Mismatch Unemployment in Production Networks

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Abstract

We explore the optimal allocation of unemployment across sectors in an economy with matching frictions and input-output linkages. We show that a social planner seeking to maximize the utility of a representative household allocates more job seekers to industries with higher matching efficiency, more vacancies, higher reliance on labor, and industries that are more important to the production of final consumption goods. The social planner allocates fewer job seekers to industries with a large existing stock of workers. Using their data on vacancies, unemployment, and matching parameters at roughly the two-digit NAICS sector level from 2001 to 2011, we replicate the mismatch index in Şahin et al. (2014) as a horizontal economy special case. We combine this data with input-output linkages from the 2007 BEA input-output tables to estimate mismatch accounting for the US network structure. Once we account for networks, the unemployment allocation from 2001 to 2011 was on average slightly closer to efficient. In addition, the sign of the mismatch index flips from positive to negative: Something that is possible in our network setup, but not in the horizontal economy with linear production technology Şahin et al. (2014) use. We find that the positive correlation between a high existing stock of workers and high vacancies at the industry level drive our finding of a negative mismatch index. We view these results as very preliminary and likely the result of abstracting from endogenous vacancy creation and sector specific productivity.

Brown University. This is our final project for ECON2416 at Harvard University.

1. Introduction

What is the optimal distribution of unemployment across sectors in an economy featuring production networks? And how far are we from this optimal allocation in the Unites States? In other words, how large is unemployment mismatch once we account for production networks? We seek to answer these questions by extending existing results on mismatch unemployment, which generally assume a horizontal economy–an economy where each sector operates in isolation–to account for production linkages. Production linkages can potentially change the optimal allocation of unemployment because employment in one sector no longer only changes output in that sector, but potentially output in other sectors as well. Sectors that are more important in the production of other goods throughout the economy should presumably receive a larger weight in optimal unemployment allocation.

Interest in unemployment mismatch–unemployed workers are potentially seeking jobs in industries or locations where there are simply not enough vacancies–goes back at least to the 1980s, when economists were attempting to understand persistently high unemployment in many European countries. Jackman and Roper (1987), for instance, show that the unemployment allocation that maximizes aggregate output in a static economy with multiple sectors equalizes labor market tightness across sectors. The slow recovery of unemployment following the great recession sparked renewed interest in the mismatch between vacant jobs and unemployed workers. Şahin et al. (2014) extend the Jackman and Roper result to a dynamic context with numerous sources of heterogeneity. In their context, the optimal unemployment allocation–the allocation that maximizes social welfare–equalizes the marginal contribution to the hiring process across industries. That is, the social planner would choose to allocate more job seekers to labor markets with higher matching efficiency and more vacancies.

In this paper, we extend the analysis further by accounting for complex input-output linkages that arise in a production network. Following Long and Plosser (1983) and Acemoglu et al. (2012), we incorporate matching frictions into a static, multisector production network model that features a representative household, production sectors, Cobb-Douglas production functions, and perfect competition. We show that the optimal unemployment allocation now depends in a complex way on the structure of the production network. In a vertical economy, for instance, it is optimal to allocate more job searchers to sectors that are more important inputs into downstream production. Once the production network becomes more complex, so does the effect on optimal unemployment. In a general network, optimal unemployment depends on how important each sector is to production in all other sectors, weighted by their contribution to social welfare. However, assuming constant returns to scale Cobb-Douglas production technologies and exogenous production networks, we derive a Hulten-like result for the impact of unemployment deviations on the economy, and show that the dependence on the production network can be summarized in by industry sales shares. The industry sales shares are jointly determined by household preferences and the network structure, and are conveniently observable. Both the horizontal and vertical economy optimality conditions are just special cases of our more general result.

Having the theoretical results for network-adjusted optimal sectoral unemployment, we want to examine the extent to which mismatch affects aggregate unemployment. We compute a mismatch index, which captures how sectoral hires in the U.S economy deviates from the optimal hires, from 2001 to 2011. We use survey-based vacancy and hire data from JOLTs, unemployment data from the CPS, and industry sales shares from the BEA. We also estimate sector-specific matching efficiencies following Borowczyk-Martins et al. (2013). All of our analyses are conducted at an industry level roughly corresponding to the NAICS two-digit codes.

We find that allowing for a production network slightly reduces how far the labor market is from efficiency from 2001 to 2011 relative to the horizontal economy benchmark. Allowing for production networks also flips the sign of the mismatch index, something that is possible once we allow for production linkages and decreasing marginal product of labor, but not in a horizontal economy with linear production technology. A negative mismatch index suggests that the optimal allocation of unemployment across sectors would actually tend to reduce and not increase the number of hires. We think it is important not to read too much into these results as they are extremely preliminary. Allowing for endogenous vacancy creation or productivity differences across sectors, neither of which we do in this draft, could flip the sign of our mismatch index.

In addition to its contribution to the mismatch literature, this paper fits closely to the recent development that attempts to bring more realism into production networks by incorporating market imperfections and inefficiencies. Jones (2013) and Bigio and La'O (2020) model frictions as an exogenous wedge between the sectors' marginal costs and marginal revenues and examine how these frictions interact with the network structure and affect aggregate output in the Cobb-Douglas economy. Bigio and La'O (2020) bring the model to data and estimate how the US input-output structure amplify financial distortions in the great recession. Baqaee and Farhi (2020) examine how productivity shocks aggregate under the presence of exogenous wedges in a CES economy and decompose output change as a component that comes from the change in technology, and a component that comes from the change in allocative efficiency. Liu (2019) assumes that market imperfections generate dead-weight losses, and these imperfections compound in the production network through demand linkages. He shows that a government should design industrial policies to target sectors based on distortions in sectoral size. To our knowledge, our paper is the first to model searching and matching in a production networks setting.

