# Unemployment in a Production Network

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**Abstract**

We develop a framework for studying unemployment in a production network by incorporating sector-specific search and matching frictions. We derive aggregation formulae for how output and unemployment respond to sector-specific productivity and labor supply shocks. Specifically, output aggregation can be decomposed into two channels, the input-output channel, which can be expressed in terms of sales shares, and the network-adjusted search-and-matching channel, which depends on the interaction between labor market frictions and an occupational-labor-share-weighted Leontief inverse. We show that the foundational theorem of Hulten (1978) is a special case of the network we study, when wages respond to exactly offset changes in the network-adjusted marginal productivity of labor. Additionally, the wage schedule assumption is essential for determining whether matching frictions amplify or dampen the impact of shocks. For example, when wages rise by less than the network-adjusted marginal product of labor in response to a positive technology shock, firms increase their hiring, amplifying the positive effect on output, and vice versa. We calibrate our model to the US data and apply it to study the impact of the Russia-Ukraine war on the US economy. We show that the shock to energy prices has x impact on inflation and y impact on labor market tightness.

*JEL Codes:* E1, J3, J6

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# 1. Introduction

Modern economies feature rich production networks. A recent literature highlights that the interactions between firms in different sectors matter for the propagation of sector level shocks (Baqaee and Rubbo 2022), how we measure productivity and the social cost of distortions (Baqaee and Farhi 2019, 2020), the optimal conduct of monetary policy (Rubbo 2020; La'O and Tahbaz-Salehi 2021), and that sector level shocks may even be a source of aggregate fluctuations and growth (Acemoglu et al. 2012; Acemoglu and Azar 2020).

We extend the existing literature by showing that production networks also matter for unemployment in a realistic frictional labor market. As shocks propagate through the network, labor demand changes, which impacts sector tightness and unemployment. On the other hand, unemployment matters for production networks. Changes in labor market tightness require firms to change their recruiting effort, and the reallocation of productive workers to recruiters alters production outcomes. In addition, unemployed workers look for jobs in other sectors, lowering tightness in those sectors, and thus affecting production across the network.

Production linkages and unemployment are important economic mechanisms, and understanding how the two interact is key to painting an accurate and granular picture of how microeconomic shocks generate macroeconomic fluctuations. In addition, understanding the labor market impact of micro shocks can help policymakers craft and evaluate policies. For example, our framework can help us dissect how energy price spikes caused by the Russia-Ukraine war impact prices and unemployment for different sectors in different countries, as well as studying how the CHIPS and Science Act, which boosts domestic research and manufacturing of semiconductors in the United States, changes aggregate output and unemployment. More generally, we use our framework to ask: What are the impacts of idiosyncratic, sector-specific technology shocks on sector and aggregate output and unemployment? How do idiosyncratic shocks generate comovements in tightness across labor markets? How do labor market inefficiencies and characteristics, such as matching frictions, mobility costs, and wage schedules, affect network propagation of shocks?

Starting with Long and Plosser (1983) and Acemoglu et al. (2012), much effort has been made on understanding how micro shocks cause macro fluctuations. The recent literature has incorporated inefficiencies into production network models, including markups and financial frictions (Liu 2019; Baqaee and Farhi 2020; Bigio and La'O 2020).

While these models offer a more realistic depiction of production linkages and the associated product market and financial inefficiencies, their treatment of labor markets is simple. These models consider labor as either perfectly inelastically supplied, or supplied according to the disutility of work in household's utility function, therefore ignoring the richness of labor market imperfections and shutting off interesting propagation channels brought forth by such imperfections.

In this paper, we extend the production network framework by incorporating matching frictions. We build a static, multisector production network model that features a representative household, many production sectors, perfect competition in product markets, and segmented frictional labor markets. The form of labor market segmentation is flexible, and our model allows for geographic, sectoral, or occupational labor market segmentation.

With this setup, we develop a general theory of how technology shocks propagate and aggregate across the production network under search frictions. In particular, we show that the aggregate impact on output and unemployment can be decomposed into impacts from real factor price changes, tightness changes, and changes in sales shares, amplified by the interaction between the Leontief inverse and labor market segmentation, which we call the occupational-labor-share-weighted Leontief inverse. We show that the aggregate output response can be decomposed into two components. The first term resembles the foundational aggregation theorem of Hulten (1978), in which the output response is a sales-share-weighted aggregation of sectoral technology shocks. The second term, which we call the search-and-matching channel, involves complex interaction between tightness, production structure, and labor market structure. We show that we can recover Hulten's Theorem in two ways. First, the economy is efficient when labor market frictions are absent, which requires a matching elasticity with respect to unemployment of 1. Second, when relative wages adjust exactly according to the occupational-labor-share-weighted Leontief inverse, wages respond to shocks in a way that keeps labor market tightness unchanged. Intuitively, when wages change by the occupational-labor-share-weighted-Leontief inverse they change by just enough to offset changes in the marginal productivity of labor across the network.

Whether the search-and-matching channel dampens or amplifies technology shocks depends largely on how wages adjust. Since the search and matching setup admits a wide range of assumptions about wages, we view this as an empirical question. Nevertheless, we use two toy examples to illustrate the determinants of the direction of amplification. First, we shut off production linkages in the model, and show that the direction of amplification depends on how wages responds relative to the marginal product of labor, as well as whether posting additional vacancies will change the number of productive workers. Since hiring an additional worker requires more and more vacancies as labor markets get tighter, and posting vacancies requires firms to allocate workers towards recruiting, there is an inflection point where an additional vacancy results in no change in the productive workforce. We then solve a simple two-sector vertical production networks model with one labor market, to show that the direction of amplification now depends on how wage responds to the network-adjusted marginal product of labor. We further show that our frictional labor market setup allows for upstream propagation of shocks, which is absent in efficient production networks with Cobb-Douglas utility and production functions.

In addition to technology shocks, our setup allows us to explore another set of shocks—shocks to the size of the labor force—which have previously received attention in the trade and spacial labor literature, but have so far been largely absent from the production networks literature. We can think of these shocks as coming from an underlying model of occupational choice, as in Humlum (Add Citation), or an underlying model of migration, as in Fernandez-Villaverde et al. (add citation). We show that incorporating search and matching gives these shocks a role in the production network context. A positive shock to the size of the labor force in one occupation reduces the tightness, and therefore the number of recruiters needed, in industries that use that type of labor. We derive similar results for labor force shocks to those described above for technology shocks. Although there is no clear parallel to Hulten's Theorem, which only allows technology shocks, we show when search and matching frictions amplify or dampen the effects of labor force shocks beyond the pure production network effects we would see when changing a fixed factor of production.

To test the empirical relevance of the theoretical channels outlined above, we calibrate our model to US data. We use survey-based vacancy and hiring data from the Job Openings and Labor Turnover Survey (JOLTS), unemployment data from the Current Population Survey (CPS), occupation data from the Occupational Employment and Wage Statistics (OEWS), and industry sales shares from the U.S Bureau of Economic Analysis (BEA). We find that, assuming nominal wage rigidity, a one-percent positive technology shock to the durables manufacturing sector increases aggregate output by [x] percent, and decreases aggregate unemployment by [y] percent. Importantly, we are able to observe how unemployment changes across sectors, with unemployment in [a] sector changing by [b], and unemployment in [c] sector changing by [d].

We demonstrate how to use our calibrated model for macroeconomic applications. We apply our model to explain the heterogeneous shifts in sectoral Beveridge Curves, and the price propagation of oil shocks, during and after COVID. In the case of Beveridge curves, we express the Beveridge elasticities for sectors as a function of idiosyncratic productivity and labor supply shocks. We show how a sectoral productivity shock and an occupational labor supply shock can generate differential changes in sectoral Beveridge elasticities through production linkages and labor market frictions. We find that some combination of shocks in some sectors / occupations are able to explain qualitatively/ quantitatively the shifts in Beveridge elasticities across sectors. For the oil price shocks, we extend our model to allow for oil as an additional factor input and express our price propagation formula in terms of oil price shocks. Using the oil shocks estimated in Kanzig (2021), we find that the oil price shocks are able to account for x percent of the observed inflation. This result is similar or different from the Harvard JMP. We find that the additional labor market tightness channel is able to dampen / amplify the inflationary impact of oil shocks.

This paper fits closely to the recent development that attempts to bring more realism into production networks by incorporating market imperfections and inefficiencies. **?** and Bigio and La'O (2020) model frictions as an exogenous wedge between the sectors' marginal costs and marginal revenues and examine how these frictions interact with the network structure and affect aggregate output in the Cobb-Douglas economy. Bigio and La'O (2020) bring the model to data and estimate how the US input-output structure amplify financial distortions in the great recession. Baqaee and Farhi (2020) examine how productivity shocks aggregate under the presence of exogenous wedges in a CES economy and decompose output changes into changes in technology and changes in allocative efficiency. Liu (2019) assumes that market imperfections generate dead-weight loss and demonstrates how these imperfections compound in the production network through demand linkages. He shows that a government should design industrial policies to target sectors based on distortions in sectoral size. To our knowledge, our paper is the first to model search and matching in a production networks setting.

This paper also contributes to the labor literature on factor reallocation. Our paper is the first to shed light on how production linkages impact reallocation and comovements in labor markets under search-and-matching frictions.

The remainder of the paper is organized as follows. Section 2 outlines our model and defines the equilibrium. Section 3 derives expressions for first order changes in output and employment in response to changes in technology and the size of the labor force. Section 4 describes the data we use to calibrate the model and presents illustrative examples to demonstrate the quantitative importance of labor markets. Section 5 works through two applications of our model to policy relevant questions: how sector level Beveridge curves shift in response to shocks and how output and employment responds to a shock to energy prices. Section 6 concludes.

# 2. Model

This section describes the basic setup of our mode. There are $J$ production sectors, indexed by $i$, with production linkages and $\mathcal{O}$ occupations index by $o$. We call $o$ an occupation, rather than sector, may be the relevant margin of choice for individuals when they search for jobs Humlum (Humlum). Individuals tend to stay in the same or similar occupations over the course of their careers **?**. However, our formulation is general enough to allow us to think of $o$ as representing sector specific employment, location specific employment, occupation-location pairs, or any other desired definition of a labor market. In addition to intermediate inputs from other sectors and labor, firms may also use $\mathcal{K}$ other factors of production indexed by $k$. These additional factors of production will allow us to explore the effects of a shock to oil, for instance. For simplicity, we keep the model static, as is the case in most of the production networks literature. Throughout, bold letters indicate vectors or matrices. We use un-bolded letters to denote functions or scalars. All vectors are column vectors.

## 2.1. Households and Final Production

A representative household inealastically supplies a labor force of size $H_o$ to occupation $o$. There is no disutility from labor. Households consume a final production good, $Y$, produced by a final goods producer with constant returns technology reflecting household preferences

$$Y = \max_{\{c_i\}_{i=1}^{J}} \mathcal{D}\left(\left\{c_i\right\}_{i=1}^{J}\right)$$

Subject to

$$\sum_{i=1}^{J} p_i c_i = \sum_{o=1}^{\mathcal{O}} w_o L_o + \sum_{k=1}^{\mathcal{K}} r_k K_k$$

Where $c_i$ is the input of sector $i$'s output into final goods production, $p_i$ is the price of the sector $i$ good, $w_o$ is the wage in occupation $o$, and $L_o$ is the labor used in sector $o$. $r_k$ is the price the additional factor $K_k$ of production. We can think of these $\mathcal{K}$ additional inputs into intermediate production as capturing capital or energy costs. The final production input choices satisfy the first order condition

$$\varepsilon^{\mathcal{D}}_{c_i} = \frac{p_i c_i}{\sum_{k=1}^{J} p_k c_k} \tag{1}$$

Where $\varepsilon^{\mathcal{D}}_{c_i}$ is the elasticity of $\mathcal{D}$ to changes in $c_i$. This is the households elasticity of utility with regards to the consumption of good $i$.

### 2.2. Labor Markets

We assume there are $\mathcal{O}$ occupations with separate labor markets, a labor force of $H_o$ possible workers, who all start out unemployed at the beginning of the single period. The exogenous recruiting cost, $r_o$, measures the units of labor required for a firm to maintain each posted vacancy in occupation $o$. When workers and firms meet there is a mutual gain from matching. There is no accepted theory for how wages are set in this context. For now we assume the nominal wage in occupation $o$, $w_o$, follows a general wage schedule that depends on productivity and the size of the labor force, and is taken as given by both firms and workers. Hires are generated by a constant returns matching function in occupation-level unemployment $U_o$ and aggregate vacancies $V_o$, which measures all vacancy postings for occupation $o$,

$$h_o = \phi_o m\left(U_o, V_o\right)$$

Let the sector-specific labor market tightness be $\theta_o = \frac{V_o}{H_o}$, the vacancy-filling rate $\mathcal{Q}_o(\theta_o) = \phi_o m\left(\frac{H_o}{V_o}, 1\right)$, and the job-finding rate $\mathcal{F}_o(\theta_o) = \phi_o m\left(1, \frac{v_o}{H_o}\right)$. Therefore, a fraction $\mathcal{F}_o(\theta_o)$ the labor force finds a job and is employed at the end of the period. Labor supply satisfies

$$L^o_i(\theta_o) = \mathcal{F}_o(\theta_o) H_o \tag{2}$$

We assume firms take the occupation level tightness as given.[1] Let $N_{io}$ denote productive employees in occupation $o$ working for sector $i$ firms. In order to hire $N_{io}$ productive

[1] One way of justifying this is with the assumption that each sector is populated by many identical competitive firms so that each firm only has an infinitesimal impact on aggregate vacancies, and therefore on aggregate tightness.

employees, the number of vacancies posted in labor market $o$ by sector $i$ firms, $v_{io}$, has to satisfy $\mathcal{Q}_o(\theta_o)v_{io} = N_{io} + r_o v_{io}$, where $r_o v_{io}$ is the total cost of posting the vacancies. Rearranging yields $v_{io} = \frac{N_{io}}{\mathcal{Q}_o(\theta_o)-r_o}$. Thus, hiring one unit of productive labor requires $\frac{1}{\mathcal{Q}_o(\theta_o)-r_o}$ vacancy postings. Factoring in hiring costs, firms need $1+\tau_o(\theta_o)$ units of total labor for each productive worker, where

$$\tau_o(\theta_o) \equiv \frac{r_o}{\mathcal{Q}_o(\theta_o) - r_o}.$$

For a given target level of occupation $o$ employment $N_{io}$, total required labor, or the labor demand, is $l^d_{io}(\theta_o) = (1+\tau_o(\theta_o))\,N_{io}$. In the language of the production networks literature, $\tau_o$ therefore acts as an endogenous wedge on firms labor costs. This endogenous wedge will turn out to play an important role in how shocks propagate through the production network. We describe how labor demand, $l^d_{io}(\theta_{io})$, is determined by firms' profit maximization in the next subsection.

Finally, we define aggregate occupation $o$ labor demand as the sum of sectoral labor demands and aggregate vacancy postings as the sum of sectoral vacancy postings.

$$\text{(3)} \qquad \begin{aligned} L^d_o(\theta_o) &= \sum_{i=1}^{J} l^d_{io}(\theta_o) \\ V_o &= \sum_{i=1}^{J} v_{io} \end{aligned}$$

Market clearing in the labor market requires labor demand equal labor supply and that the vacancy posting choices of firms in each sector are consistent with aggregate tightness.

$$\text{(4)} \qquad \begin{aligned} L^d_o &= L^s_o \\ \theta_o &= \frac{\sum_{i=1}^{J} v_{io}}{H_o} \end{aligned}$$

## 2.3. Firms

A representative firm in sector $i$ uses workers in occupation $o$, $N_{io}$, intermediate inputs from sector $j$, $x_{ij}$, and $\mathcal{K}$ additional factors $K_{ik}$ to produce output $y_i$ using constant returns production technology $f_i$.

$$y_i = A_i f_i \left( \left\{ N_{io} \right\}_{o=1}^{\mathcal{O}}, \{x_{ij}\}_{j=1}^{J}, \left\{ K_{ik} \right\}_{k=1}^{\mathcal{K}} \right) \tag{5}$$

Firms choose $\left\{ N_{io} \right\}_{o=1}^{\mathcal{O}}$, $\left\{ x_{ij} \right\}_{j=1}^{J}$, and $\left\{ K_{ik} \right\}_{ik}^{\mathcal{K}}$ to maximize profits, or equivalently to minimize costs. We assume firms are price takers in both input and output markets. Profits are given by

$$\pi_i = p_i f_i \left( \left\{ N_{io} \right\}_{o=1}^{\mathcal{O}}, \left\{ x_{ij} \right\}_{j=1}^{J}, \left\{ K_{ik} \right\}_{k=1}^{\mathcal{K}} \right) - \sum_{o=1}^{\mathcal{O}} w_o (1 + \tau_o(\theta_o)) N_{io} - \sum_{j=1}^{J} p_j x_{ij} - \sum_{k=1}^{\mathcal{K}} r_k K_{ik}$$

Firms choose inputs to solve

$$\max_{\{N_{io}\}_{o=1}^{\mathcal{O}}, \left\{ x_{ij} \right\}_{j=1}^{J}, \{K_{ik}\}_{k=1}^{\mathcal{K}}} \pi_i \left( \left\{ N_{io} \right\}_{o=1}^{\mathcal{O}}, \left\{ x_{ij} \right\}_{j=1}^{J}, \left\{ K_{ik} \right\}_{k=1}^{\mathcal{K}} \right)$$

Giving the first order conditions

$$\varepsilon_{x_{ij}}^{f_i} = \frac{p_j x_{ij}}{p_i y_i} \tag{6}$$

$$\varepsilon_{N_{io}}^{f_i} = (1 + \tau_o(\theta_o)) \frac{w_o N_{io}}{p_i y_i} \tag{7}$$

$$\varepsilon_{K_{ik}}^{f_i} = \frac{r_k K_{ik}}{p_i y_i} \tag{8}$$

Where $\varepsilon_{x_{ij}}^{f_i}$ is the elasticity of sector $i$ production to sector $j$ input, $\varepsilon_{N_{io}}^{f_i}$ is the elasticity of sector $i$ production to occupation $o$ labor, and $\varepsilon_{K_{ik}}^{f_i}$ is the elasticity of sector $i$ production to factor $k$. From Equation 7, we can derive an expression for sector $i$ and aggregate labor demand:

$$l_{io}^{d}(\theta_o) = \varepsilon_{N_{io}}^{f_i} \frac{p_i}{w_o} y_i \tag{9}$$

$$L_o^d(\theta_o) = \sum_{i=1}^{J} \varepsilon_{N_{io}}^{f_i} \frac{p_i}{w_o} y_i$$

We assume that the additional factors of production are supplied exogenously. Market clearing in the factor markets requires

$$K_k^s = \sum_{i=1}^{J} K_{ik} \tag{10}$$

### 2.4. Equilibrium

Given exogenous variables $\left\{\{A_i\}_{i=1}^{J}, \{H_o\}_{o=1}^{\mathcal{O}}, \left\{K_k^s\right\}_{k=1}^{K}\right\}$, the equilibrium is a collection of $4J + 2J^2 + 3\mathcal{O}J + \mathcal{K}J + 4\mathcal{O} + \mathcal{K}$ endogenous variables

$$\left\{\left\{p_i, y_i, \left\{x_{ij}, \varepsilon_{x_{ij}}^{f_i}\right\}_{j=1}^{J}, c_i, \varepsilon_{c_i}^{\mathcal{D}}, \left\{N_{io}, \varepsilon_{N_{io}}^{f_i}, l_{io}^{d}\right\}_{o=1}^{\mathcal{O}}, \{K_{ik}\}_{k=1}^{K}\right\}_{i=1}^{J}, \left\{\theta_o, w_o, L_o^d, L_o^s\right\}_{o=1}^{\mathcal{O}}, \{r_k\}_{k=1}^{\mathcal{K}}\right\}$$

that satisfy equations (1), (2), (3), (4), (5), (6), (7), (8), (9), and (10) for all sectors and occupations, along with constant returns to scale in final goods production

$$\sum_{i=1}^{J} \varepsilon_{c_i}^{\mathcal{D}} = 1 \tag{11}$$

constant returns to scale in secotoral production

$$\sum_{j=1}^{J} \varepsilon_{x_{ij}}^{f_i} + \sum_{o=1}^{\mathcal{O}} \varepsilon_{N_{io}}^{f_i} + \sum_{k=1}^{\mathcal{K}} \varepsilon_{K_{ik}}^{f_i} = 1 \qquad \forall i \tag{12}$$

and goods market clearing

$$y_i = c_i + \sum_{j=1}^{J} x_{ij} \qquad \forall i \tag{13}$$

Equations (1)-(13) provide $4J + J^2 + 2J\mathcal{O} + \mathcal{K}J + 3\mathcal{O} + \mathcal{K} + 1$ restrictions. To close the model, we must make an assumption about the wage schedule, providing an additional $\mathcal{O}$ restrictions, and functional form assumptions for final and sectoral production.

