

Unemployment in a Production Network

Finn Schüle and Haoyu Sheng

May 15, 2023

Abstract

We develop a framework for studying unemployment in a production network by incorporating sector-specific search and matching frictions. We derive aggregation formulae for how output and unemployment respond to sector-specific productivity and labor supply shocks. Specifically, output aggregation can be decomposed into two channels, the input-output channel, which can be expressed in terms of sales shares, and the network-adjusted search-and-matching channel, which depends on the interaction between labor market frictions and an occupational-labor-share-weighted Leontief inverse. We show that the foundational theorem of Hulten (1978) is a special case of the network we study, when wages respond to exactly offset changes in the network-adjusted marginal productivity of labor. Additionally, the wage assumption is essential for determining whether matching frictions amplify or dampen the impact of shocks. For example, when wages rise by less than the network-adjusted marginal product of labor in response to a positive technology shock, firms increase their hiring, amplifying the positive effect on output, and vice versa. We calibrate our model to the US data and show that search and matching is both quantitatively and qualitatively important for the propagation of technology and labor force shocks.

JEL Codes: E1, J3, J6

Schüle: finn_schuele@brown.edu, Brown University. Sheng: haoyu_sheng@brown.edu, Brown University.
We thank Gabriel Chodorow-Reich, Fernando Duarte, Gauti Eggertsson, Amy Handlan, Yann Koby, and Pascal Michaillat for helpful comments and advice. All errors are our own.

1. Introduction

Modern economies feature rich production networks. A recent literature highlights that the interactions between firms in different sectors matter for the propagation of sector level shocks (Baqae and Rubbo 2022), how we measure productivity and the social cost of distortions (Baqae and Farhi 2019, 2020), the optimal conduct of monetary policy (Rubbo 2020; La’O and Tahbaz-Salehi 2021), and that sector level shocks may even be a source of aggregate fluctuations and growth (Acemoglu et al. 2012; Acemoglu and Azar 2020).

We extend the existing literature by showing that production networks also matter for unemployment in a realistic frictional labor market. As shocks propagate through the network, labor demand changes, which impacts sector tightness and unemployment. On the other hand, unemployment matters for production networks. Changes in labor market tightness require firms to change their recruiting effort, and the reallocation of productive workers to recruiters alters production outcomes. In addition, unemployed workers look for jobs in other sectors, lowering tightness in those sectors, and thus affecting production across the network.

Production linkages and unemployment are important economic mechanisms, and understanding how the two interact is key to painting an accurate and granular picture of how microeconomic shocks generate macroeconomic fluctuations. In addition, understanding the labor market impact of micro shocks can help policymakers craft and evaluate policies. For example, our framework can help us dissect how energy price spikes caused by the Russia-Ukraine war impact output and unemployment for different sectors, or how the CHIPS and Science Act, which boosts domestic research and manufacturing of semiconductors in the United States, changes aggregate output and unemployment. More generally, we use our framework to ask: What are the impacts of idiosyncratic, sector-specific technology shocks on sector and aggregate output and unemployment? How do idiosyncratic shocks generate comovements in tightness across labor markets? How do labor market inefficiencies and characteristics, such as matching frictions, mobility costs, and wage schedules, affect network propagation of shocks?

Starting with Long and Plosser (1983) and Acemoglu et al. (2012), much effort has been made on understanding how micro shocks cause macro fluctuations. Since an early contribution by Jones (2011), which demonstrates that missallocation accross a production network can generate more realistic transition dynamics in a neoclassical

growth model, a recent literature has incorporated inefficiencies into production network models, ~~exogenous~~ including markups and financial frictions (Liu 2019; Baqaee and Farhi 2020; Bigio and La’O 2020) and nominal rigidities (Rubbo 2020; La’O and Tahbaz-Salehi 2021; Minton and Wheaton 2022). While these models offer a more realistic depiction of production linkages and the associated product market and financial inefficiencies, their treatment of labor markets is simple. These models consider labor as either perfectly inelastically supplied, or supplied according to the disutility of work in household’s utility function, therefore ignoring the richness of labor market imperfections and shutting off interesting propagation channels brought forth by such imperfections.

In this paper, we extend the production network framework by incorporating matching frictions. We build a static, multisector production network model that features a representative household, many production sectors, perfect competition in product markets, and segmented frictional labor markets. The form of labor market segmentation is flexible, and our model allows for geographic, sectoral, or occupational labor market segmentation.

With this setup, we develop a general theory of how technology shocks propagate and aggregate across the production network under search frictions. In particular, we show that the aggregate impact on output and unemployment can be decomposed into two components. The first term resembles the foundational aggregation theorem of Hulten (1978), in which the output response is a sales-share-weighted aggregation of sectoral technology shocks. The second term, which we call the search-and-matching channel, involves complex interaction between tightness, production structure, and labor market structure. We show that we can recover Hulten’s Theorem when relative wages adjust exactly according to the occupational-labor-share-weighted marginal product of labor, which we call the network adjusted marginal product of labor. Intuitively, when wages change by the network adjusted marginal product of labor, tightness remains unchanged, eliminating all additional propagation coming from labor market frictions.

Whether the search-and-matching channel dampens or amplifies technology shocks depends largely on how wages adjust. Since the search and matching setup admits a wide range of assumptions about wages with very different quantitative consequences, as demonstrated by Shimer (2005) and Hall (2005), we view this as an empirical question. Nevertheless, we use two toy examples to illustrate the determinants of the direction of amplification. First, we shut off production linkages in the model, and show that

the direction of amplification depends on how wages responds relative to the marginal product of labor, as well as whether posting additional vacancies will change the number of productive workers. Since hiring an additional worker requires more and more vacancies as labor markets get tighter, and posting vacancies requires firms to allocate workers towards recruiting, there is an inflection point where an additional vacancy results in no change in the productive workforce. We then solve a simple two-sector vertical production networks model with one labor market, to show that the direction of amplification now depends on how wages responds to the network-adjusted marginal product of labor. We further show that our frictional labor market setup allows for upstream propagation of shocks, which is absent in efficient production networks with Cobb-Douglas utility and production functions.

In addition to technology shocks, our setup allows us to explore another set of shocks—shocks to the size of the labor force. We can think of these shocks as coming from an underlying model of occupational choice, as in Humlum (2021), or an underlying model of migration, as in Fernandez-Villaverde (2020). Unlike in the standard production networks setup, where a positive shock to the size of the labor force mechanically increases output because firms are forced to hire all available workers, our framework allows the hiring decision of firms to vary depending on wages. For instance, if wages do not change by much in response to negative labor force shocks, firms do not reduce their hiring by much, dampening the overall effect on output but generating large changes to unemployment. In this way, our model is able to generate simultaneously tight labor markets and lower output through a combination of negative labor force and energy supply shocks.

To test the empirical relevance of the theoretical channels outlined above, we calibrate our model to US data. We use survey-based vacancy and hiring data from the Job Openings and Labor Turnover Survey (JOLTS), unemployment data from the Current Population Survey (CPS), occupation data from the Occupational Employment and Wage Statistics (OEWS), and industry sales shares from the U.S Bureau of Economic Analysis (BEA). We find that, assuming network-price-adjusted wages do not change, a one-percent positive technology shock to the durables manufacturing sector increases aggregate output by 0.56% percent, and decreases aggregate unemployment by 0.44% percent. Without search and matching frictions, the same shock would increase output by only 0.25% percent and would not affect unemployment. Importantly, we are able to observe how unemployment changes across occupations, with unemployment by 0.99% for production workers, and by 0.77% for engineers, and by 0.70% for construction

workers.

In section 5 we demonstrate how to use our calibrated model for macroeconomic applications. In particular, we examine the employment and output consequences of a simultaneous negative shock to energy supply and labor force participation to explore the consequences of the Russia-Ukraine War related reduction in global oil supply at a time when many workers still had not returned to the labor force after Covid. In a model without search frictions this generates large changes in output and no changes in labor market tightness. In our model, on the other hand, this combination of shocks generates a decline in output that is half as large, a sizeable increases in labor market tightness, and larger changes in prices in energy intensive sectors relative to the education and health services sector, a sector where prices stayed relatively constant. We view all three as more realistic given 2022 U.S. data. For simplicity and to keep the focus on labor market frictions, our current model, is static, features no nominal rigidities, and is entirely real. As a result, we can only speak to relative prices across sectors. However, in future work we hope to extend the model to incorporate dynamics and nominal rigidities. This would allow us explore the consequences of a combination of factor and labor supply shocks for price dynamics, extending Minton and Wheaton (2022) to incorporate labor market frictions and Benigno and Eggertsson (2023) to incorporate shocks to supply chain inputs. We believe a combination of a non-linear Phillips curve in labor market tightness, as in Benigno and Eggertsson (2023), and the gradual propagation of factor prices through a production network, as in Minton and Wheaton (2022), would paint a realistic picture of the current inflation episode.

This paper fits closely to the recent development that attempts to bring more realism into production networks by incorporating market imperfections and inefficiencies. Baqaee and Farhi (2020) and Bigio and La’O (2020) model frictions as an exogenous wedge between the sectors’ marginal costs and marginal revenues and examine how these frictions interact with the network structure and affect aggregate output in the Cobb-Douglas economy. Bigio and La’O (2020) bring the model to data and estimate how the US input-output structure amplify financial distortions in the great recession. Baqaee and Farhi (2020) examine how productivity shocks aggregate under the presence of exogenous wedges in a CES economy and decompose output changes into changes in technology and changes in allocative efficiency. Liu (2019) assumes that market imperfections generate dead-weight loss and demonstrates how these imperfections compound in the production network through demand linkages. He shows that a government should design industrial policies to target sectors based on distortions in

sectoral size. Like La’O and Tahbaz-Salehi (2021) and Rubbo (2020), who consider New Keynesian production network models, our model features endogenous wedges, in our case generated by search and matching frictions. To our knowledge, our paper is the first to model search and matching in a production networks setting.

This paper also contributes to the literature on factor reallocation (Adão et al. 2019; Chodorow-Reich and Wieland 2020). Our paper is the first to shed light on how production linkages impact reallocation and comovements in labor markets under search-and-matching frictions.

The remainder of the paper is organized as follows. Section 2 outlines our model and defines the equilibrium. Section 3 derives expressions for first order changes in output and employment in response to changes in technology and the size of the labor force. Section 4 describes the data we use to calibrate the model and presents illustrative examples to demonstrate the quantitative importance of labor markets. Section 5 works through an application of our model to energy supply shocks. Section 6 concludes.

2. Model

This section describes the basic setup of our mode. There are J production sectors, indexed by i , with production linkages and \mathcal{O} occupations index by o . Our definition of an occupation is flexible and can allow an occupation to respond to a sector, or to some major occupational categories, which may be the more relevant margin of choice (Hum-lum 2021). We do not allow individuals to switch occupations. This is a simplification, but reflects the fact that individuals tend to stay in the same or similar occupations over the course of their careers (Schubert et al. 2022). However, our formulation is general enough to allow us to think of o as representing sector specific employment, location specific employment, occupation-location pairs, or any other desired definition of a labor market. In addition to intermediate inputs from other sectors and labor, firms may also use K other factors of production indexed by k . These additional factors of production will allow us to explore the effects of a shock to oil, for instance. For simplicity, we keep the model static, as is the case in most of the production networks literature. Throughout, bold letters indicate vectors or matrices. We use un-bolded letters to denote functions or scalars. All vectors are column vectors.

2.1. Households and Final Production

A representative household inelastically supplies a labor force of size H_o to occupation o . There is no disutility from labor. Households consume a final production good, Y , produced by a final goods producer with constant returns technology reflecting household preferences

$$Y = \max_{\{c_i\}_{i=1}^J} \mathcal{D} \left(\{c_i\}_{i=1}^J \right),$$

subject to

$$\sum_{i=1}^J p_i c_i = \sum_{o=1}^O w_o L_o + \sum_{k=1}^K r_k K_k,$$

where c_i is the input of sector i 's output into final goods production, p_i is the price of the sector i good, w_o is the wage in occupation o , and L_o is the labor used in sector o . r_k is the price of the additional factor K_k of production. We can think of these K additional inputs into intermediate production as capturing capital or energy costs. The final production input choices satisfy the first order condition

$$(1) \quad \varepsilon_{c_i}^{\mathcal{D}} = \frac{p_i c_i}{\sum_{k=1}^J p_k c_k},$$

where $\varepsilon_{c_i}^{\mathcal{D}}$ is the elasticity of \mathcal{D} to changes in c_i . This is the households elasticity of utility with regards to the consumption of good i .

2.2. Labor Markets

We assume there are O occupations with separate labor markets, a labor force of H_o possible workers, who all start out unemployed at the beginning of the single period. The exogenous recruiting cost, r_o , measures the units of labor required for a firm to maintain each posted vacancy in occupation o . When workers and firms meet there is a mutual gain from matching. There is no accepted theory for how wages are set in this context. For now we assume the nominal wage in occupation o , w_o , follows a general wage schedule that depends on productivity and the size of the labor force, and is taken as given by both firms and workers. Hires are generated by a constant returns matching function in occupation-level unemployment U_o and aggregate vacancies V_o ,

which measures all vacancy postings for occupation o ,

$$h_o = \phi_o m(U_o, V_o).$$

Let the sector-specific labor market tightness be $\theta_o = \frac{V_o}{H_o}$, the vacancy-filling rate $\mathcal{Q}_o(\theta_o) = \phi_o m\left(\frac{H_o}{V_o}, 1\right)$, and the job-finding rate $\mathcal{F}_o(\theta_o) = \phi_o m\left(1, \frac{V_o}{H_o}\right)$. Therefore, a fraction $\mathcal{F}_o(\theta_o)$ of the labor force finds a job and is employed at the end of the period. Labor supply satisfies

$$(2) \quad L_o^S(\theta_o) = \mathcal{F}_o(\theta_o) H_o.$$

We assume firms take the occupation level tightness as given.¹ Let N_{io} denote productive employees in occupation o working for sector i firms. In order to hire N_{io} productive employees, the number of vacancies posted in labor market o by sector i firms, v_{io} , has to satisfy $\mathcal{Q}_o(\theta_o)v_{io} = N_{io} + r_o v_{io}$, where $r_o v_{io}$ is the total cost of posting the vacancies. Rearranging yields $v_{io} = \frac{N_{io}}{\mathcal{Q}_o(\theta_o) - r_o}$. Thus, hiring one unit of productive labor requires $\frac{1}{\mathcal{Q}_o(\theta_o) - r_o}$ vacancy postings. Factoring in hiring costs, firms need $1 + \tau_o(\theta_o)$ units of total labor for each productive worker, where

$$\tau_o(\theta_o) \equiv \frac{r_o}{\mathcal{Q}_o(\theta_o) - r_o}.$$

For a given target level of occupation o employment N_{io} , total required labor, or the labor demand, is $l_{io}^d(\theta_o) = (1 + \tau_o(\theta_o)) N_{io}$. In the language of the production networks literature, τ_o therefore acts as an endogenous wedge on firms' labor costs. This endogenous wedge will turn out to play an important role in how shocks propagate through the production network. We describe how labor demand, $l_{io}^d(\theta_o)$, is determined by firms' profit maximization in the next subsection.

Finally, we define aggregate occupation o labor demand as the sum of sectoral labor demands and aggregate vacancy postings as the sum of sectoral vacancy postings.

$$(3) \quad \begin{aligned} L_o^d(\theta_o) &= \sum_{i=1}^J l_{io}^d(\theta_o) \\ V_o &= \sum_{i=1}^J v_{io} \end{aligned}$$

¹We can think of each sector as being populated by many identical competitive firms so that each firm only has an infinitesimal impact on aggregate vacancies, and therefore on aggregate tightness.

Market clearing in the labor market requires labor demand equal labor supply and the vacancy posting choices of firms in each sector are consistent with aggregate tightness:

$$(4) \quad L_o^d = L_o^s, \\ \theta_o = \frac{\sum_{i=1}^J v_{io}}{H_o}.$$

2.3. Firms

A representative firm in sector i uses workers in occupation o , N_{io} , intermediate inputs from sector j , x_{ij} , and \mathcal{K} additional factors K_{ik} to produce output y_i using constant returns production technology f_i .

$$(5) \quad y_i = A_i f_i \left(\{N_{io}\}_{o=1}^O, \{x_{ij}\}_{j=1}^J, \{K_{ik}\}_{k=1}^K \right)$$

Firms choose $\{N_{io}\}_{o=1}^O$, $\{x_{ij}\}_{j=1}^J$, and $\{K_{ik}\}_{k=1}^K$ to maximize profits, or equivalently to minimize costs. We assume firms are price takers in both input and output markets. Profits are given by

$$\pi_i = p_i f_i \left(\{N_{io}\}_{o=1}^O, \{x_{ij}\}_{j=1}^J, \{K_{ik}\}_{k=1}^K \right) - \sum_{o=1}^O w_o (1 + \tau_o(\theta_o)) N_{io} - \sum_{j=1}^J p_j x_{ij} - \sum_{k=1}^K r_k K_{ik}.$$

Firms choose inputs to solve the profit maximization problem

$$\max_{\{N_{io}\}_{o=1}^O, \{x_{ij}\}_{j=1}^J, \{K_{ik}\}_{k=1}^K} \pi_i \left(\{N_{io}\}_{o=1}^O, \{x_{ij}\}_{j=1}^J, \{K_{ik}\}_{k=1}^K \right),$$

giving the first order conditions

$$(6) \quad \varepsilon_{x_{ij}}^{f_i} = \frac{p_j x_{ij}}{p_i y_i},$$

$$(7) \quad \varepsilon_{N_{io}}^{f_i} = (1 + \tau_o(\theta_o)) \frac{w_o N_{io}}{p_i y_i},$$

$$(8) \quad \varepsilon_{K_{ik}}^{f_i} = \frac{r_k K_{ik}}{p_i y_i},$$

where $\varepsilon_{x_{ij}}^{f_i}$ is the elasticity of sector i production to sector j input, $\varepsilon_{N_{io}}^{f_i}$ is the elasticity of sector i production to occupation o labor, and $\varepsilon_{K_{ik}}^{f_i}$ is the elasticity of sector i production to factor k . From Equation 7, we can derive an expression for sector i and aggregate labor demand:

$$(9) \quad l_{io}^d(\theta_o) = \varepsilon_{N_{io}}^{f_i} \frac{p_i}{w_o} y_i,$$

$$L_o^d(\theta_o) = \sum_{i=1}^J \varepsilon_{N_{io}}^{f_i} \frac{p_i}{w_o} y_i.$$

We assume that the additional factors of production are supplied exogenously. Market clearing in the factor markets requires

$$(10) \quad K_k^s = \sum_{i=1}^J K_{ik}.$$

2.4. Equilibrium

Given exogenous variables $\left\{ \{A_i\}_{i=1}^J, \{H_o\}_{o=1}^O, \{K_k^s\}_{k=1}^K \right\}$, the equilibrium is a collection of $4J + 2J^2 + 3OJ + JK + 4O + K$ endogenous variables

$$\left\{ \{p_i, y_i, \{x_{ij}, \varepsilon_{x_{ij}}^{f_i}\}_{j=1}^J, c_i, \varepsilon_{c_i}^D, \{N_{io}, \varepsilon_{N_{io}}^{f_i}, l_{io}^d\}_{o=1}^O, \{K_{ik}\}_{k=1}^K\}_{i=1}^J, \{\theta_o, w_o, L_o^d, L_o^s\}_{o=1}^O, \{r_k\}_{k=1}^K \right\}$$

that satisfy equations (1), (2), (3), (4), (5), (6), (7), (8), (9), and (10) for all sectors and occupations, along with constant returns to scale in final goods production

$$(11) \quad \sum_{i=1}^J \varepsilon_{c_i}^D = 1,$$

constant returns to scale in sectoral production

$$(12) \quad \sum_{j=1}^J \varepsilon_{x_{ij}}^{f_i} + \sum_{o=1}^O \varepsilon_{N_{io}}^{f_i} + \sum_{k=1}^K \varepsilon_{K_{ik}}^{f_i} = 1 \quad \forall i \in \{1, 2, \dots, J\}$$

and goods market clearing

$$(13) \quad y_i = c_i + \sum_{j=1}^J x_{ij} \quad \forall i \in \{1, 2, \dots, J\}.$$

Equations (1)-(13) provide $4J + J^2 + 2J\mathcal{O} + \mathcal{K}J + 3\mathcal{O} + \mathcal{K} + 1$ restrictions. To close the model, we must make an assumption about the wage schedule, providing ~~an~~ additional \mathcal{O} restrictions, and functional form assumptions for final and sectoral production.

3. Theoretical Results

In this section, we describe the main theoretical results of our paper. We first derive the first order propagation of technology, labor supply, and factor supply shocks at the dis-aggregated occupation and sector level. Specifically, we compute the first order responses of the main endogenous variables of interest in the model. We then present our aggregation theorem of idiosyncratic technology, labor supply, and factor supply shocks. The aggregate impact of idiosyncratic shocks can be summarized by sales shares and matching elasticities. We assume Cobb-Douglas production throughout. We show how to generalize the results to any constant returns production function in the case of one occupation per sector in appendix B.

