Modelling the detection chamber

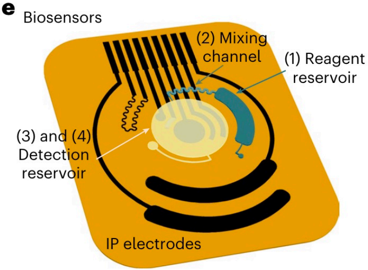

Figure 1: Inflastat chip (Tu et al. 2023).

CRP-dAb and cAb

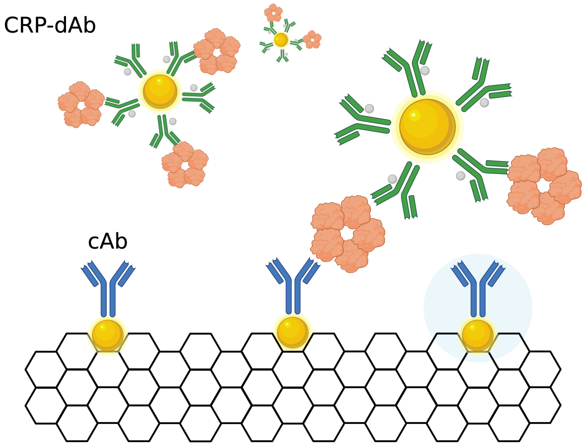

Figure 2: CRP-detector antibody and capture antibodies.

CRP-dAb concentration in the detection reservoir (Part 1)

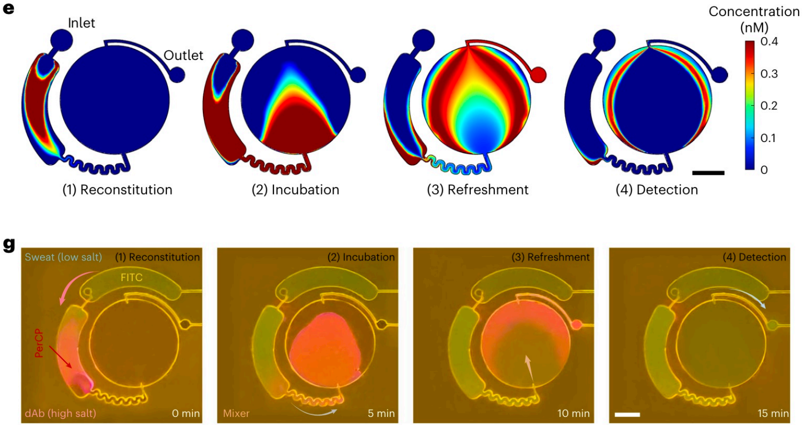

Figure 3: Numerical simulation of CRP-dAb concentrations without cAb and fluorescence flow test without dAb (Tu et al. 2023).

CRP-dAb concentration in the detection reservoir (Part 2)

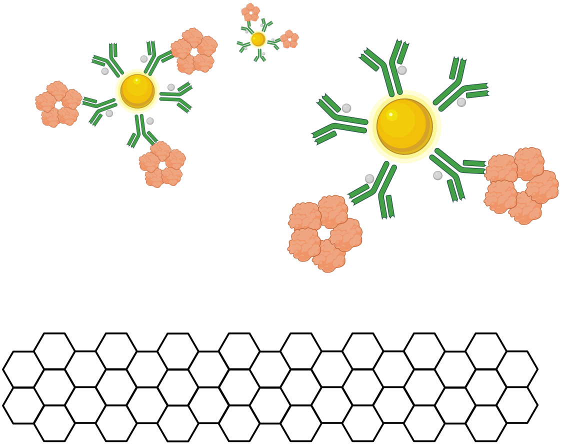

Figure 4: System studied in the literature through numerical simulation and fluorescence flow test.

Reaction-diffusion-advection numerical simulations

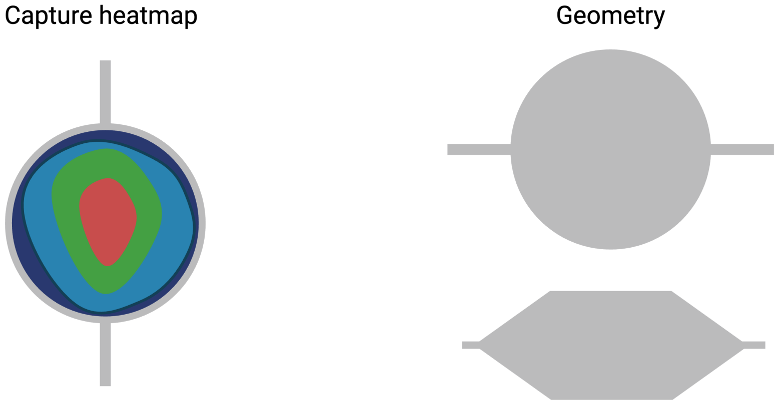

Figure 5: Objective of numerical simulations.

