

Exercise 8, Discrete Mathematics for Bioinformatics

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8.1 Bases and Basic Solutions

a)

$$x_1 + x_2 \leq 4, \quad (1)$$

$$x_1 \leq 4, \quad (2)$$

$$-x_1 \leq 0, \quad (3)$$

$$x_2 \leq 2, \quad (4)$$

$$-x_2 \leq 0. \quad (5)$$

We note that (2) is dependent on the other inequalities and can be left out. What remains is

$$Ax = \begin{pmatrix} 1 & 1 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \leq b = \begin{pmatrix} 4 \\ 0 \\ 2 \\ 0 \end{pmatrix}.$$

b) Let $M = \{1, 2, 3, 4\}$. We look for bases, i.e. cardinality 2 subsets $I \subset M$ for which A_{I*} is regular. The bases are

$$\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}.$$

The corresponding basic solutions are $A_{I*}^{-1}b_I$, i.e.

$$\begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}, \quad (6)$$

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}, \quad (7)$$

$$\begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \quad (8)$$

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}, \quad (9)$$

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \quad (10)$$

c) Feasible means $Ax \leq b$. This is the case for (7)–(10) but not for (6), since there $4 = x_2 > 2$.

- d) $I = \{1, 3\}: x = (2, 2),$
 $I = \{1, 4\}: x = (4, 0),$
 $I = \{2, 3\}: x = (0, 2),$
 $I = \{2, 4\}: x = (0, 0).$

8.2 Simplex Algorithm

x

8.3 Duality

x