Homework #5, ORF 526

Assigned on October 18; Due October 25, 2016, 5:00 pm

1. *(2 points)* Toss a fair coin 4 times. Each toss yields either H (heads) or T (tails). Consider the following information set:
   $\mathcal{G}$ = we know the outcomes of the tosses but not the order.
   Define the random variables:
   X = number of H − number of T
   Y = number of T before the first H.
   (i) Show that X is $\mathcal{G}$-measurable while Y is not $\mathcal{G}$-measurable.
   (ii) Find the expectations $\mathbb{E}[X]$, $\mathbb{E}[Y]$ and $\mathbb{E}[X|\mathcal{G}]$.

2. *(2 points)* Consider X to be a nonnegative random variable with the distribution measure μ. Let μ be absolutely continuous (w.r.t. the Lebesgue measure) and $\mathbb{E}[X|X > t] = t + a$ for some constant $a > 0$ and all $t \geq 0$. Prove that μ is the exponential distribution with parameter $c = 1/a$.

3. *(2 points)*

   Prove or disprove: For any real valued random variables X, Y and Z we have

   $$\mathbb{E}[Z|X, Y] = \mathbb{E}[\mathbb{E}[Z|X]|Y].$$

   (Note the meaning of the left side as $\mathbb{E}[Z|X, Y] = \mathbb{E}[Z|\sigma(X, Y)]$).

4. *(1 point)* Let X be a Poisson distributed random variable with parameter λ. Consider the random variable $Y = \rho^X$, with $\rho > 1$ constant. Find the expectation $\mathbb{E}(Y)$.

5. *(1.5 points)* Let $\mathcal{F}$ be a σ-algebra included in $\mathcal{H}$. If X is a $\mathcal{F}$-measurable random variable such that

   $$\int_A X\, dP = 0, \qquad \forall A \in \mathcal{F},$$

   show that $X = 0$ almost surely.

6. *(1.5 points)* If $\mathcal{F} \subset \mathcal{G}$ prove that

   $$\mathbb{E}[\mathbb{E}[X|\mathcal{F}]|\mathcal{G}] = \mathbb{E}[X|\mathcal{F}].$$

Please solve each problem on a separate sheet of paper. Staple the sheets and attach a cover page with your name and course number. The Homework will be turned in 526 mailbox in Sherred Hall, cabinet A. You can retrieve your graded Homework from your folder situated in the same room, a week later.