3.1. Notation

Before outlining our results, we define some useful notation. First, as is standard in the networks literature, we define the input-matrix Ω as the matrix containing sector level input revenue shares

$$\Omega = \begin{bmatrix} \frac{p_1 x_{11}}{p_1 y_1} & \dots & \frac{p_J x_{1J}}{p_1 y_1} \\ \vdots & \ddots & \vdots \\ \frac{p_1 x_{J1}}{p_J y_J} & \dots & \frac{p_J x_{JJ}}{p_J y_J} \end{bmatrix}_{J \times J}.$$

In our competitive goods market equilibrium setup, we can rewrite this input output matrix in terms of the elasticities of the sector level production functions. (See Equa-

tion 6)

$$\Omega = \begin{bmatrix} \varepsilon_{x_{11}}^{f_1} & \dots & \varepsilon_{x_{1J}}^{f_1} \\ \vdots & \ddots & \vdots \\ \varepsilon_{x_{J1}}^{f_J} & \dots & \varepsilon_{x_{JJ}}^{f_J} \end{bmatrix}_{J \times J} .$$

We denote the Leontief inverse by $\Psi = (\mathbf{I} - \Omega)^{-1}$. The Leontief inverse captures the importance of each sector as a direct and indirect input into production in every other sector.

Alongside the production elasticities to intermediate inputs, our model features a second set of production elasticities: elasticities to the different types of labor inputs. We collect these elasticities in the matrix ε_N^f .

$$\varepsilon_N^f = \begin{bmatrix} \varepsilon_{N_{11}}^{f_1} & \dots & \varepsilon_{N_{10}}^{f_1} \\ \vdots & \ddots & \vdots \\ \varepsilon_{N_{J1}}^{f_J} & \dots & \varepsilon_{J0}^{f_J} \end{bmatrix}_{J \times 0} .$$

Equation (7) demonstrates that this matrix is the labor input equivalent of the standard input-output matrix. In equilibrium, each entry is the revenue share of type o labor.

Similarly, we collect the factor elasticities in the matrix ε_K^f .

$$\varepsilon_K^f = \begin{bmatrix} \varepsilon_{K_{11}}^{f_1} & \dots & \varepsilon_{K_{1K}}^{f_1} \\ \vdots & \ddots & \vdots \\ \varepsilon_{K_{J1}}^{f_J} & \dots & \varepsilon_{K_{JK}}^{f_J} \end{bmatrix}_{J \times K} .$$

Equation (8) demonstrates that this matrix is the factor input equivalent of the standard input-output matrix. In equilibrium, each entry is the revenue share of factor k .

\mathcal{L} is a $O \times J$ with the share of occupation o workers employed in each sector along the rows:

$$\mathcal{L} = \begin{bmatrix} \frac{l_{11}}{L_1} & \dots & \frac{l_{J1}}{L_1} \\ \vdots & \ddots & \vdots \\ \frac{l_{1O}}{L_O} & \dots & \frac{l_{JO}}{L_O} \end{bmatrix}_{O \times J} .$$

\mathcal{F} , \mathcal{Q} , and \mathcal{T} are $O \times O$ diagonal matrices with the occupation level job-finding rate,

vacancy-filling rate, and recruiter-producer ratio along the diagonal.

$$\mathcal{F} = \begin{bmatrix} \varepsilon_{\theta_1}^{\mathcal{F}_1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \varepsilon_{\theta_O}^{\mathcal{F}_O} \end{bmatrix}_{O \times O}, \quad \mathcal{Q} = \begin{bmatrix} \varepsilon_{\theta_1}^{\mathcal{Q}_1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \varepsilon_{\theta_O}^{\mathcal{Q}_O} \end{bmatrix}_{O \times O}, \quad \mathcal{T} = \begin{bmatrix} \tau_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \tau_O \end{bmatrix}_{O \times O}.$$

Assuming a Cobb-Douglas matching function $m_o(U, V) = \phi_o U^{\eta_o} V^{1-\eta_o}$, these matrices are

$$\mathcal{F} = \begin{bmatrix} 1-\eta_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 1-\eta_O \end{bmatrix}, \quad \mathcal{Q} = \begin{bmatrix} -\eta_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & -\eta_O \end{bmatrix}.$$

3.2. The Propagation of Technology and Labor Force Shocks

We are interested in how three sets of endogenous variables—sector level relative prices, sector level output, and occupation level tightness—change in response to changes in technology and the size of the labor force. With these variables, we can compute the aggregate variables of interest: output and unemployment.

Before deriving our results, we need to specify how wages are determined. In matching models, workers and firms meet in a situation of bilateral monopoly. The resulting mutual gains from trade mean that wages are not determined by equilibrium conditions of the model², and must instead be pinned down by some wage setting norm chosen by the researcher. We introduce the following reduced-form relationship between relative wages and the shocks:

$$d \log \mathbf{w} - \mathcal{L} d \log \mathbf{p} = \boldsymbol{\Lambda}_A d \log \mathbf{A} + \boldsymbol{\Lambda}_H d \log \mathbf{H} + \boldsymbol{\Lambda}_K d \log \mathbf{K}^S,$$

where $d \log \mathbf{w}$ and $d \log \mathbf{H}$ are $O \times 1$ dimensional vectors capturing first order changes in wages and the size of the labor force, $d \log \mathbf{p}$ and $d \log \mathbf{A}$ are $J \times 1$ dimensional vectors capturing first order changes in prices and productivities, and $d \log \mathbf{K}^S$ is a $K \times 1$ dimensional vector of first order changes in factor supply. \mathcal{L} is a $O \times J$ matrix with the share of occupation o workers employed in each sector along the rows. The (o, i) -th entry

²Wages are only constrained to fall within a range where both workers and firms benefit from the match. However, this range can be wide since workers usually strongly prefer employment to unemployment and finding a new match is costly for firms.

is $\frac{l_{io}}{L_o}$. Λ_A , Λ_H , and Λ_K are $\mathcal{O} \times J$, $\mathcal{O} \times \mathcal{O}$, and $\mathcal{O} \times \mathcal{K}$ coefficient matrices that capture how wages respond to technology, labor force, and factor supply shocks in each other sector, occupation, and factor market. For instance, the (o, i) -th entry of Λ_A captures how wages in occupation o respond to technology shocks in sector i . The (o, j) -th entry in Λ_H captures how wages in occupation o respond to labor force shocks in occupation j . And the (o, k) -th entry in Λ_K captures how wages in occupation o respond to a supply shock to factor k .

Since, as we show below, relative prices are themselves determined by shocks to technology, the size of the labor force, and factor supply this assumption captures the intuition that, ultimately, wages must be driven by the fundamental shocks in our model: \mathbf{A} , \mathbf{H} and \mathbf{K}^s . While it may appear restrictive, this reduced form wage setting equation is capable of nesting many assumptions about wage setting tied to economic fundamentals for the right parameter matrices Λ_A , Λ_H , and Λ_K .

With changes in wages in hand, we can now derive how shocks propagate to labor market tightness, output, and unemployment. These first order effects are summarized in proposition 1. For a detailed derivation see appendix A.1.

PROPOSITION 1. *Assume intermediate and final goods production functions are Cobb-Douglas. Given labor supply shocks $d \log \mathbf{H} = [d \log H_1, d \log H_2, \dots, d \log H_{\mathcal{O}}]'$, productivity shocks $d \log \mathbf{A} = [d \log A_1, d \log A_2, \dots, d \log A_J]'$, and factor supply shocks $d \log \mathbf{K}^s = [d \log K_1^s, d \log K_2^s, \dots, d \log K_{\mathcal{K}}^s]'$ the first-order responses of labor market tightness $d \log \Theta = [d \log \theta_1, d \log \theta_2, \dots, d \log \theta_{\mathcal{O}}]'$ and output $d \log \mathbf{y} = [d \log y_1, d \log y_2, \dots, d \log y_J]'$ follow:*

$$d \log \Theta = \Pi_{\theta,A} d \log \mathbf{A} + \Pi_{\theta,H} d \log \mathbf{H} + \Pi_{\theta,K} d \log \mathbf{K}^s,$$

$$d \log \mathbf{y} = \Pi_{y,A} d \log \mathbf{A} + \Pi_{y,H} d \log \mathbf{H} + \Pi_{y,K} d \log \mathbf{K}^s,$$

where

$$\begin{aligned} \Pi_{\theta,A} &= [\mathcal{F} - \Xi_{\theta}]^{-1} (\mathcal{L}\Psi - \Lambda_A), \\ \Pi_{\theta,H} &= [\mathcal{F} - \Xi_{\theta}]^{-1} (\mathcal{L}\Psi \varepsilon_N^f - \mathbf{I} - \Lambda_H), \\ \Pi_{\theta,K} &= [\mathcal{F} - \Xi_{\theta}]^{-1} (\mathcal{L}\Psi \varepsilon_K^f - \Lambda_K), \\ \Pi_{y,A} &= \Psi \left[\mathbf{I} + \varepsilon_N^f (\mathcal{F} + \Omega\mathcal{T}) [\mathcal{F} - \Xi_{\theta}]^{-1} [\mathcal{L}\Psi - \Lambda_A] \right], \\ \Pi_{y,H} &= \Psi \varepsilon_N^f \left[\mathbf{I} + (\mathcal{F} + \Omega\mathcal{T}) [\mathcal{F} - \Xi_{\theta}]^{-1} [\mathcal{L}\Psi \varepsilon_N^f - \mathbf{I} - \Lambda_H] \right], \end{aligned}$$

$$\begin{aligned}\Pi_{y,K} &= \Psi \left[\varepsilon_K^f + \varepsilon_N^f (\mathcal{F} + \Omega \mathcal{T}) [\mathcal{F} - \Xi_\theta]^{-1} [\mathcal{L} \Psi \varepsilon_K^f - \Lambda_K] \right], \\ \Xi_\theta &= \mathcal{L} \Psi \varepsilon_N^f [\mathcal{F} + \Omega \mathcal{T}].\end{aligned}$$

Finally, the expression of labor supply implies that first order changes in unemployment $d \log \mathbf{U} = [d \log U_1, d \log U_2, \dots, d \log U_O]$ follow

$$d \log \mathbf{U} = \mathcal{F} d \log \theta.$$

PROOF. See appendix A.1. □

Though the expressions might seem daunting at first, we unpack the intuition behind each in turn. First, we consider the response in tightness. The equilibrium response in tightness is jointly determined by changes in labor supply and labor demand. The labor supply side is impacted directly by changes in the size of ~~the~~ labor force and job-finding rates, and the labor demand side is impacted by vacancy-filling rates, wages and prices, as well as productivity. A technology shock affects labor demand by directly altering a sector's productive capability, thus impacting prices, output, and labor usage for other sectors through production linkages. A shock to the size of the labor force impacts labor supply by changing the number of available workers, which changes sectors' production through changing the number of recruiters needed. Changes in sectors' production propagate through the network and induce other sectors to change their labor demand, thus causing tightness to adjust, so on and so forth.

Second, we consider the response in sectoral output. Sectoral output is directly impacted by changes in technology, output in other sectors, and labor market tightness. A shock to technology directly impacts a sector's productive capacity through the production function. It also indirectly impacts a sector's production through the tightness channel, which captures the equilibrium interaction between labor supply and labor demand. Such effects on the sectors' production then propagate to other sectors through production linkages, and to different labor markets through the labor network.

PROPOSITION 2. *Assume intermediate and final goods production functions are Cobb-Douglas. The first-order responses of relative sectoral and factor prices are pinned down by labor supply, technology, and factor supply shocks up to a numeraire. The first order responses in sectoral prices satisfy*

$$(\mathbf{I} - \Psi \varepsilon_N^f \mathcal{L}) d \log \mathbf{p} = \boldsymbol{\Pi}_{p,A} d \log \mathbf{A} + \boldsymbol{\Pi}_{p,H} d \log \mathbf{H} + \boldsymbol{\Pi}_{p,K} d \log \mathbf{K}^s + \Psi \varepsilon_K^f d \log \mathbf{r},$$

and

$$\begin{aligned}\Pi_{p,A} &= \Psi \left[\varepsilon_N^f \left(\Lambda_A - Q\mathcal{T} [\mathcal{F} - \Xi_\theta]^{-1} (\mathcal{L}\Psi - \Lambda_A) \right) - I \right], \\ \Pi_{p,H} &= \Psi \left[\varepsilon_N^f \left(\Lambda_H - Q\mathcal{T} [\mathcal{F} - \Xi_\theta]^{-1} (\mathcal{L}\Psi \varepsilon_N^f - I - \Lambda_H) \right) \right], \\ \Pi_{p,K} &= \Psi \left[\varepsilon_N^f \left(\Lambda_K - Q\mathcal{T} [\mathcal{F} - \Xi_\theta]^{-1} (\mathcal{L}\Psi \varepsilon_K^f - \Lambda_K) \right) \right].\end{aligned}$$

The first order responses in fixed factor prices satisfy

$$d \log \mathbf{r} = \mathbf{1} d \log y_{num} - d \log \mathbf{K}^s,$$

where $d \log y_{num}$ denotes the change in output in the numeraire sector, which is determined as per proposition 1.

PROOF. See appendix A.1. □

In our framework, relative prices are determined by market clearing in a perfectly competitive market. Given that our production functions are Cobb-Douglas, the price of a good produced by a particular sector responds to changes in prices in all other sectors, as well as the effective cost of employing workers, which relies on labor market tightness and wages. In addition, the price is also directly linked to the level of productivity in that sector. Thus, a productivity shock impacts the system of prices directly through production and indirectly through adjustments in tightness, and a labor supply shock impacts prices solely through adjustments in labor market tightness.

Therefore, if one looks closely at the propagation equations, one can see a separation between what is propagated through solely the production network, and what is propagated through the interaction between the production network, the labor usage matrix, and the matching elasticities.

3.3. Aggregation

With a Cobb-Douglas preference, it is straightforward that $d \log Y = \varepsilon_c^D d \log \mathbf{y}$. Using Proposition 1, we arrive at the following result for first-order changes in aggregate output.

THEOREM 1. *Given idiosyncratic labor supply shocks $d \log \mathbf{H}$ and productivity shocks $d \log \mathbf{A}$, the log change in real GDP is:*

$$d \log Y = \Pi_A d \log \mathbf{A} + \Pi_H d \log \mathbf{H} + \Pi_K d \log \mathbf{K}^s,$$

where

$$\begin{aligned}\Pi_A &= \lambda' \left(\mathbf{I} + \varepsilon_N^f (\mathcal{F} + \mathcal{Q}\mathcal{T}) [\mathcal{F} - \boldsymbol{\Xi}_\theta]^{-1} (\mathcal{L}\Psi - \boldsymbol{\Lambda}_A) \right), \\ \Pi_H &= \lambda' \varepsilon_N^f \left(\mathbf{I} + (\mathcal{F} + \mathcal{Q}\mathcal{T}) [\mathcal{F} - \boldsymbol{\Xi}_\theta]^{-1} \left([\mathcal{L}\Psi \varepsilon_N^f - \mathbf{I}] - \boldsymbol{\Lambda}_H \right) \right), \\ \Pi_K &= \lambda' \left(\varepsilon_K^f + \varepsilon_N^f (\mathcal{F} + \mathcal{Q}\mathcal{T}) [\mathcal{F} - \boldsymbol{\Xi}_\theta]^{-1} (\mathcal{L}\Psi \varepsilon_K^f - \boldsymbol{\Lambda}_K) \right)\end{aligned}$$

and $\boldsymbol{\lambda} = \Psi' \varepsilon_c^D$ denotes the sectors' sales shares.

PROOF. The result follows directly from $d \log Y = \varepsilon_c^D d \log \mathbf{y}$ and proposition 1. \square

Theorem 1 results from Proposition 1, as we can compute the change in real GDP by weighing the change in real output for all sectors by the household's demand elasticities. Note that the matrix product between the Leontief inverse and the demand elasticities is equal to the sales shares. This property results directly from household's maximization problem, the firms' profit maximization decision, goods market clearing, and Cobb-Douglas production functions. Therefore, the aggregate impact of productivity and labor supply shocks can be summarized as the sales-share-weighted impact of these shocks on sectoral output directly through production and indirectly through labor markets.

This result, that the aggregate impact of shocks is the sales share weighted sum of sector specific impacts, is reminiscent of Hulten's theorem (Hulten 1978). Indeed, when network price adjusted wages change exactly proportionally to the marginal product of labor, Hulten's theorem holds for technology shocks.

THEOREM 2. *Hulten's theorem holds for technology shocks whenever network price adjusted wages change exactly proportionally to the network adjusted marginal product of labor. That is when*

$$d \log \mathbf{w} - \mathcal{L} d \log \mathbf{p} = \mathcal{L} d \log MP$$

Where $d \log MP$ is a matrix of changes to the marginal product of each type of labor in each sector. Furthermore, when this condition holds, aggregate changes in response to labor force and factor supply shocks are independent of matching frictions.

PROOF. The derivative of production with respect to labor inputs, along with the firms first order conditions, imply that the network adjusted marginal product of labor satis-

fies

$$\mathcal{L}d \log MP = -\mathcal{T}\Omega d \log \theta + d \log \mathbf{w} - \mathcal{L}d \log \mathbf{p}.$$

Imposing that network price adjusted wage changes are exactly proportional to changes in the network adjusted marginal product of labor implies

$$-\mathcal{T}\Omega d \log \theta = 0.$$

$\mathcal{T}\Omega$ is a diagonal matrix with non-zero diagonal elements, therefore

$$d \log \theta = 0.$$

Since search-and-matching operates through changes in $d \log \theta$, this implies that search-and-matching has no impact on the propagation of shocks. In particular, in this case

$$d \log Y = \lambda' \left[d \log A + \varepsilon_N^f d \log H + \varepsilon_K^f d \log K^s \right]$$

The aggregate output response to technology shocks is $\lambda' d \log A$, this is exactly Hulten's theorem. The aggregate output response to the other shocks depends only on production parameters, matching frictions play no role. \square

COROLLARY 1. *When wages do not respond exactly proportionally to the network adjusted marginal product of labor, matching frictions generate deviations from Hulten's theorem captured by*

$$\begin{aligned} \Pi_{search,A} &= \lambda' \varepsilon_N^f (\mathcal{F} + \Omega \mathcal{T}) [\mathcal{F} - \Xi_\theta]^{-1} (\mathcal{L}\Psi - \Lambda_A), \\ \Pi_{search,H} &= \lambda' \varepsilon_N^f (\mathcal{F} + \Omega \mathcal{T}) [\mathcal{F} - \Xi_\theta]^{-1} \left([\mathcal{L}\Psi \varepsilon_N^f - I] - \Lambda_H \right), \\ \Pi_{search,K} &= \lambda' \varepsilon_N^f (\mathcal{F} + \Omega \mathcal{T}) [\mathcal{F} - \Xi_\theta]^{-1} \left(\mathcal{L}\Psi \varepsilon_K^f - \Lambda_K \right). \end{aligned}$$

PROOF. Follows directly from Theorem 1 and Theorem 2. \square

In general, putting a sign on these additional impacts is a quantitative question, and the direction of amplification versus dampening depends on how labor market tightness responds. Tightness, as an equilibrium object, responds to shocks in a complicated way that depends on how each sector uses different types of labor, production linkages, relative wage rigidity, and matching elasticities. In subsection 3.4, we use two simple examples to demonstrate when amplification and dampening can occur.

Finally, we are interested in the response of aggregate unemployment to sector specific shocks. Using the results in Theorem 1, we can derive changes in aggregate unemployment and relative prices.

COROLLARY 2. *Given idiosyncratic labor supply shocks $d \log \mathbf{H}$ and productivity shocks $d \log \mathbf{A}$, the first-order response in aggregate unemployment is:*

$$d \log U^{agg} = \Pi_{U^{agg}, A} d \log \mathbf{A} + \Pi_{U^{agg}, H} d \log \mathbf{H},$$

where

$$\begin{aligned}\Pi_{U^{agg}, A} &= \frac{1}{U^{agg}} \mathbf{U}' [\boldsymbol{\Lambda}_A - \mathcal{L} \boldsymbol{\Pi}_{y, A}], \\ \Pi_{U^{agg}, H} &= \frac{1}{U^{agg}} \mathbf{U}' [\mathbf{I} + \boldsymbol{\Lambda}_H - \mathcal{L} \boldsymbol{\Pi}_{y, H}].\end{aligned}$$

The first-order change in the relative price of the aggregate consumption good is:

$$d \log P = \varepsilon_{\mathbf{c}}^{\mathcal{D}'} d \log \mathbf{p},$$

where $d \log \mathbf{p}$ follows proposition 2.

PROOF. See appendix A.2 □

The change in unemployment is a weighted average of the first-order changes in the size of the labor force and employment. The employment level change for each labor market in equilibrium equates the change in labor demand in that labor market to labor supply. The change in labor demand in that labor market depends on how relative wages respond to shocks, as well as how sectoral output changes for sectors that use that particular type of labor.

