

Unemployment in a Production Network

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Motivation

Economies feature production linkages

Labor markets are frictional

Labor markets are somewhat segmented (by sector, geographic regions, and occupation)

Research questions:

- How do labor market frictions and segmentation impact network propagation of shocks?
- How do we design optimal labor market policies in a network economy?

1. Search and matching frictions
2. Segmentation in the labor market
 - [Schubert et al., 2022, Adão et al., 2019, Neffke et al., 2017, Manning and Petrongolo, 2017, Nimczik, , Parrado et al., 2007, Kambourov and Manovskii, 2008]
3. Production networks literature
 - [Acemoglu et al., 2012, Acemoglu and Azar, 2020, Baqaee, 2018, Baqaee and Farhi, 2019, Baqaee and Rubbo, 2022, La'O and Tahbaz-Salehi, 2021]

Today, we are going to present

1. a general framework for thinking about frictional labor markets in a production network economy
2. an aggregation theorem that nests the famous Hulten (1978)'s theorem + analytical characterization of deviations from Hulten's theorem due to labor market frictions
3. calibration exercises based on the US economy

Our Model

Occupation Level Labor Markets

\mathcal{O} occupations with separate labor markets.

Each labor market has H_o possible workers.

New hires governed by matching technology m :

$$h_o = \phi_o m(H_o, V_o)$$

Labor supply satisfies:

$$L_i^o(\theta_o) = \mathcal{F}_o(\theta_o) H_o,$$

where $\theta_o = \frac{V_o}{H_o}$

Recruiter-producer ratio is defined as:

$$\tau_o(\theta_o) \equiv \frac{r_o}{\mathcal{Q}_o(\theta_o) - r_o},$$

where r_o is vacancy-posting cost.

J sectors, indexed by i , produce with CRTS technology:

$$y_i = A_i f_i \left(\{N_{io}\}_{o=1}^O, \{x_{ij}\}_{j=1}^J \right)$$

Firms maximize profits:

$$\max_{\{N_{io}\}_{o=1}^O, \{x_{ij}\}_{j=1}^J} p_i f_i \left(\{N_{io}\}_{o=1}^O, \{x_{ij}\}_{j=1}^J \right) - \sum_{o=1}^O w_o (1 + \tau_o(\theta_o)) N_{io} - \sum_{j=1}^J p_j x_{ij}$$

This requires:

$$\varepsilon_{x_{ij}}^{f_i} = \frac{p_j x_{ij}}{p_i y_i}$$
$$\varepsilon_{N_{io}}^{f_i} = (1 + \tau_o(\theta_o)) \frac{w_o N_{io}}{p_i y_i}$$

A final goods producer aggregates output by CRTS technology:

$$Y = \max_{\{c_i\}_{i=1}^J} \mathcal{D}(\{c_i\}_{i=1}^J),$$

subject to

$$\sum_{i=1}^J p_i c_i = \sum_{o=1}^O w_o L_o(\theta_o) = \sum_{o=1}^O w_o \sum_{j=1}^J l_o(\theta_{io}).$$

Equilibrium Conditions (Goods market)

Firms:

$$\varepsilon_{x_{ij}}^{f_i} = \frac{p_j x_{ij}}{p_i y_i} \quad (\text{Intermediate input decision})$$

$$\varepsilon_{N_{io}}^{f_i} = (1 + \tau_o(\theta_o)) \frac{w_o N_{io}}{p_i y_i} \quad (\text{Labor input decision})$$

Households:

$$\varepsilon_{c_i}^{\mathcal{D}} = \frac{p_i c_i}{\sum_{k=1}^J p_k c_k} \quad (\text{Consumption decision})$$

Market clearing:

$$y_i = c_i + \sum_{j=1}^J x_{ji} \quad (\text{Goods market clearing})$$

Equilibrium conditions (Labor market)

Labor market:

$$l_{io}^d(\theta_o) = \varepsilon_{N_{io}}^{f_i} \frac{p_i y_i}{w_o}. \quad (\text{Labor Demand})$$

$$L_o^d(\theta_o) = \sum_{j=1}^J l_{io}^d(\theta_o)$$

$$L_o^s(\theta_o) = \mathcal{F}_o(\theta_o) H_o. \quad (\text{Labor supply})$$

Market clearing:

