Modelling the detection chamber

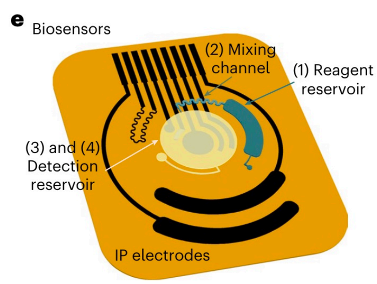

Figure 1: Inflastat chip [@tu_wireless_2023].

CRP-dAb and cAb

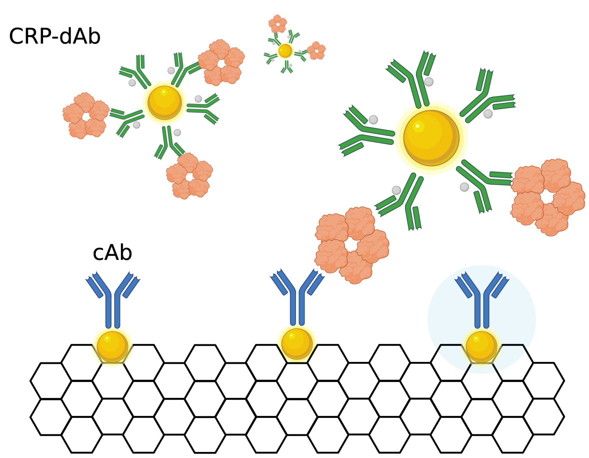

Figure 2: CRP-detector antibody and capture antibodies.

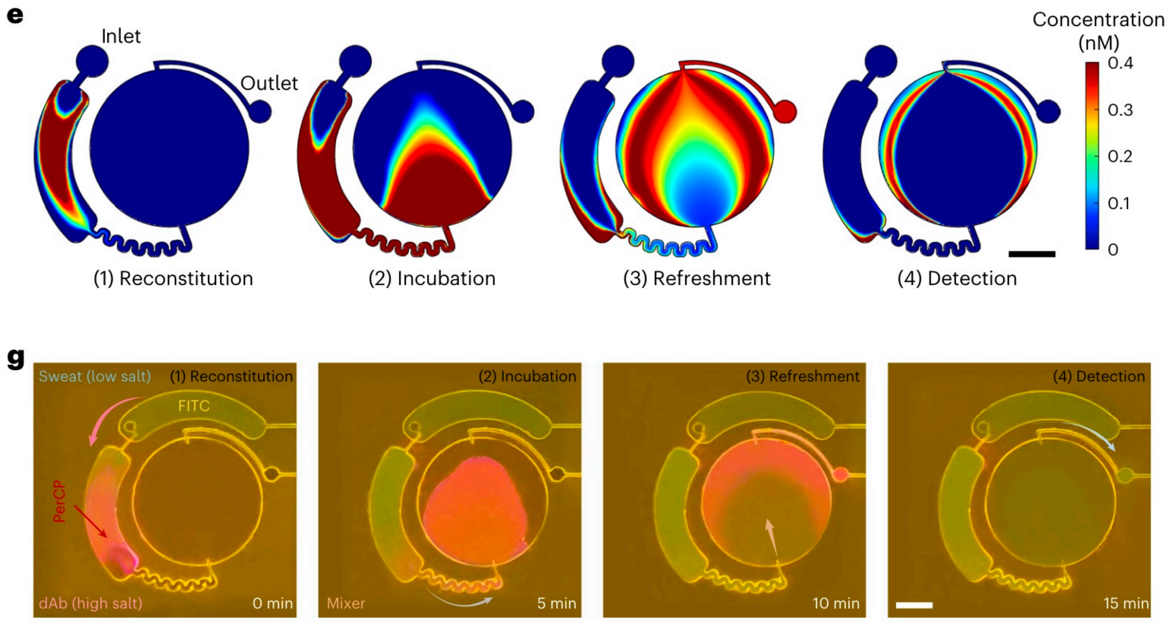

Figure 3: Numerical simulation of CRP-dAb concentrations without cAb and fluorescence flow test without dAb [@tu_wireless_2023].

CRP-dAb concentration in the detection reservoir (Part 1)

CRP-dAb concentration in the detection reservoir (Part 2)

Reaction-diffusion-advection numerical simulations

Fluid dynamics

- The velocity field $\mathbf{v}$ has no dependence on concentration because sweat is dilute.
- Solve for $\mathbf{v}$ using the Navier-Stokes equation for incompressible flow and the continuity equation.

$$\rho(\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v}) = -\nabla \mathbf{p} + \mu \nabla^2 \mathbf{v}$$

$$\nabla \cdot \mathbf{v} = \mathbf{0}$$

Non-dimensionalized Navier Stokes

- Assume steady state. Parameters aggregate into the Reynolds number.