# 3. Theoretical Results

In this section, we describe the main theoretical results of our paper. We first derive the first order propagation of technology, labor supply, and factor supply shocks at the dis-aggregated occupation and sector level. Specifically, we compute the first order responses of the main endogenous variables of interest in the model. We then present our aggregation theorem of idiosyncratic technology, labor supply, and factor supply shocks for output and unemployment. The aggregate impact of idiosyncratic shocks can be summarized by sales shares and matching elasticities. We assume Cobb-Douglas production throughout. We show how to generalize the results to any constant returns production function in the case of one occupation per sector in appendix B.

## 3.1. Notation

Before outlining our results, we define some useful notation. First, as is standard in the networks literature, we define the input-matrix $\mathbf{\Omega}$ as the matrix containing sector level input revenue shares

$$\mathbf{\Omega} = \begin{bmatrix} \frac{p_1 x_{11}}{p_1 y_1} & \cdots & \frac{p_J x_{1J}}{p_1 y_1} \\ \vdots & \ddots & \vdots \\ \frac{p_1 x_{J1}}{p_J y_J} & \cdots & \frac{p_J x_{JJ}}{p_J y_J} \end{bmatrix}_{J \times J}$$

In our competitive goods market equilibrium setup, we can rewrite this input output matrix in terms of the elasticities of the sector level production functions. (See (6))

$$\mathbf{\Omega} = \begin{bmatrix} \varepsilon^{f_1}_{x_{11}} & \cdots & \varepsilon^{f_1}_{x_{1J}} \\ \vdots & \ddots & \vdots \\ \varepsilon^{f_J}_{x_{J1}} & \cdots & \varepsilon^{f_J}_{x_{JJ}} \end{bmatrix}_{J \times J}$$

We denote the Leontief inverse by $\mathbf{\Psi} = (\boldsymbol{I} - \mathbf{\Omega})^{-1}$. The Leontief inverse captures the importance of each sector as a direct and indirect input into production in every other sector.

Alongside the production elasticities to intermediate inputs, our model features a second set of production elasticities: elasticities to the different types of labor inputs.

We collect these elasticities in the matrix $\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}$.

$$\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} = \begin{bmatrix} \varepsilon^{f_1}_{N_{11}} & \cdots & \varepsilon^{f_1}_{N_{1\mathcal{O}}} \\ \vdots & \ddots & \vdots \\ \varepsilon^{f_J}_{N_{J1}} & \cdots & \varepsilon^{f_J}_{J\mathcal{O}} \end{bmatrix}_{J\times\mathcal{O}}$$

Equation (7) demonstrates that this matrix is the labor input equivalent of the standard input-output matrix. In equilibrium, each entry is the revenue share of type $o$ labor.

Similarly, we collect the factor elasticities in the matrix $\varepsilon^{\boldsymbol{f}}_{\boldsymbol{K}}$.

$$\varepsilon^{\boldsymbol{f}}_{\boldsymbol{K}} = \begin{bmatrix} \varepsilon^{f_1}_{K_{11}} & \cdots & \varepsilon^{f_1}_{K_{1\mathcal{K}}} \\ \vdots & \ddots & \vdots \\ \varepsilon^{f_J}_{K_{J1}} & \cdots & \varepsilon^{f_J}_{K_{J\mathcal{K}}} \end{bmatrix}_{J\times\mathcal{K}}$$

Equation (8) demonstrates that this matrix is the factor input equivalent of the standard input-output matrix. In equilibrium, each entry is the revenue share of factor $k$.

$\mathcal{L}$ is a $\mathcal{O} \times J$ with the share of occupation $o$ workers employed in each sector along the rows.

$$\mathcal{L} = \begin{bmatrix} \frac{l_{11}}{L_1} & \cdots & \frac{l_{J1}}{L_1} \\ \vdots & \ddots & \vdots \\ \frac{l_{1\mathcal{O}}}{L_{\mathcal{O}}} & \cdots & \frac{l_{J\mathcal{O}}}{L_{\mathcal{O}}} \end{bmatrix}_{\mathcal{O}\times J}$$

$\mathfrak{F}$, $\mathfrak{Q}$, and $\mathfrak{T}$ are $\mathcal{O} \times \mathcal{O}$ diagonal matrices with the occupation level job-finding rate, vacancy-filling rate, and recruiter-producer ratio along the diagonal.

$$\mathfrak{F} = \begin{bmatrix} \varepsilon^{\mathcal{F}_1}_{\theta_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \varepsilon^{\mathcal{F}_{\mathcal{O}}}_{\theta_{\mathcal{O}}} \end{bmatrix}_{\mathcal{O}\times\mathcal{O}}, \quad \mathfrak{Q} = \begin{bmatrix} \varepsilon^{\mathcal{Q}_1}_{\theta_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \varepsilon^{\mathcal{Q}_{\mathcal{O}}}_{\theta_{\mathcal{O}}} \end{bmatrix}_{\mathcal{O}\times\mathcal{O}}, \quad \mathfrak{T} = \begin{bmatrix} \tau_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \tau_{\mathcal{O}} \end{bmatrix}_{\mathcal{O}\times\mathcal{O}}$$

Assuming a Cobb-Douglas matching function $m(U, V) = \phi U^{\eta} V^{1-\eta}$, these matrices are

$$\mathfrak{F} = \begin{bmatrix} 1-\eta_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1-\eta_{\mathcal{O}} \end{bmatrix}, \qquad \mathfrak{Q} = \begin{bmatrix} -\eta_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & -\eta_{\mathcal{O}} \end{bmatrix}$$

### 3.2. The Propagation of Technology and Labor Force Shocks

We are interested in how three sets of endogenous variables—sector level prices, sector level output, and occupation level tightness—change in response to change in technology and the size of the labor force. With these variables, we can compute the real aggregate variables of interest: as output and unemployment.

Before deriving our results, we need to specify how wages are determined. In matching models, workers and firms meet in a situation of bilateral monopoly. The resulting mutual gains from trade mean that wages are not determined by equilibrium conditions of the model[2], and must instead be pinned down by some wage setting norm chosen by the researcher. We introduce the following reduced-form relationship between relative wages and the shocks:

$$d\log \boldsymbol{w} - \mathcal{L} d\log \boldsymbol{p} = \boldsymbol{\Lambda_A} d\log \boldsymbol{A} + \boldsymbol{\Lambda_H} d\log \boldsymbol{H} \boldsymbol{\Lambda}_K d\log \boldsymbol{K}^S.$$

Where $d\log \boldsymbol{w}$ and $d\log \boldsymbol{H}$ are $\mathcal{O} \times 1$ dimensional vectors capturing first order changes in wages and the size of the labor force, $d\log \boldsymbol{p}$ and $d\log \boldsymbol{A}$ are $J \times 1$ dimensional vectors capturing first order changes in prices and productivities. $\mathcal{L}$ is a $\mathcal{O} \times J$ matrix with the share of occupation $o$ workers employed in each sector along the rows. The $(o, i)$th entry is $\frac{l_{io}}{L_o}$. $\boldsymbol{\Lambda}_A$, $\boldsymbol{\Lambda}_H$, and $\boldsymbol{\Lambda}_K$ are $\mathcal{O} \times J$, $\mathcal{O} \times \mathcal{O}$, and $\mathcal{O} \times \mathcal{K}$ coefficient matrices that capture how wages respond to technology, labor force, and factor supply shocks in each other sector, occupation, and factor market. For instance, the $(o, i)$th entry of $\boldsymbol{\Lambda}_A$ captures how wages in occupation $o$ respond to technology shocks in sector $i$. The $(o, j)$th entry in $\boldsymbol{\Lambda}_H$ captures how wages in occupation $o$ respond to labor force shocks in occupation $j$. And the $(o, k)$th entry in $\boldsymbol{\Lambda}_K$ captures how wages in occupation $o$ respond to a supply shock to factor $k$.

Since, as we show below, prices are themselves determined by shocks to technology, the size of the labor force, and factor supply this assumption captures the intuition that, ultimately, wages must be driven by the fundamental shocks in our model: $\boldsymbol{A}$, $\boldsymbol{H}$ and $\boldsymbol{K}^S$. While it may appear restrictive, this reduced form wage setting equation is capable of nesting any assumption about wage setting tied to economic fundamentals for the right parameter matrices $\boldsymbol{\Lambda}_A$, $\boldsymbol{\Lambda}_H$, and $\boldsymbol{\Lambda}_K$.

With changes in wages in hand, we can now derive how shocks propagate to prices,

[2] Wages are only constrained to fall within a range where both workers and firms benefit from the match. However, this range can be wide since workers usually strongly prefer employment to unemployment and finding a new match is costly for firms.

labor market tightness, and output. These first order effects are summarized in proposition 1. For a detailed derivation see appendix A.1.

PROPOSITION 1. *Assume intermediate and final goods production functions are constant returns to scale and Cobb-Douglas. Given labor supply shocks* $d\log \boldsymbol{H} = \left[d\log H_1, d\log H_2, \cdots, d\log H_{\mathcal{O}}\right]'$, *productivity shocks* $d\log \boldsymbol{A} = \left[d\log A_1, d\log A_2, \cdots, d\log A_J\right]'$, *and factor supply shocks* $d\log \boldsymbol{K}^s = \left[d\log K_1^s, d\log K_2^s, \cdots, d\log K_{\mathcal{K}}^s\right]'$ *the first-order responses of labor market tightness* $d\log \theta = \left[d\log \theta_1, d\log \theta_2, \cdots, d\log \theta_{\mathcal{O}}\right]'$ *and output* $d\log \boldsymbol{y} = \left[d\log y_1, d\log y_2, \cdots, d\log y_J\right]'$ *follow:*

$$
\begin{aligned}
d\log \theta &= \Pi_{\theta,A} d\log \boldsymbol{A} + \Pi_{\theta,H} d\log \boldsymbol{H} + \Pi_{\theta,K} d\log \boldsymbol{K}^s, \\
d\log \boldsymbol{y} &= \Pi_{y,A} d\log \boldsymbol{A} + \Pi_{y,H} d\log \boldsymbol{H} + \Pi_{y,K} d\log \boldsymbol{K}^s,
\end{aligned}
$$

*where*

$$
\begin{aligned}
\Pi_{\theta,A} &= \left[\mathcal{F} - \Xi_\theta\right]^{-1} \left(\mathcal{L}\Psi - \Lambda_A\right), \\
\Pi_{\theta,H} &= \left[\mathcal{F} - \Xi_\theta\right]^{-1} \left(\mathcal{L}\Psi \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I} - \Lambda_H\right), \\
\Pi_{\theta,K} &= \left[\mathcal{F} - \Xi_\theta\right]^{-1} \left(\mathcal{L}\Psi \varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}} - \Lambda_K\right), \\
\Pi_{y,A} &= \Psi \left[\boldsymbol{I} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left(\mathcal{F} + \mathcal{Q}\mathcal{T}\right) \left[\mathcal{F} - \Xi_\theta\right]^{-1} \left[\mathcal{L}\Psi - \Lambda_A\right]\right], \\
\Pi_{y,H} &= \Psi \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left[\boldsymbol{I} + \left(\mathcal{F} + \mathcal{Q}\mathcal{T}\right) \left[\mathcal{F} - \Xi_\theta\right]^{-1} \left[\mathcal{L}\Psi \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I} - \Lambda_H\right]\right], \\
\Pi_{y,K} &= \Psi \left[\varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left(\mathcal{F} + \mathcal{Q}\mathcal{T}\right) \left[\mathcal{F} - \Xi_\theta\right]^{-1} \left[\mathcal{L}\Psi \varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}} - \Lambda_K\right]\right], \\
\Xi_\theta &= \mathcal{L}\Psi \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left[\mathcal{F} + \mathcal{Q}\mathcal{T}\right].
\end{aligned}
$$

PROOF. See appendix A.1. □

We denote the $J \times J$ input-output matrix by $\boldsymbol{\Omega}$. The $(i, j)$th entry is the elasticity of output in sector $i$ to intermediate inputs from sector $j$. $\boldsymbol{\Psi} = (1 - \boldsymbol{\Omega})^{-1}$ is the standard $J \times J$ Leontief inverse. The Leontief inverse captures how each sectors output depends on inputs from other sectors both directly and indirectly. The $(i, j)$th entry captures how important sector $j$ is as a direct input to sector $i$ production, how important sector $j$ is as a direct input into all other sector $i$ inputs, and so on.

$\mathcal{F}$, $\mathcal{Q}$, and $\mathcal{T}$ are $\mathcal{O} \times \mathcal{O}$ diagonal matrices with, respectively, the occupation level job-finding rate, vacancy-filling rate, and recruiter-producer ratio, all evaluated at the initial equilibrium, along the diagonal. $\varepsilon_N^f$ is a $J \times \mathcal{O}$ matrix with sector $i$'s output elasticity with respect to each occupations labor input along the rows. The $(i, o)$th element is the elasticity of sector $i$ production to occupation $o$ labor. Assuming Cobb-Douglas greatly simplifies the expressions because it implies that all of the elasticities, both $\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}$ and $\boldsymbol{\Omega}$, remain constant.

Though the expressions might seem daunting at first, we unpack the intuition behind each in turn. First, we consider the response in tightness. The equilibrium response in tightness are jointly determined by changes in labor supply and labor demand. The labor supply side is impacted directly by changes in labor supply and job-finding rates, and the labor demand side is impacted by vacancy-filling rates, wages and prices, as well as productivity. A technology shock impacts labor demand by directly impacting a sector's productive capability, thus impacting prices and output and labor usage for other sectors through production linkages. A labor supply shock impacts labor supply by changing the number of available workers, which changes sectors' production through changing in the number of recruiters needed. The change in sectors' production propagate through the network and induce other sectors to change their labor demand, thus causing tightness to adjust, so on and so forth.

Second, we consider the response in sectoral output. Sectoral output is directly impacted by changes in technology, output in other sectors, and labor market tightness. A shock to technology directly impacts a sector's productive capacity through the production function. It also indirectly impacts a sector's production through the tightness channel, which documents the equilibrium interaction between labor supply and labor demand. Such impact on the sector's production then propagate to other sectors through production linkages, and to different labor markets through the labor network.

PROPOSITION 2. *Assume intermediate and final goods production functions are constant returns to scale and Cobb-Douglas. The first-order responses of relative sectoral and factor prices are pinned down by labor supply, technology, and factor supply shocks up to a numeraire. These first order responses satisfy*

$$\left(\boldsymbol{I} - \Psi \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \mathcal{L}\right) d\log \boldsymbol{p} = \boldsymbol{\Pi}_{p,A} d\log \boldsymbol{A} + \boldsymbol{\Pi}_{p,H} d\log \boldsymbol{H} + \boldsymbol{\Pi}_{p,K} d\log \boldsymbol{K}^s + \Psi \varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}} \mathbf{1} d\log y_{num},$$

*Where $d\log y_{num}$ denotes the change in output in the numeraire sector, which is determined* *as per proposition 1, and*

$$\Pi_{p,A} = \Psi\left[\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left(\Lambda_A - \mathfrak{Q}\mathcal{T}\left[\mathfrak{F} - \Xi_\theta\right]^{-1}(\mathcal{L}\Psi - \Lambda_A)\right) - \boldsymbol{I}\right],$$
$$\Pi_{p,H} = \Psi\left[\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left(\Lambda_H - \mathfrak{Q}\mathcal{T}\left[\mathfrak{F} - \Xi_\theta\right]^{-1}\left(\mathcal{L}\Psi\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I} - \Lambda_{\boldsymbol{H}}\right)\right)\right],$$
$$\Pi_{p,K} = \Psi\left[\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left(\Lambda_K - \mathfrak{Q}\mathcal{T}\left[\mathfrak{F} - \Xi_\theta\right]^{-1}\left(\mathcal{L}\Psi\varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}} - \Lambda_K\right)\right) - \varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}}\right]$$

PROOF. See appendix A.1. □

In our framework, prices are equilibrium objects determined by market clearing in a perfectly competitive market. Given that our production functions are Cobb-Douglas, the price of a good produced by a particular sector responds to changes in prices in all other sectors, as well as the effective cost of employing workers, which relies on labor market tightness and wages. In addition, the price is also directly linked to the level of productivity in that sector. Thus, a productivity shock impacts the system of prices directly through production and indirectly through adjustments in tightness, and a labor supply shock impacts prices solely through adjustments in labor market tightness.

Therefore, if one looks closely at the propagation equations, one can see a separation between what is propagated through solely the production network, and what is propagated through the interaction between the production network, the labor usage matrix, and the matching elasticities. We will return to this point in section **??**.

### 3.3. Aggregation

With a Cobb-Douglas preference, it is straightforward that $d\log Y = \varepsilon_{\boldsymbol{c}}^{\mathcal{D}'} d\log \boldsymbol{y}$. Using Proposition 1, we arrive at the following result for first-order changes in aggregate output.

THEOREM 1. *Given idiosyncratic labor supply shocks $d\log \boldsymbol{H}$ and productivity shocks $d\log \boldsymbol{A}$, the log change in real GDP is:*

$$d\log Y = \Pi_{\boldsymbol{A}} d\log \boldsymbol{A} + \Pi_{\boldsymbol{H}} d\log \boldsymbol{H} + \Pi_{\boldsymbol{K}} d\log \boldsymbol{K}^s,$$

*where*

$$
\begin{aligned}
\Pi_{\boldsymbol{A}} &= \lambda' \left( \boldsymbol{I} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left( \mathcal{F} + \mathfrak{Q}\mathfrak{T} \right) \left[ \mathcal{F} - \Xi_{\theta} \right]^{-1} \left( \mathcal{L}\Psi - \Lambda_{A} \right) \right), \\
\Pi_{\boldsymbol{H}} &= \lambda' \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left( \boldsymbol{I} + \left( \mathcal{F} + \mathfrak{Q}\mathfrak{T} \right) \left[ \mathcal{F} - \Xi_{\theta} \right]^{-1} \left( \left[ \mathcal{L}\Psi \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I} \right] - \Lambda_{H} \right) \right), \\
\Pi_{\boldsymbol{K}} &= \lambda' \left( \varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left( \mathcal{F} + \mathfrak{Q}\mathfrak{T} \right) \left[ \mathcal{F} - \Xi_{\theta} \right]^{-1} \left( \mathcal{L}\Psi \varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}} - \Lambda_{K} \right) \right)
\end{aligned}
$$

*and* $\boldsymbol{\lambda} = \boldsymbol{\Psi}' \varepsilon_{\boldsymbol{c}}^{\mathfrak{D}}$ *denotes the sectors' sales shares.*

PROOF. The result follows directly from $d \log Y = \varepsilon_{\boldsymbol{c}}^{\mathfrak{D}'} d \log \boldsymbol{y}$ and proposition 1. □

Theorem 1 results from Proposition 1, as we can compute the change in real GDP by weighing the change in real output for all sectors by the household's demand elasticities. Note that the matrix product between the Leontief inverse and the demand elasticities is equal to the sales shares. This property results directly from household's maximization problem, the firms' profit maximization decision, goods market clearing, and Cobb-Douglas production functions. Therefore, the aggregate impact of productivity and labor supply shocks can be summarized as the sales-share-weighted impact of these shocks on sectoral output directly through production and indirectly through labor markets.

This result, that the aggregate impact of shocks is the sales share weighted sum of sector specific impacts, is reminiscent of Hulten's theorem (ADD CITATION). Indeed, when network price adjusted wages change exactly proportionally to the marginal product of labor, Hulten's theorem holds for technology technology shocks.

THEOREM 2. *Hulten's theorem holds for technology shocks whenever network price adjusted wages change exactly proportionally to the network adjusted marginal product of labor. That is when*

$$
d \log \boldsymbol{w} - \mathcal{L} d \log \boldsymbol{p} = \mathcal{L} d \log MP
$$

*Where $d \log MP$ is a matrix of changes to the marginal product of each type of labor in each sector. Furthermore, when this condition holds, aggregate changes in response to labor force and factor supply shocks are independent of matching frictions.*

PROOF. The derivative of production with respect to labor inputs, along with the firms first order conditions, imply that the network adjusted marginal product of labor satisfies

$$\mathcal{L} d \log \mathrm{MP} = -\mathfrak{T}\Omega d \log \theta + d \log \boldsymbol{w} - \mathcal{L} d \log \boldsymbol{p}$$

Imposing that network price adjusted wage changes are exactly proportional to changes in the network adjusted marginal product of labor implies

$$-\mathfrak{T}\Omega d \log \theta = 0$$

Since $\mathfrak{T}\Omega$ is diagonal matrix with non-zero diagonal elements

$$d \log \theta = 0$$

Since search-and-matching operates through changes in $d \log \theta$, this implies that search-and-matching has no impact on the propagation of shocks. In particular

$$d \log Y = \lambda' \left[ d \log \boldsymbol{A} + \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} d \log \boldsymbol{H} + \varepsilon^{\boldsymbol{f}}_{\boldsymbol{K}} d \log \boldsymbol{K}^{s} \right]$$

The aggregate output response to technology shocks is $\lambda' d \log \boldsymbol{A}$, this is exactly Hulten's theorem. The aggregate output response to the other shocks depends only on production parameters, matching frictions play no role. □

COROLLARY 1. *When wages do not respond exactly proportionally to the network adjusted marginal product of labor, matching frictions generate generate deviations from Hulten's theorem captured by*

$$\begin{aligned}
\Pi_{search,A} &= \lambda' \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} (\mathcal{F} + \Omega\mathfrak{T}) \left[\mathcal{F} - \Xi_{\theta}\right]^{-1} (\mathcal{L}\Psi - \Lambda_{A}) \\
\Pi_{search,H} &= \lambda' \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} (\mathcal{F} + \Omega\mathfrak{T}) \left[\mathcal{F} - \Xi_{\theta}\right]^{-1} \left( \left[ \mathcal{L}\Psi \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} - \boldsymbol{I} \right] - \Lambda_{H} \right) \\
\Pi_{search,K} &= \lambda' \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} (\mathcal{F} + \Omega\mathfrak{T}) \left[\mathcal{F} - \Xi_{\theta}\right]^{-1} \left( \mathcal{L}\Psi \varepsilon^{\boldsymbol{f}}_{\boldsymbol{K}} - \Lambda_{K} \right)
\end{aligned}$$

PROOF. Follows directly from Theorem 1 and Theorem 2. □

In general, putting a sign on these additional impacts is a quantitative question, and the direction of amplification versus dampening depends on how labor market tightness responds. Tightness, as an equilibrium object, responds to shocks in a complicated way that depends on how each sector uses different types of labor, production linkages, relative wage rigidity, and matching elasticities. In subsection 3.4, we use two simple examples to demonstrate when amplification and dampening can occur.

