Unemployment in a Production Network

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Abstract

We develop a framework for studying unemployment in a production network by incorporating sector-specific search and matching frictions. We derive aggregation formulae for how output and unemployment respond to sector-specific productivity and labor supply shocks. We show that the foundational theorem of Hulten (1978) is a special case of the network we study—one with fixed nominal wages and Cobb-Douglas production function. Moving beyond fixed nominal wages, we show that, with Cobb-Douglas production, wage elasticities to own and cross sector tightness and productivity shocks are sufficient statistics for how productivity shocks pass through to sector level and aggregate outcomes. In addition, the assumption on wage schedules is essential for determining whether matching frictions amplify or dampen the impact of productivity shocks: when effective real wages rise in response to a positive technology shock, firms reduce their hiring, dampening some of the positive effect on output; when the effective real wage falls, firms increase hiring, amplifying the positive effect on output. Our theoretical model shows that estimating the impact of shock propagation in a production network is incomplete without accounting for labor market features, and our setup allows us to speak to how key sectoral and aggregate labor market variables, such as unemployment and tightness, change in response to shocks.

Our model allows us to answer an array of interesting policy questions, such as the aggregate impact of a sector-specific industrial subsidy. For example, we calibrate our model to the US data and show that a subsidy that results in a one-percent positive productivity shock to the durables sector generates a one percent

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positive change in aggregate output and a 0.1 percentage point change in the aggregate unemployment rate. We also apply our framework to computing the optimal unemployment allocation across sectors, holding aggregate unemployment fixed, and the efficient aggregate level of unemployment accounting for sectoral linkages. We explore when and to what extent sectoral heterogeneity matters for existing efficiency results and labor market policy prescription in the search and matching literature.

JEL Codes: E1, J3, J6

1. Introduction

Modern economies feature rich production networks. A recent literature highlights that the interactions between firms in different sectors matter for the propagation of sector level shocks (Baqaee and Rubbo 2022), how we measure productivity and the social cost of distortions (Baqaee and Farhi 2019, 2020), the optimal conduct of monetary policy (Rubbo 2020; La'O and Tahbaz-Salehi 2021), and that sector level shocks may even be a source of aggregate fluctuations and growth (Acemoglu et al. 2012; Acemoglu and Azar 2020).

We extend the existing literature by showing that production networks also matter for unemployment in a realistic frictional labor market. As shocks propagate through the network, labor demand changes, which impacts sector tightness and unemployment. On the other hand, unemployment matters for production networks. Changes in labor market tightness require firms to change their recruiting effort, and the reallocation of productive workers to recruiters alters production outcomes. In addition, unemployed workers look for jobs in other sectors, lowering tightness in those sectors, and thus affecting production.

Production linkages and unemployment are important economic mechanisms, and understanding how the two interact is key to painting an accurate and granular picture of how microeconomic shocks generate macroeconomic fluctuations. In addition, understanding the labor market impact of micro shocks can help policymakers craft and evaluate policies. For example, one can dissect how energy price spikes caused by the Russia-Ukraine war impact prices and unemployment for different sectors in different countries, as well as studying how the CHIPS and Science Act, which boosts domestic research and manufacturing of semiconductors in the United States, changes aggregate output and unemployment. More generally, we can reformat these policy questions as an interesting array of economic questions. What is the impact of an idiosyncratic, sector-specific technology shock on sector and aggregate output and unemployment? How do labor market inefficiencies and characteristics, such as matching frictions, mobility costs, and wage schedules, affect network propagation of shocks?

Starting with Long and Plosser (1983) and Acemoglu et al. (2012), much effort has been made on understanding how micro shocks cause macro fluctuations. Recent literature has incorporated inefficiencies into production network models, including markups and financial frictions (**???**). While these models offer a more realistic depiction of production linkages and the associated product market and financial inefficiencies, their treatment of labor markets are simple. These models consider labor as either perfectly inelastically supplied, or supplied according to the disutility of work in household's utility function, therefore ignoring the richness of labor market imperfections and shutting off interesting propagation channels brought forth by such imperfections.

In this paper, we extend the production network framework by incorporating matching frictions. We build a static, multisector production network model that features a representative household, many production sectors, perfect competition on product markets, and segmented frictional labor markets. The form of labor market segmentation is flexible, and our model allows for geographic, sectoral, or occupational labor market segmentation.

With this setup, we develop a general theory of how shocks propagate and aggregate across the production network under search frictions. In particular, we show that the aggregate impact on output can be decomposed into impacts from real factor price changes, tightness changes, and changes in sales shares, amplified by the interaction between the Leontief inverse and labor market segmentation. We find that the foundational aggregation theorem of Hulten (1978) is a special case of the network we study, when wages adjust to exactly offset changes in labor market tightness.

We compare our aggregation results with several benchmark models. First, we show that, compared with a production network model with inelastic labor supply, tightness and matching elasticities interact with input-output linkages to generate an additional channel of shock propagation. We then show that, compared with a search and matching model without network linkages, our aggregation results demonstrates important nonlinear interactions between the production structure and the labor market structure that are otherwise absent. [NOT SURE I TOTALLY UNDERSTAND THIS LAST SENTENCE.] [could add in additional things if we can say stuff about the direction of amplification/dampening compared to each benchmark]

We then calibrate our model to US data. We use survey-based vacancy and hiring data from JOLTs, unemployment data from the CPS, and industry sales shares from the U.S Bureau of Economic Analysis (BEA). We also estimate sector-specific matching efficiencies following **?**.[WE HAVENT ACTUALLY DONE THIS YET] We find that, assuming nominal wage rigidity, a one-percent positive technology shock to the durables manufacturing sector increases aggregate output by [x] percent, and decreases aggregate unemployment by [y] percent. Importantly, we are able to observe how unemployment changes across sectors, with unemployment in [a] sector changing by [b], and unemployment in [c] sector changing by [d].

We demonstrate how to use our calibrated model for macroeconomic applications. We apply our model to explain the heterogeneous shifts in sectoral Beveridge Curves, and the price propagation of oil shocks, both post-COVID.[I THOUGHT PASCALS IDEA WAS MORE ABOUT SHIFTING BEVERIDGE CURVES DURING COVID] In the case of Beveridge curves, we express the Beveridge elasticities for sectors as a function of idiosyncratic productivity and labor supply shocks. We show how a sectoral productivity shock and an occupational labor supply shock can generate differential changes in sectoral Beveridge elasticities through production linkages and labor market frictions. We find that some combination of shocks in some sectors / occupations are able to explain qualitatively/ quantitatively the shifts in Beveridge elasticities across sectors. For the oil price shocks, we extend our model to allow for oil as an additional factor input and express our price propagation formula in terms of oil price shocks. Using the oil shocks estimated in Kanzig (2021), we find that the oil price shocks are able to account for x percent of the observed inflation. This result is similar or different from the Harvard JMP.

This paper fits closely to the recent development that attempts to bring more realism into production networks by incorporating market imperfections and inefficiencies. **?** and **?** model frictions as an exogenous wedge between the sectors' marginal costs and marginal revenues and examine how these frictions interact with the network structure and affect aggregate output in the Cobb-Douglas economy. **?** bring the model to data and estimate how the US input-output structure amplify financial distortions in the great recession. Baqaee and Farhi (2020) examine how productivity shocks aggregate under the presence of exogenous wedges in a CES economy and decompose output changes into changes in technology and changes in allocative efficiency. **?** assumes that market imperfections generate dead-weight loss and demonstrates how these imperfections compound in the production network through demand linkages. He shows that a government should design industrial policies to target sectors based on distortions in sectoral size. To our knowledge, our paper is the first to model searching and matching in a production networks setting.

This paper also contributes to the labor literature - blah blah blah.

The remainder of the paper is organized as follows. Section 2 outlines our model and defines the equilibrium. Section 3 derives expressions for first order changes in output and employment in response to changes in technology and the size of the labor force. Section 4 describes the data we use to calibrate the model and presents illustrative examples to demonstrate the quantitative importance of labor markets. Section 5 works through two applications of our model to policy relevant questions: how sector level Beveridge curves shift in response to shocks and how output and employment responds to a shock to energy prices. Section 6 concludes.

