PIC/FLIP Fluid Simulation for Incompressible and Inviscid Liquid

Gongyi Shi
University of Toronto
Toronto, Ontario, Canada
gongyishi98@gmail.com

Yongzhen Huang
University of Toronto
Toronto, Ontario, Canada
huanggeorge48@gmail.com

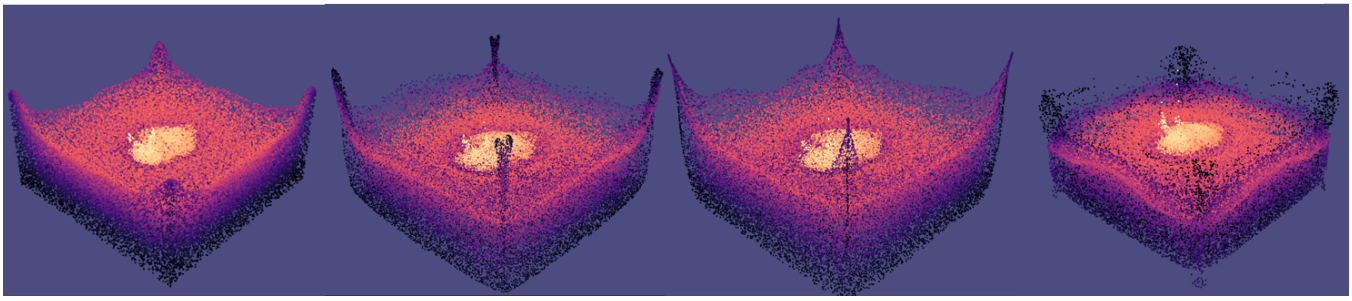

Figure 1: The Stanford Bunny simulated as water with linear combination parameter (left to right) of 0 (Pure PIC), 0.3, 0.6, 1.0 (Pure FLIP) with RK2 solver for advection

ABSTRACT

This report covers the implementation of a physics-based inviscid fluid simulation method referred to as the PIC/FLIP method based on *Fluid Simulation for Computer Graphics* by Robert Bridson[1], with improvements suggested by Zhu and Bridson[8]. The method combines the concepts of representing fluid as both Lagrangian particles and Eulerian grids, allowing efficient surface tracking, advection from particle representation, and computation of incompressibility from grid representation.

We have tested our implementation using combination of FLIP and PIC methods with free surface traced. We further address the issues of this method, and discuss improvements.

CCS CONCEPTS

• **Computing methodologies** → **Physical simulation**.

KEYWORDS

PIC, FLIP, MAC, fluids, physically-based modeling

ACM Reference Format:
Gongyi Shi and Yongzhen Huang. 2020. PIC/FLIP Fluid Simulation for Incompressible and Inviscid Liquid. In *CSC417 '20: Physics-based Animation (CSC417 '20)*. University of Toronto, Toronto, ON, Canada, 3 pages.

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CSC417 '20, September 01 – December 21, 2020, Toronto, ON
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1 INTRODUCTION

Fluid simulation has been extensively studied in mainly two representations. Representing fluids as particles borrow the intuitive concept of atoms from physics; representing fluids as grids, on the other hand, provides several complementary advantages. *Fluid Simulation for Computer Graphics* covers two efficient yet accurate methods of converting fluid representation between particles and grids: Particle-in-Cell(PIC) and Fluid-Implicit-Particle(FLIP). Thus, we can use the complementary strengths of the two to achieve a fast and reliable algorithm.

2 RELATED WORK

PARTICLE: Animating fluid as particles have been studied for decades. Desbrun and Cani[2] proposed a purely particle-based viscous fluid flow simulation named Smooth Particle Hydro-dynamics (SPH). Particle-based methods suffer from pressure projection with respect to incompressibility, while advection can be easily applied to particles by ODE solvers.
GRID: Grid-based fluid simulation is also widely explored. Stam[7] proposed the semi-Lagrangian advection method to improve the efficiency; The staggered marker and cell (MAC) grid-based method by Foster and Metaxas[3]; MAC also discretizes the partial differential equations when projecting pressure, which makes pressure convenient and efficient; The Marker-and-Cell method by Harlow [5] can be used to mark the solid, air, and fluid cells. Grids-based methods have a major problem of solving accurate advection of the flow.

3 MODELING

3.1 Governing Equations

$$\frac{\partial \mathbf{v}}{\partial t} = \rho \mathbf{g} + \tau \nabla^2 \mathbf{v} - \mathbf{v} \cdot \nabla \mathbf{v} - \frac{\nabla p}{\rho} \tag{1}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \tag{2}$$

The two equations governing the fluid flow are known as the Navier-Stokes equations, where $\frac{\partial \mathbf{v}}{\partial t}$ stands for the change of flow velocity over time, $\mathbf{g}$ denotes the body accelerations acting on the continuum, ρ represent the density of the flow, and τ denotes the deviatoric stress tensor, p is the pressure.

