Big Problems for Small Networks: Statistical Analysis of Small Networks and Team Performance

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Acknowledgements

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Network Science of Teams

a Multidisciplinary University Research Initiative

UC SANTA BARBARA · USC University of Southern California · Massachusetts Institute of Technology · NORTHWESTERN UNIVERSITY

Research Problem

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Is there any association between how team networks are structured and their performance?

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  - Social Networks: Advice Seeking, Leadership, Influence (among others).

Contents

Part I: Network Structure

Part II: Association between network structure and team performance

Part I: Network Structure

Exponential Random Graph Models (ERGMs)

Figure 1: Friendship network of a UK university faculty. Source: **igraphdata** R package (Csardi, 2015). Figure drawn using the R package **netplot** (yours truly, https://github.com/usccana/netplot)

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Where $\kappa(\theta, \mathbf{X})$ is the normalizing constant and equals $\sum_{\mathbf{g}' \in \mathcal{G}} \exp\{\theta^{\mathbf{t}} s(\mathbf{g}', \mathbf{X})\}$.

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- In the case of directed networks, $\mathcal{G}$ has $2^{n(n-1)}$ terms.
- See Wasserman, Pattison, Robins, Snijders, Handcock, Butts, and others.

Structures

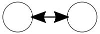
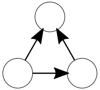
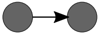
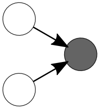
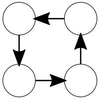
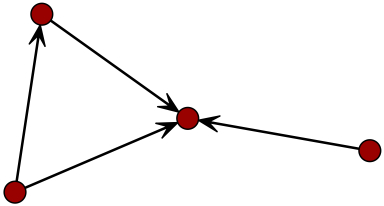
Representation	Description
	Mutual Ties (Reciprocity) $\sum_{i\neq j} y_{ij}y_{ji}$
	Transitive Triad (Balance) $\sum_{i\neq j\neq k} y_{ij}y_{jk}y_{ik}$
	Homophily $\sum_{i\neq j} y_{ij}\mathbf{1}\left(x_i = x_j\right)$
	Covariate Effect for Incoming Ties $\sum_{i\neq j} y_{ij}x_j$
	Four Cycle $\sum_{i\neq j\neq k\neq l} y_{ij}y_{jk}y_{kl}y_{li}$

Figure 2: Besides of the common edge count statistic (number of ties in a graph), ERGMs allow measuring other more complex structures that can be captured as sufficient statistics.

Example of model

In this network



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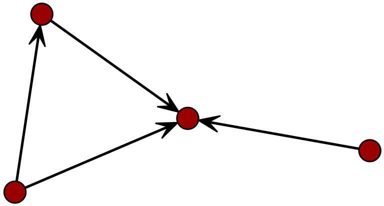

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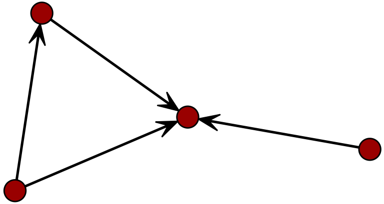

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The probability function of this model would be

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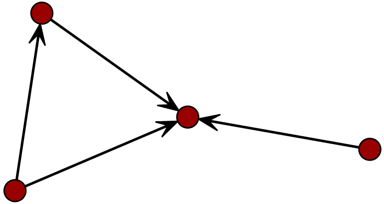

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This model has **MLE parameter estimates** of -0.19 (low density), 0.28 (high chance of ttriads), and -8.48 (low chance of mutuality) for the parameters `edges`, `ttriads`, and `mutual` respectively.

Estimation of ERGMs

- Calculating of the normalizing constant in (1), $\kappa = \sum_{\mathbf{g}' \in \mathcal{G}} \exp\left\{\theta^{\mathbf{t}} s\left(\mathbf{g}', \mathbf{X}\right)\right\}$, makes ERGMs difficult to estimate.

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- For this reason, statistical methods have focused on avoiding the direct calculation of κ; most modern methods for estimating ERGMs rely on MCMC.

