Unemployment in a Production Network

Finn Schüle and Haoyu Sheng

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Abstract

Modern economy features rich production linkages and frictional, segmented labor markets. We develop a new theoretical framework to study how they interact. Productivity shocks in relatively upstream sectors affect production in other sectors directly through the production of intermediate goods and indirectly through labor markets. Because different sectors hire similar workers, changes in labor demand in one sector affect the number of available workers in other sectors. This changes hiring costs and, therefore, output across the network. We find that, under a wide range of wage assumptions, labor market frictions amplify the response of aggregate output to sector-specific productivity shocks. We apply our model to analyze the impact of the Russia-Ukraine war during the "Great Resignation." Our model generates a modest decline in output, a pronounced increase in tightness, and relative price increases in energy-intensive sectors and their downstream sectors.

JEL Codes: E1, J3, J6

Schüle: finn_schuele@brown.edu, Brown University. Sheng: haoyu_sheng@brown.edu, Brown University. We thank Gabriel Chodorow-Reich, Eduardo Dávila, Fernando Duarte, Gauti Eggertsson, Amy Handlan, Yann Koby, and Pascal Michaillat for helpful comments and advice. We thank seminar speakers at Brown University for their helpful discussions. All errors are our own. The code for this paper can be found here.

1. Introduction

Modern economies feature rich production networks. Car manufacturers, for instance, buy steel and computer chips (along with many other inputs) from other sectors in the economy. They sell cars and trucks to transportation firms. And these transportation firms then transport goods for the steel and chip producers. As the recent chip shortage [add citation] demonstrates, because of these network connections between sectors a shock to one sector can have wide reaching consequences for firms in other sectors and ultimately consumers. A decline in the production of computer chips affects car manufacturers. As cars and trucks become more expensive, transportation firms suffer. And because chips and steel manufactures need transportation for their products, the original shock feeds back to those sectors as well.

Indeed, a recent literature highlights that these production linkages between firms in different sectors matter for the propagation of sector level shocks (Baqaee and Rubbo 2022), how we measure productivity and the social cost of distortions (Baqaee and Farhi 2019, 2020), the optimal conduct of monetary policy (Rubbo 2020; La'O and Tahbaz-Salehi 2022), and that sector level shocks may even be a source of aggregate fluctuations and growth (Acemoglu et al. 2012; Acemoglu and Azar 2020).

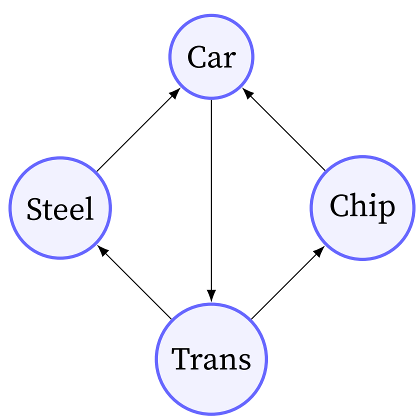

FIGURE 1. An example of a pure production network. "Car" refers to car manufacturers, "Steel" to steel manufacturer, "Chip" to computer chip manufacturers, and "Trans" to transportation firms. Modern economies are full of networks like this one, with consequences for the aggregate effects of sector level productivities and distortions.

We extend this existing literature by showing that production networks also matter for unemployment in a frictional labor market. Returning to the example above, a decline in chip production affects engineers, who are employed directly by chip producers, but also affects factory workers employed in car manufacturing. And because steel producers also hire factory workers, the shock feeds through to steel through the labor market. In addition, labor markets matter for production across the network. If, for instance, the number of available drivers declines the transportation industry will be directly affected. And even though steel and chip manufacturers do not directly employ drivers, they too will be affected because of the production network links to the transportation sector.

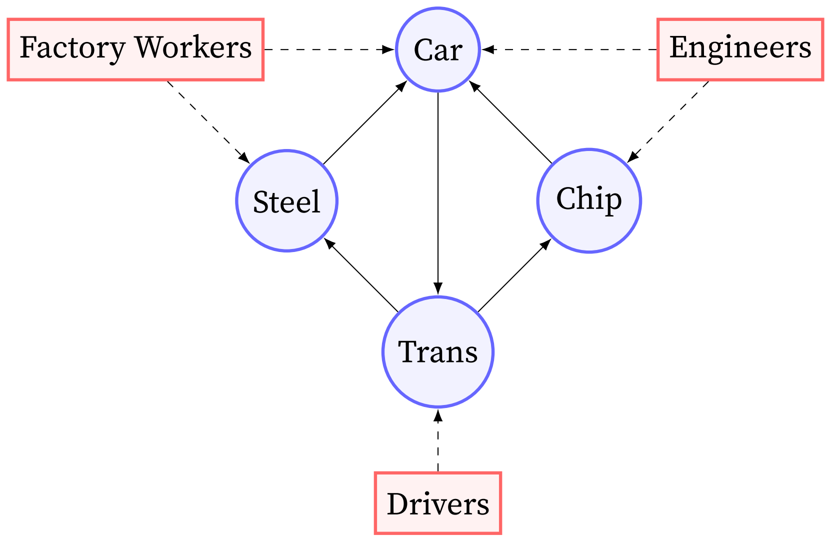

FIGURE 2. A production network with frictional labor markets. Dashed arrows denote the presence of frictions in the linkages.

We show that these kind of linkages between labor markets and production sectors are both qualitatively and quantitatively important. As shocks propagate through the network, labor demand changes. These changes in labor demand affect sector level tightness and unemployment. Furthermore, unemployment matters for production networks. Changes in labor market tightness require firms to change their recruiting effort. The resulting reallocation of productive workers to recruiters alters production outcomes. In addition, unemployed workers look for jobs in other sectors, lowering tightness in those sectors, and thus affecting production across the network.

Production linkages and labor market conditions are important economic mechanisms. Understanding how they interact is key to painting an accurate and granular picture of how microeconomic shocks generate macroeconomic fluctuations and helps policymakers craft and evaluate policies. Relative to the existing production networks literature, our framework allows us to think about how granular shocks change unemployment. And how these changes in unemployment, themselves, change the production outcomes.

For example, our framework can help us dissect how energy price spikes caused by the Russia-Ukraine war impact output and unemployment for different sectors, or how the CHIPS and Science Act, which boosts domestic research and manufacturing of semiconductors in the United States, changes aggregate output and unemployment. More generally, we use our framework to ask: What are the impacts of idiosyncratic technology or labor supply shocks on sector and aggregate output and unemployment? How do idiosyncratic shocks generate comovements in tightness across labor markets? How do labor market inefficiencies and characteristics, such as matching frictions, mobility costs, and wage schedules, affect the network propagation of shocks?

Starting with Long and Plosser (1983) and Acemoglu et al. (2012), much effort has been made on understanding how micro shocks cause macro fluctuations. Since an early contribution by Jones (2011), which demonstrates that missallocation accross a production network can generate more realistic transition dynamics in a neoclassical growth model, a recent literature has incorporated inefficiencies into production network models, including markups and financial frictions (Liu 2019; Baqaee and Farhi 2020; Bigio and La'O 2020) and nominal rigidities (Rubbo 2020; La'O and Tahbaz-Salehi 2022; Minton and Wheaton 2022). While these models offer a more realistic depiction of production linkages and the associated product market and financial inefficiencies, their treatment of labor markets is simple. These models consider labor as either perfectly inelastically supplied, or supplied according to the disutility of work in household's utility function, therefore ignoring the richness of labor market imperfections and shutting off interesting propagation channels brought forth by such imperfections.

In this paper, we extend the production network framework by incorporating matching frictions. We build a static, multisector production network model that features a representative household, many production sectors, perfect competition in product markets, and segmented frictional labor markets. The form of labor market segmentation is flexible, and our model allows for geographic, sectoral, or occupational labor market segmentation.

With this setup, we develop a general theory of how technology shocks propagate and aggregate across the production network under search frictions. In particular, we show that the aggregate impact on output and unemployment can be decomposed into two components. The first term resembles the foundational aggregation theorem of Hulten (1978), in which the aggregate output response is a sales-share-weighted sum of sectoral technology shocks. The second term, which we call the search-and-matching channel, involves complex interaction between tightness, production structure, and labor market structure. We show that we can recover Hulten's Theorem when relative wages adjust exactly according to the occupational-labor-share-weighted marginal product of labor, which we call the network-adjusted marginal product of labor. Intuitively, when wages change by the network adjusted marginal product of labor, tightness remains unchanged, eliminating all additional propagation coming from labor market frictions.

Whether the search-and-matching channel dampens or amplifies technology shocks depends largely on how wages adjust. Since the search and matching setup admits a wide range of assumptions about wages with very different quantitative consequences, as demonstrated by Shimer (2005) and Hall (2005), we view this as an empirical question. Nevertheless, we use two toy examples to illustrate the determinants of the direction of amplification. First, we shut off production linkages in the model. We show that the direction of amplification depends on how wages respond relative to the marginal product of labor, as well as whether posting additional vacancies changes the number of productive workers. Since hiring an additional worker requires more vacancies as labor markets get tighter, and posting vacancies requires firms to allocate workers towards recruiting, there is an inflection point where an additional vacancy results in no change in the productive workforce. We then solve a simple two-sector vertical production networks model with one labor market, to show that the direction of amplification now depends on how wages respond to the network-adjusted marginal product of labor. We further illustrate that our frictional labor market setup allows for upstream propagation of shocks, which is absent in efficient production networks with Cobb-Douglas utility and production functions.

In addition to technology shocks, our setup allows us to explore another set of shocks—shocks to the size of the labor force. We can think of these shocks as coming from an underlying model of occupational choice, as in Humlum (2021), or an underlying model of migration, as in Fernandez-Villaverde (2020). Unlike in the standard production networks setup, where a positive shock to the size of the labor force mechanically increases output because firms are forced to hire all available workers, our framework allows the hiring decision of firms to vary depending on wages. For instance, if wages do not change by much in response to positive labor force shocks, firms do not increase their hiring by much, dampening the overall effect on output but generating large changes to unemployment.

To test the empirical relevance of the theoretical channels outlined above, we calibrate our model to US data. We use survey-based vacancy and hiring data from the

Job Openings and Labor Turnover Survey (JOLTS), unemployment data from the Current Population Survey (CPS), occupation data from the Occupational Employment and Wage Statistics (OEWS), and industry sales shares from the U.S Bureau of Economic Analysis (BEA). We find that, assuming network price adjusted wages do not change, a one-percent positive technology shock to the durables manufacturing sector increases aggregate output by 0.56 percent, and decreases aggregate unemployment by 0.44 percent. Without search and matching frictions, the same shock would increase output by only 0.25 percent and would not affect unemployment. Importantly, we observe how unemployment changes across occupations, with unemployment decreases by 0.99 percent for production workers, by 0.77 percent for engineers, and by 0.70 percent for construction workers.