The first-order response in aggregate price level is more straightforward, and is simply the changes in sectoral prices weighted by their consumption share, which is equivalent to the demand elasticities of consumption when preferences are Cobb-Douglas. Note that one can solve out the price changes explicitly by picking a numeraire, for example, by assuming $d \log p_1 = 0$. However, we don't explicitly do it here in order to maintain a simpler expression for the relationship between the price system and productivity and labor supply shocks.

3.4. Two Toy Examples

In this subsection, we provide two toy examples to demonstrate how the search channel impacts aggregation through the production network. Since it is now well understood how production linkages amplify sector level shocks, in subsubsection 3.4.1 we focus on the search and matching channel. To clarify the exposition, we abstract from the additional factors of production, and consider a model featuring only labor and intermediate inputs. This section explores how labor market frictions can amplify or dampen the impact of idiosyncratic shocks by assuming no production linkages. Subsection 3.4.2 presents a simple vertical production network model to showcase how production linkages interact with search frictions.

3.4.1. A Model with no Production Linkage

To get an intuition for how search and matching frictions can potentially impact the response of aggregate, we first examine what happens when there are no production linkages between the sectors. This is equivalent to assuming $\Omega = \mathbf{0}$ and thus $\Psi = \mathbf{I}$. Additionally, for simplicity, we assume each sector operates its own labor market, which means $L = \varepsilon_N^f = \mathbf{I}$. In other words, we consider the case where each sector produces using linear technology in its own unique type of labor. From Theorem 1, this gives rise to the following output aggregation formula:

$$d \log Y = \Pi_A d \log A + \Pi_H d \log H,$$

where

$$\begin{aligned}\Pi_A &= \lambda' \left(\mathbf{I} + (\mathcal{F} + \Omega \mathcal{T}) [-\Omega \mathcal{T}]^{-1} (\mathbf{I} - \Lambda_A) \right), \\ \Pi_H &= \lambda' \left(\mathbf{I} - (\mathcal{F} + \Omega \mathcal{T}) [-\Omega \mathcal{T}]^{-1} \Lambda_H \right).\end{aligned}$$

Since \mathcal{F} , Ω , and \mathcal{T} are diagonal matrices, we have that:

$$(\mathcal{F} + \Omega \mathcal{T}) [-\Omega \mathcal{T}]^{-1} = \begin{pmatrix} \frac{1-\eta_1(1+\tau_1)}{\eta_1\tau_1} & 0 & \dots & 0 \\ 0 & \frac{1-\eta_2(1+\tau_2)}{\eta_2\tau_2} & \dots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & 0 & \dots & \frac{1-\eta_J(1+\tau_J)}{\eta_J\tau_J} \end{pmatrix}$$

Further, if we assume the relative wage in a sector respond only to shocks to that

sector, we have that:

$$\Lambda_A = \begin{pmatrix} \alpha_1 & 0 & \dots & 0 \\ 0 & \alpha_2 & \dots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & 0 & \dots & \alpha_J \end{pmatrix}, \quad \Lambda_H = \begin{pmatrix} -\beta_1 & 0 & \dots & 0 \\ 0 & -\beta_2 & \dots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & 0 & \dots & -\beta_J \end{pmatrix}.$$

We assume relative wages respond non-negatively to positive technology shocks, and negatively to positive labor supply shocks, that is $\alpha_i, \beta_i \geq 0$ for all i .

Therefore, the additional impact of technology shocks from search-and-matching becomes:

$$\Pi_{\text{search}, A} = \begin{pmatrix} (1 - \alpha_1) \frac{1 - \eta_1(1 + \tau_1)}{\eta_1 \tau_1} & 0 & \dots & 0 \\ 0 & (1 - \alpha_2) \frac{1 - \eta_2(1 + \tau_2)}{\eta_2 \tau_2} & \dots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & 0 & \dots & (1 - \alpha_J) \frac{1 - \eta_J(1 + \tau_J)}{\eta_J \tau_J} \end{pmatrix}.$$

Since labor market frictions operate through changes in productive employment, N_i , they cannot matter for shock propagation unless productive employment changes in response to the shocks. As a result, when $(1 - \alpha_i) \frac{1 - \eta_i(1 + \tau_i)}{\eta_i \tau_i} = 0$, search-and-matching becomes irrelevant for productivity shock propagation. This occurs when $\alpha_i = 1$ or when $1 - \eta_i(1 + \tau_i) = 0$. When $\alpha_i = 1$, the change in wages is equal to the change in the marginal product of sector i labor. Therefore, labor demand and employment remain unchanged. All of the effect of the productivity shock is absorbed into changes in wages rather than changes in employment.

$1 - \eta_i(1 + \tau_i)$ is the elasticity of the productive workforce in sector i to sector i vacancies. When $1 - \eta_i(1 + \tau_i) = 0$, posting additional vacancies does not change the number of productive workers. As a result, even if firms change the number of vacancy postings in response to productivity shocks, these changes do not have first order effects on output because they do not have first order effects on the number of productive workers.

In both cases, productive employment does not respond to changes in productivity and so search-and-matching frictions play no role in the propagation of shocks. Whenever employment does change following productivity shocks search-and-matching frictions matter.

For instance, if wages change by less than the marginal product of labor following

productivity shocks, $\alpha_i < 1$, and increasing vacancy postings increases the number of productive workers, $1 < \eta_i(1 + \tau_i)$, then employment will rise after a positive productivity shock. The resulting rise in employment boosts output further than the pure effect of the productivity shock alone. In this case, search-and-matching amplifies the effects of productivity shocks.

Similarly, when $(1 - \alpha_i) \frac{1 - \eta_i(1 + \tau_i)}{\eta_i \tau_i} < 0$, search-and-matching dampens the aggregate effect of technology shocks in sector i . In this case, the effect ~~live~~^{cost} of labor rises relative to the marginal product of labor following a positive productivity shock. The resulting decline in employment dampens the output response.

In this simple example, with linear production and one unique type of labor per sector, the effect of productivity shocks on the marginal product of labor is clear. In our full model, the effects of productivity shocks in sector i on the marginal product of type o labor is a much more complicated object. However, the intuition remains the same. If wages adjust by less than the marginal product of labor and posting additional vacancies increases the number of productive workers, then labor market frictions will amplify the effects of technology shocks. While in this example the effects remain isolated in the shocked industry i , with a full network of production linkages, the amplification would propagate to sectors that use i 's good as an input, and from those sectors to other sectors, and so on.

We can gain similar insights about the amplification of labor force shocks through the following expression:

$$\Pi_{\text{search}, H} = \begin{pmatrix} \beta_1 \frac{1 - \eta_1(1 + \tau_1)}{\eta_1 \tau_1} & 0 & \dots & 0 \\ 0 & \beta_2 \frac{1 - \eta_2(1 + \tau_2)}{\eta_2 \tau_2} & \dots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & 0 & \dots & \beta_J \frac{1 - \eta_J(1 + \tau_J)}{\eta_J \tau_J} \end{pmatrix}.$$

Again, in order for labor market frictions to change the way labor force shocks propagate, the labor market frictions must alter how productive employment changes in response to the shocks. The interpretation of the $1 - \eta_i(1 + \tau_i)$ term is the same as above, so we focus on the β_i 's. Whenever $\beta_i = 0$, wages do not respond to labor force shocks. As a result, labor demand and, therefore, productive employment do not change.

As long as increasing the number of vacancy postings increases the number of productive workers, if labor becomes cheaper when there is a positive shock to the size of the labor force ($\beta_i > 0$), employment increases in response to the labor force shock.

This increase boosts output, amplifying the effect of the labor force shock. Conversely, if labor becomes more expensive when there is a positive shock to the labor force ($\beta_i < 0$), employment falls in response to the labor force shock. This reduces output, dampening the effect of the labor force shock. Again, this intuition carries through to our full model.

3.4.2. A Two-Sector Vertical Production Network

The example above unpacks how search-and-matching can amplify or dampen shocks through wage responsiveness and the effect of increased vacancy postings on the number of productive workers. It doesn't, however, speak to how labor market propagation compounds with production linkages. The simple example below is designed to highlight how production linkages interact with labor market frictions.

Consider an economy with two sectors, a downstream sector and an upstream sector, that produce using the same type of labor:

$$y_1 = A_1 N_{11}, \\ y_2 = A_2 y_1^\gamma N_{21}^{1-\gamma}.$$

With this setup, we have that:

$$\begin{aligned} \varepsilon_N^f &= \begin{bmatrix} 1 \\ 1-\gamma \end{bmatrix}, & \Omega &= \begin{bmatrix} 0 & 0 \\ 1-\gamma & 0 \end{bmatrix}, & \Psi &= \begin{bmatrix} 1 & 0 \\ 1-\gamma & 1 \end{bmatrix} \\ \mathcal{F} &= 1 - \eta, & Q &= -\eta, & \mathcal{L} &= \begin{bmatrix} \gamma & 1-\gamma \end{bmatrix}, \\ \varepsilon_c^D &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, & \lambda &= \begin{bmatrix} 1-\gamma \\ 1 \end{bmatrix}. \end{aligned}$$

This implies that:

$$(\mathcal{F} + \Omega \mathcal{T}) [\mathcal{F} - \Xi_\theta]^{-1} = \frac{1 - \eta(1 + \tau)}{1 - \eta - [\gamma + 2(1 - \gamma)^2] (1 - \eta(1 + \tau))}.$$

First, we see that the recruiter-producer ratio that we uncover in subsubsection 3.4.1 is still at play in the numerator, that is, with a large enough recruiter-producer ratio, the direction of the impact of search and matching can be reversed. In addition, in the denominator, we now see that the production structure enters the search-and-matching part of output aggregation. Specifically, the production structure dictates how sectors

does $\mu=0$ mean sector 2 has no link to sector 1?

use labor input from different labor markets, as well as how labor demand across sectors comove. However, the direction through which the production structure impacts output aggregation is ambiguous even in this simple example. For example, with $\gamma = 0$, the denominator becomes $\eta(1 + 2\tau) - 1$, which is negative for reasonable values of η and τ . On the other hand, with $\gamma = 0.75$, the denominator becomes $1 - \eta - 0.875(1 - \eta(1 + \tau)) > 0$.

We further assume that

$$d \log w - \mathcal{L} d \log \mathbf{p} = \begin{bmatrix} \alpha_1 & \alpha_2 \end{bmatrix} \begin{bmatrix} d \log A_1 \\ d \log A_2 \end{bmatrix} + \beta_1 d \log H.$$

Applying Theorem 1, we have that:

$$\begin{aligned} \Pi_{\text{search},A} &= \frac{2(1-\gamma)(1-\eta(1+\tau))}{1-\eta - [\gamma + 2(1-\gamma)^2](1-\eta(1+\tau))} \begin{bmatrix} 1-\gamma + \gamma^2 - \alpha_1 & 1-\gamma - \alpha_2 \end{bmatrix}, \\ \Pi_{\text{search},H} &= \frac{2(1-\gamma)(1-\eta(1+\tau))}{1-\eta - [\gamma + 2(1-\gamma)^2](1-\eta(1+\tau))} \begin{pmatrix} 1-3\gamma + 2\gamma^2 - \beta_1 \end{pmatrix}. \end{aligned}$$

From this, we can see that the wage rigidity channel is still important in determining the direction and the magnitude of impacts of shock on output aggregation, with the additional insight that we are now comparing wage adjustments with network-adjusted values.

Additionally, we can examine how search frictions impact output for each of the sectors. Let $\Pi_{y,\text{search},X}$ denote the impact of search frictions on sectoral output from shocks to X, with X being either productivity or labor supply. We have that:

$$\Pi_{y,\text{search},A} = \frac{1-\eta(1+\tau)}{1-\eta - [\gamma + 2(1-\gamma)^2](1-\eta(1+\tau))} \begin{bmatrix} 1-\gamma + \gamma^2 - \alpha_1 & 1-\gamma - \alpha_2 \\ 2(1-\gamma)(1-\gamma + \gamma^2 - \alpha_1) & 2(1-\gamma)(1-\gamma - \alpha_2) \end{bmatrix}.$$

Compare to the efficient benchmark, in which the sectoral output response follows the Leontief inverse $\Psi = \begin{bmatrix} 1 & 0 \\ 1-\gamma & 1 \end{bmatrix}$, where the upper right element is 0, the sectoral output response to changes in labor market has a non-zero upper right element. What this means is that, in efficient production networks with Cobb-Douglas production function, shocks to a downstream sector does not impact production output of upstream sector. While this is usually resolved with the addition of preference shocks or CES production functions, we introduce an additional way of resolving this pitfall of classical production network models by incorporating frictional labor markets.

4. Calibration

So far, we have established that search frictions on the labor market can change the propagation of shocks in a network economy. In this section, we test the empirical relevance of the aforementioned channels by calibrating our model to the U.S. economy. We use two different labor market definitions to highlight our model's flexibility in allowing for segmentation in labor markets along several dimensions and to demonstrate the important role sector-to-sector worker transitions can play in shock propagation. The two assumptions about labor markets are

- (i) There is a unique occupation corresponding to each production sector. For instance, the durable manufacturing sector employs only durable manufacturing workers. This calibration has lower data requirements than using a more complex definition for occupations. As discussed below and in Appendix C, we can get all of the information we need to calibrate our model from JOLTs, the CPS, and the input-output accounts. Despite its simplicity, this calibration is able to generate interesting amplification and unemployment dynamics in response to technology shocks. However, because workers are constrained to stay within a single sector, this calibration leads to effects that remain more localized in the shocked sector.
- (ii) To address the limitations of the first calibration, we split the labor force into major occupation categories and allow firms in each sector to use a mix of the different major occupations in production. Workers are constrained to remain in one occupation, but because firms in different sectors use multiple occupations as an input, workers are no longer constrained to remain in just one sector. This allows for sector-to-sector labor transitions, a potentially important feature of the labor market in many countries. For instance, in the U.S. about 12% to 20% of jobs switchers also change industries ⁴ at the 1-digit level (Kambourov and Manovskii 2008; Parrado et al. 2007). Neffke et al. (2017) find that nearly 59% of German job switchers change industry at the most aggregate German industry grouping. Our major occupations calibration allows for some of this industry-to-industry flow of workers, but at the cost of higher data requirements.

We use productivity shocks to the durable manufacturing sector to demonstrate the quantitative importance of incorporating search frictions and the role wages play for the propagation of productivity shocks. In the one occupation per sector calibration,

we also use shocks to the size of the durable manufacturing workforce to demonstrate the propagation of labor force shocks. In the major occupations calibration, we instead shock the labor force of production workers, who make up the largest component of the durable manufacturing workforce, but work in many other sectors as well. We report responses to shocks to all other sectors in Appendix D.

4.1. Wage Assumptions

Wages play an important role in how shocks propagate in our model economy. In fact, as Theorem 2 shows, for the right assumption about wages, search frictions can have no effect whatsoever on shock propagation. To avoid taking a strong stance on exactly how wages change, we test the quantitative consequences of shocks in our model under a set of several different wage assumptions that cover a broad swathe of existing assumptions used in the literature.

Throughout this section, we report results for the following wage assumptions

- (i) The network price adjusted wage changes are the same as changes in the network adjusted marginal product of labor:

$$d \log \mathbf{w} - \mathcal{L} d \log \mathbf{p} = \mathcal{L} d \log \text{MP}.$$

Theorem 2 shows that Hulten's theorem holds for this assumption about wage changes. We therefore label this assumption "Hulten" in the figures below.

- (ii) The network price adjusted wage changes half as much as the network adjusted marginal product of labor:

$$d \log \mathbf{w} - \mathcal{L} d \log \mathbf{p} = 0.5 \mathcal{L} d \log \text{MP}.$$

In our setup, this is similar to assuming Nash bargaining with equal bargaining weights for firms and workers. We label this assumption "0.5MP" in the figures below.

- (iii) The network price adjusted wage does not change in response to shocks:

$$d \log \mathbf{w} - \mathcal{L} \delta d \log \mathbf{p} = 0.$$

This is akin to assuming real wages are rigid. We label this assumption "Rigid Real" in the figures below.

- (iv) The nominal wage does not change in response to shocks:

$$d \log \mathbf{w} = 0.$$

We label this assumption "Rigid Nominal" in the figures below.

- (v) Finally, as we will see below, in all the above cases search frictions amplify the effects of productivity shocks. To demonstrate the possibility that search frictions could in fact dampen the propagation of productivity shocks we use the, admittedly, somewhat unrealistic and manufactured assumption that changes in network price adjusted wages satisfy

$$d \log \mathbf{w} - \mathcal{L} d \log \mathbf{p} = 1.5 \mathcal{L} \Psi d \log \mathbf{A} + 0.5 \left[\mathcal{L} \Psi \epsilon_N^f - \mathbf{I} \right] d \log \mathbf{H} + 1.5 \mathcal{L} \Psi \epsilon_K^f d \log \mathbf{K}.$$

Corollary 1 demonstrates that this assumption about wages ensures that the additional shock propagation terms from search and matching enter negatively³. We label this assumption "Dampening" in the figures below.

4.2. Data Sources

Vacancies and Hires. We use vacancy and hiring data from The Job Openings and Labor Turnover Survey (JOLTs), which provides survey-based measures of job openings and hires at a monthly frequency. This survey data is available for 13 industries that roughly correspond to the two-digit NAICS classification. The data is available from Dec 2000 to Feb 2023 at a monthly frequency and from 2001 to 2022 at a yearly frequency.

Unemployment. We use sector level unemployment from the Current Population Survey (CPS). The data cover 13 sectors at a monthly frequency for the same range as the JOLTs data. We use the full monthly series, along with the JOLTs data, to estimate the sector level matching elasticity with respect to unemployment (η_i).

Major Occupations. We use occupation level data from the BLS's Occupational Employment and Wage Statistics(OEWS) to construct sector by sector major occupational employment and wages.

³Since $\mathcal{L} \Psi \epsilon_N^f - \mathbf{I}$ is a negative definite matrix, the coefficient of 0.5 ensures that the search and matching term enters negatively for shocks to H .

Input-output Linkages. Although JOLTs and the CPS include labor market data at a level that corresponds roughly to the two-digit NAICS classification, this correspondence is not exact. To construct input intensities that match with availability level of labor market data, we use the 2021 3-digit NAICS classification level Make and Use tables from the BEA, and aggregate back up to match the 13 CPS industries (Horowitz and Planting 2009). The BEA Make and Use tables allow us to calculate the intermediate input intensity of each sector, the labor intensity of each sector, and the elasticity of final production (demand) to each sectors output. We use employee compensation, recorded in the Use table to calculate labor elasticities.⁴

Factor Shares. In our calibration, we consider two factors: capital and energy. Capital and energy shares for the two-digit NAICS classification are available in the BEA-BLS Integrated Industry-level Production Accounts (KLEMS). We use an output-weighted mean of fixed factor shares of two-digit NAICS industries to compute the factor shares to match the corresponding CPS industries. We use these shares as energy and capital elasticities, and impose constant returns by scaling them along with the other production elasticities acquired from the input-output tables.

4.3. Effects of a 1% shock to productivity in the durable manufacturing sector

Figures 1A, 1C, and 1E show the response of output, tightness, and unemployment to a 1% shock to productivity in the durable manufacturing sector when there is one unique occupation per sector (no labor mobility across sectors). Figures 1B, 1D, and 1F show the response of output, tightness, and unemployment to the same shock assuming firms hire workers from major occupational groups (some labor mobility across sectors). The blue bars capture the effects of the productivity shock when wage changes are exactly proportional to the marginal product of labor. When this is the case, Hulten's theorem holds and labor market frictions do not impact shock propagation. The effects are therefore identical to those in a standard networks model with fully efficient labor markets. This provides a useful benchmark to which we can compare our results under different assumptions about wages.

When Hulten's theorem holds the 1% shock to productivity in durable manufacturing boosts output in the durable manufacturing sector by 1.40% and boosts aggregate output by 0.26%. Clearly, even standard production networks can amplify the effects of idiosyncratic sector level shocks and give⁴ these shocks a role in determining aggregate

⁴We provide more details on data and calibration in Appendix C.

output. When wages respond by less than the change in the marginal product of labor, labor market frictions can significantly boost the effects of the productivity shock on output. For instance, when the network price adjusted wage does not respond to the technology shock the effect on durable manufacturing output is 81% larger, at 2.54%, and the effect on aggregate output is 149% larger, at 0.65%, in the one occupation per sector case. When we allow sectors to use different major occupational labor types the own sector amplification of the productivity shock is smaller. In this case, assuming the network price adjusted wage does not change boosts the response of output in the durable manufacturing sector by just 24% to 1.75%. The amplification of aggregate output, however, remains substantial. Aggregate output responds 115% more than under Hulten's theorem, rising by 0.56%, demonstrating that our major occupation calibration increases the propagation of sector specific shocks to other sectors.

