Problem Set 2 – Industrial Organization I

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1. Given these cost and demand primitives, the firm solves:

$$\pi\left(N_t, x_t\right) = \max_{q_i} \left(10 - q_i - \sum_{j \neq i} q_j + x_t\right) \cdot q_i - 5$$

FOC yields:

$$10 - 2q_i - \sum_{j \neq i} q_j + x_t = 0$$

In equilibrium, since firms are symmetric, $q_j = q_i, \forall j$. Then:

$$10 - x_t = \left(N_t + 1\right) \cdot q_i \Rightarrow q_i = \frac{10 + x_t}{N_t + 1}$$

Then, the Nash-equilibrium single-period profits per firm are given by:

$$\pi\left(N_t, x_t\right) = \left(10 + x_t - N_t \frac{10 + x_t}{N_t + 1}\right) \cdot \frac{10 + x_t}{N_t + 1} - 5 = \left(\frac{10 + x_t}{N_t + 1}\right)^2 - 5 \tag{1}$$

2. In each period, the firms are deciding between two choices. The value of one of them is strictly monotonic, and the other is constant in the current shock. Conditional on N_t and x_t, the choice only depends on the private shock. Then, it makes sense to have a deterministic cutoff conditional on N_t and x_t, such that values above/below this threshold imply different actions.

3. $\bar{V}\left(N_t, x_t\right)$ is the present discounted value of being in the market, net of μ_{it}. It's analogous to the alternative specific value functions of Rust (1987) but it doesn't include the current period payoffs and is not defined for the other alternatives.

4. Tables 1 and 2 present the cutoffs for exit and entry, respectively.

$x_t \setminus N_t$	1	2	3	4	5
-5	13.58	6.73	5.09	4.36	5.09
0	35.30	15.29	8.63	6.31	6.04
5	72.62	32.13	17.59	11.15	8.49

Table 1: Values of $\bar{\mu}(N_t, x_t)$

$x_t \setminus N_t$	0	1	2	3	4
-5	26.76	12.32	10.34	11.43	11.85
0	34.58	15.30	8.92	7.02	7.77
5	48.01	21.37	12.13	8.24	6.87

Table 2: Values of $\bar{\gamma}(N_t, x_t)$

- **Why are the matrices** $\Pr\left(N_{t+1} \mid N_t, x_t, d_{it} = 1\right)$ **and** $\Pr\left(N_{t+1} \mid N_t, x_t, e_{it} = 1\right)$ **different (for a given** x_t**)?**

  $\Pr\left(N_{t+1} \mid N_t, x_t, e_{it} = 1\right)$ depends only on the probability that firms stay in the market, while $\Pr\left(N_{t+1} \mid N_t, x_t, d_{it} = 1\right)$ depends both on the probability of entry as well as exit.

- **Why do these equations tell us the new optimal cutoffs (given the computed values of** $\Pr\left(N_{t+1} \mid N_t, x_t, d_{it} = \mathbf{1}\right)$ **and** $\Pr\left(N_{t+1} \mid N_t, x_t, e_{it} = 1\right)$**)?**

  From the value functions, we can see that $\mu_{it} = \pi\left(N_t, x_t\right) + \Psi_1\left(N_t, x_t\right)$ and $\gamma_{it} = \Psi_2\left(N_t, x_t\right)$ would make the firms indifferent between the two choices they face. Therefore, they are exactly the cutoffs we want to find.

- **Where is the last equality coming from?** The last equality comes from the law of iterated expectations:

$$E\left[\max_{d_{it} \in \{0,1\}} \left\{\mu_{it}, \pi\left(N_t, x_t\right) + \Psi_1\left(N_t, x_t\right)\right\}\right] = E\left[E\left[\max_{d_{it} \in \{0,1\}} \left\{\mu_{it}, \pi\left(N_t, x_t\right) + \Psi_1\left(N_t, x_t\right)\right\}\right] \mid d_{it}\right]$$

  The first term corresponds to $\Pr(d_{it} = 0 \mid N_t, x_t) \cdot E[\mu_{it} \mid d_{it} = 0, N_t, x_t]$. The second term is $\Pr(d_{it} = 1 \mid N_t, x_t) \cdot E[\mu_{it} \mid d_{it} = 1, N_t, x_t]$.

5. I get the same equilibrium values for $\bar{V}\left(N_t, x_t\right)$, $\bar{\mu}\left(N_t, x_t\right)$, and $\bar{\gamma}\left(N_t, x_t\right)$ for all 5 different guesses of the initial values. Therefore, there's no evidence of multiple equilibria based on this.

6. I get: $\bar{\mu}(3, 0) = 8.63$, $\bar{\gamma}(3, 0) = 7.02$, $\bar{V}(3, 0) = 8.68$, and $V(3, 0, -2) = 8.63$

7. I get an average number of 3.45 firms in the market across these 10,000 periods.

8. The average number of firms in equilibrium decreases to 3.36. If there are multiple equilibria, it's not possible to answer this question because we could be moving to a different equilibrium with different parameter values. In that case, it wouldn't be possible to disentangle whether the observed changes in the number of firms come from the policy intervention itself or from the different parameters in this new equilibrium.

9 & 10. I follow the estimation procedure with two modifications. First, I allow the draws on the x_t process to vary across simulations, which appears to significantly improve the results. Second, I increase the number of simulations to assess its impact on the estimation accuracy. Table 3 presents the estimates for two different simulation counts. With 50 simulations, the estimates are not very accurate. However, with 1000 simulations, the results closely align with the true parameters.

Parameter	**Simulations = 1000**	**Simulations = 50**
γ	5.32	5.72
σ_γ^2	4.34	15.04
μ	5.46	7.54
σ_μ^2	4.83	11.20

Table 3: Parameter estimates for two simulation sizes.