Exercise 8, Discrete Mathematics for Bioinformatics

Sascha Meiers, Martin Seeger

Winter term 2011/2012

8.1 Bases and Basic Solutions

a)

$$x_1 + x_2 \leq 4, \tag{1}$$
$$x_1 \leq 4, \tag{2}$$
$$-x_1 \leq 0, \tag{3}$$
$$x_2 \leq 2, \tag{4}$$
$$-x_2 \leq 0. \tag{5}$$

We note that (2) is dependent on the other inequalities and can be left out. What remains is

$$Ax = \begin{pmatrix} 1 & 1 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \leq b = \begin{pmatrix} 4 \\ 0 \\ 2 \\ 0 \end{pmatrix}.$$

b) Let $M = \{1, 2, 3, 4\}$. We look for bases, i.e. cardinality 2 subsets $I \subset M$ for which A_{I*} is regular. The bases are

$$\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}.$$

The corresponding basic solutions are $A_{I*}^{-1} b_I$, i.e.

$$\begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}, \tag{6}$$

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}, \tag{7}$$

$$\begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \tag{8}$$

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}, \tag{9}$$

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{10}$$

c) Feasible means $Ax \leq b$. This is the case for (7)–(10) but not for (6), since there $4 = x_2 > 2$.

d) $I = \{1,3\}$: $x = (2,2)$,
$I = \{1,4\}$: $x = (4,0)$,
$I = \{2,3\}$: $x = (0,2)$,
$I = \{2,4\}$: $x = (0,0)$.

8.2 Simplex Algorithm

a) Let x_1 = volume of coke, and x_2 = volume of beer. Then we have

$$\text{profit} = u^T x = x_1 + 2x_2 = \max!,$$

$$\text{weight} = x_1 + 1.5x_2 \leq 150,$$

$$\text{beer availability} = x_2 \leq 35,$$

$$\text{maximum alcoholic volume} = (x_1 + x_2) \cdot 2/3 \geq x_2 \iff -2x_1 + x_2 \leq 0.$$

A and b can simply be read off.

b) Simplex algorithm: see handwritten sheet.

8.3 Duality

x