The remainder of the paper is organized as follows. Section 2 introduces the mismatch in a baseline economy with no production linkage. Section 3 computes the optimal sectoral unemployment allocation in a vertical economy. Section 4 extends the results for the vertical economy to an economy with arbitrary production linkages. Section 5 calibrates the model with U.S data and quantifies the contribution of mismatch to aggregate U.S unemployment. Section 6 concludes by discussing next steps.

2. No production networks benchmark

In this section, we derive efficient unemployment in an environment without production linkages. The setup is a static version of the baseline economy in Şahin et al. (2014). Consider an economy consisting of N industries, indexed by i, populated by a measure one of households who supply labor inelastically. Households can be either employed (e_i) or unemployed and searching (u_i) in at most one industry i. We rule out on-the-job search and searching in multiple industries. We also assume, for simplicity, that all households would choose to work if they could and have no choice of which industry to work in. The only source of unemployment are matching inefficiencies introduced by the search process.

As in the baseline economy in Şahin et al. (2014), vacancies (v_i) are exogenous. New matches between vacancies and unemployed workers are generated by a concave, homogeneous of degree one, matching function $h_i = \phi_i m(v_i, u_i)$, where h_i denotes new hires in industry i and ϕ_i is the industry specific matching efficiency.

Firms produce output using constant returns to scale production technology with homogeneous productivity across industries. For now, labor is the only input into production $y_i = L_i = e_i + h_i$.

We analyze the problem of a social planner who wants to maximize total output, which in this simple setup is equivalent to maximizing the utility of a representative household with increasing utility in total consumption. The social planner can move around unemployed workers between industries to pick an optimal level of unemployment u_i^*, given vacancies and matching efficiencies in each industry, but cannot otherwise change the economy environment.

The social planners problem is

$$\begin{aligned} &\max_{\{u_i^*\}_{i=1}^N} \sum_{i=0}^{N} L_i, \\ &\text{s.t. } L_i = e_i + h_i, \\ &h_i = \phi_i m(u_i, v_i), \\ &1 = \sum_{i=1}^{N} [e_i + u_i]. \end{aligned}$$

The first order condition of this problem implies that the social planners optimal level of unemployment in each sector satisfies

$$\phi_i m_u \left(u_i^*, v_i\right) = \phi_j m_u \left(u_j^*, v_j\right).$$

Since m is constant returns to scale, we can express this optimality condition as

$$\phi_i m_u \left(\frac{v_i}{u_i^*}\right) = \phi_j m_u \left(\frac{v_j}{u_j^*}\right).$$

where $m_u\left(u_i^*, v_i\right) = m_u\left(\frac{v_i}{u_i^*}\right)$. This is the static equivalent of the baseline optimality condition in Şahin et al. (2014)[1].

[1]Assuming Cobb-Douglas utility and defining the social planners objective as maximizing utility, the optimality condition becomes

$$\frac{\theta_i \phi_i m_u \left(u_i^*, v_i\right)}{L_i^*} = \frac{\theta_j \phi_j m_u \left(u_j^*, v_j\right)}{L_j^*}$$

In this case, the relevant expression for the mismatch index is the same as above, but with

$$\overline{\phi} = \sum_{i=1}^{N} \left[\phi_i \frac{\theta_i}{\Theta} \frac{L}{L_i^*}\right]^{\frac{1}{\eta}} \frac{v_i}{v}$$

Assuming a Cobb-Douglas matching function ${}_i m(u_i, v_i) = \phi_i v_i^{\eta} u_i^{1-\eta}$, we can write

$$\phi_i \left(\frac{v_i}{u_i^*}\right)^{\eta} = \phi_j \left(\frac{v_j}{u_j^*}\right)^{\eta}.$$

2.1. A mismatch index

Let v, u be aggregate vacancies and unemployment. Summing across markets, the aggregate number of hires h is

$$\begin{aligned} h &= \sum_{i=1}^{N} \phi_i v_i^{\eta} u_i^{1-\eta} \\ &= v^{\eta} u^{1-\eta} \sum_{i=1}^{N} \phi_i \left(\frac{v_i}{v}\right)^{\eta} \left(\frac{u_i}{u}\right)^{1-\eta} \end{aligned}$$

The number of hires that would result if unemployment were allocated optimally across sectors is

$$\begin{aligned} h^* &= \sum_{i=1}^{N} \phi_i v_i^{\eta} u_i^{*1-\eta} \\ &= v^{\eta} u^{1-\eta} \sum_{i=1}^{N} \phi_i \left(\frac{v_i}{v}\right)^{\eta} \left(\frac{u_i^*}{u}\right)^{1-\eta} \end{aligned}$$

Plugging in to this expression using the optimality condition evaluated at the aggregate levels of output, unemployment, and the matching efficiency gives

$$h^* = \overline{\phi} u^{1-\eta} v^{\eta},$$

where $\overline{\phi} = \sum_{i=1}^{N} \phi_i^{\frac{1}{\eta}} \frac{v_i}{v}$. Sahin et al. define the following mismatch index, a measure of the employment lost due to the inefficient allocation of unemployed workers across sectors.

$$\mathcal{M}_B = 1 - \frac{h}{h^*} = 1 - \sum_{i=1}^{N} \frac{\phi_i}{\overline{\phi}} \left(\frac{v_i}{v}\right)^{\eta} \left(\frac{u_i}{u}\right)^{1-\eta}$$

Notice, in principle every element in this mismatch index is observable in the data, once we have estimates of the parameters of the matching function.

3. Efficient unemployment in a vertical economy

In this section, we extend the results above to a vertical economy: an economy where industry i uses the output of industry $i - 1$ as an input into its own production. We assume that households consume only the output of the top level firm N and that the output of all other firms are entirely used up by the production of the firm immediately downstream from them in the network. Firms in each sector use constant returns to scale Cobb-Douglas technology in labor and the output of the firm in industry $i - 1$. All other aspects of the economic environment are identical to the setup above.