2. Model

This section describes the basic setup of our mode. There are J production sectors, indexed by i, with production linkages and $\mathcal{O}$ occupations index by o. We call o an occupation, rather than sector, may be the relevant margin of choice for individuals when they search for jobs Humlum (Humlum). Individuals tend to stay in the same or similar occupations over the course of their careers **?**. However, our formulation is general enough to allow us to think of o as representing sector specific employment, location specific employment, occupation-location pairs, or any other desired definition of a labor market. For simplicity, we keep the model static, as is the case in most of the production networks literature. Throughout, bold letters indicate vectors or matrices. We use un-bolded letters to denote functions or scalars. All vectors are column vectors.

2.1. Households and Final Production

A representative household inealastically supplies a labor force of size H_o to occupation o. There is no disutility from labor. Households consume a final production good, Y, produced by a final goods producer with production constant returns technology reflecting household preferences

$$Y = \max_{\{c_i\}_{i=1}^J} \mathcal{D}\left(\left\{c_i\right\}_{i=1}^J\right)$$

Subject to

$$\sum_{i=1}^{J} p_i c_i = \sum_{o=1}^{\mathcal{O}} w_o L_o$$

Where c_i is the input of sector i's output into final goods production, p_i is the price of the sector i good, w_o is the wage in occupation o, and L_o is the labor used in sector o. The final production input choices satisfy the first order condition

$$\varepsilon_{c_i}^{\mathcal{D}} = \frac{p_i c_i}{\sum_{k=1}^{J} p_k c_k} \tag{1}$$

Where $\varepsilon_{c_i}^{\mathcal{D}}$ is the elasticity of $\mathcal{D}$ to changes in c_i. This is the households elasticity of utility with regards to the consumption of good i.

2.2. Labor Markets

We assume there are $\mathcal{O}$ occupations with separate labor markets, a labor force of H_o possible workers, who all start out unemployed at the beginning of the single period. The exogenous recruiting cost, r_o, measures the units of labor required for a firm to maintain each posted vacancy in occupation o. When workers and firms meet there is a mutual gain from matching. There is no accepted theory for how wages are set in this context. For now we assume the nominal wage in occupation o, w_o, follows a general wage schedule that depends on productivity and the size of the labor force, and is taken as given by both firms and workers. Hires are generated by a constant returns matching function in occupation-level unemployment U_o and aggregate vacancies V_o, which measures all vacancy postings for occupation o,

$$h_o = \phi_o m\,(U_o, V_o)$$

Let the sector-specific labor market tightness be $\theta_o = \frac{V_o}{H_o}$, the vacancy-filling rate $\mathcal{Q}_o(\theta_o) = \phi_o m\left(\frac{H_o}{V_o}, 1\right)$, and the job-finding rate $\mathcal{F}_o(\theta_o) = \phi_o m\left(1, \frac{v_o}{H_o}\right)$. Therefore, a fraction $\mathcal{F}_o(\theta_o)$ the labor force finds a job and is employed at the end of the period. Labor supply satisfies

$$L_i^o(\theta_o) = \mathcal{F}_o(\theta_o) H_o \tag{2}$$

We assume firms take the occupation level tightness as given.[1] Let N_{io} denote productive employees in occupation o working for sector i firms. In order to hire N_{io} productive employees, the number of vacancies posted in labor market o by sector i firms, v_{io}, has to satisfy $\mathcal{Q}_o(\theta_o)v_{io} = N_{io} + r_o v_{io}$, where $r_o v_{io}$ is the total cost of posting the vacancies. Rearranging yields $v_{io} = \frac{N_{io}}{\mathcal{Q}_o(\theta_o) - r_o}$. Thus, hiring one unit of productive labor requires $\frac{1}{\mathcal{Q}_o(\theta_o) - r_o}$ vacancy postings. Factoring in hiring costs, firms need $1 + \tau_o(\theta_o)$ units of total labor for each productive worker, where

$$\tau_o(\theta_o) \equiv \frac{r_o}{\mathcal{Q}_o(\theta_o) - r_o}.$$

For a given target level of occupation o employment N_{io}, total required labor, or the labor demand, is $l^d_{io}(\theta_o) = (1 + \tau_o(\theta_o))\, N_{io}$. In the language of the production networks literature, τ_o therefore acts as an endogenous wedge on firms labor costs. This endogenous wedge will turn out to play an important role in how shocks propagate through the production network. We describe how labor demand, $l^d_{io}(\theta_{io})$, is determined by firms' profit maximization in the next subsection.

Finally, we define aggregate occupation o labor demand as the sum of sectoral labor demands and aggregate vacancy postings as the sum of sectoral vacancy postings.

$$L^d_o(\theta_o) = \sum_{i=1}^{J} l^d_{io}(\theta_o) \tag{3}$$
$$V_o = \sum_{i=1}^{J} v_{io}$$

Market clearing in the labor market requires labor demand equal labor supply and that the vacancy posting choices of firms in each sector are consistent with aggregate tightness.

$$L^d_o = L^s_o \tag{4}$$
$$\theta_o = \frac{\sum_{i=1}^{J} v_{io}}{H_o}$$

[1]One way of justifying this is with the assumption that each sector is populated by many identical competitive firms so that each firm only has an infinitesimal impact on aggregate vacancies, and therefore on aggregate tightness.

2.3. Firms

A representative firm in sector i uses workers in occupation o, N_{io}, and intermediate inputs from sector j, x_{ij}, to produce output y_i using constant returns production technology f_i.

$$y_i = A_i f_i \left(\{N_{io}\}_{o=1}^{\mathcal{O}}, \{x_{ij}\}_{j=1}^{J} \right) \tag{5}$$

Firms choose $\{N_{io}\}_{o=1}^{\mathcal{O}}$ and $\left\{x_{ij}\right\}_{j=1}^{J}$ to maximize profits, or equivalently to minimize costs. We assume firms are price takers in both input and output markets. Profits are given by

$$\pi_i = p_i f_i \left(\{N_{io}\}_{o=1}^{\mathcal{O}}, \{x_{ij}\}_{j=1}^{J} \right) - \sum_{o=1}^{\mathcal{O}} w_o(1 + \tau_o(\theta_o))N_{io} - \sum_{j=1}^{J} p_j x_{ij}$$

Firms choose inputs to solve

$$\max_{\{N_{io}\}_{o=1}^{\mathcal{O}}, \left\{x_{ij}\right\}_{j=1}^{J}} \pi_i \left(\{N_{io}\}_{o=1}^{\mathcal{O}}, \left\{x_{ij}\right\}_{j=1}^{J} \right)$$

Giving the first order conditions

$$\varepsilon_{x_{ij}}^{f_i} = \frac{p_j x_{ij}}{p_i y_i} \tag{6}$$

$$\varepsilon_{N_{io}}^{f_i} = (1 + \tau_o(\theta_o)) \frac{w_o N_{io}}{p_i y_i} \tag{7}$$

Where $\varepsilon_{x_{ij}}^{f_i}$ is the elasticity of sector i production to sector j input and $\varepsilon_{N_{io}}^{f_i}$ is the elasticity of sector i production to occupation o labor. From Equation 7, we can derive an expression for sector i and aggregate labor demand:

$$l_{io}^{d}(\theta_o) = \varepsilon_{N_{io}}^{f_i} \frac{p_i}{w_o} y_i \tag{8}$$

$$L_o^d(\theta_o) = \sum_{i=1}^{J} \varepsilon_{N_{io}}^{f_i} \frac{p_i}{w_o} y_i$$

2.4. Equilibrium

Given exogenous variables $\left\{\{A_i\}_{i=1}^{J}, \{H_o\}_{o=1}^{\mathcal{O}}\right\}$, the equilibrium is a collection of $4J + 2J^2 + 3\mathcal{O}J + 4\mathcal{O}$ endogenous variables

$$\left\{\left\{p_i, y_i, \left\{x_{ij}, \varepsilon_{x_{ij}}^{f_i}\right\}_{j=1}^{J}, c_i, \varepsilon_{c_i}^{\mathcal{D}}, \left\{N_{io}, \varepsilon_{N_{io}}^{f_i}, l_{io}^{d}\right\}_{o=1}^{\mathcal{O}}\right\}_{i=1}^{J}, \left\{\theta_o, w_o, L_o^d, L_o^s\right\}_{o=1}^{\mathcal{O}}, \right\}$$

that satisfy equations (1), (2), (3), (4), (5), (6), (7), and (8) for all sectors and occupations, along with constant returns to scale in final goods production

$$\sum_{i=1}^{J} \varepsilon_{c_i}^{\mathcal{D}} = 1 \tag{9}$$

constant returns to scale in secotoral production

$$\sum_{j=1}^{J} \varepsilon_{x_{ij}}^{f_i} + \sum_{o=1}^{\mathcal{O}} \varepsilon_{N_{io}}^{f_i} = 1 \qquad \forall i \tag{10}$$

and goods market clearing

$$y_i = c_i + \sum_{j=1}^{J} x_{ij} \qquad \forall i \tag{11}$$

Equations (1)-(11) provide $4J + J^2 + 2J\mathcal{O} + 3\mathcal{O} + 1$ restrictions. To close the model, we must make an assumption about the wage schedule, providing an additional $\mathcal{O}$ restrictions, and functional form assumptions for final and sectoral production.

