

```
In [1]: import sys
sys.path.append('.')
import numpy as np
import scipy.stats
import matplotlib
import matplotlib.pyplot as plt

matplotlib.style.use('ggplot')
%matplotlib inline
```

Inference with ELFI: the MA(3) model

The 3rd order moving average model, MA(3)

$$y_t = w_t + \theta_1 w_{t-1} + \theta_2 w_{t-2} + \theta_3 w_{t-3},$$

Where $\theta_1, \theta_2, \theta_3$ are parameters and w is iid white noise.

```
In [2]: def MA2(t1,t2,t3, batch_size=1, random_state=None):
    n_obs = 500
    if random_state is None:
        random_state = np.random.RandomState()
    w = random_state.randn(batch_size, n_obs+3) # i.i.d. sequence ~ N
    (0,1)
    y = w[:,3:] + t1 * w[:,2:-1] + t2 * w[:,1:-2] + t3 * w[:, :-3]
    return y
```

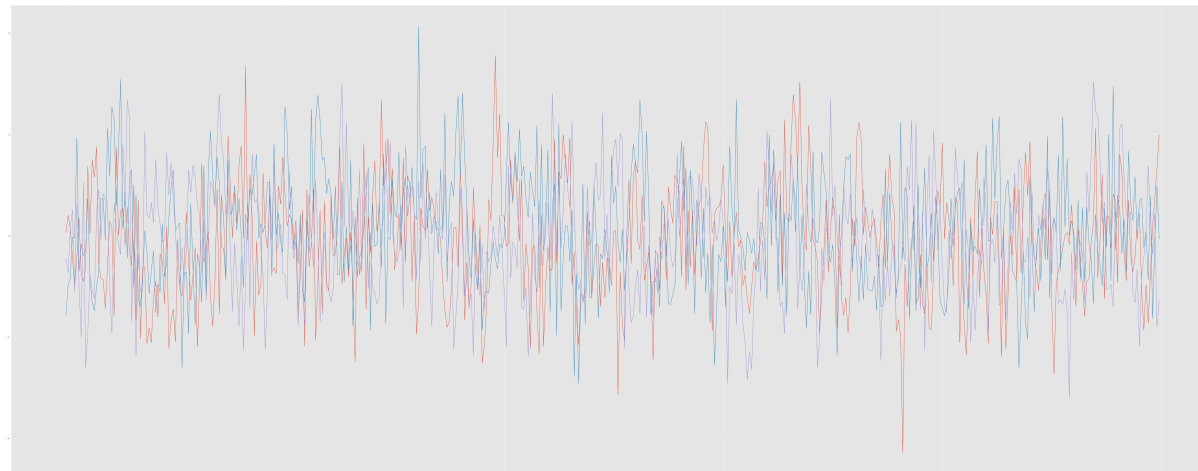
The task is to infer three parameters $t1, t2, t3$ from 500 observations from MA(3) model defined above.

```
In [3]: # true parameters
t1_0 = .2
t2_0 = .3
t3_0 = .4
```

```
# Set up observed data y with some random seed
random_state = np.random.RandomState(20170425)
y = MA2(t1_0, t2_0, t3_0, random_state=random_state)

# Plot the observed sequence
plt.figure(figsize=(100, 40));
plt.plot(y.flatten());

# To illustrate the stochasticity draw few more plots using true parameters
plt.plot(MA2(t1_0, t2_0, t3_0).flatten());
plt.plot(MA2(t1_0, t2_0, t3_0).flatten());
```



Approximate Bayesian Computation

We are going to use elfi engine for Approximate Bayesian Computation also known as likelihood free inference to estimate parameters. First, we need to define priors for these three parameters as nodes having parents (0,2), assuming that they follow uniform(0,2).

```
In [4]: import elfi

# a node is defined by giving it a name, a distribution
```

```
# from scipy.stats and its parents
t1 = elfi.Prior('t1', scipy.stats.uniform, 0, 2)

t2 = elfi.Prior('t2', 'uniform', 0, 2)

t3 = elfi.Prior('t3', 'uniform', 0, 2 )
```