Without labor market frictions, productivity shocks have no effect on tightness or unemployment. Labor market frictions are therefore the key to understanding how unemployment changes in response to the productivity shocks. Assuming the network price adjusted wage does not respond to the productivity shock, the number of unemployed workers falls by 2.5% in the durable manufacturing sector, by 1.2% in the construction sector, and aggregate unemployment falls by 0.46%, when there is one occupation per sector. Assuming sectors can hire from multiple major occupations, the unemployment effects are more spread out, with the number of unemployed production workers falling by 0.99%, the number of unemployed engineers falling by 0.77%, and the number of unemployed construction workers falling by 0.70%. In aggregate, the number of unemployed workers falls by 0.44%.

Overall, this example demonstrates that incorporating labor market frictions can have quantitatively important implications for the effects of sector specific productivity shocks on output and allows us to explore their consequences for unemployment, which would not be possible in a standard networks model.

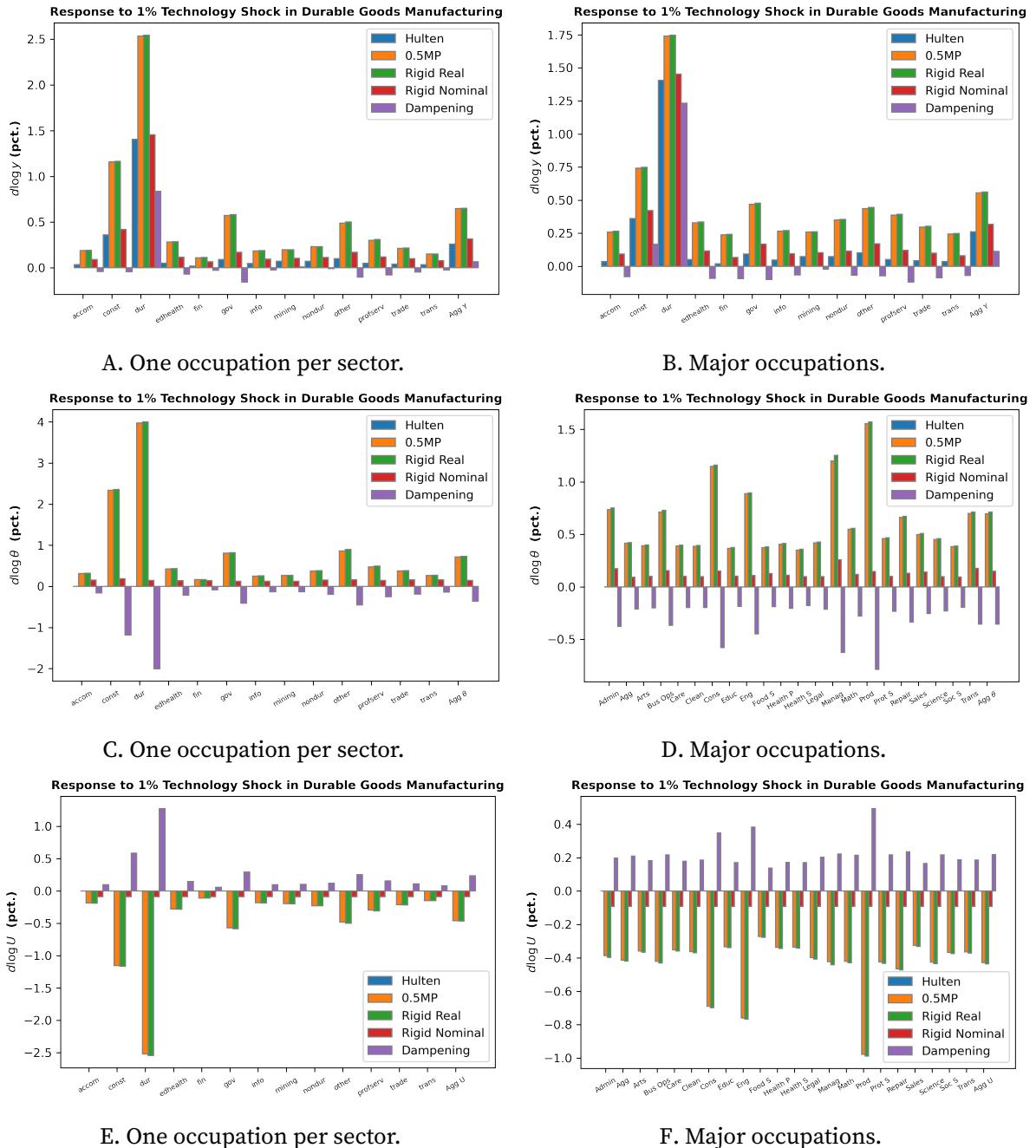


FIGURE 1. Response of output, tightness, and unemployment to a 1% shock to technology in the durable manufacturing sector.

4.4. Effects of a 1% shock to the size of the labor force

Figures 2A, 2C, and 2E show the response of output, tightness, and unemployment to a 1% shock to the size of the labor force in the durable manufacturing sector when

there is one unique occupation per sector. Figures Figures 2B, 2D, and 2F show the response of output, tightness, and unemployment to a 1% shock to the size of the labor force of production workers when firms can hire from a number of major occupational categories. There is no direct equivalent to shocking to a 1% durable manufacturing labor force shock in this setup. We choose to shock production workers since 46% of production workers work in durable manufacturing and they constitute the largest share of workers in the durable manufacturing sector. However, production workers also work in a number of other sectors: 26% of production workers work in non-durable manufacturing and 10% work in trade.

Without labor market frictions, firms are forced to hire all available workers and the increase the labor force therefore generates large changes in output. With labor market frictions, on the other hand, the effect on output depends on firms hiring decisions in response to wage changes. When wages do not change by much in response to the shock to the size of the labor force firms hiring does not change by much, and as a result output does not change by much. This is demonstrated by figures 2A and 2B. The blue bars show the response of output when there are no labor market frictions and the red bars, which are near imperceptible, show that output barely responds when the network price adjusted wages do not change at all in response to the shock.

Figure 2B also demonstrates that the propagation of the labor force shock is altered meaningfully by the production structure, and does not depend only on the labor usage matrix. The largest effects of the shock are concentrated in the two largest users of production workers, durable and non-durable manufacturing. But the change in output in construction and other services are both larger than the change in output in the trade sector even though trade uses 5 times more production workers than other services and 10 times more production workers than construction.

The bottom two panels of figure 2 demonstrate that, as was the case for productivity shocks, labor force shocks only change unemployment and tightness with labor market frictions and incomplete pass through of the shock to wages. As expected, the fact that our model does not allow workers to move across occupations keeps the effects on employment and tightness concentrated in the shocked occupation. However, since many sectors use production workers, if we aggregated the right side of figure 2 to the sector level, we would see changes across sectors in proportion to their usage of production workers. For instance, figure 3 gives a rough idea of changes in sector level tightness from a 1% shock to the production workers occupation, by aggregating up using sector level labor shares.

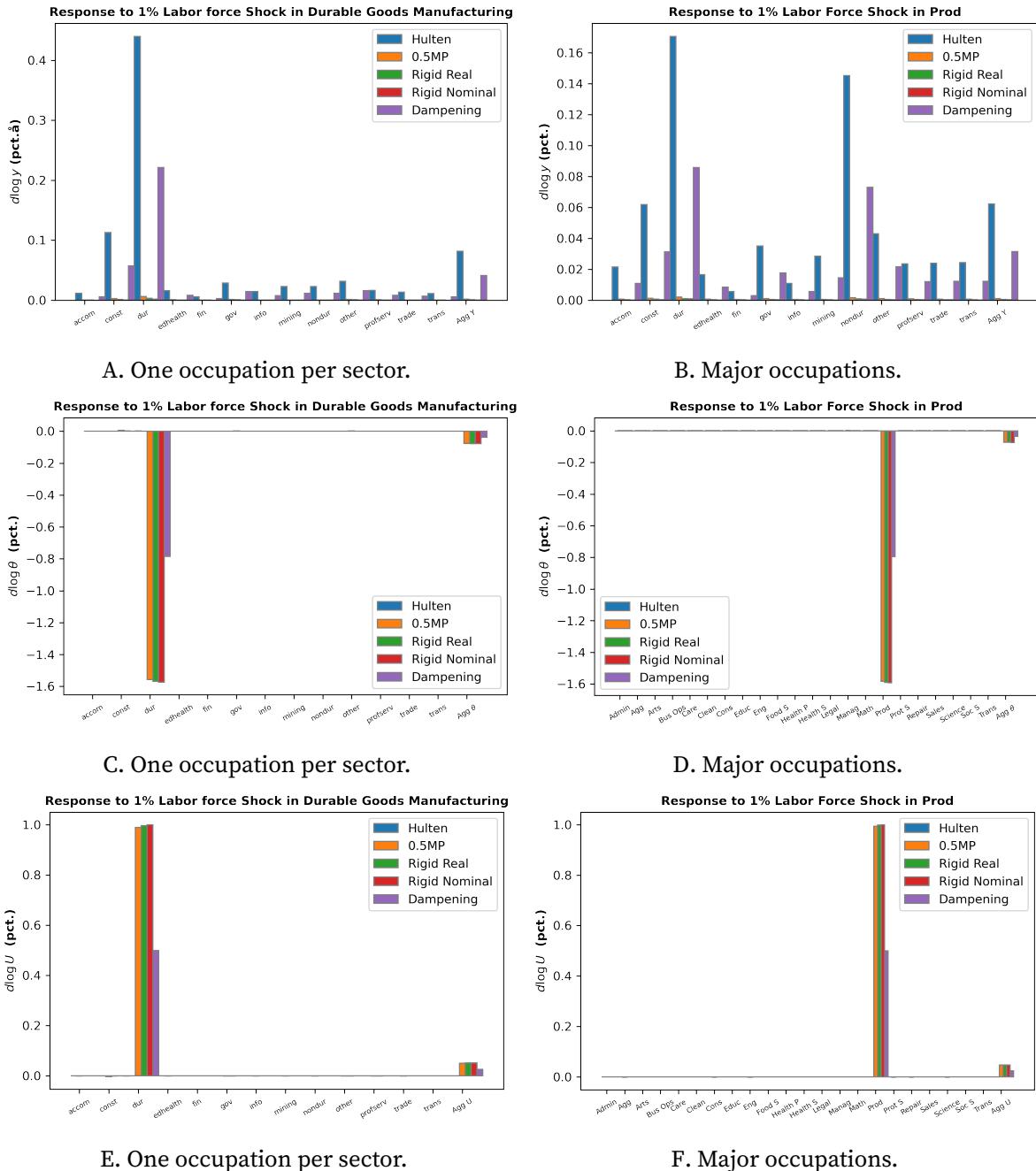


FIGURE 2. Response of output, tightness, and unemployment to a 1% shock to the size of the labor force. In the one output per sector case we shock the durable manufacturing labor force. In the major occupations case we shock the production labor force.

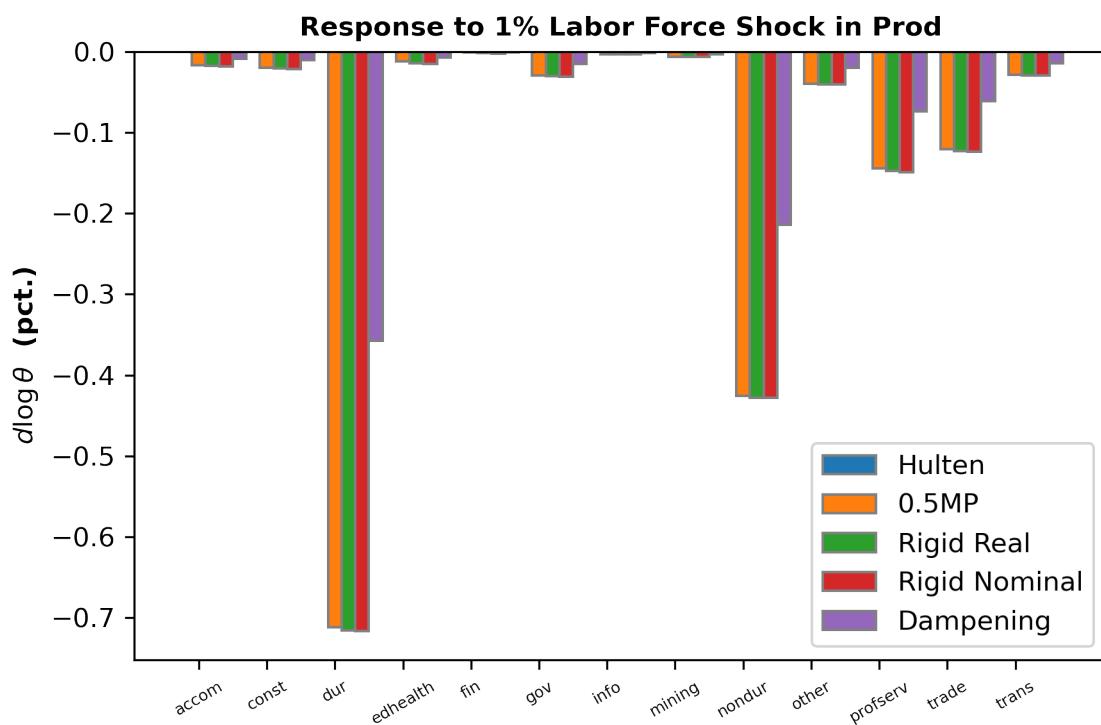


FIGURE 3. Effect of a 1% shock to labor force of production workers on sector level tightness.

5. Application: Adverse Energy and Labor Supply Shocks to the U.S Economy

Our model features two types of non-intermediate production inputs: those that are subject to search and matching, such as labor, and those that are not, such as energy. We apply our model to evaluate the effect of these two types of factor shocks. Specifically, we analyze the aggregate and granular impact of energy supply and labor supply shocks, which the US experienced in recent years due to the Russian-Ukraine war and the decrease in labor force participation during COVID. We provide a qualitative illustration of how these adverse factor supply shocks would impact sectoral production and disaggregated labor market tightness. While our model focuses on real variables, we demonstrate that relative prices move in a realistic way.

5.1. Effects of an adverse energy shock

On March 8, 2022, the White House issued a ban on imports of Russian oil, liquefied natural gas, and coal. Such ban constitutes a large adverse energy shock, as the U.S imported nearly 700,000 barrels of crude oil and refined petroleum products from Russia per day in 2021. In this section, we examine the impact of an adverse energy shock, using the major occupation setup. Figure 4A shows the response of output to the adverse energy shock. The rigid wage specifications predicts an aggregate output decline of around 1% in response to an adverse 10% energy shock, twice as large as when search and matching frictions are absent. The transportation sector is most negatively impacted, followed by the accommodation sector, the construction sector, and the trade sector, as they have high energy elasticities. Search and matching frictions amplify the impact of energy shocks significantly through the search and matching channel. These amplifications create large responses in other downstream sectors, such as government and professional services, despite their low energy elasticities.

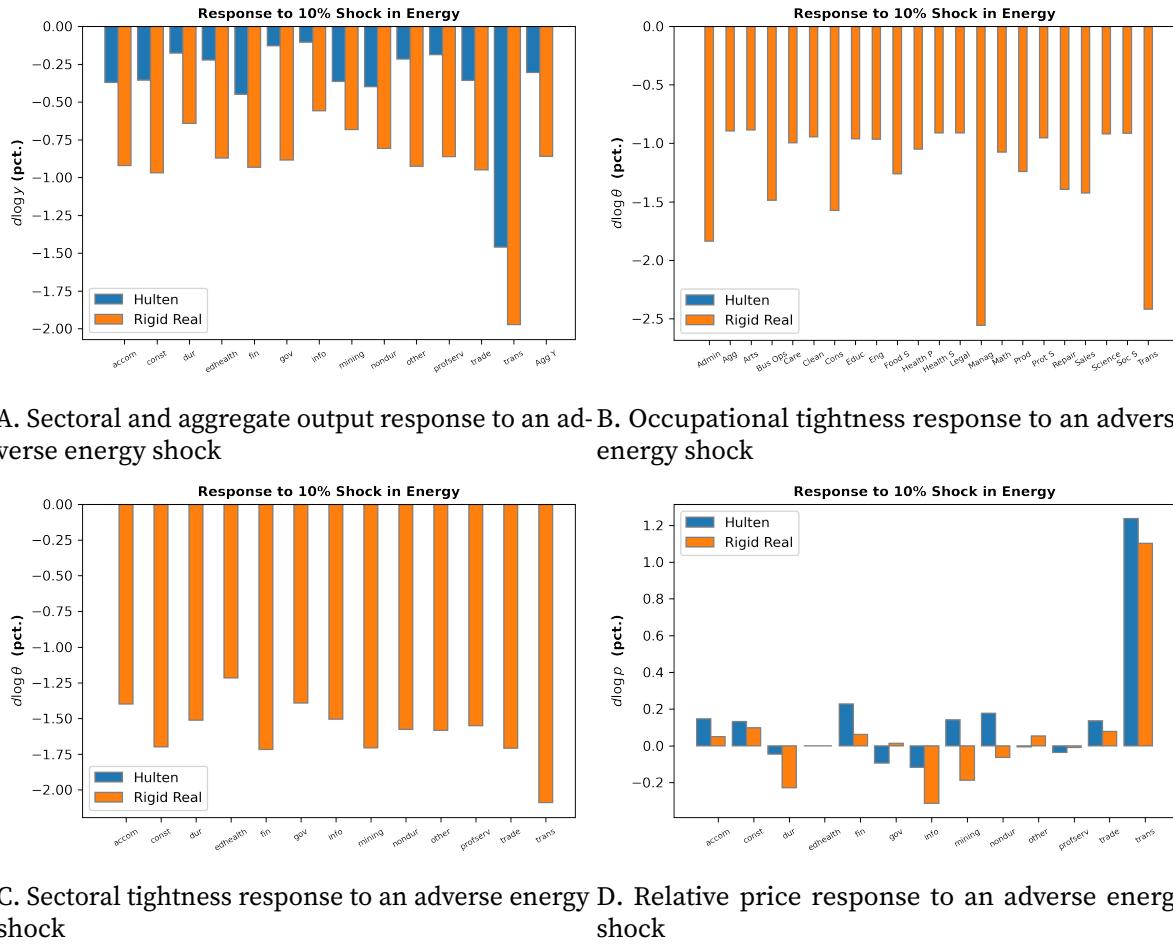


FIGURE 4. First-order responses to an adverse energy shock under sector-specific occupations specification under major occupations specification.

Figure 4B plots the response in occupational labor market tightness to the adverse energy shock. The labor market tightness for management and transportation service workers decreases the most, at around 2%. Additionally, administrative occupation also experience a large decline in labor market tightness. While the decline in labor tightness for transportation service workers is straightforward, as the transportation sector suffers the largest output drop due to their reliance on energy, it is interesting that admin and management occupations experience a large decline in tightness. The reason behind this, however, is simple. Many sectors, who are affected directly through having higher energy elasticities, or indirectly through their production linkages with energy-intensive sectors, rely on administrative workers. Therefore, even when the most affected transportation relies little on management workers, we still see a large decline in management occupation tightness. This highlights the importance of modeling

occupational linkages and production linkages jointly.

Figure 4C plots the response in sectoral labor market tightness reconstructed from occupational tightness movements. Note that this reconstruction is only valid under our calibration assumptions, and should provide a rough estimate of how sectoral tightness changes. We see the transportation sector experiences the largest decrease in tightness of around 2%. The decline in tightness for other sectors appears to be similar, since administrative workers are used intensively across many sectors.

Figure 4D documents the first-order price changes, using education and health as the numeraire sector. We choose the education and health as the numeraire for one simple reason, which is that inflation for medical care service was 0.4% from 2022 to 2023, much more stable than most other goods and services. We find that, first-order price responses differ across sectors and across wage specifications. We observe the largest price increases in sectors that suffer the largest output declines. In particular, we see a large increase in transportation prices. This is qualitatively consistent with large observed increases in transportation prices between 2022 and 2023⁵. We also see price increases in the accommodation sector and the construction sector.

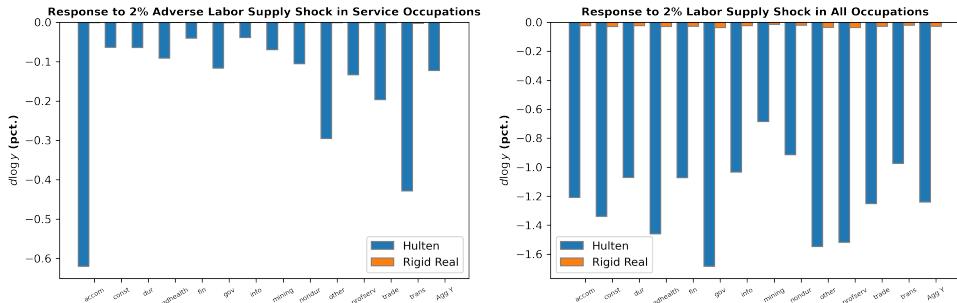
5.2. Effects of adverse labor supply shocks

Another prominent feature of the U.S labor market after the COVID-19 pandemic is the occurrence of the “Great Reshuffle” and the “Great Resignation.” These labor market phenomena signify structural labor market changes, such as more people quitting their jobs, more people retiring early, and people switching jobs for better pay and work-life balance. We focus on two aspects of this phenomenon. First, we assume that there is a reduction in labor supply for the service occupations, as suggested by the high quit rates in the accommodation sector. According to the Bureau of Labor Statistics(BLS), in September 2021, the quit rate for the accommodation sector was 6.6%, whereas the quit rate for private industries overall was 3.4%. Second, we assume there is a reduction in the overall labor participation rate, as evidenced by Hobijn and Şahin (2022). In both cases we assume the magnitude of the shock for each impacted occupation is 2%. This is consistent with Hobijn and Şahin (2022)’s finding that the labor force participation rate declined by roughly 1.2 percentage points. We examine the macroeconomic impact of these two specifications in the context of our model separately.