$$L_o^s = L_o^d \quad (\text{Labor market equilibrium})$$

Production network notation

$$\lambda_i = \frac{p_i y_i}{\sum_j p_j c_j} \quad (\text{Sales share})$$

$$\Omega = \begin{bmatrix} \varepsilon_{x11}^{f_1} & \varepsilon_{x12}^{f_1} & \cdots & \varepsilon_{x1J}^{f_1} \\ \varepsilon_{x21}^{f_2} & \varepsilon_{x22}^{f_2} & \cdots & \varepsilon_{x2J}^{f_2} \\ \vdots & \vdots & \ddots & \vdots \\ \varepsilon_{xJ1}^{f_J} & \varepsilon_{xJ2}^{f_J} & \cdots & \varepsilon_{xJJ}^{f_J} \end{bmatrix} \quad (\text{Input-output matrix})$$

$$\Psi = (I - \Omega)^{-1} \quad (\text{Leontief inverse})$$

$$\varepsilon_N^f = \begin{bmatrix} \varepsilon_{N11}^{f_1} & \varepsilon_{N12}^{f_1} & \cdots & \varepsilon_{N1\mathcal{O}}^{f_1} \\ \varepsilon_{N21}^{f_2} & \varepsilon_{N22}^{f_2} & \cdots & \varepsilon_{N2\mathcal{O}}^{f_2} \\ \vdots & \vdots & \ddots & \vdots \\ \varepsilon_{NJ1}^{f_J} & \varepsilon_{NJ2}^{f_J} & \cdots & \varepsilon_{NJ\mathcal{O}}^{f_J} \end{bmatrix}_{J \times \mathcal{O}} \quad (\text{Labor elasticity of production})$$

Labor market notation

$$\epsilon_{\theta}^{\mathcal{Q}} = \begin{bmatrix} \epsilon_{\theta_1}^{\mathcal{Q}_1} \\ \epsilon_{\theta_2}^{\mathcal{Q}_2} \\ \vdots \\ \epsilon_{\theta_{\mathcal{O}}}^{\mathcal{Q}_{\mathcal{O}}} \end{bmatrix}, \epsilon_{\theta}^{\mathcal{F}} = \begin{bmatrix} \epsilon_{\theta_1}^{\mathcal{F}_1} \\ \epsilon_{\theta_2}^{\mathcal{F}_2} \\ \vdots \\ \epsilon_{\theta_{\mathcal{O}}}^{\mathcal{F}_{\mathcal{O}}} \end{bmatrix} = \epsilon_{\theta}^{\mathcal{Q}} + \mathbf{1}$$

(Tightness elasticities of matching)

$$\mathcal{L} = \begin{bmatrix} \frac{l_{11}}{L_1} & \frac{l_{21}}{L_1} & \dots & \frac{l_{J1}}{L_1} \\ \frac{l_{12}}{L_2} & \frac{l_{22}}{L_2} & \dots & \frac{l_{J2}}{L_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{l_{1\mathcal{O}}}{L_{\mathcal{O}}} & \frac{l_{2\mathcal{O}}}{L_{\mathcal{O}}} & \dots & \frac{l_{J\mathcal{O}}}{L_{\mathcal{O}}} \end{bmatrix}_{\mathcal{O} \times J}$$

(Occupation labor share)

$$\tau = \begin{bmatrix} \tau_1(\theta_1) \\ \vdots \\ \tau_{\mathcal{O}}(\theta_{\mathcal{O}}) \end{bmatrix},$$

(Tightness)

$$\mathcal{Q} = \text{diag}(\epsilon_{\theta}^{\mathcal{Q}}), \mathcal{F} = \text{diag}(\epsilon_{\theta}^{\mathcal{F}}), \mathcal{T} = \text{diag}(\tau)$$

Theoretical Results: Propagation

Mutual gains from trade when a worker and firm match mean wage is not uniquely determined

We assume nominal wages net of sectoral employment weighted prices satisfy

$$d \log \mathbf{w} - \mathcal{L} d \log \mathbf{p} = \Lambda_{\mathbf{A}} d \log \mathbf{A} + \Lambda_{\mathbf{H}} d \log \mathbf{H} \quad (1)$$

Since $d \log \mathbf{p}$ is function of $d \log \mathbf{A}$ and $d \log \mathbf{H}$ as well, this is just a convenient way of expressing that wages changes ultimately depend on shocks.

