What is Quantum Teleportation?

Quantum teleportation is **not moving particles** physically —
It's **transferring quantum information** (a *qubit's state*) from one place (Alice) to another (Bob) using:

1. **Entanglement (EPR pair)** between Alice and Bob
2. **Classical communication (2 bits)** to complete the transfer

The original qubit's state is destroyed at Alice's side —
and **reconstructed** at Bob's side perfectly.

The Quantum State

Suppose Alice has an unknown qubit:

$$|\psi\rangle = \alpha\ |0\rangle + \beta\ |1\rangle$$

and she wants to send it to Bob.
They share an entangled pair:

$$|\Phi^{+}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

So, the **total system (3 qubits)** is:

$$|\Psi\rangle = |\psi\rangle \otimes |\Phi^{+}\rangle$$

Quantum Teleportation Steps

Step	Operation	Description
1️⃣	**Create entanglement**	Bob and Alice share an EPR pair (H + CNOT)
2️⃣	**Bell measurement**	Alice entangles her qubits and measures them
3️⃣	**Classical communication**	Alice sends 2 classical bits to Bob
4️⃣	**Conditional operations**	Bob applies $X$and/or Zgates to recover (

Mathematical Intuition (Simplified Geometry)

The process is like **rotating vectors in 2D space** to align phases:

- Alice's measurement projects Bob's qubit into a state **related to** $|\psi\rangle$
- Bob's correction "rotates" it back perfectly using X/Z gates

Geometrically:

- Measurement collapses the entangled 3D state
- Bob's corrections restore the original vector (amplitude + phase)

Quantum Communication Concept

- **Quantum entanglement** acts like a *communication channel*
- **Classical bits** transmit the correction information
- Together, they allow transfer of **quantum information**
  without sending the qubit itself

This is fundamental to:

- **Quantum internet**
- **Quantum repeaters**
- **Secure communication (QKD)**

SYSTEM SETUP

We have **3 qubits**:

- Q_0: the unknown state to send (Alice)
- Q_1: Alice's entangled qubit
- Q_2: Bob's entangled qubit

Unknown qubit

$$\mid \psi\rangle_0 = \alpha \mid 0\rangle + \beta \mid 1\rangle$$

Entangled pair (Bell state)

$$\mid \Phi^+\rangle_{12} = \frac{\mid 00\rangle + \mid 11\rangle}{\sqrt{2}}$$

Substitute:

$$\mid \Psi\rangle_{012} = (\alpha \mid 0\rangle + \beta \mid 1\rangle) \otimes \frac{\mid 00\rangle + \mid 11\rangle}{\sqrt{2}}$$

Expand:

$$\mid \Psi\rangle_{012} = \frac{1}{\sqrt{2}}(\alpha \mid 000\rangle + \alpha \mid 011\rangle + \beta \mid 100\rangle + \beta \mid 111\rangle)$$

STEP 1️⃣ — CREATE ENTANGLEMENT

Alice and Bob first prepare a Bell pair using:

- **Hadamard gate** (H)
- **CNOT gate**

Math:

$$|00\rangle_{12} \xrightarrow{H_1} \frac{|00\rangle + |10\rangle}{\sqrt{2}} \xrightarrow{CNOT_{1,2}} \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

STEP 2️⃣ — ENTANGLE MESSAGE + ALICE'S QUBIT

Alice wants to send her qubit (Q_0).

She performs **CNOT** and **Hadamard** on her two qubits (Q_0, Q_1).

Math:

We apply $CNOT_{0,1}$ then H_0.

$$\mid \Psi\rangle_{012} = \frac{1}{\sqrt{2}}(\alpha \mid 000\rangle + \alpha \mid 011\rangle + \beta \mid 100\rangle + \beta \mid 111\rangle)$$

After **CNOT(0→1)**:

$$\frac{1}{\sqrt{2}}(\alpha \mid 000\rangle + \alpha \mid 011\rangle + \beta \mid 110\rangle + \beta \mid 101\rangle)$$

After **Hadamard on qubit 0**:

$$H \mid 0\rangle = \frac{\mid 0\rangle + \mid 1\rangle}{\sqrt{2}}, H \mid 1\rangle = \frac{\mid 0\rangle - \mid 1\rangle}{\sqrt{2}}$$

Expand and simplify:

$$\mid \Psi\rangle = \frac{1}{2}[\mid 00\rangle(\alpha \mid 0\rangle + \beta \mid 1\rangle) + \mid 01\rangle(\alpha \mid 1\rangle + \beta \mid 0\rangle) + \mid 10\rangle(\alpha \mid 0\rangle - \beta \mid 1\rangle) + \mid 11\rangle(\alpha \mid 1\rangle - \beta \mid 0\rangle)]$$

STEP 3️⃣ — ALICE'S MEASUREMENT

Alice measures **Q_0 and Q_1**

So Bob's qubit (Q_2) is always related to $\mid \psi\rangle$,

but possibly rotated by X and/or Z.

STEP 4️⃣ — CLASSICAL COMMUNICATION

Alice sends her two classical bits to Bob.

Bob then applies:

- Xgate if first bit = 1
- Zgate if second bit = 1

These corrections return Bob's qubit to the **original** $\mid \psi\rangle$.