3.1. A two sector example

We begin with a simple two sector example to develop some intuition. Consider an economy with two sectors, $i = \{1, 2\}$, where sector 2 is downstream of sector 2.

$$
\begin{aligned}
y_1 &= L_1, \\
y_2 &= L_2^{\alpha_2} y_1^{a_{21}},
\end{aligned}
$$

where $\alpha_2 + a_{12} = 1$. a_{21} denotes the importance of sector 1 output in sector 2 production.

The social planner chooses unemployment to maximize the output of final goods in the economy, as these are the only goods relevant for consumption, and therefore for the utility of the representative household.

$$
\begin{gathered}
\max_{\{u_1, u_2\}} L_2^{\alpha_2} L_1^{a_{21}}, \\
\text{s.t. } L_i = e_i + h_i, \\
h_i = \phi_i m(u_i, v_i), \\
\sum_{i=1}^{2} (e_i + u_i) = 1.
\end{gathered}
$$

The first order conditions of the social planners problem imply

$$
\begin{aligned}
a_{21} L_2^{*\alpha_2} L_1^{*a_{21}-1} \phi_1 m_u(u_1^*, v_1) &= \alpha_2, L_2^{*\alpha_2 - 1} L_1^{*a_{21}} \phi_2 m_u(u_2^*, v_2), \\
\Rightarrow \frac{a_{21} y_2^*}{L_1^*} \phi_1 m_u(u_1^*, v_1) &= \frac{\alpha_2 y_2^*}{L_2^*} \phi_2 m_u(u_2^*, v_2), \\
\Rightarrow \frac{a_{21}}{L_1^*} \phi_1 m_u(u_1^*, v_1) &= \frac{\alpha_2}{L_2^*} \phi_2 m_u(u_2^*, v_2).
\end{aligned}
$$

As the importance of the upstream industry in downstream production increases (as a_{21} increases), it becomes optimal to allocate more unemployed workers to the upstream sector.

3.2. Extension to N sectors

Now, suppose that there are once again N sectors, organized in a vertical economy.

$$
\begin{aligned}
y_1 &= L_1, \\
y_2 &= L_2^{\alpha_2} y_1^{a_{21}}, \\
&\vdots \\
y_N &= L_N^{\alpha_N} y_{N-1}^{a_{N,N-1}}.
\end{aligned}
$$

The social planner chooses unemployment in each sector to maximize the production of consumption goods, y_N.

$$
\begin{aligned}
&\max_{\{u_i\}_{i=1}^N} L_N^{\alpha_N} y_{N-1}^{a_{N,N-1}}, \\
&\text{s.t. } L_i = e_i + h_i, \\
&h_i = \phi_i m(u_i, v_i), \\
&\sum_{i=1}^{N} (e_i + u_i) = 1.
\end{aligned}
$$

The FOCs are now:

$$
\frac{\alpha_i \prod_{k=i+1}^{K} a_{k,k-1}}{L_i^*} \phi_i m_u \left(u_i^*, v_i\right) = \frac{\alpha_j \prod_{k=j+1}^{K} a_{k,k-1}}{L_j^*} \phi_j m_u \left(u_j^*, v_j\right).
$$

4. Efficient Unemployment in a general production network

In this section, we derive a Hulten-theorem equivalent for labor changes. That is, we look at how aggregate output responds to changes in labor allocation. The change in labor allocation is pinned down by by changes in unemployment allocation. The social planner's task will then be to pick the unemployment allocation that maximizes household utility. We assume vacancy postings to be exogenous, and will extend postings to be endogenous in future iterations of this paper.

4.1. The Household's Problem

A representative household with Cobb-Douglas utility in the i sector outputs chooses how much of each good to consume to maximize utility

$$\begin{aligned} &\max_{\{C_i\}_{i=1}^N} u\left(\{C_i\}_{i=1}^N\right) = \max_{\{C_i\}_{i=1}^N} \prod_{i=1}^N C_i^{\theta_i}, \\ &\text{s.t. } p_i C_i = wL. \end{aligned}$$

where $\sum_{i=1}^N \theta_i = 1$, w is the wage taken as given by all households and firms, and L is a aggregate labor supply.

4.2. The Sector's Problem

Sector i produces output that can be either consumed or used as a factor of production. Goods market clearing in sector i's requires:

$$y_i = C_i + \sum_j x_{ji}.$$

We assume that the economy is populated by a measure one of risk neutral individuals who can either be employed in sector i (e_i) or be unemployed and searching in sector i (u_i). Labor markets are frictional: matches between unemployed workers and vacancies in sector i (v_i) are generated by a strictly increasing, concave, and homogeneous of degree one in unemployment and vacancies matching function m with sectoral matching efficiency ϕ_i. For now, we assume that vacancies are created exogenously. The number of new hires generated by the matching technology is $h_i = \phi_i m(u_i, v_i)$.

Firms in industry i produce output using Cobb-Douglas production technology in labor (L_i) and intermediate inputs from other sectors (x_{ij}).

$$y_i = L_i^{\alpha_i} \prod_{j=1}^{N} x_{ij}^{a_{ij}},$$

where a_{ij} is the importance of good j in the production of good i, which we assume is constant over time, and x_{ij} is the amount of good j used in the production of good i.

