ANOVA Preludes:

Basics

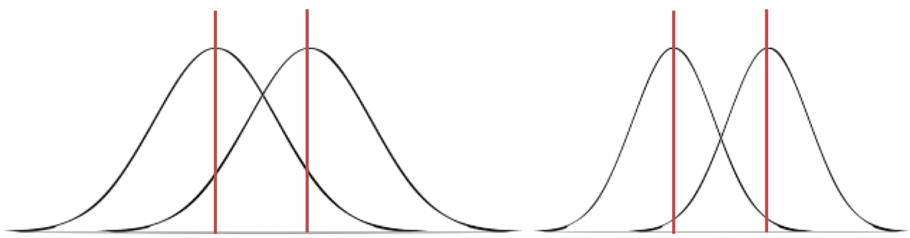
Interactions

L14 March 19, 2019

1. Basics of ANOVA

Purpose

- Comparing means in light of their variance
- Variance between means vs. around means (SS_B / SS_w)
  - Are two means different from each other relative to the variances around each mean? ("within cells" ≠ "within-subject")
  - Within-subject: difference score distribution vs. 0



Wider "within-cells" variance   Narrower "within-cells" variance

Vocabulary

- **Factor**, has **levels** (df = levels -1)
- Three or more levels ➛ **contrasts** needed
  - Omnibus has $df > 1$; difficult to interpret
- Multi-factorial (2 or more factors, various number of levels)
  - **Main effects** for each factor
    - Multiple main effect contrasts if $df > 1$
  - **Interactions**
    - Multiple interaction contrasts if at least one factor's $df > 1$

2. Interaction Effects in ANOVA

Starting Example

	Sunny	Overcast	
Drink coffee	8.5	6.8	7.7
No coffee	5.5	2.7	4.1
	7.0	4.8	5.9

Effects of caffeine and weather on *mood* (1-10 point scale)

1. **Is there an interaction effect?**
- Yes, a small one

2. **How do we interpret it?**
- Caffeine and sunny weather together are somewhat <u>less</u> mood-boosting than would be expected from main effects alone.

Two Myths and a Solution

1. "Main effects are qualified by an interaction."
   - Not when effects are orthogonal (in *n*-balanced designs)
2. "You need to run follow-up *t* tests to properly interpret an interaction."
   - Such follow-ups test ***different*** hypotheses

- Solution to gain clarity: **Teasing apart effects**
  - Remove main effects → interpret interaction effects
  - Origin: *General Linear Model*

General Linear Model

$$\boxed{\begin{aligned} X_i &= \mathbf{GM} + \mathbf{A} + \mathbf{B} + \mathbf{A} \times \mathbf{B} + \varepsilon \\ \overline{X}_{.} &= \mathbf{GM} + \mathbf{A} + \mathbf{B} + \mathbf{A} \times \mathbf{B} \end{aligned}}$$

