Exercise 1, Discrete Mathematics for Bioinformatics

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1.1 MST Approximation

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1.2 Landau Symbols

a) Let $k, l \in \mathbb{Z}$, $k > l$. $f = o(g)$ holds iff

$$\lim_{n\to\infty} \left| \frac{f(n)}{g(n)} \right| = 0. \tag{1}$$

In our case,

$$\lim_{n\to\infty} \left| \frac{f(n)}{g(n)} \right| = \lim_{n\to\infty} \left| \frac{n^l}{n^k} \right| = \lim_{n\to\infty} \left| \frac{1}{n^{k-l}} \right| = 0, \tag{2}$$

whence it follows that $n^l = o(n^k)$.□

b) Let $k, l \in \mathbb{N}$, $k > l$. In general, $f = \Theta(g)$ iff $f = O(g)$ and $g = O(f)$. We use the definition $f = O(g)$ iff

$$0 \leq \limsup_{n\to\infty} \left| \frac{f(n)}{g(n)} \right| < \infty. \tag{3}$$

In our case,

$$\limsup_{n\to\infty} \left| \frac{n^k + n^l}{n^k} \right| = \limsup_{n\to\infty} \left| 1 + \frac{1}{n^{k-l}} \right| = 1, \tag{4}$$

and

$$\limsup_{n\to\infty} \left| \frac{n^k}{n^k + n^l} \right| = \limsup_{n\to\infty} \left| 1 - \frac{n^l}{n^k + n^l} \right| = \limsup_{n\to\infty} \left| 1 - \frac{1}{n^{k-l} + 1} \right| = 1.\square \tag{5}$$

c) Counterexample: $f(n) = 2^{cn}$ with $c > 1$ is clearly $2^{O(n)}$. However,

$$\limsup_{n\to\infty} \left| \frac{2^{cn}}{2^n} \right| = \limsup_{n\to\infty} 2^{(c-1)n} = \infty, \tag{6}$$

hence $f \neq O(2^n)$.□

1.3 Amortized Analysis

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1.4 Analysis of SELECTION algorithm

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