Finally, we are interested in the response of aggregate unemployment to sector specific shocks. Using the results in Theorem 1, we can derive changes in aggregate unemployment and the relative aggregate price level.

COROLLARY 2. *Given idiosyncratic labor supply shocks $d\log \boldsymbol{H}$ and productivity shocks $d\log \boldsymbol{A}$, the first-order response in aggregate unemployment is:*

$$d\log U^{agg} = \Pi_{U^{agg},A} d\log \boldsymbol{A} + \Pi_{U^{agg},H} d\log \boldsymbol{H},$$

*where*

$$\Pi_{U^{agg},A} = \frac{1}{U^{agg}} \boldsymbol{U}' \left[\Lambda_A - \mathcal{L}\Pi_{y,A}\right],$$
$$\Pi_{U^{agg},H} = \frac{1}{U^{agg}} \boldsymbol{U}' \left[\boldsymbol{I} + \Lambda_H - \mathcal{L}\Pi_{y,H}\right].$$

*The first-order response in aggregate price is:*

$$d\log P = \varepsilon_{\boldsymbol{c}}^{\mathcal{D}'} d\log \boldsymbol{p},$$

*where*

$$\left(\boldsymbol{I} - \Psi\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\mathcal{L}\right) d\log \boldsymbol{p} = \Pi_{p,A} d\log \boldsymbol{A} + \Pi_{p,H} d\log \boldsymbol{H},$$
$$\Pi_{p,A} = \Psi\left[\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left(\Lambda_A - \Omega\mathcal{T}\left[\mathcal{F} - \Xi_\theta\right]^{-1}(\mathcal{L}\Psi - \Lambda_A)\right) - \boldsymbol{I}\right],$$
$$\Pi_{p,H} = \Psi\left[\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left(\Lambda_H - \Omega\mathcal{T}\left[\mathcal{F} - \Xi_\theta\right]^{-1}\left(\mathcal{L}\Psi\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I} - \Lambda_{\boldsymbol{H}}\right)\right)\right].$$

PROOF. See appendix A.2 □

The change in unemployment is a weighted average of the first-order changes in the size of the labor force and employment. The employment level change for each labor market in equilibrium equates the change in labor demand in that labor market to labor supply. The change in labor demand in that labor market depends on how relative wages respond to shocks, as well as how sectoral output changes for sectors that use that particular type of labor.

The first-order response in aggregate price level is more straightforward, and is simply the changes in sectoral prices weighted by their consumption share, which is equivalent to the demand elasticities of consumption when preferences are Cobb-

Douglas. Note that one can solve out the price changes explicitly by picking a numeraire, for example, by assuming $d\log p_1 = 0$. However, we don't explicitly do it here in order to maintain a simpler expression for the relationship between the price system and productivity and labor supply shocks.

### 3.4. Two Toy Examples

In this subsection, we provide two toy examples to demonstrate how the search channel impacts aggregation through the production network. Since it is now well understood how production linkages amplify sector level shocks, in subsubsection 3.4.1 we focus on the search and matching channel. To clarify the exposition, we abstract from the additional factors of production, and consider a model featuring only labor and intermediate inputs. This section explores how labor market frictions can amplify or dampen the impact of idiosyncratic shocks by assuming no production linkages. Subsection 3.4.2 presents a simple vertical production network model to showcase how production linkages interact with search frictions.

#### 3.4.1. A Model with no Production Linkage

To get an intuition for how search and matching frictions can potentially impact the response of aggregate, we first examine what happens when there are no production linkages between the sectors. This is equivalent to assuming $\boldsymbol{\Omega} = \mathbf{0}$ and thus $\boldsymbol{\Psi} = \boldsymbol{I}$. Additionally, for simplicity, we assume each sector operates its own labor market, which means $\boldsymbol{L} = \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} = \boldsymbol{I}$. In other words, we consider the case where each sector produces using linear technology in its own unique type of labor. From Theorem 1, this gives rise to the following output aggregation formula:

$$d\log Y = \Pi_{\boldsymbol{A}} d\log \boldsymbol{A} + \Pi_{\boldsymbol{H}} d\log \boldsymbol{H},$$

where

$$\begin{aligned}
\Pi_{\boldsymbol{A}} &= \lambda' \left( \boldsymbol{I} + (\mathfrak{F} + \mathfrak{Q}\mathfrak{T}) \left[-\mathfrak{Q}\mathfrak{T}\right]^{-1} (\boldsymbol{I} - \Lambda_{\boldsymbol{A}}) \right), \\
\Pi_{\boldsymbol{H}} &= \lambda' \left( \boldsymbol{I} - (\mathfrak{F} + \mathfrak{Q}\mathfrak{T}) \left[-\mathfrak{Q}\mathfrak{T}\right]^{-1} \Lambda_{\boldsymbol{H}} \right).
\end{aligned}$$

Since $\mathfrak{F}$, $\mathfrak{Q}$, and $\mathfrak{T}$ are diagonal matrices, we have that:

$$(\mathfrak{F}+\mathfrak{Q}\mathfrak{T})\,[-\mathfrak{Q}\mathfrak{T}]^{-1} = \begin{pmatrix} \frac{1-\eta_1(1+\tau_1)}{\eta_1\tau_1} & 0 & \dots & 0 \\ 0 & \frac{1-\eta_2(1+\tau_2)}{\eta_2\tau_2} & \dots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & 0 & \dots & \frac{1-\eta_J(1+\tau_J)}{\eta_J\tau_J} \end{pmatrix}$$

Further, if we assume the relative wage in a sector respond only to shocks to that sector, we have that:

$$\boldsymbol{\Lambda_A} = \begin{pmatrix} \alpha_1 & 0 & \dots & 0 \\ 0 & \alpha_2 & \dots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & 0 & \dots & \alpha_J \end{pmatrix}, \quad \boldsymbol{\Lambda_H} = \begin{pmatrix} -\beta_1 & 0 & \dots & 0 \\ 0 & -\beta_2 & \dots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & 0 & \dots & -\beta_J \end{pmatrix}.$$

We assume relative wages respond positively to positively technology shocks, and negatively to labor supply shocks, that is $\alpha_i, \beta_i \geq 0$ for all $i$.

Therefore, the additional impact of technology shocks from search-and-matching becomes:

$$\boldsymbol{\Pi}_{\text{search},\boldsymbol{A}} = \begin{pmatrix} (1-\alpha_1)\frac{1-\eta_1(1+\tau_1)}{\eta_1\tau_1} & 0 & \dots & 0 \\ 0 & (1-\alpha_2)\frac{1-\eta_2(1+\tau_2)}{\eta_2\tau_2} & \dots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & 0 & \dots & (1-\alpha_J)\frac{1-\eta_J(1+\tau_J)}{\eta_J\tau_J} \end{pmatrix}.$$

Since labor market frictions operate through changes in productive employment, $N_i$, they cannot matter for shock propagation unless productive employment changes in response to the shocks. As a result, when $(1-\alpha_i)\frac{1-\eta_i(1+\tau_i)}{\eta_i\tau_i} = 0$ search-and-matching becomes irrelevant for productivity shock propagation. This occurs when $\alpha_i = 1$ or when $1-\eta_i(1+\tau_i) = 0$. When $\alpha_i = 1$ the change in wages is equal to the change in the marginal product of sector $i$ labor. Therefore, labor demand and employment remain unchanged. All of the effect of the productivity shock is absorbed into changes in wages rather than changes in employment.

$1-\eta_i(1+\tau_i)$ is the elasticity of the productive workforce in sector $i$ to sector $i$ vacancies. When $1-\eta_i(1+\tau_i) = 0$ posting additional vacancies does not change the number of productive workers. As a result, even if firms change the number of vacancy postings in response to productivity shocks, these changes do not have first order effects on output because they do not have first order effects on the number of productive workers.

In both cases, productive employment does not respond to changes in productivity and so search-and-matching frictions play no role in the propagation of shocks. Whenever employment does change following productivity shocks search-and-matching frictions matter.

For instance, if wages change by less than the marginal product of labor following productivity shocks, $\alpha_i < 1$, and increasing vacancy postings increases the number of productive workers, $1 < \eta_i(1+\tau_i)$, then employment will rise after a positive productivity shock. The resulting rise in employment boosts output further than the pure effect of the productivity shock alone. In this case, search-and-matching amplifies the effects of productivity shocks.

Similarly, when $(1-\alpha_i)\frac{1-\eta_i(1+\tau_i)}{\eta_i\tau_i} < 0$, search-and-matching dampens the aggregate effect of technology shocks in sector $i$. In this case, the effect cost of labor rises relative the marginal product of labor following a positive productivity shock. The resulting decline in employment dampens the output response.

In this simple example, with linear production in one unique type of labor per sector, the effect of productivity shocks on the marginal product of labor is clear. In our full model, the effects of productivity shocks in sector $i$ on the marginal product of type $o$ labor is a much more complicated object. However, the intuition remains the same. If wages adjust by less than the marginal product labor and posting additional vacancies increases the number of productive workers, then labor market frictions will amplify the effects of technology shocks. While in this example the effects remain isolated in the shocked industry $i$, with a full network of production linkages the amplification would propagate to sectors that use $i$'s good as an input, and from those sectors to other sectors, and so on.

We can gain similar insights about the amplification of labor force shocks through the following expression:

$$\Pi_{\text{search},\boldsymbol{H}} = \begin{pmatrix} \beta_1\frac{1-\eta_1(1+\tau_1)}{\eta_1\tau_1} & 0 & \cdots & 0 \\ 0 & \beta_2\frac{1-\eta_2(1+\tau_2)}{\eta_2\tau_2} & \cdots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & 0 & \cdots & \beta_J\frac{1-\eta_J(1+\tau_J)}{\eta_J\tau_J} \end{pmatrix}.$$

Again, in order for labor market frictions to change the way labor force shocks propagate, productive employment must change in response to the shocks. The interpretation of the $1 - \eta_i(1 + \tau_i)$ term is the same as above, so we focus on the $\beta_i$'s. Whenever $\beta_i = 0$ wages do not respond to labor force shocks. As a result, labor demand and, therefore, productive employment do not change.

As long as increasing the number of vacancy postings increases the number of productive workers, if labor becomes cheaper when there is a positive shock to the size of the labor force ($\beta_i > 0$), employment increases in response to the labor force shock. This increase boosts output, amplifying the effect of the labor fore shock. Conversely, if labor becomes more expensive when there is a positive shock to the labor force ($\beta_i < 0$), employment falls in response to the labor force shock. This reduces output, dampening the effect of the labor force shock. Again, this intuition carries through to our full model.

#### 3.4.2. A Two-Sector Vertical Production Network

The example above unpacks how search-and-matching can amplify or dampen shocks through wage responsiveness and the effect of increased vacancy postings on the number of productive workers. It doesn't, however, speak to how labor market propagation compounds with production linkages. This simple example is designed to highlight how production linkages interact with labor market frictions.

Consider an economy with two sectors, a downstream sector and an upstream sector, that produce using the same type of labor:

$$
\begin{aligned}
y_1 &= A_1 N_{11} \\
y_2 &= A_2 y_1^{\gamma} N_{21}^{1-\gamma}
\end{aligned}
$$

With this setup, we have that:

$$
\begin{aligned}
&\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} = \begin{bmatrix} 1 \\ 1-\gamma \end{bmatrix}, \\
&\boldsymbol{\Omega} = \begin{bmatrix} 0 & 0 \\ 1-\gamma & 0 \end{bmatrix}, \boldsymbol{\Psi} = \begin{bmatrix} 1 & 0 \\ 1-\gamma & 1 \end{bmatrix} \\
&\mathcal{F} = 1 - \eta, Q = -\eta, \\
&\mathcal{L} = \begin{bmatrix} \gamma & 1-\gamma \end{bmatrix},
\end{aligned}
$$

$$\varepsilon_{\boldsymbol{c}}^{\mathcal{D}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$\lambda = \begin{bmatrix} 1-\gamma \\ 1 \end{bmatrix}.$$

This implies that:

$$(\mathcal{F} + \mathcal{Q}\mathcal{T}) \left[\mathcal{F} - \Xi_\theta\right]^{-1} = \frac{1-\eta(1+\tau)}{1-\eta-\left[\gamma+2(1-\gamma)^2\right](1-\eta(1+\tau))},$$

First, we see that the recruiter-producer ratio that we uncover in subsubsection 3.4.1 is still at play in the numerator, that is, with a large enough recruiter-producer ratio, the direction of the impact of search and matching can be reversed. In addition, in the denominator, we now see that the production structure enters the search-and-matching part of output aggregation. Specifically, the production structure dictates how sectors use labor input from different labor markets, as well as how labor demand across sectors comove. However, the direction through which the production structure impacts output aggregation is ambiguous even in this simple example. For example, with $\gamma = 0$, the denominator becomes $\eta(1+2\tau) - 1$, which is negative for reasonable values of $\eta$ and $\tau$. On the other hand, with $\gamma = 0.75$, the denominator becomes $1 - \eta - 0.875(1-\eta(1+\tau)) > 0$.

We further assume that

$$d\log w - \mathcal{L} d\log \boldsymbol{p} = \begin{bmatrix} \alpha_1 & \alpha_2 \end{bmatrix} \begin{bmatrix} d\log A_1 \\ d\log A_2 \end{bmatrix} + \beta_1 d\log H.$$

Applying Theorem 1, we have that:

$$\Pi_{\text{search},\boldsymbol{A}} = \frac{2(1-\gamma)(1-\eta(1+\tau))}{1-\eta-\left[\gamma+2(1-\gamma)^2\right](1-\eta(1+\tau))} \begin{bmatrix} 1-\gamma+\gamma^2-\alpha_1 & 1-\gamma-\alpha_2 \end{bmatrix},$$

$$\Pi_{\text{search},\boldsymbol{H}} = \frac{2(1-\gamma)(1-\eta(1+\tau))}{1-\eta-\left[\gamma+2(1-\gamma)^2\right](1-\eta(1+\tau))} \left(1-3\gamma+2\gamma^2-\beta_1\right).$$

From this, we can see that the wage rigidity channel is still important in determining the direction and the magnitude of impacts of shock on output aggregation, with the additional insight that we are now comparing wage adjustments with network-adjusted values.

Additionally, we can examine how search frictions impact output for each of the sectors. Let $\Pi_{y,\text{search},X}$ denote the impact of search frictions on sectoral output from shocks to X, with X being either productivity or labor supply. We have that:

$$\Pi_{y,\text{search},A} = \frac{1-\eta(1+\tau)}{1-\eta-\left[\gamma+2(1-\gamma)^2\right](1-\eta(1+\tau))}\begin{bmatrix} 1-\gamma+\gamma^2-\alpha_1 & 1-\gamma-\alpha_2 \\ 2(1-\gamma)(1-\gamma+\gamma^2-\alpha_1) & 2(1-\gamma)(1-\gamma-\alpha_2) \end{bmatrix}.$$

Compare to the efficient benchmark, in which the sectoral output response follows the Leontief inverse $\Psi = \begin{bmatrix} 1 & 0 \\ 1-\gamma & 1 \end{bmatrix}$, where the upper right element is 0, the sectoral output response to changes in labor market has a non-zero upper right element. What this means is that, in efficient production networks with Cobb-Douglas production function, shocks to a downstream sector does not impact production output of upstream sector. While this is usually resolved with the addition of preference shocks or CES production functions, we introduce an additional way of resolving this pitfall of classical production network models by incorporating frictional labor markets.

# 4. Calibration

[ADD transition from previous sections] We test the empirical relevance of the channels outline above by calibrating our model to the U.S. economy. We use shocks to the durables sector to demonstrative the quantitative importance of incorporating search frictions and the role that wages play. We report responses to shocks to all other sectors in Appendix D.

## 4.1. Wage Assumptions

Wages play a large role in how shocks propagate in our model economy. In fact, as 2 shows, for the right assumption about wages, search frictions can have no effect whatsoever on shock propagation. To avoid taking a strong stance on exactly how wages change, we test the quantitative consequences of shocks in our model under a several different wage assumptions that cover a broad swathe of existing assumptions used in the literature.

Throughout this section we report results for the following assumptions about wages

(i) The network price adjusted wage changes are perfectly proportional to changes in the network adjusted marginal product of labor.

$$d \log \boldsymbol{w} - \mathcal{L}\delta \log \boldsymbol{p} = \mathcal{L} d \log \mathrm{MP}$$

Theorem 2 shows that Hulten's theorem holds for this assumption about wage changes. We therefore label this assumption "Hulten" in the figures below.

(ii) The network price adjusted wage changes half as much as the network adjusted marginal product of labor.

$$d \log \boldsymbol{w} - \mathcal{L}\delta \log \boldsymbol{p} = 0.5\mathcal{L} d \log \mathrm{MP}$$

In our setup, this is similar to assuming Nash bargaining with equal bargaining weights for firms and workers. We label this assumption "0.5MP" in figures below.

(iii) The network price adjusted wage does not change in response to shocks

$$d \log \boldsymbol{w} - \mathcal{L}\delta \log \boldsymbol{p} = 0$$

This is akin to assuming real wages are rigid. We label this assumption "Rigid Real" in figures below.

(iv) The nominal wage does not change in response to shocks

$$d \log \boldsymbol{w} = 0$$

We label this assumption "Rigid Nominal" in figures below.

(v) Finally, as we will see below, in all the above cases search frictions amplify the effects of productivity shocks. To demonstrate the possibility that search frictions could in fact dampen the propagation of productivity shocks we use the, admittedly, somewhat unrealistic and manufactured assumption that changes in network price adjusted wages satisfy

$$d \log \boldsymbol{w} - \mathcal{L} d \log \boldsymbol{p} = 1.5\mathcal{L}\boldsymbol{\Psi} d \log \boldsymbol{A} + 0.5\left[\mathcal{L}\boldsymbol{\Psi}\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} - \boldsymbol{I}\right] d \log \boldsymbol{H} + 1.5\mathcal{L}\boldsymbol{\Psi}\varepsilon^{\boldsymbol{f}}_{\boldsymbol{K}} d \log \boldsymbol{K}$$

Corollary 1 demonstrates that this assumption about wages ensures that the additional shock propagation terms from search and matching enter negatively. We label this assumption "Dampening" in figures below.

## 4.2. One occupation per sector

### 4.2.1. Data Sources

*Vacancies and Hires.* We use vacancy and hire data from The Job Openings and Labor Turnover Survey (JOLTs), which provides survey-based measures of job openings and hires at a monthly frequency. This survey data is available for 17 industries that roughly correspond to the two-digit NAICS classification. The data covers Dec 2000 to Feb 2023 at a monthly frequency and 2001 to 2022 at a yearly frequency.

*Unemployment.* We use sector level unemployment from the Current Population Survey (CPS). The data cover 13 sectors at a monthly frequency for the same range as the JOLTs data. We use the full monthly series, along with the JOLTs data, to estimate the sector level matching elasticity with respect to unemployment $(\eta_i)$.

*Input-output Linkages.* Although JOLTs and the CPS include labor market data at a level that corresponds roughly to the two-digit NAICS classification, this correspondence is not exact. To construct input intensity that matches with labor market data availability level, we use the 2021 3-digit NAICS classification level Make and Use tables from the BEA, and aggregate back up to match the 13 CPS industries. The BEA Make and Use tables allow us to calculate the intermediate input intensity of each sector, the labor intensity of each sector, and the elasticity of final production (demand) to each sectors output. We assume that employee compensation, recorded in the Use table to calculate labor elasticities. We then impose constant returns by scaling all input elasticities proportionally.