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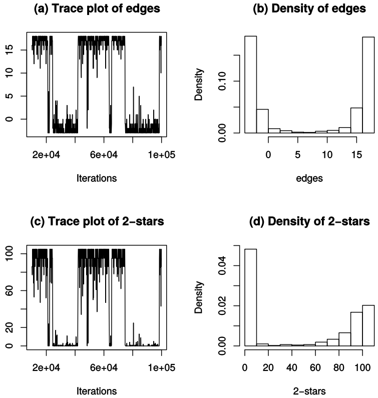

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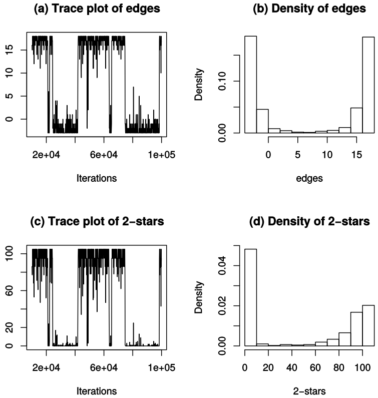

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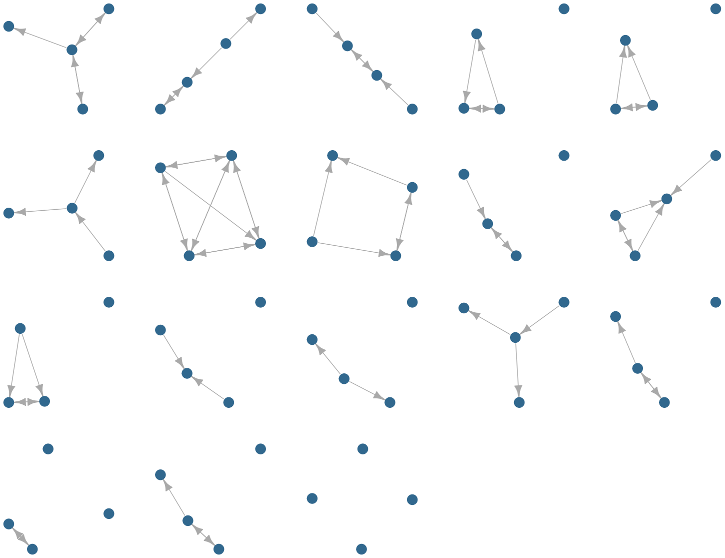
- Inference degeneracy is particularly problematic with small networks. . . (says anyone who has tried to fit one).

ERGMs for Small Networks



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How different is this from the "normal" way to fit ERGMs?

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We have implemented this and more in the `ergmito` R package

Sidetrack...

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% Screen shot of ERGMito tweet.

Special thanks to George Barnett who proposed the name during the 2018 NASN!

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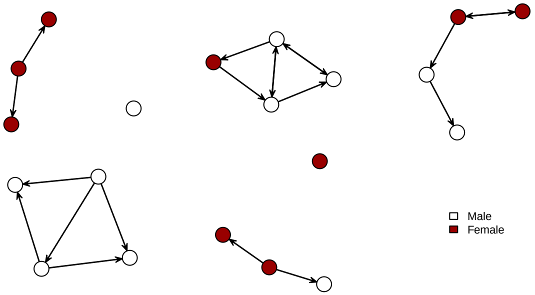
- Implements pooled ERGM models.
- Includes a simulation function for efficiently drawing samples of small networks, and by **efficiently** we mean **fast**.

[1] A directed graph of size 6 has a support set with $2^{6\times(6-1)} = 1,073,741,824$ elements.

ergmito example

```r
library(ergmito)
data(fivenets, package = "ergmito")
```



```r
# Looking at one of the five networks
fivenets[[1]]
```

```
##  Network attributes:
##   vertices = 4
##   directed = TRUE
##   hyper = FALSE
##   loops = FALSE
##   multiple = FALSE
##   bipartite = FALSE
##   total edges= 2
##     missing edges= 0
##     non-missing edges= 2
##
##  Vertex attribute names:
##     female name
##
## No edge attributes
```

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fivenets[[1]]
```

```
##  Network attributes:
##   vertices = 4
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##   total edges= 2
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##
##  Vertex attribute names:
##     female name
##
## No edge attributes
```

How can we fit an ERGMito to this 5 networks?

ergmito example (cont'd)

The same as you would do with the ergm package:

