FRC 566.010 — UCC PDE: Existence, Uniqueness, and Dissipation

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Abstract

We establish well-posedness for the Universal Coherence Condition (UCC) $\partial_t \ln C = D_C \Delta \ln C + S_C$ under standard boundary conditions and prove a dissipation inequality $\sigma(t) = k_* D_C \int \|\nabla \ln C\|^2 \, dV \geq 0$. Fractional variants are briefly discussed. Numerical 1D/2D demos illustrate convergence to stationary states.

1. Statement and assumptions

Define functional and spatial setting; S_C regularity; domains and BCs (Neumann/Dirichlet).

2. Existence and uniqueness

Outline standard parabolic theory (energy estimates, semigroup framework); uniqueness from Gronwall-type arguments.

3. Dissipation inequality and stationary solutions

Prove $\sigma(t) \geq 0$ and characterize stationary solutions for simple S_C.

4. Numerical demos (reproducible)

Run `python code/566.010/pde_demo.py` to regenerate 1D/2D figures.

References

To be added (parabolic PDE, semigroup, fractional Laplacians).