*Factor Shares.* In our calibration, we consider two factors: capital and energy. Capital and energy shares for the two-digit NAICS classification are available in the BEA-BLS Integrated Industry-level Production Accounts (KLEMS). We use an output-weighted mean of fixed factor shares of two-digit NAICS industries to compute the factor shares to match the corresponding CPS industries. We use these shares as energy and capital elasticities, and impose constant returns by scaling them along with the other production elasticities acquired from the input-output tables.

*Details.* We provide more details on data and calibration in Appendix C.

### 4.2.2. Effects of a 1% shock to productivity in the durable manufacturing sector

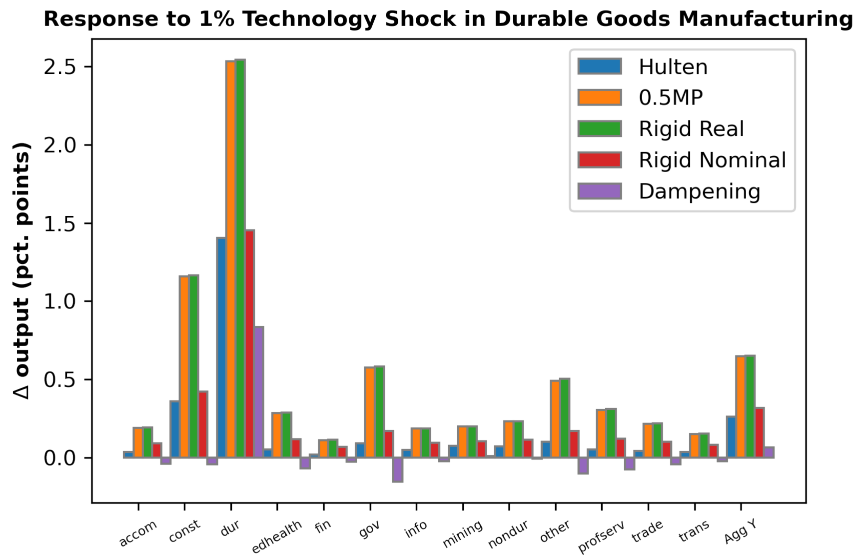


FIGURE 1. Effect of 1% shock to productivity in durables manufacturing on real output.

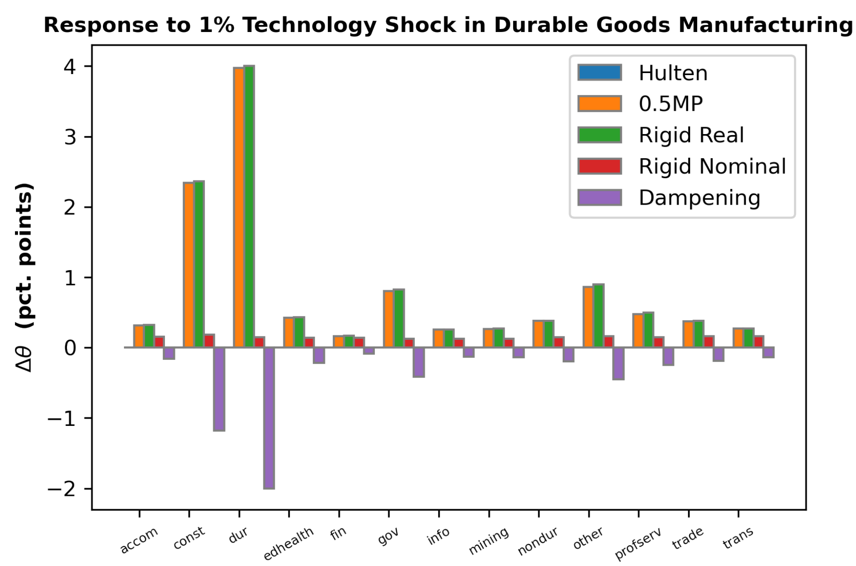


FIGURE 2. Effect of 1% shock to productivity in durables manufacturing on tightness.

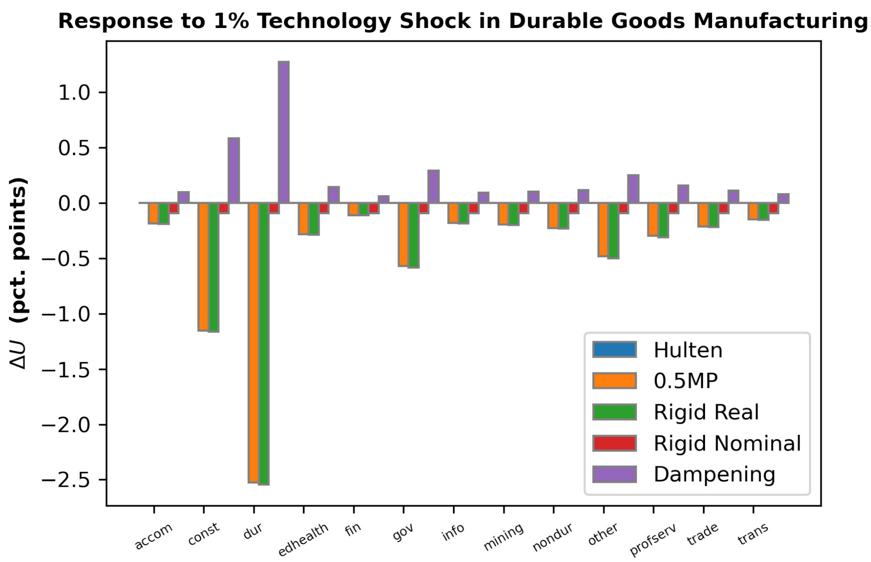


FIGURE 3. Effect of 1% shock to productivity in durables manufacturing on unemployment.

#### 4.2.3. Effects of a 1% shock to the size of the durable manufacturing labor force

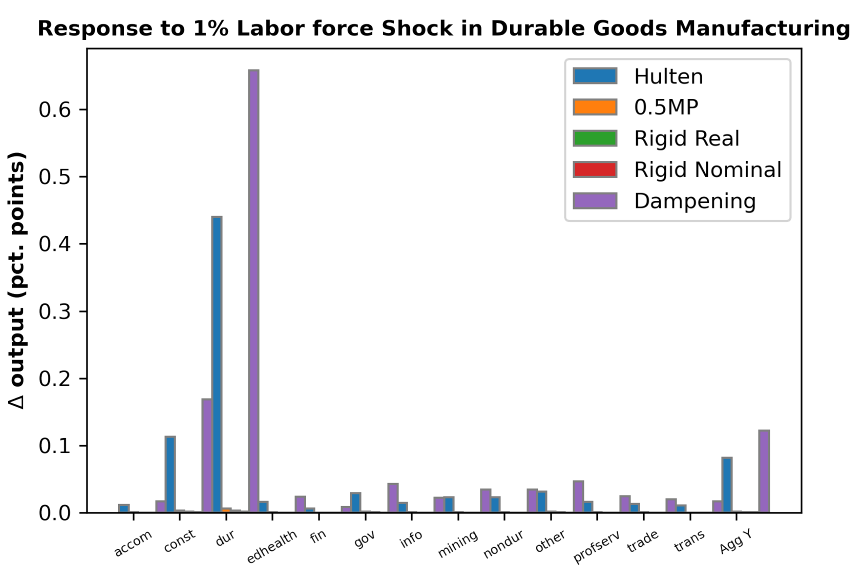


FIGURE 4. Effect of 1% shock to productivity in durables manufacturing on real output.

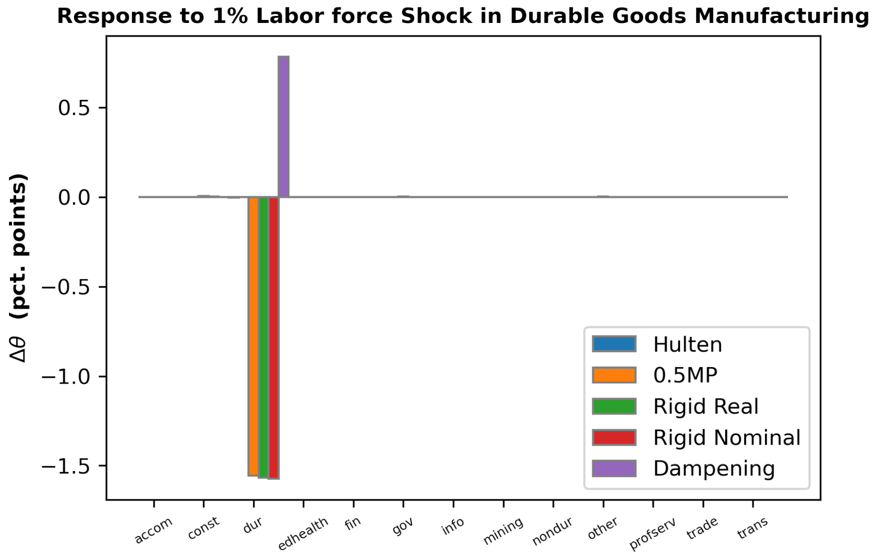


FIGURE 5. Effect of 1% shock to productivity in durables manufacturing on tightness.

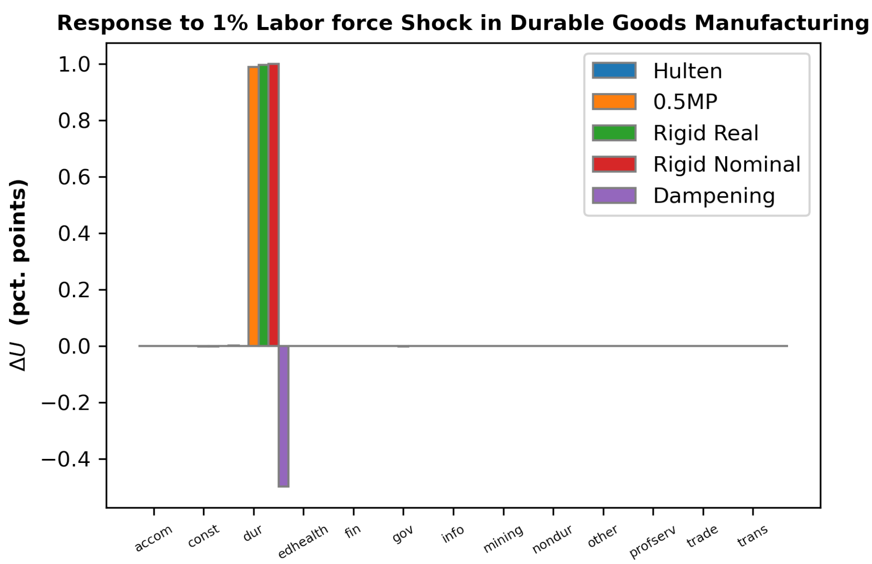


FIGURE 6. Effect of 1% shock to productivity in durables manufacturing on unemployment.

### 4.3. Separate Occupations

#### 4.3.1. Data Sources

#### 4.3.2. Effects of a 1% shock in the durable manufacturing sector

## 5. Application: The Effects of Energy Shocks

In this section, we apply our model to evaluate the effect of an adverse energy shock to sectoral output, tightness and relative price levels. We provide a qualitative illustration of how the recent Russian oil embargo would impact sectoral production and disaggregated labor market tightness. While our model focuses on real variables, we attempt to address the relative price movements in response to the adverse oil shock up to our choice of numeraire goods.

### 5.1. One occupation per sector

In this section, we examine the impact of an adverse energy shock following and occupational setup in subsection 4.2. Figure 8 shows the response of output to the adverse energy shock. The rigid wage specifications predict an aggregate output decline of 0.1% in response to an adverse 1% energy shock, twice as large as when search and matching frictions are present. The transportation sector is negatively impacted the most, followed by the accomodation sector, the construction sector, and the trade sector, as they have high energy elasticities. Search and matching frictions amplify the impact of energy shocks significantly through the search and matching channel. These amplifications create large responses in other downstream sectors, such as government and professional services, despite their low energy elasticities.

Figure 8 plots the response in labor market tightness to the adverse energy shock. The responses are similar to the output responses, in that sectors that suffer the most decline in output also experience the largest increase in slackness, leading to an increase in unemployment.

Figure 9 documents the first-order price changes, using education and health as the numeraire sector. We choose the education and health as the numeraire for one simple reason, which is that inflation for medical care service was 0.4% from 2022 to 2023, much more stable than other goods and services[3]. We find that, first-order price responses differ across sectors and across wage specifications. We observe the

[3] We will probably plot sector prices over time to pick a better numeraire

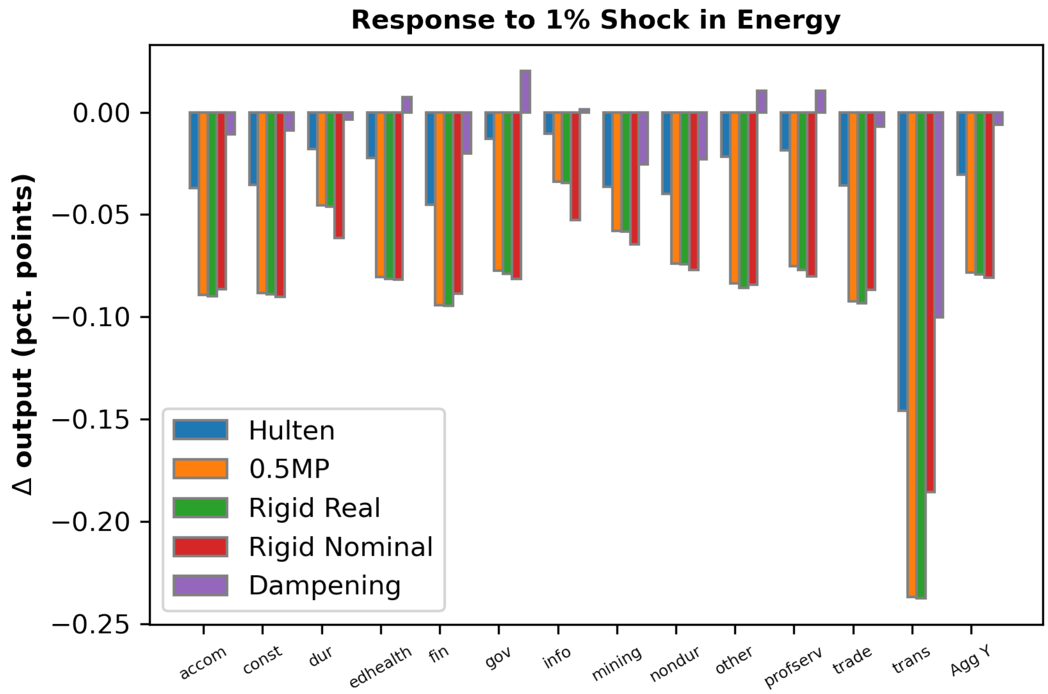


FIGURE 7. Effect of 1% adverse energy shock on output

largest price increases in sectors that suffer the largest output declines. In particular, we see a large increase in transportation prices[4]. We also see price increases in the accomodation sector and the construction sector.

How does this correspond to reality? We find that oil shocks have a substantially larger impact on output than what conventional models would have predicted. We also find a decrease in labor market tightness across sectors, and increase in relative prices for sectors that have large energy elasticities, as well as sectors that are linked to them. Under rigid wage assumptions, we can generate price increases in certain sectors that are quantitatively substantive. Energy shocks might have helped explain why the U.S economy is growing at a rate much slower than the assumed 2%, but are unable in explaining why the labor market is tight. Under the specification that wages respond more than sectoral prices (the dampening specification), we are able to uncover 1) muted output response, 2) tighter labor markets, and 3) higher relative prices.

[4]According to the Bureau of Transportation Statistics, transportation prices rose 15.5% overall between 2022 and 2023

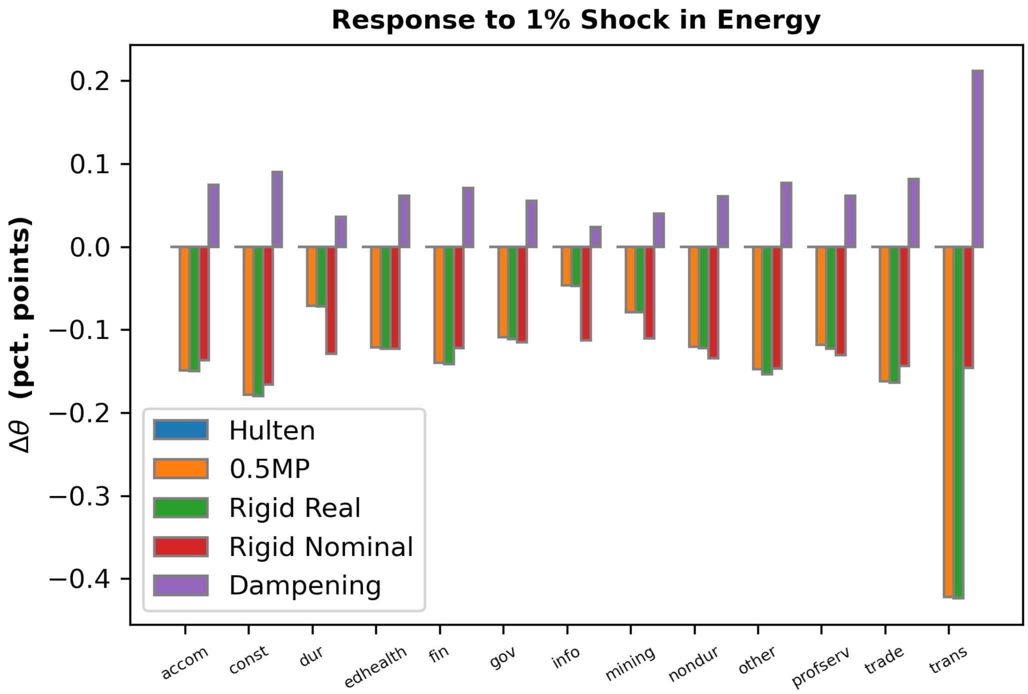


FIGURE 8. Effect of 1% adverse energy shock on labor market tightness

## 6. Conclusion

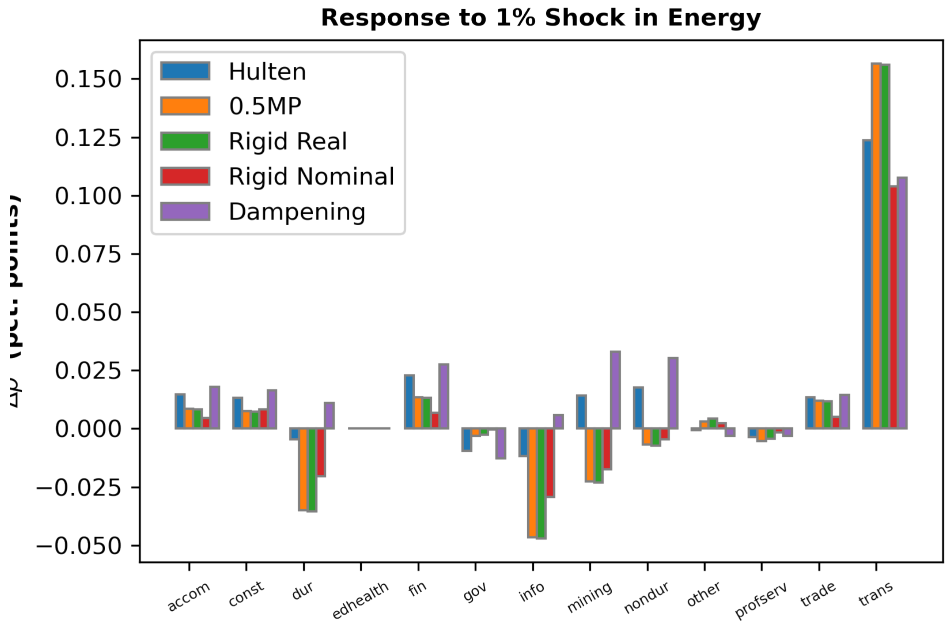


FIGURE 9. Effect of 1% adverse energy shock on relative first-order price changes

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# Appendix A. Proofs

## A.1. Proof for Propositions 1 and 2

We first show how to prove Proposition 2 by assuming Cobb-Douglas preferences and production.

