**Huybrechts 2.1**

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**2.1.3**

- The algebraic dimension of $\mathbb{P}^n$ is $n$. For consider the function $[x_0 : .... : x_n] \mapsto x_i/x_0$ for $i = 1, ..., n$. I claim that this is a rational function on $\mathbb{P}^n$. Consider any point $p = [p_0 : ... : p_n]$. If $p_0 \neq 0$ then $p \in U_0$ and on $U_0$, this function is represented by the obviously holomorphic coordinate function $x_i$. If $p_0 = 0$, then $p_j \neq 0$ for some $j > 0$, so $p \in U_j$, and $z_i/z_j$ and $z_0/z_j$ are well-defined on $U_j$ and $z_0/z_j$ is invertible. (If $i = j$, then $z_i/z_j = 1$, but the argument still goes through.)

  These $n$ functions are algebraically independent. For say

  $$F(x_1/x_0, ..., x_n/x_0) = 0.$$

  Then $x_0^m F(x_1/x_0, ..., x_n/x_0) = x_0^m G(x_0, x_1, ..., x_n) = 0$ for some polynomial $G$ and some $m$. This implies that $G = 0$, so $F = 0$.

  The result then follows from the upper bound of Proposition 2.1.9.

- The Weierstrass $p$ function is a nontrivial rational function on the torus. Therefore by Proposition 2.1.9, the algebraic dimension of $\mathbb{T}$ is 1.

- Proposition 2.1.9 does not apply to (non-compact) $\mathbb{C}$. In fact, $K(\mathbb{C})$ has infinite transcendence degree. For $K(\mathbb{C})$ contains the holomorphic functions on $\mathbb{C}$, and $e^{z^k}$ for $k \in \mathbb{N}$ is an algebraically independent set of holomorphic functions on $\mathbb{C}$. This can be seen by considering the asymptotics along the positive real axis.