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Chapter 2.5: Blow-ups

Huybrechts, *Complex Geometry*

2.5.1 (See the first answer to https://math.stackexchange.com/questions/245965/pull-back-of-sections-of-vector-bundles for a useful discussion of pullbacks of sections. After you draw all the diagrams, you find that $f^*\sigma = (y \mapsto \sigma(f(y)) \in f^*E)$. Apply this *away from the exceptional divisor.*)

Here is how to define the map. Take $\alpha \in H^0(X, K_x)$. Let σ be the blow-up map, so $\sigma|_{\hat{X}\setminus E}$ is an isomorphism $\hat{X} \setminus E \simeq X \setminus \{x\}$. Now $a|_{\hat{X}\setminus E} \circ \sigma|_{\hat{X}\setminus E}$ defines a section $\hat{X} \setminus E \to K_{\hat{X}\setminus E}$, because $K_{\hat{X}\setminus E} \simeq K_X$. To show that this section extends across the exceptional divisor E, it is sufficient to show that it is bounded in some local trivialization at each point in E (Riemann Extension Theorem Prop 1.17).

But this follows because α is "bounded" on X and σ gives a surjection onto $\hat{X} \setminus E$. α is bounded in the sense that there are finitely many trivializations of X and α is bounded in each of these trivializations. This follows because X is compact, so we can cover it in compact sets contained in trivializations.