Given the matching technology m, exogenously created vacancies, and unemployed searching workers in each sector, the labor input in sector i is $L_i = e_i + h_i$. Firms choose how much of each good to use in production, taking their labor input and prices in sector j, (p_j) as given. Profit maximization on the part of firms pins down x_{ij}.

$$\max_{\{x_{ij}\}} p_i L_i^{\alpha_i} \prod_{j=1}^{N} x_{ij}^{a_{ij}} - \sum_j p_j x_{ij} - wL_i$$
$$\Rightarrow x_{ij} = a_{ij} \frac{p_i}{p_j} y_i$$

Or:

$$p_j x_{ij} = a_{ij} p_i y_i$$

4.3. Summary and Price Determination

An equilibrium is a collection of consumption choices, input choices, and price $\left\{C_i, \{x_{ij}\}_{j=1}^{K}, p_i\right\}_{i=1}^{K}$ such that given $\left\{\theta_i, \{a_{ij}\}_{j=1}^{K}, \alpha_i, L_i\right\}_{i=1}^{K}$

(i) Firms are profit maximizing, which requires the following FOCs hold (n^2 restrictions)

$$a_{ij} = \frac{p_j x_{ij}}{p_i y_i} \qquad \forall i, j$$

(ii) Households' are utility maximizing, which requires the following FOCs hold ($n-1$

restrictions)

$$\frac{\theta_i}{\theta_j} = \frac{p_i C_i}{p_j C_j} \qquad \forall i, j$$

(iii) Market clearing holds (n restrictions)

$$y_i = C_i + \sum_{j=1}^{K} x_{ji} \qquad \forall i$$

Together, these conditions provide $n^2 + 2n - 1$ restrictions on the $n^2 + 2n$ unknowns: they pin down all but one of the prices. We arbitrarily pick p_1 as the numeraire and set $p_1 = 1$.

4.4. Sales Shares

Let $\gamma_i = \frac{p_i y_i}{G}$, where $G = \sum_j p_j C_j = GDP = wL$, denote the final sales share of GDP for sector i. We have that:

$$p_j x_{ij} = a_{ij} \gamma_i G.$$

From household's maximization problem, I have that $p_i C_i = \theta_i G$. Combining the two gives:

$$\gamma_j G = p_j y_j = p_j (C_j + \sum_i x_{ij}) = \left(\theta_j + \sum_i a_{ij} \gamma_i \right) G$$
$$\Rightarrow \gamma = (I - A')^{-1} \theta,$$

which means that the γs are fixed.

4.5. Response to Labor Shocks

Let $\hat{p}$ denote the price adjustment. Given a fixed $\gamma_i = \frac{p_i Y_i}{G}$, I have that:

$$\hat{p}_i + \hat{y}_i = \hat{G}.$$

Also, from $p_j x_{ij} = a_{ij}\gamma_i G$, I have that:

$$\begin{aligned} \hat{p}_j + \hat{x}_{ij} = \hat{G} = \hat{p}_j + \hat{y}_j, \\ \Rightarrow \hat{y}_j = \hat{x}_{ij}. \end{aligned}$$

Thus, holding productivity fixed:

$$\begin{aligned} \hat{y}_i &= \sum_j a_{ij}\hat{x}_{ij} + \alpha_i \hat{L}_i = \sum_j a_{ij}\hat{y}_j + \alpha_i \hat{L}_i, \\ \Rightarrow \hat{y} &= (I - A)^{-1}\alpha \hat{L}, \end{aligned}$$

where

$$\alpha = \text{diag}(\alpha_1, \alpha_2, \cdots, \alpha_K).$$

Note that $L_i = e_i + h_i$. This gives that $\hat{L_i} = \frac{e_i}{L_i}\hat{e}_i + \frac{h_i}{L_i}\hat{h}_i$.

$$\begin{aligned} \hat{L_i} &= \frac{e_i}{L_i}\hat{e}_i + \frac{h_i}{L_i}\hat{h}_i \\ &= \frac{\Phi\phi_i m(u_i, v_i)}{L_i}\hat{h}_i \\ &= \frac{\eta\Phi\phi_i m(u_i, v_i)}{L_i}\hat{u}_i, \quad \text{where } m(u_i, v_i) = u_i^{\eta} v_i^{1-\eta}. \end{aligned}$$

This gives us that:

$$\hat{y} = \eta(I - A)^{-1}B\hat{u},$$

where

$$B = \text{diag}\left(\frac{\alpha_1 h_1}{L_1}, \frac{\alpha_2 h_2}{L_2}, \ldots, \frac{\alpha_K h_K}{L_K}\right).$$

4.6. The Labor Shock Variant of Hulten's Theorem

From the household's problem, we have:

$$dU = \sum_i \theta_i \hat{C}_i = \sum_i \theta_i(\hat{G} - \hat{p}_i) = \sum_i \theta_i \hat{y}_i = \theta'\eta(I - A)^{-1}B\hat{u} = \eta\gamma' B\hat{u}. \tag{1}$$

Equation 1 is the labor shock variant of Hulten's theorem, which expresses the change in aggregate utility as a function of log unemployment deviations $\hat{u}$, their weight on deviations in labor B, and domar weights γ. The socially optimal allocation of unemployment must satisfy that the marginal contribution of unemployment changes in all sectors to the utility of the representative household are equal. In other words,

$$\frac{dU}{du_i} = \frac{dU}{du_j}$$

for all i, j's.