Fluid dynamics

- The velocity field $\mathbf{v}$ has no dependence on concentration because sweat is dilute.
- Solve for $\mathbf{v}$ using the Navier-Stokes equation for incompressible flow and the continuity equation.

$$\rho(\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v}) = -\nabla \mathbf{p} + \mu \nabla^2 \mathbf{v}$$

$$\nabla \cdot \mathbf{v} = \mathbf{0}$$

Non-dimensionalized Navier Stokes

- Assume steady state. Parameters aggregate into the Reynolds number.

$$\frac{\rho U L}{\mu}(\tilde{\mathbf{v}} \cdot \tilde{\nabla})\tilde{\mathbf{v}} = -\frac{\Pi L}{U\mu}\tilde{\nabla}\tilde{\mathbf{p}} + \tilde{\nabla}^2\tilde{\mathbf{v}}$$

- Use the viscous pressure scale

$$\Pi = \frac{U\mu}{L}$$

$$\mathrm{Re}(\tilde{\mathbf{v}} \cdot \tilde{\nabla})\tilde{\mathbf{v}} = -\tilde{\nabla}\tilde{\mathbf{p}} + \tilde{\nabla}^2\tilde{\mathbf{v}}$$

Creeping (Stokes) flow

- Using sweat volumetric flowrate, the mean chamber chord length, and the width of the PET layer ($\dot{V},\ \bar{C},\ W$).

$$\mathrm{Re} = \frac{\rho U L}{\mu} = \frac{10^3 \cdot 10^{-4} \cdot 10^{-4}}{10^{-3}} = 10^{-2} \ll 1$$

- Fluid flow in the microfluidic channel is in the Stokes flow regime.

$$-\tilde{\nabla}\tilde{\mathbf{p}} + \tilde{\nabla}^2\tilde{\mathbf{v}} = \mathbf{0}$$

- Solve alongside the non-dimensionalized continuity equation.

$$\tilde{\nabla} \cdot \tilde{\mathbf{v}} = \mathbf{0}$$

Rate of CRP-dAb and cAb binding

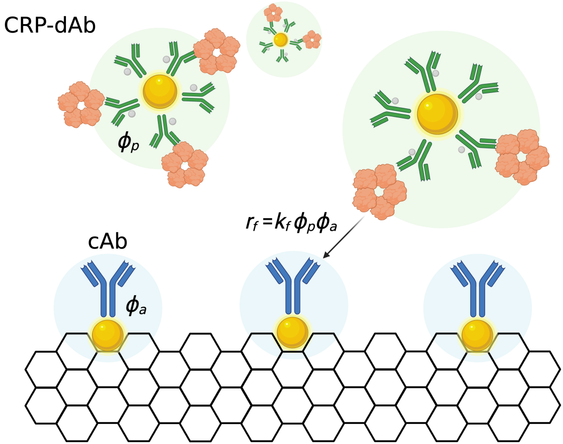

Figure 6: Binding rate first order to CRP-dAb and cAb concentration.

Rate of complex dissociation

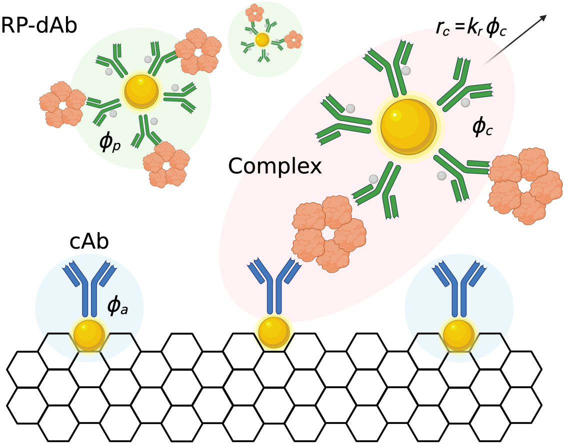

Figure 7: Dissociation rate first order to complex concentration.