In section 6 we demonstrate how to use our calibrated model for macroeconomic applications. In particular, we examine the employment and output consequences of a simultaneous negative shock to energy supply and labor force participation to explore the consequences of the Russia-Ukraine War related reduction in global oil supply at a time when many workers still had not returned to the labor force after Covid. In a model without search frictions, this generates large changes in output and no changes in labor market tightness. In our model, on the other hand, this combination of shocks generates a decline in output that is half as large, sizeable increases in labor market tightness, and larger increases in relative prices in energy intensive sectors relative to the education and health services sector, a sector where prices stayed relatively constant. We view all three as more realistic given 2022 U.S. data. For simplicity and to keep the focus on labor market frictions, our current model is static, features no nominal rigidities, and is entirely real. As a result, we can only speak to relative prices across sectors. However, in future work we hope to extend the model to incorporate dynamics and nominal rigidities. This would allow us explore the consequences of a combination of factor and labor supply shocks for price dynamics, extending Minton and Wheaton (2022) to incorporate labor market frictions and Benigno and Eggertsson (2023) to incorporate shocks to supply chain inputs. We believe a combination of a non-linear Phillips curve in labor market tightness, as in Benigno and Eggertsson (2023), and the gradual propagation of factor prices through a production network, as in Minton and Wheaton (2022), would paint a realistic picture of the current inflation episode.

This paper fits closely to the recent development that attempts to bring more realism into production networks by incorporating market imperfections and inefficiencies. Baqaee and Farhi (2020) and Bigio and La'O (2020) model frictions as an exogenous wedge between the sectors' marginal costs and marginal revenues and examine how these frictions interact with the network structure and affect aggregate output in the Cobb-Douglas economy. Bigio and La'O (2020) bring the model to data and estimate how the US input-output structure amplify financial distortions in the great recession. Baqaee and Farhi (2020) examine how productivity shocks aggregate under the presence of exogenous wedges in a CES economy and decompose output changes into changes in technology and changes in allocative efficiency. Liu (2019) assumes that market imperfections generate dead-weight loss and demonstrates how these imperfections compound in the production network through demand linkages. He shows that a government should design industrial policies to target sectors based on distortions in sectoral size. Like La'O and Tahbaz-Salehi (2022) and Rubbo (2020), who consider New Keynesian production network models, our model features endogenous wedges, in our case generated by search and matching frictions. To our knowledge, our paper is the first to model search and matching in a production network setting.

This paper also contributes to the literature on factor reallocation and the aggregate impact of differential regional responses to shocks. Like Adão et al. (2019), who study the differential and aggregate impact of trade shocks on U.S. labor markets, our model allows us to explore both the aggregate and regional affects of shocks in a unified framework. Our model differs in that we incorporate labor market frictions, allowing us to speak to involuntary unemployment. Labor market frictions also naturally generate the high sensitivity of employment to wages documented empirically by Adão et al. (2019). Like Chodorow-Reich and Wieland (2020), who study the impact of labor reallocation across industries over the business, we build a multi-sector search-and-matching model of the labor market. We demonstrate that incorporating production linkages to the multi-sector model qualitatively and quantitatively changes the propagation of shocks to technology. Therefore, although this is not the focus of our paper, production linkages would presumably alter labor reallocation patterns as well. To the best of our knowledge, our paper is the first to shed light on how production linkages impact reallocation and co-movements in labor markets under search-and-matching frictions.

The remainder of the paper is organized as follows. Section 3 outlines our model and defines the equilibrium. Section 4 derives expressions for first order changes in output and employment in response to changes in technology and the size of the labor force. Section 5 describes the data we use to calibrate the model and presents illustrative examples to demonstrate the quantitative importance of labor markets. Section 6 works through an application of our model to energy supply shocks. Section 7 concludes.

2. Simple Two-Sector Model

In this section we present the simplest possible model featuring both production networks and labor market frictions to build intuition for why their interactions are important for the aggregate response of output and unemployment. This model features a two sector vertical production network and one type of labor. We can think of this model as the car-chip sub-block of the production network example from the introduction. The intuition this model provide will turn out to follow through to our full model presented in section 3.

2.1. Model Setup

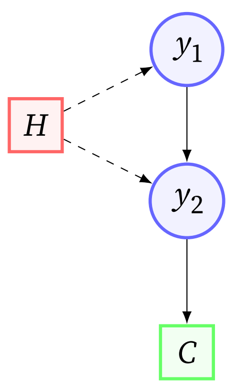

FIGURE 3. A two-sector vertical network. We can think of this simple example as the chips-cars sub-block of the network example in the introduction. y_1 is the chips sector, it sells to car manufactures, y_2, who sell to households C. For simplicity, we assume cars are the only direct consumption good. H are engineers, who are employed in both the car and chip manufacturing sectors. Dashed arrows denote the presence of frictions.

Setting. We consider a static two sector vertical economy with a single type of labor. We denote the output of the upstream sector by y_1 and the output of the downstream sector by y_2. A represenative household inelastically supplies a labor force of size H and entirely consumes the output of the downstream sector. The entire labor force starts the single period unemployed.

We assume firms in the upstream sector produce output y_1 using linear technology in labor.

$$y_1 = A_1 N_1$$

And firms in the downstream sector produce output y_2 using Cobb-Douglas technology in labor and the output from the upstream sector.

$$y_2 = A_2 y_1^{\gamma} N_2^{1-\gamma}$$

N_1 and N_2 denote the number of productive workers in each of the two sectors. A_1 and A_2 are sector specific productivity shifters. Firms in both sectors take prices and wages as given. We normalizing the price of the consumption good, y_2, to 1 and denote the sector 1 price by p.

Labor market frictions. The labor market is frictional. Firms must post vacancies to hire workers. Each posted vacancy requires r recruiters to maintain. We assume the recruiters receive the same wage as the production workers. Hires, h, are generated by a Cobb-Douglas matching function in unemployment, H, and total vacancy postings V.

$$h = H^{\eta} V^{1-\eta}$$

We can write the vacancy-filling rate, $\mathcal{Q}(\theta) = \frac{h}{V}$, and the job-finding rate, $\frac{h}{H} = \frac{h}{H}$, in terms of tightness $\theta = \frac{V}{U}$.

$$\mathcal{Q}(\theta) = \theta^{-\eta}, \ \mathcal{F}(\theta) = \theta^{1-\eta}$$

Labor supply. We define labor supply as the fraction of workers that find a job by the end of the period.

$$L^s = \mathcal{F}(\theta) H$$

Labor demand. If the sector i firm wants to hire N_i productive workers it must post $v_i = \frac{N_i}{\mathcal{Q}(\theta) - r}$ vacancies. Posting this many vacancies, in turn, requires $\frac{r}{\mathcal{Q}(\theta) - r} N_i$ recruiters. We let $\tau(\theta) = \frac{r}{\mathcal{Q}(\theta) - r}$ denote the recruiter producer ratio. This ratio will be the same across the two sectors since they face identical matching frictions and recruiting costs.

The total amount of labor a firm needs to maintain a productive workforce of size N_i is then $(1+\tau(\theta))N_i$. We define the sector i labor demand as

$$L_i^d = (1+\tau(\theta))N_i$$

for the profit maximizing choice of N_i. Furthermore, we define aggregate labor demand as

$$L^d = L_1^d + L_2^d$$

2.2. Equilibrium and Shock Propagation Without Labor Market Frictions

Absent labor market frictions, the equilibrium in this economy is a set of quantities, y_1, y_2, N_1, N_2; and prices, p and w; Such that the following conditions hold.

(i) Output in the two sectors satisfies the technology constraint

$$y_1 = A_1 N_1$$
$$y_2 = A_2 y_1^{\gamma} N_2^{1-\gamma}$$

(ii) The zero profit condition holds in the upstream sector

$$p y_1 = w N_1$$

(iii) Firms in the downstream sector choose inputs to minimize costs

$$\gamma y_2 = p y_1$$
$$(1-\gamma) y_2 = w N_2$$

(iv) Labor market clearing holds.

$$H = N_1 + N_2$$

These equilibrium conditions imply that changes in the output of the final consumption good in this frictionless economy are given by

$$d \log y_2 = \gamma d \log A_1 + d \log A_2 + d \log H$$

Notice that since y_2 is the final consumption good, GDP in this economy is exactly equal to y_2. Therefore, γ is sector 1's sales as a share of GDP and 1 is sector 2's sales as a share of GDP. Hulten's theorem holds in this frictionless economy: Changes in aggregate output in response to sector level productivity shocks are given by the sales share weighted sum of the shocks. Intuitively, productivity shocks to each sector are rescaled by the sectors' importance as a direct and indirect supplier of final consumption. In efficient economies each sector's importance is fully captured by sales as a share of GDP. This intuition continues to hold in more complex economies. Furthermore, productivity shocks have no effect on aggregate employment.

2.3. Equilibrium and Shock Propagation With Labor Market Frictions

We define an equilibrium in the frictional economy analogously to the frictionless case above. There is one additional endogenous variable, θ, but no additional equilibrium condition.[1] As a result, and as is typically true in models with frictional labor markets, the wage is not pinned down endogenously.

An equilibrium is now a set of quantities, y_1, y_2, N_1, and N_2; prices, p and w; and a labor market tightness, θ; such that the following conditions hold.

(i) Output in the two sectors satisfies the technology constraints

$$\begin{aligned} y_1 &= A_1 N_1 \\ y_2 &= A_2 y_1^{\gamma} N_2^{1-\gamma} \end{aligned}$$

(ii) The zero profit condition holds in the upstream sector

$$p y_1 = w \frac{\theta^{-\eta}}{\theta^{-\eta} - r} N_1$$

(iii) The firms in the downstream sector choose inputs to minimize costs

$$\begin{aligned} \gamma y_2 &= p y_1 \\ (1-\gamma) y_2 &= w \frac{\theta^{-\eta}}{\theta^{-\eta} - r} N_2 \end{aligned}$$

[1]Note that (ii),(iii), and, (iv) jointly imply that market clearing holds in the final goods sector. That is, that household income equals final expenditures on the consumption good $y_2 = w\theta^{1-\eta} H$. Goods market clearing therefore does not provide any additional restrictions we could use to pin down wages.