Thus, from equation 1, since $\hat{u}_i = \frac{du_i}{u_i}$, we have that:

$$dU = \eta \sum_{i=1}^{N} \gamma_i \frac{\alpha_i h_i}{L_i} \frac{du_i}{u_i}.$$

The socially optimal allocation of unemployment requires:

$$\frac{dU}{du_i} = \eta \gamma_i \frac{\alpha_i h_i}{L_i u_i},$$

and $\frac{dU}{du_i} = \frac{dU}{du_j}$ implies that:

$$\frac{\gamma_i \alpha_i \phi_i m(u_i^*, v_i)}{L_i^* u_i^*} = \frac{\gamma_j \alpha_j \phi_j m(u_j^*, v_j)}{u_j^* L_j^*},$$

$$\Rightarrow \frac{\gamma_i \alpha_i \phi_i m_u \left(u_i^*, v_i\right)}{L_i^*} = \frac{\gamma_j \alpha_j \phi_j m_u \left(u_j^*, v_j\right)}{L_j^*}. \tag{2}$$

For a Cobb-Douglas matching function, equation 2 is increasing in γ_i, α_i, ϕ_i, and v_i and decreasing in u_i^* and e_i. Therefore, the optimal allocation of unemployment assigns more searchers to sectors that rely more on labor (high α_i), have higher sales share (higher λ_i), have more efficient labor markets (higher ϕ_i), and more vacancies. The optimal allocation of unemployment assigns fewer searchers to sectors with a large existing stock of worker e_i. Unlike in a horizontal economy, the optimal unemployment allocation does not just reassign workers to maximize total hires. Instead, because output in each sector potentially depends on output in every other sector, the optimal unemployment allocation assigns more workers to those sectors that are most important to the network. As a result, it is possible for u^* to lead to fewer hires and lower employment.

Alternatively, let ρ_{ji} denote the element in the $j-th$ row and $i-th$ column in the Leontief inverse $(I-A)^{-1}$, where

$$(I-A)^{-1}=\sum_{k=0}^{\infty}A^k,$$

which implies that

$$\rho_{ji}=a_{ji}+\sum_{r=1}^{N}a_{jr}a_{ri}+\ldots,$$

with the first element corresponding to i's role as a direct supplier to j, and the second term capturing i's role to j's suppliers, so on and so forth.

From the second-to-last equality of equation , we have that:

$$dU=\eta\sum_{i=1}^{N}\lambda_i\frac{\alpha_i h_i}{L_i}\frac{du_i}{u_i}$$

where

$$\lambda_i=\sum_{j=1}^{K}\theta_j\rho_{ji}.$$

We can rewrite equation 2 as the following, connecting input-output linkages with sales shares:

$$\frac{\lambda_i\alpha_i\phi_i m_u\left(u_i^*,v_i\right)}{L_i^*}=\frac{\lambda_j\alpha_j\phi_j m_u\left(u_j^*,v_j\right)}{L_j^*},$$

The optimal allocation of unemployment assigns more searchers to sectors that more important to the overall production network weighted by importance to final consumption (higher λ_i). Connecting it back to equation 2, optimal unemployment allocations responds to changes in sales share, which can be driven by either changes in consumer tastes or changes in production linkages.

4.7. Mismatch index

The mismatch index is

$$\mathcal{M}_G = 1 - \frac{h}{h^*} = 1 - \sum_{i=1}^{N} \frac{\phi_i}{\overline{\phi}} \left(\frac{v_i}{v}\right)^{\eta} \left(\frac{u_i}{u}\right)^{1-\eta} \tag{3}$$

where

$$\overline{\phi} = \sum_{i=1}^{N} \left(\phi_i \frac{\gamma_i}{\Lambda} \frac{\alpha_i}{\alpha_{ag}} \frac{L}{L_i^*}\right)^{\frac{1}{\eta}} \frac{v_i}{v}$$

Unlike in the horizontal economy case, where $\mathcal{M}_B$ falls between 0 and 1, $\mathcal{M}_G$ can be negative.

4.8. Special Cases

Now that we derive the optimal unemployment allocation for the general setup, we now verify if our general formula applies to the special cases of vertical and horizontal economies.

4.8.1. Vertical Economy

In the vertical economy case $\theta_N = 1$ and $\theta_i = 0$ for all $i \neq N$, and the matrix A is:

$$A = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 \\ a_{21} & 0 & 0 & \cdots & 0 & 0 \\ 0 & a_{32} & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & a_{N,N-1} & 0 \end{bmatrix}.$$

$(I-A)^{-1}$ is therefore:

$$(I-A)^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ a_{21} & 1 & 0 & 0 & \cdots & 0 & 0 \\ a_{21}a_{32} & a_{32} & 1 & 0 & \cdots & 0 & 0 \\ a_{21}a_{32}a_{43} & a_{32}a_{43} & a_{43} & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \prod_{j=1}^{N} a_{j+1,j} & \prod_{j=2}^{N} a_{j+1,j} & \prod_{j=3}^{N} a_{j+1,j} & \prod_{j=4}^{N} a_{j+1,j} & \cdots & a_{N,N-1} & 1 \end{bmatrix}.$$

Define $\lambda_i = \sum_{j=1}^{N} \theta_i \rho_{ji}$. Then, in the vertical economy

$$\lambda_i = \theta_N \rho_{Ni} = \theta_N \prod_{j=i}^{N} a_{j+1,j}.$$

Using the notation above, the social planner's optimal unemployment allocation satisfies

$$\frac{\lambda_i \alpha_i \phi_i m_u(u_i^*, v_i)}{L_i^*} = \frac{\lambda_j \alpha_j \phi_j m_u(u_j^*, v_j)}{L_j^*}.$$

4.9. Horizontal Economy

In a horizontal economy, $(I-A)^{-1} = I$. This implies that $\lambda_i = \theta_i$. The general formula now implies:

$$\frac{\theta_i \alpha_i \phi_i m_u(u_i^*, v_i)}{L_i^*} = \frac{\theta_j \alpha_j \phi_j m_u(u_j^*, v_j)}{L_j^*}.$$

If we assume linear production technology, this implies that $\alpha_i = 1$ for all $i's$, the optimal unemployment requires:

$$\frac{\theta_i \phi_i m_u(u_i^*, v_i)}{L_i^*} = \frac{\theta_j \phi_j m_u(u_j^*, v_j)}{L_j^*}.$$

5. Data and Estimation

This section outlines our data requirements and suggested estimation strategy. The empirical statistics we wish to estimate is the mismatch index, given by equation 3. To compute our mismatch index, we need to compute the optimal number of hires, which is pinned down by the optimal allocation of sectoral unemployment, as shown in equation 2. Despite our results are generalized for arbitrary production networks, we can conveniently bypass the structure of the networks by looking at the observable sales shares. Some other key economic variables includes vacancies, hires, unemployment, and matching parameters.

