ASTR 150

Concepts in Modern Astronomy

Lab 2: Mars Orbit (*or*) Mars Orbit

Date of lab: January 20, 2021

Report due date & time: January 27, 2021 6:30pm

1. Objective

The key learning objectives in this lab. The goal of this lab is to Learn about Kepler's laws of planetary motion. This intern will help us better understand planetary motion. In addition, we will learn how to utilize triangulation and the locate planets in our solar system through use of the heliocentric, and geocentric systems.

2. Introduction

2.1. Context

Before the more common understanding of our solar system's orbit the common conception was a geocentric view of the solar system. In the geocentric view the solar system revolved around the Earth. An alternative view of the universe was a heliocentric view where the planets revolve around the Sun.

Using observations made before telescopes Johannes Kepler devised three laws of planetary motion which reinforced the notion that instead the solar system revolved around the Sun. This view of the solar system revolving around the Sun is known as the ecliptic view of the solar system. Kepler also identified the fact that the orbit of planets followed an elliptical shape. He also identified that the planets travel faster as they move closer to the Sun. Finally he determined that the planets semi-major axis and it's period are proportional.

3. Procedure

For this lab the only equipment used was a sheet of graph paper, two triangular rulers, a protractor, and a compass. Using the compass I drew a circle with a five centimetre radius at the centre of the graph paper to represent Earth's orbit. I then drew a horizontal line from the centre of the circle to represent the first point of Aries. Then using a table containing positions of Earth in heliocentric coordinates and Mars in geocentric coordinates on a specialized date. We then use these coordinates to triangulate the position of Mars. First I placed the matching locations of Earth. Then using 2 triangular rulers a parallel line to the first point of Aries at the location of Earth using 2 was used as 0° for the geocentric coordinates of Mars. Then from there I used the geocentric coordinates for Mars from each of the locations of Earth to triangulate Mars. This triangulation should be accurate since the measurements where taken at a multiple of orbital period of Mars (687 days). After marking each of the triangulated positions of Mars I drew an approximation of Mars's orbit. I then measured the distance between each calculated position of Mars. These measurements where marked on the paper in purple ink. The longest distance was drawn in red representing the major axis. The distance was then measured from the Sun to each end of the major axis, as well as the distance to the middle of the major axis.

4. Observations

Using table 1 we see the longest distance is determined to be from M_A to M_D with a distance of 16.2 cm. Contrary to the procedure described in the lab book this line did not cross the position of the Sun. The semi-major distance is found to be 8.1cm is figure 0. The distance from the focal length was determined as to be 1.1cm. Finally the perihelion measured in at 7cm while the aphelion was 9.2cm.

4.1. Tables

location 1	location 2	Distance (cm)	Distance (AU)
M_A	M_B	6.1	1.22
M_A	M_C	13.0	2.6
M_A	M_D	16.2	3.24
M_A	M_E	15.9	3.18
M_A	M_F	8.0	1.6
M_B	M_C	8.0	1.6
M_B	M_D	13.7	2.74
M_B	M_E	16.1	3.22
M_B	M_F	11.5	2.3
M_C	M_D	7.7	1.54
M_C	M_E	13.2	2.64
M_C	M_F	13.9	2.78
M_D	M_E	7.0	1.4
M_D	M_F	12.8	2.56
M_E	M_F	8.2	1.64

Table 1: Distances Between Locations Of Mars

4.2. Graphs

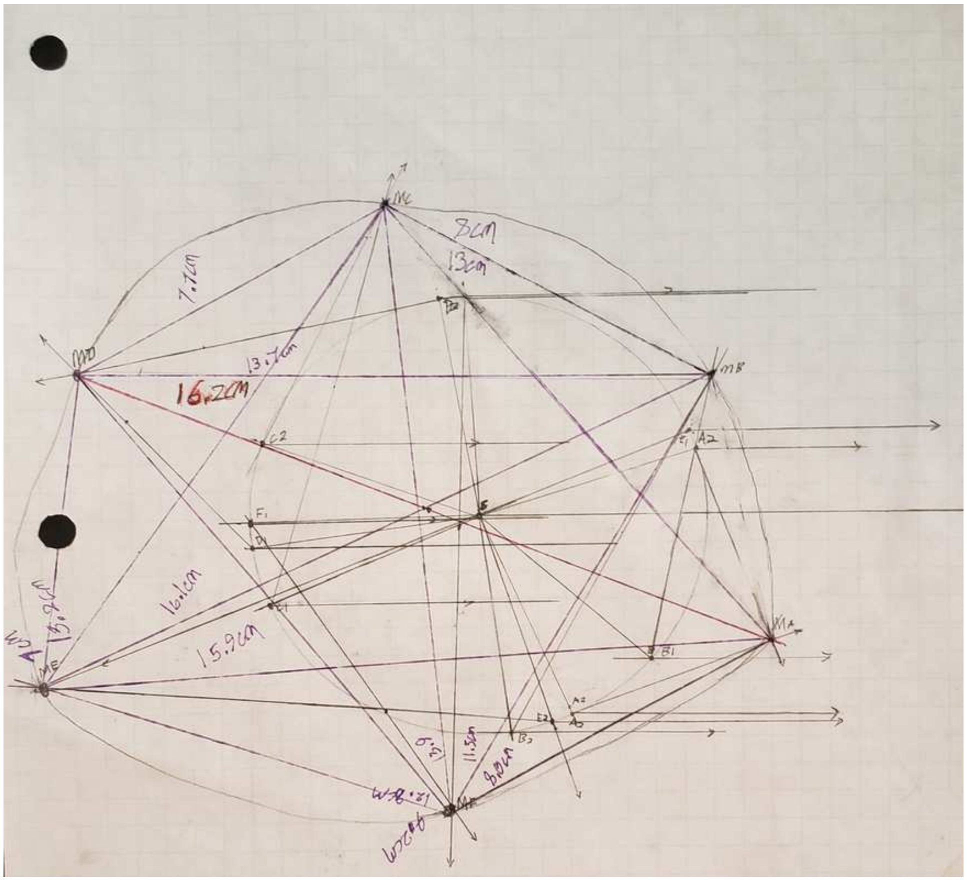

Figure 1: Triangulated Orbit of Mars

5. Calculations

$$Scaled\ Distance\ To\ Sun\ (AU) = \frac{Distance\ To\ Sun\ In\ Diagram\ (cm)}{5} \tag{1}$$

$$Distance\ from\ M_A\ To\ Sun\ = \frac{7cm}{5} = 1.4AU \tag{2}$$

$$Average\ Distance\ (AU) = \frac{Total\ Scaled\ Distance\ (AU)}{Locations\ Of\ Mars} \tag{3}$$

$$Average\ Distance\ (AU) = \frac{(1.4 + 1.2 + 1.46 + 1.84 + 2.04 + 1.34)}{6} \tag{4}$$

$$\approx 1.546666667\ AU$$

$$\frac{Focal\ length}{Semi\ major\ axis\ length},\ \frac{1.1cm}{8.1cm} \approx 0.1358024691[-] \tag{5}$$

$$\frac{r_1^3}{T_1^2} = \frac{r_2^3}{T_1^2} \tag{6}$$

$$\frac{1.62AU^3}{687days^2} = \frac{1AU^3}{365days^2}$$

$$\frac{4.251528}{471969}[-] = \frac{1}{133225}[-]$$

$$0.000009008066208[-] = 0.000007506098705[-] \tag{7}$$

6. Answers

6.1. (1)

The following definition of Kepler's are from NASA Solar System Exploration[1]

Kepler's First Law:
each planet's orbit about the Sun is an ellipse. The Sun's center is always located at one focus of the orbital ellipse. The Sun is at one focus. The planet follows the ellipse in its orbit, meaning that the planet to Sun distance is constantly changing as the planet goes around its orbit.

Kepler's Second Law:
the imaginary line joining a planet and the sons sweeps equal areas of space during equal time intervals as the planet orbits. Basically, that planets do not move with constant speed along their orbits. Rather, their speed varies so that the line joining the centers of the Sun and the planet sweeps out equal parts of an area in equal times. The point of nearest approach of the planet to the Sun is termed perihelion. The point of greatest separation is aphelion, hence by Kepler's Second Law, a planet is moving fastest when it is at perihelion and slowest at aphelion.

Kepler's Third Law:
the squares of the orbital periods of the planets are directly proportional to the cubes of the semi major axes of their orbits. Kepler's Third Law implies that the period for a planet to orbit the Sun increases rapidly with the radius of its orbit. Thus we find that Mercury, the innermost planet, takes only 88 days to orbit the Sun. The Earth takes 365 days, while Saturn requires 10,759 days to do the same. Though Kepler hadn't known about gravitation when he came up with his three laws, they were instrumental in Isaac Newton deriving his theory of universal gravitation, which explains the unknown force behind Kepler's Third Law. Kepler and his theories were crucial in the better understanding of our solar system dynamics and as a springboard to newer theories that more accurately approximate our planetary orbits.