(iv) The labor market clears

$$\theta^{1-\eta} H = \frac{\theta^{-\eta}}{\theta^{-\eta} - r} (N_1 + N_2)$$

(ii) and (iii) together imply that

$$N_2 = \frac{1-\gamma}{\gamma} N_1$$

Which, together with (iv) means that

$$N_1 = \gamma \left(\theta^{1-\eta} - r\theta\right) H$$
$$N_2 = (1-\gamma) \left(\theta^{1-\eta} - r\theta\right) H$$

Plugging back into the conditions gives the following expressions for output in sector 2 and labor market tightness.

$$y_2 = \gamma^{\gamma} (1-\gamma)^{1-\gamma} A_1^{\gamma} A_2 \left(\theta^{1-\eta} - r\theta\right) H$$
$$\theta = \left(\frac{1}{r}\left(1 - \frac{w}{\gamma^{\gamma}(1-\gamma)^{1-\gamma} A_1^{\gamma} A_2}\right)\right)^{\frac{1}{\eta}}$$

The following expression captures how productivity and labor force shocks feed through to changes in labor market tightness, to first order.

$$d \log \theta = \frac{1}{\eta}$$

And first order effects of productivity shocks on output in the downstream sector are

$$d \log y_2 = \gamma d \log A_1 + d \log A_2 + d \log H$$

3. Model

Our model is a static multi-sector production networks model (Baqaee and Farhi 2020; Bigio and La'O 2020). The model features J production sectors, indexed by i, and $\mathcal{O}$ occupations index by o. Production requires intermediate inputs, labor, and fixed factors. We embed search-and-matching frictions into labor markets the production sectors hire from. In this section, we present the model and characterize its equilibrium. For exposition clarity, the model we present features Cobb-Douglas production functions and preferences. We derive general results for constant return-to-scale technology in Appendix B.

3.1. Environment

3.1.1. Households

Preferences. A representative household consumes goods produced by sector i, with utility

$$\mathcal{U}(\{c_i\}_{i=1}^{J}) = \sum_{i=1}^{J} \sigma_i \log c_i,$$

where

$$\sum_i \sigma_i = 1.$$

Budget constraint. The representative household inealastically supplies a labor force of size H_o to occupation o. In addition, the household owns the fixed factors required for production. The household's consumption equates its labor income and fixed factor income. The household's budget constraint is:

$$\sum_{i=1}^{J} p_i c_i = \sum_{o=1}^{\mathcal{O}} w_o L_o + \sum_{k=1}^{\mathcal{K}} r_k K_k,$$

where c_i is the final consumption of sector i's output, p_i is the price of the sector i good, w_o is the wage in occupation o, and L_o is the labor used in sector o. r_k are the prices of the additional factors K_k of production. We can think of these $\mathcal{K}$ additional inputs into production as capital or energy.

Optimization. The household faces the following optimization problem:

$$\max_{\{c_i\}_{i=1}^{J}} \mathcal{U}(\{c_i\}_{i=1}^{J}),$$

subject to

$$\sum_{i=1}^{J} p_i c_i = \sum_{o=1}^{\mathcal{O}} w_o L_o + \sum_{k=1}^{\mathcal{K}} r_k K_k.$$

The consumption choices satisfy the first order condition

$$\sigma_i = \frac{p_i c_i}{\sum_{j=1}^{J} p_j c_j}. \tag{1}$$

3.1.2. Sectors

Production. A representative firm in sector i uses workers in occupation o, N_{io}, intermediate inputs from sector j, x_{ij}, and $\mathcal{K}$ additional factors K_{ik} to produce output y_i using Cobb-Douglas technology. We refer to the representative firm directly as a sector.

$$y_i = A_i \Pi_{j=1}^{J} \Pi_{o=1}^{\mathcal{O}} \Pi_{k=1}^{\mathcal{K}} x_{ij}^{\alpha_{ij}} N_{io}^{\beta_{io}} K_{ik}^{\kappa_{ik}}, \tag{2}$$

where

$$\sum_{j=1}^{J} \alpha_{ij} + \sum_{o=1}^{\mathcal{O}} \beta_{io} + \sum_{k=1}^{\mathcal{K}} \kappa_{ik} = 1.$$

The goods produced by a sector can be either an intermediate good for other sectors, or a consumption good for the household.

Profits. A sector's profit is given by the difference between its revenue and costs. The production costs include labor cost, intermediate input cost, and fixed factor cost. The profit π_i for sector i is:

$$\pi_i = p_i y_i - \sum_{o=1}^{\mathcal{O}} w_o L_{io} - \sum_{j=1}^{J} p_j x_{ij} - \sum_{k=1}^{\mathcal{K}} r_k K_{ik},$$

where

$$L_{io} = (1 + \tau_o(\theta_o)) N_{io}.$$

Here L_{io} denotes the total number of workers from occupation o hired by sector i. $\tau_o = \frac{L_{io}-N_{io}}{N_{io}}$ is the recruiter-producer ratio. We explain the distinction between recruiters and producers in subsubsection 3.1.3.

Optimization. We assume sectors are price takers in both input and output markets. Sectors choose $\left\{N_{io}\right\}_{o=1}^{\mathcal{O}}$, $\left\{x_{ij}\right\}_{j=1}^{J}$, and $\left\{K_{ik}\right\}_{ik}^{\mathcal{K}}$ to maximize profits, or equivalently to minimize costs

$$\max_{\{N_{io}\}_{o=1}^{\mathcal{O}},\left\{x_{ij}\right\}_{j=1}^{J},\{K_{ik}\}_{k=1}^{\mathcal{K}}} \pi_i\left(\left\{N_{io}\right\}_{o=1}^{\mathcal{O}}, \left\{x_{ij}\right\}_{j=1}^{J}, \left\{K_{ik}\right\}_{k=1}^{\mathcal{K}}\right).$$

The profit maximization problem gives the first order conditions

$$\alpha_{ij} = \frac{p_j x_{ij}}{p_i y_i}, \tag{3}$$

$$\beta_{io} = (1+\tau_o(\theta_o))\,\frac{w_o N_{io}}{p_i y_i}, \tag{4}$$

$$\kappa_{ik} = \frac{r_k K_{ik}}{p_i y_i}. \tag{5}$$

3.1.3. Labor Markets

We assume there are $\mathcal{O}$ occupations with separate labor markets, a labor force of H_o possible workers, who all start out unemployed at the beginning of the single period. The exogenous recruiting cost, r_o, measures the units of labor required for a firm to maintain each posted vacancy in occupation o.

Matching Functions. Hires are generated by a Cobb-Douglas matching function in occupation-level unemployment U_o and vacancies V_o, which measures all vacancy postings for occupation o,

$$h_o = \phi_o U_o^{\eta_o} V_o^{1-\eta_o}.$$

Since our model is static, we have $U_o = H_o$. The number of aggregate vacancy postings V_o is the sum of sectoral vacancy postings $\sum_{i=1}^{J} v_{io}$. The occupational labor market tightness is $\theta_o = \frac{V_o}{U_o} = \frac{V_o}{H_o}$[2].

The vacancy-filling rate $\mathcal{Q}_o$ and the job-finding rate $\mathcal{F}_o$ are thus:

$$\mathcal{Q}_o(\theta_o) = \frac{h_o}{V_o} = \phi_o\theta_o^{-\eta_o}, \quad \mathcal{F}_o(\theta_o) = \frac{h_o}{U_o} = \phi_o\theta_o^{1-\eta_o}.$$

Labor Supply. A fraction $\mathcal{F}_o(\theta_o)$ of the labor force finds a job and is employed at the end of the period. Labor supply satisfies

$$L_o^s(\theta_o) = \mathcal{F}_o(\theta_o)H_o. \tag{6}$$

Labor Demand. We assume firms take the occupation level tightness as given.[3] Let L_{io}^d denote the total workers sector i wants to hire from occupation o and N_{io} the number of productive employees it wants to hire. Since posting each vacancy costs r_o units of occupation o labor, L_{io}^d is not equal to N_{io}, and sector i has to dedicate a portion of its occupation o labor to recruiting.

In order to hire N_{io} productive employees, sector i firms has to post v_{io} vacancies in labor market o, such that $\mathcal{Q}_o(\theta_o)v_{io} = N_{io}+r_o v_{io}$. This relationship states that the number of vacancies filled equals the number of productive workers hired plus the number of recruiters. Rearranging this expression yields $v_{io} = \frac{N_{io}}{\mathcal{Q}_o(\theta_o)-r_o}$, which implies that hiring one unit of productive labor requires $\frac{1}{\mathcal{Q}_o(\theta_o)-r_o}$ vacancy postings. Since posting each vacancy requires r_o recruiters, τ_o, the recruiter-producer ratio for occupation o, is

$$\tau_o(\theta_o) \equiv \frac{L_o^d - N_o}{N_o} = \frac{r_o}{\mathcal{Q}_o(\theta_o) - r_o}.$$

In the language of the production networks literature, τ_o acts as an endogenous wedge on sectors' labor costs. This endogenous wedge plays an important role in how shocks propagate through the production network through labor demand. The labor demand, $L_{io}^d(\theta_{io}) = (1 + \tau_o(\theta_o))N_{io}$, is determined by sectors' profit maximization problem.

Finally, we define aggregate occupation o labor demand as the sum of sectoral labor

[2] We define the occupational labor market tightness to be $\theta_o = \frac{V_o}{H_o}$ for internal model consistency, as our model is static. However, we yield similar expressions for labor demand and labor supply as a function of θ, if we instead define $\theta_o = \frac{V_o}{U_o}$ and assume balanced labor market flows.

[3] We can think of each sector as being populated by many identical competitive firms so that each firm only has an infinitesimal impact on aggregate vacancies, and therefore on aggregate tightness.

demands.

$$L_o^d(\theta_o) = \sum_{i=1}^{J} L_{io}^d(\theta_o) = \sum_{i=1}^{J}(1 + \tau_o(\theta_o))N_{io} \tag{7}$$

Wages. In matching models, workers and firms meet in a situation of bilateral monopoly. The resulting mutual gains from trade mean that wages are not determined by equilibrium conditions of the model[4], and must instead be pinned down by some wage setting norm chosen by the researcher. For now, we assume a general functional form for wages:

$$\boldsymbol{w} = g(\boldsymbol{p}, \boldsymbol{A}, \boldsymbol{H}, \boldsymbol{K^s}). \tag{8}$$

This wage assumption is general enough and nests both nominally rigid and real rigid wages. For example, nominal wages can be a function of productivity as in Blanchard and Galí (2010) and Michaillat (2012), or a constant. In addition, we can adjust for price changes and introduce real wage rigidity. Whether wage rigidity is real or nominal determines whether the model is real, since how wages change relative to prices are key to the determination of labor market equilibrium.

3.1.4. Equilibrium

Given exogenous variables $\left\{\{A_i\}_{i=1}^J, \{H_o\}_{o=1}^{\mathcal{O}}, \left\{K_k^s\right\}_{k=1}^K\right\}$ and a wage function g, the equilibrium is a collection of allocations $\left\{\left\{y_i, \{x_{ij}\}_{j=1}^J, c_i, \left\{N_{io}\right\}_{o=1}^{\mathcal{O}}, \left\{K_{ik}\right\}_{k=1}^K\right\}_{i=1}^J, \{\theta_o\}_{o=1}^{\mathcal{O}}\right\}$ and prices $\left\{\{p_i\}_{i=1}^J, \{w_o\}_{o=1}^{\mathcal{O}}, \{r_k\}_{k=1}^{\mathcal{K}}\right\}$ such that

(i) the allocations solve the household's problem (Equation 1),

(ii) the allocations solve the firm's problem (Equations 2 - 5),

(iii) goods markets clear

$$y_i = c_i + \sum_{j=1}^{J} x_{ij} \qquad \forall i \in \{1, 2, \ldots, J\}. \tag{9}$$

[4]Wages are only constrained to fall within a range where both workers and firms benefit from the match. However, this range can be wide since workers usually strongly prefer employment to unemployment and finding a new match is costly for firms.