### A.1.1. Price Propagation

Log-linearizing the production function, for each sector $i$, we have:

$$d\log y_i = \underbrace{\varepsilon^{f_i}_{A_i}}_{=1} d\log A_i + \sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} d\log N_{io} + \sum_{j=1}^{N} \varepsilon^{f_i}_{x_{ij}} d\log x_{ij} + \sum_{k=1}^{\mathcal{K}} \varepsilon^{f_i}_{K_{ik}} d\log K_{ik}$$

Plugging in Equation 7 and Equation 6, the first order conditions for optimal input usage, into the log-linearized production function gives

$$\begin{aligned}
d\log y_i = &\sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} \left[d\log \varepsilon^{f_i}_{N_{io}} + d\log y_i + d\log p_i - d\log w_o - d\log\left(1+\tau_o(\theta_o)\right)\right] \\
&+ \sum_{j=1}^{N} \varepsilon^{f_i}_{x_{ij}} \left[d\log \varepsilon^{f_i}_{x_{ij}} + d\log y_i + d\log p_i - d\log p_j\right] \\
&+ \sum_{k=1}^{\mathcal{K}} \varepsilon^{f_i}_{K_{ik}} \left[d\log \varepsilon^{f_i}_{K_{ik}} + d\log y_i + d\log p_i - d\log r_k\right] + d\log A_i \\
&= \left[d\log y_i + d\log p_i\right] \underbrace{\left[\sum_{k=1}^{\mathcal{K}} \varepsilon^{f_i}_{K_{ik}} + \sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} + \sum_{j=1}^{N} \varepsilon^{f_i}_{x_{ij}}\right]}_{=1 \text{ by crts}} + \underbrace{\left[\sum_{k=1}^{\mathcal{K}} d\varepsilon^{f_i}_{k_{ik}} + \sum_{o=1}^{\mathcal{O}} d\varepsilon^{f_i}_{N_{io}} + \sum_{j=1}^{N} d\varepsilon^{f_i}_{x_{ij}}\right]}_{=0 \text{ by crts}} \\
&- \sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} \left[d\log w_o + d\log(1+\tau_o(\theta_o))\right] - \sum_{j=1}^{N} \varepsilon^{f_i}_{x_{ij}} \left[d\log p_j\right] - \sum_{k=1}^{\mathcal{K}} \varepsilon^{f_i}_{K_{ik}} d\log r_k + d\log A_i,
\end{aligned}$$

where the second equality holds because the sum of elasticities equals one for constant returns to scale technology and $\varepsilon^{f_i}_{x_{ij}} d\log \varepsilon^{f_i}_{x_{ij}} = d\varepsilon^{f_i}_{x_{ij}}$.

Rearranging terms gives

$$
\begin{aligned}
d\log p_i &= \sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} \left[d\log w_o + d\log(1+\tau_o(\theta_o))\right] + \sum_{j=1}^{N} \varepsilon^{f_i}_{x_{ij}} \left[d\log p_j\right] + \sum_{k=1}^{\mathcal{K}} \varepsilon^{f_i}_{K_{ik}} d\log r_k - d\log A_i \\
&= \sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} \left[d\log w_o + \varepsilon^{1+\tau_o}_{\theta_o} d\log\theta_o\right] + \sum_{j=1}^{J} \varepsilon^{f_i}_{x_{ij}} \left[d\log p_j\right] + \sum_{k=1}^{\mathcal{K}} \varepsilon^{f_i}_{K_{ik}} d\log r_k - d\log A_i \\
&= \sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} \left[d\log w_o - \tau_o(\theta_o)\varepsilon^{\Omega_o}_{\theta_o} d\log\theta_o\right] + \sum_{j=1}^{J} \varepsilon^{f_i}_{x_{ij}} \left[d\log p_j\right] + \sum_{k=1}^{\mathcal{K}} \varepsilon^{f_i}_{K_{ik}} d\log r_k - d\log A_i
\end{aligned}
$$

Stacking equations over sectors, we can write

$$
d\log \boldsymbol{p} = \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} \left[d\log \boldsymbol{w} - \mathfrak{Q}\mathfrak{T} d\log\theta\right] + \boldsymbol{\Omega} d\log \boldsymbol{p} + \varepsilon^{\boldsymbol{f}}_{\boldsymbol{K}} d\log \boldsymbol{r} - d\log \boldsymbol{A}
$$

Which implies

$$
d\log \boldsymbol{p} = \boldsymbol{\Psi} \left[\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} \left[d\log \boldsymbol{w} - \mathfrak{Q}\mathfrak{T} d\log\theta\right] + \varepsilon^{\boldsymbol{f}}_{\boldsymbol{K}} d\log \boldsymbol{r} - d\log \boldsymbol{A}\right]
$$

Or equivalently

$$
\left(\boldsymbol{I} - \boldsymbol{\Psi}\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}\mathcal{L}\right) d\log \boldsymbol{p} = \boldsymbol{\Psi} \left[\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} \left[d\log \boldsymbol{w} - \mathcal{L} d\log \boldsymbol{p} - \mathfrak{Q}\mathfrak{T} d\log\theta\right] + \varepsilon^{\boldsymbol{f}}_{\boldsymbol{K}} d\log \boldsymbol{r} - d\log \boldsymbol{A}\right]
$$

Where $d\log \boldsymbol{r}$ is pinned down by market clearing in the additional factor market for $K_k$. Assuming Cobb-Douglas production, for any $i$

$$
d\log r_k = d\log y_i + d\log p_i - d\log K_{ik}
$$

Which means that $d\log k_{ik} = d\log K_{jk}$ for all $i, j$. Furthermore, this means that $d\log K_{ik} = d\log K^s_k$ for all $i$. So we can write,

$$
d\log r_k = d\log y_i + d\log p_i - d\log K^s_k
$$

Which pins down $d\log r_k$ given a numeraire sector.

### A.1.2. Output Propagation

Since the log-linearized expression for the Domar weight must hold for every sector, we can write

$$
\begin{aligned}
d\log\lambda_i - d\log\lambda_j &= d\log p_i - d\log p_j + d\log y_i - d\log y_j \\
&= d\log x_{ij} - d\log \varepsilon^{f_i}_{x_{ij}} - d\log y_j \\
\Rightarrow d\log x_{ij} &= d\log\lambda_i - d\log\lambda_j + d\log y_j + d\log \varepsilon^{f_i}_{x_{ij}}
\end{aligned}
$$

Plugging back into the production function,

$$
\begin{aligned}
d\log y_i &= d\log A_i + \sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} d\log N_{io} + \sum_{j=1}^{J} \varepsilon^{f_i}_{x_{ij}} + d\log y_j + \sum_{k=1}^{\mathcal{K}} \varepsilon^{f_i}_{K_{ik}} d\log K_{ik} \\
&= d\log A_i + \sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} d\log N_{io} + \sum_{j=1}^{J} \varepsilon^{f_i}_{x_{ij}} d\log y_j + \sum_{k=1}^{\mathcal{K}} \varepsilon^{f_i}_{K_{ik}} d\log K^s_K
\end{aligned}
$$

Using the definition of labor demand,

$$
\begin{aligned}
\sum_i \frac{l_{io}}{L_o} d\log N_{io} &= d\log L^d_o - d\log(1+\tau_o(\theta_o)) \\
&= d\log L^d_o + \tau_o(\theta_o)\varepsilon^{Q_o}_{\theta_o} d\log\theta_o
\end{aligned}
$$

From output labor usage, we have that labor usage ratio for an occupation by two different sectors as:

$$
\frac{l_{io}}{l_{jo}} = \frac{\varepsilon^{f}_{N_{io}}\lambda_i}{\varepsilon^{f}_{N_{jo}}\lambda_j}
$$

for any $l_{io}, l_{jo} > 0$

Log-linearizing it, assuming Cobb-Douglas preferences, yields:

$$
d\log l_{io} = d\log l_{jo}
$$

Also since, $d\log l_{io} = d\log N_{io} + d\log(1+\tau_o(\theta_o)) = d\log l_{jo} = d\log N_{jo} + d\log(1+\tau_o(\theta_o))$, we have that $d\log N_{io} = d\log N_{jo}$

Using the labor market clearing condition, and the definition of labor supply,

$$\sum_k \frac{l_{ko}}{L_o} d\log N_{ko} = \left(\varepsilon_{\theta_o}^{\mathcal{F}_o} + \tau_o(\theta_o)\varepsilon_{\theta_o}^{\mathcal{Q}_o}\right) d\log\theta_o + d\log H_o$$

$$\Rightarrow d\log N_{io} \underbrace{\sum_k \frac{l_{ko}}{L_o}}_{=1} = \left(\varepsilon_{\theta_o}^{\mathcal{F}_o} + \tau_o(\theta_o)\varepsilon_{\theta_o}^{\mathcal{Q}_o}\right) d\log\theta_o + d\log H_o$$

Plugging this back into the linearized production function gives:

$$d\log y_i = d\log A_i + \sum_{o=1}^{\mathcal{O}} \varepsilon_{N_{io}}^{f_i} \left[\left(\varepsilon_{\theta_o}^{\mathcal{F}_o} + \tau_o(\theta_o)\varepsilon_{\theta_o}^{\mathcal{Q}_o}\right) d\log\theta_o + d\log H_o\right]$$
$$+ \sum_{j=1}^{J} \varepsilon_{x_{ij}}^{f_i} d\log y_j + \sum_{k=1}^{\mathcal{K}} \varepsilon_{K_{ik}}^{f_i} d\log K_k^s$$

Stacking over sectors gives,

$$d\log \boldsymbol{y} = d\log \boldsymbol{A} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left(\mathfrak{F} + \mathfrak{Q}\mathfrak{T}\right) d\log\theta + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} d\log \boldsymbol{H} + \boldsymbol{\Omega} d\log \boldsymbol{y} + \varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}} d\log \boldsymbol{K}^s$$

Which implies

$$d\log \boldsymbol{y} = \boldsymbol{\Psi}\left(d\log \boldsymbol{A} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left(\mathfrak{F} + \mathfrak{Q}\mathfrak{T}\right) d\log\theta + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} d\log \boldsymbol{H} + \varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}} d\log \boldsymbol{K}^s\right)$$

### A.1.3. Tightness Propagation

Labor market clearing implies that changes in labor demand have to equal changes in labor supply:

$$d\log L_o^s(\theta, \boldsymbol{H}) = d\log L_o^d(\theta, \boldsymbol{A}).$$
$$\varepsilon_{\theta_o}^{\mathcal{F}_o} d\log\theta_o + d\log H_o = \sum_{i=1}^{J} \frac{l_{io}}{L_o^d} d\log l_{io}(\theta_o)$$

Where $\frac{l_i o}{L_o^d} = \frac{\varepsilon_{N_{io}}^{f_i} p_i y_i}{\sum_{j=1}^{J} \varepsilon_{N_{jo}}^{f_j} p_j y_j}$[5]. For every sector $i$ we have

$$d \log l_{io}(\theta_o) = d \log \varepsilon_{N_{io}}^{f_i} - d \log w_o + d \log p_i + d \log y_i$$

Which implies that

$$d \log \boldsymbol{L^d}(\theta) = \operatorname{diag}\left(\mathcal{L} d \log \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\right) - \left[d \log \boldsymbol{w} - \mathcal{L} d \log \boldsymbol{p}\right] + \mathcal{L} d \log \boldsymbol{y}$$

since $\sum_{i=1}^{J} \frac{l_{io}}{L_o^d} = 1$ for all $o$. Plugging in for $d \log \boldsymbol{y}$ gives

$$\begin{aligned} d \log \boldsymbol{L^d}(\theta) = {} & \operatorname{diag}\left(\mathcal{L} d \log \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\right) - \left[d \log \boldsymbol{w} - \mathcal{L} d \log \boldsymbol{p}\right] \\ & + \mathcal{L}\Psi \left[d \log \boldsymbol{A} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left(\mathcal{F} + \mathcal{Q}\mathcal{T}\right) d \log \theta + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} d \log \boldsymbol{H} + \varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}} d \log \boldsymbol{K}^s\right] \end{aligned}$$

Labor market clearing implies

$$\mathcal{F} d \log \theta + d \log \boldsymbol{H} = \mathcal{L}\Psi \left[d \log \boldsymbol{A} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left(\mathcal{F} + \mathcal{Q}\mathcal{T}\right) d \log \theta + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} d \log \boldsymbol{H} + \varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}} d \log \boldsymbol{K}^s\right] - \left[d \log \boldsymbol{w} - \mathcal{L} d \log \boldsymbol{p}\right]$$

Which pins down first order changes in log tightness as

$$d \log \theta = \left[\mathcal{F} - \Xi_\theta\right]^{-1} \left[\mathcal{L}\Psi d \log \boldsymbol{A} - \left[d \log \boldsymbol{w} - \mathcal{L} d \log \boldsymbol{p}\right] + \left[\mathcal{L}\Psi \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I}\right] d \log \boldsymbol{H} + \mathcal{L}\Psi \varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}} d \log \boldsymbol{K}^s\right]$$

Where $\Xi_\theta = \mathcal{L}\Psi \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left[\mathcal{F} + \mathcal{Q}\mathcal{T}\right]$.

Therefore, combining these propagation expressions, we are able to expres $d \log \theta$, $d \log \boldsymbol{y}$, and $d \log \boldsymbol{p}$ in terms of exogenous shocks.

### A.2. Proof for Theorem 1 and corrollaries

Let $\lambda_i = \frac{p_i y_i}{G}$, where $G = \sum_j p_j c_j = GDP = \sum_{o=1}^{\mathcal{O}} w_o L_o$, denote the final sales share of GDP for sector $i$. We have that:

$$p_j x_{ij} = \varepsilon^{f_i} x_{ij} \lambda_i G.$$

[5]One implication of this formula is that I think we should be able to check whether the elasticities $\left\{\left\{\varepsilon_{N_{io}}^{f}\right\}_{o=1}^{\mathcal{O}}\right\}_{i=1}^{J}$ are consistent with the Domar weights.

From household's maximization problem, I have that $p_i c_i = \varepsilon^{\mathcal{D}}_{c_i} G$. Combining the two gives:

$$\varepsilon^{\mathcal{D}}_{c_j} G = p_j y_j = p_j (c_j + \sum_i x_{ij}) = \left( \varepsilon^{\mathcal{D}}_{c_j} + \sum_i \varepsilon^{f_i} x_{ij} \lambda_i \right) G$$
$$\Rightarrow \boldsymbol{\lambda} = \boldsymbol{\Psi}' \varepsilon^{\boldsymbol{D}}_{\boldsymbol{c}}.$$

The aggregate labor force, employment, and unemployment are $H^{agg} = \sum_{o=1}^{\mathcal{O}} H_o$, $L^{agg} = \sum_{o=1}^{\mathcal{O}} L_o$, and $U^{agg} = \sum_{o=1}^{\mathcal{O}} U_o$. Changes in aggregates are therefore given by

$$dH^{agg} = \sum_{o=1}^{\mathcal{O}} dH_o$$
$$dL^{agg} = \sum_{o=1}^{\mathcal{O}} dL_o$$
$$dU^{agg} = \sum_{o=1}^{\mathcal{O}} dU_o$$

Or in terms of log changes

$$d\log H^{agg} = \frac{1}{H^{agg}} \sum_{o=1}^{\mathcal{O}} H_o d\log H_o$$
$$d\log L^{agg} = \frac{1}{L^{agg}} \sum_{o=1}^{\mathcal{O}} L_o d\log L_o$$
$$d\log U^{agg} = \frac{1}{U^{agg}} \sum_{o=1}^{\mathcal{O}} U_o d\log U_o$$

In matrix notation

$$d\log H^{agg} = \frac{1}{H^{agg}} \boldsymbol{H}' d\log \boldsymbol{H}$$
$$d\log L^{agg} = \frac{1}{L^{agg}} \boldsymbol{L}' d\log \boldsymbol{L}$$
$$d\log U^{agg} = \frac{1}{U^{agg}} \boldsymbol{U}' d\log \boldsymbol{U}$$

Substituting in for $d\log \boldsymbol{L}$

$$
\begin{aligned}
d\log L^{agg} &= \frac{1}{L^{agg}}\boldsymbol{L'}\left[\mathcal{L} d\log \boldsymbol{y} - \left[d\log \boldsymbol{w} - \mathcal{L} d\log \boldsymbol{p}\right]\right] \\
&= \frac{1}{L^{agg}}\boldsymbol{L'}\left[\mathcal{L}\left[\boldsymbol{\Pi}_{y,A} d\log \boldsymbol{A} + \boldsymbol{\Pi}_{y,H} d\log \boldsymbol{H} - \boldsymbol{\Lambda}_A d\log \boldsymbol{A} - \boldsymbol{\Lambda}_H d\log \boldsymbol{H}\right]\right] \\
&= \boldsymbol{\Pi}_{L^{agg},A} d\log \boldsymbol{A} + \boldsymbol{\Pi}_{L^{agg},H} d\log \boldsymbol{H}
\end{aligned}
$$

Where

$$
\begin{aligned}
\boldsymbol{\Pi}_{L^{agg},A} &= \frac{1}{L^{agg}}\boldsymbol{L}'\left[\mathcal{L}\boldsymbol{\Pi}_{y,A} - \boldsymbol{\Lambda}_A\right] \\
\boldsymbol{\Pi}_{L^{agg},H} &= \frac{1}{L^{agg}}\boldsymbol{L}'\left[\mathcal{L}\boldsymbol{\Pi}_{y,H} - \boldsymbol{\Lambda}_H\right]
\end{aligned}
$$

And

$$
\begin{aligned}
d\log U^{agg} &= \frac{1}{U^{agg}}\boldsymbol{U}'\left[d\log \boldsymbol{H} - d\log \boldsymbol{L}\right] \\
&= \frac{1}{U^{agg}}\boldsymbol{U}'\left[d\log \boldsymbol{H} - \left[\mathcal{L}\left[\boldsymbol{\Pi}_{y,A} d\log \boldsymbol{A} + \boldsymbol{\Pi}_{y,H} d\log \boldsymbol{H} - \boldsymbol{\Lambda}_A d\log \boldsymbol{A} - \boldsymbol{\Lambda}_H d\log \boldsymbol{H}\right]\right]\right] \\
&= \boldsymbol{\Pi}_{U^{agg},A} d\log \boldsymbol{A} + \boldsymbol{\Pi}_{U^{agg},H} d\log \boldsymbol{H}
\end{aligned}
$$

Where

$$
\begin{aligned}
\boldsymbol{\Pi}_{U^{agg},A} &= \frac{1}{U^{agg}}\boldsymbol{U}'\left[\boldsymbol{\Lambda}_A - \mathcal{L}\boldsymbol{\Pi}_{y,A}\right] \\
\boldsymbol{\Pi}_{U^{agg},H} &= \frac{1}{U^{agg}}\boldsymbol{U}'\left[\boldsymbol{I} + \boldsymbol{\Lambda}_H - \mathcal{L}\boldsymbol{\Pi}_{y,H}\right]
\end{aligned}
$$

## Appendix B. Results for general CRTS production functions and one occupation per sector

In this section we generalize our results to any constant returns to scale production function, under the assumption that there is one type of labor per sector. This generalization results in additional terms that capture how the production elasticities change when shocks hit the economy. The expressions are otherwise similar to above. The model setup is identical, we just do not impose Cobb-Douglas technology and instead impose $\mathcal{O} = J$,

[FILL IN IF TIME]

# Appendix C. Data and Calibration Details

This appendix describes our data in greater detail.

## C.1. One occupation per sector.

### C.1.1. Input-output matrix

We use the 3-digit 2021 BEA Make and Use tables accessible at https://www.bea.gov/industry/input-output-accounts-data to calculate the relevant production elasticities[6]. The 3-digit Make and Use tables record the nominal amount of each 71 commodities made by and used by each of 71 industries. The commodities are denoted using the same codes as the industries, but they are conceptually distinct as each industry can produce more than one commodity.

For consistency with the industry classifications in JOLTs and the CPS unemployment by sector series, we collapse the 3-digit tables to a 13 sector table. The table below outlines the mapping from the NAICS 2-digit classification codes to our industry classifications. The mapping from 2-digit codes to 3-digit codes is readily available online.

[6]See https://www.bea.gov/sites/default/files/methodologies/IOmanual_092906.pdf for a detailed description of how these tables are generated.

| Industry Name | Short Name | 2-digit codes |
|---|---|---|
| Leisure and Hospitality | accom | 71, 72 |
| Construction | const | 33 |
| Durable goods | dur | 33DG |
| Education and Health Services | edhealth | 61, 62 |
| Financial Activities | fin | 52, 53 |
| Government | gov | G |
| Information | info | 51 |
| Mining | mining | 21 |
| Nondurable good | nondur | 11, 31ND |
| Other services, except government | other | 81 |
| Professional and business services | profserv | 54, 55, 56 |
| Wholesale and Retail trade | trade | 42, 44RT |
| Transportation and Utilities | trans | 22, 48TW |

TABLE A1. Mapping from NAICS classification to our industries.

