Numerical simulation and Scientific Computing II – Fluid mechanics. General informations:

- **Submission is due on Wednesday, May 26^{th} 2021 11 am**. Please submit the material to *nssc@iue.tuwien.ac.at*. Use Dropbox, Wetransfer or owncloud in case you are willing to submit files of large size.
- Include the name of all group members (max. 3) in your submission
- Submit everything (source files, results, animations – if any – and final reports) as one zip-file per exercise. Include a README file to explain the content and the usage of the different files in the archives.
- The number of points corresponding to each task of the exercises is indicated within square brackets, e.g. [2 POINTS]
- You can write your codes in Fortran, Matlab, Python, C/C++. Note: assistance can be given in case you use Fortran or Matlab.
- In case you are willing to use Matlab to solve the exercises, but you do not have it already installed in your computer, then you can download and install it via the following link: http://www.sss.tuwien.ac.at/sss/mla/. There is a free Matlab version for TU students.

Exercise 1: Numerical solution of the diffusion equation in a finite domain

Consider the 1D unsteady diffusion equation in cartesian coordinates:

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C_i}{\partial x^2}, \tag{1}$$

with $D = 10^{-6}$ the binary diffusion coefficient. The size of the domain along x is h (i.e. the domain is characterized by a finite length). The grid spacing is Δx (obtained discretizing the domain length h using N_x points), and the time step is Δt. Initial condition is $C = 0$ everywhere inside the domain.

Questions

1. Assume Dirichlet/Neumann boundary conditions: $C = 1$ at $x = 0$ and $\partial C/\partial x = 0$ at $x = h$. Solve Eq. 1 by an *explicit* finite difference approach that is 2^{nd} order accurate in space and 1^{st} order accurate in time. Compare the results with the analytical solution (see Lecture for reference). Show that the numerical solution is not unconditionally stable. In particular, show that it is unstable for $d = D\Delta t/\Delta x^2 > 0.5$. [2 POINTS]

2. Assume Dirichlet boundary conditions at both boundaries: $C = 1$ at $x = 0$ and $C = 0$ at $x = h$. Solve Eq. 1 for the same cases and using the same discretization strategy (i.e. *explicit* finite difference approach, 2^{nd} order in space and 1^{st} order in time) analyzed at point 1. Compare the long term behavior, $\lim_{t\to\infty} C(x)$, of the present case with that of the previous case, point 1. [2 POINTS]

3. Assume Dirichlet/Neumann boundary conditions: i.e. $C = 1$ at $x = 0$ and $\partial C/\partial x = 0$ at $x = h$. Solve the equation using an *implicit* scheme (2^{nd} order in space and 1^{st} order in time) and discuss the stability of the solution. Compare the new results with those obtained at point 1. [2 POINTS]

4. Assume Dirichlet/Neumann boundary conditions: i.e. $C = 1$ at $x = 0$ and $\partial C/\partial x = 0$ at $x = h$. Solve the equation using an *implicit* scheme that is 2^{nd} order in space and $2nd$ order in time. Discuss the accuracy of the solution in comparison with that obtained at point 3. [2 POINTS]

Produce plots and movies to support your conclusions (for your convenience, you can take plots and movies presented during the lecture as reference for the purpose).

Exercise 2: Numerical solution of the linear advection equation in a periodic domain

Consider the 1D linear advection equation in cartesian coordinates:

$$\frac{\partial C_i}{\partial t} = U \frac{\partial C_i}{\partial x}, \tag{2}$$

where $U = 1$ is a given convection velocity. Assume $\Delta x = 0.01$.

Questions

1. Solve Eq. 2 with the *upwind* scheme (please refer to the lecture for details). Assume two different initial conditions: a square pulse ($C = 1$ for $0.1 < x < 0.3$, $C = 0$ elsewhere) and a Gauss signal (for example $C = \exp(-10(4x - 1)^2)$). The shape of these initial conditions is given in Fig. 1. Discuss the stability and accuracy of the solution for varying Courant number Co. In particular, what happens for $Co > 1$? And for $Co < 1$? And for $Co = 1$? [2 POINTS]

Produce plots and movies to support your conclusions (for your convenience, you can take plots and movies presented during the lecture as reference for the purpose).

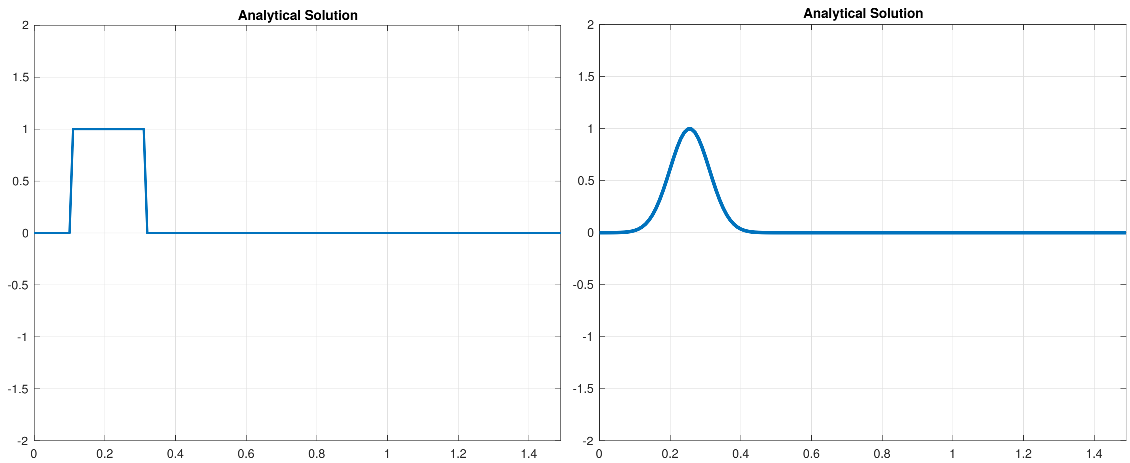

Figure 1: Initial condition for the linear advection equation: square pulse (left), Gauss signal (right).