5.0000000e-01
x4 3 5.0000000e-01
All other variables are zero (within 1.0e-09).
```

b) Input file:
```
DIM = 4

LOWER_BOUNDS
0 0 0 0
UPPER_BOUNDS
2 2 1 1         \ We derived these bounds from the inequalities.

INEQUALITIES_SECTION
x1 + x2 + x3      <= 2
x1 + x2      + x4 <= 2
          x3 + x4 <= 1
END
```

Output file of *vint*:
```
DIM =  4

CONV_SECTION
(   1) 0 0 0 0
(   2) 0 0 0 1
(   3) 0 0 1 0
(   4) 0 1 0 0
(   5) 0 1 0 1
(   6) 0 1 1 0
(   7) 0 2 0 0
(   8) 1 0 0 0
(   9) 1 0 0 1
```

```
( 10) 1 0 1 0
( 11) 1 1 0 0
( 12) 2 0 0 0

END
```

These are all feasible points.

c) *traf* transforms these points into a inequality-representation:

```
DIM = 4

VALID
2 0 0 0

INEQUALITIES_SECTION
(  1) -x1          <= 0
(  2)    -x2       <= 0
(  3)       -x3    <= 0
(  4)          -x4 <= 0
(  5)       +x3+x4 <= 1
(  6) +x1+x2+x3+x4 <= 2

END
```

d) We solve the new LP using *soplex* and get:

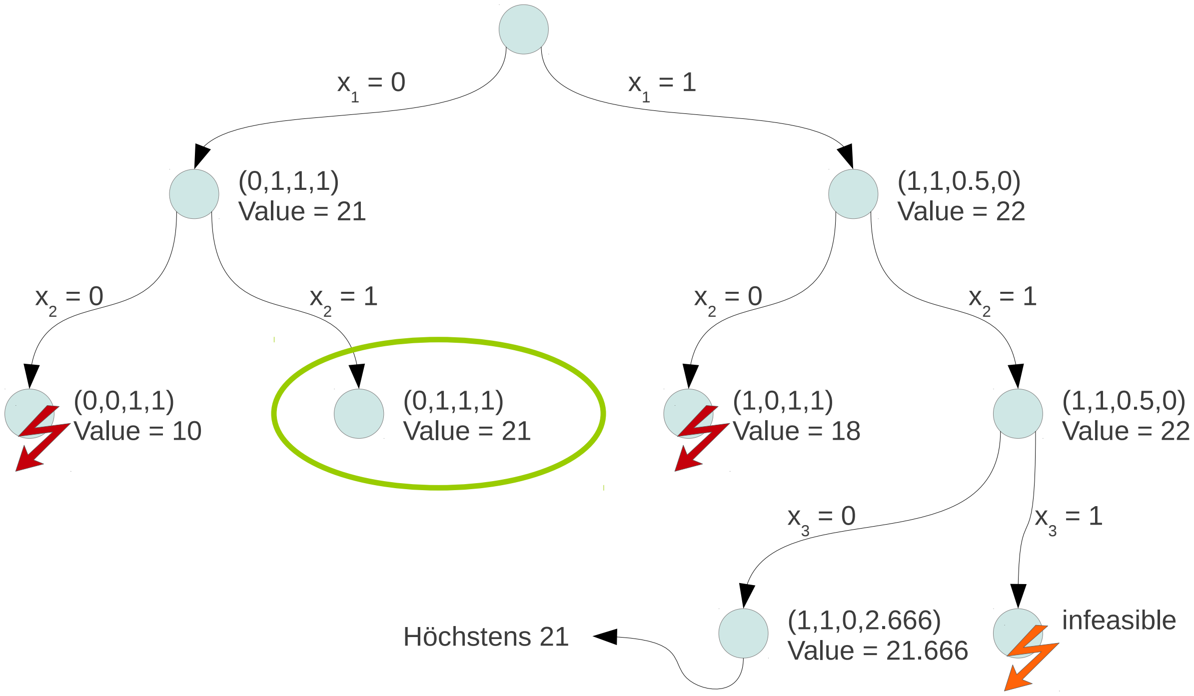
```
Solution value is: 2.0000000e+00

Primal solution (name, id, value):
x1 0 2.0000000e+00
All other variables are zero (within 1.0e-09).
```

9.2 Branch and Bound

a)
```
$ lp_solve -S3 example2.lp

Value of objective function: 22.4

Actual values of the variables:
x1                            2.8
x2                              0
x3                              0
x4                              0

Actual values of the constraints:
c1                             14
```

b) We manually find a solution $(0, 1, 1, 1)$, which is feasible, since $7 \cdot 1 + 4 \cdot 1 + 3 \cdot 1 \leq 14$, and gives a **global lower bound** for the ILP of 21.

The solution of the LP-relaxation (with all variables bound to $0 \leq x_i \leq 1$) gives an optimal value of 22 in the point $(1, 1, 0.5, 0)$. Since the variables are not integral we start branching now:


x1 = 0
x1 = 1
(0,1,1,1)
Value = 21
(1,1,0.5,0)
Value = 22
x2 = 0
x2 = 1
x2 = 0
x2 = 1
(0,0,1,1)
Value = 10
(0,1,1,1)
Value = 21
(1,0,1,1)
Value = 18
(1,1,0.5,0)
Value = 22
x3 = 0
x3 = 1
Höchstens 21
(1,1,0,2.666)
Value = 21.666
infeasible