

**Economics 600a**  
**Fall 2024**  
**Prof. P. Haile**  
**Homework Assignment<sup>1</sup>**  
**Due before midnight October 11**

## 1 Overview

You will estimate demand and supply in a stylized model of the market for pay-TV services. You will use a matrix programming language (Matlab is recommended) to create your own fake data set for the industry and do some relatively simple estimation. Then, using the pyBLP package of Conlon and Gortmaker, you will estimate the model and perform some merger simulations.<sup>2</sup> The pyBLP package has excellent documentation and a very helpful tutorial (which covers merger simulation), both easy to find via Google.

Please submit (on canvas) a single PDF document presenting your answers to the questions below, requested results, and code. Write this up cleanly with nice tables where appropriate. You may work in groups on the coding. However, your write-ups should be your own work.

## 2 Model

There are  $T$  markets, each with four inside goods  $j \in \{1, 2, 3, 4\}$  and an outside option. Goods 1 and 2 are satellite television services (e.g., DirecTV and Dish); goods 3 and 4 are wired television services (e.g., Frontier and Comcast in New Haven). The conditional indirect utility of consumer  $i$  for good  $j$  in market  $t$  is given by

$$u_{ijt} = \beta^{(1)} x_{jt} + \beta_i^{(2)} \text{satellite}_{jt} + \beta_i^{(3)} \text{wired}_{jt} + \alpha p_{jt} + \xi_{jt} + \epsilon_{ijt} \quad j > 0$$
$$u_{i0t} = \epsilon_{i0t},$$

where  $x_{jt}$  is a measure of good  $j$ 's quality,  $p_{jt}$  is its price,  $\text{satellite}_{jt}$  is an indicator equal to 1 for the two satellite services, and  $\text{wired}_{jt}$  is an indicator equal to 1 for the two wired services. The remaining notation is as usual in the class notes, including the i.i.d. type-1 extreme value  $\epsilon_{ijt}$ . Each consumer purchases the good giving them the highest conditional indirect utility.

Goods are produced by single-product firms. Firm  $j$ 's (log) marginal cost in market  $t$  is

$$\ln mc_{jt} = \gamma^0 + w_{jt}\gamma^1 + \omega_{jt}/8,$$

where  $w_{jt}$  is an observed cost shifter. Firms compete by simultaneously choosing prices in each market under complete information. Firm  $j$  has profit

$$\pi_{jt} = \max_{p_{jt}} M_t(p_{jt} - mc_{jt}) s_{jt}(p_t).$$

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<sup>1</sup>Thanks to Jaewon Lee for coding tips and other highly useful feedback on this assignment.

<sup>2</sup>Using data you generate yourself gives you a way to check whether the estimation is working; this is a good thing to try whenever you code up an estimator!

### 3 Generate Fake Data

Generate a data set from the model above. Start with

$$\begin{aligned}\beta^{(1)} &= 1, \beta_i^{(k)} \sim \text{iid } N(4, 1) \text{ for } k = 2, 3 \\ \alpha &= -2 \\ \gamma^{(0)} &= 1/2, \gamma^{(1)} = 1/4.\end{aligned}$$

1. Draw the exogenous product characteristic  $x_{jt}$  for  $T = 600$  geographically defined markets (e.g., cities). Assume each  $x_{jt}$  is equal to the absolute value of an iid standard normal draw, as is each  $w_{jt}$ . Simulate demand and cost unobservables as well, specifying

$$\begin{pmatrix} \xi_{jt} \\ \omega_{jt} \end{pmatrix} \sim N \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0.25 \\ 0.25 & 1 \end{pmatrix} \right) \text{ iid across } j, t.$$

2. Solve for the equilibrium prices for each good in each market.<sup>3</sup>
  - (a) Start by writing a procedure to approximate the derivatives of market shares with respect to prices (taking prices, shares,  $x$ , and demand parameters as inputs). The key steps are:
    - (i) For each  $jt$ , write the choice probability for good  $j$ ,  $s_{jt}$ , as a weighted average (integral) of the (multinomial logit) choice probabilities conditional on the value of each consumer's random coefficients;
    - (ii) Anticipating differentiation under the integral sign, derive the analytical expression for the derivative of the *integrand* with respect to each  $p_{kt}$ ;
    - (iii) Use the expression you obtained in (2) and simulation draws of the random coefficients to approximate the integral that corresponds to  $\partial s_{jt} / \partial p_{kt}$  for each  $j$  and  $k$  (i.e., replace the integral with the mean over the values at each simulation draw). Recall the advice in the lecture regarding "jittering."
    - (iv) Experiment to see how many simulation draws you need to get precise approximations and check this again at the equilibrium shares and prices you obtain below.
  - (b) The FOC for firm  $j$ 's profit maximization problem in market  $t$  is

