

# FRC 100.007 - The Lambda-Field: Scalar Completion of Coherence Dynamics

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DOI: [10.5281/zenodo.17968952](https://doi.org/10.5281/zenodo.17968952)

## Abstract

The Fractal Resonance Cognition (FRC) framework requires coherence to be locally defined, transportable, and dynamically influential. This paper introduces the Lambda-field - a real scalar field on spacetime - as the minimal mathematical object satisfying these requirements. Coherence is defined logarithmically as a scalar density, an explicit action is postulated, and field equations are derived. The coherence drift term introduced in FRC 100.004 is shown to arise as Lambda-gradient coupling on state space, with trace preservation explicitly maintained. Born-rule statistics emerge as equilibrium states of Lambda-dynamics rather than axioms. Standard quantum mechanics is recovered exactly in the vanishing-coupling limit. We explicitly constrain the scope of this field to mesoscopic coherence dynamics; the Lambda-field couples to quantum information states, not to macroscopic stress-energy. This paper establishes the minimal field-theoretic closure required for the FRC 100-series.

## 1. Why a Field Is Required

The preceding FRC papers establish that coherence must be locally defined (FRC 100.001), thermodynamically conjugate to entropy (FRC 566.001), dynamically active through gradients (FRC 100.004), and equilibrate to Born-rule statistics (FRC 100.006). The minimal mathematical object satisfying all these requirements is a real scalar field on spacetime.

Paper	Result	Implication
FRC 100.001	Fractal resonance organizes systems	Coherence has spatial structure
FRC 566.001	$dS + k^* d \ln C = 0$	Entropy-coherence conjugacy
FRC 100.004	$d\rho/dt = L[\rho] + \alpha \text{grad}_\rho \ln C$	Gradient-driven dynamics
FRC 100.006	Born rule as equilibrium	Attractors, not axioms

## 2. Definition of the Lambda-Field

### 2.1 Fundamental Identification

Let  $C(x) \geq 0$  be a dimensionless local coherence density. We define the Lambda-field through the logarithmic identification:

$$\text{Lambda}(x) = \text{Lambda}_0 * \ln C(x)$$

where  $\text{Lambda}(x)$  is a real scalar field with dimensions of energy,  $\text{Lambda}_0$  is the coherence scale, and  $C(x) = \exp(\text{Lambda}/\text{Lambda}_0)$  inverts the relation. The logarithmic form ensures additivity ( $\text{Lambda}_{AB} =$

$\Lambda_A + \Lambda_B$  for independent systems), thermodynamic conjugacy with entropy, positivity of coherence, and information-theoretic consistency.

### 3. Action and Field Equations

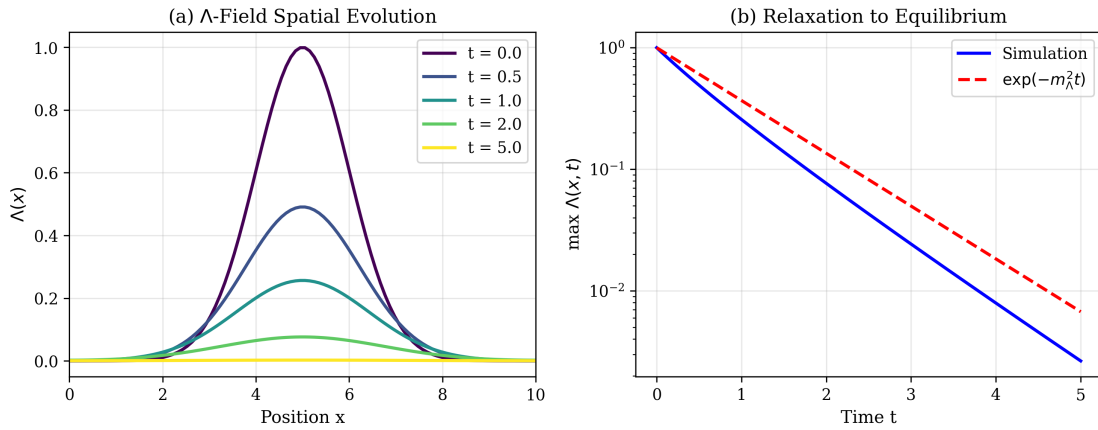
We postulate the minimal Lorentz-invariant action with a quadratic potential:

$$S = \text{Integral} \left[ \frac{1}{2} (\partial_\mu \Lambda)^2 - \frac{1}{2} m_\Lambda^2 (\Lambda - \Lambda_{\text{inf}})^2 \right] d^4x$$

Variation yields the massive Klein-Gordon equation:

$$\Box \Lambda + m_\Lambda^2 (\Lambda - \Lambda_{\text{inf}}) = J_\Lambda$$

where  $m_\Lambda$  is the coheron mass,  $\Lambda_{\text{inf}}$  is the vacuum value, and  $J_\Lambda$  is the source from quantum system coupling. The equilibration time is  $T_{\text{eq}} \sim 1/m_\Lambda$ .



