Homework #1, ORF 526

Assigned on Sept 20; Due Sept 27, 2016

1. *(1 point)* Let $\mathcal{E}$ be a σ-algebra and consider a sequence $(A_n)_n \subset \mathcal{E}$. Prove that $\bigcap_{n\geq 1} A_n \in \mathcal{E}$.

2. *(2 points)* Let $\mathcal{D}$ be a d-system on E. Fix D in $\mathcal{D}$ and define

$$\widehat{\mathcal{D}} = \{A \in \mathcal{D}; A \cap D \in \mathcal{D}\}.$$

   Prove that $\widehat{\mathcal{D}}$ is a d-system.

3. *(2 points)* Let E be a set and $(F, \mathcal{F})$ a measurable space. Consider a function $f : E \to F$. Define $f^{-1}(\mathcal{F}) = \{f^{-1}(B); B \in \mathcal{F}\}$. Prove that:

   (i) $f^{-1}(\mathcal{F})$ is a σ-algebra.

   (ii) $f^{-1}(\mathcal{F})$ is the smallest σ-algebra on E such that f is measurable relative to it and $\mathcal{F}$.

4. *(1 point)* Let $f : \mathbb{R} \to \overline{\mathbb{R}}$ be an increasing function. Show that f is Borel measurable.

5. *(1 point)* Let $\mathcal{C}, \mathcal{D} \subset 2^E$. Show that $\mathcal{C} \subset \mathcal{D} \Rightarrow \sigma\mathcal{C} \subset \sigma\mathcal{D}$.

6. *(3 points)* Let $(E, \mathcal{E})$ be a measurable space and $f : E \to \overline{\mathbb{R}}$ a Borel measurable function.
   (i) Show that $|f|$ is measurable;
   (ii) Let $f^+ = \max\{f, 0\}$ and $f^- = -\min\{f, 0\}$. Show that $|f| = f^+ + f^-$;
   (iii) Use (i) and (ii) to show that f^+ and f^- are measurable.

Please solve each problem on a separate sheet of paper. Staple the sheets and attach a cover page with your name and course number. The Homework will be turned in 526 mailbox in Sherred Hall, cabinet A. You can retrieve your graded Homework from your folder situated in the same room, a week later.