Unemployment in a Production Network

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Motivation

Motivation

Two points of departure

1. Mismatch literature, e.g. [Şahin et al., 2014]
   - Linear production independent sectors, doesn't capture spillovers from moving people
2. Production networks literature
   -

Research Question

1. How does sector level output $d \log \boldsymbol{y}$ and unemployment $d \log \boldsymbol{u}$ respond to shocks to productivity $d \log \boldsymbol{A}$ and to the size of labor force $d \log \boldsymbol{H}$?
2. How do sector level responses aggregate to real GDP $d \log Y$ and unemployment $d \log U$?

This requires:

- $d \log \boldsymbol{p}$: How sector level prices change.
- $d \log \boldsymbol{w}$: How sector level wages change.
- $d \log \boldsymbol{\theta}$: How sector level tightnesses change.

Our model

Sectors

J sectors indexed by i produce with CRTS technology:

$$y_i = A_i f_i \left(N_i, \{x_{ij}\}_{j=1}^{J}\right)$$

Firms maximize profits:

$$\max_{N_i, \{x_{ij}\}_{j=1}^{J}} p_i f_i \left(N_i, \{x_{ij}\}_{j=1}^{J}\right) - w_i(1+\tau_i(\theta_i))N_i - \sum_{j=1}^{J} p_j x_{ij}$$

This requires:

$$p_i f_{i,x_{ij}} = p_j \qquad \text{(FOC } x_{ij}\text{)}$$

$$p_i f_{i,N_i} = w_i \left(1+\tau_i(\theta_i)\right) \qquad \text{(FOC } N_i\text{)}$$

Labor Markets

Each sector has separate labor market, with new hires governed by matching technology m:

$$h_i = \phi_i m\left(u_i, v_i\right)$$

We assume balanced flows which implies:

$$L_i^s(\theta_i) = \frac{\mathcal{F}_i(\theta_i)}{s_i + \mathcal{F}_i(\theta_i)} H_i$$
$$\tau_i(\theta_i) = \frac{r_i s_i}{\mathcal{Q}_i(\theta_i) - r_i s_i}$$

Households

A final goods producer aggregates output by CRTS technology:

$$Y = \max_{\{c_i\}_{i=1}^J} \mathcal{D}\left(\{c_i\}_{i=1}^J\right)$$

subject to

$$\sum_{i=1}^J p_i c_i = \sum_{i=1}^J \left[w_i L_i + \pi_i\right].$$

Equilibrium Conditions (Goods market)

Firms:

$$\varepsilon^{f_i}_{x_{ij}} = \frac{p_j x_{ij}}{p_i y_i} \qquad \text{(Intermediate input decision)}$$

$$\varepsilon^{f_i}_{N_i} = (1 + \tau_i(\theta_i)) \frac{w_i N_i}{p_i y_i} \qquad \text{(Labor input decision)}$$

Households:

$$\varepsilon^{\mathcal{D}}_{c_i} = \frac{p_i c_i}{\sum_{k=1}^{J} p_k c_k} \qquad \text{(Consumption decision)}$$

Market clearing:

$$y_i = c_i + \sum_{j=1}^{J} x_{ji} \qquad \text{(Goods market clearing)}$$

Equilibrium conditions (Labor market)

Labor market:

$$L_i^d = \varepsilon_{N_i}^{f_i} \frac{p_i y_i}{w_i} \qquad \text{(Labor demand)}$$

$$L_i^s = \frac{\mathcal{F}_i(\theta_i)}{s_i + \mathcal{F}_i(\theta_i)} H_i \qquad \text{(Labor supply)}$$

Market clearing:

$$L_i^s = L_i^d \qquad \text{(Labor market equilibrium)}$$

Production network notation

$$\lambda_i = \frac{p_i y_i}{\sum_j p_j c_j} \qquad \text{(Sales share)}$$

$$\boldsymbol{\Omega} = \begin{bmatrix} \varepsilon^{f_1}_{x_{11}} & \varepsilon^{f_1}_{x_{12}} & \cdots & \varepsilon^{f_1}_{x_{1J}} \\ \varepsilon^{f_2}_{x_{21}} & \varepsilon^{f_2}_{x_{22}} & \cdots & \varepsilon^{f_2}_{x_{2J}} \\ \vdots & \vdots & \ddots & \vdots \\ \varepsilon^{f_J}_{x_{J1}} & \varepsilon^{f_J}_{x_{J2}} & \cdots & \varepsilon^{f_J}_{x_{JJ}} \end{bmatrix} \qquad \text{(Input-output matrix)}$$

$$\boldsymbol{\Psi} = (\boldsymbol{I} - \boldsymbol{\Omega})^{-1} \qquad \text{(Leontief inverse)}$$

$$\boldsymbol{\varepsilon}^{\boldsymbol{f}}_{\boldsymbol{N}} = \begin{bmatrix} \varepsilon^{f_1}_{N_1} & \cdots & \varepsilon^{f_J}_{N_J} \end{bmatrix}', \ \boldsymbol{\varepsilon}^{\boldsymbol{\mathfrak{Q}}}_{\boldsymbol{\theta}} = \begin{bmatrix} \varepsilon^{\mathfrak{Q}_1}_{\theta_1} & \cdots & \varepsilon^{\mathfrak{Q}_J}_{\theta_J} \end{bmatrix}'$$

$$\boldsymbol{\tau}(\boldsymbol{\theta}) = \begin{bmatrix} \tau_1(\theta_1) & \cdots & \tau_J(\theta_J) \end{bmatrix}', \ \boldsymbol{\mathfrak{F}} = \begin{bmatrix} \frac{s_1}{s_1 + \mathfrak{F}_1(\theta_1)} \varepsilon^{\mathfrak{F}_1}_{\theta_1} & \cdots & \frac{s_J}{s_J + \mathfrak{F}_J(\theta_J)} \varepsilon^{\mathfrak{F}_J}_{\theta_J} \end{bmatrix}'$$

Theoretical Results

Our goal

Express $d \log Y$, where

$$d \log Y = \varepsilon_{\boldsymbol{c}}^{\mathcal{D}\prime} d \log \boldsymbol{c}$$

in terms of shocks $d \log \boldsymbol{A}$ and $d \log \boldsymbol{H}$ and primitives

This requires:

- $d \log \boldsymbol{p}$
- $d \log \boldsymbol{w}$
- $d \log \boldsymbol{\theta}$
- $d \log \boldsymbol{y}$

Price propagation

From the production function and the input decisions, sector level log price changes satisfy

$$
\begin{aligned}
d\log \boldsymbol{p} = \underbrace{\boldsymbol{\Psi}}_{\text{production linkages}} \Big(& \underbrace{\operatorname{diag}\left(\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\right) d\log \boldsymbol{w}}_{\text{factor prices}} \\
& - \underbrace{\left(\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\right) \operatorname{diag}\left(\boldsymbol{\tau}\right) \operatorname{diag}\left(\varepsilon_{\boldsymbol{\theta}}^{\Omega}\right) d\log \boldsymbol{\theta}}_{\text{searching and matching}} \\
& - \underbrace{d\log \boldsymbol{A}}_{\text{productivity}} \Big) \qquad \text{(Pricing Equation)}
\end{aligned}
$$

Sales shares

From goods market clearing, changes in sales shares satisfy

$$d\boldsymbol{\lambda} = \left[d\varepsilon_{\boldsymbol{c}}^{\mathcal{D}\prime} + \boldsymbol{\lambda}' d\boldsymbol{\Omega}\right] \boldsymbol{\Psi} \qquad \text{(Sales Shares)}$$

If production technology and utility are Cobb-Douglas, sales shares satisfy

$$d\boldsymbol{\lambda} = \mathbf{0}$$

Sector level output

With sales shares and prices, we can back out sector level log output changes:

$$d \log \boldsymbol{y} = -d \log \boldsymbol{p} + d \log \boldsymbol{\lambda} + \boldsymbol{\Xi} d \log \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}, \qquad \text{(Sector Level Output)}$$

where $\boldsymbol{\Xi} = \left(\boldsymbol{I} - \operatorname{diag}\left(\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}\right) \boldsymbol{\Psi}\right)^{-1} \operatorname{diag}\left(\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}\right)$.

If production technology is Cobb-Douglas

$$d \log \boldsymbol{y} = -d \log \boldsymbol{p}$$

Sector level tightness

Given shocks $d\log \boldsymbol{A}$ and $d\log \boldsymbol{H}$, and a general wage schedule $\boldsymbol{w}(\boldsymbol{\theta},\boldsymbol{A})$, labor market clearing implies

$$\overbrace{\operatorname{diag}(\mathcal{F})\, d\log \boldsymbol{\theta} + d\log \boldsymbol{H}}^{d\log \boldsymbol{L}^s} = \overbrace{-d\log \boldsymbol{w}(\boldsymbol{\theta},\boldsymbol{A}) + d\log \boldsymbol{\lambda}(\boldsymbol{\theta},\boldsymbol{A})}^{d\log L^d}$$
$$+ (\boldsymbol{I} + \Xi)\, d\log \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}(\boldsymbol{\theta},\boldsymbol{A})$$

If the labor schedule $\boldsymbol{w}$ is differentiable in $\boldsymbol{A}$ and $\boldsymbol{\theta}$

$$d\log \boldsymbol{\theta} = \left(\operatorname{diag}(\mathcal{F}) - \varepsilon^{\boldsymbol{L^d}}_{\boldsymbol{\theta}}\right)^{-1} \varepsilon^{\boldsymbol{L^d}}_{\boldsymbol{A}}\, d\log \boldsymbol{A} - \left(\operatorname{diag}(\mathcal{F}) - \varepsilon^{\boldsymbol{L^d}}_{\boldsymbol{\theta}}\right)^{-1} d\log \boldsymbol{H}$$

Sector level tightness - Cobb-Douglas

If production technology is Cobb-Douglas $\boldsymbol{\varepsilon}_{\boldsymbol{\theta}}^{\boldsymbol{L}^d} = \boldsymbol{\varepsilon}_{\boldsymbol{\theta}}^{\boldsymbol{w}}$ and $\boldsymbol{\varepsilon}_{\boldsymbol{A}}^{\boldsymbol{L}^d} = \boldsymbol{\varepsilon}_{\boldsymbol{A}}^{\boldsymbol{w}}$

$$d \log \boldsymbol{\theta} = \left(\operatorname{diag}(\mathcal{F}) - \boldsymbol{\varepsilon}_{\boldsymbol{\theta}}^{\boldsymbol{w}}\right)^{-1} \boldsymbol{\varepsilon}_{\boldsymbol{A}}^{\boldsymbol{w}} d \log \boldsymbol{A} - \left(\operatorname{diag}(\mathcal{F}) - \boldsymbol{\varepsilon}_{\boldsymbol{\theta}}^{\boldsymbol{w}}\right)^{-1} d \log \boldsymbol{H}$$

Assuming Cobb-Douglas production matrices of wage elasticities $\boldsymbol{\varepsilon}_{\boldsymbol{\theta}}^{\boldsymbol{w}}$, $\boldsymbol{\varepsilon}_{\boldsymbol{A}}^{\boldsymbol{w}}$ are sufficient statistics for changes in tightness.

In this case, if wages are rigid, changes in tightness are not related to production linkages. This is a special property for Cobb-Douglas production functions, since no expenditure switching occurs.

With CES, however, network linkages will still come into play through $d \log \boldsymbol{\lambda}$ and $d \log \boldsymbol{\varepsilon}_{\boldsymbol{N}}^{\boldsymbol{f}}$

Summary

We now have a system of sector-level prices, quantities, and tightness:

$$d\log \boldsymbol{p} = \boldsymbol{\Psi}\left(\operatorname{diag}\left(\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}\right)\left(d\log \boldsymbol{w} - \operatorname{diag}\left(\boldsymbol{\tau}\right)\operatorname{diag}\left(\varepsilon^{\Omega}_{\boldsymbol{\theta}}\right)d\log\boldsymbol{\theta}\right) - d\log \boldsymbol{A}\right) \quad \text{(pricing)}$$

$$d\boldsymbol{\lambda} = \left[d\varepsilon^{\mathcal{D}\prime}_{\boldsymbol{c}} + \boldsymbol{\lambda}' d\boldsymbol{\Omega}\right]\boldsymbol{\Psi} \quad \text{(sales shares)}$$

$$d\log\boldsymbol{\theta} = \left(\operatorname{diag}\left(\mathcal{F}\right) - \varepsilon^{\boldsymbol{L}^d}_{\boldsymbol{\theta}}\right)^{-1}\varepsilon^{\boldsymbol{L}^d}_{\boldsymbol{A}} d\log\boldsymbol{A} - \left(\operatorname{diag}\left(\mathcal{F}\right) - \varepsilon^{\boldsymbol{L}^d}_{\boldsymbol{\theta}}\right)^{-1} d\log\boldsymbol{H} \quad \text{(tightness)}$$

$$d\log\boldsymbol{y} = -d\log\boldsymbol{p} + d\log\boldsymbol{\lambda} + \Xi d\log\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}, \quad \text{(sector-level output)}$$

with assumptions on wage schedules determining $d\log \boldsymbol{w}$

General aggregation formula

$$
\begin{aligned}
d \log Y &= d \log \sum_i p_i c_i - \sum_i \frac{p_i c_i}{\sum_j p_j c_j} d \log p_i \\
&= \varepsilon_{\boldsymbol{c}}^{\mathcal{D}\prime} d \log \boldsymbol{c} \\
&= \boldsymbol{\varepsilon}_{\boldsymbol{c}}^{\mathcal{D}\prime} \left(d \log \varepsilon_{\boldsymbol{c}}^{\mathcal{D}} + d \log \boldsymbol{y} - d \log \boldsymbol{\lambda} \right)
\end{aligned}
$$