Tightness

Given $d \log \mathbf{w} - \mathcal{L} d \log \mathbf{p}$, labor market clearing in each occupation implies

$$\varepsilon_{\theta_o}^{f_o} d \log \theta_o + d \log H_o = \sum_{i=1}^J \frac{l_{io}}{L_o} [d \log y_i - (d \log w_o - d \log p_i)].$$

Plugging in the production function, stacking over occupations, sectors, and solving for $d \log \boldsymbol{\theta}$ gives

$$d \log \boldsymbol{\theta} = [\mathcal{F} - \boldsymbol{\Xi}_{\boldsymbol{\theta}}]^{-1} \underbrace{\left[\mathcal{L} \boldsymbol{\Psi} d \log \mathbf{A} - (d \log \mathbf{w} - \mathcal{L} d \log \mathbf{p}) + \mathcal{L} \boldsymbol{\Psi} \varepsilon_N^f d \log \mathbf{H} \right]}_{\text{Demand-side pressure}} \\ - [\mathcal{F} - \boldsymbol{\Xi}_{\boldsymbol{\theta}}]^{-1} \underbrace{d \log \mathbf{H}}_{\text{Supply-side pressure}}$$

where $\boldsymbol{\Xi}_{\boldsymbol{\theta}} = \mathcal{L} \boldsymbol{\Psi} \varepsilon_N^f [\mathcal{F} + \mathcal{Q}\mathcal{T}]$. ► General case

Given $d \log \theta$, the production function, and the input decisions, the price system satisfies:

$$\begin{aligned}
 (I - \Psi \epsilon_N^f \mathcal{L}) d \log \mathbf{p} = & \underbrace{\Psi \epsilon_N^f (d \log \mathbf{w} - \mathcal{L} d \log \mathbf{p})}_{\text{Relative factor prices}} \\
 & - \underbrace{\Psi \epsilon_N^f \mathcal{Q} \mathcal{T} d \log \theta}_{\text{Search frictions}} \\
 & - \underbrace{\Psi d \log \mathbf{A}}_{\text{Productivity}}
 \end{aligned} \tag{2}$$

Note that the price system is in terms of real variables. We can pin down the price system by picking a numeraire.

Sector level output

With sales given tightness, we can use the production function to back out changes in output:

$$d \log \mathbf{y} = \Psi \left(d \log \mathbf{A} + \underbrace{\varepsilon_N^f (\mathcal{F} + \mathcal{Q}\mathcal{T}) d \log \boldsymbol{\theta} + \varepsilon_N^f d \log \mathbf{H}}_{\text{Labor usage}} \right)$$

► General case

Theoretical Results: Aggregation

Aggregation formula

With Cobb-Douglas assumptions,

$$\begin{aligned}d \log Y &= \varepsilon_c^{\mathcal{D}'} d \log \mathbf{c} \\&= \varepsilon_c^{\mathcal{D}'} d \log \mathbf{y} \\&= \varepsilon_c^{\mathcal{D}'} \Psi \left(d \log \mathbf{A} + \varepsilon_N^f (\mathcal{F} + \mathcal{Q}\mathcal{T}) d \log \boldsymbol{\theta} + \varepsilon_N^f d \log \mathbf{H} \right) \\&= \Pi_A d \log \mathbf{A} + \Pi_H d \log \mathbf{H},\end{aligned}$$

where

$$\begin{aligned}\Pi_A &= \varepsilon_c^{\mathcal{D}'} \Psi \left(\mathbf{I} + \varepsilon_N^f (\mathcal{F} + \mathcal{Q}\mathcal{T}) [\mathcal{F} - \Xi_\theta]^{-1} (\mathcal{L}\Psi - \Lambda_A) \right) \\ \Pi_H &= \varepsilon_c^{\mathcal{D}'} \Psi \varepsilon_N^f \left(\mathbf{I} + (\mathcal{F} + \mathcal{Q}\mathcal{T}) [\mathcal{F} - \Xi_\theta]^{-1} ([\mathcal{L}\Psi \varepsilon_N^f - \mathbf{I}] - \Lambda_H) \right)\end{aligned}$$

► General case

What happens without search frictions?

Without search frictions, wages are given by:

$$d \log \mathbf{w} = (d \log \mathbf{p} + d \log \mathbf{y}) - d \log \mathbf{H}$$

Linearizing the production functions gives:

$$d \log y_i = d \log A_i + \sum_{o=1}^O \varepsilon_{N_{io}}^{f_i} d \log N_{io} + \sum_{j=1}^J \varepsilon_{x_{ij}}^{f_i} d \log y_j$$

Stacking the equations yields:

$$\begin{aligned} d \log \mathbf{y} &= d \log \mathbf{A} + \varepsilon_N^f d \log \mathbf{N} + \Omega d \log \mathbf{y} \\ &= \Psi (d \log \mathbf{A} + \varepsilon_N^f d \log \mathbf{H}), \end{aligned}$$

so

$$d \log Y = \varepsilon_c^{\mathcal{D}'} \Psi (d \log \mathbf{A} + \varepsilon_N^f d \log \mathbf{H})$$

