ERGMito

Statistical Models for Small Team Social Networks

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What makes a successful team?

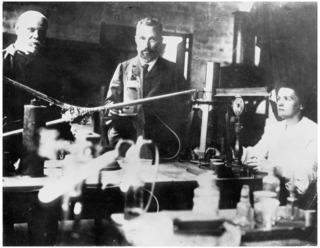

Figure 1 Nobel in Physics 1903: Henri Becquerel, Pierre Curie, and Marie Curie

Figure 2 Nobel in Economics 2019: Abhijit Banerjee, Esther Duflo, and Michael Kremer

Figure 3 US 2019 Women's Soccer team

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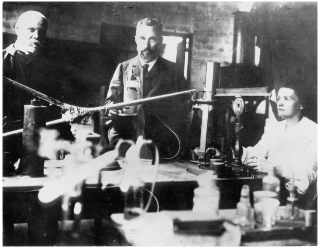

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A lot depends on human relations...

What makes a successful team?

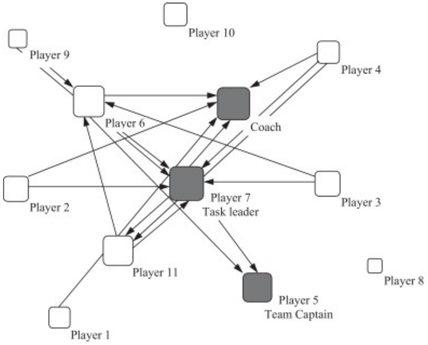

Figure 4 A volleyball team leadership network (Fransen et al., 2015)

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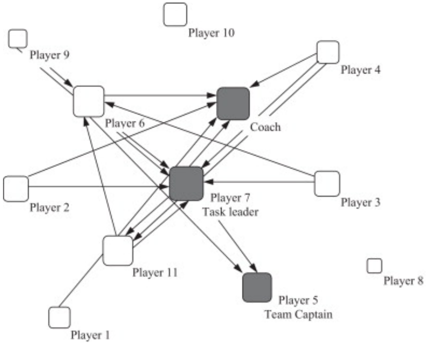
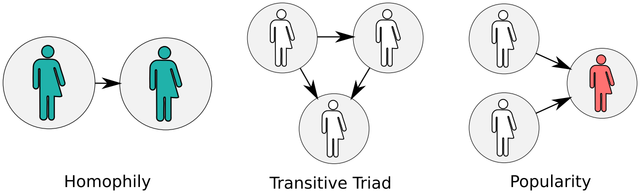


Figure 4 A volleyball team leadership network (Fransen et al., 2015)

What processess govern the formation of social networks?

- Is it *homophily*, *social balance* (transitivity), *popularity*?



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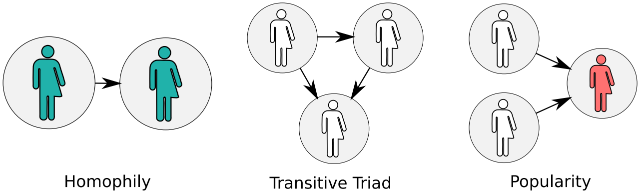

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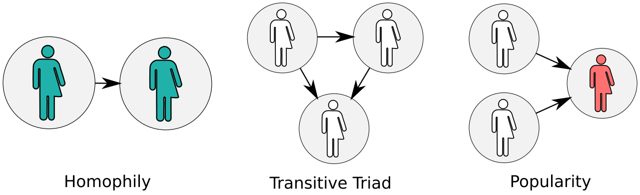

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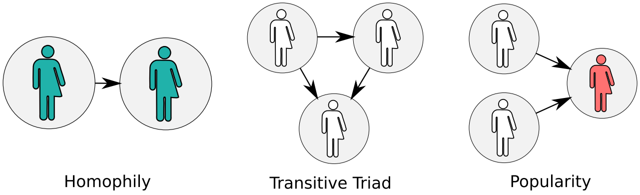
  statistical inference on what network patterns/structures/motifs govern social networks



Fairly complex models for studying networks

A vector of model parameters

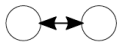
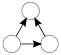
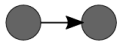
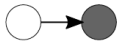
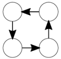
A vector of sufficient statistics

$$\Pr\left(\mathbf{Y} = \mathbf{y} \mid \theta, \mathbf{X}\right) = \frac{\exp\left\{\theta^{\mathbf{t}} s\left(\mathbf{y}, \mathbf{X}\right)\right\}}{\sum_{\mathbf{y}' \in \mathcal{Y}} \exp\left\{\theta^{\mathbf{t}} s\left(\mathbf{y}', \mathbf{X}\right)\right\}}, \quad \forall \mathbf{y} \in \mathcal{Y}$$