To start off, we examine the impact of adverse labor supply shocks to the service

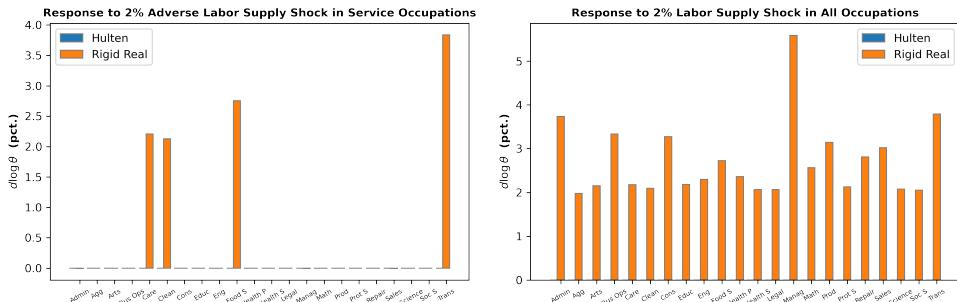
⁵According to the Bureau of Transportation Statistics, transportation prices rose 15.5% overall between 2022 and 2023

occupations. Figures 5A, 5C, 5E and 5G show the output, tightness, and relative price responses to a 2% adverse labor supply shock to the following ONET major occupation classification: building and grounds cleaning and maintenance occupations, food preparation and serving related occupations, personal care and service occupations, and transportation and material moving occupations. Overall, while the efficient benchmark exhibits large movements in output and prices, our specifications show very small changes. However, when it comes to labor market tightness, we can see clear responses in the occupations we shock. From Figure 5E can also see that sectoral tightness increases for all sectors, but increases the most for sectors such as accommodation, transportation, retail, and other services.



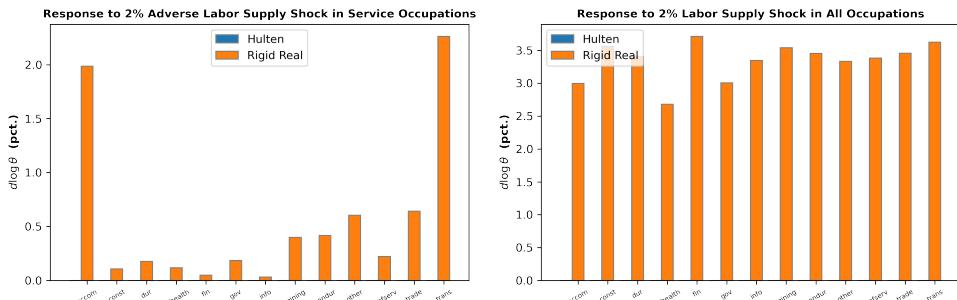
A. Output response to service occupation shocks

B. Output response to labor force participation shocks



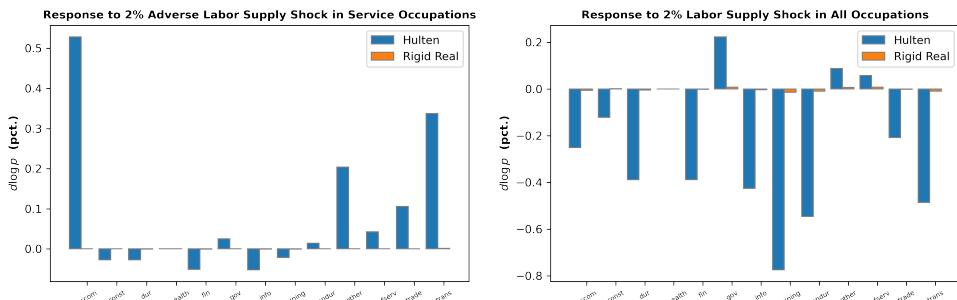
C. Occupational tightness vice occupation shocks

D. Occupational tightness response to labor force participation shocks



E. Sectoral tightness occupation shocks

F. Sectoral tightness response to labor force participation shocks



G. Relative price response to an adverse H. Relative price response to an adverse labor supply shock in service occupations labor supply shock in all occupations

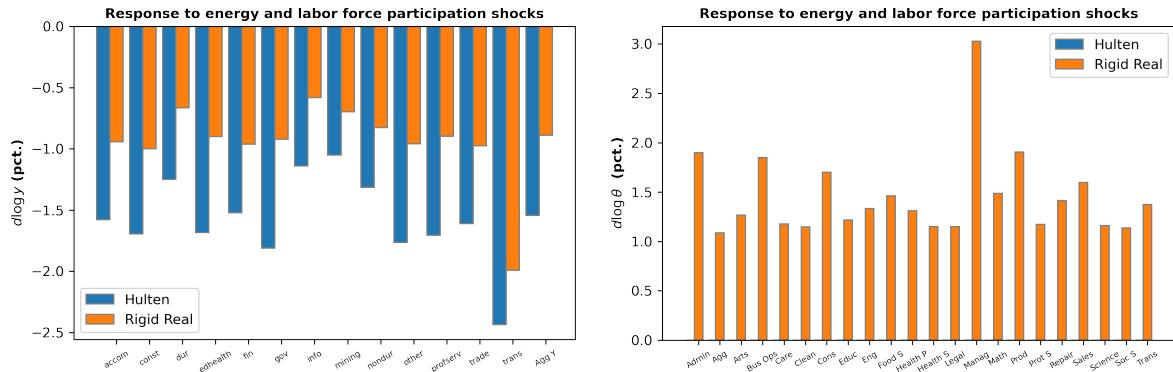
FIGURE 5. Left: First-order responses to an adverse labor supply shock in service occupations. Right: First-order responses to an adverse labor supply shock in all occupations.

Why is the output response to labor supply shock so muted? The effect of output depends on how much the cost of productive workers changes. Since the recruiter producer ratio in the US economy tends to be fairly small, when wages do not change directly an adverse labor supply shock to service occupations does not alter the cost of productive workers substantially. Therefore, the shock has negligible effects on output. The small effects of occupational labor supply shocks on output and prices means that changes in labor demand for other types of occupation are small as well, which is why we see very little movement in tightness for other occupations.

Next, we analyze the impact of adverse labor supply shocks to all occupations. Figures 5B, 5D, 5F, and 5H document the output, tightness, and relative price responses to an adverse shock to all occupations. Overall, the patterns of responses are similar to what we observe for service occupation shocks. The only difference is that, the output responses, while still small compared to the efficient benchmark, are visible on the graphs. This suggests that the change in labor demand induced by negative output responses can dampen the increase in tightness we observe, although the effect are quantitatively negligible.

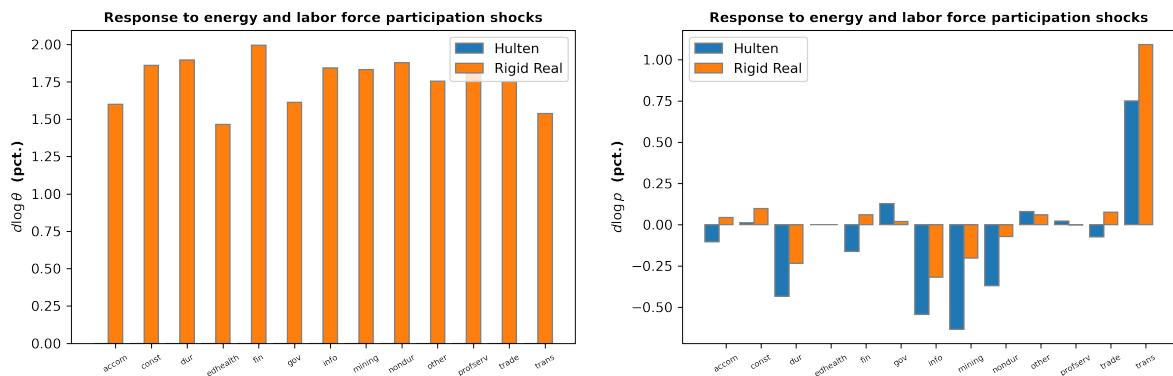
5.3. Combination Shocks

Now, we combine the energy shock and the labor participation shock. Although the first-order impact of these shocks are linearly additive, it is helpful to see how they affect the economy together. Figures 6A, 6B, 6C, and 6D show the combined output, tightness and relative price responses to the shocks that we believe are relevant for the U.S economy in the past few years. Overall, we see a moderate drop in real output, an increase in tightness across all occupations and sectors, and increases in relative prices for sectors such as transportation, accommodation, and construction.



A. Output response to energy and labor force participation shocks

B. Occupational tightness response to energy and labor force participation shocks



C. Sectoral tightness response to energy and labor force participation shocks

D. Relative price response to energy and labor force participation shocks

FIGURE 6. First-order responses to a shock in labor force participation rate combined with an adverse energy shock.

Compared to the efficient production networks model, does our model do a better job at capturing reality? We find that combined energy and labor supply shocks have an impact on aggregate output that is half of what the conventional model would have predicted. This is mainly due to the smaller effect of labor supply shocks on output in our model. While conventional production network models don't feature unemployment, we find that our model predicts an increase in labor market tightness across sectors. In addition, our model predicts a larger increase in relative prices for sectors that have large energy elasticities, as well as sectors that are linked to them through production and occupations. While our results should only be viewed as qualitative as we have only chosen approximate shock values, our models predictions of a small output contraction and tighter labor markets seem consistent with the U.S. data. U.S. real GDP did decline from Q4 2021 to Q2 2022, but the decline was muted, while labor markets have been

exceptionally tight. The standard production networks model would instead predict a much larger decline in output and no movement in tightness.

6. Conclusion

Modern economies feature production networks and frictional, segmented labor markets. We show that accounting for both can alter our understanding of how shocks propagate in a quantitative and qualitatively meaningful way. The impact of matching frictions on network propagation depends on the assumption about wages. We show that under a wide range of assumptions about wages, labor market frictions amplify shocks to productivity. But it is nevertheless possible for the market frictions to dampen shock propagation. This highlights that determining the empirical response of wages to shocks across the network is an import avenue for future work. In addition, a combination of negative energy supply shocks and negative labor force shocks generates simultaneously tight labor markets and depressed output in our framework. While our current model cannot speak to the effect on the price level, in future work we hope to extend the model to incorporate dynamics and nominal rigidities to help paint a realistic picture of the current inflation episode.

References

- Acemoglu, D. and P. D. Azar (2020). Endogenous Production Networks. *Econometrica* 88(1), 33–82. _eprint: <https://onlinelibrary.wiley.com/doi/pdf/10.3982/ECTA15899>.
- Acemoglu, D., V. M. Carvalho, A. Ozdaglar, and A. Tahbaz-Salehi (2012). The Network Origins of Aggregate Fluctuations. *Econometrica* 80(5), 1977–2016. _eprint: <https://onlinelibrary.wiley.com/doi/pdf/10.3982/ECTA9623>.
- Adão, R., C. Arkolakis, and F. Esposito (2019, February). General Equilibrium Effects in Space: Theory and Measurement. Technical Report w25544, National Bureau of Economic Research, Cambridge, MA.
- Baqae, D. and E. Rubbo (2022). Micro Propagation and Macro Aggregation. pp. 53.
- Baqae, D. R. and E. Farhi (2019). The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten's Theorem. pp. 58.
- Baqae, D. R. and E. Farhi (2020). Productivity and Misallocation in General Equilibrium. pp. 99.
- Benigno, P. and G. Eggertsson (2023, April). It's Baaack: The Surge in Inflation in the 2020s and the Return of the Non-Linear Phillips Curve. Technical Report w31197, National Bureau of Economic Research, Cambridge, MA.
- Bigio, S. and J. La’O (2020, November). Distortions in Production Networks*. *The Quarterly Journal of Economics* 135(4), 2187–2253.
- Chodorow-Reich, G. and J. Wieland (2020). Secular Labor Reallocation. *Journal of Political Economy* 128(6).
- Fernandez-Villaverde, J. (2020). Solving High-Dimensional Dynamic Programming Problems using Deep Learning.
- Hall, R. E. (2005, March). Employment Fluctuations with Equilibrium Wage Stickiness. *The American Economic Review* 95(1), 50–65. Num Pages: 16 Place: Nashville, United States Publisher: American Economic Association.
- Hobijn, B. and A. Şahin (2022, December). Missing Workers and Missing Jobs Since the Pandemic.
- Horowitz, K. J. and M. A. Planting (2009). Concepts and Methods of the U.S. Input-Output Accounts. *Bureau of Economic Analysis*.
- Hulten, C. R. (1978, October). Growth Accounting with Intermediate Inputs. *The Review of Economic Studies* 45(3), 511–518.
- Humlum, A. (2021). Robot Adoption and Labor Market Dynamics. pp. 69.
- Jones, C. I. (2011). Missallocation, Economic Growth, and Input-Output Economics.

- Kambourov, G. and I. Manovskii (2008, February). Rising Occupational and Industry Mobility in the United States: 1968–97. *International Economic Review* 49(1), 41–79. Publisher: Wiley-Blackwell.
- La’O, J. and A. Tahbaz-Salehi (2021). Optimal Monetary Policy in Production Networks. pp. 65.
- Liu, E. (2019). Industrial Policies in Production Networks.
- Long, J. B. and C. I. Plosser (1983). Real Business Cycles. *Journal of Political Economy* 91(1), 39–69. Publisher: University of Chicago Press.
- Minton, R. and B. Wheaton (2022). Hidden Inflation in Supply Chains: Theory and Evidence.
- Neffke, F. M., A. Otto, and A. Weyh (2017, October). Inter-industry labor flows. *Journal of Economic Behavior & Organization* 142, 275–292.
- Parrado, E., A. Caner, and E. N. Wolff (2007, June). Occupational and industrial mobility in the United States. *Labour Economics* 14(3), 435–455.
- Rubbo, E. (2020). Networks, Phillips Curves, and Monetary Policy. pp. 42.
- Schubert, G., A. Stansbury, and B. Taska (2022, December). Employer Concentration and Outside Options.
- Shimer, R. (2005, March). The Cyclical Behavior of Equilibrium Unemployment and Vacancies. *The American Economic Review* 95(1), 25–49. Num Pages: 25 Place: Nashville, United States Publisher: American Economic Association.

Appendix A. Proofs

A.1. Proof for Propositions 1 and 2

We first show how to prove Proposition 2 by assuming Cobb-Douglas preferences and production.

A.1.1. Price Propagation

Log-linearizing the production function, for each sector i , we have:

$$d \log y_i = \underbrace{\varepsilon_{A_i}^{f_i}}_{=1} d \log A_i + \sum_{o=1}^O \varepsilon_{N_{io}}^{f_i} d \log N_{io} + \sum_{j=1}^N \varepsilon_{x_{ij}}^{f_i} d \log x_{ij} + \sum_{k=1}^K \varepsilon_{K_{ik}}^{f_i} d \log K_{ik}$$

Plugging in Equation 7 and Equation 6, the first order conditions for optimal input usage, into the log-linearized production function gives

$$\begin{aligned} d \log y_i &= \sum_{o=1}^O \varepsilon_{N_{io}}^{f_i} \left[d \log \varepsilon_{N_{io}}^{f_i} + d \log y_i + d \log p_i - d \log w_o - d \log (1 + \tau_o(\theta_o)) \right] \\ &\quad + \sum_{j=1}^N \varepsilon_{x_{ij}}^{f_i} \left[d \log \varepsilon_{x_{ij}}^{f_i} + d \log y_i + d \log p_i - d \log p_j \right] \\ &\quad + \sum_{k=1}^K \varepsilon_{K_{ik}}^{f_i} \left[d \log \varepsilon_{K_{ik}}^{f_i} + d \log y_i + d \log p_i - d \log r_k \right] + d \log A_i \\ &= [d \log y_i + d \log p_i] \underbrace{\left[\sum_{k=1}^K \varepsilon_{K_{ik}}^{f_i} + \sum_{o=1}^O \varepsilon_{N_{io}}^{f_i} + \sum_{j=1}^N \varepsilon_{x_{ij}}^{f_i} \right]}_{=1 \text{ by crts}} + \underbrace{\left[\sum_{k=1}^K d \varepsilon_{K_{ik}}^{f_i} + \sum_{o=1}^O d \varepsilon_{N_{io}}^{f_i} + \sum_{j=1}^N d \varepsilon_{x_{ij}}^{f_i} \right]}_{=0 \text{ by crts}} \\ &\quad - \sum_{o=1}^O \varepsilon_{N_{io}}^{f_i} [d \log w_o + d \log (1 + \tau_o(\theta_o))] - \sum_{j=1}^N \varepsilon_{x_{ij}}^{f_i} [d \log p_j] - \sum_{k=1}^K \varepsilon_{K_{ik}}^{f_i} d \log r_k + d \log A_i, \end{aligned}$$

where the second equality holds because the sum of elasticities equals one for constant returns to scale technology and $\varepsilon_{x_{ij}}^{f_i} d \log \varepsilon_{x_{ij}}^{f_i} = d \varepsilon_{x_{ij}}^{f_i}$.

Rearranging terms gives

$$\begin{aligned}
d \log p_i &= \sum_{o=1}^O \varepsilon_{N_{io}}^{f_i} [d \log w_o + d \log(1 + \tau_o(\theta_o))] + \sum_{j=1}^N \varepsilon_{x_{ij}}^{f_i} [d \log p_j] + \sum_{k=1}^K \varepsilon_{K_{ik}}^{f_i} d \log r_k - d \log A_i \\
&= \sum_{o=1}^O \varepsilon_{N_{io}}^{f_i} [d \log w_o + \varepsilon_{\theta_o}^{1+\tau_o} d \log \theta_o] + \sum_{j=1}^J \varepsilon_{x_{ij}}^{f_i} [d \log p_j] + \sum_{k=1}^K \varepsilon_{K_{ik}}^{f_i} d \log r_k - d \log A_i \\
&= \sum_{o=1}^O \varepsilon_{N_{io}}^{f_i} [d \log w_o - \tau_o(\theta_o) \varepsilon_{\theta_o}^{Q_o} d \log \theta_o] + \sum_{j=1}^J \varepsilon_{x_{ij}}^{f_i} [d \log p_j] + \sum_{k=1}^K \varepsilon_{K_{ik}}^{f_i} d \log r_k - d \log A_i
\end{aligned}$$

Stacking equations over sectors, we can write

$$d \log \mathbf{p} = \varepsilon_{\mathbf{N}}^f [d \log \mathbf{w} - \mathbf{Q}\mathbf{T}d \log \boldsymbol{\theta}] + \mathbf{\Omega}d \log \mathbf{p} + \varepsilon_{\mathbf{K}}^f d \log \mathbf{r} - d \log \mathbf{A}$$

Which implies

$$d \log \mathbf{p} = \Psi \left[\varepsilon_{\mathbf{N}}^f [d \log \mathbf{w} - \mathbf{Q}\mathbf{T}d \log \boldsymbol{\theta}] + \varepsilon_{\mathbf{K}}^f d \log \mathbf{r} - d \log \mathbf{A} \right]$$

Or equivalently

$$(\mathbf{I} - \Psi \varepsilon_{\mathbf{N}}^f \mathcal{L}) d \log \mathbf{p} = \Psi \left[\varepsilon_{\mathbf{N}}^f [d \log \mathbf{w} - \mathcal{L}d \log \mathbf{p} - \mathbf{Q}\mathbf{T}d \log \boldsymbol{\theta}] + \varepsilon_{\mathbf{K}}^f d \log \mathbf{r} - d \log \mathbf{A} \right]$$

Where $d \log \mathbf{r}$ is pinned down by market clearing in the additional factor market for K_k . Assuming Cobb-Douglas production, for any i

$$d \log r_k = d \log y_i + d \log p_i - d \log K_{ik}$$

Which means that $d \log k_{ik} = d \log K_{jk}$ for all i, j . Furthermore, this means that $d \log K_{ik} = d \log K_k^s$ for all i . So we can write,

$$d \log r_k = d \log y_i + d \log p_i - d \log K_k^s$$

Which pins down $d \log r_k$ given a numeraire sector.

