Lecture 2: Stat/Math 395

6.1

Multinomial Distribution

Consider a situation with:

- r categories
- n "trials"

ex) a box with 3 balls (R, G, B) and 10 draws (trials) with replacement

Let $X_1, \ldots, X_r$ be the categories and $k_1, \ldots, k_r$ be the occurrences of the X_i's so $k_1 + k_2 + \ldots + k_r = n$

How many permutations are there for this situation?

$$\binom{n}{k_1} \cdot \binom{n-k_1}{k_2} \cdots \binom{n-k_1-\ldots-k_{n-1}}{k_n}$$

$$= \frac{n!}{k_1!(n-k_1)!} \cdot \frac{(n-k_1)!}{k_2!(n-k_1-k_2)!} \cdots$$

$$= \frac{n!}{k_1!\, k_2! \cdots k_n!} = \binom{n}{k_1\, k_2 \cdots k_n}$$

Multinomial coefficient

Now, let $p_1, \ldots, p_r$ be probability values

s.t. $p_1 + \ldots + p_r = 1$ and the prob of success for category i is p_i

Then:

$$(X_1, \ldots, X_r) \sim \text{Multinom}(n, p_1, \ldots, p_r)$$

s.t. $$p(x_1, \ldots, x_r) = \binom{n}{x_1 x_2 \ldots x_r} p_1^{x_1} \cdots p_r^{x_r}$$

If $r = 2$, then we have a binomial distribution

Ex)

Consider a 6-sided die. Roll it 100 times and record the rolls.

Let $X_1 = \#$ of 1's

$\vdots$

$X_6 = \#$ of 6's

What is $p(X_1, \ldots, X_6)$?

$\Rightarrow (X_1, \ldots, X_6) \sim Multi(100, \frac{1}{6}, \frac{1}{6}, \ldots, \frac{1}{6})$

$$p(X_1, \ldots, X_6) = \binom{100}{X_1 X_2 \ldots X_6} \left(\frac{1}{6}\right)^{\sum X_i}$$

What is $p(X_1, X_2 + X_3, X_4, X_5, X_6)$?

$(X_1, X_2 + X_3, X_4, X_5, X_6) \sim Multi(100, \frac{1}{6}, \frac{2}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6})$

$P(X_2 \cup X_3) = P(X_2) + P(X_3) = \frac{2}{6}$