Aggregation under Cobb-Douglas

With Cobb-Douglas utility and production function,

$$
\begin{aligned}
d\log Y &= \boldsymbol{\varepsilon}_{\boldsymbol{c}}^{\mathcal{D}'}\left(d\log \boldsymbol{y}\right) \\
&= -\boldsymbol{\varepsilon}_{\boldsymbol{c}}^{\mathcal{D}'}\left[\boldsymbol{\Psi}(\operatorname{diag}\left(\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\right)\left(d\log \boldsymbol{w} - \operatorname{diag}\left(\boldsymbol{\tau}\right)\operatorname{diag}\left(\varepsilon_{\boldsymbol{\theta}}^{\Omega}\right) d\log \boldsymbol{\theta}\right) - d\log \boldsymbol{A})\right]
\end{aligned}
$$

where

$$
d\log \boldsymbol{\theta} = \left(\operatorname{diag}\left(\mathcal{F}\right) - \varepsilon_{\boldsymbol{\theta}}^{\boldsymbol{w}}\right)^{-1}\varepsilon_{\boldsymbol{A}}^{\boldsymbol{w}} d\log \boldsymbol{A} - \left(\operatorname{diag}\left(\mathcal{F}\right) - \varepsilon_{\boldsymbol{\theta}}^{\boldsymbol{w}}\right)^{-1} d\log \boldsymbol{H}
$$

With fixed wages and labor force participation, this reduces to Hulten's theorem:

$$
d\log Y = \varepsilon_{\boldsymbol{c}}^{\mathcal{D}'}\boldsymbol{\Psi}\, d\log \boldsymbol{A}
$$

Applications: Cobb-Douglas

Calibration: Labor Market

Cobb-Douglas matching technology

$$h_i = \phi_i U_i^{\eta} V_i^{1-\eta}$$

set

- $\eta = 0.5$
- $r_i = 0.1$ for all i
- $s_i = 0.03$ for all i
- ϕ_i from [Şahin et al., 2014]

Calibration: Product Market

Input output tables from [fill in]

- We use the 2007 6-digit NAICS classification level table, and aggregate back up to match the JOLTs industries

Response of output and unemployment to technology shocks

What is the effect of a -1% productivity shock in all sectors?

Set $d\log \boldsymbol{H} = \mathbf{0}$, $d\log \boldsymbol{w} = \mathbf{0}$ (Nominal wages are constant)

- $\varepsilon_{\boldsymbol{\theta}}^{\boldsymbol{w}} = \varepsilon_{\boldsymbol{A}}^{\boldsymbol{w}} = \mathbf{0} \Rightarrow$
  - $d\log \boldsymbol{L} = d\log \boldsymbol{U} = 0$
  - $d\log Y = \varepsilon_{\boldsymbol{c}}^{\mathcal{D}'}\Psi d\log \boldsymbol{A}$ (Standard networks result)

Suppose $d\log \boldsymbol{H} = \mathbf{0}$, $d\log \boldsymbol{w} = d\log P$ (Real wages are constant)

- $\varepsilon_{\boldsymbol{\theta}}^{\boldsymbol{w}} =$
  $-\left(\boldsymbol{I} - \boldsymbol{e}\varepsilon_{\boldsymbol{c}}^{\boldsymbol{D}'}\Psi \mathrm{diag}\left(\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\right)\right)^{-1} \boldsymbol{e}\varepsilon_{\boldsymbol{c}}^{\boldsymbol{D}'}\Psi \mathrm{diag}\left(\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\right)\mathrm{diag}\left(\boldsymbol{\tau}\right)\mathrm{diag}\left(\varepsilon_{\boldsymbol{\theta}}^{\Omega}\right)$
- $\varepsilon_{\boldsymbol{A}}^{\boldsymbol{\theta}} = -\left(\boldsymbol{I} - \boldsymbol{e}\varepsilon_{\boldsymbol{c}}^{\boldsymbol{D}'}\Psi \mathrm{diag}\left(\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\right)\right)^{-1} \boldsymbol{e}\varepsilon_{\boldsymbol{c}}^{\boldsymbol{D}'}\Psi$

Response of output and unemployment to shocks to the size of the labor force

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