With the 13-sector make and use tables in hand, we can construct production elasticities in intermediate inputs and to labor, and demand elasticities. Let $M_{ij}$ denote the nominal value of commodity $i$ made by industry $j$. Let $U_{ij}$ denote the nominal amount of commodity $i$ used by industry $j$. The two tables below demonstrate the elements of the Make and Use tables.

| | Sector 1 | Sector 2 | $\cdots$ | Sector J | Total Industry Output |
|---|---|---|---|---|---|
| Sector 1 | $M_{11}$ | $M_{21}$ | $\cdots$ | $M_{J1}$ | $\sum_{i=1}^{J} M_{i1}$ |
| Sector 2 | $M_{12}$ | $M_{22}$ | $\cdots$ | $M_{J2}$ | $\sum_{i=1}^{J} M_{i2}$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ | $\vdots$ |
| Sector J | $M_{1J}$ | $M_{2J}$ | $\cdots$ | $M_{JJ}$ | $\sum_{i=1}^{J} M_{iJ}$ |
| Total Commodity Output | $\sum_{j=1}^{J} M_{1j}$ | $\sum_{j=1}^{J} M_{2j}$ | $\cdots$ | $\sum_{j=1}^{J} M_{Jj}$ | — |

TABLE A2. Make table

| | Sector 1 | Sector 2 | $\cdots$ | Sector J | Total Intermediate Uses | Total Final Uses |
|---|---|---|---|---|---|---|
| Sector 1 | $U_{11}$ | $U_{12}$ | $\cdots$ | $U_{1J}$ | $\sum_{j=1}^{J} U_{1j}$ | $\sum_{j=1}^{J} U_{1j} + p_1 c_1$ |
| Sector 2 | $U_{21}$ | $U_{22}$ | $\cdots$ | $U_{2J}$ | $\sum_{j=1}^{J} U_{2j}$ | $\sum_{j=1}^{J} U_{2j} + p_2 c_2$ |
| $\vdots$ | $\vdots$ | $\vdots$ | $\ddots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| Sector J | $U_{J1}$ | $U_{J2}$ | $\cdots$ | $U_{JJ}$ | $\sum_{j=1}^{J} U_{Jj}$ | $\sum_{j=1}^{J} U_{Jj} + p_J c_J$ |
| Total Intermediate Inputs | $\sum_{i=1}^{J} U_{i1}$ | $\sum_{i=1}^{J} U_{i2}$ | $\cdots$ | $\sum_{i=1}^{J} U_{iJ}$ | — | — |
| Total industry output | $\sum_{i=1}^{J} U_{i1} + w_1(1+\tau_1)N_1$ | $\sum_{i=1}^{J} U_{i2} + w_2(1+\tau_2)N_2$ | $\cdots$ | $\sum_{i=1}^{J} U_{iJ} + w_J(1+\tau_J)N_J$ | — | — |

TABLE A3. Use table

First, we calculate the fraction of commodity $i$ produced by industry $j$ by dividing the entry in along each row by the corresponding "total industry output"

$$m_{ij} = \frac{M_{ij}}{\sum_{j=1}^{J} M_{ji}}$$

Second, we calculate the share of commodity $i$ in industry $j$'s total uses as by dividing each entry in the column corresponding to industry $j$ by the corresponding "Total industry output"

$$u_{ij} = \frac{U_{ij}}{\sum_{i=1}^{J} U_{ij} + w_j(1+\tau_j)N_j}$$

We form the two matrices

$$\boldsymbol{M} = \begin{bmatrix} m_{11} & m_{21} & \cdots & m_{J1} \\ m_{12} & m_{22} & \cdots & m_{J2} \\ \vdots & \vdots & \ddots & \vdots \\ m_{1J} & m_{2J} & \cdots & m_{JJ} \end{bmatrix}, \boldsymbol{U} = \begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1J} \\ u_{21} & u_{22} & \cdots & u_{2J} \\ \vdots & \vdots & \ddots & \vdots \\ u_{J1} & u_{J2} & \cdots & u_{JJ} \end{bmatrix}$$

Then, we can calculate our input output matrix by

$$\boldsymbol{\Omega} = [\boldsymbol{MU}]'$$

Given our assumption of constant returns to scale and zero profits, the difference between total intermediate inputs and total industry output is the nominal income paid to workers in each sector. We abstract from the other components of total industry output in the IO accounts, taxes and gross operating surplus, as they have no model counterpart in our setup. We can therefore calculate the labor elasticities from the Use table as "Total industry output" – "Total intermediate inputs" ÷ "Total industry output."

$$\varepsilon^{f_j}_{N_j} = \frac{w_j(1+\tau_j)N_j}{\sum_{i=1}^{J} U_{ij} + w_j(1+\tau_j)N_j}$$

Finally, we can back out the demand elasticities from "Total intermediate uses" and "Total final uses" columns of the Uses table.

$$p_i c_i = \sum_{j=1}^{J} U_{ij} + p_i c_i - \sum_{j=1}^{J} U_{ij}$$

We can the work out the elasticities by

$$\varepsilon^{\mathcal{D}}_{c_i} = \frac{p_i c_i}{\sum_{i=1}^{J} p_i c_i}$$

Finally, to ensure that constant returns to scale holds we rescale our elasticities proportionally to ensure they sum to one. This adjustment is minor and is only needed because we drop the small "Used" and "rest of world adjustment" categories. It does not change any elasticity by more than 3 percent.

We report the resulting estimates of the production elasticities, labor elasticities, and demand elasticities in the tables below. In tables A4 and A5 we assume all non-intermediate spending goes to labor income, which automatically imposes constant returns but leads to large labor shares. In tables A6 and A7 we instead use reported employee compensation to measure the labor share. We then rescale all elasticities proportionally to ensure we still have constant returns to scale. This rescaling leads us to overestimate the importance of intermediate inputs, but reduces how much we overestimate the labor share. Our results are qualitatively robust to either specification.

| Sector | Labor Elasticity $\left(\varepsilon_N^f\right)$ | Demand Elasticity $\left(\varepsilon_c^{\mathcal{D}}\right)$ |
|---|---|---|
| accom | 0.510 | 0.051 |
| const | 0.474 | 0.065 |
| dur | 0.434 | 0.138 |
| edhealth | 0.616 | 0.129 |
| fin | 0.617 | 0.165 |
| gov | 0.626 | 0.132 |
| info | 0.571 | 0.043 |
| mining | 0.518 | 0.008 |
| nondur | 0.352 | 0.151 |
| other | 0.608 | 0.024 |
| profserv | 0.591 | 0.071 |
| trade | 0.523 | 0.000 |
| trans | 0.492 | 0.022 |

TABLE A4. Labor elasticities and demand elasticities according the BEA make use tables for 13-industry classification, rounded to 3 decimal places. We assume all non-intermediate spending goes to labor income.

| | accom | const | dur | edhealth | fin | gov | info | mining | nondur | other | profserv | trade | trans |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| accom | 0.029 | 0.002 | 0.017 | 0.003 | 0.099 | 0.007 | 0.036 | 0.008 | 0.103 | 0.014 | 0.143 | 0.000 | 0.030 |
| const | 0.001 | 0.000 | 0.282 | 0.000 | 0.033 | 0.001 | 0.016 | 0.022 | 0.089 | 0.006 | 0.072 | 0.000 | 0.004 |
| dur | 0.001 | 0.001 | 0.393 | 0.000 | 0.016 | 0.001 | 0.012 | 0.013 | 0.056 | 0.003 | 0.055 | 0.004 | 0.012 |
| edhealth | 0.019 | 0.000 | 0.030 | 0.015 | 0.103 | 0.006 | 0.028 | 0.004 | 0.056 | 0.009 | 0.101 | 0.000 | 0.012 |
| fin | 0.011 | 0.023 | 0.007 | 0.000 | 0.195 | 0.005 | 0.020 | 0.001 | 0.008 | 0.005 | 0.083 | 0.001 | 0.022 |
| gov | 0.006 | 0.023 | 0.048 | 0.011 | 0.055 | 0.004 | 0.032 | 0.013 | 0.081 | 0.012 | 0.072 | 0.000 | 0.017 |
| info | 0.017 | 0.001 | 0.040 | 0.000 | 0.047 | 0.003 | 0.125 | 0.002 | 0.015 | 0.005 | 0.160 | 0.001 | 0.013 |
| mining | 0.001 | 0.005 | 0.010 | 0.000 | 0.060 | 0.002 | 0.017 | 0.128 | 0.056 | 0.001 | 0.090 | 0.000 | 0.022 |
| nondur | 0.001 | 0.003 | 0.044 | 0.000 | 0.020 | 0.002 | 0.008 | 0.124 | 0.377 | 0.003 | 0.040 | 0.004 | 0.021 |
| other | 0.013 | 0.005 | 0.066 | 0.010 | 0.113 | 0.005 | 0.033 | 0.003 | 0.032 | 0.013 | 0.086 | 0.000 | 0.012 |
| nprofserv | 0.020 | 0.000 | 0.029 | 0.001 | 0.070 | 0.004 | 0.054 | 0.002 | 0.026 | 0.008 | 0.176 | 0.000 | 0.017 |
| trade | 0.006 | 0.002 | 0.025 | 0.002 | 0.104 | 0.009 | 0.038 | 0.002 | 0.031 | 0.015 | 0.159 | 0.021 | 0.063 |
| trans | 0.014 | 0.007 | 0.022 | 0.000 | 0.080 | 0.015 | 0.025 | 0.030 | 0.080 | 0.011 | 0.093 | 0.001 | 0.129 |

TABLE A5. Production elasticities to intermediate inputs at 13-sector level ($\mathbf{\Omega}$), rounded to 3 decimal places. We assume all non-intermediate spending goes to labor income.

| Sector | Labor Elasticity $\left(\varepsilon_N^f\right)$ | Demand Elasticity $\left(\varepsilon_c^{\mathcal{D}}\right)$ |
|---|---|---|
| accom | 0.411 | 0.051 |
| const | 0.378 | 0.065 |
| dur | 0.300 | 0.138 |
| edhealth | 0.574 | 0.129 |
| fin | 0.281 | 0.165 |
| gov | 0.564 | 0.132 |
| info | 0.348 | 0.043 |
| mining | 0.184 | 0.008 |
| nondur | 0.163 | 0.151 |
| other | 0.545 | 0.024 |
| profserv | 0.521 | 0.071 |
| trade | 0.394 | 0.000 |
| trans | 0.344 | 0.022 |

TABLE A6. Labor elasticities and demand elasticities according the BEA make use tables for 13-industry classification, rounded to 3 decimal places. We measure labor income using reported employee compensation, and rescale elasticities proportionally to ensure constant returns to scale.

| | accom | const | dur | edhealth | fin | gov | info | mining | nondur | other | profserv | trade | trans |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| accom | 0.035 | 0.002 | 0.021 | 0.004 | 0.118 | 0.008 | 0.044 | 0.009 | 0.123 | 0.016 | 0.171 | 0.000 | 0.036 |
| const | 0.001 | 0.000 | 0.333 | 0.000 | 0.039 | 0.001 | 0.019 | 0.026 | 0.105 | 0.007 | 0.086 | 0.000 | 0.005 |
| dur | 0.002 | 0.001 | 0.486 | 0.000 | 0.020 | 0.002 | 0.015 | 0.016 | 0.069 | 0.003 | 0.067 | 0.005 | 0.015 |
| edhealth | 0.022 | 0.000 | 0.033 | 0.017 | 0.115 | 0.006 | 0.031 | 0.004 | 0.062 | 0.010 | 0.112 | 0.000 | 0.013 |
| fin | 0.021 | 0.044 | 0.012 | 0.000 | 0.366 | 0.010 | 0.038 | 0.001 | 0.016 | 0.009 | 0.156 | 0.003 | 0.042 |
| gov | 0.007 | 0.027 | 0.056 | 0.012 | 0.064 | 0.005 | 0.038 | 0.015 | 0.095 | 0.014 | 0.084 | 0.000 | 0.020 |
| info | 0.025 | 0.002 | 0.061 | 0.000 | 0.072 | 0.005 | 0.190 | 0.003 | 0.022 | 0.008 | 0.243 | 0.002 | 0.020 |
| mining | 0.001 | 0.009 | 0.169 | 0.000 | 0.101 | 0.003 | 0.029 | 0.216 | 0.094 | 0.002 | 0.152 | 0.001 | 0.038 |
| nondur | 0.002 | 0.004 | 0.057 | 0.000 | 0.026 | 0.003 | 0.011 | 0.160 | 0.487 | 0.004 | 0.051 | 0.005 | 0.027 |
| other | 0.015 | 0.006 | 0.077 | 0.011 | 0.131 | 0.006 | 0.039 | 0.004 | 0.037 | 0.015 | 0.100 | 0.000 | 0.014 |
| profserv | 0.023 | 0.000 | 0.034 | 0.001 | 0.083 | 0.005 | 0.064 | 0.003 | 0.031 | 0.010 | 0.207 | 0.000 | 0.020 |
| trade | 0.008 | 0.002 | 0.031 | 0.003 | 0.132 | 0.011 | 0.048 | 0.003 | 0.040 | 0.020 | 0.202 | 0.027 | 0.081 |
| ntrans | 0.018 | 0.010 | 0.028 | 0.000 | 0.104 | 0.020 | 0.032 | 0.039 | 0.104 | 0.014 | 0.120 | 0.001 | 0.167 |

TABLE A7. Production elasticities to intermediate inputs at 13-sector level ($\mathbf{\Omega}$), rounded to 3 decimal places. We measure labor income using reported employee compensation, and rescale elasticities proportionally to ensure constant returns to scale.

We estimate the parameters of the sector specific matching function from monthly data on hires and vacancies from JOLTs and unemployment from the CPS. In particular, we estimate

$$\log H_{i,t} = \log \phi_i + \eta_i \log U_{i,t} + (1 - \eta_i) \log V_{i,t} + \epsilon_{i,t}$$

by least squares. $\phi_i$ is the matching efficiency in sector $i$ and $\eta_i$ is the matching elasticity with respect to unemployment in sector $i$. We report the resulting estimates in the table below.

| | Matching Efficiency $\left(\widehat{\phi}_i\right)$ | Unemployment Elasticity $(\widehat{\eta}_i)$ |
|---|---|---|
| accom | 1.185 | 0.401 |
| const | 1.106 | 0.507 |
| dur | 0.688 | 0.364 |
| edhealth | 0.703 | 0.336 |
| fin | 0.705 | 0.329 |
| gov | 0.640 | 0.291 |
| info | 0.703 | 0.275 |
| mining | 1.236 | 0.262 |
| nondur | 0.779 | 0.391 |
| other | 0.848 | 0.441 |
| profserv | 1.077 | 0.372 |
| trade | 1.009 | 0.430 |
| trans | 0.862 | 0.439 |

TABLE A8. Matching function parameter estimates. Based on monthly hiring, unemployment, and vacancy data from Jan 2000 to Feb 2023.

Finally, we use the sector level proportion of HR workers as a proxy for the recruiter producer ratio. The resulting recruiter producer ratios are reported below

| | $\tau_i$ |
|---|---|
| accom | 0.002 |
| const | 0.002 |
| dur | 0.007 |
| edhealth | 0.005 |
| fin | 0.008 |
| gov | 0.011 |
| info | 0.013 |
| mining | 0.005 |
| nondur | 0.007 |
| other | 0.018 |
| profserv | 0.020 |
| trade | 0.003 |
| trans | 0.001 |

TABLE A9. Estimated recruiter producer ratios based on the number of HR workers in industry $i$ over total employment in industry $i$.

### C.1.2. Wages

We estimate the response of wages to technology shocks using data on sector level productivity, labor input, labor costs, nominal output and real output at the 2-digit NAICS code level. This data is available from the BLS.

We need to aggregate certain sector series to get to the 13-sector specification we use. We combine labor related index series by weighting by the labor costs of each industry. We combine output indices by weighting by nominal output shares of each industry. We then regress changes in nominal real wages, $d\log w_i - d\log p_i$ on changes in TFP in each industry plus a constant.

$$d\log w_i - d\log p_i = \lambda_{i,0} + \sum_{j=1}^{J} \lambda_{i,j} d\log A_j + \epsilon_i$$

The resulting estimates are reported in the table below

| | accom | const | dur | edhealth | fin | info | mining | nondur | other | profserv | trade | trans | constant |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| accom | 0.857 | 0.858 | 0.095 | 0.226 | -0.012 | -0.183 | 0.217 | -0.339 | -0.169 | 0.171 | 0.182 | -0.071 | 0.005 |
| const | -0.024 | 1.138 | -0.001 | 0.209 | 0.062 | -0.097 | 0.084 | -0.050 | -0.118 | -0.005 | 0.119 | 0.040 | -0.007 |
| dur | -0.690 | 1.735 | 0.714 | 0.996 | 0.284 | -0.262 | 0.272 | 0.244 | 0.186 | -0.304 | 0.277 | -0.076 | 0.017 |
| edhealth | 0.043 | -0.052 | -0.174 | 1.261 | 0.023 | 0.045 | -0.022 | 0.178 | -0.023 | -0.065 | 0.094 | -0.029 | -0.001 |
| fin | 0.474 | 6.181 | 1.687 | -0.908 | 1.530 | -0.422 | 0.850 | 1.453 | -0.379 | -0.225 | -0.561 | -0.535 | -0.003 |
| info | -0.505 | 1.556 | 0.885 | 0.441 | 0.768 | 0.189 | 0.217 | 1.627 | 0.041 | 0.150 | -0.780 | -0.361 | 0.039 |
| mining | -2.182 | 5.179 | -0.688 | 2.606 | 0.789 | -0.184 | 2.248 | -2.031 | 0.873 | -1.520 | 1.415 | 0.385 | 0.006 |
| nondur | -0.953 | 3.000 | -0.150 | 1.037 | 0.351 | -0.200 | 0.536 | 0.769 | 0.238 | -0.393 | 0.495 | 0.041 | 0.015 |
| other | -0.601 | 1.013 | 0.326 | 0.823 | 0.201 | -0.037 | 0.240 | -0.112 | 1.639 | -0.012 | 0.159 | 0.095 | 0.006 |
| profserv | 0.008 | 0.333 | 0.020 | -0.142 | -0.020 | -0.090 | 0.084 | 0.072 | -0.042 | 1.152 | 0.032 | -0.045 | 0.007 |
| trade | -0.248 | 1.430 | 0.282 | 0.216 | -0.013 | -0.209 | 0.178 | 0.108 | -0.005 | 0.139 | 1.054 | -0.076 | 0.007 |
| trans | -1.013 | 3.992 | 0.680 | 2.327 | 0.914 | -0.470 | 1.252 | -1.234 | 0.425 | -0.533 | 0.943 | 0.463 | 0.018 |

TABLE A10. Estimated response of wages to productivity shocks in different sectors.

# Appendix D. Additional Results on Calibrated Shock Propagation.

## D.1. Responses to technology shocks across sectors

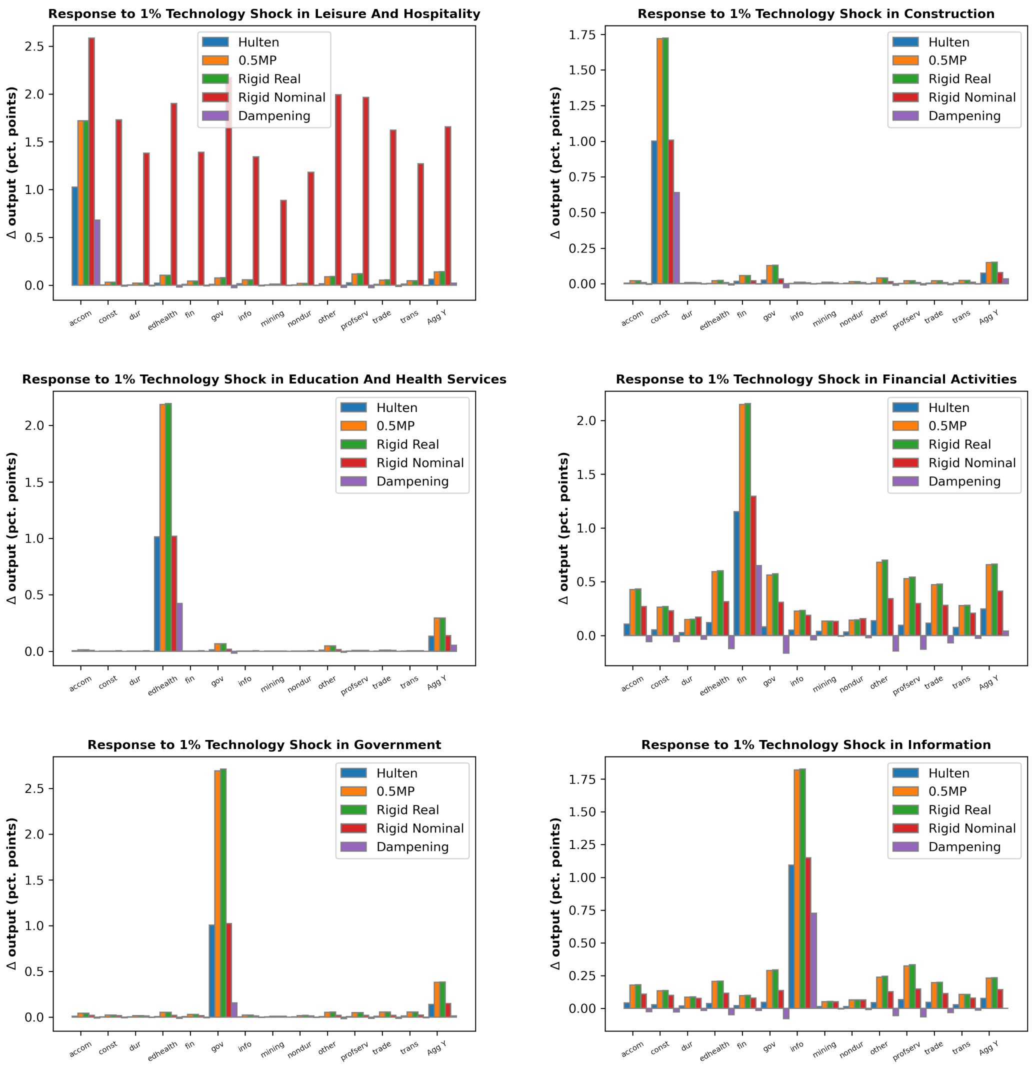


FIGURE A1. Responses of output to 1% technology shocks other sectors.