For this iteration of the project, we are computing the network-adjusted mismatch index for the same period as Şahin et al. (2014), as many sectoral labor market statistics are readily available from their online appendix. We augment the data set with sector-level data on production linkages from the 2007 input-output tables published by the Bureau of Economic Analysis (BEA). We will extend the period and the scope of our analysis in future iterations.

5.1. Data Sources

Vacancies and Hires. We use vacancy and hire data from The Job Openings and Labor Turnover Survey (JOLTs), which provides survey-based measures of job openings and hires at a monthly frequency. This survey data is available for 17 industries that roughly correspond to the two-digit NAICS classification. The data covers 2001 to 2011, at a monthly and a yearly frequency.

Unemployment. We calculate unemployment using the Current Population Survey (CPS). We use the same industry classification as JOLTs. The data covers 2001 to 2011, at a monthly and a yearly frequency.

Matching Functions. We use $\eta = 0.5$ for the vacancy share in matching functions. This value is roughly in the middle of a range of literature estimates (Petrongolo and Pissarides (2001)). We use the heterogeneous matching efficiency estimated by Şahin et al. (2014), which follows the methodology proposed by Borowczyk-Martins et al. (2013).

Input-output Linkages. Although JOLTs include labor market data at the level that corresponds roughly to the two-digit NAICS classification, this correspondence is not exact. To construct input intensity that matches with JOLTs' data availability level, we use the 2007 6-digit NAICS classification level table, and aggregate back up to match the JOLTs industries. Assuming constant-return-to-scale, we calculate the input intensity of each input sector as its share of the consumer sector's labor-inclusive total expenditure. Note that even though our results doesn't require examining the explicit structure of the U.S production networks, we still compute the input-intensity for the future when we deviate from Cobb-Douglas, or look at impacts of idiosyncratic unemployment shocks.

5.2. Results

Estimates of the mismatch index quantify how far we are from the efficient allocation of unemployment. A value of 0 corresponds to efficiency. In a horizontal economy with linear production technology, $\mathcal{M} \geq 0$, with $\mathcal{M} > 0$ indicating a departure from the efficient allocation of unemployment. In an economy featuring a non-trivial production network $\mathcal{M}$ can be either smaller or larger than zero, with $\mathcal{M} \neq 0$ indicating a departure from efficiency.

Figure 1 plots our estimated mismatch index assuming a horizontal economy with linear technology (the black solid line, as in Şahin et al. (2014)) and accounting for the input-output between two-digit NAICS industries with Cobb-Douglas technology (the red dashed line). Our results for the horizontal econonmy with linear technology are identical to those in Şahin et al. (2014). The unemployment allocation is never efficient in either case. On average, accounting for production networks slightly reduces absolute deviations from efficiency: the average value of $\mathcal{M}$ from 2001 to 2011 is 0.045 assuming a horizontal economy and –0.038 once we account for production networks.

Of course the implications of a negative mismatch index are quite different from the implications of a positive index. A negative index value suggest that the optimal allocation of unemployment actually reduces the number of hires rather than increases them. How is this possible? Once we account for production linkages and decreasing marginal productivity of labor, optimal unemployment no longer mechanically maximizes the number of new hires by assigning more searchers to industries with more vacancies and higher matching efficiency. Instead, the social planner now chooses to assign more searchers to industries that rely more on labor, are more important to the production of final consumption goods, and have a smaller existing stock of workers, captured by the parameters α_i, λ_i, and ϕ_i respectively, while only matching efficiency

ϕ_i and vacancies v_i matter for the number of new hires generated. As a result, if or λ_i or α_i are negatively correlated with v_i or ϕ_i, or if e_i is positively correlated with v_i or ϕ_i, then mismatch can be negative. In the following subsection we show that decreasing productivity of labor is the key to understand why we find negative mismatch in the production networks case.

When the allocation of unemployment is furthest from optimal also differs once we account for production networks. In the horizontal economy case, mismatch is highest during the Great Recession, whereas in the production networks case this is when mismatch is closes to 0. At this point, we don't think its wise to read to much into this result, as we feel it is most likely due to several simplifying assumptions we made along in this iteration of the paper.

