COSC 420/527: Biologically-Inspired Computation
Lab 1: "Edge of Chaos" in 1D Cellular Automata

Due: February 10, 2023, 11:59 PM

Introduction

In this project you will explore "Edge of Chaos" phenomena (Wolfram class IV behavior) in 1D cellular automata. You will do this by systematically modifying randomly generated transition tables and observing the lambda and entropy values associated with phase changes in the behavior of the automata.

Experimental Setup

You will use an off-the-shelf simulator for this project. We have adapted a Java program used for this class previously into a Python script. This script automates nearly everything for you. This script takes the following as arguments:

- -e or –experiments : Number of experiments (required)
- –seed : RNG seed. Default is the current time
- –exp_dir : Experiment file directory. Default is experiments.
- –master_file : Master results file name. Default is MasterExperiments.csv

This script will generate 13 cellular automata and their corresponding images as PNGs per experiment in the experiments directory. The corresponding experimental data will be recorded in MasterExperiments.csv.

Table-Walk-Through Procedure

As explained in class, we cannot simply allow the simulator to pick a random rule string with a certain λ value, since there is too much variability between unrelated rule strings. Therefore we will use Chris Langton's table-walk-through method, which involves generating a series of rule strings differing only in the number of quiescent entries. The following describes the operational procedure.

Each experiment is conducted on a random rule string. This rule string is recorded in the master experiments file for each experiment. The script will now decimate the rule string by zeroing one non-zero entry at a time. For the original rule string and for each of the decimated rule strings the following values will be computed: λ, λ_T, H, and H_T value. You will also observe and record the behavior (I, II, III, or IV) of the original string and each of its decimations. There are columns for the class behavior, as well as notes or observations for each experiment and each step in the experiment.

For each decimation, one of the non-zero entries is set to 0. This is repeated until all of the values are zeroed. You will make observations for each of these different rules (the different CAs as they are run).

Lambda and Entropy Calculations

You will be investigating lambda and entropy values computed in two different ways. The simulator computes these for you, but you still need to understand what they mean.

The simplest way gives the totalistic parameters λ_T and H_T, where are based on how frequently each state appears in the totalistic rule table (that is, the rule string that you have recorded). The minimum sum of the neighborhood states is 0 and the maximum is $N(K-1)$, where K is the number of states and $N = 2r+1$ is the size of the neighborhood for radius r.

Therefore the size of the totalistic table is $S = N(K-1)+1$. In the case for this lab, we will be using $r = 1$ and $K = 5$, so $S = 13$. Let R be the rule table so that R_k gives the new state for the sum k of the neighborhood states. We are interested in the distribution new-state values, so let $m_s = |\{R_k|R_k = s\}|$ that is, m_s is the number of table entries that map into the state s. The totalistic lambda value λ_T is the fraction of totalistic rule table entries that do not map into the quiescent state $s = 0$.

$$\lambda_T = \frac{S - m_0}{S} = 1 - \frac{m_0}{S}$$

The entropy of the totalistic rule table is a function of its probability distribution. Therefore, let $p_s = \frac{m_s}{S}$, so that the entropy H_T of the totalistic transition table is defined:

$$H_T = -\sum_s p_s \lg p_s$$

where $\lg x$ is the logarithm of x to the base 2. (Note that $0 \lg 0 = 0$, so there is a special case for that.)

To facilitate comparison with non-totalistic CAs, the λ and H values are usually defined over the complete (non-totalistic) transition table, which has $T = K^N$ entries. However, we have an abbreviated rule table of length $N(K-1)+1$, which the new state R_k for a sum k; in most cases, there are multiple configurations having the same sum. Therefore let C_k be the number of neighborhood configurations having the sum k. For $K = 5$ and $r = 1$, C_k is given by the following table:

