Extensions of Factor Analysis

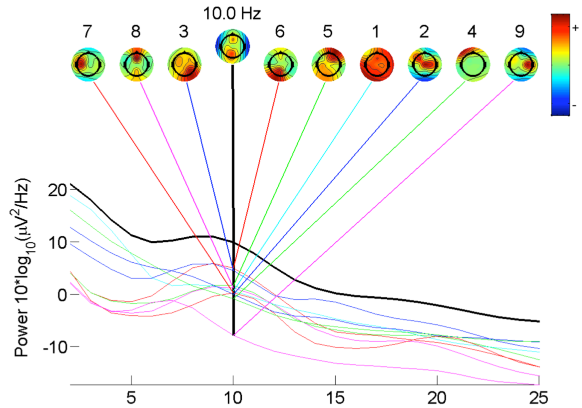
- Lecture 13
  March 14, 2019

1. Independent Component Analysis (ICA)

Very Brief Note

ICA

- Signal processing, engineering, physics, CS
  - Source separation



ICA

- Signal processing, engineering, physics, CS
  - Source separation
- More similar to FA than PCA
- $\mathbf{x} = \Lambda \boldsymbol{f} + \mathbf{e}$,
- $\mathbf{x} = \Lambda(\boldsymbol{f})$,
  - where f are estimated by $\hat{\boldsymbol{f}}$, a function of $\mathbf{x}$
- Goal is to make $\hat{\boldsymbol{f}}$ maximally statistically independent (not just uncorrelated)
  - E.g., overall cost function based on entropy
  - Often even nonlinear separation

2. Comparing Factor Solutions

Stepwise SPSS Procedure

We compare loading matrices

Sample 1

```
FACTOR VAR = DISTANT TO EASYGOIN
 /MATRIX OUT(FAC=*)
 /plot = eigen
 /crit = fact(5)
 /print = default
 /extr = paf
 /rota = varimax.
```

Rotated Factor Matrix[a]

	Factor				
	1	2	3	4	5
distant	.589	.084	.063	-.106	.283
talkativ	-.756	.050	.024	.138	.167
careless	.026	.595	.096	-.014	.271
hardwork	-.180	-.617	.149	.090	.127
anxious	.160	.103	.687	.136	.215
agreeabl	-.019	.066	-.056	.636	-.141
tense	.147	.013	.767	-.009	.262
kind	-.090	-.299	.036	.620	-.169
opposing	-.036	.091	.082	-.117	.616
relaxed	-.002	.151	-.683	.360	-.078
disorgan	.006	.715	.027	.045	.138
outgoing	-.821	-.064	-.053	.214	.032
approvin	-.231	-.105	-.135	.523	-.087
shy	.699	.190	.168	.015	-.115
discipli	.059	-.625	.067	.081	.093
harsh	.050	.087	.068	-.198	.648
persever	-.125	-.547	.170	.202	.098
friendly	-.479	-.148	.064	.553	-.129
worrying	.164	.072	.741	.029	.115
responsi	.010	-.703	.064	.228	.049
contrary	.055	.132	.147	-.139	.718
sociable	-.710	.038	-.061	.255	-.074
lazy	.182	.661	.095	.040	.112
cooperat	-.099	-.192	-.098	.591	-.248
quiet	.790	.103	.177	.165	-.012
organize	-.057	-.740	-.041	.039	-.038
critical	.067	-.219	.172	-.107	.578
lax	.030	.451	-.225	.244	.103
laidback	-.012	.282	-.567	.271	.073
withdraw	.734	.149	.123	-.091	.261
givingup	.345	.458	.210	-.114	.122
easygoin	-.111	.157	-.437	.428	-.041

Extraction Method: Principal Axis Factoring.
Rotation Method: Varimax with Kaiser Normalization.

a. Rotation converged in 7 iterations.

The “out matrix” that SPSS produces has *r x p* dimensionality *(r* factors in rows and *p* variables in columns). However, our familiar loading matrix should be *p x r*. Therefore we have to FLIP (transpose) the output matrix, as shown below. Once it is flipped, we can alphabetically label the original variables (referred to after the flip as “CASE_LBL”) and directly name the columns as what they are: loadings within factors 1, 2, and 3 (across 32 original variables) in sample 1.

```
FLIP VAR = DISTANT TO EASYGOIN.

LIST VAR = all.

SORT CASES BY CASE_LBL.

SAVE OUTFILE = load1  /RENAME =
(VAR001=samp1f1) (VAR002=samp1f2)
(VAR003=samp1f3) (VAR004=samp1f4)
(VAR005=samp1f5).
```

CASE_LBL	var001	var002	var003	var004	var005
agreeabl	-.02	.07	-.06	.64	-.14
anxious	.16	.10	.69	.14	.22
approvin	-.23	-.11	-.14	.52	-.09
careless	.03	.60	.10	-.01	.27
contrary	.05	.13	.15	-.14	.72
cooperat	-.10	-.19	-.10	.59	-.25
critical	.07	-.22	.17	-.11	.58
discipli	.06	-.62	.07	.08	.09
disorgan	.01	.72	.03	.04	.14
distant	.59	.08	.06	-.11	.28
easygoin	-.11	.16	-.44	.43	-.04
friendly	-.48	-.15	.06	.55	-.13
givingup	.35	.46	.21	-.11	.12
hardwork	-.18	-.62	.15	.09	.13
harsh	.05	.09	.07	-.20	.65
kind	-.09	-.30	.04	.62	-.17
laidback	-.01	.28	-.57	.27	.07
lax	.03	.45	-.22	.24	.10
lazy	.18	.66	.10	.04	.11
opposing	-.04	.09	.08	-.12	.62
organize	-.06	-.74	-.04	.04	-.04
outgoing	-.82	-.06	-.05	.21	.03
persever	-.12	-.55	.17	.20	.10
quiet	.79	.10	.18	.16	-.01
relaxed	.00	.15	-.68	.36	-.08
responsi	.01	-.70	.06	.23	.05
shy	.70	.19	.17	.01	-.11
sociable	-.71	.04	-.06	.25	-.07
talkativ	-.76	.05	.02	.14	.17
tense	.15	.01	.77	-.01	.26
withdraw	.73	.15	.12	-.09	.26
worrying	.16	.07	.74	.03	.11

Number of cases read: 32   Number of cases listed: 32

Sample 2

```
FACTOR VAR = DISTANT TO EASYGOIN
 /MATRIX OUT(FAC=*)
 /plot = eigen
 /crit = fact(5)
 /print = default
 /extr = paf
 /rota = oblim.
```

Factor Correlation Matrix

Factor	1	2	3	4	5
1	1.000	-.188	.113	-.171	-.188
2	-.188	1.000	-.141	.034	-.100
3	.113	-.141	1.000	-.138	-.240
4	-.171	.034	-.138	1.000	.127
5	-.188	-.100	-.240	.127	1.000

Extraction Method: Principal Axis Factoring.
Rotation Method: Oblimin with Kaiser Normalization.

Pattern Matrix[a]

	Factor				
	1	2	3	4	5
distant	.587	.003	.268	-.020	.021
talkativ	-.789	-.118	.190	.085	-.073
careless	-.042	-.586	.244	.038	-.091
hardwork	-.146	.610	.149	.076	-.126
anxious	.088	-.135	.141	.231	-.685
agreeabl	.025	-.065	-.120	.629	.029
tense	.064	-.052	.179	.091	-.758
kind	-.027	.286	-.142	.599	-.060
opposing	-.075	-.039	.625	-.062	-.010
relaxed	.082	-.080	.005	.306	.696
disorgan	-.060	-.718	.111	.083	-.045
outgoing	-.831	-.014	.067	.134	-.011
approvin	-.181	.099	-.046	.487	.110
shy	.696	-.145	-.162	.093	-.140
discipli	.113	.651	.119	.080	-.023
harsh	.011	-.022	.656	-.135	.018
persever	-.089	.542	.117	.197	-.151
friendly	-.454	.087	-.105	.505	-.125
worrying	.085	-.123	.027	.119	-.752
responsi	.079	.724	.083	.218	-.027
contrary	.006	-.067	.720	-.061	-.057
sociable	-.718	-.116	-.047	.180	-.013
lazy	.121	-.653	.074	.097	-.102
cooperat	-.032	.181	-.214	.555	.065
quiet	.805	-.032	-.048	.260	-.124
organize	.014	.752	.000	.003	.069
critical	.052	.275	.587	-.048	-.085
lax	.029	-.416	.122	.254	.228
laidback	.039	-.213	.140	.241	.587
withdraw	.727	-.055	.232	.014	-.036
givingup	.287	-.441	.070	-.040	-.196
easygoin	-.053	-.119	.020	.389	.432

Extraction Method: Principal Axis Factoring.
Rotation Method: Oblimin with Kaiser Normalization.
a. Rotation converged in 18 iterations.

```
FLIP VAR = DISTANT TO EASYGOIN.

LIST VAR = all.

SORT CASES BY CASE_LBL.

SAVE OUTFILE = load2  /RENAME =
(VAR001=samp2f1) (VAR002=samp2f2)
(VAR003=samp2f3) (VAR004=samp2f4) (VAR005=samp2f5).



MATCH FILES FILE=load1 /FILE=load2 /BY CASE_LBL.

CORR VAR = samp1f1 WITH samp2f1 /VAR = samp1f2 WITH samp2f2
/VAR = samp1f3 WITH samp2f3.
```

Correlations for Analysis 1

		samp2f1
samp1f1	Pearson Correlation	.990
	Sig. (2-tailed)	.000
	N	32

Correlations for Analysis 2

		samp2f2
samp1f2	Pearson Correlation	-.991
	Sig. (2-tailed)	.000
	N	32

Correlations for Analysis 3

		samp2f3
samp1f3	Pearson Correlation	.157
	Sig. (2-tailed)	.391
	N	32

```
CORR VAR = samp1f1 WITH samp2f1
/VAR = samp1f2 WITH samp2f2
/VAR = samp1f3 WITH samp2f5.
```

Correlations for Analysis 1

		samp2f1
samp1f1	Pearson Correlation	.990
	Sig. (2-tailed)	.000
	N	32

Correlations for Analysis 2

		samp2f2
samp1f2	Pearson Correlation	-.991
	Sig. (2-tailed)	.000
	N	32

Correlations for Analysis 3

		samp2f5
samp1f3	Pearson Correlation	-.988
	Sig. (2-tailed)	.000
	N	32

3. Confirmatory Factor Analysis

Logic and JAMOVI implementation

Basic Considerations

- Model of Factor Analysis: $\mathbf{Y} = \mathbf{L} \cdot \mathbf{F} + \boldsymbol{\varepsilon}$
- Pre-define $\mathbf{F}$ (previous study, theory)
- Compare how well $\mathbf{Y} = \mathbf{L} \cdot \mathbf{F}$ can be described by the predefined $\mathbf{F}$
- Fit between the actual $\mathbf{F}$ and the predefined $\mathbf{F}$

Implementation

- SPSS for Mac doesn’t offer anything
- SPSS for Windows offers AMOS
- AMOS is said to be cumbersome

- JAMOVI
  - Free, easy to use
  - Under the hood is R
  - Other things it can do: Parallel analysis, mixed models, power, meta-analysis
  - Reproducibility: data, analyses, results all in one file

https://www.jamovi.org/download.html