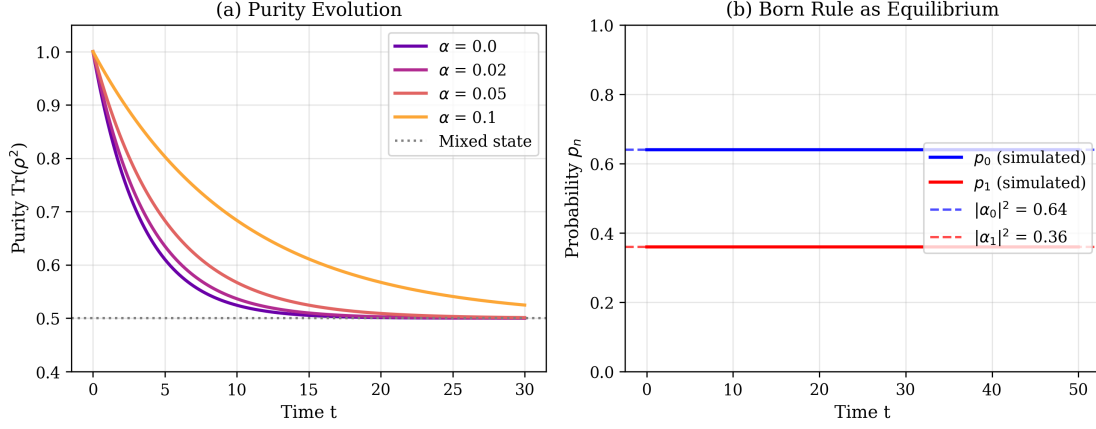
**Figure 1:**  $\Lambda$ -field evolution. (a) Spatial profiles showing relaxation from initial Gaussian perturbation to equilibrium  $\Lambda_{\text{inf}} = 0$ . (b) Maximum field value decaying exponentially with characteristic time  $1/m_\Lambda^2$ .

### 4. Coupling to Quantum Dynamics

The coherence drift term from FRC 100.004 becomes, in  $\Lambda$ -field language:

$$d\rho/dt = L[\rho] + (\alpha/\Lambda_0) * (\text{grad}_\rho \Lambda - \text{Tr}[\text{grad}_\rho \Lambda * \rho] * I)$$

The trace-subtraction term ensures complete positivity and trace preservation (CPTP). The drift biases quantum evolution toward states of higher coherence. When  $\alpha \rightarrow 0$ , standard quantum mechanics is recovered exactly.



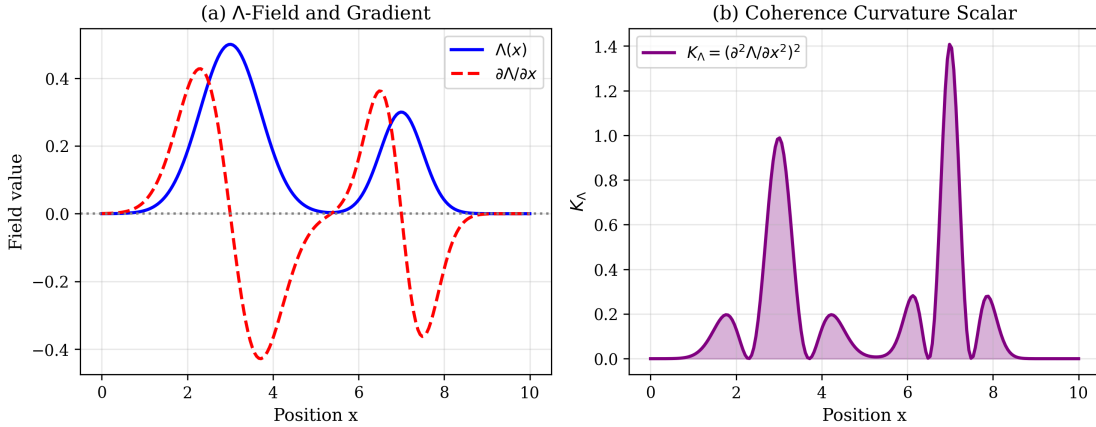
**Figure 2:** Quantum dynamics with Lambda-coupling. (a) Purity evolution showing competition between dephasing ( $\gamma$ ) and coherence-enhancing drift ( $\alpha$ ). (b) Diagonal probabilities converging to Born rule values  $p_n = |\alpha_n|^2$  at equilibrium.