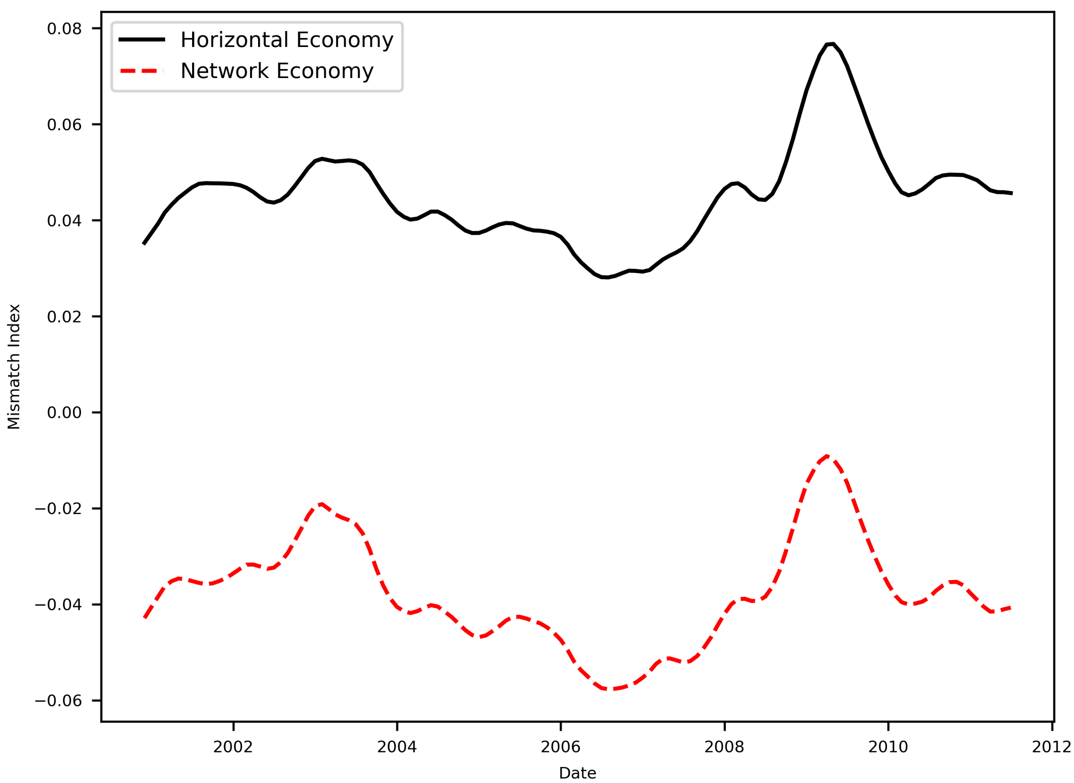

FIGURE 1. Mismatch Index in a Horizontal and a Network Economy

5.2.1. Digging into the negative mismatch index

In this section we explore what is driving the negative mismatch index values once we account for production networks. Figure 2 and Figure 3 plot the relationship between network and production function inputs λ_i, α_i, and e_i and matching function inputs v_i and ϕ_i. There is no particularly clear relationship between λ_i, α_i, e_i and ϕ_i, λ_i, while α_i, and e_i appear to be strongly positively correlated with v_i. The strong positive correlation between e_i and v_i in particular would tend to produce negative matching index estimates because u_i^* is declining e_i due to the decreasing marginal productivity of labor. As a result, the social planner allocates fewer workers to high vacancy industries than they would otherwise, reducing the number of hires generated by the matching process. The positive correlation between $\lambda_i \alpha_i$ and v_i should counteract this effect somewhat.

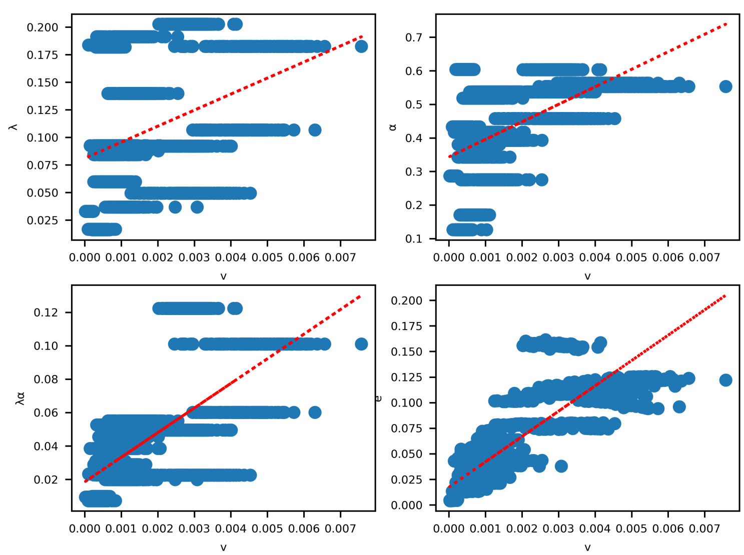

FIGURE 2. Relationship between the distribution of network parameters and vacancies across industries

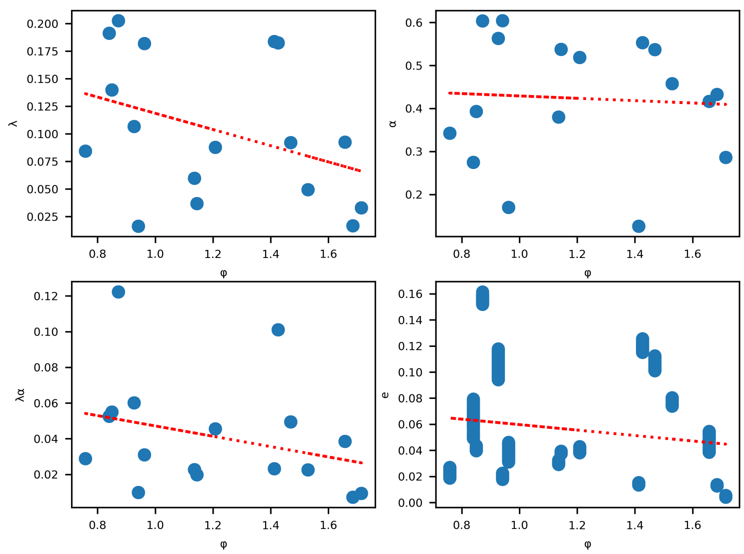

FIGURE 3. Relationship between the distribution of network parameters and matching efficiency across industries

This intuition is confirmed by Figure 4, which decomposes mismatch index into the effects of different parameters by holding one (or two in the case of $\lambda \times \alpha$) parameter fixed at a time. The comparison between the red, blue, and black lines is particularly informative. Holding $\lambda \times \alpha$ fixed highlights the impact of e_i: as suggested by the correlations above, differences in the stock of existing workers across industries seems to be driving the negative mismatch index. Assigning the same stock of workers to each industry leads to the blue line and completely undoes the negative mismatch. In addition, the network parameter λ_i and the relative importance of labor in production α_i undo some, but not all, of the effects of e_i, as captured by the blue and yellow lines. Interestingly, the red dashed line is exactly what would result in a horizontal economy while allowing decreasing productivity of labor. Figure 4 therefore highlights the importance of the assuming linear production to the results in Şahin et al. (2014).