Now we would define simulator node with prior nodes as parents.

```
In [5]: Y = elfi.Simulator('MA2', MA2, t1, t2, t3, observed=y)
```

Now we would define autocovariances with lags 1, 2 and 3 as the summary statistics. After that we would calculate the distance using Euclidean L2-distance.

```
In [6]: def autocov(x, lag=1):
        mu = np.mean(x, axis=1, keepdims=True)
        C = np.mean(x[:,lag:] * x[:, :-lag], axis=1, keepdims=True) - mu**2
        return C

        def distance(x, y):
            d = np.linalg.norm( np.array(x) - np.array(y), ord=2, axis=0)
            return d
```

Defining summary nodes with autocovariances and observed data as parents.

```
In [7]: S1 = elfi.Summary('S1', autocov, Y)
```

```
In [8]: from functools import partial

        autocov2 = partial(autocov, lag=2)

        S2 = elfi.Summary('S2', autocov2, Y)
```

```
In [9]: from functools import partial

        autocov3 = partial(autocov, lag=3)
```

```
S3 = elfi.Summary('S3', autocov3, Y)
```

Similarly, defining distance node with summery statistic parents.

```
In [10]: d = elfi.Discrepancy('d', distance, S1, S2, S3)
```

Redefining priors with ranges [-b, b],[-a,a],[-c,c]

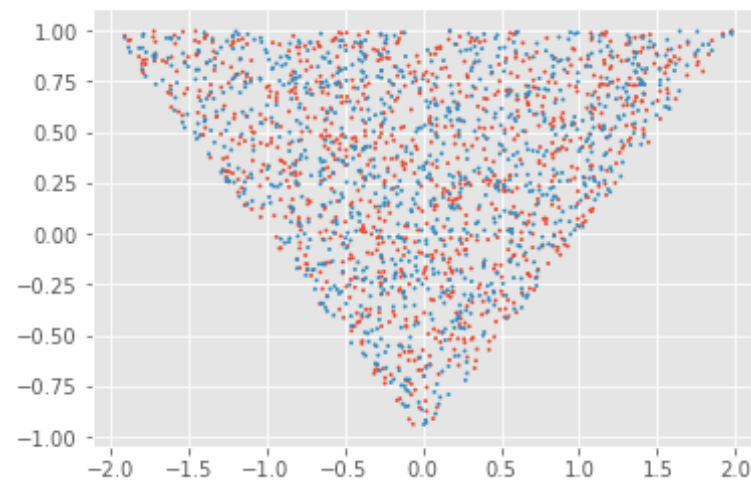
```
In [11]: class MarinPrior_t1(elfi.Distribution):
    def rvs(b, size=1, random_state=None):
        u = scipy.stats.uniform.rvs(loc=0, scale=1, size=size, random_s
tate=random_state)
        t1 = np.where(u<0.5, np.sqrt(2.*u)*b-b, -np.sqrt(2.*(1.-u))*b+b
)
        return t1

# define prior for t2 conditionally on t1 as in Marin et al., 2012, in
range [-a, a]
class MarinPrior_t2(elfi.Distribution):
    def rvs(t1, a, size=1, random_state=None):
        locs = np.maximum(-a-t1, t1-a)
        scales = a - locs
        t2 = scipy.stats.uniform.rvs(loc=locs,scale=scales,size=size,ra
ndom_state=random_state)
        return t2
class MarinPrior_t3(elfi.Distribution):
    def rvs(t1, c, size=1, random_state=None):
        locs = np.maximum(-c-t1, t1-c)
        scales = c - locs
        t3 = scipy.stats.uniform.rvs(loc=locs,scale=scales,size=size,ra
ndom_state=random_state)
        return t3
```

