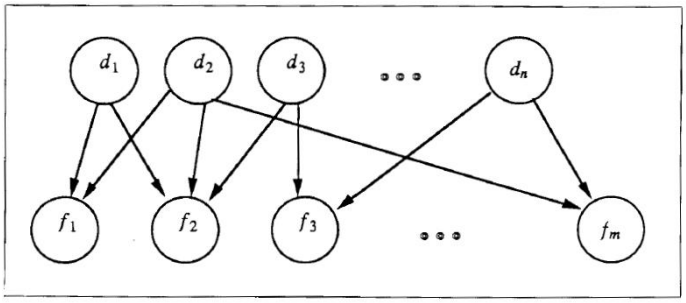
Small Problem 2: Disease Diagnosis

Version 3. November 20, 2014

The file "problem-2-generator.R" contains R code to generate random bipartite networks relating diseases to findings inspired by the famous QMR-DT medical diagnosis system (Shwe, Middleton, Heckerman, Henrion, Horvitz, Lehmann, Cooper (1991). Probabilistic diagnosis using a reformulation of the INTERNIST-1/QMR knowledge base: I. The probabilistic model and inference algorithms. *Methods of Information in Medicine*, Schattauer).



The conditional probability of $P(f_j|d_1, \ldots, d_n)$ is parameterized as a Noisy-OR. In the published work, a leakage probability (corresponding to a species disease node that is always observed to be true) was included. However, the code does not include this.

The following files are provided:

Name	*Description*
problem-2-disease-priors.csv	Prior probability of occurrence of each disease
problem-2-edges.csv	Weights on each edge. A 0 weight means no edge
problem-2-cases-findings.csv	Findings for four cases
problem-2-cases-ground-truth.csv	Ground truth for the four cases
problem-2-treatment-costs.csv	Cost of treating each disease
problem-2-observation-costs.csv	Cost of observing each finding
problem-2-cases-partial-findings.csv	Partial findings for query 3

For this network, I have manually tweaked the model to ensure that the rarest disease is the most expensive to treat. But I have not adjusted observation costs to reflect the informativeness of the findings.

Query 1: Posterior distribution over the disease state variables. Metric: Total variation distance between the true posterior and the posterior output by the probabilistic program.

Query 2: Joint MAP value of the disease state variables. Metric: Hamming distance between the true disease states and the predicted MAP disease states.

Query 3: For each case, for each unobserved finding, compute the one-step expected value of information for observing that finding. Let F_p be the given set of partial findings and their values and let $F_p \cup f_j$ be the revised set of findings after observing finding j. Let $T(d)$ be the cost of treating disease d and $O(j)$ be the cost of observing finding j. Then the value of information is the expected cost of treating the disease(s) without observing j less the cost of observing j plus the expected cost of treating the disease(s) after observing j.

$$VOI(j) = \sum_d P(d|F_0)T(d) - \left[O(j) + \sum_{v \in \{0,1\}} P\left(f_j = v \middle| F_0\right) \sum_d P\left(d \middle| F_0 \cup \{f_j = v\}\right) T(d) \right]$$

$$P\left(f_j = v \middle| F_0\right) = \sum_d P(d|F_0) P(f_j = v|d)$$

Metric: $\sum_j \left[\widehat{VOI}(j) - VOI(j)\right]^2$.