3. Theoretical Results

In this section, we describe the main theoretical results of our paper. We first derive the first order propagation of technology and labor supply shocks at the dis-aggregated occupation and sector level. Specifically, we compute the first order response of the main endogenous variables of interest in the model. We then present our aggregation theorem of idiosyncratic labor supply and technology shocks for output and unemployment. The aggregate impact of idiosyncratic shocks can be summarized by sales shares and matching elasticities. In the remaining sections, we assume production to be Cobb-

Douglas. We show how to generalize the results to any constant returns production function in the case of one occupation per sector in appendix B.

3.1. Notation

Before outlining our results, we define some useful notation. First, as is standard in the networks literature, we define the input-matrix $\mathbf{\Omega}$ as the matrix containing sector level input revenue shares

$$\mathbf{\Omega} = \begin{bmatrix} \frac{p_1 x_{11}}{p_1 y_1} & \cdots & \frac{p_J x_{1J}}{p_1 y_1} \\ \vdots & \ddots & \vdots \\ \frac{p_1 x_{J1}}{p_J y_J} & \cdots & \frac{p_J x_{JJ}}{p_J y_J} \end{bmatrix}_{J \times J}$$

In our competitive goods market equilibrium setup, we can rewrite this input output matrix in terms of the elasticities of the sector level production functions. (See (6))

$$\mathbf{\Omega} = \begin{bmatrix} \varepsilon^{f_1}_{x_{11}} & \cdots & \varepsilon^{f_1}_{x_{1J}} \\ \vdots & \ddots & \vdots \\ \varepsilon^{f_J}_{x_{J1}} & \cdots & \varepsilon^{f_J}_{x_{JJ}} \end{bmatrix}_{J \times J}$$

We denote the Leontief inverse by $\mathbf{\Psi} = (\boldsymbol{I} - \mathbf{\Omega})^{-1}$. The Leontief inverse captures the importance of each sector as a direct and indirect input into production in every other sector.

Alongside the production elasticities to intermediate inputs, our model features a second set of production elasticities: elasticities to the different types of labor inputs. We collect these elasticities in the matrix $\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}$.

$$\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} = \begin{bmatrix} \varepsilon^{f_1}_{N_{11}} & \cdots & \varepsilon^{f_1}_{N_{1\mathcal{O}}} \\ \vdots & \ddots & \vdots \\ \varepsilon^{f_J}_{N_{J1}} & \cdots & \varepsilon^{f_J}_{J\mathcal{O}} \end{bmatrix}_{J \times \mathcal{O}}$$

Equation (7) demonstrates that this matrix is the labor input equivalent of the standard input-output matrix. In equilibrium, each entry is the revenue share of type o labor.

Below $\mathcal{L}$ is a $\mathcal{O} \times J$ with the share of occupation o workers employed in each sector along the rows.

$$\mathcal{L} = \begin{bmatrix} \frac{l_{11}}{L_1} & \cdots & \frac{l_{J1}}{L_1} \\ \vdots & \ddots & \vdots \\ \frac{l_{1\mathcal{O}}}{L_{\mathcal{O}}} & \cdots & \frac{l_{J\mathcal{O}}}{L_{\mathcal{O}}} \end{bmatrix}_{\mathcal{O}\times J}$$

$\mathfrak{F}$, $\mathfrak{Q}$, and $\mathfrak{T}$ are $\mathcal{O} \times \mathcal{O}$ diagonal matrices with the occupation level job-finding rate, vacancy-filling rate, and recruiter-producer ratio along the diagonal.

$$\mathfrak{F} = \begin{bmatrix} \varepsilon^{\mathcal{F}_1}_{\theta_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \varepsilon^{\mathcal{F}_{\mathcal{O}}}_{\theta_{\mathcal{O}}} \end{bmatrix}_{\mathcal{O}\times\mathcal{O}}, \quad \mathfrak{Q} = \begin{bmatrix} \varepsilon^{\mathcal{Q}_1}_{\theta_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \varepsilon^{\mathcal{Q}_{\mathcal{O}}}_{\theta_{\mathcal{O}}} \end{bmatrix}_{\mathcal{O}\times\mathcal{O}}, \quad \mathfrak{T} = \begin{bmatrix} \tau_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \tau_{\mathcal{O}} \end{bmatrix}_{\mathcal{O}\times\mathcal{O}}$$

Assuming a Cobb-Douglas matching function $m(U,V) = \phi U^{\eta} V^{1-\eta}$, these matrices are

$$\mathfrak{F} = \begin{bmatrix} 1-\eta_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1-\eta_{\mathcal{O}} \end{bmatrix}, \qquad \mathfrak{Q} = \begin{bmatrix} -\eta_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & -\eta_{\mathcal{O}} \end{bmatrix}$$

3.2. The Propagation of Technology and Labor Force Shocks

We are interested in how three sets of endogenous variables—sector level prices, sector level output, and occupation level tightness—change in response to change in technology and the size of the labor force. With these variables, we can compute the real aggregate variables of interest: as output and unemployment.

Before deriving our results, we need to specify how wages are determined. In matching models, workers and firms meet in a situation of bilateral monopoly. The resulting mutual gains from trade mean that wages are not determined by equilibrium conditions of the model[2], and must instead be pinned down by some wage setting norm chosen by the researcher. We introduce the following reduced-form relationship between relative wages and the shocks:

$$d\log \boldsymbol{w} - \mathcal{L} d\log \boldsymbol{p} = \boldsymbol{\Lambda_A} d\log \boldsymbol{A} + \boldsymbol{\Lambda_H} d\log \boldsymbol{H}.$$

[2]Wages are only constrained to fall within a range where both workers and firms benefit from the match. However, this range can be wide since workers usually strongly prefer employment to unemployment and finding a new match is costly for firms.

Where $d\log \boldsymbol{w}$ and $d\log \boldsymbol{H}$ are $\mathcal{O} \times 1$ dimensional vectors capturing first order changes in wages and the size of the labor force, $d\log \boldsymbol{p}$ and $d\log \boldsymbol{A}$ are $J \times 1$ dimensional vectors capturing first order changes in prices and productivities. $\mathcal{L}$ is a $\mathcal{O} \times J$ matrix with the share of occupation o workers employed in each sector along the rows. The (o, i)th entry is $\frac{l_{io}}{L_o}$. $\boldsymbol{\Lambda}_A$ and $\boldsymbol{\Lambda}_H$ are $\mathcal{O} \times J$ and $\mathcal{O} \times \mathcal{O}$ coefficient matrices that capture how wages respond to technology and labor force shocks in each other sector and occupation. For instance, the (o, i)th entry of $\boldsymbol{\Lambda}_A$ captures how wages in occupation o respond to technology shocks in sector i. The (o, k)th entry in $\boldsymbol{\Lambda}_H$ captures how wages in occupation o respond to labor force shocks in occupation k.

Since, as we show below, prices are themselves determined by shocks to technology and the size of the labor force, this assumption captures the intuition that, ultimately, wages must be driven by the only two fundamental shocks in our model: $\boldsymbol{A}$ and $\boldsymbol{H}$. While it may appear restrictive, this reduced form wage setting equation is capable of nesting any assumption about wage setting tied to economic fundamentals for the right parameter matrices $\boldsymbol{\Lambda}_A$ and $\boldsymbol{\Lambda}_H$.