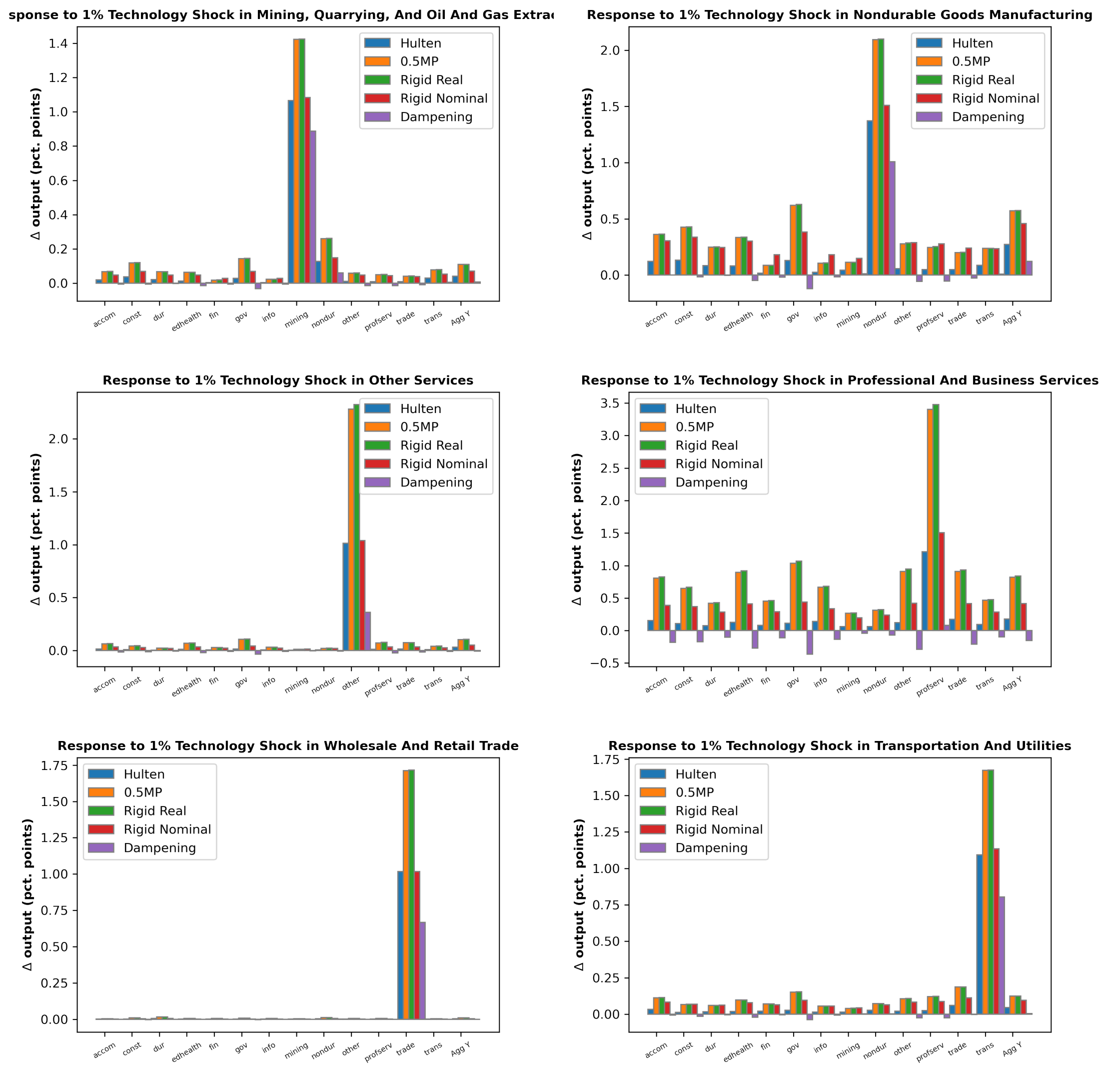


FIGURE A2. Responses of output to 1% technology shocks other sectors.

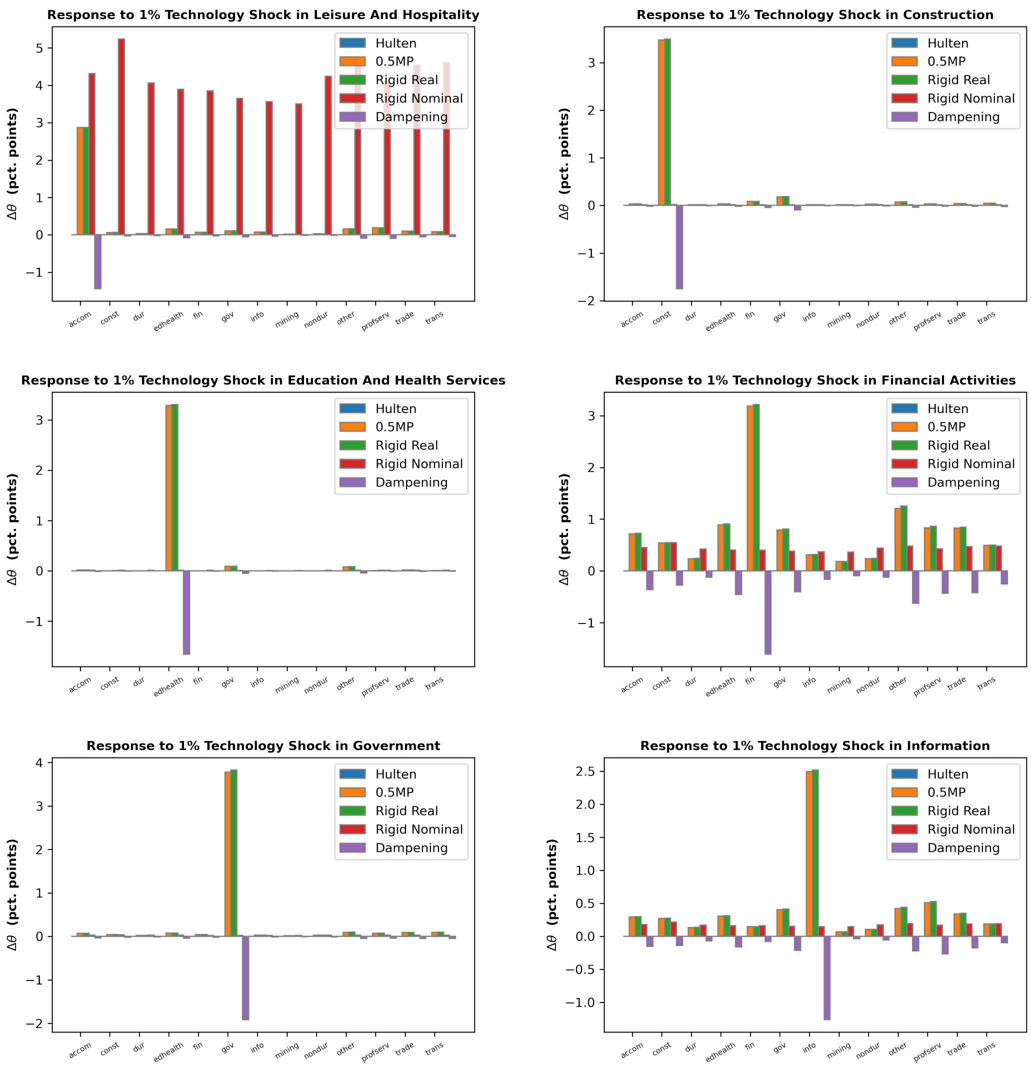


FIGURE A3. Responses of tightness to 1% labor force shocks other sectors.

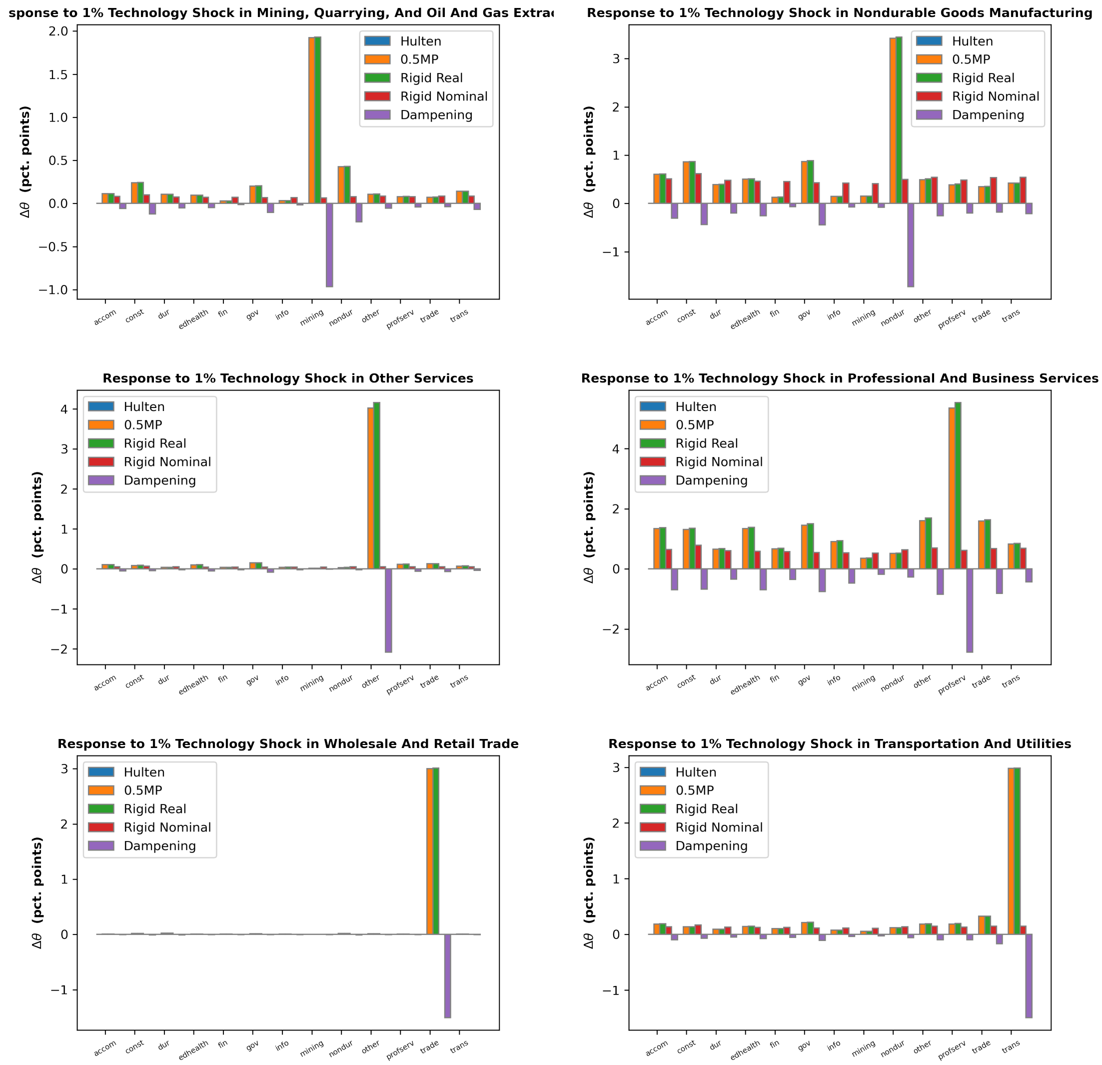


FIGURE A4. Responses of tightness to 1% labor force shocks other sectors.

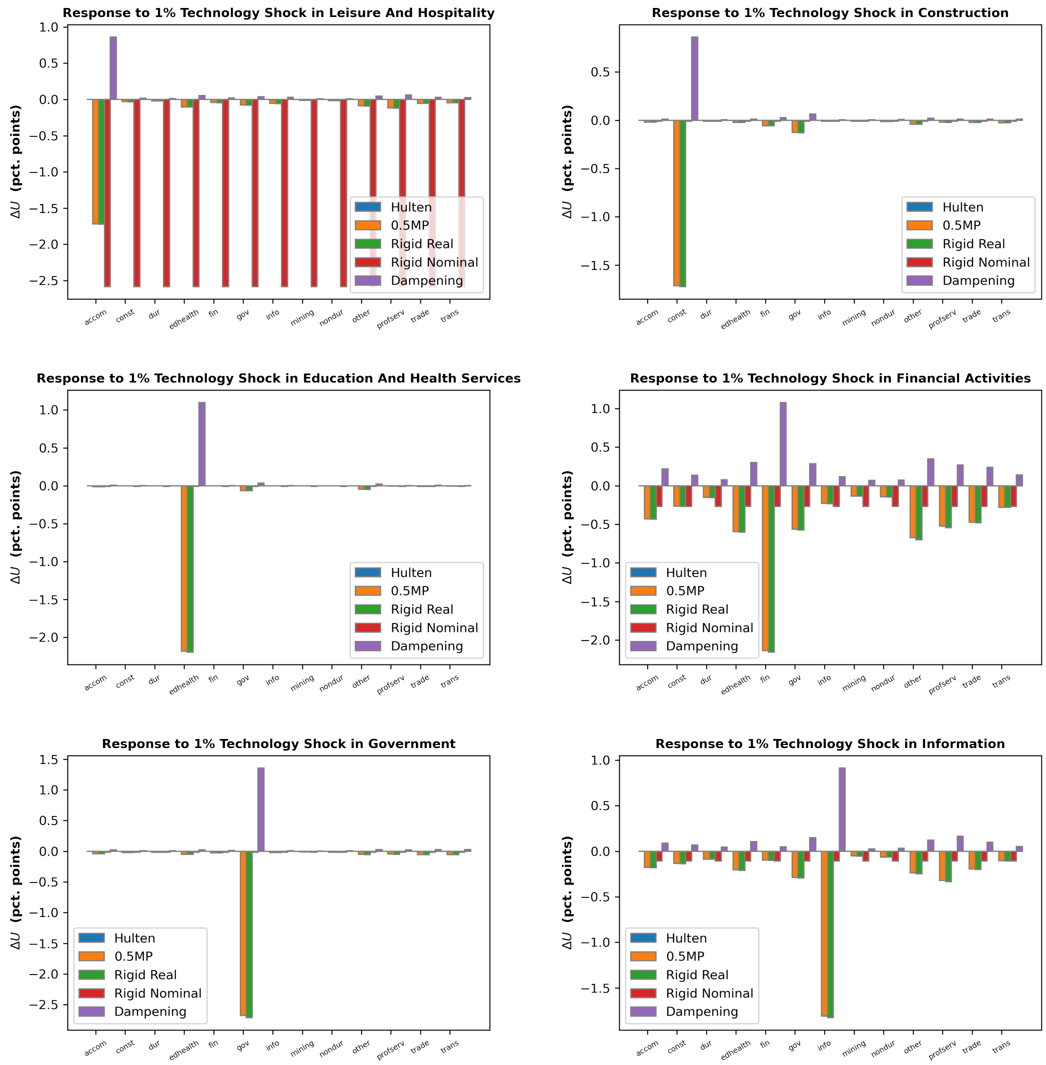


FIGURE A5. Responses of unemployment to 1% labor force shocks other sectors.

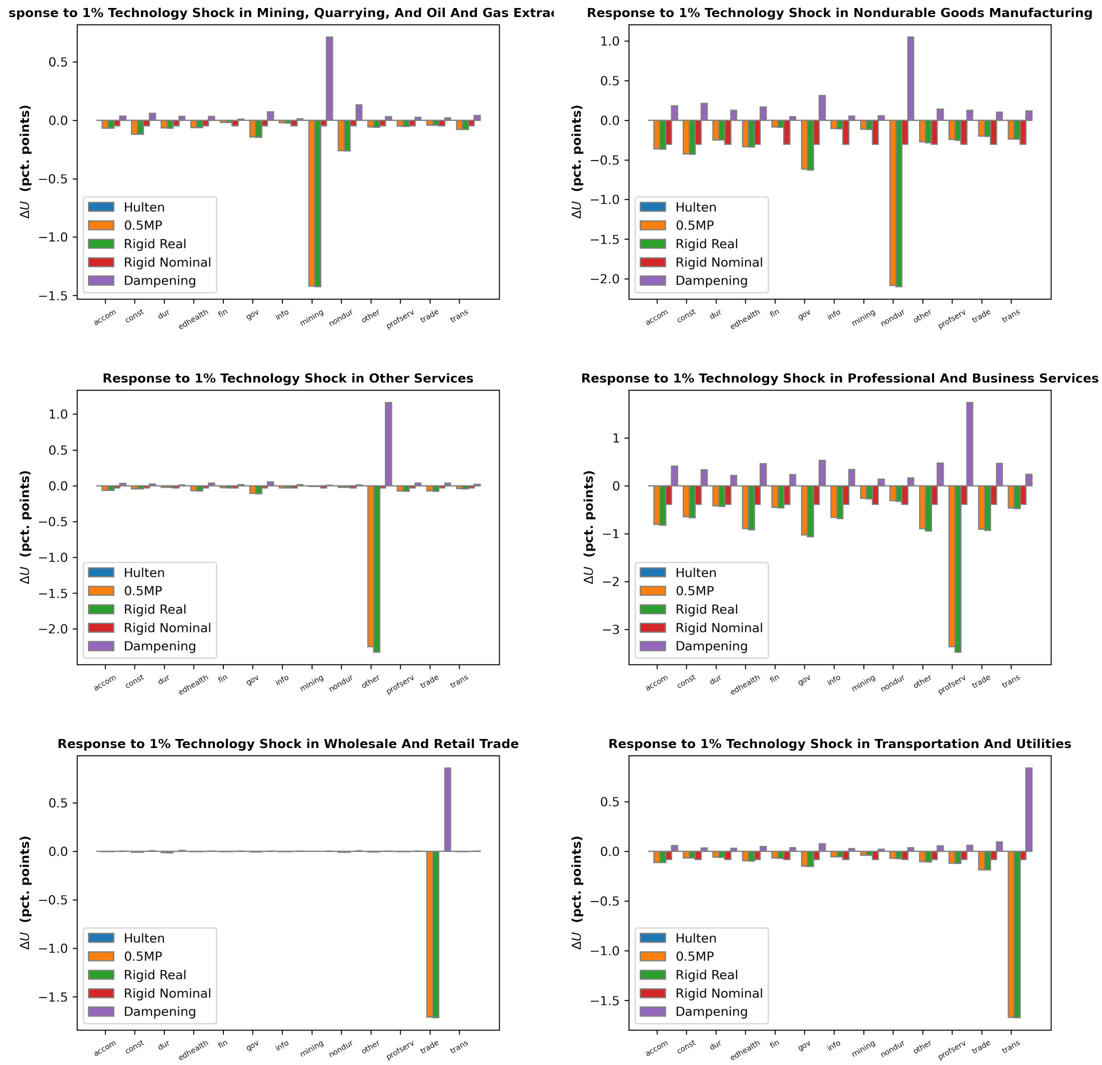


FIGURE A6. Responses of unemployment to 1% labor force shocks other sectors.