Cell means

Main effect **A**
e.g., A1 – A2

Therefore:
Main effects are ***not*** "qualified" by interactions

Exception: unequal cell sizes, some w/s designs

Teasing Apart Effects I

	Conserv	Liberal	
A1	2	5	3.5
A2	2	1	1.5
	2	3	2.5

Remove row effect 0.5*(3.5-1.5) = +/-1

	Conserv	Liberal		
A1	1	4	2.5	[-1]
A2	3	2	2.5	[+1]
	2	3	2.5	

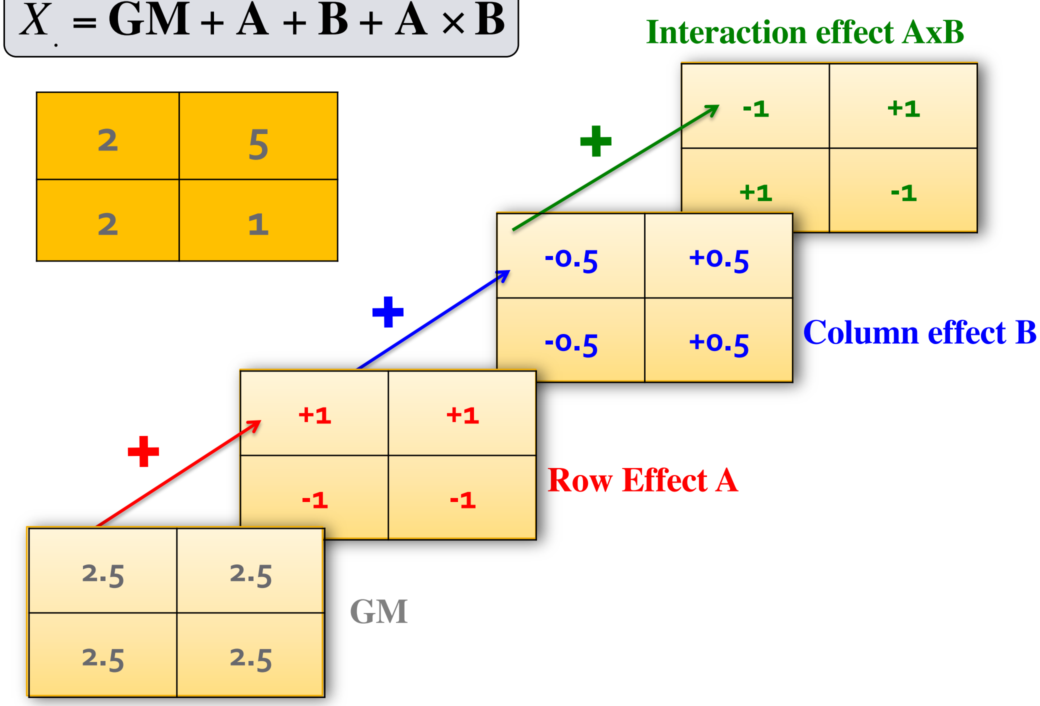
Remove column effect 0.5*(3-2) = +/-0.5

	Conserv	Liberal	
A1	1.5	3.5	2.5
A2	3.5	1.5	2.5
	2.5	2.5	2.5
	[+0.5]	[-0.5]	

Remove grand mean

	Conserv	Liberal	
A1	-1	+1	0
A2	+1	-1	0
	0	0	0

$$\bar{X}_{.} = \mathbf{GM} + \mathbf{A} + \mathbf{B} + \mathbf{A} \times \mathbf{B}$$



Teasing Apart Effects II

	Sunny	Overcast	
Drink coffee	8.5	6.8	7.7
No coffee	5.5	2.7	4.1
	7.0	4.8	5.9

	Sunny	Overcast		
Drink coffee	6.7	5	5.9	[-1.8]
No coffee	7.3	4.5	5.9	[+1.8]
	7.0	4.8	5.9	

	Sunny	Overcast	
Drink coffee	5.6	6.1	5.9
No coffee	6.2	5.6	5.9
	5.9	5.9	5.9
	[-1.1]	[+1.1]	

	Sunny	Overcast	
Drink coffee	-0.3	+0.3	0
No coffee	+0.3	-0.3	0
	0	0	0

Do *t* tests help interpret interaction effects?

	Stereotype threat		
	low	high	
easy	5.2	5.1	5.2
diffic	4.9	3.3	4.1
	5.1	4.2	4.6

if interaction significant…

			t tests
easy	5.2	5.1	*ns*
diffic	4.9	3.3	$p < .01$

What does interaction show?

If this is not what the theory predicted, then one should not use the interaction term to test the hypothesis. Better to examine **simple effects** or **specific contrasts**, tested against a "pooled" error term. For example:

0	+1
0	-1

-1	+1
-1	3

Excel Interaction Identification

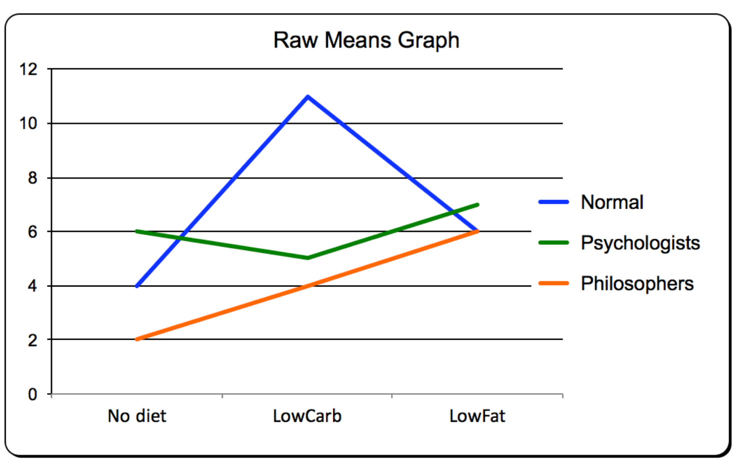
Cell means:

	B1	B2	
A1	5.2	2.1	3.7
A2	6.0	6.2	6.1
	5.6	4.2	4.9

Interaction:

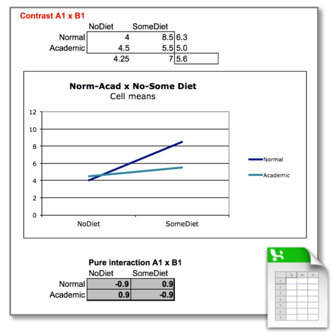
0.8	-0.8
-0.8	0.8

What about 3+level factors?



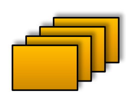
What about 3+level factors?

- Break down into single-*df* contrasts (main & interaction)
  - A(3) x B(2): define two orthogonal contrasts for A
    - E.g., Helmert: A1 vs. A23; A2 vs. A3
  - Cross with B factor: (A1 vs. A23) × B; (A2 vs. A3) × B
  - Graph and interpret each contrast
  - Plot actual contrast values onto new scale
- Good contrasts to use
  - Helmert (or reverse Helmert)
  - Polynomials (LIN, QUAD…)

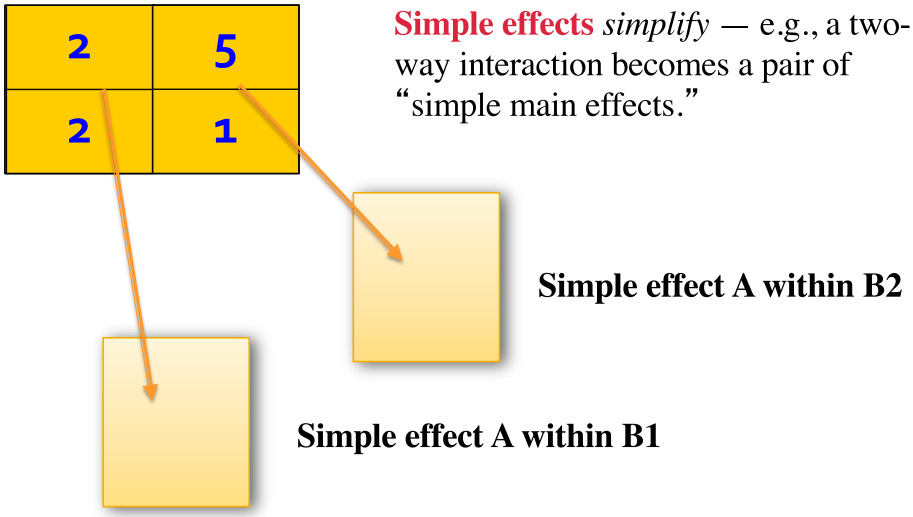


Interactions vs. Simple Effects

- **Simple effects** work on cell means *as they appear*
  - Direct comparisons among any cell means of interest
  - No removal of whole effects,
    effects are "jointly operating" to produce what we see



Simple effects *simplify* — e.g., a two-way interaction becomes a pair of "simple main effects."



Which one—interaction or simple effects?

- Interaction tests are traditionally packaged with main effect tests
  - Does collapsing across one factor to test the other (i.e., main effect) make sense? *(e.g., experimental + control group vs. two experimental groups; two diagnoses)*
  - Do main effects have a meaningful interpretation?
- Interactions are cross-over patterns (reversals); is this really the hypothesized pattern?
  - Interactions model joint process of both factors (mutual dampening or amplifying)
  - Simple effects model two separate, parallel processes

3. Unequal Cell Sizes in ANOVA

Basic Problem

	A1	A2
B1	N = 6	N = 5
B2	N = 8	N = 8

A	B	AxB
1	1	1
1	1	1
1	1	1
1	1	1
1	1	1
1	1	1
1	-1	-1
1	-1	-1
1	-1	-1
1	-1	-1
1	-1	-1
-1	1	-1
-1	1	-1
-1	1	-1
-1	1	-1
-1	1	-1
-1	1	-1
-1	1	-1
-1	1	-1
-1	-1	1
-1	-1	1
-1	-1	1
-1	-1	1

Problem: Effects are not orthogonal

	A	B
B	-0.12	
AxB	0.21	-0.02

The one situation in which interactions *can* statistically qualify main effects.

Regression vs. Sequential

- Two different ways of decomposing explained variance (SSQs)
- Decision arises when design is unbalanced.
- Because effects are correlated, they can (and usually do) share explained variance.
- The default approach (Type III, "unique," "regression") is like multiple regression:
  - All effects (main, interactions…) are corrected for all other effects.
  - This means: Results are computed as if cell sizes were equal.
  - Analyzed means are *unweighted* (ignoring their different cell sizes)
  - SSQs don't add up.
- Alternative: sequentially partition SSQs, one effect after another.

Types of SSQ Decompositions

Type I (sequential) sums of squares are determined by considering each effect (e.g., main effect A) sequentially, in the order they are listed in the model. Effects are corrected (adjusted) for earlier effects but not for subsequent ones. Type I SS are useful to determine shared variance patterns. They are also useful to find parsimonious polynomial models, allowing the simpler components (e.g. linear) to explain as much variation as possible before resorting to terms of higher complexity (e.g. quadratic, cubic, etc.).

Type III (regression or **unique)** sums of squares correct every effect for every other effect, like in classic multiple regression.

Type II (hierarchical) sums of squares adjust higher-order terms for lower-order terms but not vice versa. So the A x B interaction is adjusted the main effects A and B, but the main effects are not adjusted for the interaction. In addition, effects at the same level are adjusted for each other. So A and B are adjusted for each other. If there were three factors, A, B, and C, then the three two-way interactions would also be adjusted for each other.

Output Example

Multiple Sequential Anova tests.pdf

Output is annotated for Example 1; you'll be able to interpret the other examples by analogy.