(iv) factor markets clear

$$K_k^s = \sum_{i=1}^{J} K_{ik} \qquad \forall k \in \{1, 2, \ldots, \mathcal{K}\}. \tag{10}$$

(v) wages are set according to Equation 8,

(vi) labor markets, specified by Equations 6 and 7, clear

$$L_o^d = L_o^s \qquad \forall o \in \{1, 2, \ldots, \mathcal{O}\}. \tag{11}$$

4. Theoretical Results: The Propagation, Aggregation, and Amplification of Shocks

In this section, we describe the main theoretical results of our paper. We define shocks to be small proportional changes in the exogenous variables. We first derive the propagation of technology, labor supply, and factor supply shocks at the dis-aggregated occupation and sector level. We compute the first-order proportional responses of the main endogenous variables of interest in the model, such as output and unemployment. We then present our aggregation theorem of idiosyncratic shocks. The first-order approximation we present is exact under the Cobb-Douglas assumptions outlined in section 3. We show how to generalize the results to any constant returns production function in the case of one occupation per sector in appendix B.

The goal of this section is also to highlight the theoretical importance of modeling labor market frictions and production linkages in conjunction. How much a sector's production depends on intermediate goods produced by other sectors, and how much sectors rely on a particular occupation has an impact on that occupation's labor demand. The effect production linkages and occupational usage have on occupational labor demand affects tightness in a frictional labor market. Such effect creates an endogenous matching wedge for the propagation of shocks. In the following section, we show that the effects of this wedge are salient for output and prices.

4.1. Notation

Before outlining our results, we define some useful notations. First, we define the input-output matrix $\mathbf{\Omega}$ as the matrix containing sector level input revenue shares

$$\mathbf{\Omega} = \begin{bmatrix} \frac{p_1 x_{11}}{p_1 y_1} & \cdots & \frac{p_J x_{1J}}{p_1 y_1} \\ \vdots & \ddots & \vdots \\ \frac{p_1 x_{J1}}{p_J y_J} & \cdots & \frac{p_J x_{JJ}}{p_J y_J} \end{bmatrix}_{J \times J} .$$

In our competitive goods market equilibrium setup, we can rewrite this input-output matrix in terms of the elasticities of the sector level production functions. (See Equation 3)

$$\text{(Input-Output Matrix)} \qquad \mathbf{\Omega} = \begin{bmatrix} \alpha_{11} & \cdots & \alpha_{1J} \\ \vdots & \ddots & \vdots \\ \alpha_{J1} & \cdots & \alpha_{JJ} \end{bmatrix}_{J \times J} .$$

We denote the Leontief inverse by $\mathbf{\Psi} = (\boldsymbol{I} - \mathbf{\Omega})^{-1}$. The Leontief inverse captures the importance of each sector as a direct and indirect input into production in every other sector.

Alongside the production elasticities to intermediate inputs, our model features a second set of production elasticities: elasticities to the different types of labor inputs. We collect these elasticities in the matrix $\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}$.

$$\text{(Labor Elasticity Matrix)} \qquad \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} = \begin{bmatrix} \beta_{11} & \cdots & \beta_{1\mathcal{O}} \\ \vdots & \ddots & \vdots \\ \beta_{J1} & \cdots & \beta_{J\mathcal{O}} \end{bmatrix}_{J \times \mathcal{O}} .$$

Equation (4) demonstrates that this matrix is the labor input equivalent of the standard input-output matrix. In equilibrium, each entry is the revenue share of type o labor.

Similarly, we collect the factor elasticities in the matrix $\varepsilon^{\boldsymbol{f}}_{\boldsymbol{K}}$.

$$\text{(Factor Elasticity Matrix)} \qquad \varepsilon^{\boldsymbol{f}}_{\boldsymbol{K}} = \begin{bmatrix} \kappa_{11} & \cdots & \kappa_{1\mathcal{K}} \\ \vdots & \ddots & \vdots \\ \kappa_{J1} & \cdots & \kappa_{J\mathcal{K}} \end{bmatrix}_{J \times \mathcal{K}} .$$

Equation (5) demonstrates that this matrix is the factor input equivalent of the standard input-output matrix. In equilibrium, each entry is the revenue share of factor k.

$\mathcal{L}$ is an $\mathcal{O} \times J$ matrix with the share of occupation o workers employed in each sector along the rows:

$$\text{(Occupation-Share Matrix)} \qquad \mathcal{L} = \begin{bmatrix} \frac{L_{11}}{L_1} & \cdots & \frac{L_{J1}}{L_1} \\ \vdots & \ddots & \vdots \\ \frac{L_{1\mathcal{O}}}{L_{\mathcal{O}}} & \cdots & \frac{L_{J\mathcal{O}}}{L_{\mathcal{O}}} \end{bmatrix}_{\mathcal{O} \times J}.$$

$\mathcal{M}$ is an $\mathcal{O} \times \mathcal{O}\mathcal{H}$ with the matching elasticities along the diagonal

$$\text{(Matching Elasticity Matrix)} \qquad \mathcal{M} = \begin{bmatrix} \eta_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \eta_{\mathcal{O}} \end{bmatrix}_{\mathcal{O} \times \mathcal{O}}.$$

$\mathcal{T}$ is an $\mathcal{O} \times \mathcal{O}$ with occupational recruiter-producer ratios along the diagonal.

$$\text{(Recruiter-Producer Ratio Matrix)} \qquad \mathcal{T} = \begin{bmatrix} \tau_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \tau_{\mathcal{O}} \end{bmatrix}_{\mathcal{O} \times \mathcal{O}}.$$

4.2. The Propagation of Shocks

We are interested in how three sets of endogenous variables—sector level relative prices, sector level output, and occupation level tightness—change in response to changes in technology and the size of the labor force. With these variables, we can compute the aggregate variables of interest: output and unemployment.

Before deriving our results, we introduce the following reduced-form relationship between relative wages and the shocks:

$$d \log \boldsymbol{w} - \mathcal{L} d \log \boldsymbol{p} = \boldsymbol{\Lambda}_{\boldsymbol{A}} d \log \boldsymbol{A} + \boldsymbol{\Lambda}_{\boldsymbol{H}} d \log \boldsymbol{H} + \boldsymbol{\Lambda}_K d \log \boldsymbol{K}^S,$$

where $d \log \boldsymbol{w}$ and $d \log \boldsymbol{H}$ are $\mathcal{O} \times 1$ dimensional vectors capturing first order changes in wages and the size of the labor force, $d \log \boldsymbol{p}$ and $d \log \boldsymbol{A}$ are $J \times 1$ dimensional vectors capturing first order changes in prices and productivities, and $d \log \boldsymbol{K}^S$ is a $\mathcal{K} \times 1$ dimensional vector of first order changes in factor supply. $\mathcal{L}$ is the occupational-share matrix. $\boldsymbol{\Lambda}_A$, $\boldsymbol{\Lambda}_H$, and $\boldsymbol{\Lambda}_K$ are $\mathcal{O} \times J$, $\mathcal{O} \times \mathcal{O}$, and $\mathcal{O} \times \mathcal{K}$ coefficient matrices that capture how wages respond to technology, labor force, and factor supply shocks in each other sector, occupation, and factor market. For instance, the (o, i)-th entry of $\boldsymbol{\Lambda}_A$ captures how wages in occupation o respond to technology shocks in sector i.

This relationship is a general first-order approximation of the wage function in Equation 8. A first-order approximation of the wage function allows us to represent first-order changes in wages as a function of first-order changes in prices and exogenous variables. We choose to represent the relative wages as the difference between change in nominal occupational wages and change in occupation-share weighted prices, because this is the effective wage that determines labor demand for workers in each occupation. As we show below, relative prices are themselves determined by changes in exogenous variables such as technology shocks, labor force shocks, and factor supply shocks. This means that the relative wages are also determined by changes in exogenous variables.

We now derive how shocks propagate to labor market tightness, output, and unemployment. These first order effects are summarized in proposition 1. For a detailed derivation see appendix A.1.

Proposition 1. *Given occupational labor supply shocks* $d\log \boldsymbol{H} = \left[d\log H_1, \cdots, d\log H_{\mathcal{O}}\right]'$, *sectoral productivity shocks* $d\log \boldsymbol{A} = \left[d\log A_1, \cdots, d\log A_J\right]'$, *and factor supply shocks* $d\log \boldsymbol{K}^s = \left[d\log K_1^s, \cdots, d\log K_{\mathcal{K}}^s\right]'$, *the first-order responses of labor market tightness* $d\log \theta = \left[d\log \theta_1, \cdots, d\log \theta_{\mathcal{O}}\right]'$ *and output* $d\log \boldsymbol{y} = \left[d\log y_1, \cdots, d\log y_J\right]'$ *follow:*

$$
\begin{aligned}
d\log \theta &= \Pi_{\theta,A}\, d\log \boldsymbol{A} + \Pi_{\theta,H}\, d\log \boldsymbol{H} + \Pi_{\theta,K}\, d\log \boldsymbol{K}^s, \\
d\log \boldsymbol{y} &= \Pi_{y,A}\, d\log \boldsymbol{A} + \Pi_{y,H}\, d\log \boldsymbol{H} + \Pi_{y,K}\, d\log \boldsymbol{K}^s,
\end{aligned}
$$

where

$$
\begin{aligned}
\Pi_{\theta,A} &= \left[\boldsymbol{I} - \mathcal{M} - \Xi_\theta\right]^{-1} \left(\mathcal{L}\boldsymbol{\Psi} - \boldsymbol{\Lambda}_A\right), \\
\Pi_{\theta,H} &= \left[\boldsymbol{I} - \mathcal{M} - \Xi_\theta\right]^{-1} \left(\mathcal{L}\boldsymbol{\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I} - \boldsymbol{\Lambda}_H\right), \\
\Pi_{\theta,K} &= \left[\boldsymbol{I} - \mathcal{M} - \Xi_\theta\right]^{-1} \left(\mathcal{L}\boldsymbol{\Psi}\varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}} - \boldsymbol{\Lambda}_K\right), \\
\Pi_{y,A} &= \boldsymbol{\Psi} \left[\boldsymbol{I} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left(\boldsymbol{I} - \underbrace{\mathcal{M}\left(\boldsymbol{I} + \mathcal{T}\right)}_{\text{search cost}}\right) \underbrace{\left[\boldsymbol{I} - \mathcal{M} - \Xi_\theta\right]^{-1} \left[\mathcal{L}\boldsymbol{\Psi} - \boldsymbol{\Lambda}_A\right]}_{\Pi_{\theta,\boldsymbol{A}}}\right],
\end{aligned}
$$

$$
\begin{aligned}
\Pi_{y,H} &= \Psi\left[\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left(\boldsymbol{I} - \underbrace{\mathcal{M}\left(\boldsymbol{I} + \mathcal{T}\right)}_{\text{search cost}}\right)\underbrace{\left[\boldsymbol{I} - \mathcal{M} - \Xi_{\theta}\right]^{-1}\left[\mathcal{L}\Psi\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I} - \Lambda_{H}\right]}_{\Pi_{\theta,\boldsymbol{H}}}\right], \\
\Pi_{y,K} &= \Psi\left[\varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left(\boldsymbol{I} - \underbrace{\mathcal{M}\left(\boldsymbol{I} + \mathcal{T}\right)}_{\text{search cost}}\right)\underbrace{\left[\boldsymbol{I} - \mathcal{M} - \Xi_{\theta}\right]^{-1}\left[\mathcal{L}\Psi\varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}} - \Lambda_{K}\right]}_{\Pi_{\theta,\boldsymbol{K}}}\right], \\
\Xi_{\theta} &= \mathcal{L}\Psi\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left[\boldsymbol{I} - \mathcal{M}\left(\boldsymbol{I} + \mathcal{T}\right)\right].
\end{aligned}
$$

Finally, the expression of labor supply implies that first order changes in occupational unemployment level $d\log\boldsymbol{U} = \left[d\log U_1, d\log U_2, \cdots, d\log U_{\mathcal{O}}\right]$ *follow*

$$d\log\boldsymbol{U} = -\left(\boldsymbol{I} - \mathcal{M}\right)d\log\theta + d\log\boldsymbol{H}.$$

PROOF. See appendix A.1. □

We now unpack the intuition behind these expressions. First, we consider the response in tightness. The equilibrium response in tightness is jointly determined by changes in labor supply and labor demand. The labor supply side is impacted directly by changes in the size of the labor force and job-finding rates, and the labor demand side is impacted by vacancy-filling rates, wages and prices, as well as productivity. Intuitively, a technology shock affects labor demand by directly altering a sector's productive capability, thus impacting prices, output, and labor usage for other sectors through production linkages. A shock to the size of the labor force impacts labor supply by changing the number of available workers, which changes sectors' production through changing the number of recruiters needed. Changes in sectors' production propagate through the network and induce other sectors to change their labor demand, thus causing tightness to adjust, so on and so forth. Algebraically, the difference between the occupation-share adjusted Leontief inverse $\mathcal{L}\Psi$ and the relative wage coefficients Λ captures the positive net effect an exogenous shock has on labor demand. The multiplicative constant $\left[\boldsymbol{I} - \mathcal{M} - \Xi_{\theta}\right]^{-1}$ represents the equilibrium interaction between labor supply and labor demand.

Second, we analyze the response in sectoral output. The first-order impact of a shock to any of the exogenous variables on sectoral output can be split into two terms: a term based on the exogenous variable's elasticity of production, such as $(\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}})$ or $(\varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}})$, and a term based on tightness adjustments, both multiplied by the Leontief inverse. The first term, which describes how much production relies on the variable being shocked, captures the impact of the shock directly through the production function. The second term captures the shock's indirect impact through the tightness channel, since these shocks alter labor supply and demand through the production process. A search cost $\mathcal{M}(\boldsymbol{I}+\mathcal{T})$ accompanies the indirect impact from labor market tightness, since sectors dedicate a fraction of the labor they hire to recruiting. Such direct and indirect effects propagate to impact production in other sectors through production linkages, hence the Leontief inverse in front.

Last, we examine the response in occupational unemployment levels. Unemployment is defined as the number workers who search for jobs and cannot find one. Algebraically, occupational unemployment level U_o is equal to $(1-f_o(\theta_o))H_o$. Therefore, a change in exogenous variables impacts unemployment level through changes in tightness. If a labor market becomes tighter, then the unemployment level goes down. In addition, a change in labor force participation for an occupation impacts its unemployment level directly, as more workers look for jobs in that labor market.

PROPOSITION 2. *The first-order responses of relative sectoral and factor prices are pinned down by labor supply, technology, and factor supply shocks up to a numeraire. The first-order responses in sectoral prices satisfy*

$$\left(\boldsymbol{I}-\boldsymbol{\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\mathcal{L}\right)d\log\boldsymbol{p}=\Pi_{p,A}d\log\boldsymbol{A}+\Pi_{p,H}d\log\boldsymbol{H}+\Pi_{p,K}d\log\boldsymbol{K}^{s}+\boldsymbol{\Psi}\varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}}d\log\boldsymbol{r},$$

and

$$\Pi_{p,A}=\boldsymbol{\Psi}\left[\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left(\Lambda_A+\mathcal{M}\mathcal{T}\underbrace{\left[\boldsymbol{I}-\mathcal{M}-\Xi_\theta\right]^{-1}\left[\mathcal{L}\boldsymbol{\Psi}-\Lambda_A\right]}_{\Pi_{\theta,\boldsymbol{A}}}\right)-\boldsymbol{I}\right],$$

$$\Pi_{p,H}=\boldsymbol{\Psi}\left[\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left(\Lambda_H+\mathcal{M}\mathcal{T}\underbrace{\left[\boldsymbol{I}-\mathcal{M}-\Xi_\theta\right]^{-1}\left(\mathcal{L}\boldsymbol{\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}-\boldsymbol{I}-\Lambda_{\boldsymbol{H}}\right)}_{\Pi_{\theta,\boldsymbol{H}}}\right)\right],$$

$$\Pi_{p,K}=\boldsymbol{\Psi}\left[\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left(\Lambda_K+\mathcal{M}\mathcal{T}\underbrace{\left[\boldsymbol{I}-\mathcal{M}-\Xi_\theta\right]^{-1}\left(\mathcal{L}\boldsymbol{\Psi}\varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}}-\Lambda_K\right)}_{\Pi_{\theta,\boldsymbol{K}}}\right).\right]$$

The first-order responses in fixed factor prices satisfy

$$d \log \boldsymbol{r} = \mathbf{1} d \log y_{num} - d \log \boldsymbol{K^s},$$

where $d \log y_{num}$ denotes the change in output in the numeraire sector, which is determined as per proposition 1.

PROOF. See appendix A.1. □

In our framework, relative prices are determined by market clearing in a perfectly competitive market. The price of a good produced by a particular sector responds to changes in prices in all other sectors, and $\boldsymbol{\Psi}$ captures the co-movement and interaction of prices throughout the production network. Prices also depend on the effective cost of employing workers. In models without search frictions, prices respond directly to change in wages, which corresponds to the $\boldsymbol{\Lambda}$ terms in the expressions above. In our model, wage adjustments impact prices through an additional tightness channel, which corresponds to the product of the matching elasticity matrix, the recruiter-producer matrix, and the $\boldsymbol{\Pi}_\theta$ matrices. This is because wage adjustments impact labor market tightness, and changes in labor market tightness changes the number of recruiters needed to hire a certain number of workers. In addition, the price is also directly linked to the level of productivity in that sector. Thus, a productivity shock impacts the system of prices directly through production and indirectly through adjustments in tightness, and a labor supply shock impacts prices solely through adjustments in labor market tightness. Additionally, the first-order responses in factor prices follow from the numeraire sector's optimal factor usage (Equation 5).

At the dis-aggregate level, labor market structure, frictions, and production linkages interact to contribute meaningfully to how prices and allocations respond. In particular, changes in labor demand depend on how sectors connect to each other through the use of intermediate goods, and what common occupations they hire from. The changes in labor demand lead to changes in labor market tightness, which acts as an endogenous wedge that impacts output and price responses since sectors change their allocation of recruiters, as in both Proposition 1 and Proposition 2.

4.3. The Aggregate Impact of Shocks

We are interested in how the impact of idiosyncratic shocks aggregate to impact output. With a Cobb-Douglas preference, the first-order response of aggregate output is given by $d\log Y = \boldsymbol{\sigma}' d\log \boldsymbol{y}$. Using Proposition 1, we arrive at the following result for first-order changes in aggregate output.

THEOREM 1. *Given idiosyncratic labor supply shocks $d\log \boldsymbol{H}$ and productivity shocks $d\log \boldsymbol{A}$, the log change in real GDP is:*

$$d\log Y = \boldsymbol{\Pi_A} d\log \boldsymbol{A} + \boldsymbol{\Pi_H} d\log \boldsymbol{H} + \boldsymbol{\Pi_K} d\log \boldsymbol{K}^s,$$

where

$$\begin{aligned}
\boldsymbol{\Pi_A} &= \lambda' \left(\boldsymbol{I} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left(\boldsymbol{I} - \mathcal{M}\left(\boldsymbol{I} + \mathcal{T}\right)\right) \left[\boldsymbol{I} - \mathcal{M} - \Xi_\theta\right]^{-1} \left(\mathcal{L}\Psi - \Lambda_A\right) \right), \\
\boldsymbol{\Pi_H} &= \lambda' \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left(\boldsymbol{I} + \left(\boldsymbol{I} - \mathcal{M}\left(\boldsymbol{I} + \mathcal{T}\right)\right) \left[\boldsymbol{I} - \mathcal{M} - \Xi_\theta\right]^{-1} \left(\left[\mathcal{L}\Psi\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I}\right] - \Lambda_H \right) \right), \\
\boldsymbol{\Pi_K} &= \lambda' \left(\varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left(\boldsymbol{I} - \mathcal{M}\left(\boldsymbol{I} + \mathcal{T}\right)\right) \left[\boldsymbol{I} - \mathcal{M} - \Xi_\theta\right]^{-1} \left(\mathcal{L}\Psi\varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}} - \Lambda_K \right) \right)
\end{aligned}$$

and $\boldsymbol{\lambda} = \boldsymbol{\Psi}'\boldsymbol{\sigma}$ denotes the sectors' sales shares.

PROOF. The result follows from $d\log Y = \boldsymbol{\sigma}' d\log \boldsymbol{y}$ and proposition 1. □

Theorem 1 follows from Proposition 1, as we can compute the change in real GDP by weighing the change in real output for all sectors by the household's demand elasticities. Note that the matrix product between the Leontief inverse and the demand elasticities is equal to the sales shares. This property results from household's maximization problem, the firms' profit maximization decision, goods market clearing, and Cobb-Douglas production functions (see appendix A.2). Therefore, the aggregate impact of productivity and labor supply shocks can be summarized as the sales-share-weighted impact of these shocks on sectoral output directly through production and indirectly through labor markets.

Finally, we are interested in the response of aggregate unemployment to sector specific shocks. Using the results in Theorem 1, we can derive changes in aggregate unemployment and relative prices.

COROLLARY 1. *Given idiosyncratic labor supply shocks $d\log \boldsymbol{H}$ and productivity shocks $d\log \boldsymbol{A}$, the first-order response in aggregate unemployment is:*

$$d\log U^{agg} = \boldsymbol{\Pi}_{U^{agg},A} d\log \boldsymbol{A} + \boldsymbol{\Pi}_{U^{agg},H} d\log \boldsymbol{H},$$

where

$$\Pi_{U^{agg},A} = \frac{1}{U^{agg}}\boldsymbol{U}'\left[\Lambda_A - \mathcal{L}\Pi_{y,A}\right],$$
$$\Pi_{U^{agg},H} = \frac{1}{U^{agg}}\boldsymbol{U}'\left[\boldsymbol{I} + \Lambda_H - \mathcal{L}\Pi_{y,H}\right].$$

The first-order change in the relative price of the aggregate consumption good is:

$$d\log P = \sigma' d\log \boldsymbol{p},$$

where $d\log \boldsymbol{p}$ *follows proposition 2.*

PROOF. See appendix A.2 □

The change in unemployment is a weighted average of the first-order changes in the size of the labor force and employment. The employment level change for each labor market in equilibrium equates the change in labor demand in that labor market to labor supply. The change in labor demand in that labor market depends on how relative wages respond to shocks, as well as how sectoral output changes for sectors that use that particular type of labor.

The first-order response in aggregate price level is the changes in sectoral prices weighted by their consumption share, which is equivalent to the demand elasticities of consumption when preferences are Cobb-Douglas. Note that one can solve out the price changes explicitly by picking a numeraire, for example, by assuming $d\log p_1 = 0$. However, we don't explicitly do it here in order to maintain a simpler expression for the relationship between the price system and productivity, fixed factor, and labor force shocks.

4.4. Labor Market Frictions and Amplification

Hulten (1978)'s theorem states that in an efficient economy, the first-order effect of an idiosyncratic productivity shock to an industry on aggregate output is equal to that industry's sales share. We compare our results to Hulten's foundational theorem and examine how the interaction between production linkages and labor market frictions impacts aggregate output. In particular, we analyze the conditions under which Hulten's theorem holds, as well as when labor market frictions interact with production linkages to amplify the aggregation of shocks.

In the discussion following Proposition 1, labor market inefficiencies impact output propagation through adjustments in tightness. Tightness generates an additional wedge between wages and the marginal product of labor. This additional wedge exists because firms have to dedicate more resources to recruit in tighter labor markets, increasing the marginal cost of labor. When network price adjusted wages change exactly proportionally to the marginal product of labor, tightness remains unchanged, the first-order impact of search costs is eliminated, and Hulten's theorem holds.

Corollary 2. *Hulten's theorem holds for technology shocks whenever network price adjusted wages change exactly proportionally to the network adjusted marginal product of labor. That is when*

$$d\log \boldsymbol{w} - \mathcal{L}\, d\log \boldsymbol{p} = \mathcal{L}\, d\log MP,$$

where $d\log MP$ is a matrix of changes to the marginal product of each type of labor in each sector. Furthermore, when this condition holds, aggregate changes in response to labor force and factor supply shocks are independent of matching frictions.

Proof. The derivative of production with respect to labor inputs, along with the firms first order conditions, imply that the network adjusted marginal product of labor satisfies

$$\mathcal{L}\, d\log \mathrm{MP} = \mathfrak{TM}\, d\log \theta + d\log \boldsymbol{w} - \mathcal{L}\, d\log \boldsymbol{p}.$$

Imposing that network price adjusted wage changes are exactly proportional to changes in the network adjusted marginal product of labor implies

$$\mathfrak{TM}\, d\log \theta = 0.$$

$\mathfrak{TM}$ is a diagonal matrix with non-zero diagonal elements, therefore

$$d\log \theta = 0.$$

Since search-and-matching operates through changes in $d\log \theta$, this implies that search-and-matching has no impact on the propagation of shocks. In particular, in this case

$$d\log Y = \lambda' \left[d\log \boldsymbol{A} + \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}\, d\log \boldsymbol{H} + \varepsilon^{\boldsymbol{f}}_{\boldsymbol{K}}\, d\log \boldsymbol{K}^{s} \right]$$

The aggregate output response to technology shocks is $\lambda' d \log \boldsymbol{A}$, this is exactly Hulten's theorem. The aggregate output response to the other shocks depends only on production parameters, matching frictions play no role. □

When wages do not respond to shut off changes in tightness, labor market frictions impact the aggregation of idiosyncratic shocks. We can formally characterize the search channel of idiosyncratic shocks as the following:

COROLLARY 3. *When wages do not respond exactly proportionally to the network adjusted marginal product of labor, matching frictions generate deviations from Hulten's theorem captured by*

$$\begin{aligned}
\Pi_{search,A} &= \lambda' \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} \left(\boldsymbol{I} - \mathcal{M}\left(\boldsymbol{I} + \mathcal{T}\right)\right) \left[\boldsymbol{I} - \mathcal{M} - \Xi_{\theta}\right]^{-1} \left(\mathcal{L}\Psi - \Lambda_A\right), \\
\Pi_{search,H} &= \lambda' \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} \left(\boldsymbol{I} - \mathcal{M}\left(\boldsymbol{I} + \mathcal{T}\right)\right) \left[\boldsymbol{I} - \mathcal{M} - \Xi_{\theta}\right]^{-1} \left(\left[\mathcal{L}\Psi \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} - \boldsymbol{I}\right] - \Lambda_H\right), \\
\Pi_{search,K} &= \lambda' \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} \left(\boldsymbol{I} - \mathcal{M}\left(\boldsymbol{I} + \mathcal{T}\right)\right) \left[\boldsymbol{I} - \mathcal{M} - \Xi_{\theta}\right]^{-1} \left(\mathcal{L}\Psi \varepsilon^{\boldsymbol{f}}_{\boldsymbol{K}} - \Lambda_K\right).
\end{aligned}$$

PROOF. Follows from Theorem 1 and 2. □

The search channel is a product of sales shares λ, the labor elasticity matrix $\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}$, the search cost term $(\boldsymbol{I} - \mathcal{M}(\boldsymbol{I} + \mathcal{T}))$, and the first-order response in labor market tightness. We want to examine whether the search channel amplifies the aggregate impact of shocks. Before we proceed, we first define amplification:

DEFINITION 1. *For a shock $x \in \{A, H, K\}$, the search channel* ***amplifies*** *the impact of the shock if $\Pi_{search,x} > 0$, and* ***dampens*** *if $\Pi_{search,x} < 0$.*

In theory, whether amplification occurs depends on the magnitudes of the matching elasticities matrix $\mathcal{M}$, the recruiter-producer ratio matrix $\mathcal{T}$, production structures, occupational structure, as well as the wage schedules. We show that under common and nonrestrictive parametrization, rigid wages are enough to guarantee amplification of shocks through the search channel.

ASSUMPTION 1. *For occupation $o \in \{1, 2, ..., \mathcal{O}\}$, the matching elasticity η_o and recruiter-producer ratio satisfy $\frac{1}{(1+\tau_o)} \geq \eta_o$.*

This assumption states that ratio between productive workers and total workers is greater than or equal to the matching elasticity. Petrongolo and Pissarides (2001) claims that a matching elasticity between 0.5 and 0.7 is plausible and Landais et al. (2018) finds that the share of recruiters in the workforce is around 2.3%. For this inequality to break, the share of recruiters would have to be at least 30%, which is unrealistic.

PROPOSITION 3. *Under Assumption 1, we have the following amplification results:*

- *If* $\mathbf{\Gamma}_{\boldsymbol{A}} < \mathcal{L}\mathbf{\Psi}$*,* $\mathbf{\Pi}_{search,A} > 0$.
- *If* $\mathbf{\Gamma}_{\boldsymbol{H}} < \mathcal{L}\mathbf{\Psi}\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} - \boldsymbol{I}$*,* $\mathbf{\Pi}_{search,H} > 0$.
- *If* $\mathbf{\Gamma}_{\boldsymbol{K}} < \mathcal{L}\mathbf{\Psi}\varepsilon^{f}_{K}$*,* $\mathbf{\Pi}_{search,K} > 0$.

PROOF. When $\frac{1}{(1+\tau_o)} \geq \eta_o$, $(\boldsymbol{I} - \mathcal{M}\,(\boldsymbol{I} + \mathcal{T}))$ is greater than 0 on the diagonals, and the equilibrium adjustment coefficients $\left[\boldsymbol{I} - \mathcal{M} - \Xi_\theta\right]^{-1}$ is non-negative element-wise(shown in Appendix C). □

Proposition 3 shows that, when wages adjust less than the occupation and network-adjusted total/labor/factor productivity, the search channel amplifies the aggregate impact of idiosyncratic shocks on output.

5. Calibration

So far, we have established that search frictions on the labor market can change the propagation of shocks in a network economy. In this section, we test the empirical relevance of the aforementioned channels by calibrating our model to the U.S. economy. We use two different labor market definitions to highlight our model's flexibility in allowing for segmentation in labor markets along several dimensions and to demonstrate the important role sector-to-sector worker transitions can play in shock propagation. The two assumptions about labor markets are

(i) There is a unique occupation corresponding to each production sector. For instance, the durable manufacturing sector employs only durable manufacturing workers. This calibration has lower data requirements than using a more complex definition for occupations. As discussed below and in Appendix E, we can get all of the information we need to calibrate our model from JOLTs, the CPS, and the input-output accounts. Despite its simplicity, this calibration is able to generate interesting amplification and unemployment dynamics in response to technology shocks. However, because workers are constrained to stay within a single sector, this calibration leads to effects that remain more localized in the shocked sector.

(ii) To address the limitations of the first calibration, we split the labor force into major occupation categories and allow firms in each sector to use a mix of the different major occupations in production. Workers are constrained to remain in one occupation, but because firms in different sectors use multiple occupations as an input, workers are no longer constrained to remain in just one sector. This allows for sector-to-sector labor transitions, a potentially important feature of the labor market in many countries. For instance, in the U.S. about 12% to 20% of jobs switchers also change industries at the 1-digit level (Kambourov and Manovskii 2008; Parrado et al. 2007). Neffke et al. (2017) find that nearly 59% of German job switchers change industry at the most aggregate German industry grouping. Our major occupations calibration allows for some of this industry-to-industry flow of workers, but at the cost of higher data requirements.

We use productivity shocks to the durable manufacturing sector to demonstrate the quantitative importance of incorporating search frictions and the role wages play for the propagation of productivity shocks. In the one occupation per sector calibration, we also use shocks to the size of the durable manufacturing workforce to demonstrate the propagation of labor force shocks. In the major occupations calibration, we instead shock the labor force of production workers, who make up the largest component of the durable manufacturing workforce, but work in many other sectors as well. We report responses to shocks to all other sectors in Appendix F.

5.1. Wage Assumptions

Wages play an important role in how shocks propagate in our model economy. In fact, as 2 shows, for the right assumption about wages, search frictions can have no effect whatsoever on shock propagation. To avoid taking a strong stance on exactly how wages change, we test the quantitative consequences of shocks in our model under a set of several different wage assumptions that cover a broad swathe of existing assumptions used in the literature.

Throughout this section, we report results for the following wage assumptions

(i) The network price adjusted wage changes are the same as changes in the network adjusted marginal product of labor:

$$d\log \boldsymbol{w} - \mathcal{L} d\log \boldsymbol{p} = \mathcal{L} d\log \mathrm{MP}.$$

Theorem 2 shows that Hulten's theorem holds for this assumption about wage changes. We therefore label this assumption "Hulten" in the figures below.

(ii) The network price adjusted wage changes half as much as the network adjusted marginal product of labor:

$$d\log \boldsymbol{w} - \mathcal{L} d\log \boldsymbol{p} = 0.5\mathcal{L} d\log \mathrm{MP}.$$

In our setup, this is similar to assuming Nash bargaining with equal bargaining weights for firms and workers. We label this assumption "0.5MP" in the figures below.

(iii) The network price adjusted wage does not change in response to shocks:

$$d\log \boldsymbol{w} - \mathcal{L}\delta \log \boldsymbol{p} = 0.$$

This is akin to assuming real wages are rigid. We label this assumption "Rigid Real" in the figures below.

(iv) The nominal wage does not change in response to shocks:

$$d\log \boldsymbol{w} = 0.$$

We label this assumption "Rigid Nominal" in the figures below.

5.2. Data Sources

Vacancies and Hires. We use vacancy and hiring data from The Job Openings and Labor Turnover Survey (JOLTs), which provides survey-based measures of job openings and hires at a monthly frequency. This survey data is available for 13 industries that roughly correspond to the two-digit NAICS classification. The data is available from Dec 2000 to Feb 2023 at a monthly frequency and from 2001 to 2022 at a yearly frequency.

Unemployment. We use sector level unemployment from the Current Population Survey (CPS). The data cover 13 sectors at a monthly frequency for the same range as the JOLTs data. We use the full monthly series, along with the JOLTs data, to estimate the sector level matching elasticity with respect to unemployment (η_i).

Major Occupations. We use occupation level data from the BLS's Occupational Employment and Wage Statistics(OEWS) to construct sector by sector major occupational employment and wages.

Input-output Linkages. Although JOLTs and the CPS include labor market data at a level that corresponds roughly to the two-digit NAICS classification, this correspondence is not exact. To construct input intensities that match with availability level of labor market data, we use the 2021 3-digit NAICS classification level Make and Use tables from the BEA, and aggregate back up to match the 13 CPS industries (Horowitz and Planting 2009). The BEA Make and Use tables allow us to calculate the intermediate input intensity of each sector, the labor intensity of each sector, and the elasticity of final consumption demand to each sectors output. We use employee compensation, recorded in the Use table to calculate labor elasticities. [5]

Factor Shares. In our calibration, we consider two factors: capital and energy. Capital and energy shares for the two-digit NAICS classification are available in the BEA-BLS Integrated Industry-level Production Accounts (KLEMS). We use an output-weighted mean of fixed factor shares of two-digit NAICS industries to compute the factor shares to match the corresponding CPS industries. We use these shares as energy and capital elasticities, and impose constant returns by scaling them along with the other production elasticities acquired from the input-output tables.

5.3. Effects of a 1% shock to productivity in the durable manufacturing sector

Figures 4A, 4C, and 4E show the response of output, tightness, and unemployment to a 1% shock to productivity in the durable manufacturing sector when there is one unique occupation per sector (no labor mobility across sectors). Figures 4B, 4D, and 4F show the response of output, tightness, and unemployment to the same shock assuming firms hire workers from major occupational groups (some labor mobility across sectors). The blue bars capture the effects of the productivity shock when wage changes are exactly proportional to the marginal product of labor. When this is the case, Hulten's theorem holds and labor market frictions do not impact shock propagation. The effects are therefore identical to those in a standard networks model with fully efficient labor markets. This provides a useful benchmark to which we can compare our results under different assumptions about wages.

[5] We provide more details on data and calibration in Appendix E.

When Hulten's theorem holds the 1% shock to productivity in durable manufacturing boosts output in the durable manufacturing sector by 1.40% and boosts aggregate output by 0.26%. Clearly, even standard production networks can amplify the effects of idiosyncratic sector level shocks and give these shocks a role in determining aggregate output. When wages respond by less than the change in the marginal product of labor, labor market frictions can significantly boost the effects of the productivity shock on output. For instance, when the network price adjusted wage does not respond to the technology shock the effect on durable manufacturing output is 81% larger, at 2.54%, and the effect on aggregate output is 149% larger, at 0.65%, in the one occupation per sector case. When we allow sectors to use different major occupational labor types the own sector amplification of the productivity shock is smaller. In this case, assuming the network price adjusted wage does not change boosts the response of output in the durable manufacturing sector by just 24% to 1.75%. The amplification of aggregate output, however, remains substantial. Aggregate output responds 115% more than under Hulten's theorem, rising by 0.56%, demonstrating that our major occupation calibration increases the propagation of sector specific shocks to other sectors.

Without labor market frictions, productivity shocks have no effect on tightness or unemployment. Labor market frictions are therefore the key to understanding how unemployment changes in response to the productivity shocks. Assuming the network price adjusted wage does not respond to the productivity shock, the number of unemployed workers falls by 2.5% in the durable manufacturing sector, by 1.2% in the construction sector, and aggregate unemployment falls by 0.46%, when there is one occupation per sector. Assuming sectors can hire from multiple major occupations, the unemployment effects are more spread out, with the number of unemployed production workers falling by 0.99%, the number of unemployed engineers falling by 0.77%, and the number of unemployed construction workers falling by 0.70%. In aggregate, the number of unemployed workers falls by 0.44%.

Overall, this example demonstrates that incorporating labor market frictions can have quantitatively important implications for the effects of sector specific productivity shocks on output and allows us to explore their consequences for unemployment, which would not be possible in a standard networks model.

figures/calib2/dur_AshockY.png

A. One occupation per sector.

figures/calib3/dur_AshockY.png

B. Major occupations.

figures/calib2/dur_AshockT.png

C. One occupation per sector.

figures/calib3/dur_AshockT.png

D. Major occupations.

figures/calib2/dur_AshockU.png

E. One occupation per sector.

figures/calib3/dur_AshockU.png

F. Major occupations.

FIGURE 4. Response of output, tightness, and unemployment to a 1% shock to technology in the durable manufacturing sector.

5.4. Effects of a 1% shock to the size of the labor force

Figures 5A, 5C, and 5E show the response of output, tightness, and unemployment to a 1% shock to the size of the labor force in the durable manufacturing sector when there is one unique occupation per sector. Figures 5B, 5D, and 5F show the response of output, tightness, and unemployment to a 1% shock to the size of the labor force of production workers when firms can hire from a number of major occupational categories. There is no direct equivalent to shocking to a 1% durable manufacturing labor force shock in this setup. We choose to shock production workers since 46% of production workers work in durable manufacturing and they constitute the largest share of workers in the durable manufacturing sector. However, production workers also work in a number of other sectors: 26% of production workers work in non-durable manufacturing and 10% work in trade.

Without labor market frictions, firms are forced to hire all available workers and the increase the labor force therefore generates large changes in output. With labor market frictions, on the other hand, the effect on output depends on firms hiring decisions in response to wage changes. When wages do not change by much in response to the shock to the size of the labor force firms hiring does not change by much, and as a result output does not change by much. This is demonstrated by figures 5A and 5B. The blue bars show the response of output when there are no labor market frictions and the red bars, which are near imperceptible, show that output barely responds when the network price adjusted wages do not change at all in response to the shock.

Figure 5B also demonstrates that the propagation of the labor force shock is altered meaningfully by the production structure, and does not depend only on the labor usage matrix. The largest effects of the shock are concentrated in the two largest users of production workers, durable and non-durable manufacturing. But the change in output in construction and other services are both larger than the change in output in the trade sector even though trade uses 5 times more production workers than other services and 10 times more production workers than construction.

The bottom two panels of figure 2 demonstrate that, as was the case for productivity shocks, labor force shocks only change unemployment and tightness with labor market frictions and incomplete pass through of the shock to wages. As expected, the fact that our model does not allow workers to move across occupations keeps the effects on employment and tightness concentrated in the shocked occupation. However, since many sectors use production workers, if we aggregated the right side of figure 2 to the sector level, we would see changes across sectors in proportion to their usage of production workers. For instance, figure 6 gives a rough idea of changes in sector level tightness from a 1% shock to the production workers occupation, by aggregating up using sector level labor shares.

figures/calib2/dur_HshockY.png

A. One occupation per sector.

figures/calib3/Prod_HshockY.png

B. Major occupations.

figures/calib2/dur_HshockT.png

C. One occupation per sector.

figures/calib3/Prod_HshockT.png

D. Major occupations.

figures/calib2/dur_HshockU.png

E. One occupation per sector.

figures/calib3/Prod_HshockU.png

F. Major occupations.

FIGURE 5. Response of output, tightness, and unemployment to a 1% shock to the size of the labor force. In the one output per sector case we shock the durable manufacturing labor force. In the major occupations case we shock the production labor force.

figures/calib3/prod_HshockTsec.png

FIGURE 6. Effect of a 1% shock to labor force of production workers on sector level tightness.

6. Application: Adverse Energy and Labor Supply Shocks to the U.S Economy

Our model features two types of non-intermediate production inputs: those that are subject to search and matching, such as labor, and those that are not, such as energy. We apply our model to evaluate the effect of these two types of factor shocks. Specifically, we analyze the aggregate and granular impact of energy supply and labor supply shocks, which the US experienced in recent years due to the Russian-Ukraine war and the decrease in labor force participation during COVID. We provide a qualitative illustration of how these adverse factor supply shocks would impact sectoral production and disaggregated labor market tightness. While our model focuses on real variables, we demonstrate that relative prices move in a realistic way.

6.1. Effects of an adverse energy shock

On March 8, 2022, the White House issued a ban on imports of Russian oil, liquefied natural gas, and coal. Such ban constitutes a large adverse energy shock, as the U.S imported nearly 700, 000 barrels of crude oil and refined petroleum products from Russia per day in 2021. In this section, we examine the impact of an adverse energy shock, using the major occupation setup. Figure 7A shows the response of output to the adverse energy shock. The rigid wage specifications predicts an aggregate output decline of around 1% in response to an adverse 10% energy shock, twice as large as when search and matching frictions are absent. The transportation sector is most negatively impacted, followed by the accommodation sector, the construction sector, and the trade sector, as they have high energy elasticities. Search and matching frictions amplify the impact of energy shocks significantly through the search and matching channel. These amplifications create large responses in other downstream sectors, such as government and professional services, despite their low energy elasticities.

figures/energy_shock/energy_KshockY.png

A. Sectoral and aggregate output response to an adverse energy shock

figures/energy_shock/energy_KshockT.png

B. Occupational tightness response to an adverse energy shock

figures/energy_shock/energy_KshockTsec.png

C. Sectoral tightness response to an adverse energy shock

figures/energy_shock/energy_KshockP.png

D. Relative price response to an adverse energy shock

FIGURE 7. First-order responses to an adverse energy shock under sector-specific occupations specification under major occupations specification.

