Exercise 9, Discrete Mathematics for Bioinformatics

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9.1 PORTA — Polyhedron Representation Transformation Algorithm

a)
```
$ soplex -x problem.lp

Solution value is: 2.5000000e+00

Primal solution (name, id, value):
x2 1 1.5000000e+00
x3 2 5.0000000e-01
x4 3 5.0000000e-01
All other variables are zero (within 1.0e-09).
```

b) Input file:

```
DIM = 4

LOWER_BOUNDS
0 0 0 0
UPPER_BOUNDS
1 1 1 1         \ We assume to have boolean variables.

INEQUALITIES_SECTION
x1 + x2 + x3      <= 2
x1 + x2      + x4 <= 2
          x3 + x4 <= 1
END
```

Output file of *vint*:

```
DIM =  4

CONV_SECTION
(  1) 0 0 0 0
(  2) 0 0 0 1
(  3) 0 0 1 0
(  4) 0 1 0 0
(  5) 0 1 0 1
(  6) 0 1 1 0
(  7) 1 0 0 0
(  8) 1 0 0 1
(  9) 1 0 1 0
```

```
( 10) 1 1 0 0

END
```

These are all feasible points.

c) *traf* transforms these points into a inequality-representation:

```
DIM = 4

VALID
1 1 0 0

INEQUALITIES_SECTION
(  1) -x1          <= 0
(  2)    -x2       <= 0
(  3)       -x3    <= 0
(  4)          -x4 <= 0
(  5)    +x2       <= 1
(  6) +x1          <= 1
(  7)       +x3+x4 <= 1
(  8) +x1+x2+x3+x4 <= 2

END
```

d) We solve the new LP using *soplex* and get:

```
Solution value is: 2.0000000e+00

Primal solution (name, id, value):
x1 0 1.0000000e+00
x2 1 1.0000000e+00
All other variables are zero (within 1.0e-09).
```

9.2 Branch and Bound

a)
```
$ lp_solve -S3 example2.lp

Value of objective function: 22.4

Actual values of the variables:
x1                            2.8
x2                              0
x3                              0
x4                              0

Actual values of the constraints:
c1                             14
```

b) Let us call the original problem P_0. We branch the problem into two problems: $P_{11} = P_0 \wedge x_1 \leq 2$ and $P_{12} = P_0 \wedge x_1 \geq 3$.

Result of solving P_{11}:

```
$ lp_solve -S3 example21.lp
```

```
Value of objective function: 22.2857

Actual values of the variables:
x1                              2
x2                       0.571429
x3                              0
x4                              0

Actual values of the constraints:
c1                             14
c2                              2
```

Result of solving P_{12}:

```
$ lp_solve -S3 example22.lp
This problem is infeasible
```

Hence, we branch further into $P_{21} = P_{11} \wedge x_2 \leq 0$ and $P_{22} = P_{11} \wedge x_2 \geq 1$.

Result of solving P_{21}:

```
$ lp_solve -S3 example221.lp

Value of objective function: 22

Actual values of the variables:
x1                              2
x2                              0
x3                              1
x4                              0

Actual values of the constraints:
c1                             14
c2                              2
c3                              0
```

Result of solving P_{22}:

```
$ lp_solve -S3 example222.lp

Value of objective function: 22.2

Actual values of the variables:
x1                            1.4
x2                              1
x3                              0
x4                              0

Actual values of the constraints:
c1                             14
c2                            1.4
c3                              1
```

Since $22 = \lfloor 22.4 \rfloor$, and the solutions of P_{21} are integers, this is the solution of the branch and bound ILP algorithm.