Homework #2, ORF 526

Assigned on Sept 27; Due October 4, 2016

1. *(1 point)* Let $(E, \mathcal{E})$ be a measurable space and $f : E \to \mathbb{R}$ a Borel-measurable function taking finitely many real values. Prove that f is a simple function.

2. *(3 points)* Let $f, g \in \mathcal{E}_+$. Show that:

   (a) $f \wedge g, f \vee g \in \mathcal{E}_+$;
   (b) $f + g \in \mathcal{E}_+$;
   (c) $fg \in \mathcal{E}_+$.

3. *(1.5 point)* Let $\mu_1, \mu_2, \cdots$ be measures on $(E, \mathcal{E})$ and denote $\mu = \sum_{n \geq 1} \mu_n$. Prove that μ is also a measure on $(E, \mathcal{E})$.

4. *(1.5 point)* If δ_{x_0} denotes the Dirac measure sitting at x_0, show that $\delta_{x_0} f = f(x_0)$, for any $f \in \mathcal{E}$.

5. *(1.5+1.5 points)* Let $(E, \mathcal{E}, \mu)$ be a measure space, and $p \in \mathcal{E}_+$. Define

$$\nu(A) = \int_A p(x)\, d\mu(x), \qquad \forall A \in \mathcal{E}.$$

   (a) Show that ν is a measure on $(E, \mathcal{E})$;

   (b) Prove that for any $f \in \mathcal{E}_+$ we have

$$\int_E f(x)\, d\nu(x) = \int_E f(x) p(x)\, d\mu(x).$$

Please solve each problem on a separate sheet of paper. Staple the sheets and attach a cover page with your name and course number. The Homework will be turned in 526 mailbox in Sherred Hall, cabinet A. You can retrieve your graded Homework from your folder situated in the same room, a week later.