Analytical analysis of timescales of seawater intrusion and retreat

Tianyuan Zheng[a,b], Bo Guo[c,*]

[a] *Department of Environmental Informatics, Helmholtz Centre for Environmental Research - UFZ, Permoserstraße 15, 04318 Leipzig, Germany*
[b] *Applied Environmental Systems Analysis, Dresden University of Technology, Germany*
[c] *Civil and Environmental Engineering, Princeton University, NJ, US, 08544*

Abstract

Keywords: Seawater intrusion, Timescale, Analytical analysis

---

1. Introduction

2. Methodology

2.1. Conceptual model

The conceptual model used in this study was a confined coastal aquifer with length L and thickness B which is shown is Fig. 1. The left boundary is coast and the right boundary is the inland aquifer. $h_\mathbf{s}$ is the initial seawater level and $h_\mathbf{f}$ is the initial inland freshwater head. The initial condition is regarded to be steady state.

2.2. Theoretical method

Basic assumptions [2]:

- Darcy's flow is valid.
- The standard expression for specific storage in a confined aquifer is applicable.
- The diffusive approach to dispersive transport is based on Fick's law.

*Corresponding author. Civil and Environmental Engineering, Princeton University
Email addresses: `tianyuan.zheng@ufz.de` (Tianyuan Zheng), `boguo@princeton.edu` (Bo Guo)

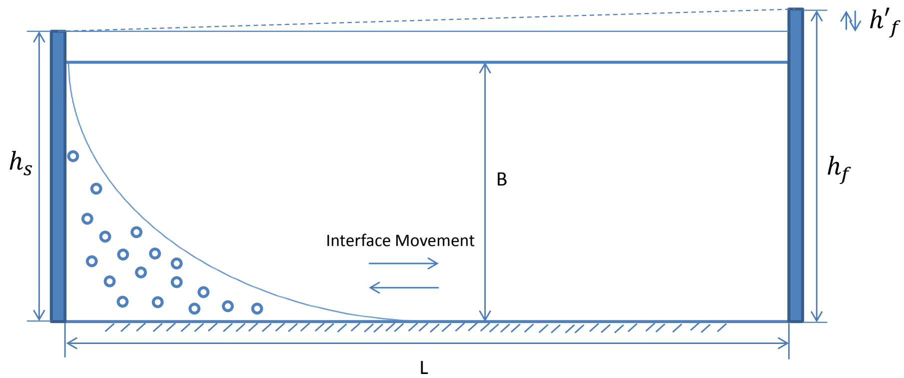

Figure 1: Conceptual model of the seawater intrusion and retreat. Following the same concept of [1]

- Isothermal conditions prevails.
- The porous medium is fully saturated with water.
- A single, fully miscible liquid phase of very small compressibility is taken in to account.

The governing equations [2]: Conversions from native aquifer water and equivalent freshwater head is in Eq. 1 and 2

$$h_f = \frac{\rho}{\rho_f} - \frac{\rho - \rho_f}{\rho_f} Z \tag{1}$$

$$h = \frac{\rho_f}{\rho} + \frac{\rho - \rho_f}{\rho} Z \tag{2}$$

where h_f is the equivalent freshwater head, h is the native aquifer water head, ρ is the density of saline ground water and ρ_f is the density of fresh water.

The governing equation for the conservation of mass is in 3

$$-\nabla \cdot (\rho \mathbf{q}) + \bar{\rho} q_{\mathbf{s}} = \frac{\partial(\rho\theta)}{\partial t} \tag{3}$$

where $\mathbf{q}$ is the specific discharge vector, $\bar{\rho}$ is the density of water entering from a source or leaving through a sink, $q_{\mathbf{s}}$ is the volumetric flow rate per unit volume of aquifer representing sources and sinks, θ is porosity and t is time.

The left hand side of Eq. 3 is the net flux of mass through the REV and the right hand side is the time rate of change in the mass stored in the REV which can be expanded with the chain rule:

$$\frac{\partial \rho \theta}{\partial t} = \rho \frac{\partial \theta}{\partial t} + \theta \frac{\partial \rho}{\partial t} = \rho \frac{\partial \theta}{\partial P} \frac{\partial P}{\partial t} + \theta \frac{\partial \rho}{\partial P} \frac{\partial P}{\partial t} + \theta \frac{\partial \rho}{\partial C} \frac{\partial C}{\partial t} \tag{4}$$

where C is the solute concentration and P is the fluid pore pressure.

3. Result and discussion

References

[1] C. Lu, A. D. Werner, Timescales of seawater intrusion and retreat, Advances in Water Resources 59 (2013) 39–51.

[2] W. Guo, C. D. Langevin, User's guide to seawat; a computer program for simulation of three-dimensional variable-density ground-water flow, Tech. rep. (2002).