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2.1

a

A nonlinear transfer function gives the neural network a universal property: Given enough layers and neurons, the network can model any function within a certain accuracy. In a network with a linear transfer function we can only compute a linear function. A network with a linear transfer function of n layers will always be equivalent to a network with only one layer: linear functions can always be concatenated.

Whenever the function that we are trying to model isn't a linear function, it is useful to use a nonlinear transfer function. Examples include image classification or speech recognition.

b

Consider a simple neural network with two input neurons that can both either be 0 or 1 and one output layer. We want to construct an AND gate with our network, so without bias our quest would be to find $w = (w_1, w_2)$ such that:

$$0 \le 0$$

$$w_1 \le 0$$

$$w_2 \le 0$$

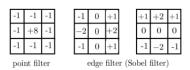
$$w_1 + w_2 > 0$$

| w_0 | | | | | | 1.5 |
|-------|---|---|---|---|---|-----|
| w_1 | | 0 | 1 | 0 | 1 | 1 |
| w_2 | | 0 | 0 | 1 | 1 | 1 |
| 0.0 | - | - | - | - | - | - |
| 0 1 | - | _ | - | + | + | - |
| 10 | - | _ | + | - | + | - |
| 11 | + | _ | + | + | + | + |

which is impossible. We can easily however create the network with a bias, if we have the weights w=(1,1) and the bias $\theta=\frac{3}{2}$. Then $sgn(w^Tx-\theta)$ would give us AND.

 \mathbf{c}

Point and edge filters are for example a connectionist neuron which gets values of a scalar field as input, that represent the color of each pixel or the color gradient or even a higher derivative and has weights in the following form: In the simple case of two colors (0 and 1) this point filter would return



zero for no point, 1 or -1 for a point in the outer region and 8 or -8 if the point is in the middle. This goes analogously for the other filter.

d

The first is deterministic and the second has a noise parameter and can return different states for set parameters and a given input.

2.2

a

b

 \mathbf{c}

Using numpy's linspace to take 999 samples from [-3,3], we find that the best weight vector is [0.93969262, 0.34202014]. It classifies 76% correctly.

 \mathbf{d}

 \mathbf{e}

2.3

 \mathbf{a}

A MLP could decide between a horizontal and a vertical edge, whereas a perceptionist neuron would either be able to differ between vertical edge or no vertical edge OR horizontal edge or no horizontal edge.

b

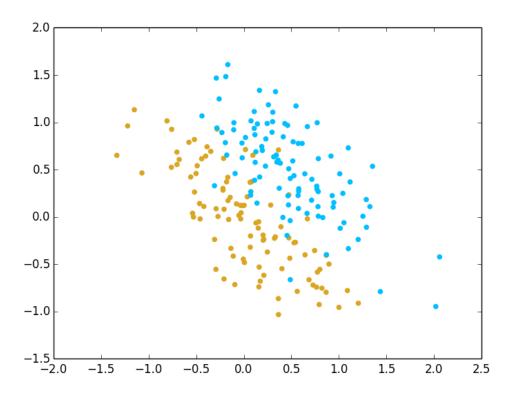


Figure 1: Plot where Y=1 (blue) and Y=0 (gold).

c + bonus

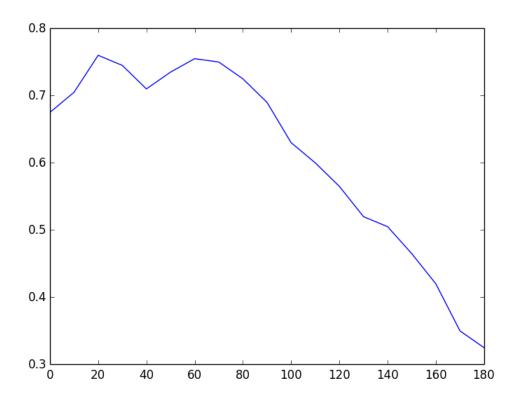


Figure 2: Classification performance for different alphas.

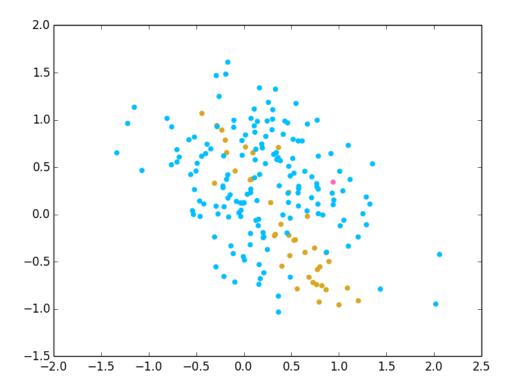


Figure 3: Classified data points. Blue = correctly classified, Gold = other data points, Pink = weight.

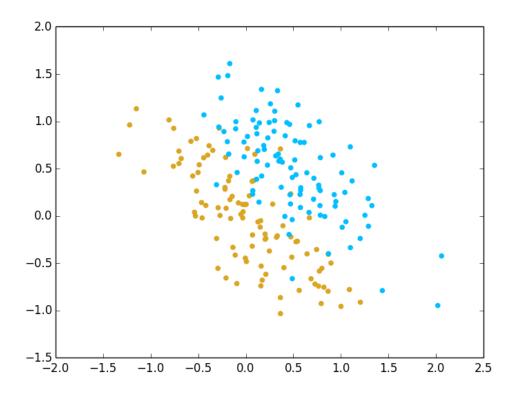


Figure 4: Plot where Y=1 (blue) and Y=0 (gold).

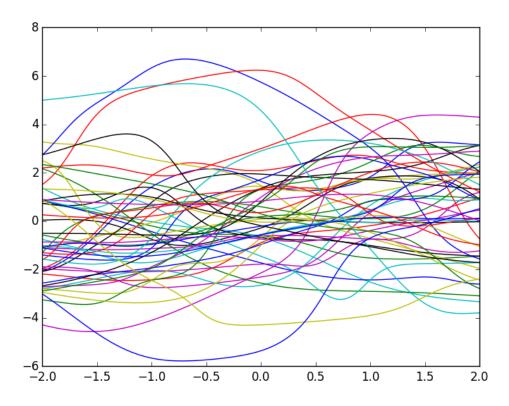


Figure 5: functions computed with normally distributed a_i with a standard deviation of 2

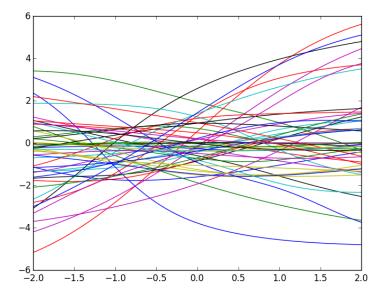


Figure 6: functions computed with normally distributed a_i with a standard deviation of 0.5

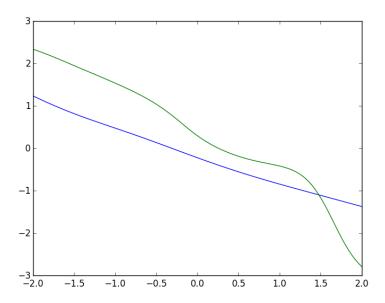


Figure 7: functions with least mean square error from f(x)=-x, computed with an std of 2 (green) and 0.5 (blue)