ELE 535: Machine Learning and Pattern Recognition
Homework 2

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Exercise 1: Let $u \in \mathbb{R}^m$, $v \in \mathbb{R}^n$, and $A \in \mathbb{R}^{m \times n}$. Find the orthogonal projection of A onto $\text{span}(uv^T)$.

Answer:

Exercise 2: Norm Invariance under Orthogonal Transformations. Show that for any $A \in \mathbb{R}^{m\times n}$, $Q \in \mathcal{O}_m$, $R \in \mathcal{O}_n$, $\|QAR\|_F = \|A\|_F$. Thus the Frovenius norm is invariant under orthogonal transformations. Similarly, show the induced 2-norm of $A \in \mathbb{R}^{m\times n}$ is invariant under orthogonal transformations.

Answer:

Exercise 3: Let A, B be matrices of appropriate size and $x \in \mathbb{R}^n$. Prove that

a) $\|Ax\|_2 \leq \|A\|_2 \|x\|_2$;

b) $\|AB\|_2 \leq \|A\|_2 \|B\|_2$.

Answer:

Exercise 4: For $A, B \in \mathbb{R}^{m \times n}$. Show that $\sigma_1(A + B) \leq \sigma_1(A) + \sigma_1(B)$.

Answer:

Exercise 5: The Moore-Penrose pseudo-inverse. The Moore-Penrose pseudo-inverse of a matrix $A \in \mathbb{R}^{m\times n}$ is the unique matrix $A^+ \in \mathbb{R}^{n\times m}$ satisfying the following four properties:

a) $A(A^+A) = A$

b) $(A^+A)A^+ = A^+$

c) $(A^+A)^T = A^+A$

d) $(AA^+)^T = AA^+$

Let A have compact SVD $A = U\Sigma V^T$. Show that $A^+ = V\Sigma^{-1}U^T$. Give an interpretation of A^+ in terms of $\mathcal{N}(A)$, $\mathcal{N}(A)^\perp$, and $\mathcal{R}(A)$.

Answer: