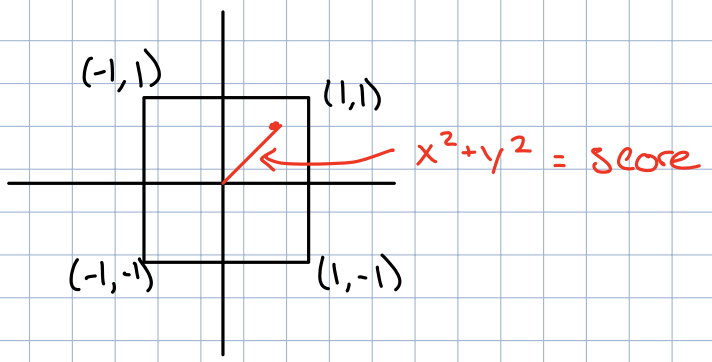
Ex) Imagine a square dart board with edge size 2, such that it is centered around 0. You throw darts that land uniformly on the square, and your score is the squared distance of your dart from the center. What is your average score?



$X \sim Unif(-1,1)$
$Y \sim Unif(-1,1)$

So $f(x,y) = \frac{1}{4} \cdot I_{(-1,1)\times(-1,1)}$

So:

$$\text{Average score} = E(x^2+y^2) = \int_{-1}^{1}\int_{-1}^{1} \frac{x^2+y^2}{4}\, dx\, dy$$

$$\Rightarrow \frac{1}{4}\iint x^2+y^2\, dx\, dy$$

$$= \frac{1}{4}\int_{-1}^{1} \frac{x^3}{3} + xy^2 \Big|_{-1}^{1} dy = \frac{1}{4}\int_{-1}^{1} \frac{2}{3} + 2y^2\, dy$$

$$= \frac{1}{4}\left(\frac{2}{3}(y + y^3)\right)\Big|_{-1}^{1} = \boxed{\frac{2}{3}}$$