1. Random variable ***X*** has probability distribution given by probability mass function in following table.

x	0	1	2
$P(X=x)$	0,3	0,4	0,3

Random variable ***Y*** has probability distribution given by probability mass function in following table.

y	0	1	2	3
$P(Y=y)$	0.1	0.4	0.4	0.1

V = X·Y under assumption, that the random variables ***X, Y*** are independent.

Support of random variable V, S_V={0,1,2,3,4,6}

Then to find probability distribution of random variable V we need to compute probabilities for all items from the support or random variable V.

The event {V=0} may be consider as exceptional

P(V=0)=P(X·Y=0)=P(X=0 or Y=0)=P(X=0)+P(Y=0)-P(X=0 and Y=0)=P(X=0)+P(Y=0)-P(X=0)·P(Y=0)=

=0.3+0.1-0.3·0.1=0.37

(because of the P(A∪B)=P(A)+P(B)-P(A∩B))

For events {V=1}, {V=2} and so on natural approach works.

P(V=1)=P(X·Y=1)=P(X=1 and Y=1)=0.4·0.4=0.16

P(V=2)=P(X·Y=2)=P((X=1 and Y=2) or (X=2 and Y=1))=P(X=1 and Y=2) + P(X=2 and Y=1)=0.4·0.4+0.3·0.4

And so on…

2. Joint probability distribution of random vector (***X,Y***) is given in the following table.

$X \setminus Y$	-1	1
0	0,125	0,125
2	0,5	c

Find constant c, marginal and conditional distributions for random variables ***X*** and ***Y***. Additionally find the value of correlation coefficient for random variables ***X*** and ***Y***.

In order to find value of parameter c it's necessary to apply the basic condition about probability distribution:

0,125+0,125+0,5+c=1

c=0,25

Now we have joint probability distribution:

X \ Y	-1	1	P(X=x)
0	0,125	0,125	0,25
2	0,5	0,25	0,75
P(Y=y)	0,625	0,375	1

Last column consists of marginal probability distribution for random variable X, in last row there is marginal probability distribution for random variable Y.

Conditional probability distribution for random variable X given {Y=-1}

P(X=0/Y=-1)=$\frac{P(X=0,Y=-1)}{P(Y=-1)} = \frac{0{,}125}{0{,}625} = 0{,}2$

P(X=2/Y=-1)=$\frac{P(X=2,Y=-1)}{P(Y=-1)} = \frac{0{,}5}{0{,}625} = 0{,}8$

Conditional probability distribution for random variable X given {Y=1}

P(X=0/Y=1)=$\frac{P(X=0,Y=1)}{P(Y=1)} = \frac{0{,}125}{0{,}375} = 0{,}33$

P(X=2/Y=1)=$\frac{P(X=2,Y=1)}{P(Y=1)} = \frac{0{,}25}{0{,}375} = 0{,}66$

Conditional probability distribution for random variable Y given {X=0}

P(Y=-1/X=0)=$\frac{P(X=0,Y=-1)}{P(X=0)} = \frac{0{,}125}{0{,}25} = 0{,}5$

P(Y=1/X=0)=$\frac{P(X=0,Y=1)}{P(X=0)} = \frac{0{,}125}{0{,}25} = 0{,}5$

Conditional probability distribution for random variable Y given {X=2}

P(Y=-1/X=2)=$\frac{P(X=2,Y=-1)}{P(X=2)} = \frac{0{,}5}{0{,}75} = 0{,}66$

P(Y=1/X=2)=$\frac{P(X=2,Y=1)}{P(X=1)} = \frac{0{,}25}{0{,}75} = 0{,}33$

Examination of independence:

X \ Y	-1	1	P(X=x)
0	1/8	1/8	¼
2	1/2	1/4	¾
P(Y=y)	5/8	3/8	1

Random variables X and Y are not independent!

