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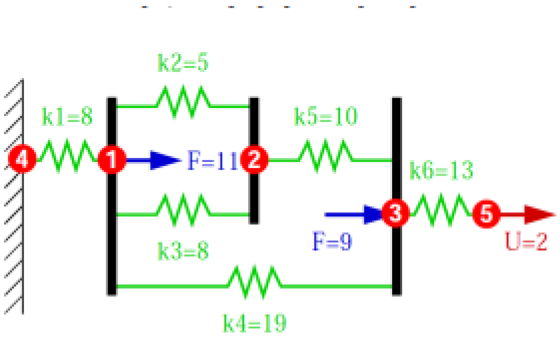
Deadline: 17.03.2021 - 13:00

In [65]:
```
from IPython.display import Image
Image(filename='Angabe.png')
```

Out[65]:



Setting up the stiffness matrix

by creating the stiffness matrix for each element. Then we combine all matrices to describe the complete system

k_1

$$\begin{pmatrix} k_1 & 0 & 0 & -k_1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -k_1 & 0 & 0 & k_1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} u_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} F_1 = 11 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

k_2

$$\begin{pmatrix} k_2 & -k_2 & 0 & 0 & 0 \\ -k_2 & k_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} u_1 \\ u_2 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} F_1 = 11 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

k_3

$$\begin{pmatrix} k_3 & -k_3 & 0 & 0 & 0 \\ -k_3 & k_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} u_1 \\ u_2 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} F_1 = 11 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

k_4

$$\begin{pmatrix} k_4 & 0 & -k_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -k_4 & 0 & k_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} u_1 \\ 0 \\ u_3 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ F_3 = 9 \\ 0 \\ 0 \end{pmatrix}$$

k_5

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & k_5 & -k_5 & 0 & 0 \\ 0 & -k_5 & k_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ u_2 \\ u_3 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ F_3 = 9 \\ 0 \\ 0 \end{pmatrix}$$

k_6

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_6 & 0 & -k_6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -k_6 & 0 & k_6 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ u_3 \\ 0 \\ u_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ F_3 = 9 \\ 0 \\ F_5 \end{pmatrix}$$

combining everything...

$K \cdot u = F$:

$$\begin{pmatrix} k_1 + k_2 + k_4 + k_3 & -k_2 - k_3 & -k_4 & -k_1 & 0 \\ -k_2 - k_3 & k_2 + k_3 + k_5 & -k_5 & 0 & 0 \\ -k_4 & -k_5 & k_6 + k_5 + k_4 & 0 & -k_6 \\ -k_1 & 0 & 0 & k_1 & 0 \\ 0 & 0 & -k_6 & 0 & k_6 \end{pmatrix} \cdot \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 = 0 \\ u_5 = 2 \end{pmatrix} = \begin{pmatrix} F_1 = 11 \\ F_2 = 0 \\ F_3 = 9 \\ F_4 \\ F_5 \end{pmatrix}$$

we have 5 unknowns: u_1, u_2, u_3, F_4, F_5

Calculating the matrix product using the respective values for k_i leads to

$$\begin{pmatrix} 40 \cdot \mathrm{u}_1 - 13 \cdot \mathrm{u}_2 - 19 \cdot \mathrm{u}_3 \\ -13 \cdot \mathrm{u}_1 + 23 \cdot \mathrm{u}_2 - 10 \cdot \mathrm{u}_3 \\ -19 \cdot \mathrm{u}_1 - 10 \cdot \mathrm{u}_2 + 42 \cdot \mathrm{u}_3 - 26 \\ -8 \cdot \mathrm{u}_1 \\ 26 - 13 \cdot \mathrm{u}_3 \end{pmatrix} = \begin{pmatrix} 11 \\ 0 \\ 9 \\ \mathrm{F}_4 \\ \mathrm{F}_5 \end{pmatrix}$$

We can split this system of equations into 2 parts:

The first three rows can be solved using numpy.linalg.solve(). The last 2 rows will be solved by hand

Rows 1-3

In [66]:

```
import numpy as np

# K Matrix without 4th node
K = np.array([
    [40, -13, -19],
    [-13, 23, -10],
    [-19, -10, 42] # 26 has to be brought to the RHS as it is a force
    ])

F = [11, 0, 9+26]

print(f"\nK:\n{K}")
print(f"\nF:\n{F}")

u = list(np.linalg.solve(K, F))   # solve LSE

# adding boundary conditions by hand just for completeness
u.append(0)
u.append(2)


print("\nSolutions for u:")
for i in range(len(u)) : print(f"u{i+1} = {u[i]}")
```

```
K:
[[ 40 -13 -19]
 [-13  23 -10]
 [-19 -10  42]]

F:
[11, 0, 35]

Solutions for u:
u1 = 2.0540597244562564
u2 = 2.1498706203231004
u3 = 2.2744247849499972
u4 = 0
u5 = 2
```

Rows 4-5

In [67]:

```
F = [11, 0, 9]
F.append(-8*u[0])
F.append( 26 - 13*u[2])

print("\nSolutions for F:")
for i in range(len(u)) : print(f"F{i+1} = {F[i]}")
```

```
Solutions for F:
F1 = 11
F2 = 0
F3 = 9
F4 = -16.43247779565005
F5 = -3.567522204349963
```

Sanity check:

$\sum F_i = 0$

In [68]:
```
print(f"Sum of all forces: {np.sum(F)}")
```

```
Sum of all forces: -1.4210854715202004e-14
```