These draw samples from distribution of zero mean and variance 1.

```
In [12]: t1_1000 = MarinPrior_t1.rvs(2, 1000)
t2_1000 = MarinPrior_t2.rvs(t1_1000, 1, 1000)
t3_1000 = MarinPrior_t3.rvs(t1_1000, 1, 1000)

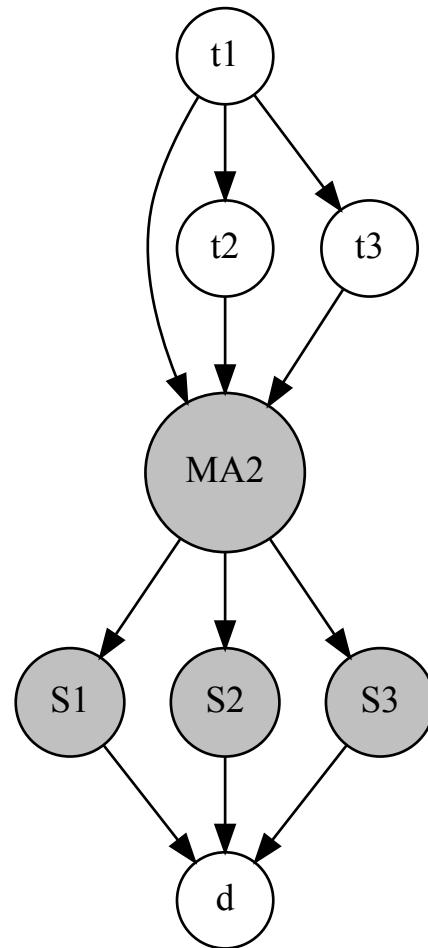
plt.scatter(t1_1000, t2_1000, s=4, edgecolor='none');
plt.scatter(t1_1000, t3_1000, s=4, edgecolor='none');
```



Inferencee Model:

```
In [13]: t1.redefine(MarinPrior_t1, 2)
t2.redefine(MarinPrior_t2, t1, 1)
t3.redefine(MarinPrior_t3, t1, 1)
elfi.draw_model(d)
```

Out[13]:



inference with rejection sampling

```
In [14]: rej = elfi.Rejection(d, [t1, t2, t3], batch_size=10000)
         rej = elfi.Rejection(d, [t1, t2, t3], batch_size=10000)
```

```
In [15]: N = 10000
         %time result = rej.sample(N, quantile=0.01)
```

```
elfi.methods - INFO - Rejection: Running with 1000000 proposals.
```

```
CPU times: user 48.4 s, sys: 12.9 s, total: 1min 1s  
Wall time: 25.6 s
```

```
In [16]: result.summary()
```

```
Method: Rejection  
Number of posterior samples: 10000  
Number of simulations: 1000000  
Threshold: 0.235  
Posterior means: t1: 0.328, t2: 0.257, t3: 0.479
```

```
In [17]: %time result2 = rej.sample(N, threshold=0.2)
```

```
print(result2)
```

```
elfi.methods - INFO - Rejection: Running with 10000 proposals.  
elfi.methods - INFO - Rejection: Running with 1890000 proposals.
```

```
CPU times: user 1min 30s, sys: 22.1 s, total: 1min 52s  
Wall time: 41 s  
Method: Rejection  
Number of posterior samples: 10000  
Number of simulations: 1890000  
Threshold: 0.2  
Posterior means: t1: 0.31, t2: 0.273, t3: 0.474
```

```
In [18]: %time result3 = rej.reject(threshold=0.3)
```

```
result3
```

```
elfi.methods - INFO - Rejection: Running with 1890000 proposals.
```