6.2. (2)

Using equation (1) we get

Position Of Mars	Diagram Distance (cm)	Scaled Distance (AU)
M_A	7.0	1.4
M_B	6.0	1.2
M_C	7.3	1.46
M_D	9.2	1.84
M_E	10.2	2.04
M_F	6.7	1.34

Table 2: Distance From Each Position Of Mars To The Sun

$\boxed{\text{Using equation (3) the average distance between the Sun and Mars is } 1.546666667 \text{ AU}}$

6.3. (3)

$\boxed{\therefore \text{ using equation (5) the eccentricity of Mars is } 0.1358024691\ [-]}$

6.4. (4)

Using the measurements for perihelion (7cm) and aphelion (9.2cm) with equation(1)

$\boxed{\text{We find that the perihelion is 1.4AU and the aphelion is 1.84AU}}$

6.5. (5)

The orbital period of Earth is 365 and the orbital period of Mars is 687.

Based on the fact that the Earth's orbital period is close to half the orbital period of Mars it should be once every two years

6.6. (6)

In figure 1 we can see that the closest Earth gets to Mars is at M_A with Earth at position A_1 it can be seen that Based on the given dates and diagram that was created that Earth and Mars likely are the closest in October.

6.7. (7)

In figure 1 we can see that the closest Earth gets to Mars is at M_F with Earth at F_2 it can be seen that Based on the given dates and diagram that was created it can be assumed that they are in conjunction in December.

6.8. (8)

Recall that: The semi-major distance of Mars is found to be 8.1cm aka 1.62AU
Recall that: The semi-major distance of Mars is found to be 5.0cm aka 1AU

The calculated ratio can be seen in equation (7) While these numbers are not identical they are close considering the possible error caused by miss aligning the ruler and considering the scale that these values are at. Therefore, we can conclude that the square of the orbital period is proportional to the cube of semi-major distance. This justifies Kepler's third law.

7. Discussion

The actual average distance from the Sun is 142,000,000 miles which is 1.527803343 AU. This is fairly close to our expected 1.546666667 AU . The real eccentricity of Mars[3] is 0.0935 which is very different from our calculated 0.1358024691. This deviation is likely due to the major-axis of Mars did not cross the Sun as expected. The fact that our major axis did not cross the Sun would add extra length to the eccentricity. As expected Mars and Earth come close to each other every 2 years[4] . The actual perihelion of Mars[3] is 1.381326291 AU which could be just a rounding error compared to 1.4 AU but considering that the expected Aphelion of 1.84 Au is very different from the actual Aphelion[3] 1.6662064122 AU we can see that this representation is not perfectly accurate. This is likely caused by the incorrect orbit of Earth we used. now using the equation (6) with the real semi-major axis of Mars (227923000 km) and Earth (149596000 km)[3] we get

$$0.000007496208588 = 0.000007508702436$$

These values are so close that this discrepancy is likely a rounding error.

Earth in reality is intended to have a slight elliptical with a small eccentricity of 0.0167. Since we removed this elliptical shape this in theory would cause the Mars's orbit to be improperly shaped. This most likely would cause our measured eccentricity of Mars to be less accurate since our geocentric measure counts on the location of the Earth. If instead we used the geocentric measurements with the actual orbit of the Earth our findings would have been more accurate.

The last time Earth and Mars where last at their closest was on October 13th 2020[5] . This is the closest they have been but they are actually in opposition every 26 months. Since Earth and Mars's period is not an exact multiple of each other it is understandable that they opposition would not occur on the same month every time. This falls into reason that our expected month of October is possible. The next time they will be this close is not until the year 2035[6] .

8. Conclusions

From the results found in this lab we have justified Kepler's laws. Now we know how to utilize triangulation, planetary motion, and how the heliocentric, and geocentric coordinates systems work.

References

1. "Orbits and Kepler's Laws," *NASA Solar System Exploration,* https://solarsystem.nasa.gov/resources/310/orbits-and-keplers-laws/ (26 June 2008). Accessed 25 Jan. 2021.
2. "Mars Facts," *All About Mars NASAs Mars Exploration Program,* https://mars.nasa.gov/all-about-mars/facts/. Accessed 27 Jan. 2021.
3. David R. Williams, *Mars Fact Sheet,* https://nssdc.gsfc.nasa.gov/planetary/factsheet/marsfact.html (25 November 2020). Accessed 27 Jan. 2021.
4. "Mars Exploration Program," *Mars in our Night Sky NASAs Mars Exploration Prog,* https://mars.nasa.gov/all-about-mars/night-sky/close-approach/mars.nasa.gov/all-about-mars/night-sky/close-approach/. Accessed 27 Jan. 2021.
5. Mathewson, Samantha., "Mars at opposition shines extra bright in the night sky tonight," *Space,* https://www.space.com/mars-opposition-october-2020 (13 Oct. 2020). Accessed 27

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6. Sinnott, Roger W., “Mars at Its Biggest and Brightest Until 2035,” *Sky & Telescope,* https://skyandtelescope.org/press-releases/mars-at-its-biggest-and-brightest-until-2035/. (6 Oct. 2020). Accessed 27 Jan. 2021.