A.1.2. Output Propagation

Since the log-linearized expression for the Domar weight must hold for every sector, we can write

$$\begin{aligned} d \log \lambda_i - d \log \lambda_j &= d \log p_i - d \log p_j + d \log y_i - d \log y_j \\ &= d \log x_{ij} - d \log \varepsilon_{x_{ij}}^{f_i} - d \log y_j \\ \Rightarrow d \log x_{ij} &= d \log \lambda_i - d \log \lambda_j + d \log y_j + d \log \varepsilon_{x_{ij}}^{f_i} \end{aligned}$$

Plugging back into the production function,

$$\begin{aligned} d \log y_i &= d \log A_i + \sum_{o=1}^O \varepsilon_{N_{io}}^{f_i} d \log N_{io} + \sum_{j=1}^J \varepsilon_{x_{ij}}^{f_i} + d \log y_j + \sum_{k=1}^K \varepsilon_{K_{ik}}^{f_i} d \log K_{ik} \\ &= d \log A_i + \sum_{o=1}^O \varepsilon_{N_{io}}^{f_i} d \log N_{io} + \sum_{j=1}^J \varepsilon_{x_{ij}}^{f_i} d \log y_j + \sum_{k=1}^K \varepsilon_{K_{ik}}^{f_i} d \log K_K^s \end{aligned}$$

Using the definition of labor demand,

$$\begin{aligned} \sum_i \frac{l_{io}}{L_o} d \log N_{io} &= d \log L_o^d - d \log(1 + \tau_o(\theta_o)) \\ &= d \log L_o^d + \tau_o(\theta_o) \varepsilon_{\theta_o}^{Q_o} d \log \theta_o \end{aligned}$$

From output labor usage, we have that labor usage ratio for an occupation by two different sectors as:

$$\frac{l_{io}}{l_{jo}} = \frac{\varepsilon_{N_{io}}^f \lambda_i}{\varepsilon_{N_{jo}}^f \lambda_j}$$

for any $l_{io}, l_{jo} > 0$

Log-linearizing it, assuming Cobb-Douglas preferences, yields:

$$d \log l_{io} = d \log l_{jo}$$

Also since, $d \log l_{io} = d \log N_{io} + d \log(1 + \tau_o(\theta_o)) = d \log l_{jo} = d \log N_{jo} + d \log(1 + \tau_o(\theta_o))$, we have that $d \log N_{io} = d \log N_{jo}$

Using the labor market clearing condition, and the definition of labor supply,

$$\begin{aligned} \sum_k \frac{l_{ko}}{L_o} d \log N_{ko} &= \left(\varepsilon_{\theta_o}^{\mathcal{F}_o} + \tau_o(\theta_o) \varepsilon_{\theta_o}^{\mathcal{Q}_o} \right) d \log \theta_o + d \log H_o \\ \Rightarrow d \log N_{io} \underbrace{\sum_k \frac{l_{ko}}{L_o}}_{=1} &= \left(\varepsilon_{\theta_o}^{\mathcal{F}_o} + \tau_o(\theta_o) \varepsilon_{\theta_o}^{\mathcal{Q}_o} \right) d \log \theta_o + d \log H_o \end{aligned}$$

Plugging this back into the linearized production function gives:

$$\begin{aligned} d \log y_i &= d \log A_i + \sum_{o=1}^O \varepsilon_{N_{io}}^{f_i} \left[\left(\varepsilon_{\theta_o}^{\mathcal{F}_o} + \tau_o(\theta_o) \varepsilon_{\theta_o}^{\mathcal{Q}_o} \right) d \log \theta_o + d \log H_o \right] \\ &\quad + \sum_{j=1}^J \varepsilon_{x_{ij}}^{f_i} d \log y_j + \sum_{k=1}^K \varepsilon_{K_{ik}}^{f_i} d \log K_k^s \end{aligned}$$

Stacking over sectors gives,

$$d \log \mathbf{y} = d \log \mathbf{A} + \varepsilon_{\mathbf{N}}^f (\mathcal{F} + \mathcal{Q}\mathcal{T}) d \log \theta + \varepsilon_{\mathbf{N}}^f d \log \mathbf{H} + \Omega d \log \mathbf{y} + \varepsilon_{\mathbf{K}}^f d \log \mathbf{K}^s$$

Which implies

$$d \log \mathbf{y} = \Psi \left(d \log \mathbf{A} + \varepsilon_{\mathbf{N}}^f (\mathcal{F} + \mathcal{Q}\mathcal{T}) d \log \theta + \varepsilon_{\mathbf{N}}^f d \log \mathbf{H} + \varepsilon_{\mathbf{K}}^f d \log \mathbf{K}^s \right)$$

A.1.3. Tightness Propagation

Labor market clearing implies that changes in labor demand have to equal changes in labor supply:

$$\begin{aligned} d \log L_o^s(\theta, \mathbf{H}) &= d \log L_o^d(\theta, \mathbf{A}). \\ \varepsilon_{\theta_o}^{\mathcal{F}_o} d \log \theta_o + d \log H_o &= \sum_{i=1}^J \frac{l_{io}}{L_o^d} d \log l_{io}(\theta_o) \end{aligned}$$

Where $\frac{l_{io}}{L_o^d} = \frac{\varepsilon_{N_{io}}^{f_i} p_i y_i}{\sum_{j=1}^J \varepsilon_{N_{jo}}^{f_j} p_j y_j}$ ⁶. For every sector i we have

$$d \log l_{io}(\theta_o) = d \log \varepsilon_{N_{io}}^{f_i} - d \log w_o + d \log p_i + d \log y_i$$

Which implies that

$$d \log L^d(\theta) = \text{diag} \left(\mathcal{L} d \log \varepsilon_N^f \right) - [d \log \mathbf{w} - \mathcal{L} d \log \mathbf{p}] + \mathcal{L} d \log \mathbf{y}$$

since $\sum_{i=1}^J \frac{l_{io}}{L_o^d} = 1$ for all o . Plugging in for $d \log \mathbf{y}$ gives

$$\begin{aligned} d \log L^d(\theta) &= \text{diag} \left(\mathcal{L} d \log \varepsilon_N^f \right) - [d \log \mathbf{w} - \mathcal{L} d \log \mathbf{p}] \\ &\quad + \mathcal{L} \Psi \left[d \log \mathbf{A} + \varepsilon_N^f (\mathcal{F} + \mathcal{Q}\mathcal{T}) d \log \theta + \varepsilon_N^f d \log \mathbf{H} + \varepsilon_K^f d \log \mathbf{K}^s \right] \end{aligned}$$

Labor market clearing implies

$$\mathcal{F} d \log \theta + d \log \mathbf{H} = \mathcal{L} \Psi \left[d \log \mathbf{A} + \varepsilon_N^f (\mathcal{F} + \mathcal{Q}\mathcal{T}) d \log \theta + \varepsilon_N^f d \log \mathbf{H} + \varepsilon_K^f d \log \mathbf{K}^s \right] - [d \log \mathbf{w} - \mathcal{L} d \log \mathbf{p}]$$

Which pins down first order changes in log tightness as

$$d \log \theta = [\mathcal{F} - \Xi_\theta]^{-1} \left[\mathcal{L} \Psi d \log \mathbf{A} - [d \log \mathbf{w} - \mathcal{L} d \log \mathbf{p}] + [\mathcal{L} \Psi \varepsilon_N^f - \mathbf{I}] d \log \mathbf{H} + \mathcal{L} \Psi \varepsilon_K^f d \log \mathbf{K}^s \right]$$

Where $\Xi_\theta = \mathcal{L} \Psi \varepsilon_N^f [\mathcal{F} + \mathcal{Q}\mathcal{T}]$.

Therefore, combining these propagation expressions, we are able to express $d \log \theta$, $d \log \mathbf{y}$, and $d \log \mathbf{p}$ in terms of exogenous shocks.

A.2. Proof for Theorem 1 and corollaries

Let $\lambda_i = \frac{p_i y_i}{G}$, where $G = \sum_j p_j c_j = GDP = \sum_{o=1}^O w_o L_o$, denote the final sales share of GDP for sector i . We have that:

$$p_j x_{ij} = \varepsilon_{N_{io}}^{f_i} x_{ij} \lambda_i G.$$

⁶One implication of this formula is that I think we should be able to check whether the elasticities $\left\{ \left\{ \varepsilon_{N_{io}}^{f_i} \right\}_{o=1}^O \right\}_{i=1}^J$ are consistent with the Domar weights.

From household's maximization problem, I have that $p_i c_i = \varepsilon_{c_i}^D G$. Combining the two gives:

$$\begin{aligned}\varepsilon_{c_j}^D G &= p_j y_j = p_j(c_j + \sum_i x_{ij}) = \left(\varepsilon_{c_j}^D + \sum_i \varepsilon^{f_i} x_{ij} \lambda_i \right) G \\ \Rightarrow \boldsymbol{\lambda} &= \boldsymbol{\Psi}' \varepsilon_{\mathbf{c}}^D.\end{aligned}$$

The aggregate labor force, employment, and unemployment are $H^{agg} = \sum_{o=1}^O H_o$, $L^{agg} = \sum_{o=1}^O L_o$, and $U^{agg} = \sum_{o=1}^O U_o$. Changes in aggregates are therefore given by

$$\begin{aligned}dH^{agg} &= \sum_{o=1}^O dH_o \\ dL^{agg} &= \sum_{o=1}^O dL_o \\ dU^{agg} &= \sum_{o=1}^O dU_o\end{aligned}$$

Or in terms of log changes

$$\begin{aligned}d \log H^{agg} &= \frac{1}{H^{agg}} \sum_{o=1}^O H_o d \log H_o \\ d \log L^{agg} &= \frac{1}{L^{agg}} \sum_{o=1}^O L_o d \log L_o \\ d \log U^{agg} &= \frac{1}{U^{agg}} \sum_{o=1}^O U_o d \log U_o\end{aligned}$$

In matrix notation

$$\begin{aligned}d \log H^{agg} &= \frac{1}{H^{agg}} \mathbf{H}' d \log \mathbf{H} \\ d \log L^{agg} &= \frac{1}{L^{agg}} \mathbf{L}' d \log \mathbf{L} \\ d \log U^{agg} &= \frac{1}{U^{agg}} \mathbf{U}' d \log \mathbf{U}\end{aligned}$$

Substituting in for $d \log \mathbf{L}$

$$\begin{aligned} d \log L^{agg} &= \frac{1}{L^{agg}} \mathbf{L}' [\mathcal{L} d \log \mathbf{y} - [d \log \mathbf{w} - \mathcal{L} d \log \mathbf{p}]] \\ &= \frac{1}{L^{agg}} \mathbf{L}' [\mathcal{L} [\Pi_{y,A} d \log \mathbf{A} + \Pi_{y,H} d \log \mathbf{H} - \Lambda_A d \log \mathbf{A} - \Lambda_H d \log \mathbf{H}]] \\ &= \Pi_{L^{agg},A} d \log \mathbf{A} + \Pi_{L^{agg},H} d \log \mathbf{H} \end{aligned}$$

Where

$$\begin{aligned} \Pi_{L^{agg},A} &= \frac{1}{L^{agg}} \mathbf{L}' [\mathcal{L} \Pi_{y,A} - \Lambda_A] \\ \Pi_{L^{agg},H} &= \frac{1}{L^{agg}} \mathbf{L}' [\mathcal{L} \Pi_{y,H} - \Lambda_H] \end{aligned}$$

And

$$\begin{aligned} d \log U^{agg} &= \frac{1}{U^{agg}} \mathbf{U}' [d \log \mathbf{H} - d \log \mathbf{L}] \\ &= \frac{1}{U^{agg}} \mathbf{U}' [d \log \mathbf{H} - [\mathcal{L} [\Pi_{y,A} d \log \mathbf{A} + \Pi_{y,H} d \log \mathbf{H} - \Lambda_A d \log \mathbf{A} - \Lambda_H d \log \mathbf{H}]]] \\ &= \Pi_{U^{agg},A} d \log \mathbf{A} + \Pi_{U^{agg},H} d \log \mathbf{H} \end{aligned}$$

Where

$$\begin{aligned} \Pi_{U^{agg},A} &= \frac{1}{U^{agg}} \mathbf{U}' [\Lambda_A - \mathcal{L} \Pi_{y,A}] \\ \Pi_{U^{agg},H} &= \frac{1}{U^{agg}} \mathbf{U}' [\mathbf{I} + \Lambda_H - \mathcal{L} \Pi_{y,H}] \end{aligned}$$

Appendix B. Results for general CRTS production functions and one occupation per sector

In this section we generalize our results to any constant returns to scale production function, under the assumption that there is one type of labor per sector. This generalization results in additional terms that capture how the production elasticities change when shocks hit the economy. The expressions are otherwise similar to above. The model setup is identical, we just do not impose Cobb-Douglas technology and instead impose $\mathcal{O} = J$.

B.1. Price changes

First order changes in prices remain largely unchanged and satisfy

$$(\mathbf{I} - \Psi \varepsilon_N^f) d \log \mathbf{p} = \Psi \left[\varepsilon_N^f [d \log \mathbf{w} - d \log \mathbf{p} - \Omega \mathcal{T} d \log \theta] - d \log \mathbf{A} \right]$$

B.2. Sales Share Propagation

We can rewrite the goods market clearing condition in terms of Domar weights:

$$\begin{aligned} y_i &= c_i + \sum_{j=1}^J x_{ji} \\ \Rightarrow \frac{p_i y_i}{\sum_{k=1}^J p_k c_k} &= \frac{p_i c_i}{\sum_{k=1}^J p_k c_k} + \sum_{j=1}^J \frac{p_i x_{ji}}{p_j x_j} \frac{p_j x_j}{\sum_{k=1}^J p_k c_k} \\ (A1) \qquad \qquad \qquad \Rightarrow \lambda_i &= \varepsilon_{c_i}^D + \sum_{j=1}^J \varepsilon_{x_{ji}}^{f_j} \lambda_j, \end{aligned}$$

where $\lambda_i = \frac{p_i y_i}{\sum_{k=1}^J p_k c_k}$ is the Domar weight of sector i .

By stacking (A1) for each sector, we get the following expression for Domar weights across the production network.

$$\lambda' = \varepsilon_c^D + \lambda' \Omega$$

We can see how Domar weights change across the production network by totally differentiating

$$\begin{aligned} d\lambda' &= d\varepsilon_c^D + d\lambda' \Omega + \lambda' d\Omega \\ (A2) \qquad \qquad \Rightarrow d\lambda' &= [d\varepsilon_c^D + \lambda' d\Omega] \Psi \end{aligned}$$

The Domar weights will help us express how shocks propagate to output.

B.3. Output changes

We can now write changes in output in terms of changes in tightness, technology, the size of the labor force, and changes in production elasticities, including changes in

Domar weights, as

$$d \log \mathbf{y} = \Psi \left(d \log A + \varepsilon_N^f (\mathcal{F} + \mathcal{Q}\mathcal{T}) d \log \theta + \varepsilon_N^f d \log \mathbf{H} \right) \\ - \Psi d \log \mathcal{E} + \Psi (\text{diag}(\Omega \mathbf{1}) - \Omega) d \log \lambda$$

Where $\mathbf{1}$ is a $J \times 1$ vector of ones and $d \log \mathcal{E}$ is the $J \times 1$ vector of diagonal elements of $\varepsilon_N^f d \log \varepsilon_N^{f'}$.

B.4. Tightness changes

Much like output, changes in tightness now also depends on changes in the elasticities of the production functions. The expressiong for changes in tightness is

$$d \log \theta = [\mathcal{F} - \Xi_\theta]^{-1} \left[\Psi d \log A - [d \log \mathbf{w} - d \log \mathbf{p}] + [\Psi \varepsilon_N^f - \mathbf{I}] d \log \mathbf{H} \right] \\ + [\mathcal{F} - \Xi_\theta]^{-1} \left[\text{diag}(\mathcal{L} d \log \varepsilon_N^f) + \Psi [(\text{diag}(\Omega \mathbf{1}) - \Omega) d \log \lambda - d \log \mathcal{E}] \right]$$

Where $\Xi_\theta = \Psi \varepsilon_N^f [\mathcal{F} + \mathcal{Q}\mathcal{T}]$. Notice, all terms in the second line are zero assuming Cobb-Douglas production technology.

B.5. Aggregation

Aggregate output now satisfies

$$d \log Y = \varepsilon_c^{\mathcal{D}'} d \log \mathbf{c} \\ = \varepsilon_c^{\mathcal{D}'} (d \log \varepsilon_c^{\mathcal{D}} d \log \mathbf{y} - d \log \lambda)$$

Appendix C. Data and Calibration Details

This appendix describes our data in greater detail.

C.1. Input-output matrix

We use the 3-digit 2021 BEA Make and Use tables accessible at <https://www.bea.gov/industry/input-output-accounts-data> to calculate the relevant production elasticities⁷. The 3-digit Make

⁷See https://www.bea.gov/sites/default/files/methodologies/IOmanual_092906.pdf for a detailed de-
scription of how these tables are generated.

and Use tables record the nominal amount of each 71 commodities made by and used by each of 71 industries. The commodities are denoted using the same codes as the industries, but they are conceptually distinct as each industry can produce more than one commodity.

For consistency with the industry classifications in JOLTs and the CPS unemployment by sector series, we collapse the 3-digit tables to a 13 sector table. The table below outlines the mapping from the NAICS 2-digit classification codes to our industry classifications. The mapping from 2-digit codes to 3-digit codes is readily available online.

Industry Name	Short Name	2-digit codes
Leisure and Hospitality	accom	71, 72
Construction	const	33
Durable goods	dur	33DG
Education and Health Services	edhealth	61, 62
Financial Activities	fin	52, 53
Government	gov	G
Information	info	51
Mining	mining	21
Nondurable good	nondur	11, 31ND
Other services, except government	other	81
Professional and business services	profserv	54, 55, 56
Wholesale and Retail trade	trade	42, 44RT
Transportation and Utilities	trans	22, 48TW

TABLE A1. Mapping from NAICS classification to our industries.

With the 13-sector make and use tables in hand, we can construct production elasticities in intermediate inputs and to labor, and demand elasticities. Let M_{ij} denote the nominal value of commodity i made by industry j . Let U_{ij} denote the nominal amount of commodity i used by industry j . The two tables below demonstrate the elements of the Make and Use tables.

	Sector 1	Sector 2	...	Sector J	Total Industry Output
Sector 1	M_{11}	M_{21}	...	M_{J1}	$\sum_{i=1}^J M_{i1}$
Sector 2	M_{12}	M_{22}	...	M_{J2}	$\sum_{i=1}^J M_{i2}$
⋮	⋮	⋮	⋮	⋮	⋮
Sector J	M_{1J}	M_{2J}	...	M_{JJ}	$\sum_{i=1}^J M_{ij}$
Total Commodity Output	$\sum_{j=1}^J M_{ij}$	$\sum_{j=1}^J M_{2j}$...	$\sum_{j=1}^J M_{jj}$	—

TABLE A2. Make table

	Sector 1	Sector 2	...	Sector J	Total Intermediate Uses	Total Final Uses
Sector 1	U_{11}	U_{12}	...	U_{1J}	$\sum_{j=1}^J U_{1j}$	$\sum_{j=1}^J U_{1j} + p_1 c_1$
Sector 2	U_{21}	U_{22}	...	U_{2J}	$\sum_{j=1}^J U_{2j}$	$\sum_{j=1}^J U_{2j} + p_2 c_2$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
Sector J	U_{J1}	U_{J2}	...	U_{JJ}	$\sum_{j=1}^J U_{jj}$	$\sum_{j=1}^J U_{jj} + p_J c_J$
Total Intermediate Inputs	$\sum_{i=1}^J U_{ii}$	$\sum_{i=1}^J U_{i2}$...	$\sum_{i=1}^J U_{ij}$	—	—
Total industry output	$\sum_{i=1}^J U_{ii} + w_1(1 + \tau_1)N_1$	$\sum_{i=1}^J U_{i2} + w_2(1 + \tau_2)N_2$...	$\sum_{i=1}^J U_{ij} + w_j(1 + \tau_j)N_j$	—	—

TABLE A3. Use table

First, we calculate the fraction of commodity i produced by industry j by dividing the entry in along each row by the corresponding "total industry output"

$$m_{ij} = \frac{M_{ij}}{\sum_{j=1}^J M_{ji}}$$

Second, we calculate the share of commodity i in industry j 's total uses as by dividing each entry in the column corresponding to industry j by the corresponding "Total industry output"

$$u_{ij} = \frac{U_{ij}}{\sum_{i=1}^J U_{ij} + w_j(1 + \tau_j)N_j}$$

We form the two matrices

$$\mathbf{M} = \begin{bmatrix} m_{11} & m_{21} & \cdots & m_{J1} \\ m_{12} & m_{22} & \cdots & m_{J2} \\ \vdots & \vdots & \ddots & \vdots \\ m_{1J} & m_{2J} & \cdots & m_{JJ} \end{bmatrix}, \mathbf{U} = \begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1J} \\ u_{21} & u_{22} & \cdots & u_{2J} \\ \vdots & \vdots & \ddots & \vdots \\ u_{J1} & u_{J2} & \cdots & u_{JJ} \end{bmatrix}$$

Then, we can calculate our input output matrix by

$$\Omega = [\mathbf{MU}]'$$

Given our assumption of constant returns to scale and zero profits, the difference between total intermediate inputs and total industry output is the nominal income paid to workers in each sector. We abstract from the other components of total industry output in the IO accounts, taxes and gross operating surplus, as they have no model counterpart in our setup. We can therefore calculate the labor elasticities from the Use table as "Total industry output" – "Total intermediate inputs" ÷ "Total industry output."

$$\varepsilon_{N_j}^{f_j} = \frac{w_j(1 + \tau_j)N_j}{\sum_{i=1}^J U_{ij} + w_j(1 + \tau_j)N_j}$$

Finally, we can back out the demand elasticities from "Total intermediate uses" and "Total final uses" columns of the Uses table.

$$p_i c_i = \sum_{j=1}^J U_{ij} + p_i c_i - \sum_{j=1}^J U_{ij}$$

We can then work out the elasticities by

$$\varepsilon_{c_i}^D = \frac{p_i c_i}{\sum_{i=1}^J p_i c_i}$$

Finally, to ensure that constant returns to scale holds we rescale our elasticities proportionally to ensure they sum to one. This adjustment is minor and is only needed because we drop the small "Used" and "rest of world adjustment" categories. It does not change any elasticity by more than 3 percent.