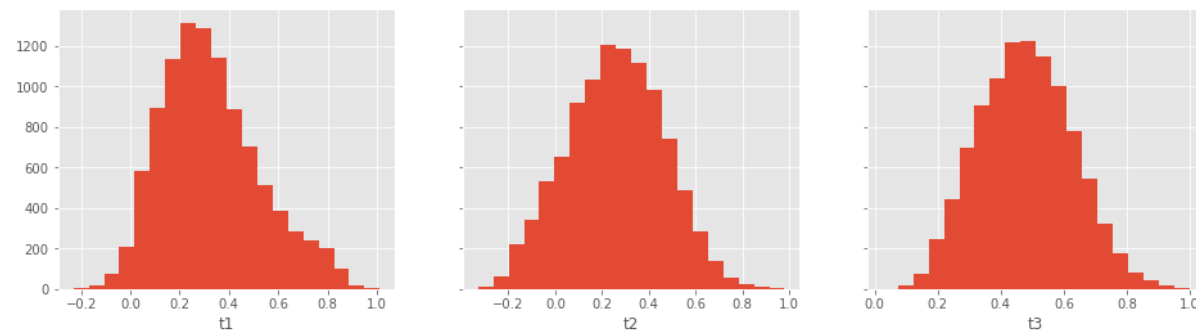
```
CPU times: user 1min 31s, sys: 22.4 s, total: 1min 53s  
Wall time: 42.4 s
```

```
Out[18]: Method: Rejection  
Number of posterior samples: 39070
```

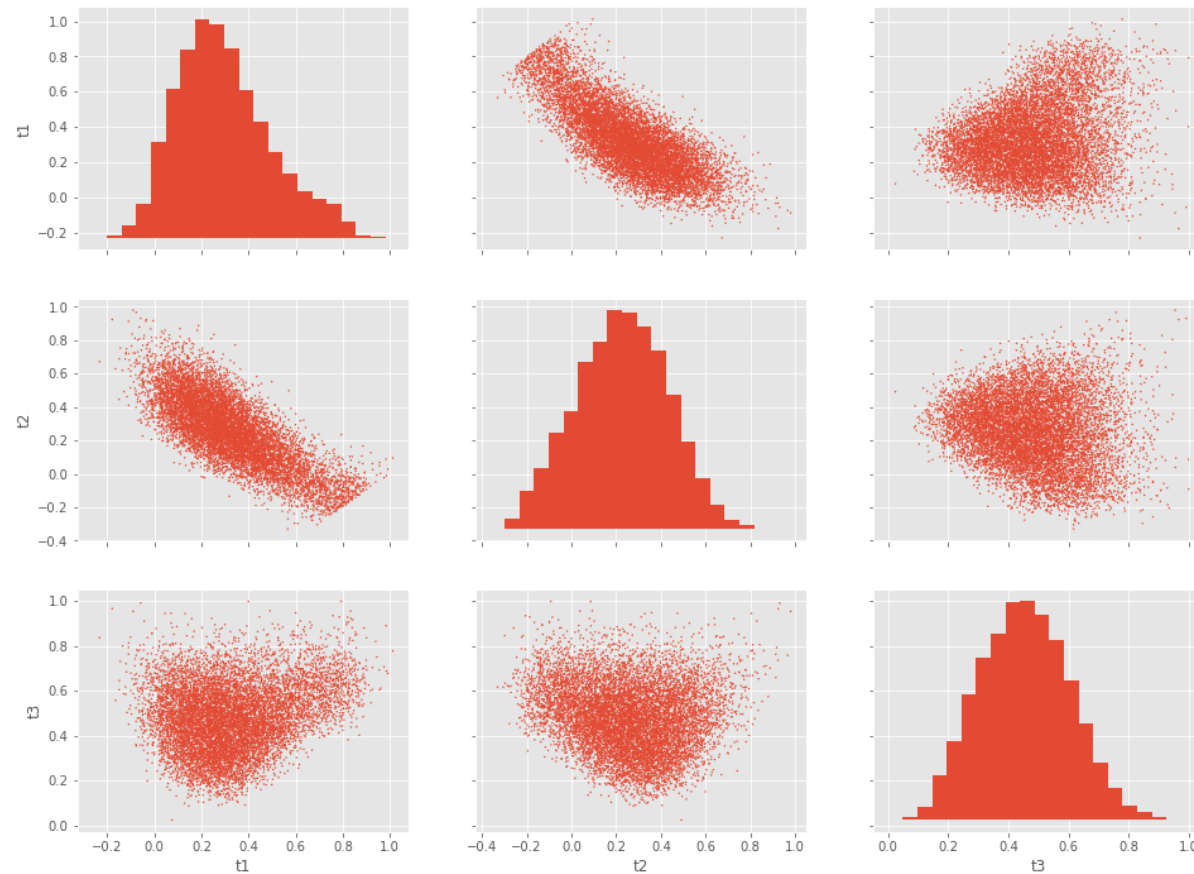
Number of simulations: 1890000
Threshold: 0.3
Posterior means: t1: 0.349, t2: 0.239, t3: 0.482