Equation 1 describes the change in a velocity field over time. The first term $\rho \mathbf{g}$ indicates the body accelerations due to external forces, which in our simulation is the gravitational force. The second term $\nu \nabla^2 V - V$ models the viscosity of the fluid. Since we focuses on inviscid fluid, this term is dropped. The third term models the self-advection within the velocity field. The last term models the action of the pressure that attempts to keep the fluid from compression. Equation 2 conserves the mass of the fluid. Since we enforces incompressibility, ρ is not a function of time or space. Thus, the equations can be further simplified:

$$\frac{\partial \mathbf{v}}{\partial t} = \rho \mathbf{g} - \mathbf{v} \cdot \nabla \mathbf{v} - \frac{\nabla p}{\rho} \tag{3}$$

$$\nabla \cdot \mathbf{v} = 0 \tag{4}$$

$\mathbf{g}$ now denotes the gravitational acceleration in our simulation. Notice that, equation 4 enforces the velocity field to be divergence-free, which conserves the volume of the fluid.

3.2 Representations and Converters

The method uses particles and staggered grids in different computation steps.

PIC[5] and FLIP[6] are two methods that allow representation conversion between particles and grids. Due to interpolation, PIC introduces dissipations that cause viscosity effects, while FLIP is almost free of numerical dissipations. In addition, for grids, we use the Marker-and-Cell method [5] to mark them as solid, air, and fluid cells. We introduce a linear combination of both converters to manipulate the viscosity. θ is the linear combination parameter

$$\mathbf{v} = \mathbf{v}_{\text{PIC}} \cdot \theta + \mathbf{v}_{\text{FLIP}} \cdot (1 - \theta) \tag{5}$$

3.3 Boundary conditions

In this method, we consider two boundary conditions: solid condition and free surface condition.

A solid condition is when the fluid interacts with solid objects. Intuitively, the normal component of the fluid velocity pointing to the solid surface must be the same as the solid surface's velocity in the same direction, without any sticky effects. The equation can be generated as:

$$\mathbf{v} \cdot \hat{n} = \mathbf{v}_{\text{solid}} \cdot \hat{n} \tag{6}$$

where $\hat{n}$ is the normal vector to the solid boundary, $\vec{v}$ and $\vec{v}_{\text{solid}}$ are the velocities of the fluid and solid respectively.

For the fluid-air free surface condition, we use the "ghost pressure" trick introduced by Gibou and Fedkiw[4] to enforce the $p = 0$ condition at the surface.

3.4 Splitting Equations

At the stage of computation, splitting the incompressible equations and applying each step separately can still achieve a first-order accuracy, while significantly reduce the complexity of solving such a system.

To avoid any change in volume, self-advection must be applied in a divergence-free velocity field. Therefore, we apply the advection after the pressure projection. The order we choose to update the velocity field is: external forces, and pressure projection, self-advection.

3.5 Semi-Lagrangian

For the advection step, the method uses the semi-Lagrangian method introduced to graphics by Stam[7] on each particle. In particular, we can choose Forward Euler to update every particle's location:

$$\vec{x}_{t+1} = \vec{x}_t + \Delta t \vec{u}_t$$

Semi-Lagrangian methods are stable over any length of the time step we choose. Thus, we can get rid of the CFL condition and be able to choose a small time step to achieve higher accuracy, or a big time step for the sake of efficiency.

4 METHOD

A complete procedure of the method can be described as follow:

- Set up a box surrounding by solid cells, and put liquid with divergence-free velocity field.
- For each frame: 1, 2, 3...
  - Transfer particle velocity to staggered grids velocity
  - Save the grid velocity for FLIP
  - Apply external forces
  - Apply boundary conditions
  - Apply pressure projection
  - Update the grid velocity
  - Find the particle velocity using FLIP and PIC
  - Update the particle velocity with a linear combination of both
  - Update the particle position with an ODE solver like Forward Euler, Second Order Runge Kutta, etc.
  - Move fluid particles that moves into solid grid back to fluid grid.

5 EXAMPLES

We used Libigl for rendering. In figure 1 We use the Stanford bunny example simulated with 23,824 particles in total (10,000 inside the bunny) inside a 10x10x10 grid between a linear combination of PIC and FLIP; the coefficients for PIC/FLIP blend is 100%, 60%, 30%, and 0% PIC. The average run-time for a time step is less than 0.4 seconds on 12-core Intel-i7 CPU; we are only running single-core in this simulation.

6 CONCLUSION

We have presented the inviscid version of the fluid simulation solver proposed by Zhu and Bridson[8]. This method combines the strengths of not only particle- and grid-representations, but also FLIP and PIC. It provides the flexibility of controlling viscosity in a certain level even if the viscosity term is dropped in all other steps. It can be further extend to multiple fluid simulation, and increase the accuracy of the boundary conditions.

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