Define n_s to be the number of configurations leading to state s:

k	0	1	2	3	4	5	6	7	8	9	10	11	12
C_k	1	3	6	10	15	18	19	18	15	10	6	3	1

$$n_s = \sum_{\{k|R_k=s\}} C_k$$

That is, n_s is the total number of configurations that are mapped into state s by the corresponding non-totalistic rule table. The probability of a new state in the complete transition table is given by $p_s = n_s/T$. (Note: these p_s are different from the p_s defined above for H_T.) Then, Langton's λ is defined:

$$\lambda = \frac{T - n_0}{T} = 1 - \frac{n_0}{T} = 1 - p_0$$

The entropy of the complete transition table is defined:

$$H = -\sum_s p_s \lg p_s$$

Additional Work for CS 527 Students

If you are taking this course for graduate credit (CS 527), in addition to the forementioned four parameters, you will need to compute a fifth parameter for the rules you test. But you get to pick or design the parameter! Think about what property of the transition tables might be correlated with the behavior class of a CA and figure out a way to compute it. Your writeup (see below) should include a short (one paragraph) motivation for your parameter (i.e., what makes you think that it might be correlated with behavior) and a description of its computation. Your parameter might not turn out to be very good, but that doesn't matter; the point is to make a reasonable conjecture and to test it. In your writeup, evaluate your new parameter in the same ways you do the others (λ, λ_T, H, H_T).

You will add a function to the Python script provided to calculate your new parameter in the same place that the other parameters are calculated. You should also add it to print to the master experiments file.

Students taking this course for undergraduate credit (CS 420) can also do this part, which will earn you extra credit.

Report Write-Up

Your report write-up should include the following information:

Calculations

Compute the average and standard deviation of the λ, λ_T, H, H_T values for all simulation instances that exhibit class IV behavior. Include these values in a table in the report. Which of these (λ, λ_T, H, H_T) seems to be a more reliable indicator of class IV behavior?

Graphs

Make four graphs of behavior vs. λ, λ_T, H, H_T. That is, use λ, λ_T, H, H_T for the x-axes of the four graphs (CS 527 students should make graphs of their fifth parameters). For the y-axes of the graphs, use the following numerical values to indicate qualitative behavior: 0 for classes I and II, 1 for class IV, and 2 for class III. Each of your graphs should show all of your experiments as separate curves; try to use colors or other ways of making the curves distinguishable.

Discussion

Draw some conclusions about the range of values of λ, λ_T, H, H_T that lead to class IV behavior. Note any anomalies. Did you ever observe class I or II behavior at high λ, λ_T, H, H_T values? Did you ever observe complex (IV) or chaotic (III) behavior at λ, λ_T, H, H_T that were otherwise in the simple (I, II) region? How do you explain these anomalies?

Submission

You will submit your write-up as a PDF, your completed master CSV file, with all class labels and observation notes for each experiment. For CS 527 students (or CS 420 students who are going for extra credit), you will also submit your updated code to show how you calculated the fifth parameter. **DO NOT** submit all of the generated image files. However, keep them on hand, in case we request to see them.

Grading

Undergraduate Grading Breakdown:

- **30 points** for 30 total table-walk throughs (including class labels and observations for each experiment)
- **5 points** for report introduction, which should at least include a description of the number of experiments conducted and the number of total class IV behaviors observed. Should also include a few example images of class IV behavior observed.

- **15 points** for table of averages for each of the simulation instances and discussion of which ones are a more reliable indicator
- **40 points total**: 10 points per graph/discussion of each graph for λ, λ_T, H, H_T
- **10 points** for anomaly discussion.

For undergraduates, if you define and calculate the fifth parameter, that will be worth up to an additional 15 points. **The maximum score you can receive is 100.**

Graduate Grading Breakdown:

- **30 points** for 30 total table-walk throughs (including class labels and observations for each experiment)
- **5 points** for report introduction, which should at least include a description of the number of experiments conducted and the number of total class IV behaviors observed. Should also include a few example images of class IV behavior observed.
- **5 points** for fifth parameter description and implementation.
- **10 points** for table of averages for each of the simulation instances and discussion of which ones are a more reliable indicator
- **40 points total**: 8 points per graph/discussion of each graph for λ, λ_T, H, H_T, and your fifth parameter
- **10 points** for anomaly discussion.