

## Lecture 12

8.4

### Correlation

- Correlation gives a scaled covariance from  $[-1, 1]$
- $$\text{Corr}(X, Y) = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X) \cdot \text{Var}(Y)}} = \frac{\text{Cov}(X, Y)}{\text{sd}(X) \text{sd}(Y)}$$

$\hookrightarrow \rho$   
 $\rho(X, Y)$

### Lemma

- $$\begin{aligned} \text{Corr}(aX + b, Y) &= \frac{a}{|a|} \text{Corr}(X, Y) \\ &= \text{sign}(a) \cdot \text{Corr}(X, Y) \end{aligned}$$

### Correlation = $\pm 1$

- $$\text{Corr} = 1 \iff \exists a > 0, b \in \mathbb{R} \text{ s.t. } Y = aX + b$$

$$\bullet \text{Corr} = -1 \iff \exists a < 0, b \in \mathbb{R} \text{ s.t.} \\ Y = aX + b$$