With changes in wages in hand, we can now derive how shocks propagate to prices, labor market tightness, and output. These first order effects are summarized in proposition 1. For a detailed derivation see appendix A.1.

Proposition 1. *Assume intermediate and final goods production functions are constant returns to scale and Cobb-Douglas. Given labor supply shocks* $d\log \boldsymbol{H} = \left[d\log H_1, d\log H_2, \cdots, d\log H_{\mathcal{O}}\right]^T$ *and productivity shocks* $d\log \boldsymbol{A} = \left[d\log A_1, d\log A_2, \cdots, d\log A_J\right]^T$*, the first-order responses in sectoral prices* $d\log \boldsymbol{p} = \left[d\log p_1, d\log p_2, \cdots, d\log p_J\right]^T$*, labor market tightness* $d\log \theta = \left[d\log \theta_1, d\log \theta_2, \cdots, d\log \theta_{\mathcal{O}}\right]^T$ *and sectoral output* $d\log \boldsymbol{y} = \left[d\log y_1, d\log y_2, \cdots, d\log y_J\right]^T$ *follow:*

$$\begin{aligned}
\left(\boldsymbol{I} - \Psi \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \mathcal{L}\right) d\log \boldsymbol{p} &= \boldsymbol{\Pi}_{p,A} d\log \boldsymbol{A} + \boldsymbol{\Pi}_{p,H} d\log \boldsymbol{H}, \\
d\log \theta &= \boldsymbol{\Pi}_{\theta,A} d\log \boldsymbol{A} + \boldsymbol{\Pi}_{\theta,H} d\log \boldsymbol{H}, \\
d\log \boldsymbol{y} &= \boldsymbol{\Pi}_{y,A} d\log \boldsymbol{A} + \boldsymbol{\Pi}_{y,H} d\log \boldsymbol{H},
\end{aligned}$$

where

$$\begin{aligned}
\boldsymbol{\Pi}_{p,A} &= \Psi \left[\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left(\boldsymbol{\Lambda}_A - \mathfrak{Q}\mathfrak{T} \left[\mathfrak{F} - \Xi_\theta\right]^{-1} (\mathcal{L}\Psi - \boldsymbol{\Lambda}_A)\right) - \boldsymbol{I}\right], \\
\boldsymbol{\Pi}_{p,H} &= \Psi \left[\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left(\boldsymbol{\Lambda}_H - \mathfrak{Q}\mathfrak{T} \left[\mathfrak{F} - \Xi_\theta\right]^{-1} \left(\mathcal{L}\Psi \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I} - \boldsymbol{\Lambda}_{\boldsymbol{H}}\right)\right)\right],
\end{aligned}$$

$$
\begin{aligned}
\Pi_{\theta,A} &= \left[\mathcal{F} - \Xi_\theta\right]^{-1} \left(\mathcal{L}\Psi - \Lambda_A\right), \\
\Pi_{\theta,H} &= \left[\mathcal{F} - \Xi_\theta\right]^{-1} \left(\mathcal{L}\Psi\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I} - \Lambda_{\boldsymbol{H}}\right), \\
\Pi_{y,A} &= \Psi \left[\boldsymbol{I} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left(\mathcal{F} + \mathfrak{Q}\mathfrak{T}\right) \left[\mathcal{F} - \Xi_\theta\right]^{-1} \left[\mathcal{L}\Psi - \Lambda_{\boldsymbol{A}}\right]\right], \\
\Pi_{y,H} &= \Psi\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left[\boldsymbol{I} + \left(\mathcal{F} + \mathfrak{Q}\mathfrak{T}\right) \left[\mathcal{F} - \Xi_\theta\right]^{-1} \left[\mathcal{L}\Psi\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I} - \Lambda_{\boldsymbol{H}}\right]\right], \\
\Xi_\theta &= \mathcal{L}\Psi\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left[\mathcal{F} + \mathfrak{Q}\mathfrak{T}\right].
\end{aligned}
$$

We denote the $J \times J$ input-output matrix by $\boldsymbol{\Omega}$. The (i, j)th entry is the elasticity of output in sector i to intermediate inputs from sector j. $\boldsymbol{\Psi} = (1 - \boldsymbol{\Omega})^{-1}$ is the standard $J \times J$ Leontief inverse. The Leontief inverse captures how each sectors output depends on inputs from other sectors both directly and indirectly. The (i, j)th entry captures how important sector j is as a direct input to sector i production, how important sector j is as a direct input into all other sector i inputs, and so on.

$\mathcal{F}$, $\mathfrak{Q}$, and $\mathfrak{T}$ are $\mathcal{O} \times \mathcal{O}$ diagonal matrices with, respectively, the occupation level job-finding rate, vacancy-filling rate, and recruiter-producer ratio, all evaluated at the initial equilibrium, along the diagonal. ε_N^f is a $J \times \mathcal{O}$ matrix with sector i's output elasticity with respect to each occupations labor input along the rows. The (i, o)th element is the elasticity of sector i production to occupation o labor. Assuming Cobb-Douglas greatly simplifies the expressions because it implies that all of the elasticities, both $\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}$ and $\boldsymbol{\Omega}$, remain constant.

Though the expressions might seem daunting at first, we unpack the intuition behind each in turn. First, we consider the response in tightness. The equilibrium response in tightness are jointly determined by changes in labor supply and labor demand. The labor supply side is impacted directly by changes in labor supply and job-finding rates, and the labor demand side is impacted by vacancy-filling rates, wages and prices, as well as productivity. A technology shock impacts labor demand by directly impacting a sector's productive capability, thus impacting prices and output and labor usage for other sectors through production linkages. A labor supply shock impacts labor supply by changing the number of available workers, which changes sectors' production through changing in the number of recruiters needed. The change in sectors' production propagate through the network and induce other sectors to change their labor demand, thus causing tightness to adjust, so on and so forth.

Second, we consider the response in sectoral output. Sectoral output is directly impacted by changes in technology, output in other sectors, and labor market tightness. A shock to technology directly impacts a sector's productive capacity through the production function. It also indirectly impacts a sector's production through the tightness channel, which documents the equilibrium interaction between labor supply and labor demand. Such impact on the sector's production then propagate to other sectors through production linkages, and to different labor markets through the labor network.

Last, we consider the response in prices. In our framework, prices are equilibrium objects determined by market clearing in a perfectly competitive market. Given that our production functions are Cobb-Douglas, the price of a good produced by a particular sector responds to changes in prices for all other sectors, as well as the effective cost of employing workers, which relies on labor market tightness and wages. In addition, the price is also directly linked to the level of productivity in that sector. Thus, a productivity shock impacts the system of prices directly through production and indirectly through adjustments in tightness, and a labor supply shock impacts prices solely through adjustments in labor market tightness.

Therefore, if one looks closely at the propagation equations, one can see a separation between what is propagated through solely the production network, and what is propagated through the interaction between the production network, the labor usage matrix, and the matching elasticities.

3.3. Aggregation

First, we derive the first-order change in aggregate output. With a Cobb-Douglas preference, it is straightforward that $d\log Y = \varepsilon_{\boldsymbol{c}}^{\mathcal{D}\prime} d\log \boldsymbol{y}$. Using results from Proposition 1, we arrive at the following result.

THEOREM 1. *Given idiosyncratic labor supply shocks $d\log \boldsymbol{H}$ and productivity shocks $d\log \boldsymbol{A}$, the log change in real GDP is:*

$$d\log Y = \Pi_{\boldsymbol{A}} d\log \boldsymbol{A} + \Pi_{\boldsymbol{H}} d\log \boldsymbol{H},$$

where

$$\begin{aligned}
\Pi_{\boldsymbol{A}} &= \lambda' \left(\boldsymbol{I} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} (\mathcal{F} + \mathcal{Q}\mathcal{T}) \left[\mathcal{F} - \Xi_\theta\right]^{-1} (\mathcal{L}\Psi - \Lambda_{\boldsymbol{A}}) \right), \\
\Pi_{\boldsymbol{H}} &= \lambda' \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left(\boldsymbol{I} + (\mathcal{F} + \mathcal{Q}\mathcal{T}) \left[\mathcal{F} - \Xi_\theta\right]^{-1} \left(\left[\mathcal{L}\Psi \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I}\right] - \Lambda_{\boldsymbol{H}} \right) \right),
\end{aligned}$$

and $\lambda = \Psi' \varepsilon_{\boldsymbol{c}}^{\mathcal{D}}$ denotes the sectors' sales shares.