$$\begin{aligned}(p_{jt} - mc_{jt}) \frac{\partial s_{jt}(p_t)}{\partial p_{jt}} + s_{jt} &= 0 \\ \implies p_{jt} - mc_{jt} &= - \left( \frac{\partial s_{jt}(p_t)}{\partial p_{jt}} \right)^{-1} s_{jt}\end{aligned}\tag{1}$$

- (c) Substituting in your approximation of each  $\left( \frac{\partial s_{jt}(p_t)}{\partial p_{jt}} \right)$ , solve the system of equations (1) ( $J$  equations per market) for the equilibrium prices in each market.
  - i. First do this using Matlab's "fsolve" operator. Check the exit flag from fsolve to be sure whether you found a solution for each market.

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<sup>3</sup>Although the satellite services set national list prices, they offer local discounts, so we will model them as setting separate prices for each market.

- ii. Do this again (still in Matlab) using the algorithm of Morrow and Skerlos (2011), discussed in section 3.6 of Conlon and Gortmaker (2019) (and in the pyBLP “problem simulation tutorial”). Use the numerical integration approach you used in step (a) to approximate the terms defined in equation (25) of Conlon and Gortmaker. If you get different results using this method, resolve this discrepancy either by correcting your code or explaining why your preferred method is the one to be trusted.
3. Calculate “observed” market shares for your fake data set using your parameters, your draws of  $x, w, \beta_i, \omega, \xi$ , and your equilibrium prices.
4. Below you’ll be using  $x$  and  $w$  as instruments in the demand estimation. Check whether these appear to be good instruments in your fake data using some regressions of prices and market shares on the exogenous variables (or some function of them; see the related discussion in the coding tips). If you believe the instruments are not providing enough variation, modify the parameter choices above until you are satisfied. Report your final choice of parameters and the results you rely on to conclude that the instruments seem good enough.

## 4 Estimate Some Mis-specified Models

5. Estimate the plain multinomial logit model of demand by OLS (ignoring the endogeneity of prices).
6. Re-estimate the multinomial logit model of demand by two-stage least squares, instrumenting for prices with the exogenous demand shifters  $x$  and excluded cost shifters  $w$ . Discuss the predictive value of the instruments in first stage, and how the second-stage results differ from those obtained by OLS.
7. Now estimate a nested logit model by linear IV, treating “satellite” and “wired” as the two nests for the inside goods. You will probably want to review the discussion of the nested logit in Berry (1994) regarding this model and which variables require instruments. Note that Berry focuses on the special case in which all the “nesting parameters” are the same; you should allow a different nesting parameter for each nest.<sup>4</sup>
8. Using the nested logit results, provide a table comparing the estimated own-price elasticities to the true own-price elasticities.<sup>5</sup> Provide two additional tables showing the true matrix of diversion ratios and the diversion ratios implied by your estimates.

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<sup>4</sup>In Berry’s notation, this means letting the parameter  $\sigma$  become  $\sigma_{g(j)}$ , where  $g(j)$  indicates the group (satellite or wired) to which each inside good  $j$  belongs.

<sup>5</sup>The procedure you developed above for approximating derivatives cannot be used for your estimates based on the nested logit model. But because we have analytic expressions for market shares in the nested logit model, you could either differentiate these or use “finite difference” approximation of derivatives.

## 5 Estimate the Correctly Specified Model

Use the pyBLP software to estimate the correctly specified model. Allow pyBLP to construct approximations to the optimal instruments, using the exogenous demand shifters and exogenous cost shifters.<sup>6</sup>

9. Report a table with the estimates of the demand parameters and standard errors. Do this twice: once when you estimate demand alone, then again when you estimate jointly with supply.
10. Using your preferred estimates from the prior step (explain your preference), provide a table comparing the estimated own-price elasticities to the true own-price elasticities. Provide two additional tables showing the true matrix of diversion ratios and the diversion ratios implied by your estimates.

