

Small Problem 3: Discrete-time Discrete-observation HMM

The file “problem-3-generator.R” contains R code to generate a simple discrete-time, discrete-observation HMM. The specific instance for this small problem has 5 states and 5 observations and sequences of length 20. The transition distribution has probability $\frac{1}{3}$ of staying in the same state s , $\frac{1}{3}$ of transitioning to state $s + 1$ and $\frac{1}{3}$ of transitioning to state $s + 2$. The starting state is $s = 1$, and the states wrap around mod 5.

The observation for state s is equal to s with probability 0.6 and 0.1 of being equal to one of the other observation values.

<i>Name</i>	<i>Description</i>
problem-3-true-state.csv	The true state of the MDP for each time step 1:20
problem-3-outputs.csv	The sequence of observations 1:20

Note that this problem involves no learning. The task is just to perform probabilistic inference with a given model and data.

Query 1: The MAP state sequence. Metric: Minimum Hamming distance between the predicted and the true MAP state sequences.

Query 2: Smoothing: For each time step $t > 1$, the marginal distribution $P(s_t|O)$, where O is the output sequence. Metric: Total variation distance between the true and computed marginal posteriors at each time step. The mean and variance of the per-step total variation distance computed across all time steps.

Query 3: Filtering: For each time step $t > 1$, compute the marginal distribution $P(s_t|O_{1:t})$, where $O_{1:t}$ is the vector of outputs from time 1 up to the current time t . Metrics: Same as for Query 2.