(the condition for independence says that marginal probability for X times by marginal probability for Y supposed to be joint probability – it's not hold)

There is a need to compute coefficient of correlation.

First step – computation of expected values for X and Y.

$EX=0\cdot 1/4+2\cdot 3/4=6/4=3/2$ $\quad$ $EX^2=0^2\cdot 1/4+2^2\cdot 3/4=3$

$EY=(-1)\cdot 5/8+1\cdot 3/8=-2/8=-1/4$ $\quad$ $EY^2=(-1)^2\cdot 5/8+1^2\cdot 3/8=1$

Variance of X

$VarX= EX^2 -(EX)^2=3-(3/2)^2=3/4$

Variance of Y

$VarY= EY^2 -(EY)^2=1-(-1/4)^2=15/16$

Preparation for computation of covariance

$E(X\cdot Y)=1/8\cdot 0\cdot(-1)+1/8\cdot 0\cdot 1+1/2\cdot 2\cdot(-1)+1/4\cdot 2\cdot 1=-1+1/2=-1/2$

cov(X,Y)=E(X·Y)-EX·EY=-1/2-3/2·(-1/4)=-1/2+3/8=-1/8

Coefficient of correlation

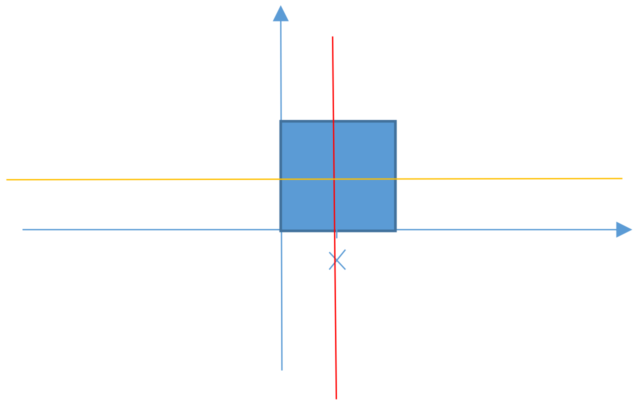
corr(X,Y)=$\frac{cov(X,Y)}{\sqrt{VarX \cdot \mathrm{VarY}}} = \frac{-1/8}{\sqrt{\frac{3}{4}\cdot\frac{15}{16}}}$.

3. Joint distribution of random vector (***X***,***Y***) is given by two variables density function.

$$f(x,y)=\begin{cases} c & for \quad 0<x<1,\ 0<y<1 \\ 0 & \qquad in\ other\ cases \end{cases}$$

Find constant c, marginal and conditional distributions for random variables ***X*** and ***Y***. Additionally find the value of correlation coefficient for random variables ***X*** and ***Y***.

$$\iint_{-\infty}^{+\infty} f(x,y)dxdy = \iint_{0}^{1} c\ dxdy = c\iint_{0}^{1} 1\ dxdy = c \cdot 1$$



f(x,y) supposed to be joint density => c· $1 = 1 => c = 1$
Joint probability distribution for random vector (X,Y) is given by 2 dimensional density function:

$$f(x,y) = \begin{cases} 1 & 0 < x,y < 1 \\ 0 & in\ other\ cases \end{cases}$$

Marginal density for random variable X (by definition)

$$f_x(x) = \int_{-\infty}^{+\infty} f(x,y)dy$$

For 0<x<1

$$f_x(x) = \int_{-\infty}^{+\infty} f(x,y)dy = \int_{0}^{1} 1dy = 1$$

Finally marginal density for X

$$fx(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & in\ other\ cases \end{cases}$$

Marginal density for random variable Y (by definition)

$$f_y(y) = \int_{-\infty}^{+\infty} f(x,y)dx$$

For 0<y<1

$$f_y(y) = \int_{-\infty}^{+\infty} f(x,y)dx = \int_0^1 1dx = 1$$

Marginal density for Y

$$fy(y) = \begin{cases} 1 & 0 < y < 1 \\ 0 & in\ other\ cases \end{cases}$$

Examination of independence:

$f(x,y) = fx(x) \cdot fy(y)$ => random variables X and Y are independent! => corr(X,Y)=0

$f(x,y) = fx(x) \cdot fy(y)$ => conditional prob. distributions are exactly the same like marginal prob. distributions (no need to compute conditional prob. distributions)

Conditional prob. distribution for random variable X given {Y=y}

$$f(x/y) = \begin{cases} 1 & 0 < x, y < 1 \\ 0 & in\ other\ cases \end{cases}$$

Conditional prob. distribution for random variable Y given {X=x}

$$f(y/x) = \begin{cases} 1 & 0 < x, y < 1 \\ 0 & in\ other\ cases \end{cases}$$

4. Random variable ***X*** has probability distribution given by probability mass function in following table.

x	1	2	3
$P(X = x)$	0,5	0,25	0,25

Random variable ***Y*** has probability distribution given by probability mass function in following table.

y	0	1	2
$P(Y = y)$	0,25	0,5	0,25

Find the joint probability distribution of random vector (***X***,***Y***) under the condition that ***X*** and ***Y*** are independent.

The available information about probability distribution is represented by information about marginal probability distributions for the components X and Y.

In terms of joint probability distribution for random vector (X,Y) we have the knowledge about last row and last column of following table:

X \ Y	0	1	2	**P(X=x)**
1	p_{11}	p_{12}	p_{13}	$\frac{1}{2}$
2	p_{21}	p_{22}	p_{23}	$\frac{1}{4}$
3	p_{31}	p_{32}	p_{33}	$\frac{1}{4}$
P(Y=y)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	

The "grey part" isn't available directly. Concerning the procedure for computation of marginal probabilities we can consider unknown joint probabilities as unknown parameters p_{11}, p_{12}, p_{13}, p_{21}, p_{22}, p_{23}, p_{31}, p_{32}, p_{33} . The concept of above mentioned parameters let us to build system of equations:

$$\begin{cases} p_{11}+p_{12}+p_{13}=\frac{1}{2} \\ p_{21}+p_{22}+p_{23}=\frac{1}{4} \\ p_{31}+p_{32}+p_{33}=\frac{1}{4} \\ p_{11}+p_{21}+p_{31}=\frac{1}{4} \\ p_{12}+p_{22}+p_{32}=\frac{1}{2} \\ p_{13}+p_{23}+p_{33}=\frac{1}{4} \\ p_{11}+p_{12}+p_{13}+p_{21}+p_{22}+p_{23}+p_{31}+p_{32}+p_{33}=1 \end{cases}$$

This part corresponds to marginal probability distribution of X

This part corresponds to marginal probability distribution of Y

General condition for all probability distributions.

Above system of equations has endless number of solutions (Kronecker-Capelli Theorem).

In order to find joint probabilities is necessary to use independence of random variables X and Y.

Because of independence P(X=1,Y=0)=P(X=1)·P(Y=0), P(X=1,Y=1)=P(X=1)·P(Y=1) and so on.

Independence of random variables X and Y let us to derive joint probability distribution for random vector (X,Y)

X \ Y	0	1	2	**P(X=x)**
1	$\frac{1}{2}\cdot\frac{1}{4}$	$\frac{1}{2}\cdot\frac{1}{2}$	$\frac{1}{2}\cdot\frac{1}{4}$	$\frac{1}{2}$
2	$\frac{1}{4}\cdot\frac{1}{4}$	$\frac{1}{2}\cdot\frac{1}{4}$	$\frac{1}{4}\cdot\frac{1}{4}$	$\frac{1}{4}$
3	$\frac{1}{4}\cdot\frac{1}{4}$	$\frac{1}{2}\cdot\frac{1}{4}$	$\frac{1}{4}\cdot\frac{1}{4}$	$\frac{1}{4}$
P(Y=y)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	