COSC 420/527: Biologically-Inspired Computation
Lab 3: Hopfield Networks

Due: March 10, 11:59 PM

Introduction

In this lab, you will be investigating the associative memory capacity of a Hopfield network. You will be implementing the software to run the experiments. You may write the code in whatever programming language you prefer. You will turn in the code as part of your submission, along with the report PDF.

Experimental Setup

You will be investigating the associative memory aspect of Hopfield networks with $N = 100$ neurons and no self-coupling. To examine the memory capacity of the network, you will be completing a series of experiments.

In each experiment, you will do the following:

1. Generate 50 random bipolar vectors ($x \in \{-1, 1\}^{100}$).
2. For $p = 1, ..., 50$, imprint the first p patterns onto the Hopfield network using the following rule. Let $x^1, ..., x^p$ be the patterns we want to imprint. Then, $W_{ij} = \frac{1}{n} \sum_{k=1}^{p} x_i^k x_j^k$. Note that $W_{ii} = 0$.
3. Test the p imprinted patterns for stability. For each of the p patterns:
   (a) Set the state of the network to that pattern.
   (b) For each of the 100 neurons, first compute its new state using the following formulas:

$$h_i = \sum_{j=1}^{N} w_{ij} s_j \tag{1}$$

$$s_i' = \sigma(h_i) \tag{2}$$

$$\sigma(h_i) = \begin{cases} -1 & h_i < 0 \\ 1 & h_i \geq 0 \end{cases} \tag{3}$$

   If any of $s_i' \neq s_i$, then the imprinted pattern is NOT stable. Otherwise, it is stable.

(c) Keep a counter of how many of these imprinted patterns are stable for each value p. You can keep track of each counter with an array of 50 elements (one for each p).

(d) To compute the probability of stable imprints for each p, just divide the number of stable imprints for that p by that number p. To get the probability of UNstable imprints for that p, subtract the probability of stable imprints for that p from 1.

4. Generate data needed for the following two graphs:

   (a) The fraction of unstable imprints as a function of the number of imprints (p=1 to 50). That is, the x-axis will be the number of imprints (p) and the y-axis will be the fraction of unstable imprints.

   (b) The number of stable imprints as a function of the number of imprints. That is, the x-axis will be the number of imprints and the y-axis will be the number of stable imprints.

You will repeat this process 5 times for different random collections of 50 patterns.

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You will also estimate the size of the basins of attraction for each imprinted pattern of each value of p. In the part above where you test for stability of each of the p imprinted patterns, add the following:

1. If the pattern is unstable, set its basin size to 0.

2. If the pattern is stable:

   (a) Generate a permutation of the numbers 1 to 100 by creating an array of the numbers 1 to 100 in random order. This list of numbers will indicate which bits of the pattern to flip and in which order to flip them. You will only need to consider the first 50 elements of this permutation since the maximum basin size is 50.

   (b) Letting i be the loop variable, go through each of the first 50 elements in the permutation array and:

      i. Initialize the neural network to the current pattern of the p imprinted patterns.

      ii. Flip the states of the positions of the neural network given by the first i permutation array elements. That is, change a 1 to a -1 and a -1 to a 1 in these i positions of the neural network.

      iii. Go through 10 iterations of updating the neural network. Change each network element according to sigma of its local field h.

iv. Check to see if the network is equal to the current imprinted pattern after these 10 iterations.

(c) The first iteration of the permutation array (as given by i) where the network does not converge to the current imprinted pattern is the number that estimates the size of the basin of attraction for that imprinted pattern. It is equivalent to the number of bits in the pattern you need to flip until the network does not converge to that pattern. If the network converges for all 50 iterations of the permutation array (that is, it never doesn't converge), then let the size of the basin of attraction for that pattern be 50, since that is the maximum size of a basin of attraction.

3. Increment the counter of the array keeping track of the basin sizes. You might have a two dimensional array where the first dimension indicates the number of imprinted patterns, and the second dimension gives the size of the basin of attraction. This will keep track of a histogram of basin sizes for each value p.

4. Produce a graph of the histograms for various values of p (even values of p should be enough). This graph should have on the x-axis the basin size. On the y-axis, you should calculate the fraction of imprinted patterns that have that basin size. There will be separate line for each even value of p. You should use colors and markers to indicate differences in the lines and include a legend for the plot.

Report Write-Up

Your report write-up should include the following information:

Graphs

You will include two graphs where the x-axis will be the number of imprints (p) and the y-axis will be the fraction of unstable imprints. The first graph should show each of the lines of the 5 experiments separately. The second graph should show the line representing the average across all experiments, along with lines showing the average plus the standard deviation and average minus the standard deviation of the experimental results.

You will also include two graphs where the x-axis will be the number of imprints and the y-axis will be the number of stable imprints. Again, the first graph should show each of the lines of the 5 experiments separately. The second graph should show the line representing the average across all experiments, along with lines showing the average plus the standard deviation and average minus the standard deviation of the experimental results.

For CS 527: You will also include graphs showing the histograms for various values of p in terms of the basin size, as described above. Because these graphs will already be cluttered, you should have separate graphs for each of the 5 experiments. All 5 of these graphs should be included in the report.

Discussion

The questions/discussion points you should address in your report are:

- How much variation did you see in capacity across the different experiments? Were there significant differences across experiments?
- Does it appear that there's a maximum capacity of the network? When does the maximum capacity occur?
- **CS 527**: How do the basins of attraction sizes change as the number of patterns imprinted increases? Does this change significantly across experiments?

You should have at least 4 sentences (a good size paragraph) for each of these discussion questions.

Submission

You will submit your report PDF, as well as the code you wrote to perform the experiments and any files that are required to compile and/or run the code. You should include instructions for compiling and/or running your code in a README file. Your code should be documented.

Grading

Undergraduate Grading Breakdown:

- **60 points** for the code you will create and the associated documentation (README to compile and/or run the code).
- **20 points total**: 10 points for each of the two types of plots showing capacity of the network.
- **20 points:** 10 points for each of the discussion questions listed above. You should include at least a paragraph (of at least 4 sentences) for each discussion point.

Total: 100 points

For undergraduates, if you complete the basin of attraction analysis required for CS 527, that will be an additional 15 points.

Graduate Grading Breakdown:

- **50 points** for the code you will create and the associated documentation (README to compile and/or run the code).
- **10 points total**: 5 points for each of the two types of plots showing capacity of the network.
- **10 points:** Graph/plot of the histograms for various values of p in terms of basin size.
- **30 points:** 10 points for each of the discussion questions listed above. You should include at least a paragraph (of at least 4 sentences) for each discussion point.

Total: 100 points