Calibration exercise

Recall,

$$d \log \mathbf{w} - \mathcal{L} d \log \mathbf{p} = \Lambda_{\mathbf{A}} d \log \mathbf{A} - \Lambda_{\mathbf{H}} d \log \mathbf{H}$$

We now test the quantitative implications of deviations from Hulten's theorem by assuming

$$\Lambda_{\mathbf{A}} = \gamma_{\mathbf{A}} \mathcal{L} \Psi$$

$$\Lambda_{\mathbf{H}} = \gamma_{\mathbf{H}} [\mathcal{L} \Psi \epsilon_N^f - I]$$

When $d \log \mathbf{H} = 0$, Hulten's theorem corresponds to $\gamma_{\mathbf{A}} = 1$.

Calibration: Aggregate Deviations from Hulten

For the given assumption about wages, we can analytically characterize the importance of deviations from Hulten's theorem.

$$d \log Y = \Pi_A d \log \mathbf{A} + \Pi_H d \log \mathbf{H},$$

Where

$$\Pi_A = \varepsilon_c^{\mathcal{D}'} \Psi \left(I + (1 - \gamma_A) \varepsilon_N^f (\mathcal{F} + \mathcal{Q}\mathcal{T}) [\mathcal{F} - \Xi_\theta]^{-1} \mathcal{L} \Psi \right)$$

$$\Pi_H = \varepsilon_c^{\mathcal{D}'} \Psi \varepsilon_N^f \left(I + (1 - \gamma_H) (\mathcal{F} + \mathcal{Q}\mathcal{T}) [\mathcal{F} - \Xi_\theta]^{-1} [\mathcal{L} \Psi \varepsilon_N^f - I] \right)$$

Cobb-Douglas matching technology

$$h_i = \phi_i U_i^\eta V_i^{1-\eta}$$

set

- $\eta = 0.5$
- Pick r_i so that recruiter producer ratio is about 2.3% in aggregate.
- U_i, V_i from JOLTS

Input output tables from BEA

- We use the 2007 6-digit NAICS classification level table, and aggregate back up to match the JOLTs industries

Sector gross output from BEA

Key Parameters

Sector	λ	ε_N^f	ϕ
Accommodation and food services	0.05	0.46	1.53
Arts, entertainment, and recreation	0.02	0.43	1.68
Construction	0.09	0.42	1.66
Durable goods	0.19	0.28	0.84
Educational services	0.02	0.60	0.94
Finance and insurance	0.14	0.39	0.85
Government	0.20	0.60	0.87
Health care and social assistance	0.11	0.56	0.93
Information	0.08	0.34	0.76
Mining	0.03	0.29	1.71
Nondurable goods	0.18	0.17	0.96
Other services, except government	0.04	0.54	1.14
Professional and business services	0.18	0.55	1.43
Real estate and rental and leasing	0.18	0.13	1.41
Retail trade	0.09	0.54	1.47
Transportation and warehousing	0.06	0.38	1.14
Wholesale trade	0.09	0.52	1.21