Plotting of results

```
In [19]: result.plot_marginals();
```



```
In [20]: result.plot_pairs();
```

Inference with Sequential Monte Carlo

Redefining priors for SMC.

```
In [21]: class MarinPrior_t1(elfi.Distribution):
          def rvs(b, size=1, random_state=None):
              u = scipy.stats.uniform.rvs(loc=0, scale=1, size=size, random_s
              tate=random_state)
              t1 = np.where(u<0.5, np.sqrt(2.*u)*b-b, -np.sqrt(2.*(1.-u))*b+b
              )
```

```

        return t1

    def pdf(x, b):
        p = 1./b - np.abs(x) / (b*b)
        p = np.where(p < 0., 0., p)
        return p

class MarinPrior_t2(elfi.Distribution):
    def rvs(t1, a, size=1, random_state=None):
        locs = np.maximum(-a-t1, t1-a)
        scales = a - locs
        t2 = scipy.stats.uniform.rvs(loc=locs, scale=scales, size=size, random_state=random_state)
        return t2

    def pdf(x, t1, a):
        locs = np.maximum(-a-t1, t1-a)
        scales = a - locs
        p = scipy.stats.uniform.pdf(x, loc=locs, scale=scales)
        p = np.where(scales>0., p, 0.)
        return p

class MarinPrior_t3(elfi.Distribution):
    def rvs(t1, a, size=1, random_state=None):
        locs = np.maximum(-a-t1, t1-a)
        scales = a - locs
        t2 = scipy.stats.uniform.rvs(loc=locs, scale=scales, size=size, random_state=random_state)
        return t2

    def pdf(x, t1, c):
        locs = np.maximum(-c-t1, t1-c)
        scales = c - locs
        p = scipy.stats.uniform.pdf(x, loc=locs, scale=scales)
        p = np.where(scales>0., p, 0.)
        return p

# Redefine the priors

