Horizon Penetrating Coordinate for Spherically Symmetric Metric

Hyun Lim

We present a horizon penetrating coordinate for spherically symmetric metric. We adopt $G = c = 1$ unit system. Consider usual Schwarzschild line element in BL coordinate for practice

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \tag{1}$$

where M is a mass. Compare this with usual 3+1 line element form $ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^2 + \beta^i dt)(dx^j \beta^j dt)$ we can identify the lapse $\alpha^2 = \left(1 - \frac{2M}{r}\right)$. As we know, the line element(Eqn. 1) is singular at the horizon $(r = 2M)$ and lapse collapse to zero. This can be problematic because equations of motion for the metric can be become exponentially unstable in the presence of a coordinate singularity without some regularization technique.

One way to resolve this problem is to move to a horizon penetrating coordinate system where this singularity is not present. The Kerr-Schild coordinates are one such coordinate system. General spherical symmetric line element in polar-areal form

$$ds^2 = -\alpha(r,t)^2 dt^2 + a(r,t)^2 dr^2 + r^2 d\Omega^2 \tag{2}$$

where α is referred as lapse function. Compare with above Schwarzschild solution, $\alpha = 1/a$.

I. MODEL

Here, we first use the perfect fluid approximation for the matted model of our neutron stars. So the stress-energy tensor takes form

$$T_{ab} = (\rho + P)u_a u_b + P g_{ab} \tag{3}$$