Rate of reaction

$$\mathrm{CRP{-}dAb} + \mathrm{cAb} \underset{\mathrm{k}_r}{\overset{\mathrm{k}_f}{\rightleftharpoons}} \mathrm{Complex}$$

- $p,\ a,\ c$ represent CRP-dAb, cAb, and Complex respectively.

$$\mathbf{R}(\phi_p, \phi_a, \phi_c) = k_f(T)\phi_p\phi_a - k_r(T)\phi_c$$

Reaction-diffusion-advection dynamics (Part 1)

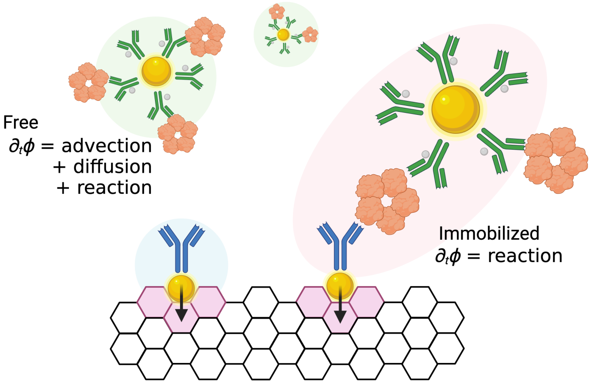

Figure 8: Mobility of different species.

Reaction-diffusion-advection dynamics (Part 2)

$$\partial_t \phi_p(\mathbf{x}, t) = D\nabla^2 \phi_p - \mathbf{v} \cdot \nabla \phi_p - \mathbf{R}$$

$$\partial_t \phi_a(\mathbf{x}, t) = -\mathbf{R}$$

$$\partial_t \phi_c(\mathbf{x}, t) = \mathbf{R}$$

Non-dimensionalized reaction-diffusion-advection

- Parameters aggregate into two time scales.

$$\partial_{\tilde{t}}\tilde{\phi}_p = \frac{\tau D}{L^2}\tilde{\nabla}^2\tilde{\phi}_p - \frac{\tau U}{L}\tilde{\mathbf{v}} \cdot \tilde{\nabla}\tilde{\phi}_p - \tilde{\mathbf{R}}$$

$$\partial_{\tilde{t}}\tilde{\phi}_a = -\tilde{\mathbf{R}} \quad \text{and} \quad \partial_{\tilde{t}}\tilde{\phi}_c = \tilde{\mathbf{R}}$$

$$\tilde{\mathbf{R}} = \tilde{k}_f(T)\tilde{\phi}_p\tilde{\phi}_a - \tilde{k}_r(T)\tilde{\phi}_c$$

- The Peclet number compares diffusion and advection contribution.

$$\mathrm{Pe} = \frac{\tau_{\mathrm{diff}}}{\tau_{\mathrm{adv}}} = \frac{L^2/D}{L/U} = \frac{LU}{D} = \frac{(10^{-4})(10^{-4})}{(10^{-12})} = 10^4$$

- Advection dominates the flow. Use $\tau = \tau_{\mathrm{adv}} = \frac{L}{U}$.

$$\partial_{\tilde{t}}\tilde{\phi}_p = \frac{1}{\mathrm{Pe}}\tilde{\nabla}^2\tilde{\phi}_p - \tilde{\mathbf{v}} \cdot \tilde{\nabla}\tilde{\phi}_p - \tilde{\mathbf{R}}$$

S1: Characteristic scales (Part 1)

- Define dimensionless variables using characteristic length, pressure, velocity, and time scales (L, Π, U, τ).

$$\tilde{\mathbf{v}} = \frac{\mathbf{v}}{U} \implies \mathbf{v} = U\tilde{\mathbf{v}}$$

$$\tilde{\mathbf{p}} = \frac{\mathbf{p} - \mathbf{p}_0}{\Pi} \implies \mathbf{p} = \Pi\tilde{\mathbf{p}} + \mathbf{p}_0$$

$$\tilde{t} = \frac{t}{\tau} \implies t = \tau\tilde{t}$$

$$\tilde{\nabla} = L\nabla \implies \nabla = \frac{1}{L}\tilde{\nabla}$$

S2: Characteristic scales (Part 2)