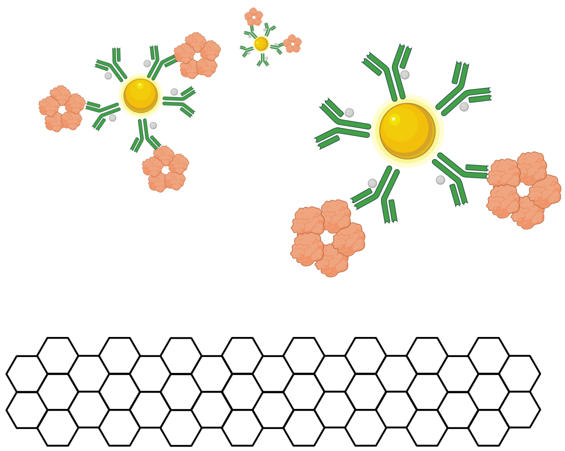

Figure 4: System studied in the literature through numerical simulation and fluorescence flow test.

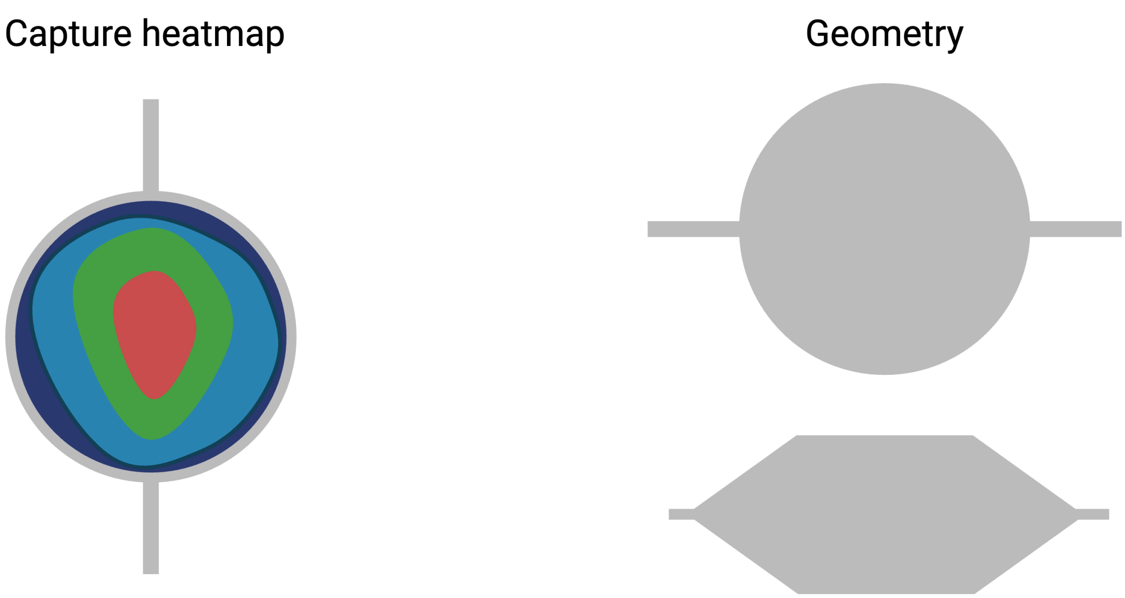

Figure 5: Objective of numerical simulations.

$$\frac{\rho U L}{\mu}(\tilde{\mathbf{v}} \cdot \tilde{\nabla})\tilde{\mathbf{v}} = -\frac{\Pi L}{U\mu}\tilde{\nabla}\tilde{\mathbf{p}} + \tilde{\nabla}^2\tilde{\mathbf{v}}$$

- Use the viscous pressure scale

$$\Pi = \frac{U\mu}{L}$$
$$\mathrm{Re}(\tilde{\mathbf{v}} \cdot \tilde{\nabla})\tilde{\mathbf{v}} = -\tilde{\nabla}\tilde{\mathbf{p}} + \tilde{\nabla}^2\tilde{\mathbf{v}}$$

Creeping (Stokes) flow

- Using sweat volumetric flowrate, the mean chamber chord length, and the width of the PET layer ($\dot{V},\ \bar{C},\ W$).