Observed data

The normalizing constant

All possible networks

Representation	Description
	Mutual Ties (Reciprocity) $\sum_{i \neq j} y_{ij} y_{ji}$
	Transitive Triad (Balance) $\sum_{i \neq j \neq k} y_{ij} y_{jk} y_{ik}$
	Homophily $\sum_{i \neq j} y_{ij} \mathbf{1}\left(x_i = x_j\right)$
	Attribute-receiver effect $\sum_{i \neq j} y_{ij} x_j$
	Four Cycle $\sum_{i \neq j \neq k \neq l} y_{ij} y_{jk} y_{kl} y_{li}$

Exponential Random Graph Models

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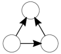
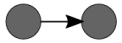
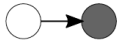
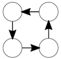
	Transitive Triad (Balance) $\sum_{i \neq j \neq k} y_{ij} y_{jk} y_{ik}$
	Homophily $\sum_{i \neq j} y_{ij} \mathbf{1}\left(x_i = x_j\right)$
	Attribute-receiver effect $\sum_{i \neq j} y_{ij} x_j$
	Four Cycle $\sum_{i \neq j \neq k \neq l} y_{ij} y_{jk} y_{kl} y_{li}$

We will be using a novel extension that focuses on small networks (**ERGMitos**)

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ERGMs for small networks

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Small means that we can calculate the normalizing constant exactly, and thus:

ERGMs for small networks

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Observed data

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- Improved accuracy (bias),

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ERGMs for small networks

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- Smaller Type I Error rates,
- Faster estimation (about 20 times faster)

ERGMs for small networks

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Observed data

The normalizing constant

All possible networks

Small means that we can calculate the normalizing constant exactly, and thus:

- Improved accuracy (bias),
- Smaller Type I Error rates,
- Faster estimation (about 20 times faster)
- A world of possibilities...

(Vega Yon, Salughter and de la Haye (2019))

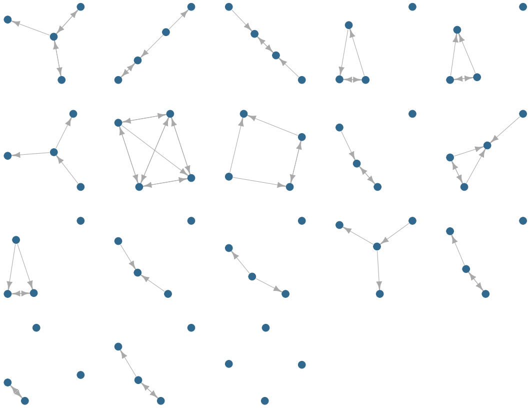
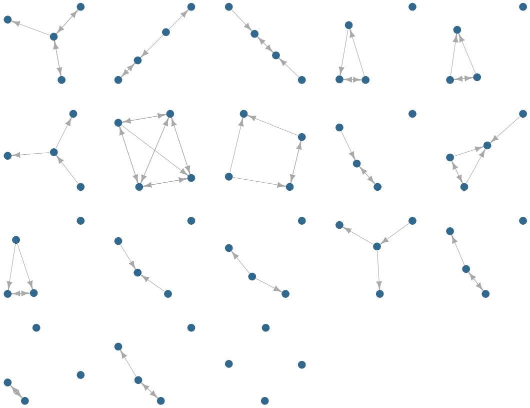


Figure 5 Advice seeking networks observed in an experiment.

Some results on the emergence of advice-seeking networks in small teams

1. Higher Social Perceptiveness *means* more connections.
2. The friend of my friend is my friend.



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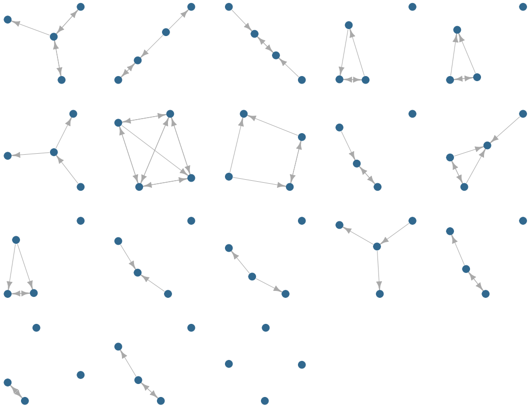
Some results on the emergence of advice-seeking networks in small teams

1. Higher Social Perceptiveness *means* more connections.
2. The friend of my friend is my friend.
3. Women ask for advice.



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- Understanding local-macro dependencies is key.
- We can use ERGMitos to elucidate the processes that govern small networks.
- Insight into **team networks** is valuable for understanding team **process and performance**, and improving **team design**

ERGMito

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Thanks!