```
(model1 <- ergmito(fivenets ~ edges + nodematch("female")))
```

```
##
## ERGMito estimates
##
##  Coefficients:
##            edges  nodematch.female
##           -1.705             1.587
```

	Model 1
edges	-1.70^{**}
	(0.54)
nodematch.female	1.59^{*}
	(0.64)
AIC	73.34
BIC	77.53
Log Likelihood	-34.67
Num. networks	5

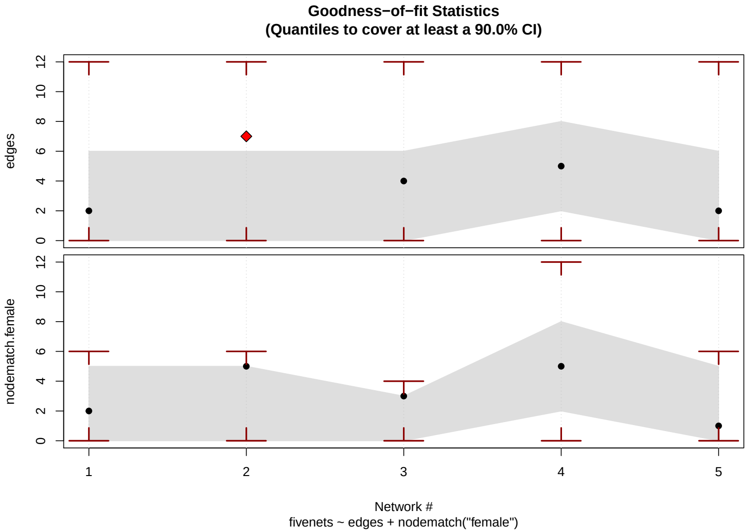
$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table 1: Statistical models

```r
(gof1 <- gof_ergmito(model1))
```

```
##
## Goodness-of-fit for edges
##
##        obs min mean max lower upper lower prob. upper prob.
## net 1   2   0  3.7  12     0     6      0.0081        0.96
## net 2   7   0  3.7  12     0     6      0.0081        0.96
## net 3   4   0  3.1  12     0     6      0.0206        0.99
## net 4   5   0  5.6  12     2     8      0.0309        0.95
## net 5   2   0  3.7  12     0     6      0.0081        0.96
##
##
## Goodness-of-fit for nodematch.female
##
##        obs min mean max lower upper lower prob. upper prob.
## net 1   2   0  2.8   6     0     5       0.022        0.99
## net 2   5   0  2.8   6     0     5       0.022        0.99
## net 3   3   0  1.9   4     0     3       0.079        0.95
## net 4   5   0  5.6  12     2     8       0.031        0.95
## net 5   1   0  2.8   6     0     5       0.022        0.99
##
## Note: Exact confidence intervals where used. This implies that the requestes CI may differ from the one used (se
```

