ORF522 Assignment 1

Due 11:59 PM Sep. 29 2016

1 Problem 1 (20 pts)

Consider a graph with n nodes. We denote the set of nodes by $V = \{1, ..., n\}$. For each pair of nodes (i, j), there is a edge connecting them with a weight $w_{ij} \geq 0$. Now we want to separate the nodes into two disjoint set S and T such that $S \bigcap T = \phi$ (empty set) and $S \bigcup T = V$. And we want to maximize the weights that are in the cut, i.e.,

$$\sum_{i \in S} \sum_{j \in T} w_{ij}.$$

This is called the *maximum cut problem* and has wide applications in problems such as circuit design (See Figure 1 for an illustration of a cut). Now using decision variables

$$x_i = \begin{cases} 1 & \text{if } x_i \in S \\ -1 & \text{if } x_i \in T \end{cases}$$

write an optimization problem for max-cut problem.

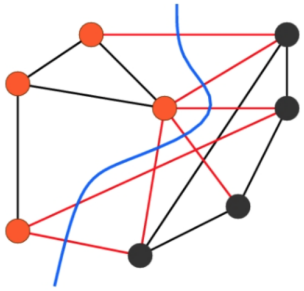

Figure 1: Illustration of a cut: The red edges are cut (therefore, there weights are counted)

2 Problem 2 (25 pts)

Consider the following linear program:

$$\begin{aligned} \text{maximize} \quad & x_1 + 4x_2 + x_3 \\ \text{s.t.} \quad & 2x_1 + 3x_2 + x_3 \leq 4 \\ & x_1 - 2x_3 \geq 1 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

- Transform it into standard form;
- Argue without solving this LP that there must exist an optimal solution with no more than 2 positive variables;
- List all the basic solutions and basic feasible solutions (of the standard form);

- Find the optimal solution by using the results in step 3.

3 Problem 3 (30 pts)

Consider an LP problem of standard form

$$\begin{aligned} \text{minimize}_{\mathbf{x}} \quad & \mathbf{c}^T\mathbf{x} \\ \text{subject to} \quad & A\mathbf{x} = \mathbf{b} \\ & \mathbf{x} \geq 0 \end{aligned}$$

where $\mathbf{x} \in \mathbb{R}^n$, A is $m \times n$. Assume that the feasible set, i.e.,

$$P = \{\mathbf{x} \mid A\mathbf{x} = \mathbf{b}, \mathbf{x} \geq 0\},$$

is nonempty and that A has full rank m. We denote the nullspace of A by

$$\mathbf{N}(A) = \{x \mid Ax = 0\}.$$

1. Show that P is closed.
2. Show that P is convex.
3. Show that P is bounded if and only if $\mathbf{N}(A) \cap \{x \geq 0\} = \{0\}$.

4 Problem 4 (25 pts)

The Kitty Railroad is in the process of planning relocations of freight cars among the 5 regions of the country to get ready for the fall harvest. Table 1 shows the cost of moving a car between each pair of regions. And Table 2 shows the current number of cars in each region and the number needed for harvest shipping.

From/To	1	2	3	4	5
1	-	10	12	17	35
2	10	-	18	8	30
3	12	18	-	9	27
4	17	8	9	-	20
5	35	30	27	20	-

Table 1: Costs of moving a car

	1	2	3	4	5
Present	115	385	410	480	610
Need	180	500	800	200	300

Table 2: Number of current and needed cars

Write down a linear program to compute the least costly way to move the cars such us the need is met. And solve the problem using MATLAB or AMPL.