Problem Set 1 – Industrial Organization I

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1. The dynamic programming problem for a firm maximizing the PDV of future profits is given by:

$$V(a_t, \epsilon_t; \theta) = \max_{i_t} \left(\Pi(a_t, i_t, \epsilon_{0t}, \epsilon_{1t}) + \beta E\left[V(a_{t+1}, \epsilon_{t+1}; \theta) \mid a_t, i_t; \theta\right]\right)$$

subject to

$$a_{t+1} = \begin{cases} \min\{5, a_t + 1\} & \text{if } i_t = 0 \\ 1 & \text{if } i_t = 1 \end{cases}$$

and

$$\Pi(a_t, i_t, \epsilon_{0t}, \epsilon_{1t}) = \begin{cases} \theta_1 a_t + \epsilon_{0t} & \text{if } i_t = 0 \\ R + \epsilon_{1t} & \text{if } i_t = 1 \end{cases}$$

2 & 3. Figure 1 presents the estimated alternative-specific value functions for the parameters $(\theta_1 = -1, R = -3)$.

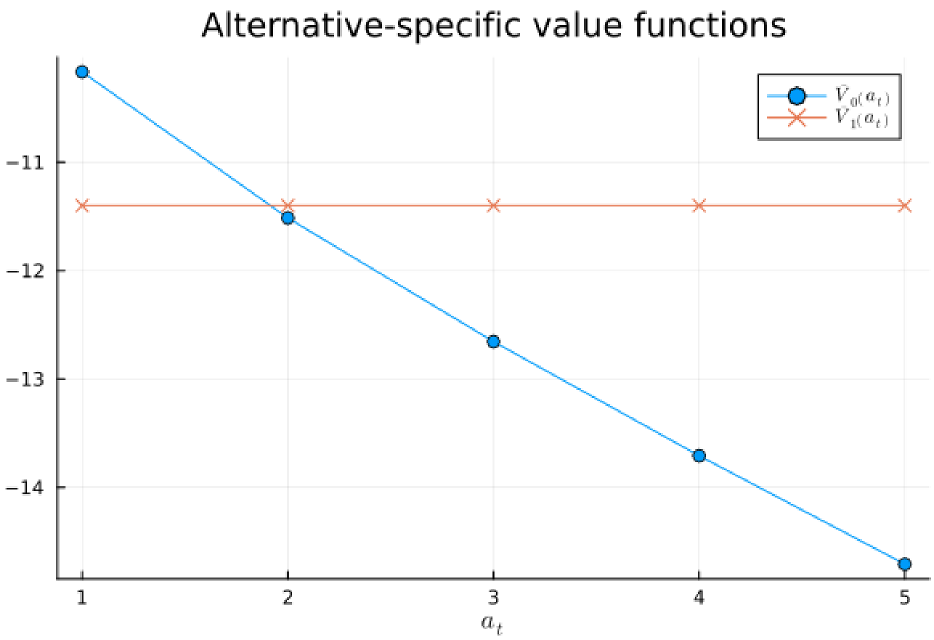

Figure 1: Plot of $\bar{V}_0(a_t)$ and $\bar{V}_1(a_t)$

From the policy function definition, $f(a_t, \epsilon_t) = 1$ iff $\bar{V}_1(a_t; \theta) + \epsilon_{1,t} > \bar{V}_0(a_t; \theta) + \epsilon_{0,t}$. We have that: $\bar{V}_0(2) = -11.515$ and $\bar{V}_1(2) = -11.4$. Therefore, for $\epsilon_{0t} - \epsilon_{1t} = 0.115$, the firm is indifferent between replacing its machine or not.

Under the given assumptions, the probability that this firm will replace its machine is:

$$P(i_t = 1 | a_t) = \frac{\exp \bar{V}_1(a_t)}{\exp \bar{V}_0(a_t) + \exp \bar{V}_1(a_t)}$$

Then, $P(i_t = 1 | a_t = 2) = 52.9\%$.