```r
plot(gof1)
```


Goodness−of−fit Statistics
(Quantiles to cover at least a 90.0% CI)
edges
nodematch.female
0
2
4
6
8
10
12
1
2
3
4
5
Network #
fivenets ~ edges + nodematch("female")

Simulation Study

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3. Use the **drawn parameters** and **group sizes** to **generate random graphs** using an ERGM data generating process.

We simulated 100,000 samples, each one composed of an average of 30 networks.

Simulation Study (cont'd)

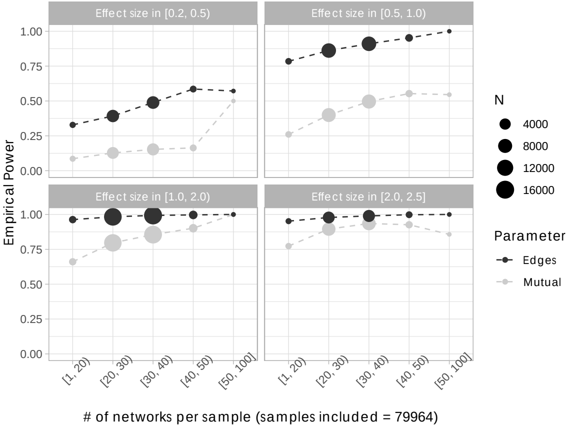

Figure 4: Empirical power of Pooled-ERGM estimates at various levels of effect size. As expected, power increases significantly with sample size (# of networks per sample). Interestingly, the discovery rate of an effect size within $[1, 2)$ is very high even with a sample size of 20-30 networks. More extreme points have higher volatility due to small number of samples included.

So now that we can estimate ERGMs for small networks (cool!). . .

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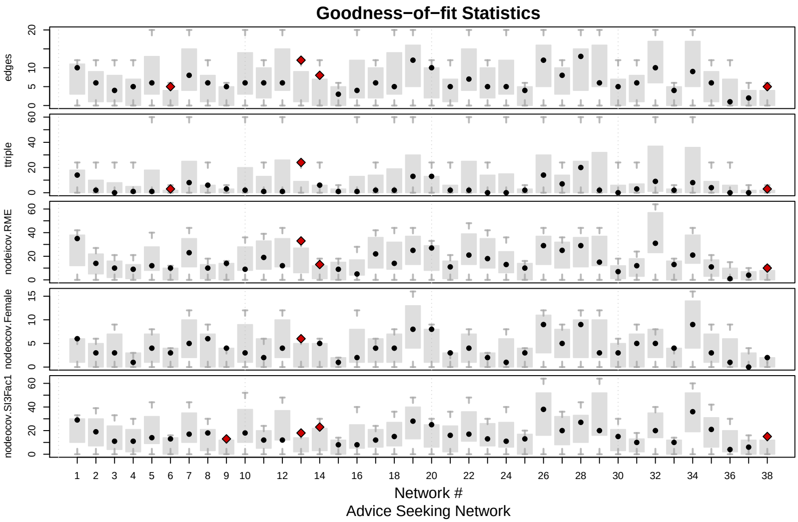
... what can this tell us about our 42 teams?

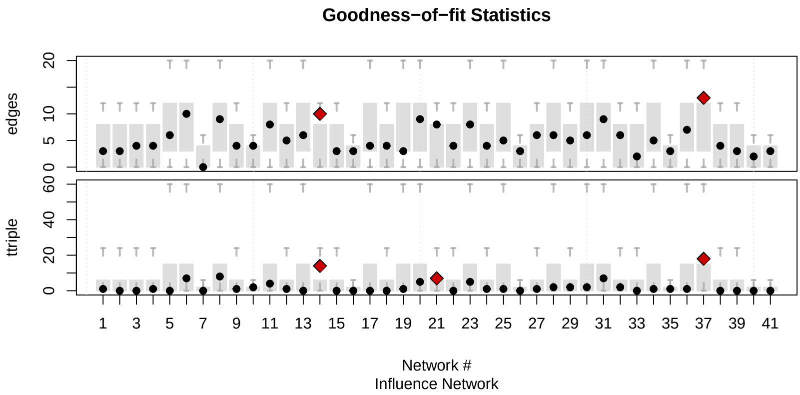
Preliminary results

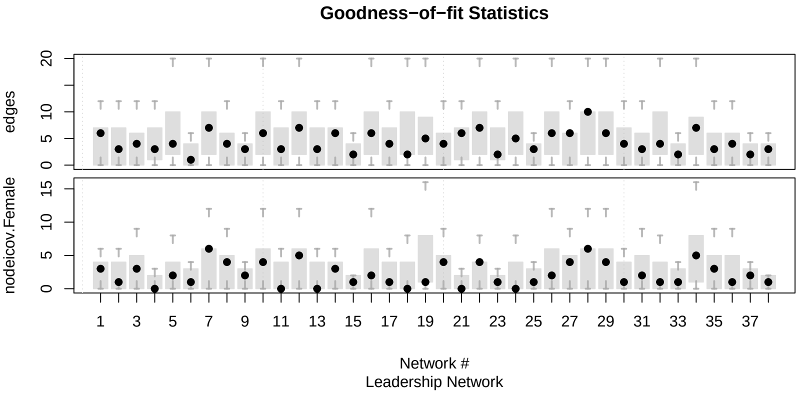
	Advice	Influence	Leadership
Edges	-1.87^{***}	-0.78^{***}	-0.57^{***}
	(0.30)	(0.13)	(0.14)
Transitive Triads	0.24^{***}	0.21^{**}	
	(0.06)	(0.08)	
Indeg. RME	0.35^{***}		
	(0.08)		
Outdeg. Female	0.43^{*}		
	(0.19)		
Outdeg. Social Accomodation	0.11		
	(0.08)		
Indeg. Female			-0.38^{*}
			(0.19)
AIC	693.18	760.40	655.78
BIC	714.50	769.12	664.32
Log Likelihood	-341.59	-378.20	-325.89
Num. networks	38	41	38

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table 2: The two statistics that showed to be the most robust were **Indeg. RME** and **Outdeg. Female**. These two effects can be described as (1) individuals with high levels of RME receive more ties, and (2) female subjects were more likely of seeking advice than male. Other statistics such as GPA, religiousness, age, and ethnicity were not significant.


Goodness−of−fit Statistics
edges
ttriple
nodeicov.RME
nodeocov.Female
nodeocov.SI3Fac1
Network #
Advice Seeking Network


Goodness−of−fit Statistics
edges
ttriple
0
5
10
20
0
20
40
60
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
31
33
35
37
39
41
Network #
Influence Network



Goodness−of−fit Statistics
edges
nodeicov.Female
0
5
10
15
20
1
3
5
7
9
11
13
15
17
19
21
23
25
27
29
31
33
35
37
Network #
Leadership Network

Part II: Association between network structure and team performance

Testing effects of social network structure on group performance

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BTW: Talking about Degree sequence leads directly to the now controvelsial Scale-free networks.

“Scale-free networks are rare”

> “*The **structural diversity of real-world networks** uncovered here presents both a puzzle and an opportunity. The strong focus in the scientific literature on **explaining and exploiting scale-free patterns has meant relatively less is known about mechanisms that produce non-scale-free structural patterns**, e.g., those with degree distributions better fitted by a log-normal. Two important directions of future work will be the **development and validation of novel mechanisms for generating more realistic degree structure in networks**, and novel statistical techniques for identifying or untangling them given empirical data*”
