# Analytical analysis of timescales of seawater intrusion and retreat

Tianyuan Zheng[a,b], Bo Guo[c,*]

[a] *Department of Environmental Informatics, Helmholtz Centre for Environmental Research - UFZ, Permoserstraße 15, 04318 Leipzig, Germany*
[b] *Applied Environmental Systems Analysis, Dresden University of Technology, Germany*
[c] *Civil and Environmental Engineering, Princeton University, NJ, US, 08544*

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**Abstract**

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## 1. Introduction

## 2. Methodology

### *2.1. Conceptual model*

The conceptual model used in this study was a confined coastal aquifer with length L and thickness B which is shown is Fig. 1. The left boundary is coast and the right boundary is the inland aquifer. $h_\mathbf{s}$ is the initial seawater level and $h_\mathbf{f}$ is the initial inland freshwater head. The initial condition is regarded to be steady state.

### *2.2. Theoretical method*

For this kind of problem, normally two kinds of models are introduced, miscible fluid model and immiscible fluid model [2]. Miscible fluid model takes the mixing zone of density and viscosity into account when salt water intrudes while Immiscible fluid model assumes a line interface between fresh and salt water and in either fluid, the concentration and density are constant. First Miscible model is introduced. Miscible model is widely implemented in numerical models but not appropriate for analytical analysis.

*Corresponding author. Civil and Environmental Engineering, Princeton University
*Email addresses:* tianyuan.zheng@ufz.de (Tianyuan Zheng), boguo@princeton.edu (Bo Guo)

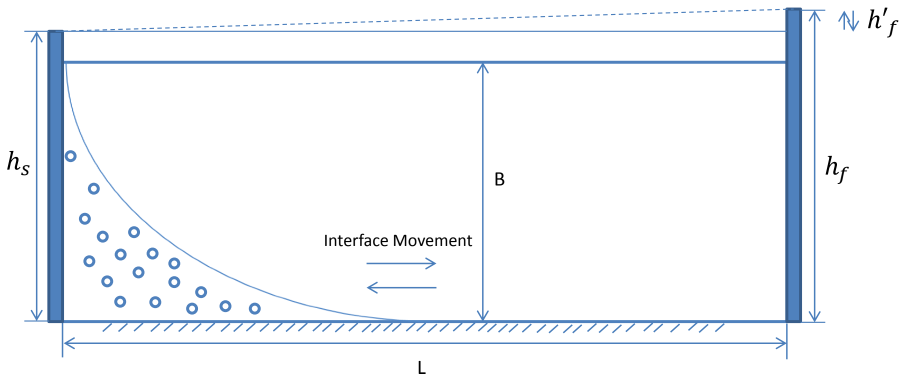


Figure 1: Conceptual model of the seawater intrusion and retreat. Following the same concept of [1]

Basic assumptions [3]:

- Darcy's flow is valid.
- The standard expression for specific storage in a confined aquifer is applicable.
- The diffusive approach to dispersive transport is based on Fick's law.
- Isothermal conditions prevails.
- The porous medium is fully saturated with water.
- A single, fully miscible liquid phase of very small compressibility is taken in to account.

The governing equations is based on [4].

The mass balance equation:

$$\rho S \frac{\partial P}{\partial t} + \phi \frac{\partial \rho}{\partial C_{\mathbf{m}}} \frac{C_{\mathbf{m}}}{\partial t} + \nabla \cdot (\rho \mathbf{q}) = \rho Q_{\mathbf{s}} \tag{1}$$

where $S$ is the specific storativity of the porous medium, $\rho$ is the mass density of the fluid, $\mathbf{q}$ is the Darcy velocity, $Q_{\mathbf{s}}$ is the source/sink term, $\phi$ is the porosity and $C_{\mathbf{m}}$ is the solute mass fraction.

The specific storativity $S$ is defined as:

$$S = \alpha(1-\phi) + \phi \beta \tag{2}$$

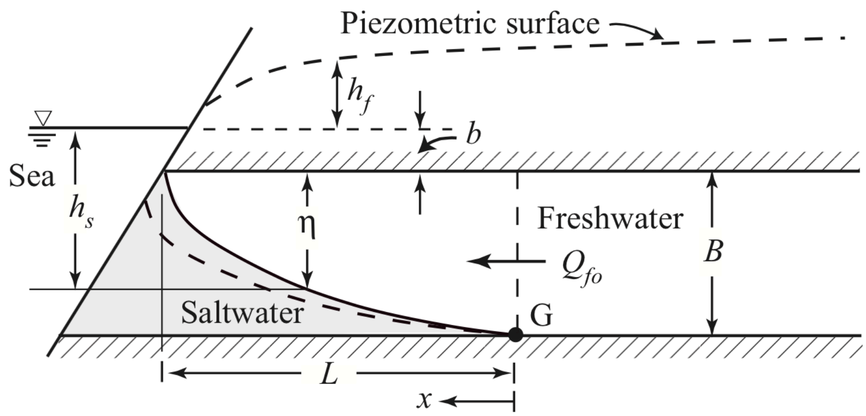


Figure 2: Conceptual model of the sharp interface seawater intrusion

where $\alpha$ is the coefficient of compressibility of porous medium, $\beta$ is the coefficient of compressibility of the fluid:

$$\beta = \frac{1}{\rho}\frac{\partial \rho}{\partial P}, \qquad \alpha = \frac{1}{1-\phi}\frac{\partial \phi}{\partial P} \tag{3}$$

The specific discharge is defined by the generalized Darcy's law:

$$\mathbf{q} = -\frac{1}{\mu}\mathbf{k}\cdot(\nabla P + \rho g \nabla z) \tag{4}$$

where $\mu$ is the dynamic viscosity of the fluid, $\mathbf{k}$ is the permeability tensor and $g$ is the gravity acceleration.

The solute transport equation:

$$\phi\rho\frac{\partial C_{\mathbf{m}}}{\partial t} + \rho\mathbf{q}\cdot\nabla C_{\mathbf{m}} = \nabla\cdot(\phi\rho\,\mathbf{D}\cdot\nabla C_{\mathbf{m}}) \tag{5}$$

In which $\mathbf{D}$ is the dispersion tensor.

While immiscible model is more simplified, several analytical solution for steady state is existed [5]. Let the interface toe to be located at the origin x = 0 (see Fig. 2), The seaward freshwater flow at this point is $Q_{fo}$; it is the difference between the total inflow to the aquifer through the right side boundary. We assume that the Dupuit assumption of essentially horizontal flow is valid for the freshwater domain above the interface.

$$-\mathbf{K}_f \eta(x) \frac{dh_f(X)}{dx} = Q_{fo}, \tag{6}$$

where $\eta$ is the thickness of freshwater above the saltwater wedge. Based on the Ghy-Herzberg approximation, the freshwater head is

$$h_f = \frac{\eta + b}{\delta} \tag{7}$$

where $\eta = \rho_f/(\rho_s - \rho_f)$ and using the above equation in Eq. 6, we can easily obtain

$$\eta^2 = \frac{2\delta Q_{fo}}{\mathbf{K}_f} L \tag{8}$$

In the above, we have used the boundary condtion $\eta = 0$ at $x = L$, Eq. 8 shows that the interface has the shape of a parabolic. Using the condition $\eta = B$ at $x = 0$, we can get

$$B^2 = \frac{2\delta Q_{fo}}{\mathbf{K}_f} L \tag{9}$$

This equation clearly indicates that the length of the seawater edge, $L$ and the discharge of freshwater to the sea. $Q_{fo}$. As $Q_{fo}$ increases, $L$ decreases.

## 3. Result and discussion

## References

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