$$\mathrm{Re} = \frac{\rho U L}{\mu} = \frac{10^3 \cdot 10^{-4} \cdot 10^{-4}}{10^{-3}} = 10^{-2} \ll 1$$

- Fluid flow in the microfluidic channel is in the Stokes flow regime.

$$-\tilde{\nabla}\tilde{\mathbf{p}} + \tilde{\nabla}^2\tilde{\mathbf{v}} = \mathbf{0}$$

- Solve alongside the non-dimensionalized continuity equation.

$$\tilde{\nabla} \cdot \tilde{\mathbf{v}} = \mathbf{0}$$

- The Poisson pressure equation

$$\frac{\Pi}{L^2}\tilde{\nabla}^2\tilde{\mathbf{p}} = -\rho\frac{U^2}{L^2}(\tilde{\nabla} \cdot \tilde{\mathbf{v}})^2$$
$$\tilde{\nabla}^2\tilde{\mathbf{p}} = -\frac{\rho U L}{\mu}(\tilde{\nabla} \cdot \tilde{\mathbf{v}})^2$$
$$\tilde{\nabla}^2\tilde{\mathbf{p}} = 0$$

Reaction-diffusion-advection dynamics (Part 1)

Reaction-diffusion-advection dynamics (Part 2)

$$\partial_t\phi_p(\mathbf{x}, t) = D\nabla^2\phi_p - \mathbf{v} \cdot \nabla\phi_p - \mathbf{R}$$
$$\partial_t\phi_a(\mathbf{x}, t) = -\mathbf{R}$$
$$\partial_t\phi_c(\mathbf{x}, t) = \mathbf{R}$$
$$\mathbf{R}(\phi_p, \phi_a, \phi_c) = k_f(T)\phi_p\phi_a - k_r(T)\phi_c$$

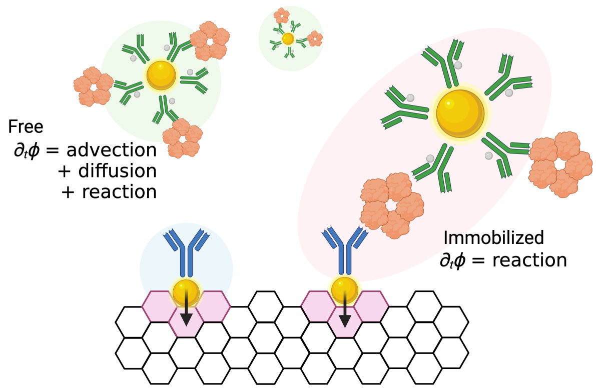

Figure 6: Mobility of different species.

Non-dimensionalized reaction-diffusion-advection

- Parameters aggregate into two time scales.

$$\partial_{\tilde{t}}\tilde{\phi}_p = \frac{\tau D}{L^2}\tilde{\nabla}^2\tilde{\phi}_p - \frac{\tau U}{L}\tilde{\mathbf{v}} \cdot \tilde{\nabla}\tilde{\phi}_p - \tilde{\mathbf{R}}$$

$$\partial_{\tilde{t}}\tilde{\phi}_a = -\tilde{\mathbf{R}} \quad \text{and} \quad \partial_{\tilde{t}}\tilde{\phi}_c = \tilde{\mathbf{R}}$$

$$\tilde{\mathbf{R}} = \tilde{k}_f(T)\tilde{\phi}_p\tilde{\phi}_a - \tilde{k}_r(T)\tilde{\phi}_c$$

- The Peclet number compares diffusion and advection contribution.

$$\mathrm{Pe} = \frac{\tau_{\text{diff}}}{\tau_{\text{adv}}} = \frac{L^2/D}{L/U} = \frac{LU}{D} = \frac{(10^{-4})(10^{-4})}{(10^{-12})} = 10^4$$

- Advection dominates the flow. Use $\tau = \tau_{\text{adv}} = \frac{L}{U}$.

$$\partial_{\tilde{t}}\tilde{\phi}_p = \frac{1}{\mathrm{Pe}}\tilde{\nabla}^2\tilde{\phi}_p - \tilde{\mathbf{v}} \cdot \tilde{\nabla}\tilde{\phi}_p - \tilde{\mathbf{R}}$$

$$\partial_{\tilde{t}}\tilde{\phi}_p = -\tilde{\mathbf{v}} \cdot \tilde{\nabla}\tilde{\phi}_p - \tilde{\mathbf{R}}$$

$$\partial_{\tilde{t}}\tilde{\phi}_a = -\tilde{\mathbf{R}}$$