>
> **– p. 8, Broido and Clauset (2019)**

See Holme (2019) for a recent reference on the Scale-free issue.

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  - On average, sampled networks will **look like the original** graph (addressing Broido and Clauset (2019)'s comments).

In principle, this would be equivalent to a revised rewiring test...

Algorithm

1. Estimate an ERGM (estimates can come from a single graph or pooled estimates). We denote the data-generating-process of this model as $\mathcal{D} : \Theta \times \mathcal{X} \mapsto \mathcal{G}$.

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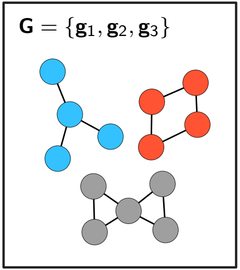
Note An important distinction to make is that structures that gave origin to the graph need not to be relevant for the team's performance per se.

Illustrated example

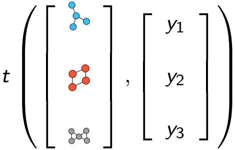
Suppose that we have a 3 networks of sizes 4, 4, and 5 respectively. The

Step 1:
Fit the ERGMito



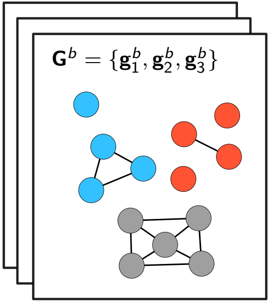
Fit the ERGMito,
This will give us $\mathcal{D}(\hat{\theta}, X_j)$

Step 2:
Calculate $t_0 =$



Throughout the simulations the only part that changes is the networks, not Y

Step 3:
For $b \in 1, \dots, B$ do



3.1) For $j \in \{1, 2, 3\}$ draw a new network from $\mathcal{D}$
3.2) Use the new sample to calculate $t_b = t(\mathbf{G}^b, Y)$

We can use the distribution of the sequence $\{t_1, \dots, t_B\}$ as null to compare against t_0

Extended example with `fivenets`

Going back to our `fivenets` example:

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- Our test statistic t is the correlation between y and $s(\mathbf{g})$

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- The data looks something like this

y	s(g)
1.0138091	2
0.6051448	1
4.3085153	2
0.9547600	0
-0.1330788	1

We will try to assess the presence/absence of an association between the outcome y and the number of ties received by women, `nodeicov("female")`, using both a linear regression model and our test

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- OLS approach In the following model:

$$
y = \alpha + \theta^{OLS} s\left(\mathbf{g}\right) + \varepsilon, \quad \varepsilon \sim N(0,1)
$$

is the θ_{OLS} parameter significantly different from zero?

Extended example with `fivenets` (cont'd)

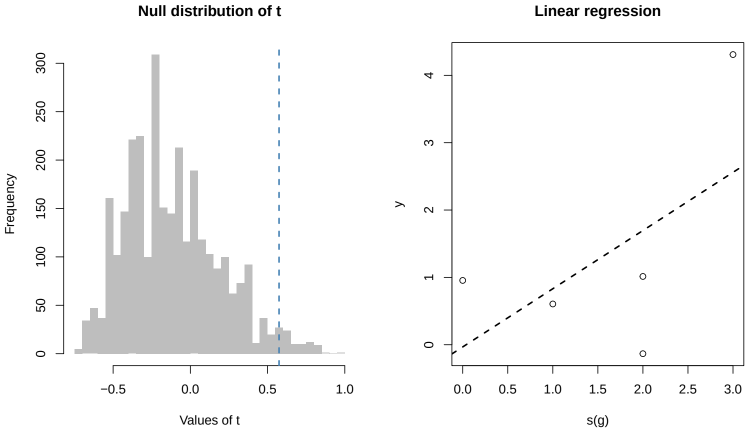

Figure 5: Comparing our method against a linear regression. Our proposed method returned a two sided p-value of 0.045, while the pvalue for the OLS coefficient was 0.311.

Concluding remarks

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Exponential Random Graph Models for Small Networks:

1. Not a new thing, what's new is the tool to do so in a smooth way.

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3. Working on a more formal statistical framework (when is it a good/bad idea to use this kind of method).