## 6 Merger Simulation

11. Suppose two of the four firms were to merge. Give a brief intuition for what theory tells us is likely to happen to the equilibrium prices of each good  $j$ .
12. Suppose firms 1 and 2 are proposing to merge. Use the pyBLP merger simulation procedure to provide a prediction of the post-merger equilibrium prices.
13. Now suppose instead that firms 1 and 3 are the ones to merge. Re-run the merger simulation. Provide a table comparing the (average across markets) predicted merger-induced price changes for this merger and that in part 11. Interpret the differences between the predictions for the two mergers.
14. Thus far you have assumed that there are no “efficiencies” resulting from the merger. Explain briefly why a merger-specific reduction in marginal cost could mean that a merger is welfare-enhancing.
15. Consider the merger between firms 1 and 2, and suppose the firms demonstrate that by merging they would reduce marginal cost of each of their products by 15%. Furthermore, suppose that they demonstrate that this cost reduction could not be achieved without merging. Using the pyBLP software, re-run the merger simulation with the 15% cost saving. Show the predicted post-merger price changes (again, for each product, averaged across markets). What is the predicted impact of the merger on consumer welfare,<sup>7</sup> assuming that the total measure of consumers  $M_t$  is the same in each market  $t$ ? Explain why this additional assumption (or data on the correct values of  $M_t$ ) is needed here, whereas up to this point it was without loss to assume  $M_t = 1$ . What is the predicted impact of the merger on total welfare.

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<sup>6</sup>For your own benefit, you may want to see what happens without the approximation of the optimal instruments.

<sup>7</sup>Note that because we have quasilinear preferences, consumer surplus is a valid measure of aggregate consumer welfare under the usual assumption of optimal redistribution.

## 7 Coding Tips

- You can draw from a multivariate normal with variance  $\Sigma$  by drawing independent standard normal random variables and using the Cholesky decomposition of  $\Sigma$  (the latter obtained with a command in Matlab). In particular, if  $z = (z_1, \dots, z_k)' \sim N(0, I_k)$  and  $A = \text{Chol}(\Sigma)$ , then  $Az$  is distributed  $N(0, \Sigma)$ .
- When you estimate the logit and nested logit models, you will have to choose which functions of the exogenous variables to use as instruments. One option would be to use all of them—the exogenous demand shifters (own and competing products) and the exogenous cost shifters. Alternatively, you might want to use something more like the BLP approximation of the optimal instruments. For example, for good  $j$  in market  $t$ , the instruments might be  $x_{jt}, w_{jt}, \text{satellite}_{jt}, \text{wired}_{jt}$ , the quality index summed over competing goods  $-j$  in market  $t$ , the quality index of the other good in the same nest as good  $j$ .<sup>8</sup> Of course, you do not want to feed an approximation of the optimal IV into pyBLP if you are going to have pyBLP compute a (better) approximation of the optimal instruments.
- To migrate your data from Matlab to Python, try exporting and importing a csv (i.e., comma separated) file. To export the data to a csv file, look into the `writematrix` or `writetable` functions in Matlab. To import the csv file, look into `read_csv` in Python (pandas package).
- To display the average prices, use the following (where `changed_prices` is the output of `compute_prices` as in Post-Estimation Tutorial of pyBLP).  
T, J= 600, 4  

```
print(changed_prices.reshape((T, J)).mean(axis= 0))
```
- To display the average elasticities and diversion ratios, use the following (where `elasticities`, for example, is the output of `compute_elasticities` in Post-Estimation Tutorial of pyBLP).  
T, J= 600, 4  

```
print(elasticities.reshape((T, J, J)).mean(axis= 0))
```

(These resemble what one would write in Matlab, but there are subtle issues behind it, including row-major order (Python) vs column-major order (Matlab).)
- To apply 15% cost reduction by the merged firms, use the following.  

```
merger_costs= costs.copy()
merger_costs[product_data.merger_ids== 1]=
0.85*merger_costs[product_data.merger_ids== 1]
```

where `costs` and `merger_ids` are as in Post-Estimation Tutorial of pyBLP. (Using `merger_costs= costs` instead of using `copy` could lead to an unexpected behavior.)

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<sup>8</sup>Note that on the supply side, there is an intercept in the marginal cost function, which will be collinear with the two dummies for satellite and wired.