We report the resulting estimates of the production elasticities, labor elasticities, and demand elasticities in the tables below. In tables A4 and A5 we assume all non-

intermediate, non-energy, and non-capital, spending goes to labor income, which automatically imposes constant returns but leads to large labor shares.

Sector	Labor Elasticity (ε_N^f)	Demand Elasticity (ε_c^D)
accom	0.510	0.051
const	0.474	0.065
dur	0.434	0.138
edhealth	0.616	0.129
fin	0.617	0.165
gov	0.626	0.132
info	0.571	0.043
mining	0.518	0.008
nondur	0.352	0.151
other	0.608	0.024
profserv	0.591	0.071
trade	0.523	0.000
trans	0.492	0.022

TABLE A4. Labor elasticities and demand elasticities according the BEA make use tables for 13-industry classification, rounded to 3 decimal places. We assume all non-intermediate spending goes to labor income.

	accom	const	dur	edhealth	fin	gov	info	mining	nondur	other	profserv	trade	trans
accom	0.029	0.002	0.017	0.003	0.099	0.007	0.036	0.008	0.103	0.014	0.143	0.000	0.030
const	0.001	0.000	0.282	0.000	0.033	0.001	0.016	0.022	0.089	0.006	0.072	0.000	0.004
dur	0.001	0.001	0.393	0.000	0.016	0.001	0.012	0.013	0.056	0.003	0.055	0.004	0.012
edhealth	0.019	0.000	0.030	0.015	0.103	0.006	0.028	0.004	0.056	0.009	0.101	0.000	0.012
fin	0.011	0.023	0.007	0.000	0.195	0.005	0.020	0.001	0.008	0.005	0.083	0.001	0.022
gov	0.006	0.023	0.048	0.011	0.055	0.004	0.032	0.013	0.081	0.012	0.072	0.000	0.017
info	0.017	0.001	0.040	0.000	0.047	0.003	0.125	0.002	0.015	0.005	0.160	0.001	0.013
mining	0.001	0.005	0.010	0.000	0.060	0.002	0.017	0.128	0.056	0.001	0.090	0.000	0.022
nondur	0.001	0.003	0.044	0.000	0.020	0.002	0.008	0.124	0.377	0.003	0.040	0.004	0.021
other	0.013	0.005	0.066	0.010	0.113	0.005	0.033	0.003	0.032	0.013	0.086	0.000	0.012
nprofserv	0.020	0.000	0.029	0.001	0.070	0.004	0.054	0.002	0.026	0.008	0.176	0.000	0.017
trade	0.006	0.002	0.025	0.002	0.104	0.009	0.038	0.002	0.031	0.015	0.159	0.021	0.063
trans	0.014	0.007	0.022	0.000	0.080	0.015	0.025	0.030	0.080	0.011	0.093	0.001	0.129

TABLE A5. Production elasticities to intermediate inputs at 13-sector level (Ω), rounded to 3 decimal places. We assume all non-intermediate spending goes to labor income.

C.2. Matching Parameters

We estimate the parameters of the sector specific matching function from monthly data on hires and vacancies from JOLTs and unemployment from the CPS. In particular, we estimate

$$\log H_{i,t} = \log \phi_i + \eta_i \log U_{i,t} + (1 - \eta_i) \log V_{i,t} + \epsilon_{i,t}$$

by least squares. ϕ_i is the matching efficiency in sector i and η_i is the matching elasticity with respect to unemployment in sector i . We report the resulting estimates in the table below.

	Matching Efficiency ($\hat{\phi}_i$)	Unemployment Elasticity ($\hat{\eta}_i$)
accom	1.185	0.401
const	1.106	0.507
dur	0.688	0.364
edhealth	0.703	0.336
fin	0.705	0.329
gov	0.640	0.291
info	0.703	0.275
mining	1.236	0.262
nondur	0.779	0.391
other	0.848	0.441
profserv	1.077	0.372
trade	1.009	0.430
trans	0.862	0.439

TABLE A6. Matching function parameter estimates. Based on monthly hiring, unemployment, and vacancy data from Jan 2000 to Feb 2023.

Finally, we use the sector level proportion of HR workers as a proxy for the recruiter producer ratio. The resulting recruiter producer ratios are reported below

	τ_i
accom	0.002
const	0.002
dur	0.007
edhealth	0.005
fin	0.008
gov	0.011
info	0.013
mining	0.005
nondur	0.007
other	0.018
profserv	0.020
trade	0.003
trans	0.001

TABLE A7. Estimated recruiter producer ratios based on the number of HR workers in industry i over total employment in industry i .

C.3. Computing Occupational Worker Share

For our occupational labor market calibration, we need to compute ε_N^f , which is the occupational worker elasticity of production. To do this, we obtain wage and employment data for ONET major occupations at 3-digit sector level from the Occupational Employment and Wage Statistics (OES). For each sector i , we compute $\varepsilon_{N_{io}}^f$ as:

$$\varepsilon_{N_{io}}^f = \varepsilon_{Ni}^f \frac{w_{io} L_{io}}{\sum_o w_{io} L_{io}},$$

where ε_N^f is the labor share we obtained earlier from the input-output table.

Table A8 contains our calibration estimates.

	Admin	Agg	Arts	Bus Ops	Care	Clean	Cons	Educ	Eng	Food S	Health P	Health S	Legal	Manag	Math	Prod	Prot S	Repair	Sales	Science	Soc S	Trans
accom	1.7	0.0	0.9	0.8	1.9	1.6	0.0	0.2	0.0	23.9	0.1	0.0	0.0	3.5	0.1	0.3	0.4	0.7	1.4	0.0	0.0	0.8
const	2.8	0.0	0.1	3.0	0.0	0.1	22.9	0.0	0.7	0.0	0.0	0.0	0.0	5.6	0.1	0.6	0.0	3.4	1.0	0.1	0.0	1.1
dur	1.9	0.0	0.2	2.3	0.0	0.1	0.6	0.0	4.0	0.0	0.0	0.0	0.1	4.6	1.9	11.5	0.0	1.3	1.2	0.2	0.0	1.4
edhealth	4.1	0.0	0.4	1.4	0.6	0.8	0.1	12.9	0.0	0.7	17.8	5.3	0.0	4.3	0.7	0.1	0.3	0.4	0.1	0.6	2.1	0.4
fin	7.2	0.0	0.2	10.6	0.1	0.3	0.1	0.0	0.1	0.0	0.3	0.0	0.6	8.4	3.2	0.0	0.1	1.3	6.3	0.0	0.1	0.3
gov	6.8	0.1	0.5	8.6	0.6	0.8	2.3	1.0	2.6	0.2	4.0	0.5	2.7	6.6	2.4	0.9	12.0	2.3	0.2	2.5	2.9	2.1
info	2.4	0.0	4.4	4.0	0.2	0.0	0.1	0.1	0.6	0.1	0.0	0.0	0.3	7.2	10.3	0.1	0.0	2.0	3.2	0.0	0.0	0.2
mining	1.3	0.0	0.0	1.4	0.0	0.0	7.3	0.0	1.8	0.0	0.0	0.0	0.1	3.7	0.4	1.4	0.0	1.9	0.6	0.7	0.0	2.4
nondur	1.8	0.1	0.2	1.2	0.0	0.2	0.1	0.0	1.0	0.3	0.0	0.0	0.0	3.3	0.3	9.8	0.0	1.7	1.2	0.8	0.0	2.3
other	6.4	0.0	1.7	6.1	8.0	0.8	0.2	0.8	0.3	0.5	0.3	0.7	0.3	9.4	1.0	2.4	0.3	9.5	1.9	0.3	1.6	3.4
profserv	5.4	0.0	1.1	8.6	0.1	2.3	0.7	0.1	3.4	0.1	1.2	0.3	3.1	10.5	7.9	1.0	1.1	0.7	2.2	1.2	0.1	1.7
trade	3.9	0.1	0.5	1.7	0.1	0.2	0.1	0.0	0.2	0.7	1.6	0.1	0.0	5.0	0.8	1.1	0.1	2.3	15.2	0.1	0.0	6.0
trans	6.2	0.0	0.0	0.4	0.0	0.2	0.4	0.0	0.4	0.0	0.0	0.0	0.0	0.5	0.2	0.9	0.0	2.9	0.5	0.1	0.0	18.2

TABLE A8. Occupational worker elasticity of output, in percentage terms, rounded to 1 decimal place.

C.4. Imputing Occupation Labor Market Parameters

For the occupational labor market specification, we need to calibrate unemployment, vacancy, and tightness of each occupation. We currently don't have access to occupational labor market characteristics, so we instead impute these parameters. For simplicity, we assume the total number of unemployment and vacancy for an occupation is the sum of unemployment and vacancy across sectors, weighted by the sectors' wage shares of that particular occupation:

$$V_o = \sum_i \frac{\varepsilon_{N_i o}^f}{\varepsilon_{N_i}^f} V_i,$$

$$H_o = \sum_i \frac{\varepsilon_{N_i o}^f}{\varepsilon_{N_i}^f} H_i,$$

where V denotes vacancy and H denotes labor supply, which is equivalent to unemployment in our static setup. The intuition behind this is that each sector's contribution to vacancy postings and the number of people looking for jobs in that sector for an occupation is proportional to how much the sector relies on that occupation.

Note that, with this imperfect simplifying assumption, we can back out changes in sectoral tightness. For sector j , the first-order response in tightness is: Tightness for sector j is:

$$\begin{aligned} \theta_j &= \frac{V_j}{H_j} = \frac{\sum_o \frac{V_{jo}}{V_o} V_o}{\sum_o \frac{H_{jo}}{H_o} H_o} \\ \Rightarrow d \log \theta_j &= d \log V_j - d \log H_j \\ &= \sum_o \frac{V_o}{V_j} \frac{V_{jo}}{V_o} d \log V_o - \frac{H_o}{H_j} \frac{H_{jo}}{H_o} d \log H_o \\ &= \sum_o \frac{V_{jo}}{V_j} d \log V_o - \frac{H_{jo}}{H_j} d \log H_o \\ &= \sum_o \frac{\varepsilon_{N_j o}^f}{\varepsilon_{N_j}^f} d \log \theta_o. \end{aligned}$$

Additionally, we compute the recruiter-producer ratio, the matching elasticities through a similar weighting approach.

Appendix D. Additional Results on Calibrated Shock Propagation.

Below, we report the responses of output and unemployment to technology shocks in each of the 13 sectors in our calibration.

D.1. Responses to technology shocks across sectors: One Occupation Per Sector

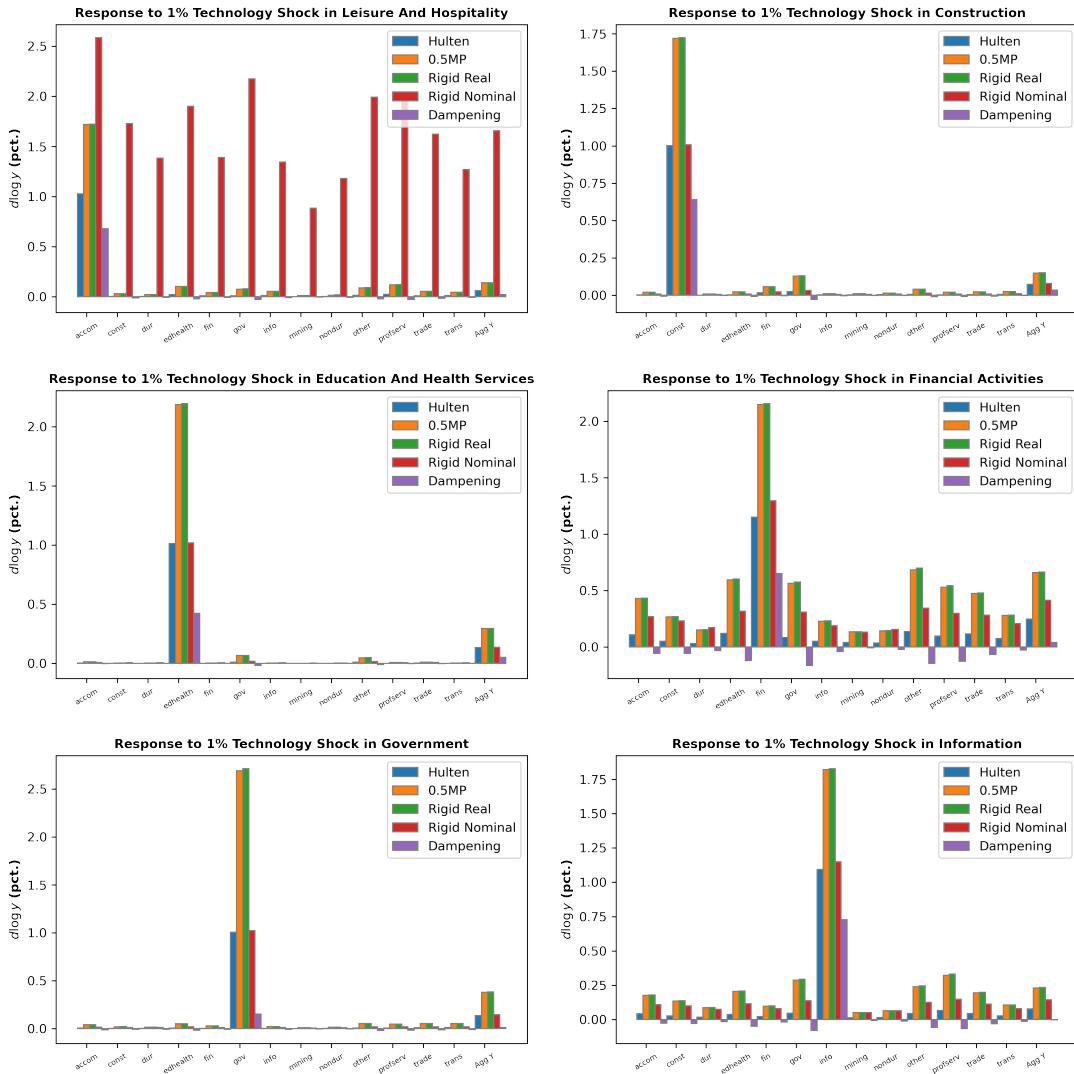


FIGURE A1. Responses of output to 1% technology shocks other sectors.

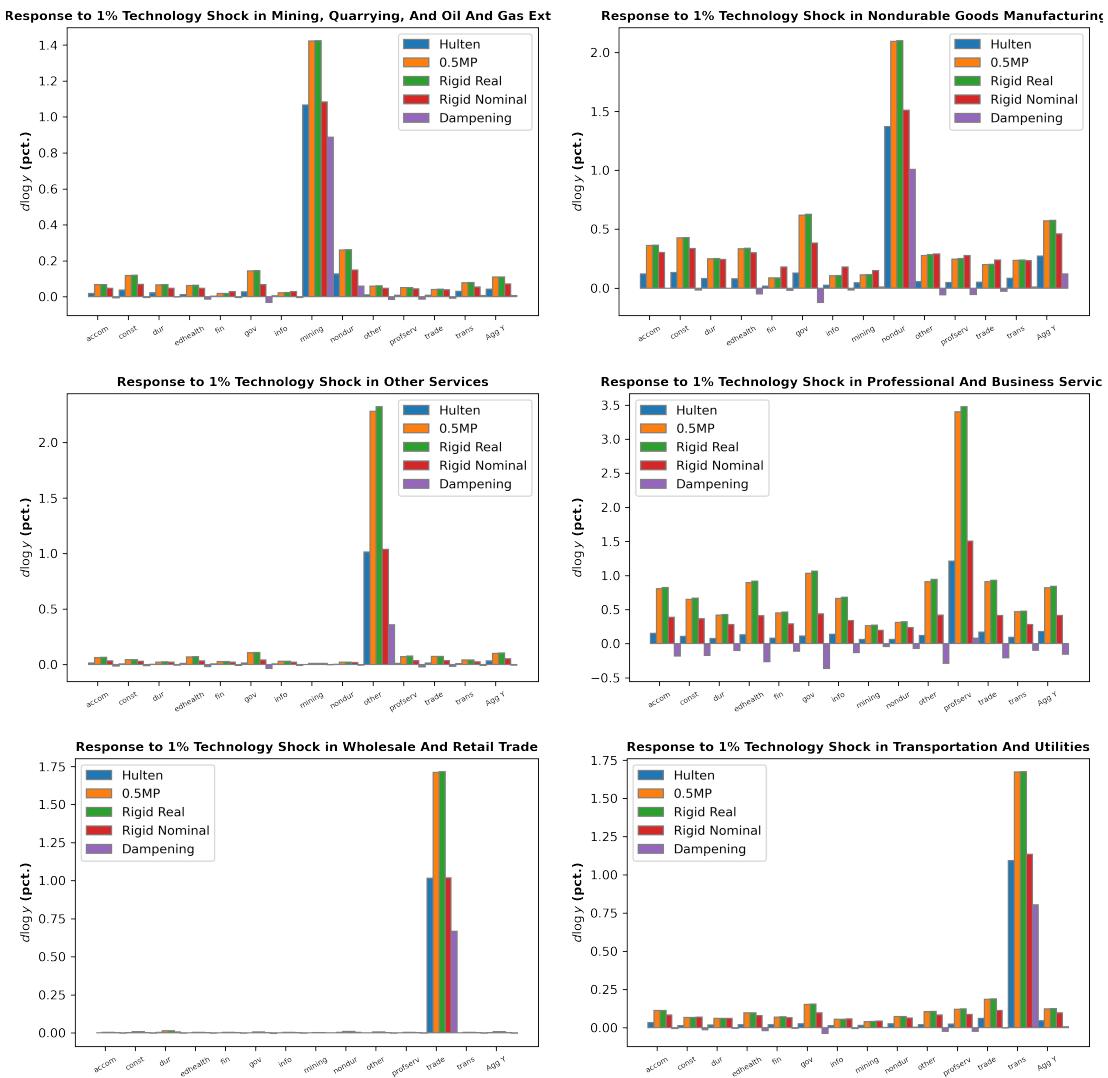


FIGURE A2. Responses of output to 1% technology shocks other sectors.

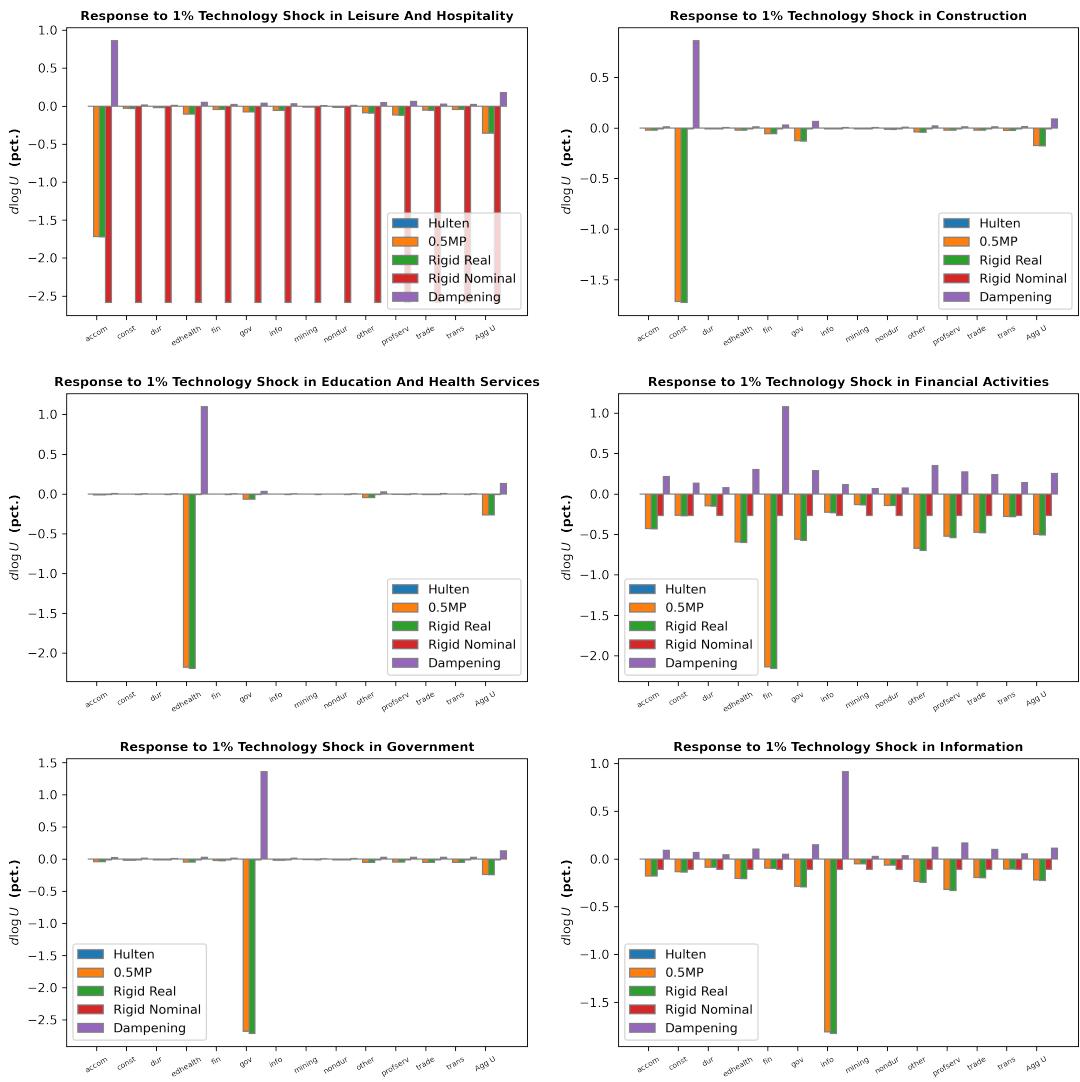


FIGURE A3. Responses of unemployment to 1% labor force shocks other sectors.