Theorem 1 results from Proposition 1, as we can compute the change in real GDP by weighing the change in real output for all sectors by the household's demand elasticities. Note that the matrix product between the Leontief inverse and the demand elasticities is equal to the sales shares. This property results directly from household's maximization problem, the firms' profit maximization decision, goods market clearing, and Cobb-Douglas production functions. Therefore, the aggregate impact of productivity and labor supply shocks can be summarized as the sales-share-weighted impact of these shocks on sectoral output directly through production and indirectly through labor markets. We derive Theorem 1 in appendix subsection A.2.

Similarly, we can derive changes in aggregate unemployment and price levels.

COROLLARY 1. *Given idiosyncratic labor supply shocks $d\log \boldsymbol{H}$ and productivity shocks $d\log \boldsymbol{A}$, the first-order response in aggregate unemployment is:*

$$d\log U^{agg} = \Pi_{U^{agg},A}\, d\log \boldsymbol{A} + \Pi_{U^{agg},H}\, d\log \boldsymbol{H},$$

where

$$\begin{aligned}\Pi_{U^{agg},A} &= \frac{1}{U^{agg}}\boldsymbol{U}'\left[\Lambda_A - \mathcal{L}\Pi_{y,A}\right],\\ \Pi_{U^{agg},H} &= \frac{1}{U^{agg}}\boldsymbol{U}'\left[\boldsymbol{I} + \Lambda_H - \mathcal{L}\Pi_{y,H}\right].\end{aligned}$$

The first-order response in aggregate price is:

$$d\log P = \varepsilon_{\boldsymbol{c}}^{\mathcal{D}\prime}\, d\log\, \boldsymbol{p},$$

where

$$\begin{aligned}\left(\boldsymbol{I} - \Psi\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\mathcal{L}\right) d\log\, \boldsymbol{p} &= \Pi_{p,A}\, d\log \boldsymbol{A} + \Pi_{p,H}\, d\log \boldsymbol{H},\\ \Pi_{p,A} &= \Psi\left[\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left(\Lambda_A - \mathfrak{Q}\mathcal{T}\left[\mathcal{F} - \Xi_\theta\right]^{-1}(\mathcal{L}\Psi - \Lambda_A)\right) - \boldsymbol{I}\right],\\ \Pi_{p,H} &= \Psi\left[\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left(\Lambda_H - \mathfrak{Q}\mathcal{T}\left[\mathcal{F} - \Xi_\theta\right]^{-1}\left(\mathcal{L}\Psi\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I} - \Lambda_{\boldsymbol{H}}\right)\right)\right].\end{aligned}$$

The change in unemployment is a weighted average of the first-order changes in the size of the labor force and employment. The employment level change for each labor market in equilibrium equates the change in labor demand in that labor market to labor supply. The change in labor demand in that labor market depends on how relative wages respond to shocks, as well as how sectoral output changes for sectors that use that particular type of labor.

The first-order response in aggregate price level is more straightforward, and is simply the changes in sectoral prices weighted by their consumption share, which is equivalent to the demand elasticities of consumption when preferences are Cobb-Douglas. Note that one can solve out the price changes explicitly by picking a numeraire, for example, by assuming $d\log p_1 = 0$. However, we don't explicitly do it here in order to maintain a simpler expression for the relationship between the price system and productivity and labor supply shocks.

3.4. Deviation from Hulten's Theorem

In this section, we show that Theorem 1 and our general model setup nests the famous Hulten's Theorem [ADD CITATION]. We also show how our results deviate from Hulten's Theorem when search and matching frictions emerge. Let us consider the case where no search frictions are present, which is equivalent to assuming every unemployed person is able to get a job:

$$h_o = \phi_o m(U_o, V_o) = U_o.$$

In this case, job-finding rate $\mathcal{F}_o(\theta_o)$ becomes 1, which means $\boldsymbol{\mathcal{F}} = \mathbf{0}$. Since there are no hiring costs without matching frictions, firms do not need to hire any recruiters, implying $\tau_i = 0$. Together, this implies that $\boldsymbol{\mathcal{F}} + \boldsymbol{\mathfrak{Q}}\boldsymbol{\mathfrak{T}} = \mathbf{0}$. As a result, our aggregation formula becomes:

$$d\log Y = \lambda' d\log \boldsymbol{A} + \lambda' \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} d\log \boldsymbol{H}.$$

which illustrates that, when labor markets are efficient, our model nests the canonical result of Hulten's Theorem: the first-order impact on output of a technology shock to a sector is equal to that sector's Domar weight.

Therefore, comparing the aggregation result in an efficient production network with what we obtain in Theorem 1, it is easy to see that the additional impact from search and matching on the aggregate response of output to labor supply and technology shocks are:

$$\Pi_{\text{search},\boldsymbol{A}} = \lambda' \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left(\boldsymbol{\mathcal{F}} + \boldsymbol{\mathfrak{Q}}\boldsymbol{\mathfrak{T}}\right) \left[\boldsymbol{\mathcal{F}} - \Xi_\theta\right]^{-1} \left(\boldsymbol{\mathcal{L}}\boldsymbol{\Psi} - \boldsymbol{\Lambda}_{\boldsymbol{A}}\right),$$

$$\Pi_{\text{search},\boldsymbol{H}} = \lambda' \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} (\mathfrak{F} + \mathfrak{Q}\mathfrak{T}) \left[\mathfrak{F} - \Xi_\theta\right]^{-1} \left(\left[\mathcal{L}\Psi\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I}\right] - \Lambda_{\boldsymbol{H}}\right).$$

In general, putting a sign on these additional impacts is a quantitative question, and the direction of amplification versus dampening depends on how labor market tightness responds. Tightness, as an equilibrium object, responds to shocks in a complicated way that depends on how each sector uses different types of labor, production linkages, relative wage rigidity, and matching elasticities. In general, we leave this as a quantitative question that we answer in section 4. In subsection 3.5, we use two simple examples to demonstrate when amplification and dampening can occur.

3.5. Two Toy Examples

In this subsection, we provide two toy examples to demonstrate how the search channel impacts aggregation through the production network. Since it is now well understood how production linkages amplify sector level shocks, in subsubsection 3.5.1 we focus on the search and matching channel. This section explores how labor market frictions can amplify or dampen the impact of idiosyncratic shocks by assuming no production linkages. Subsection 3.5.2 presents a simple vertical production network model to showcase how production linkages interact with search frictions.

3.5.1. A Model with no Production Linkage

To get an intuition for how search and matching frictions can potentially impact the response of aggregate, we first examine what happens when there are no production linkages between the sectors. This is equivalent to assuming $\boldsymbol{\Omega} = \boldsymbol{0}$ and thus $\boldsymbol{\Psi} = \boldsymbol{I}$. Additionally, for simplicity, we assume each sector operates its own labor market, which means $\boldsymbol{L} = \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} = \boldsymbol{I}$. In other words, we consider the case where each sector produces using linear technology in its own unique type of labor. From Theorem 1, this gives rise to the following output aggregation formula:

$$d\log Y = \Pi_{\boldsymbol{A}} d\log \boldsymbol{A} + \Pi_{\boldsymbol{H}} d\log \boldsymbol{H},$$

where

$$\Pi_{\boldsymbol{A}} = \lambda' \left(\boldsymbol{I} + (\mathfrak{F} + \mathfrak{Q}\mathfrak{T}) \left[-\mathfrak{Q}\mathfrak{T}\right]^{-1} (\boldsymbol{I} - \Lambda_{\boldsymbol{A}})\right),$$
$$\Pi_{\boldsymbol{H}} = \lambda' \left(\boldsymbol{I} - (\mathfrak{F} + \mathfrak{Q}\mathfrak{T}) \left[-\mathfrak{Q}\mathfrak{T}\right]^{-1} \Lambda_{\boldsymbol{H}}\right).$$