- Define dimensionless parameters using characteristic concentration and time scales $(\phi_{a,0}, \tau)$.

$$\tilde{\phi}_p = \frac{\phi_p}{\phi_{a,0}} \implies \phi_p = \phi_{a,0}\tilde{\phi}_p$$

$$\tilde{\phi}_a = \frac{\phi_a}{\phi_{a,0}} \implies \phi_a = \phi_{a,0}\tilde{\phi}_a$$

$$\tilde{k}_f = \tau\phi_{a,0}k_f \implies k_f = \frac{\tilde{k}_f}{\phi_{a,0}\tau}$$

$$\tilde{k}_r = \tau k_r \implies k_r = \frac{\tilde{k}_r}{\tau}$$

$$\tilde{\mathbf{R}} = \frac{\tau}{\phi_{a,0}}\mathbf{R} \implies \mathbf{R} = \frac{\phi_{a,0}}{\tau}\tilde{\mathbf{R}}$$

S3: Deriving non-dimensional Navier-Stokes

- Navier-Stokes

$$\rho(\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v}) = -\nabla \mathbf{p} + \mu \nabla^2 \mathbf{v}$$

- Substitute with non-dimensional variables

$$\rho(\frac{U}{\tau}\partial_{\tilde{t}}\tilde{\mathbf{v}} + \frac{U^2}{L}(\tilde{\mathbf{v}} \cdot \tilde{\nabla})\tilde{\mathbf{v}}) = -\frac{\Pi}{L}\tilde{\nabla}\tilde{\mathbf{p}} + \frac{U}{L^2}\mu\tilde{\nabla}^2\tilde{\mathbf{v}}$$

- Assume steady state

$$\rho\frac{U^2}{L}(\tilde{\mathbf{v}} \cdot \tilde{\nabla})\tilde{\mathbf{v}} = -\frac{\Pi}{L}\tilde{\nabla}\tilde{\mathbf{p}} + \frac{U}{L^2}\mu\tilde{\nabla}^2\tilde{\mathbf{v}}$$

S4: Deriving non-dimensional reaction-diffusion-advection

- Substitute with non-dimensional variables

$$\frac{\phi_{a,0}}{\tau}\partial_{\tilde{t}}\tilde{\phi}_p = \frac{\phi_{a,0}D}{L^2}\tilde{\nabla}^2\tilde{\phi}_p - \frac{U\phi_{a,0}}{L}\tilde{\mathbf{v}}\cdot\tilde{\nabla}\tilde{\phi}_p - \frac{\phi_{a,0}}{\tau}\tilde{\mathbf{R}}$$

$$\frac{\phi_{a,0}}{\tau}\partial_{\tilde{t}}\tilde{\phi}_a = -\frac{\phi_{a,0}}{\tau}\tilde{\mathbf{R}}$$

$$\frac{\phi_{a,0}}{\tau}\tilde{\mathbf{R}} = \frac{\phi_{a,0}}{\tau}\tilde{k}_f(T)\tilde{\phi}_p\tilde{\phi}_a - \frac{\phi_{a,0}}{\tau}\tilde{k}_r(T)\tilde{\phi}_c$$

S5: Characteristic length and velocity estimations

- Use sweat volumetric flowrate, the mean detection chamber chord length, and the width of the PET layer ($\dot{V}$, $\bar{C}$, W).

$$\dot{V} \approx 2 \text{ L/min} \approx 3 \times 10^{-11}\,\mathrm{m\,s^{-1}}$$
$$\bar{C} = 450 \times 10^{-6}\,\mathrm{m}$$
$$W = 50 \times 10^{-6}\,\mathrm{m}$$

- Use effective diameter as characteristic length. Use mean velocity as characteristic velocity.

$$L = d_{\mathrm{eff}} = \frac{4\bar{C}W}{2(\bar{C}+W)} \approx 1 \times 10^{-4}\,\mathrm{m}$$
$$U = \frac{\dot{V}}{\bar{C}W} \approx 1 \times 10^{-4}\,\mathrm{m}$$

References

Tu, Jiaobing, Jihong Min, Yu Song, Changhao Xu, Jiahong Li, Jeff Moore, Justin Hanson, et al. 2023. "A Wireless Patch for the Monitoring of C-Reactive Protein in Sweat." *Nature Biomedical Engineering*, June, 1–14. https://doi.org/10.1038/s41551-023-01059-5.