Finally, we know that the value function is given by:

$$V(a_t, \epsilon_t; \theta) = \max_{i_t} \left\{ \bar{V}_0(a_t, \theta) + \epsilon_{0t}, \bar{V}_1(a_t, \theta) + \epsilon_{1t} \right\}$$

Then, at state $\{a_t = 4, \epsilon_{0t} = 1, \epsilon_{1t} = -1.5\}$, the PDV of future profits is -12.71.

4 & 5. Table 1 presents estimates for parameters and standard errors using maximum likelihood.

Parameter	Estimate	Standard Error
θ_1	-1.15	0.076
R	-4.45	0.325

Table 1: Parameter Estimates and Standard Errors

6. (a) Assume that the data is a random sample of machines. If we observe $a_t = 1$ for firms that did not replace this period ($i_t = 0$), that gives information about λ. More specifically, $P(a_t = 1 \mid i_t = 0)$ is strictly increasing in λ, so this provides the variation we need to identify this parameter. Formally, I would need to change the transition probability matrix $P(x_{t+1}|x_t)$ so it depends on λ. This affects the term $E[V(a_{t+1}, \epsilon_{t+1}; \theta) \mid a_t, i_t = 0; \theta]$.

(b) Since we don't have information on the type of the firms, we cannot separately identify θ_{1A} and θ_{1B} but we can recover $\tilde{\theta}_1 = \alpha \cdot \theta_{1A} + (1 - \alpha) \cdot \theta_{1B}$. In the dynamic program problem, we would have a different value function for each type of firm. The likelihood function does not change.

(c) Take f_j as the firm id. In this case, we can write the likelihood function of observation j as:

$$L_j = \prod_{f=1}^{F} \mathbf{1}\{f_j = f\} \left[i_j \, \text{Prob}(i_j = 1 \mid a_j, f_j) + (1 - i_j)(1 - \text{Prob}(i_j = 1 | a_j, f_j)) \right]$$

(d) The expected profits would now be given by:

$$\Pi\left(a_t, i_t, \epsilon_{0t}, \epsilon_{1t}\right) = \begin{cases} \left[\alpha \cdot \theta_{1A} + (1-\alpha) \cdot \theta_{1B}\right] \cdot a_t + \epsilon_{0t} & \text{if } i_t = 0 \\ R + \epsilon_{1t} & \text{if } i_t = 1 \end{cases}$$

but the other objects would remain the same.

7. For the Hotz and Miller algorithm, we normalize $u\left(a_t, i_t = 1; \theta\right) = 0$. Then, we have: $u\left(a_t, i_t = 0; \theta\right) = \theta_1 a_t - R$. In this case, $i = 1$ is a renewal action.

Define $\tilde{v}_0\left(a_t\right) \equiv u\left(a_t, i_t = 0; \theta\right) + \beta E\left[\log \hat{P}\left(0 \mid a_{t+1}\right) \mid a_t, i_t = 1\right] - \beta E\left[\log \hat{P}\left(0 \mid a_{t+1}\right) \mid a_t, i_t = 0\right]$ and $\hat{\tilde{v}}_0\left(a_t\right) \equiv \ln\left(\hat{P}\left(i_t = 0 \mid a_t\right)\right) - \ln\left(\hat{P}\left(i_t = 1 \mid a_t\right)\right)$. I compute the CCPs non-parametrically.

I then estimate θ by minimum distance:

$$\hat{\theta} = \arg\min_{\theta} \left\| \hat{\tilde{v}}_0\left(a_t\right) - \tilde{v}_0\left(a_t; \theta\right) \right\|$$

Table 2 presents the results. We can see that the estimates are close to the previous ones, but not exactly the same.

Parameter	Estimate
θ_1	-1.11
R	-4.34

Table 2: Parameter Estimates using the Hotz and Miller algorithm