Since $\mathfrak{F}$, $\mathfrak{Q}$, and $\mathfrak{T}$ are diagonal matrices, we have that:

$$(\mathfrak{F} + \mathfrak{Q}\mathfrak{T})\,[-\mathfrak{Q}\mathfrak{T}]^{-1} = \begin{pmatrix} \frac{1-\eta_1(1+\tau_1)}{\eta_1\tau_1} & 0 & \dots & 0 \\ 0 & \frac{1-\eta_2(1+\tau_2)}{\eta_2\tau_2} & \dots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & 0 & \dots & \frac{1-\eta_J(1+\tau_J)}{\eta_J\tau_J} \end{pmatrix}$$

Further, if we assume the relative wage in a sector respond only to shocks to that sector, we have that:

$$\boldsymbol{\Lambda_A} = \begin{pmatrix} \alpha_1 & 0 & \dots & 0 \\ 0 & \alpha_2 & \dots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & 0 & \dots & \alpha_J \end{pmatrix}, \quad \boldsymbol{\Lambda_H} = \begin{pmatrix} -\beta_1 & 0 & \dots & 0 \\ 0 & -\beta_2 & \dots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & 0 & \dots & -\beta_J \end{pmatrix}.$$

We assume relative wages respond positively to positively technology shocks, and negatively to labor supply shocks, that is $\alpha_i, \beta_i \geq 0$ for all i.

Therefore, the additional impact of technology shocks from search-and-matching becomes:

$$\boldsymbol{\Pi}_{\text{search},\boldsymbol{A}} = \begin{pmatrix} (1-\alpha_1)\frac{1-\eta_1(1+\tau_1)}{\eta_1\tau_1} & 0 & \dots & 0 \\ 0 & (1-\alpha_2)\frac{1-\eta_2(1+\tau_2)}{\eta_2\tau_2} & \dots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & 0 & \dots & (1-\alpha_J)\frac{1-\eta_J(1+\tau_J)}{\eta_J\tau_J} \end{pmatrix}.$$

Since labor market frictions operate through changes in productive employment, N_i, they cannot matter for shock propagation unless productive employment changes in response to the shocks. As a result, when $(1-\alpha_i)\frac{1-\eta_i(1+\tau_i)}{\eta_i\tau_i} = 0$ search-and-matching becomes irrelevant for productivity shock propagation. This occurs when $\alpha_i = 1$ or when $1 - \eta_i(1+\tau_i) = 0$. When $\alpha_i = 1$ the change in wages is equal to the change in the marginal product of sector i labor. Therefore, labor demand and employment remain unchanged. All of the effect of the productivity shock is absorbed into changes in wages rather than changes in employment.

$1-\eta_i(1+\tau_i)$ is the elasticity of the productive workforce in sector i to sector i vacancies. When $1-\eta_i(1+\tau_i) = 0$ posting additional vacancies does not change the number of productive workers. As a result, even if firms change the number of vacancy postings in response to productivity shocks, these changes do not have first order effects on output because they do not have first order effects on the number of productive workers.

In both cases, productive employment does not respond to changes in productivity and so search-and-matching frictions play no role in the propagation of shocks. Whenever employment does change following productivity shocks search-and-matching frictions matter.

For instance, if wages change by less than the marginal product of labor following productivity shocks, $\alpha_i < 1$, and increasing vacancy postings increases the number of productive workers, $1 < \eta_i(1+\tau_i)$, then employment will rise after a positive productivity shock. The resulting rise in employment boosts output further than the pure effect of the productivity shock alone. In this case, search-and-matching amplifies the effects of productivity shocks.

Similarly, when $(1-\alpha_i)\frac{1-\eta_i(1+\tau_i)}{\eta_i\tau_i} < 0$, search-and-matching dampens the aggregate effect of technology shocks in sector i. In this case, the effect cost of labor rises relative the marginal product of labor following a positive productivity shock. The resulting decline in employment dampens the output response.

In this simple example, with linear production in one unique type of labor per sector, the effect of productivity shocks on the marginal product of labor is clear. In our full model, the effects of productivity shocks in sector i on the marginal product of type o labor is a much more complicated object. However, the intuition remains the same. If wages adjust by less than the marginal product labor and posting additional vacancies increases the number of productive workers, then labor market frictions will amplify the effects of technology shocks. While in this example the effects remain isolated in the shocked industry i, with a full network of production linkages the amplification would propagate to sectors that use i's good as an input, and from those sectors to other sectors, and so on.

We can gain similar insights about the amplification of labor force shocks through the following expression:

$$\Pi_{\text{search},\boldsymbol{H}} = \begin{pmatrix} \beta_1\frac{1-\eta_1(1+\tau_1)}{\eta_1\tau_1} & 0 & \cdots & 0 \\ 0 & \beta_2\frac{1-\eta_2(1+\tau_2)}{\eta_2\tau_2} & \cdots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & 0 & \cdots & \beta_J\frac{1-\eta_J(1+\tau_J)}{\eta_J\tau_J} \end{pmatrix}.$$

Again, in order for labor market frictions to change the way labor force shocks propagate, productive employment must change in response to the shocks. The interpretation of the $1-\eta_i(1+\tau_i)$ term is the same as above, so we focus on the β_i's. Whenever $\beta_i = 0$ wages do not respond to labor force shocks. As a result, labor demand and, therefore, productive employment do not change.

As long as increasing the number of vacancy postings increases the number of productive workers, if labor becomes cheaper when there is a positive shock to the size of the labor force ($\beta_i > 0$), employment increases in response to the labor force shock. This increase boosts output, amplifying the effect of the labor fore shock. Conversely, if labor becomes more expensive when there is a positive shock to the labor force ($\beta_i < 0$), employment falls in response to the labor force shock. This reduces output, dampening the effect of the labor force shock. Again, this intuition carries through to our full model.

3.5.2. A Two-Sector Vertical Production Network

The example above unpacks how search-and-matching can amplify or dampen shocks through wage responsiveness and the effect of increased vacancy postings on the number of productive workers. It doesn't, however, speak to how labor market propagation compounds with production linkages.

Consider an economy with two sectors, a downstream sector and an upstream sector, that produce using the same type of labor:

$$\begin{aligned} y_1 &= A_1 N_{11} \\ y_2 &= A_2\, y_1^{\gamma} N_{21}^{1-\gamma} \end{aligned}$$

With this setup, we have that:

$$\begin{aligned} \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} &= \begin{bmatrix} 1 \\ 1-\gamma \end{bmatrix}, \\ \boldsymbol{\Omega} &= \begin{bmatrix} 0 & 0 \\ 1-\gamma & 0 \end{bmatrix}, \boldsymbol{\Psi} = \begin{bmatrix} 1 & 0 \\ 1-\gamma & 1 \end{bmatrix} \\ \mathcal{F} &= 1-\eta, Q = -\eta, \\ \mathcal{L} &= \begin{bmatrix} \gamma & 1-\gamma \end{bmatrix}, \end{aligned}$$

$$\varepsilon_{\boldsymbol{c}}^{\mathcal{D}} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$\lambda = \begin{bmatrix} 1-\gamma \\ 1 \end{bmatrix}.$$

This implies that:

$$(\mathcal{F} + \mathfrak{Q}\mathcal{T}) \left[\mathcal{F} - \Xi_\theta\right]^{-1} = \frac{1-\eta(1+\tau)}{1-\eta-\left[\gamma+2(1-\gamma)^2\right](1-\eta(1+\tau))},$$

First, we see that the recruiter-producer ratio that we uncover in subsubsection 3.5.1 is still at play in the numerator, that is, with a large enough recruiter-producer ratio, the direction of the impact of search and matching can be reversed. In addition, in the denominator, we now see that the production structure enters the search-and-matching part of output aggregation. Specifically, the production structure dictates how sectors use labor input from different labor markets, as well as how labor demand across sectors comove. However, the direction through which the production structure impacts output aggregation is ambiguous even in this simple example. For example, with $\gamma = 0$, the denominator becomes $\eta(1+2\tau)-1$, which is negative for reasonable values of η and τ. On the other hand, with $\gamma = 0.75$, the denominator becomes $1-\eta-0.875(1-\eta(1+\tau)) > 0$.