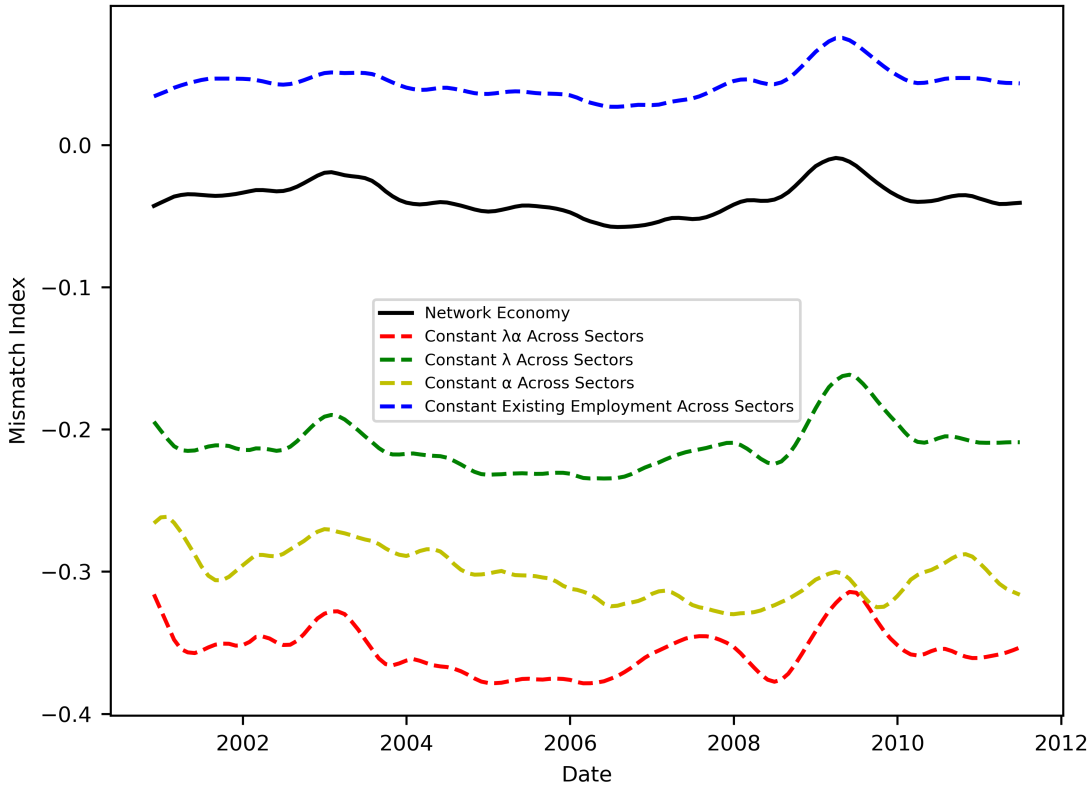

FIGURE 4. Decomposition of Network Mismatch: Effect of Existing Stock of Workers vs Effect of Network Parameters (λ, α)

The discussion above highlights why it is possible for mismatch to be negative once we account for production networks, but we think it is important not to read too much into these preliminary results. Allowing for several realistic extensions to the model could reduce or totally undo the finding of negative implied mismatch. We discuss two below:

Allowing for endogenous vacancy creation. Increasing the number of searchers a sector increases the likelihood of filling a vacancy posting. As a result, industries the social planner assigns more searchers too would tend to increase the number of vacancies they post. Thus, allowing for endogenous vacancy creation would increase the number of vacancies in favored industries, and therefore increase the number of hires. In addition, as production in high u^* industries rises, sectors that use those industry outputs as inputs into their own production benefit, effectively increasing their productivity. This could lead to further increases in vacancy postings and even more hires, and so on.

Allowing for sector specific productivity differences. Once we allow for sector specific productivity, the social planner would tend to assign more searchers to high productivity industries. If industries large industries, which also have large existing stocks of workers, are also more productive this would tend to counteract the effects of the decreasing marginal product of labor.

6. Conclusion

In this paper, we investigate the impact of mismatch on aggregate unemployment in a production network. We build a static, multi-sector, competitive networks model with Cobb-Douglas utilities, production and matching functions. We assume vacancy posting to be exogenous, and social planner maximizes household utility by picking the optimal unemployment allocation for sectors. We derive a Hulten-like result that states that change in household utility can be expressed as a sum of unemployment shocks weighted by labor productivity, hire-to-workforce ratio, and industry sales ratio. We take advantage of this result and find that optimal sectoral allocation requires equalizing sector's labor-share and network-weighted contribution to the hiring process.

We then take our model to the data to quantify the extent of mismatch in the U.S economy between 2005 and 2011. We derive a series of mismatch index that is similar to Şahin et al. (2014) in trend, but different in signs. The negative mismatch we get can be attributed to decreasing marginal return to labor and the interaction between a sector's importance in the network and the size of its workforce.

For future steps, there are a few directions we want to go. On the theoretical front, we want to endogenize firms' vacancy posting decisions and incorporate idiosyncratic productivity. We are hoping that these features address the negative mismatch we compute. On the empirical front, we would like to extend our analysis to more sectors and a longer time period. We are evaluating whether the Burning Glass dataset would be a good direction to go, or if we can find publicly available vacancy data at a more granular level. We also want to compare the optimal unemployment allocation from our model with the one from Şahin et al. (2014) on a sector level. This allows us to more closely examine how and where the change in mismatch takes place. In addition, we are thinking of relaxing our Cobb-Douglas assumptions, and write our model using CES functions (Baqaee and Farhi 2019, 2020; Liu 2019). We are not yet sure what this is going to bring us, but we are interested in seeing if our results will still hold if we deviate from the Cobb-Douglas world.

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