CLPS 2908: Multivariate Statistical Techniques

Homework 2

January 31, 2019

1

For the matrices below, obtain the following matrices, and state their dimensions: (1) $\mathbf{A}+\mathbf{B}$, (2) $\mathbf{A}-\mathbf{B}$, (3) $\mathbf{AC}$, (4) $\mathbf{AB}'$, (5) $\mathbf{B}'\mathbf{A}$

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 2 & 6 \\ 3 & 8 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 1 & 3 \\ 1 & 4 \\ 2 & 5 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 3 & 8 & 1 \\ 5 & 4 & 0 \end{bmatrix}$$

2

Let $\mathbf{B}$ be defined as follows:

$$\mathbf{B} = \begin{bmatrix} 1 & 5 & 0 \\ 1 & 0 & 5 \\ 1 & 0 & 5 \end{bmatrix}$$

a Are the column vectors of $\mathbf{B}$ linearly dependent?

b What is the rank of $\mathbf{B}$?

c What must be the determinant of $\mathbf{B}$?

3

a Find the inverse of $\mathbf{A}$:

$$\mathbf{A} = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$$

b Check that your resulting matrix is indeed the inverse.

4

Diagonal matrices have non-zero elements on the main diagonal, but zeroes in the off-diagonals. Show that the following is true:

a If all diagonal elements d_{ij} of $\mathbf{D}$ are the *same*, then pre- or postmultiplying any matrix $\mathbf{X}$ with $\mathbf{D}$ gives the same result, namely, a scalar multiplication of $\mathbf{X}$ with d.

b If the diagonal elements d_{ij} of a diagonal matrix $\mathbf{D}$ are *different* from each other, then premultiplying a matrix $\mathbf{X}$ with $\mathbf{D}$ gives a different result than post-multiplying $\mathbf{X}$ with with $\mathbf{D}$.

c Try to explain why both (a) and (b) have to be true.

5

Compute the results of the following operations by hand, if they have a solution.

$$\mathbf{a} = \begin{bmatrix} 1 \\ 8 \\ 4 \\ 2 \end{bmatrix} \quad \mathbf{y} = \begin{bmatrix} 7 \\ 5 \\ 10 \\ 2 \end{bmatrix} \quad \mathbf{X} = \begin{bmatrix} 7 & 3 \\ 5 & 3 \\ 10 & 3 \\ 2 & 1 \end{bmatrix} \quad \underline{\mathbf{1}} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$\frac{1}{4}\mathbf{a} =$

$\mathbf{a}' =$

$\mathbf{a}'\mathbf{y} =$

$\underline{\mathbf{1}}'\mathbf{a} =$

$\mathbf{a}'\mathbf{a} =$

$\mathbf{a} - \frac{1}{4}(\mathbf{a}'\underline{\mathbf{1}}) =$

$\mathbf{a}'\mathbf{X} =$

$\mathbf{X}'\mathbf{a} =$

$\mathbf{X}'\mathbf{X} =$

$\mathbf{X}\mathbf{X}^{-1} =$

6

Using matrix algebra, derive a correlation matrix for the following data:

Cases	X1	X2
1	8	4
2	7	3
3	2	6
4	2	7
5	6	0

Show your work at each of the following steps:

a Pre-multiply your nxp data matrix by the transpose of an appropriate unit vector ($1'$); then multiply by the scalar n^{-1} to compute a vector of means. Thus you compute $1'\mathbf{X}n^{-1}$.

b Premultiply the above $1xp$ vector of means by an appropriate unit vector such that you expand the means vector to nxp dimensionality. Then subtract this expanded matrix of means from the raw data matrix to produce mean-deviated (centered) scores: $\mathbf{X} - 11'\mathbf{X}n^{-1}$, often written as $\mathbf{Y}$.

c Generate a pxp Sums-of-Squares and Cross-Products matrix from your $\mathbf{Y}$ matrix. Then multiply this SSQ/CP matrix by the scalar $(n-1)^{-1}$ to compute $\mathbf{S_{yy}}$, the variance-covariance matrix of the mean-deviated scores, which is equivalent to $\mathbf{S_{xx}}$.

d Pre- and post-multiply $\mathbf{S_{yy}}$ with an appropriate diagonal matrix that contains the reciprocals of the corresponding standard deviations, s_{x1} and s_{x2}. Briefly explain what this step accomplishes.