We further assume that

$$d\log w - \mathcal{L} d\log \boldsymbol{p} = \begin{bmatrix} \alpha_1 & \alpha_2 \end{bmatrix} \begin{bmatrix} d\log A_1 \\ d\log A_2 \end{bmatrix} + \beta_1 d\log H.$$

Applying Theorem 1, we have that:

$$\Pi_{\text{search},\boldsymbol{A}} = \frac{2(1-\gamma)(1-\eta(1+\tau))}{1-\eta-\left[\gamma+2(1-\gamma)^2\right](1-\eta(1+\tau))} \begin{bmatrix} 1-\gamma+\gamma^2-\alpha_1 & 1-\gamma-\alpha_2 \end{bmatrix},$$

$$\Pi_{\text{search},\boldsymbol{H}} = \frac{2(1-\gamma)(1-\eta(1+\tau))}{1-\eta-\left[\gamma+2(1-\gamma)^2\right](1-\eta(1+\tau))} \left(1-3\gamma+2\gamma^2-\beta_1\right).$$

From this, we can see that the wage rigidity channel is still important in determining the direction and the magnitude of impacts of shock on output aggregation, with the additional insight that we are now comparing wage adjustments with network-adjusted values.

Additionally, we can examine how search frictions impact output for each of the sectors. Let $\Pi_{y,\text{search},X}$ denote the impact of search frictions on sectoral output from shocks to X, with X being either productivity or labor supply. We have that:

$$\Pi_{y,\text{search},A} = \frac{1-\eta(1+\tau)}{1-\eta-\left[\gamma+2(1-\gamma)^2\right](1-\eta(1+\tau))}\begin{bmatrix} 1-\gamma+\gamma^2-\alpha_1 & 1-\gamma-\alpha_2 \\ 2(1-\gamma)(1-\gamma+\gamma^2-\alpha_1) & 2(1-\gamma)(1-\gamma-\alpha_2) \end{bmatrix}.$$

Compare to the efficient benchmark, in which the sectoral output response follows the Leontief inverse $\Psi = \begin{bmatrix} 1 & 0 \\ 1-\gamma & 1 \end{bmatrix}$, where the upper right element is 0, the sectoral output response to changes in labor market has a non-zero upper right element. What this means is that, in efficient production networks with Cobb-Douglas production function, shocks to a downstream sector does not impact production output of upstream sector. While this is usually resolved with the addition of preference shocks or CES production functions, we introduce an additional way of resolving this pitfall of classical production network models by incorporating frictional labor markets.

4. Calibration

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This section outlines our data requirements and suggested estimation strategy. The empirical statistics we wish to estimate is the mismatch index, given by equation **??**. To compute our mismatch index, we need to compute the optimal number of hires, which is pinned down by the optimal allocation of sectoral unemployment, as shown in equation **??**. Despite our results are generalized for arbitrary production networks, we can conveniently bypass the structure of the networks by looking at the observable sales shares. Some other key economic variables includes vacancies, hires, unemployment, and matching parameters.

For this iteration of the project, we are computing the network-adjusted mismatch index for the same period as Şahin et al. (2014), as many sectoral labor market statistics are readily available from their online appendix. We augment the data set with sector-level data on production linkages from the 2007 input-output tables published by the Bureau of Economic Analysis (BEA). We will extend the period and the scope of our analysis in future iterations.

4.1. One occupation per sector

4.1.1. Data Sources

Vacancies and Hires. We use vacancy and hire data from The Job Openings and Labor Turnover Survey (JOLTs), which provides survey-based measures of job openings and hires at a monthly frequency. This survey data is available for 17 industries that roughly correspond to the two-digit NAICS classification. The data covers 2001 to 2011, at a monthly and a yearly frequency.

Unemployment. We calculate unemployment using the Current Population Survey (CPS). We use the same industry classification as JOLTs. The data covers 2001 to 2011, at a monthly and a yearly frequency.

Matching Functions. We use $\eta = 0.5$ for the vacancy share in matching functions. This value is roughly in the middle of a range of literature estimates (**?**). We use the heterogeneous matching efficiency estimated by Şahin et al. (2014), which follows the methodology proposed by **?**.

Input-output Linkages. Although JOLTs include labor market data at the level that corresponds roughly to the two-digit NAICS classification, this correspondence is not exact. To construct input intensity that matches with JOLTs' data availability level, we use the 2007 6-digit NAICS classification level table, and aggregate back up to match the JOLTs industries. Assuming constant-return-to-scale, we calculate the input intensity of each input sector as its share of the consumer sector's labor-inclusive total expenditure. Note that even though our results doesn't require examining the explicit structure of the U.S production networks, we still compute the input-intensity for the future when we deviate from Cobb-Douglas, or look at impacts of idiosyncratic unemployment shocks.

4.2. Separate Occupations

4.2.1. Data Sources

5. Applications

5.1. Shifts in Sectoral Beveridge Curves

5.2. Explaining Inflation with Oil Price Shocks

6. Conclusion

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Appendix A. Proofs

A.1. Proof for Proposition 1

A.2. Proof for Theorem Theorem 1

Let $\lambda_i = \frac{p_i y_i}{G} =$, where $G = \sum_j p_j c_j = GDP = \sum_{o=1}^{\mathcal{O}} w_o L_o$, denote the final sales share of GDP for sector i. We have that:

$$p_j x_{ij} = \varepsilon^{f_i}{}_{x_{ij}} \lambda_i G.$$

From household's maximization problem, I have that $p_i c_i = \varepsilon^{\mathcal{D}}_{c_i} G$. Combining the two gives:

$$\begin{aligned}
&\varepsilon^{\mathcal{D}}_{c_j} G = p_j y_j = p_j(c_j + \sum_i x_{ij}) = \left(\varepsilon^{\mathcal{D}}_{c_j} + \sum_i \varepsilon^{f_i}{}_{x_{ij}} \lambda_i\right) G \\
&\Rightarrow \boldsymbol{\lambda} = \boldsymbol{\Psi}' \varepsilon^{\boldsymbol{D}}_{\boldsymbol{c}}.
\end{aligned}$$

The aggregate labor force, employment, and unemployment are $H^{agg} = \sum_{o=1}^{\mathcal{O}} H_o$, $L^{agg} = \sum_{o=1}^{\mathcal{O}} L_o$, and $U^{agg} = \sum_{o=1}^{\mathcal{O}} U_o$. Changes in aggregates are therefore given by

$$\begin{aligned}
dH^{agg} &= \sum_{o=1}^{\mathcal{O}} dH_o \\
dL^{agg} &= \sum_{o=1}^{\mathcal{O}} dL_o \\
dU^{agg} &= \sum_{o=1}^{\mathcal{O}} dU_o
\end{aligned}$$

Or in terms of log changes

$$\begin{aligned}
d\log H^{agg} &= \frac{1}{H^{agg}} \sum_{o=1}^{\mathcal{O}} H_o d\log H_o \\
d\log L^{agg} &= \frac{1}{L^{agg}} \sum_{o=1}^{\mathcal{O}} L_o d\log L_o
\end{aligned}$$

$$d\log U^{agg} = \frac{1}{U^{agg}} \sum_{o=1}^{\mathcal{O}} U_o d\log U_o$$

In matrix notation

$$\begin{aligned}
d\log H^{agg} &= \frac{1}{H^{agg}} \boldsymbol{H}' d\log \boldsymbol{H} \\
d\log L^{agg} &= \frac{1}{L^{agg}} \boldsymbol{L}' d\log \boldsymbol{L} \\
d\log U^{agg} &= \frac{1}{U^{agg}} \boldsymbol{U}' d\log \boldsymbol{U}
\end{aligned}$$

