Small Problem 5: Probabilistic Context-Free Grammar

Queries:
Query 1: P(y = “bdcb”| prefix(y) = “bd”)
Query 2: P(y = “cddb”| prefix(y) = “cd”)

Metric 1:
Square of the difference in negative log probability (“surprise”) between the true and the computed conditional probability: $(-\log P(xy|x\ ) \ + \ \log \hat{P}(xy|x))^2$

Ground Truth:
Query 1: P(y = “bdcb”| prefix(y) = “bd”) = 0.00047541/0.231537598204265 = 0.002053273
Query 2: P(y = “cddb”| prefix(y) = “cd”) = 0.0004670596875/0.054882154882155 = 0.008510229

TODO:
Compute Metric 1 using the ground truth. Submit the metric and your code as described in the main CP4 problem description document, e.g. PPAML Challenge Problem 4-v9.pdf.

Ground Truth Details
This problem can be solved using the PRogramming In Statistical Modeling (PRISM) system (Sato, 2009). Here are the required PRISM programs and commands.

Step 1: Save the following code as a PRISM file (attached as “`pcfg_ppaml.psm`”)

```
nonterminal(s).
nonterminal(x).
nonterminal(y).
nonterminal(z).

values(s,[[x,y],[y,z],[x,z],[z,x]]).
values(x,[[a],[b],[s]]).
values(y,[[b],[c],[d]]).
values(z,[[d],[e],[s]]).

:- set_sw(s,[0.25,0.2,0.4,0.15]).
:- set_sw(x,[0.05,0.3,0.65]).
:- set_sw(y,[0.5,0.3,0.2]).
:- set_sw(z,[0.35,0.1,0.55]).

:- p_not_table proj/2.
pcfg(L):- pcfg(s,L-[]).

pcfg(LHS,L0-L1):-
( nonterminal(LHS) -> msw(LHS,RHS),proj(RHS,L0-L1) ; L0 = [LHS|L1])
. proj([],L-L).
proj([X|Xs],L0-L1):- pcfg(X,L0-L2),proj(Xs,L2-L1).
```

Step 2: Save the following code as a PRISM file (attached as “`pcfg_prefix_ppaml.psm`”)

```
nonterminal(s).
nonterminal(x).
nonterminal(y).
nonterminal(z).

values(s,[[x,y],[y,z],[x,z],[z,x]]).
values(x,[[a],[b],[s]]).
values(y,[[b],[c],[d]]).
values(z,[[d],[e],[s]]).

:- set_sw(s,[0.25,0.2,0.4,0.15]).
:- set_sw(x,[0.05,0.3,0.65]).
:- set_sw(y,[0.5,0.3,0.2]).
:- set_sw(z,[0.35,0.1,0.55]).

:- set_prism_flag(error_on_cycle,off).
prefix_pcfg(L):- prefix_pcfg([s],L,[]).
prefix_pcfg([A|R],L0,L2):-
      ( get_values(A,_) -> msw(A,RHS),
            prefix_pcfg(RHS,L0,L1)
      ; L0=[A|L1] ),
      ( L1=[] -> L2=[]
      ; prefix_pcfg(R,L1,L2) ).
prefix_pcfg([],L1,L1).
```

Step 3: Run PRISM and type in the following commands:

```
| ?- prism(pcfg_ppaml)
loading::pcfg_ppaml.psm.out
yes

//query P(y = “bdcb”)
| ?- prob(pcfg([b,d,c,b])).
Probability of pcfg([b,d,c,b]) is: 0.000475410000000
yes

//query P(y = “cddb”)
| ?- prob(pcfg([c,d,d,b])).
Probability of pcfg([c,d,d,b]) is: 0.000467059687500
yes

| ?- prism(pcfg_prefix_ppaml)
loading::pcfg_prefix_ppaml.psm.out
yes

| ?- lin_prob(prefix_pcfg([b,d]))
Probability of prefix_pcfg([b,d]) is: 0.231537598204265
yes

| ?- lin_prob(prefix_pcfg([c,d]))
Probability of prefix_pcfg([c,d]) is: 0.054882154882155
yes
```

The final conditional probabilities are computed manually using the values above.