## D.2. Responses to labor force shocks across sectors

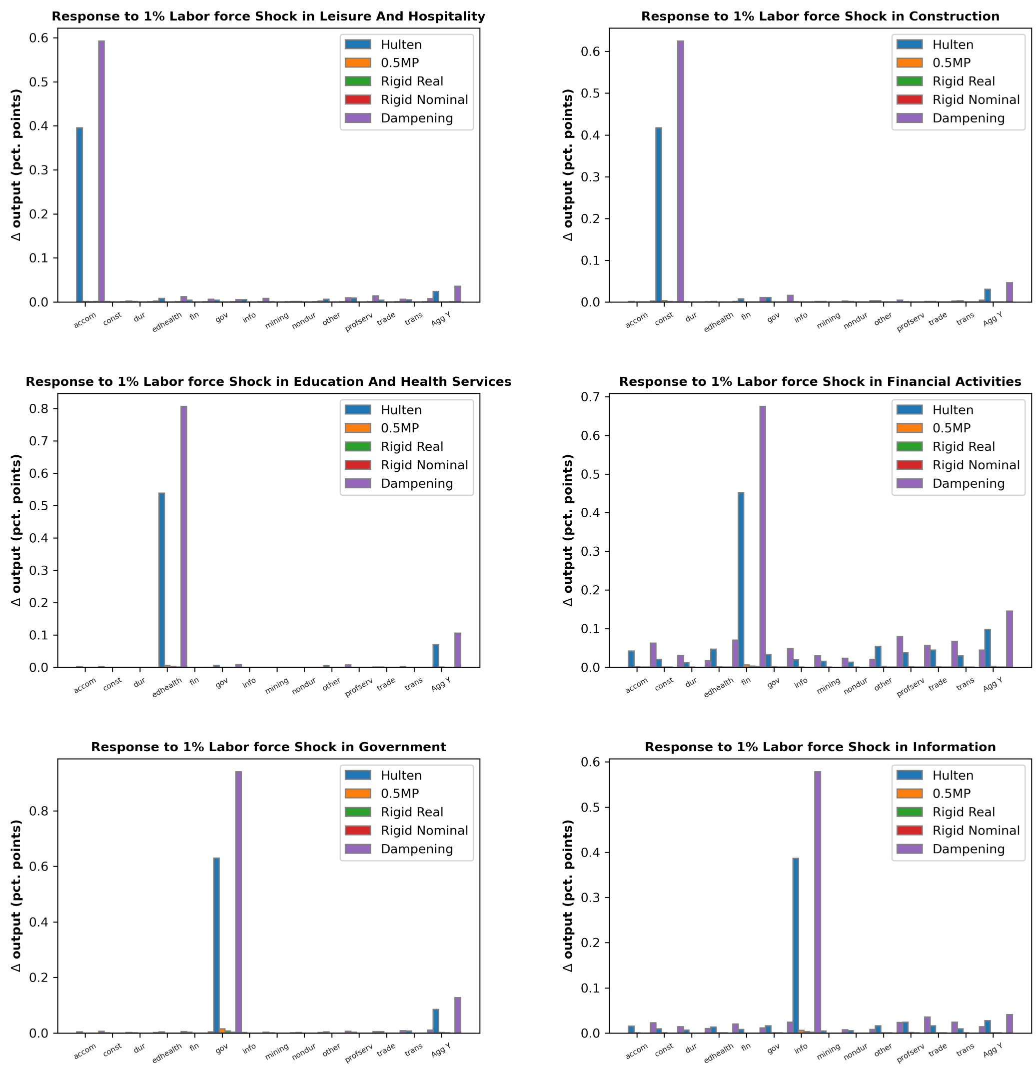


FIGURE A7. Responses of output to 1% technology shocks other sectors.

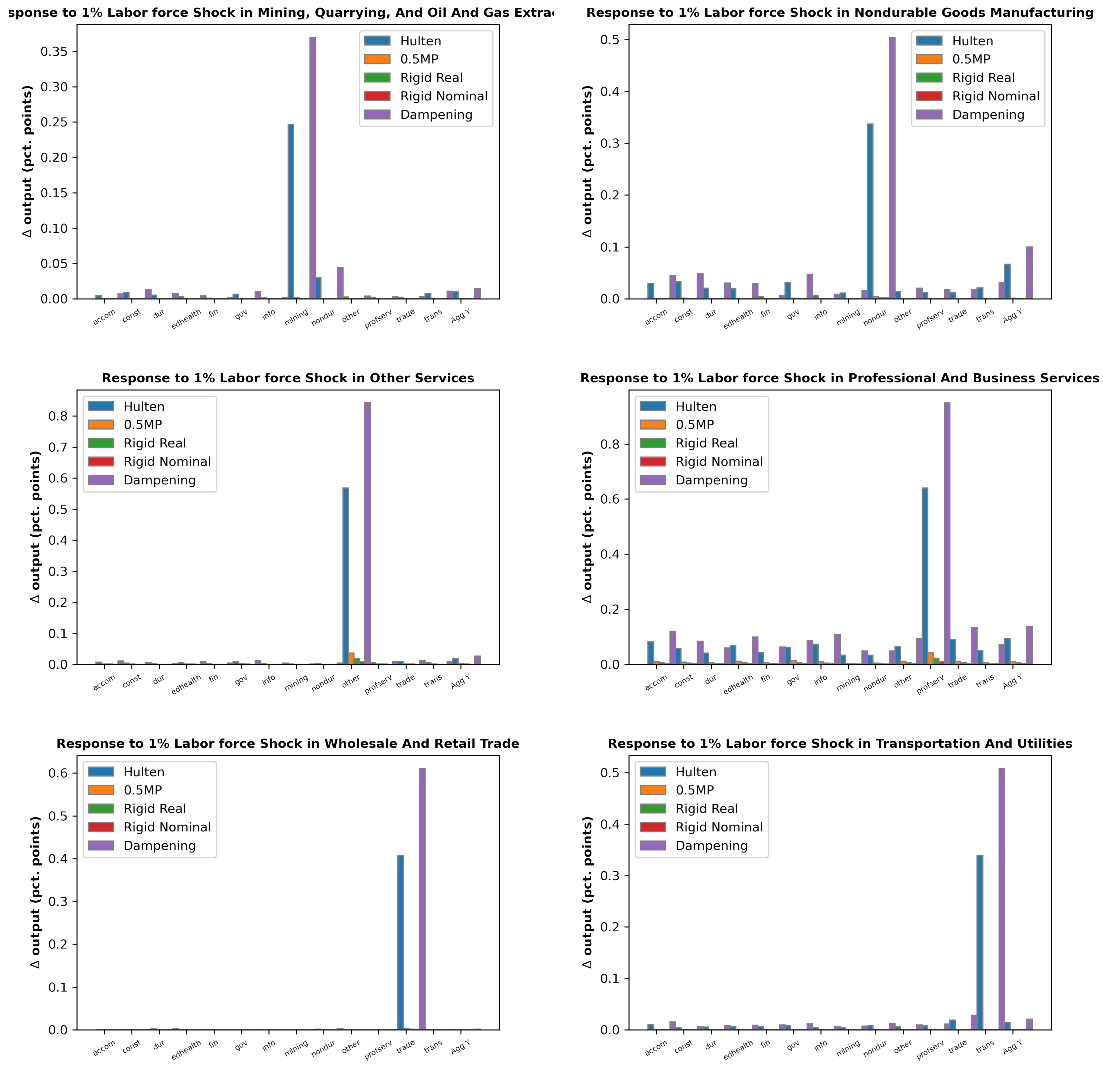


FIGURE A8. Responses of output to 1% technology shocks other sectors.

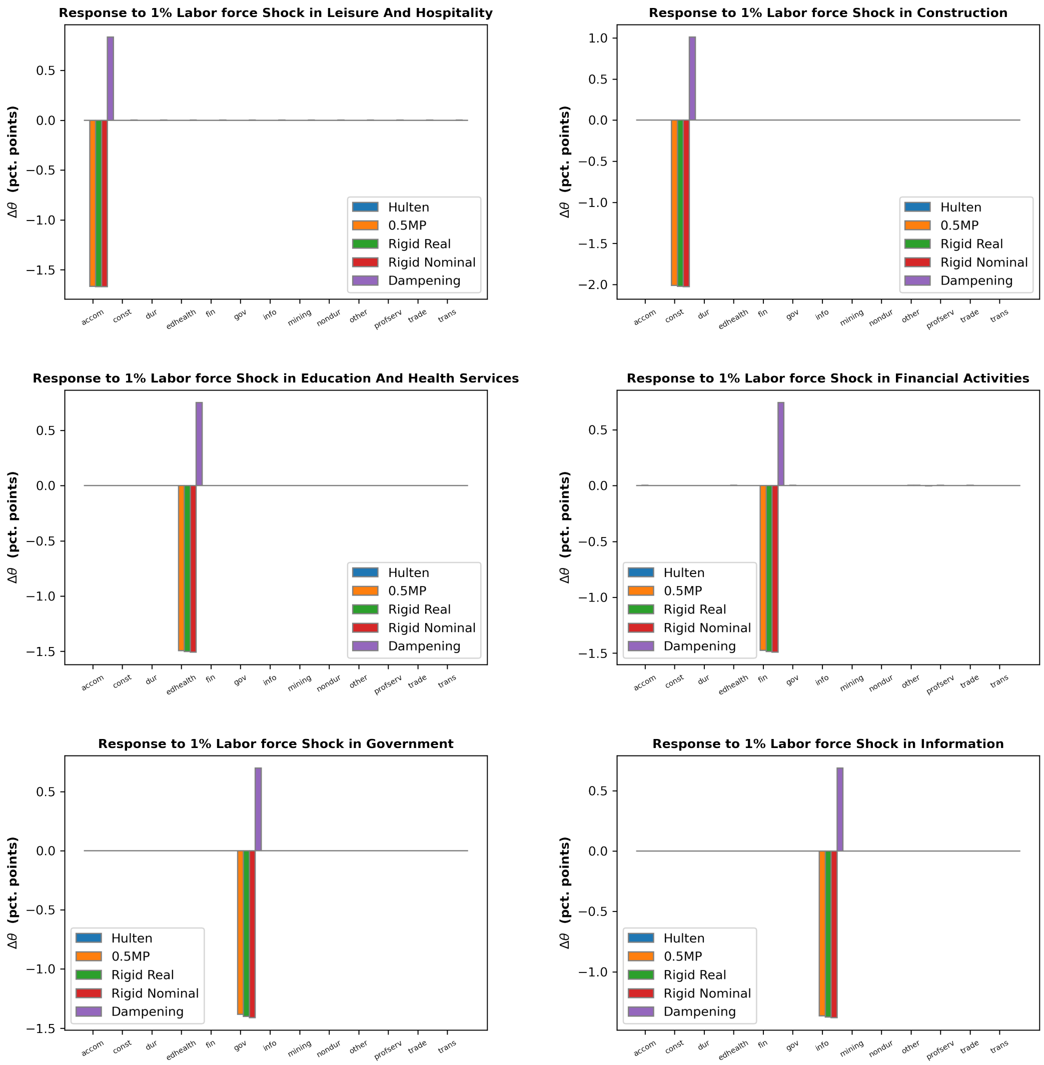


FIGURE A9. Responses of tightness to 1% technology shocks other sectors.

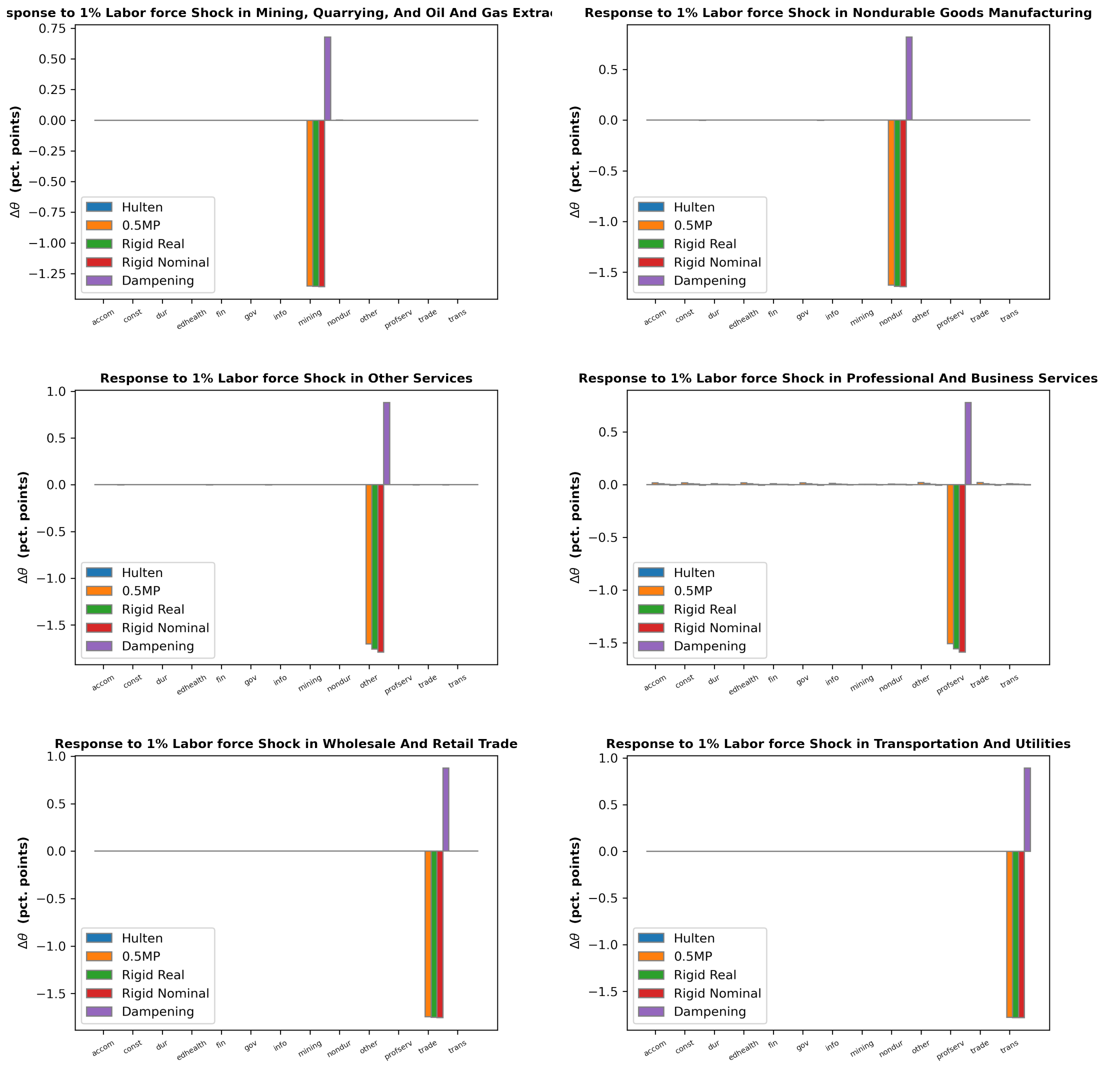


FIGURE A10. Responses of tightness to 1% technology shocks other sectors.

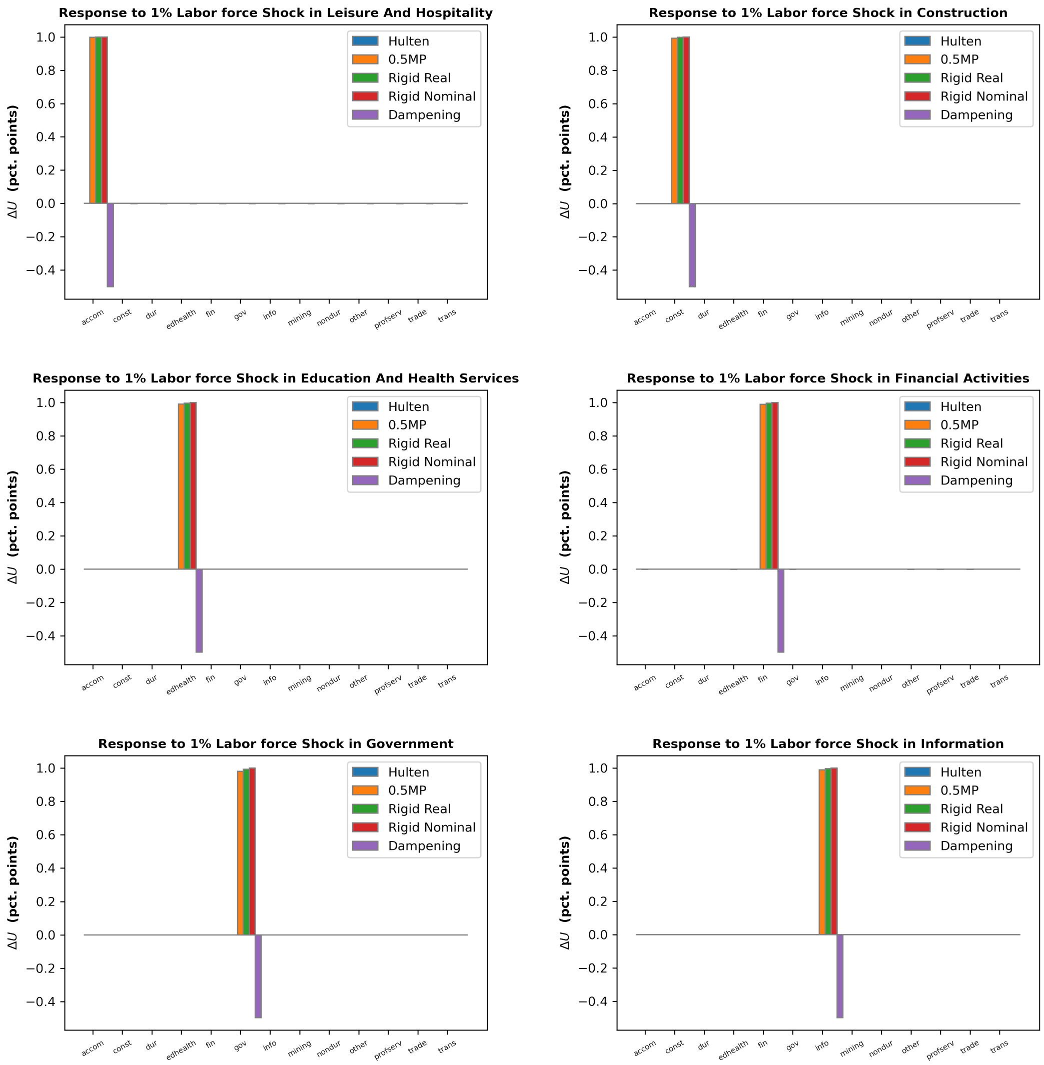


FIGURE A11. Responses of unemployment to 1% technology shocks other sectors.

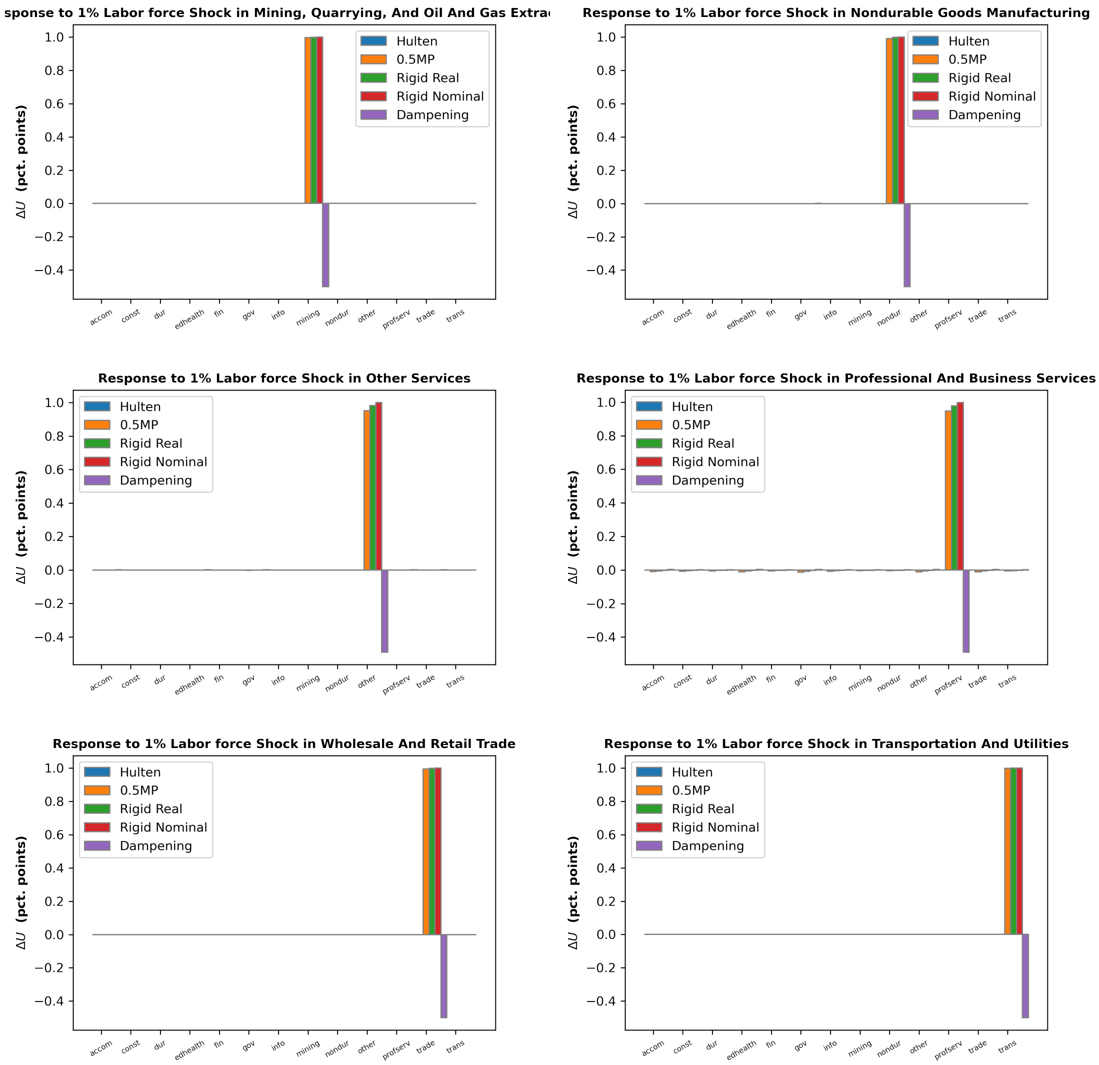


FIGURE A12. Responses of unemployment to 1% technology other sectors.