Substituting in for $d\log \boldsymbol{L}$

$$\begin{aligned}
d\log L^{agg} &= \frac{1}{L^{agg}} \boldsymbol{L'} \left[\mathcal{L} d\log \boldsymbol{y} - \left[d\log \boldsymbol{w} - \mathcal{L} d\log \boldsymbol{p} \right] \right] \\
&= \frac{1}{L^{agg}} \boldsymbol{L'} \left[\mathcal{L} \left[\boldsymbol{\Pi}_{y,A} d\log \boldsymbol{A} + \boldsymbol{\Pi}_{y,H} d\log \boldsymbol{H} - \boldsymbol{\Lambda}_A d\log \boldsymbol{A} - \boldsymbol{\Lambda}_H d\log \boldsymbol{H} \right] \right] \\
&= \boldsymbol{\Pi}_{L^{agg},A} d\log \boldsymbol{A} + \boldsymbol{\Pi}_{L^{agg},H} d\log \boldsymbol{H}
\end{aligned}$$

Where

$$\begin{aligned}
\boldsymbol{\Pi}_{L^{agg},A} &= \frac{1}{L^{agg}} \boldsymbol{L}' \left[\mathcal{L} \boldsymbol{\Pi}_{y,A} - \boldsymbol{\Lambda}_A \right] \\
\boldsymbol{\Pi}_{L^{agg},H} &= \frac{1}{L^{agg}} \boldsymbol{L}' \left[\mathcal{L} \boldsymbol{\Pi}_{y,H} - \boldsymbol{\Lambda}_H \right]
\end{aligned}$$

And

$$\begin{aligned}
d\log U^{agg} &= \frac{1}{U^{agg}} \boldsymbol{U}' \left[d\log \boldsymbol{H} - d\log \boldsymbol{L} \right] \\
&= \frac{1}{U^{agg}} \boldsymbol{U}' \left[d\log \boldsymbol{H} - \left[\mathcal{L} \left[\boldsymbol{\Pi}_{y,A} d\log \boldsymbol{A} + \boldsymbol{\Pi}_{y,H} d\log \boldsymbol{H} - \boldsymbol{\Lambda}_A d\log \boldsymbol{A} - \boldsymbol{\Lambda}_H d\log \boldsymbol{H} \right] \right] \right] \\
&= \boldsymbol{\Pi}_{U^{agg},A} d\log \boldsymbol{A} + \boldsymbol{\Pi}_{U^{agg},H} d\log \boldsymbol{H}
\end{aligned}$$

Where

$$\begin{aligned}
\boldsymbol{\Pi}_{U^{agg},A} &= \frac{1}{U^{agg}} \boldsymbol{U}' \left[\boldsymbol{\Lambda}_A - \mathcal{L} \boldsymbol{\Pi}_{y,A} \right] \\
\boldsymbol{\Pi}_{U^{agg},H} &= \frac{1}{U^{agg}} \boldsymbol{U}' \left[\boldsymbol{I} + \boldsymbol{\Lambda}_H - \mathcal{L} \boldsymbol{\Pi}_{y,H} \right]
\end{aligned}$$

Appendix B. Results for general CRTS production functions and one occupation per sector

In this section we generalize our results to any constant returns to scale production function, under the assumption that there is one type of labor per sector. This generalization results in additional terms that capture how the production elasticities change when shocks hit the economy. The expressions are otherwise similar to above. The model setup is identical, we just do not impose Cobb-Douglas technology and instead impose $\mathcal{O} = J$,

B.1.

Appendix C. Data and Calibration Details

This appendix describes our data in greater detail.

C.1. One occupation per sector.

We use the 3-digit 2021 BEA Make and Use tables accessible at https://www.bea.gov/industry/input-output-accounts-data to calculate the relevant production elasticities[3]. The 3-digit Make and Use tables record the nominal amount of each 71 commodities made by and used by each of 71 industries. The commodities are denoted using the same codes as the industries, but they are conceptually distinct as each industry can produce more than one commodity. Let M_{ij} denote the nominal value of commodity i made by industry j. Let U_{ij} denote the nominal amount of commodity i used by industry j. The two tables below demonstrate the elements of the Make and Use tables.

[3] See https://www.bea.gov/sites/default/files/methodologies/IOmanual_092906.pdf for a detailed description of how these tables are generated.

	Sector 1	Sector 2	$\cdots$	Sector J	Total Industry Output
Sector 1	M_{11}	M_{21}	$\cdots$	M_{J1}	$\sum_{i=1}^{J} M_{i1}$
Sector 2	M_{12}	M_{22}	$\cdots$	M_{J2}	$\sum_{i=1}^{J} M_{i2}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
Sector J	M_{1J}	M_{2J}	$\cdots$	M_{JJ}	$\sum_{i=1}^{J} M_{iJ}$
Total Commodity Output	$\sum_{j=1}^{J} M_{1j}$	$\sum_{j=1}^{J} M_{2j}$	$\cdots$	$\sum_{j=1}^{J} M_{Jj}$	—

TABLE A1. Make table

	Sector 1	Sector 2	$\cdots$	Sector J	Total Intermediate Uses	Total Final Uses
Sector 1	U_{11}	U_{12}	$\cdots$	U_{1J}	$\sum_{j=1}^{J} U_{1j}$	$\sum_{j=1}^{J} U_{1j} + p_1 c_1$
Sector 2	U_{21}	U_{22}	$\cdots$	U_{2J}	$\sum_{j=1}^{J} U_{2j}$	$\sum_{j=1}^{J} U_{2j} + p_2 c_2$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
Sector J	U_{J1}	U_{J2}	$\cdots$	U_{JJ}	$\sum_{j=1}^{J} U_{Jj}$	$\sum_{j=1}^{J} U_{Jj} + p_J c_J$
Total Intermediate Inputs	$\sum_{i=1}^{J} U_{i1}$	$\sum_{i=1}^{J} U_{i2}$	$\cdots$	$\sum_{i=1}^{J} U_{iJ}$	—	—
Total industry output	$\sum_{i=1}^{J} U_{i1} + w_1(1+\tau_1)N_1$	$\sum_{i=1}^{J} U_{i2} + w_2(1+\tau_2)N_2$	$\cdots$	$\sum_{i=1}^{J} U_{iJ} + w_J(1+\tau_J)N_J$	—	—

TABLE A2. Use table

First, we calculate the fraction of commodity i produced by industry j by dividing the entry in along each row by the corresponding "total industry output"

$$m_{ij} = \frac{M_{ij}}{\sum_{j=1}^{J} M_{ji}}$$

Second, we calculate the share of commodity i in industry j's total uses as by dividing each entry in the column corresponding to industry j by the corresponding "Total industry output"

$$u_{ij} = \frac{U_{ij}}{\sum_{i=1}^{J} U_{ij} + w_j(1+\tau_j)N_j}$$

We form the two matrices

$$\boldsymbol{M} = \begin{bmatrix} m_{11} & m_{21} & \cdots & m_{J1} \\ m_{12} & m_{22} & \cdots & m_{J2} \\ \vdots & \vdots & \ddots & \vdots \\ m_{1J} & m_{2J} & \cdots & m_{JJ} \end{bmatrix}, \boldsymbol{U} = \begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1J} \\ u_{21} & u_{22} & \cdots & u_{2J} \\ \vdots & \vdots & \ddots & \vdots \\ u_{J1} & u_{J2} & \cdots & u_{JJ} \end{bmatrix}$$

Then, we can calculate our input output matrix by

$$\boldsymbol{\Omega} = [\boldsymbol{MU}]'$$

Given our assumption of constant returns to scale and zero profits, the difference between total intermediate inputs and total industry output is the nominal income paid to workers in each sector. We abstract from the other components of total industry output in the IO accounts, taxes and gross operating surplus, as they have no model counterpart in our setup. We can therefore calculate the labor elasticities from the Use table as "Total industry output" – "Total intermediate inputs" ÷ "Total industry output."

$$\varepsilon_{N_j}^{f_j} = \frac{w_j(1+\tau_j)N_j}{\sum_{i=1}^{J} U_{ij} + w_j(1+\tau_j)N_j}$$

Finally, we can back out the demand elasticities from "Total intermediate uses" and "Total final uses" columns of the Uses table.

$$p_i c_i = \sum_{j=1}^{J} U_{ij} + p_i c_i - \sum_{j=1}^{J} U_{ij}$$

We can the work out the elasticities by

$$\varepsilon_{c_i}^{\mathcal{D}} = \frac{p_i c_i}{\sum_{i=1}^{J} p_i c_i}$$

We report the resulting estimates of the production elasticities, labor elasticities, and demand elasticities in the tables below.