```

```
t1.redefine(MarinPrior_t1, 2)
t2.redefine(MarinPrior_t2, t1, 1)
t3.redefine(MarinPrior_t3, t1, 1)
```

```
In [22]: smc = elfi.SMC(d, [t1, t2, t3], batch_size=10000)
```

```
In [23]: N = 1000
n_populations = 3
schedule = [0.7, 0.2, 0.05]
%time result_smc = smc.sample(N, n_populations, schedule)
```

```
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elfi.methods - INFO - Rejection: Running with 10000 proposals.
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CPU times: user 1min 10s, sys: 8.25 s, total: 1min 18s

Wall time: 2min 23s

In [24]: `result_smc.summary()`

Method: SMC

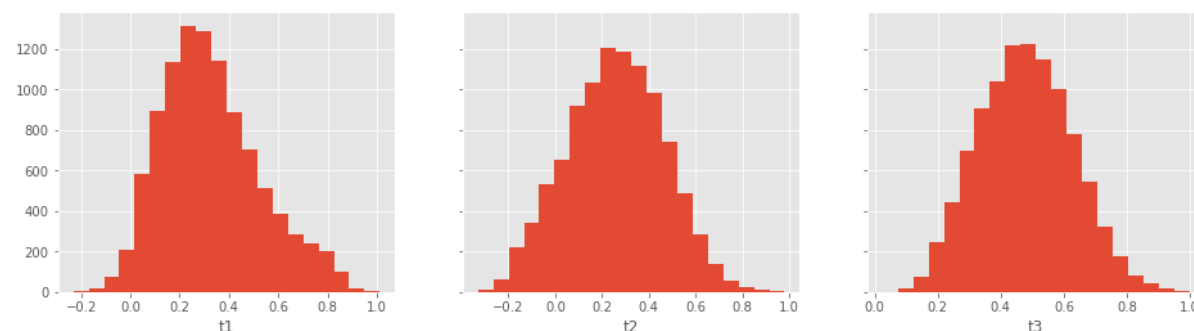
Number of posterior samples: 1000

Number of simulations: 5000000

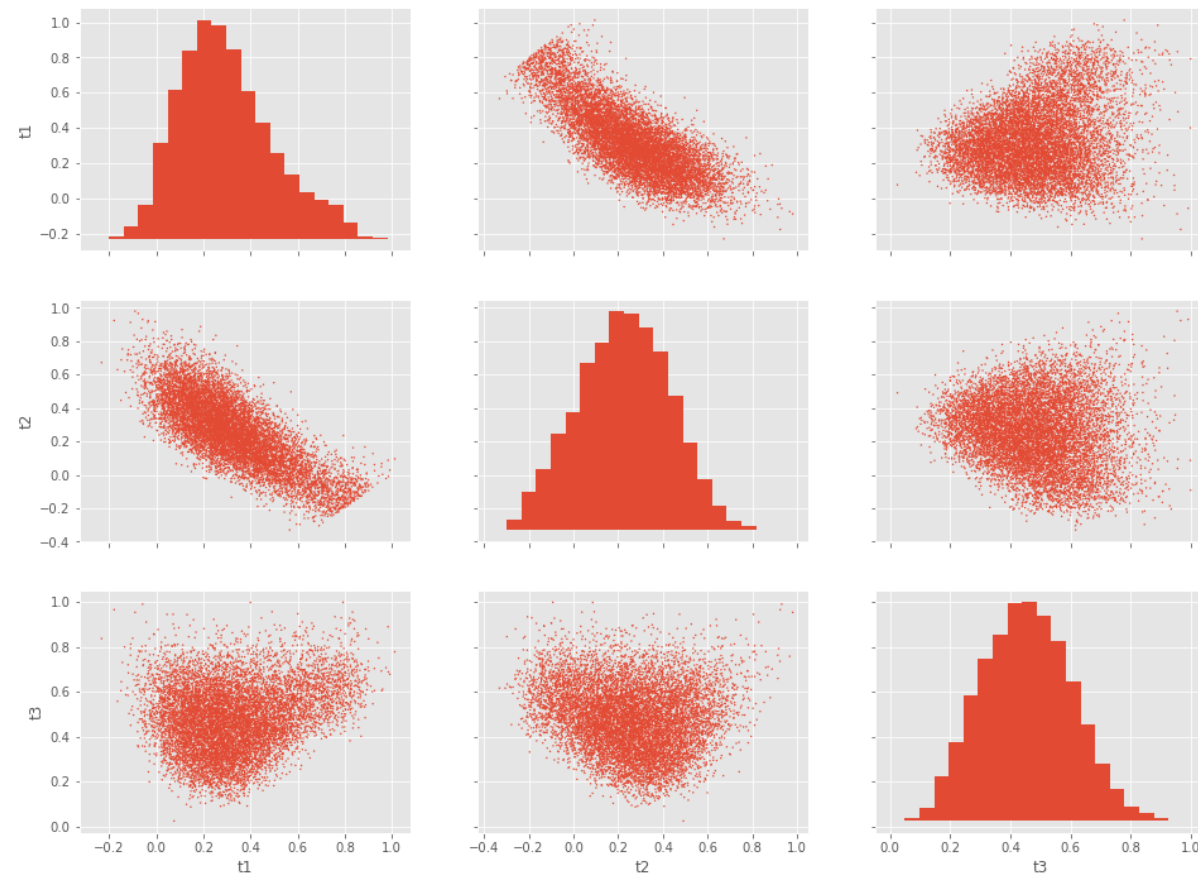
Threshold: 0.05

Posterior means: t1: 0.273, t2: 0.31, t3: 0.452

In [25]: `result.plot_marginals();`




```
In [26]: result.plot_pairs();
```



Conclusion:

It is obvious from results that SMC infer the parameters better then rejection sampling by using the importance sampling.