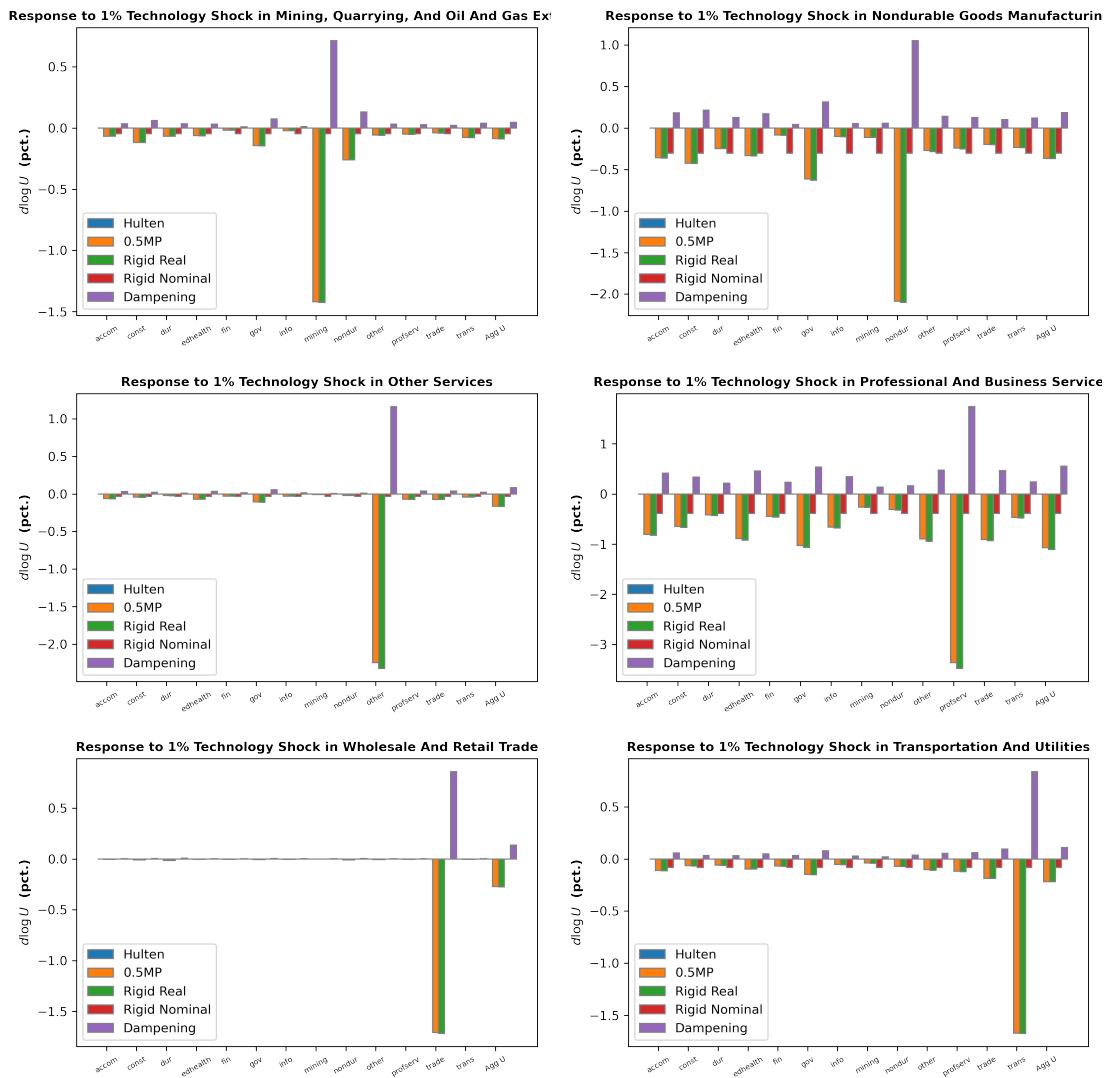


FIGURE A4. Responses of unemployment to 1% labor force shocks other sectors.

D.2. Responses to technology shocks across sectors: Major Occupations

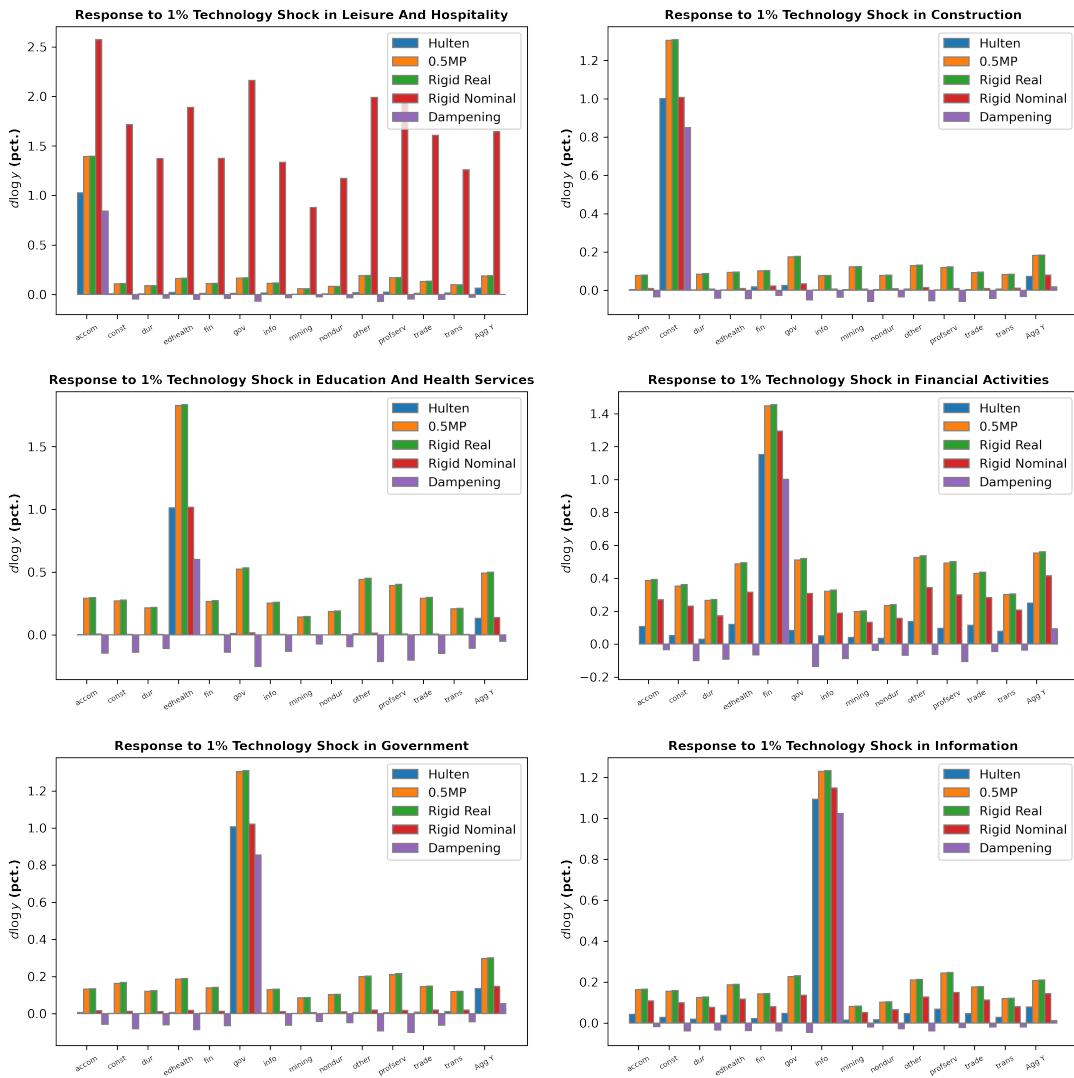


FIGURE A5. Responses of output to 1% technology shocks other sectors.

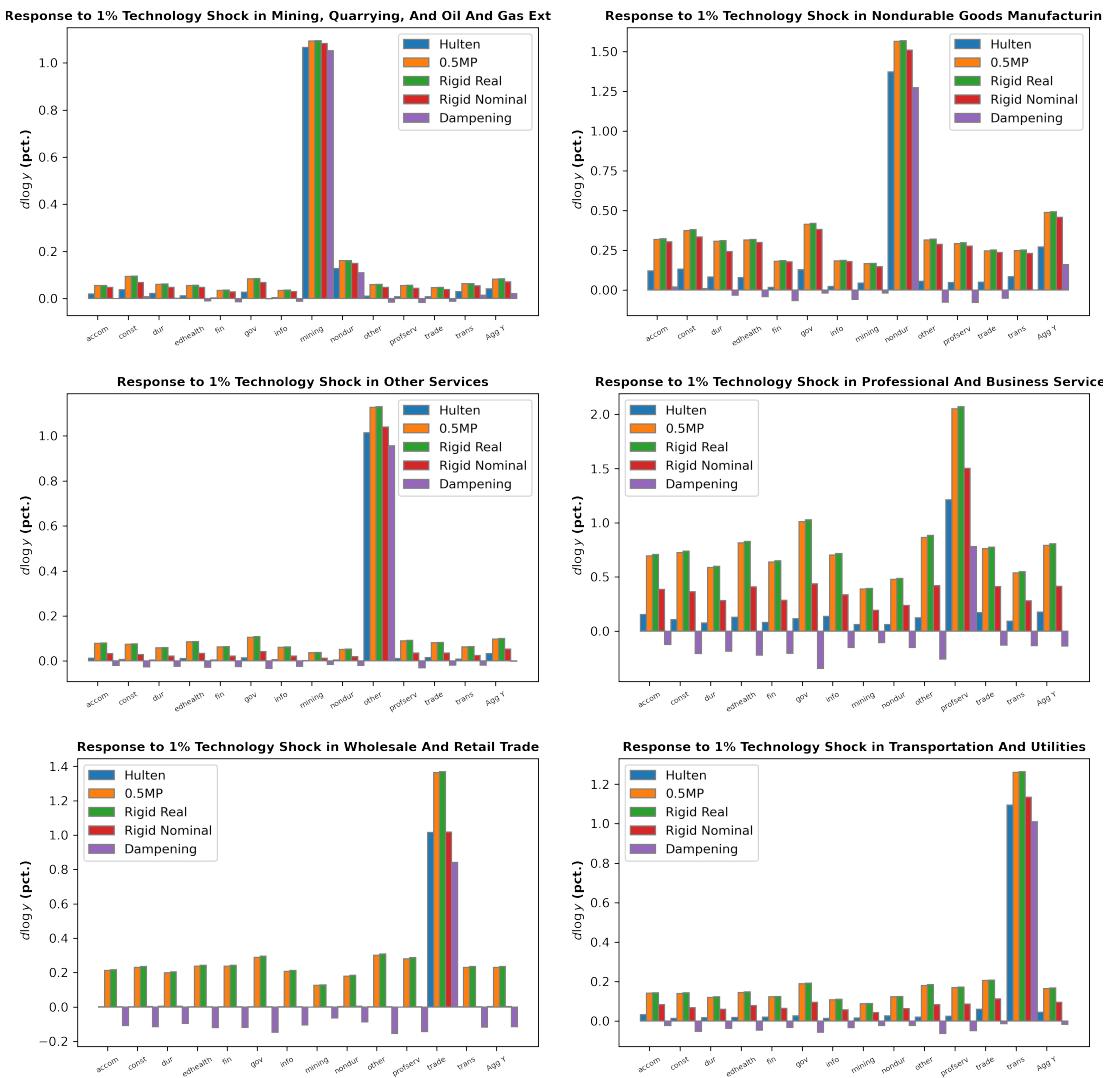


FIGURE A6. Responses of output to 1% technology shocks other sectors.

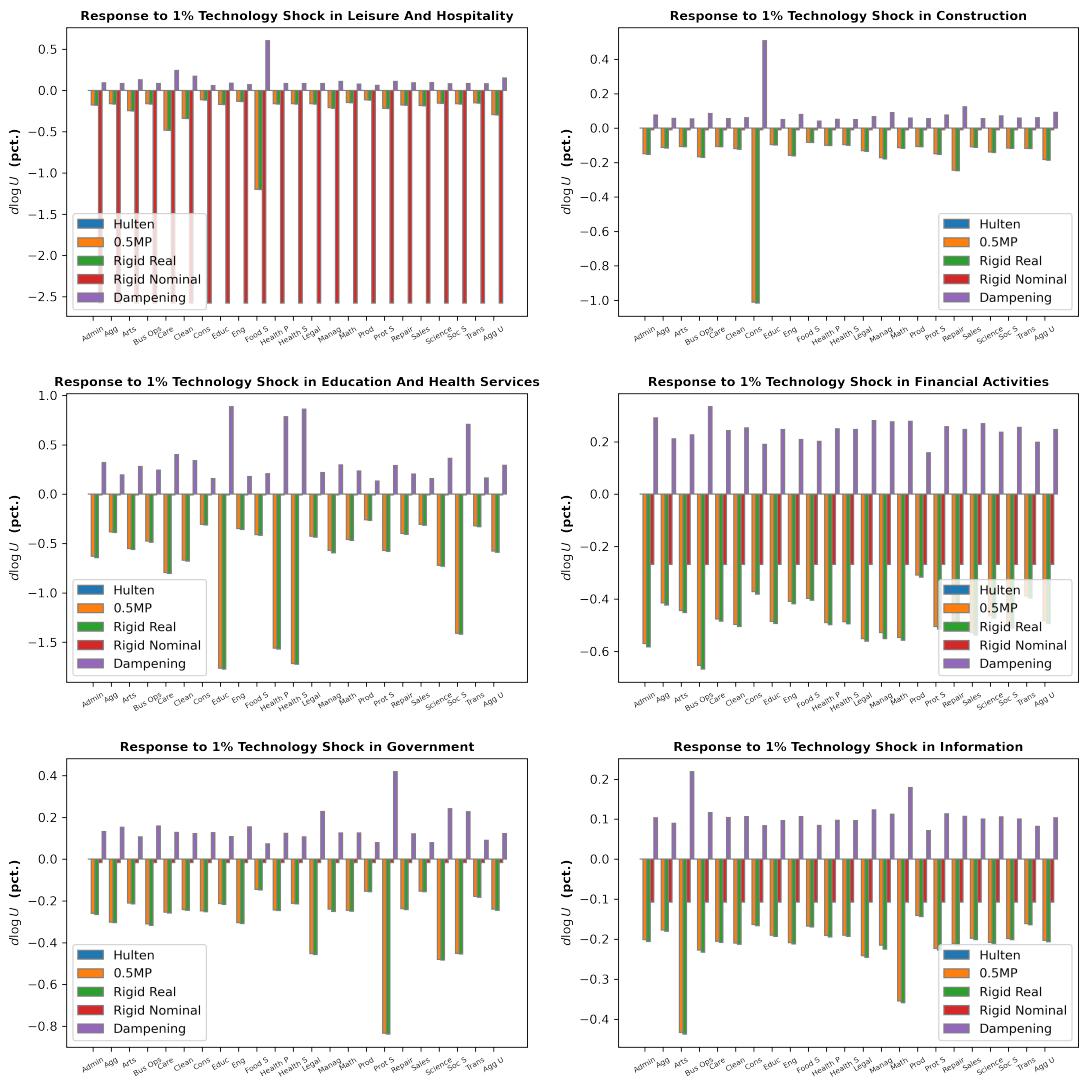


FIGURE A7. Responses of unemployment to 1% labor force shocks other sectors.

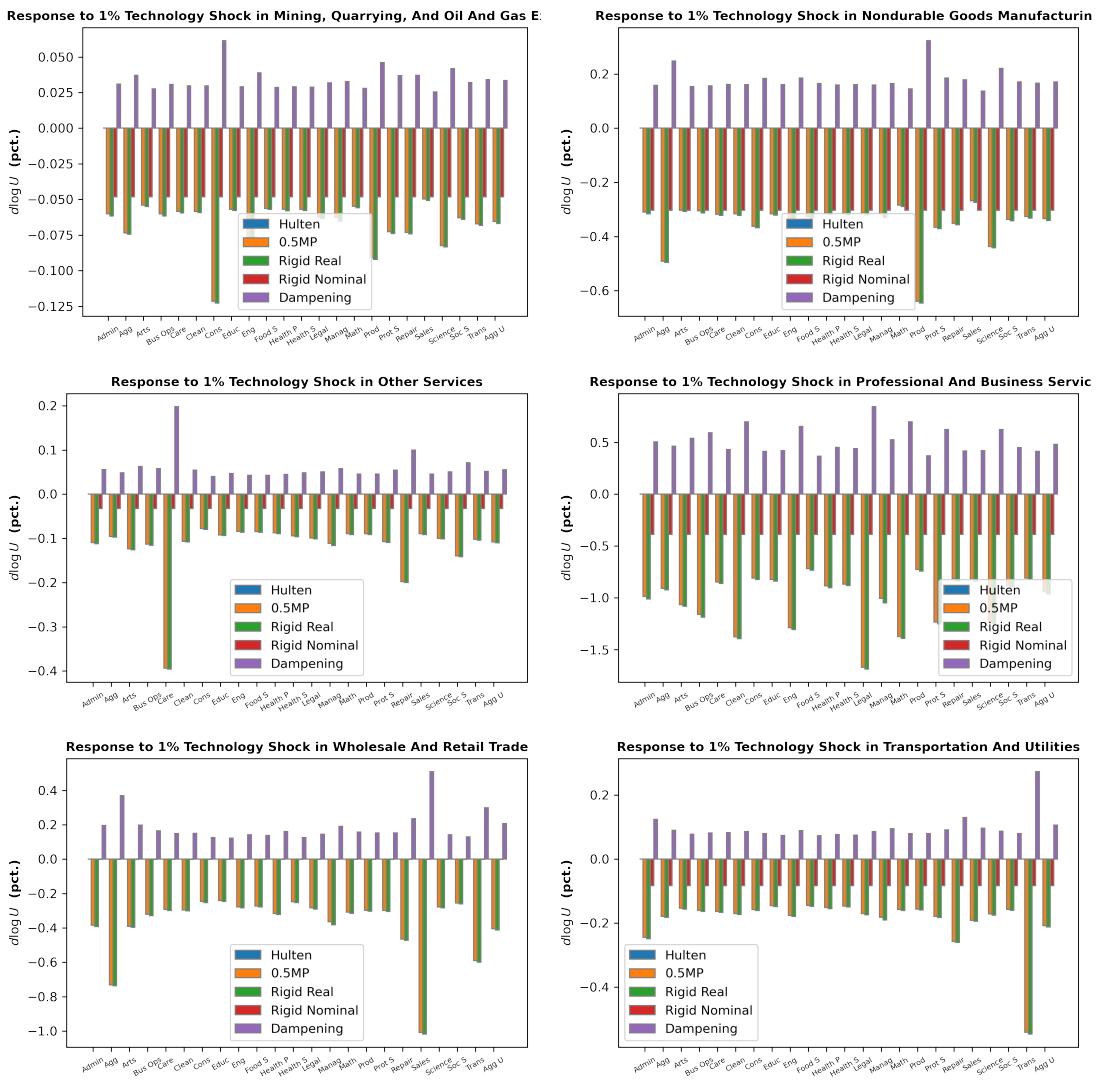


FIGURE A8. Responses of unemployment to 1% labor force shocks other sectors.