Thanks!

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Let's chat!

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References I

Allaire, JJ, Yihui Xie, Jonathan McPherson, Javier Luraschi, Kevin Ushey, Aron Atkins, Hadley Wickham, Joe Cheng, Winston Chang, and Richard Iannone. 2018. Rmarkdown: Dynamic Documents for R. https://rmarkdown.rstudio.com.

Broido, Anna D, and Aaron Clauset. 2019. "Scale-Free Networks Are Rare." Nature Communications 10 (1): 1017.

Csardi, Gabor. 2015. Igraphdata: A Collection of Network Data Sets for the 'Igraph' Package. https://CRAN.R-project.org/package=igraphdata.

Geyer, Charles J., and Elizabeth A. Thompson. 1992. "Constrained Monte Carlo Maximum Likelihood for Dependent Data." Journal of the Royal Statistical Society. Series B (Methodological) 54 (3): 657–99. http://www.jstor.org/stable/2345852.

Handcock, Mark S. 2003. "Assessing Degeneracy in Statistical Models of Social Networks." Working Paper No. 39 76 (39): 33–50. https://doi.org/10.1.1.81.5086.

References II

Handcock, Mark S., David R. Hunter, Carter T. Butts, Steven M. Goodreau, Pavel N. Krivitsky, and Martina Morris. 2018. Ergm: Fit, Simulate and Diagnose Exponential-Family Models for Networks. The Statnet Project (http://www.statnet.org). https://CRAN.R-project.org/package=ergm.

Handcock, Mark, Peng Wang, Garry Robins, Tom Snijders, and Philippa Pattison. 2006. "Recent developments in exponential random graph (p*) models for social networks." Social Networks 29 (2): 192–215. https://doi.org/10.1016/j.socnet.2006.08.003.

Holme, Petter. 2019. "Rare and everywhere: Perspectives on scale-free networks." Nature Communications 10 (1): 1016. https://doi.org/10.1038/s41467-019-09038-8.

Hunter, David R., Mark S. Handcock, Carter T. Butts, Steven M. Goodreau, and Martina Morris. 2008. "Ergm: A Package to Fit, Simulate and Diagnose Exponential-Family Models for Networks." Journal of Statistical Software 24 (3): 1–29.

References III

Kim, Young Ji, David Engel, Anita Williams Woolley, Jeffrey Yu-Ting Lin, Naomi McArthur, and Thomas W. Malone. 2017. "What Makes a Strong Team?: Using Collective Intelligence to Predict Team Performance in League of Legends." In Proceedings of the 2017 Acm Conference on Computer Supported Cooperative Work and Social Computing, 2316–29. CSCW '17. New York, NY, USA: ACM. https://doi.org/10.1145/2998181.2998185.

Milo, R, N Kashtan, S Itzkovitz, M E J Newman, and U Alon. 2004. "On the uniform generation of random graphs with prescribed degree sequences." Arxiv Preprint Condmat0312028 cond-mat/0: 1–4. http://arxiv.org/abs/cond-mat/0312028.

R Core Team. 2018. R: A Language and Environment for Statistical Computing. Vienna, Austria: R Foundation for Statistical Computing. https://www.R-project.org/.

Wasserman, Stanley, and Philippa Pattison. 1996. "Logit models and logistic regressions for social networks: I. An introduction to Markov graphs andp." Psychometrika 61 (3): 401–25. https://doi.org/10.1007/BF02294547.

Xie, Yihui. 2018. Knitr: A General-Purpose Package for Dynamic Report Generation in R. https://yihui.name/knitr/.