Table 1: Key parameter values

For now we have two baseline assumptions about occupations

1. Each sector has a corresponding occupation
2. There is a single occupation (one type of labor)

Case 1: Response of output to technology shocks

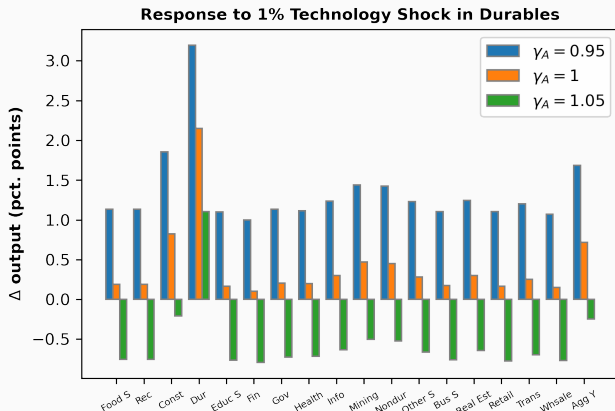


Figure 1: Response of output

Case 1: Response of unemployment to technology shocks

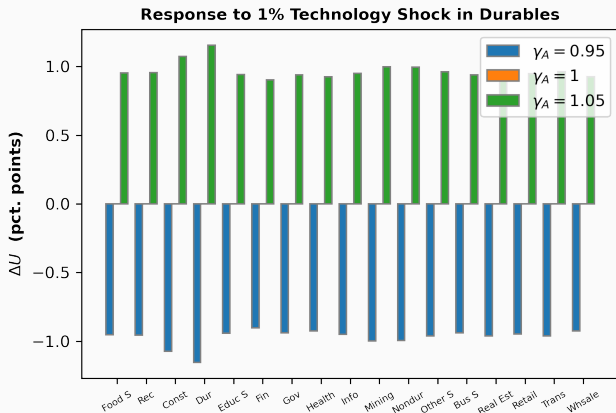


Figure 2: Response of unemployment rate

Case 1: Response of output to labor force shocks

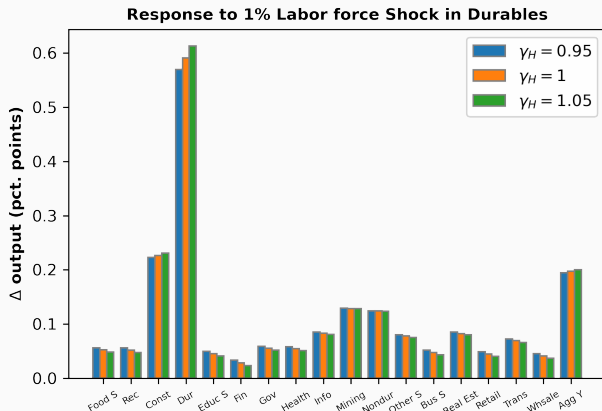


Figure 3: Response of output

Case 1: Response of unemployment to labor force shocks

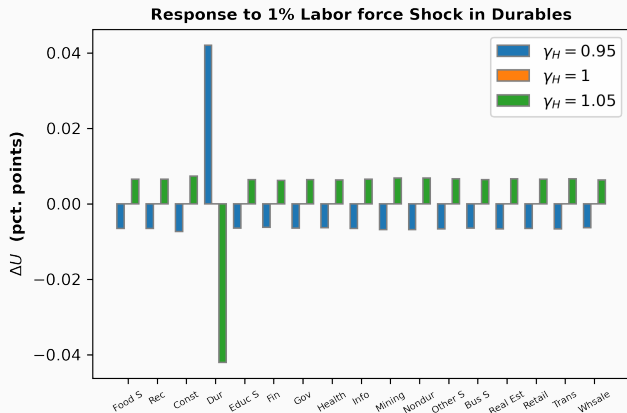


Figure 4: Response of unemployment rate

Case 2: Response of output to technology shocks

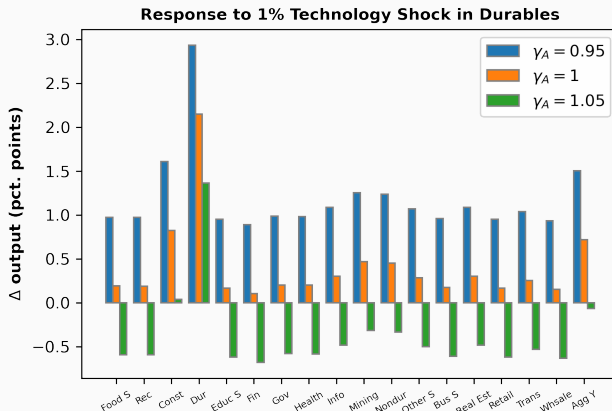


Figure 5: Response of output

Case 2: Response of unemployment to technology shocks

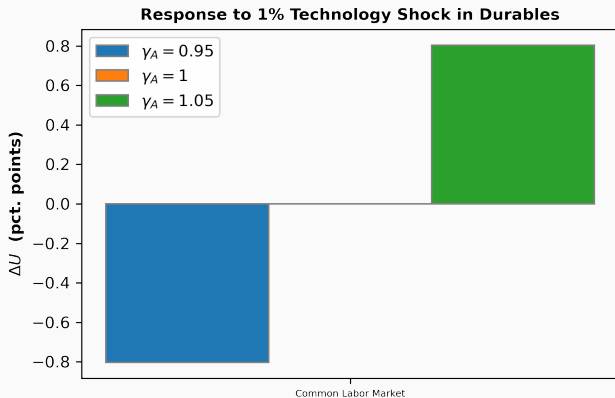


Figure 6: Response of unemployment rate

Case 2: Response of output to labor force shocks

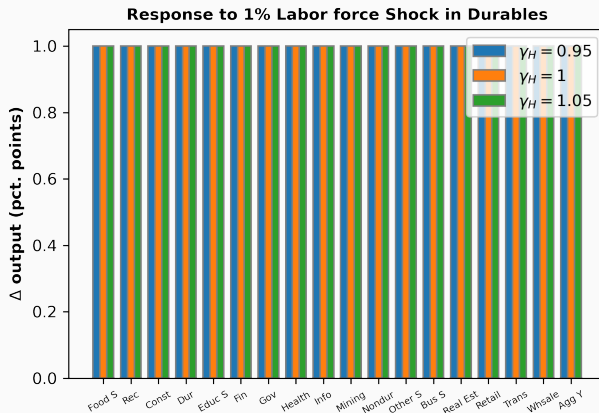


Figure 7: Response of output

Case 2: Response of unemployment to labor force shocks

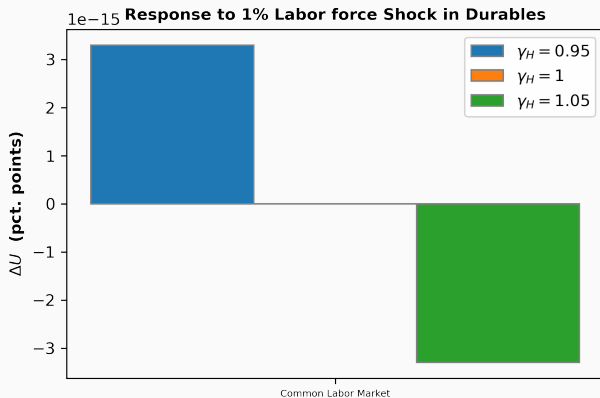


Figure 8: Response of unemployment rate

Appendix

Results for General Production Functions

All results presented above generalize to any constant returns production function and preferences.

To pin down first order changes in a general setting, we just need to work out how elasticities respond to shocks.

Elasticity changes will be pinned down precisely by the exact functional form we assume for production.