## 5. Born Rule as Equilibrium

Equilibrium states satisfy  $\text{grad}_\rho \text{Lambda}[\rho^*] = 0$ . For an initial superposition  $|\psi\rangle = \sum_n \alpha_n |n\rangle$ , the equilibrium density matrix under dephasing is diagonal with weights  $p_n = |\alpha_n|^2$ . This demonstrates that Born-rule statistics emerge as dynamical attractors of Lambda-driven dynamics, not as fundamental axioms.

## 6. Derived Geometric Quantities

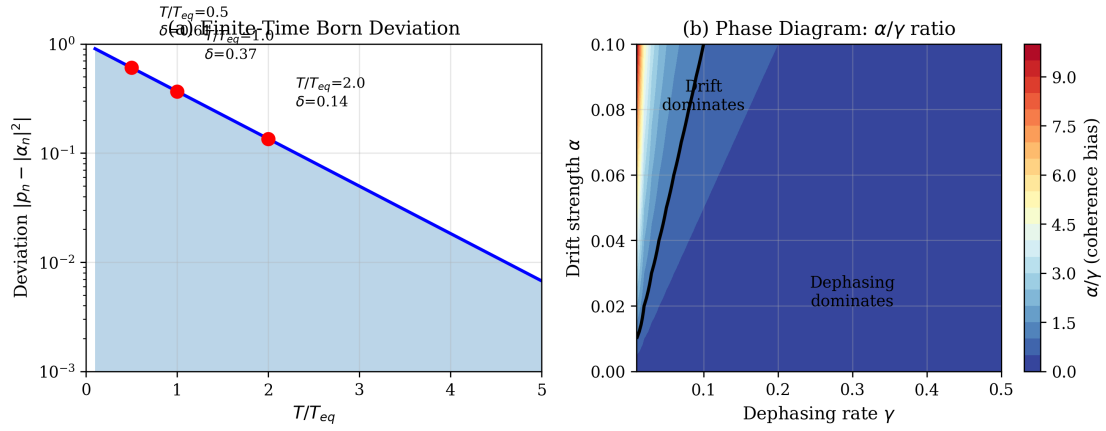
The coherence tensor  $C_{\mu\nu} = d_\mu d_\nu \text{Lambda} - g_{\mu\nu} \Box \text{Lambda}$  measures Lambda-curvature in spacetime. The coherence curvature scalar  $K_\text{Lambda} = C_{\mu\nu} C^{\mu\nu}$  quantifies rapid coherence variation. These quantities provide geometric characterization of coherence structure.



**Figure 3:** Coherence geometry. (a) Lambda-field with two Gaussian peaks and its spatial gradient. (b) Coherence curvature scalar  $K_\text{Lambda}$  showing localized structure.

## 7. Predictions

The framework makes three testable predictions: (1) Finite-time Born deviations  $|\rho_n - |\alpha_n|^2| \sim \exp(-T/T_{eq})$  for measurements completed in time  $T < T_{eq}$ ; (2) Pre-collapse entropy reduction observable in weak-measurement sequences; (3) Protocol dependence of outcome statistics on temporal measurement structure.



**Figure 4:** Predictions. (a) Finite-time deviation from Born rule decreasing exponentially with measurement time. (b) Phase diagram showing  $\alpha/\gamma$  ratio determining whether dephasing or coherence drift dominates.

## 8. Scope and Constraints

This paper establishes: Lambda as a real scalar coherence field; explicit action with quadratic potential; CPTP-preserving quantum coupling; Born rule as equilibrium; standard QM as limit. Explicitly excluded: Higgs physics, cosmology, gravitational coupling (deferred to FRC 821.xxx); computational implementation (FRC 100.008); consciousness models.

**No Fifth-Force Coupling:** The Lambda-field couples to quantum information states (coherence/entropy), not to macroscopic stress-energy  $T_{\mu\nu}$ . It does not mediate forces between macroscopic bodies. Gravitational extensions require screening mechanisms treated in FRC 821.xxx.

### 8.1 Parameters

Parameter	Symbol	Status	Constraint
Coherence scale	Lambda_0	Free	Set by domain
Coheron mass	m_Lambda	Constrained	$T_{eq}$ measurements
Vacuum value	Lambda_inf	Boundary	System preparation
Drift strength	alpha	Bounded	$\leq 0.01$

## 9. Conclusion

The Lambda-field provides the minimal field-theoretic completion of coherence dynamics implied by the FRC 100-series. It transforms coherence from an abstract functional into a physical scalar quantity with transport (field equation with source), dynamics (CPTP-preserving gradient coupling), and equilibrium structure (Born rule as attractor). All prior results are recovered, and standard quantum mechanics remains intact as  $\alpha \rightarrow 0$ . No fifth-force or gravitational coupling is assumed. This paper closes the 100-series and provides a mathematically well-posed foundation for extensions.

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