General Tightness Propagation

$$\begin{aligned}d \log \boldsymbol{\theta} = & [\mathcal{F} - \boldsymbol{\Xi}_{\boldsymbol{\theta}}]^{-1} [\mathcal{L} \Psi d \log \mathbf{A} - [d \log \mathbf{w} - \mathcal{L} d \log \mathbf{p}]] \\& + [\mathcal{F} - \boldsymbol{\Xi}_{\boldsymbol{\theta}}]^{-1} [[\mathcal{L} \Psi \boldsymbol{\varepsilon}_N^f - \mathbf{I}] d \log \mathbf{H}] \\& + [\mathcal{F} - \boldsymbol{\Xi}_{\boldsymbol{\theta}}]^{-1} [\text{diag}(\mathcal{L} d \log \boldsymbol{\varepsilon}_N^f) + \mathcal{L} \Psi (\text{diag}(\boldsymbol{\Omega} \mathbf{1}) - \boldsymbol{\Omega}) d \log \boldsymbol{\lambda}] \\& - [\mathcal{F} - \boldsymbol{\Xi}_{\boldsymbol{\theta}}]^{-1} d \log \mathcal{E}\end{aligned}$$

► Back

General Sectoral Output Propagation

$$\begin{aligned}d \log \mathbf{y} = & \Psi \left(d \log \mathbf{A} + \varepsilon_N^f (\mathcal{F} + \mathcal{Q}\mathcal{T}) d \log \boldsymbol{\theta} + \varepsilon_N^f d \log \mathbf{H} \right) \\ & - \Psi d \log \mathcal{E} + \Psi (\text{diag}(\mathbf{\Omega}\mathbf{1}) - \mathbf{\Omega}) d \log \lambda\end{aligned}$$

► Back

Sales Share Propagation

$$\begin{aligned}d\lambda' &= d\varepsilon_c^{\mathcal{D}'} + d\lambda'\Omega + \lambda'd\Omega \\ \Rightarrow d\lambda' &= \left[d\varepsilon_c^{\mathcal{D}'} + \lambda'd\Omega \right] \Psi\end{aligned}$$

General Aggregation Formula

$$\begin{aligned}d \log Y &= \epsilon_c^{\mathcal{D}'} d \log \mathbf{c} \\ &= \epsilon_c^{\mathcal{D}'} (d \log \epsilon_c^{\mathcal{D}} + d \log \mathbf{y} - d \log \boldsymbol{\lambda})\end{aligned}$$

► Back



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