Figure 7B plots the response in occupational labor market tightness to the adverse energy shock. The labor market tightness for management and transportation service workers decreases the most, at around 2%. Similarly, the construction occupation also experience a large decline in tightness. The decline in labor tightness for transportation service workers and construction workers is straightforward, as the construction and transportation sectors are energy-intensive. Most other occupations experience a similar and sizable decline in labor market tightness. This is because sectors, who are affected directly through having higher energy elasticities, or indirectly through their production linkages with energy-intensive sectors, lower their labor demand for the occupations they employ, thus changing labor market tightness across occupations.

Figure 7C plots the response in sectoral labor market tightness reconstructed from occupational tightness movements. Note that this reconstruction is only valid under our calibration assumptions, and should provide a rough estimate of how sectoral tightness changes. We see the transportation sector experiences the largest decrease in tightness of around 2%. The decline in tightness for other sectors appears to be similar.

Figure 7D documents the first-order price changes, using education and health as the numeraire sector. We choose the education and health as the numeraire for one simple reason, which is that inflation for medical care service was 0.4% from 2022 to 2023, much more stable than most other goods and services. We find that, first-order price responses differ across sectors and across wage specifications. We observe the largest price increases in sectors that suffer the largest output declines. In particular, we see a large increase in transportation prices. This is qualitatively consistent with large observed increases in transportation prices between 2022 and 2023[6]. We also see price increases in the accommodation sector and the construction sector.

6.2. Effects of adverse labor supply shocks

Another prominent feature of the U.S labor market after the COVID-19 pandemic is the occurrence of the “Great Reshuffle" and the “Great Resignation." These labor market phenomena signify structural labor market changes, such as more people quitting their jobs, more people retiring early, and people switching jobs for better pay and work-life balance. We focus on two aspects of this phenomenon. First, we assume that there is a reduction in labor supply for the service occupations, as suggested by the high quit rates in the accommodation sector. According to the Bureau of Labor Statistics(BLS), in September 2021, the quit rate for the accommodation sector was 6.6%, whereas the quit rate for private industries overall was 3.4%. Second, we assume there is a reduction in the overall labor participation rate, as evidenced by Hobijn and Şahin (2022). In both cases we assume the magnitude of the shock for each impacted occupation is 2%. This is consistent with Hobijn and Şahin (2022)’s finding that the labor force participation rate declined by roughly 1.2 percentage points. We examine the macroeconomic impact of these two specifications in the context of our model separately.

To start off, we examine the impact of adverse labor supply shocks to the service occupations. Figures 8A,8C, 8E and8G show the output, tightness, and relative price responses to a 2% adverse labor supply shock to the following ONET major occupation classification: building and grounds cleaning and maintenance occupations, food preparation and serving related occupations, personal care and service occupations,

[6]According to the Bureau of Transportation Statistics, transportation prices rose 15.5% overall between 2022 and 2023

and transportation and material moving occupations. Overall, while the efficient benchmark exhibits large movements in output and prices, our specifications show very small changes. However, when it comes to labor market tightness, we can see clear responses in the occupations we shock. From Figure 8E can also see that sectoral tightness increases for all sectors, but increases the most for sectors such as accommodation, transportation, retail, and other services.

figures/covid_labor_shocks/energy_HsershockY.png

figures/covid_labor_shocks/energy_HallshockY.png

A. Output response to service occupation shocks

B. Output response to labor force participation shocks

figures/covid_labor_shocks/energy_HsershockT.png

figures/covid_labor_shocks/energy_HallshockT.png

C. Occupational tightness response to service occupation shocks

D. Occupational tightness response to labor force participation shocks

figures/covid_labor_shocks/energy_HsershockTsec.png

figures/covid_labor_shocks/energy_HallshockTsec.p

E. Sectoral tightness response to service occupation shocks

F. Sectoral tightness response to labor force participation shocks

figures/covid_labor_shocks/energy_HsershockP.png

figures/covid_labor_shocks/energy_HallshockP.png

G. Relative price response to an adverse labor supply shock in service occupations

H. Relative price response to an adverse labor supply shock in all occupations

FIGURE 8. Left: First-order responses to an adverse labor supply shock in service occupations. Right: First-order responses to an adverse labor supply shock in all occupations.

Why is the output response to labor supply shock so muted? The effect of output depends on how much the cost of productive workers changes. Since the recruiter producer ratio is the US economy tends to be fairly small, when wages do not change directly an adverse labor supply shock to service occupations does not alter the cost of productive workers substantially. Therefore, the shock has negligible effects on output. The small effects of occupational labor supply shocks on output and prices means that changes in labor demand for other types of occupation are small as well, which is why we see very little movement in tightness for other occupations.

Next, we analyze the impact of adverse labor supply shocks to all occupations. Figures 8B,8D, 8F, and 8H document the output, tightness, and relative price responses to an adverse shock to all occupations. Overall, the patterns of responses are similar to what we observe for service occupation shocks. The only difference is that, the output responses, while still small compared to the efficient benchmark, are visible on the graphs. This suggests that the change in labor demand induced by negative output responses can dampen the increase in tightness we observe, although the effect are quantitatively negligible.

6.3. Combination Shocks

Now, we combine the energy shock and the labor participation shock. Although the first-order impact of these shocks are linearly additive, it is helpful to see how they affect the economy together. Figures 9A, 9B, 9C, and 9D show the combined output, tightness and relative price responses to the shocks that we believe are relevant for the U.S economy in the past few years. Overall, we see a moderate drop in real output, an increase in tightness across all occupations and sectors, and increases in relative prices for sectors such as transportation, accommodation, and construction.

figures/combo_shocks/energy_HKshockY.png

A. Output response to energy and labor force participation shocks

figures/combo_shocks/energy_HKshockT.png

B. Occupational tightness response to energy and labor force participation shocks

figures/combo_shocks/energy_HKshockTs.png

C. Sectoral tightness response to energy and labor force participation shocks

figures/combo_shocks/energy_HKshockP.png

D. Relative price response to energy and labor force participation shocks

FIGURE 9. First-order responses to a shock in labor force participation rate combined with an adverse energy shock.

Compared to the efficient production networks model, does our model do a better job at capturing reality? We find that combined energy and labor supply shocks have an impact on aggregate output that is half of what the conventional model would have predicted. This is mainly due to the smaller effect of labor supply shocks on output in our model. While conventional production network models don't feature unemployment, we find that our model predicts an increase in labor market tightness across sectors. In addition, our model predicts a larger increase in relative prices for sectors that have large energy elasticities, as well as sectors that are linked to them through production and occupations. While our results should only be viewed as qualitative as we have only chosen approximate shock values,our models predictions of a small output contraction and tighter labor markets seem consistent with the U.S. data. U.S. real GDP did decline from Q4 2021 to Q2 2022, but the decline was muted, while labor markets have been exceptionally tight. The standard production networks model would instead predict a much larger decline in output and no movement in tightness.

7. Conclusion

Modern economies feature production networks and frictional, segmented labor markets. We show that accounting for both can alter our understanding of how shocks propagate in a quantitative and qualitatively meaningful way. The impact of matching frictions on network propagation depends on the assumption about wages. We show that under a wide range of assumptions about wages, labor market frictions amplify shocks to productivity. But it is nevertheless possible for the market frictions to dampen shock propagation. This highlights that determining the empirical response of wages to shocks across the network is an import avenue for future work. In addition, a combination of negative energy supply shocks and negative labor force shocks generates simultaneously tight labor markets and depressed output in our framework. While our current model cannot speak to the effect on the price level, in future work we hope to extend the model to incorporate dynamics and nominal rigidities to help paint a realistic picture of the current inflation episode.

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Appendix A. Proofs

A.1. Proof for Propositions 1 and 2

We first show how to prove Proposition 2 by assuming Cobb-Douglas preferences and production.

A.1.1. Price Propagation

Log-linearizing the production function, for each sector i, we have:

$$d\log y_i = \underbrace{\varepsilon^{f_i}_{A_i}}_{=1} d\log A_i + \sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} d\log N_{io} + \sum_{j=1}^{N} \varepsilon^{f_i}_{x_{ij}} d\log x_{ij} + \sum_{k=1}^{\mathcal{K}} \varepsilon^{f_i}_{K_{ik}} d\log K_{ik}$$

Plugging in Equation 4 and Equation 3, the first order conditions for optimal input usage, into the log-linearized production function gives

$$\begin{aligned}
d\log y_i &= \sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} \left[d\log \varepsilon^{f_i}_{N_{io}} + d\log y_i + d\log p_i - d\log w_o - d\log(1+\tau_o(\theta_o))\right] \\
&+ \sum_{j=1}^{N} \varepsilon^{f_i}_{x_{ij}} \left[d\log \varepsilon^{f_i}_{x_{ij}} + d\log y_i + d\log p_i - d\log p_j\right] \\
&+ \sum_{k=1}^{\mathcal{K}} \varepsilon^{f_i}_{K_{ik}} \left[d\log \varepsilon^{f_i}_{K_{ik}} + d\log y_i + d\log p_i - d\log r_k\right] + d\log A_i \\
&= \left[d\log y_i + d\log p_i\right] \underbrace{\left[\sum_{k=1}^{\mathcal{K}} \varepsilon^{f_i}_{K_{ik}} + \sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} + \sum_{j=1}^{N} \varepsilon^{f_i}_{x_{ij}}\right]}_{=1 \text{ by crts}} + \underbrace{\left[\sum_{k=1}^{\mathcal{K}} d\varepsilon^{f_i}_{k_{ik}} + \sum_{o=1}^{\mathcal{O}} d\varepsilon^{f_i}_{N_{io}} + \sum_{j=1}^{N} d\varepsilon^{f_i}_{x_{ij}}\right]}_{=0 \text{ by crts}} \\
&- \sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} \left[d\log w_o + d\log(1+\tau_o(\theta_o))\right] - \sum_{j=1}^{N} \varepsilon^{f_i}_{x_{ij}} \left[d\log p_j\right] - \sum_{k=1}^{\mathcal{K}} \varepsilon^{f_i}_{K_{ik}} d\log r_k + d\log A_i, \qquad \blacksquare
\end{aligned}$$

where the second equality holds because the sum of elasticities equals one for constant returns to scale technology and $\varepsilon^{f_i}_{x_{ij}} d\log \varepsilon^{f_i}_{x_{ij}} = d\varepsilon^{f_i}_{x_{ij}}$.

Rearranging terms gives

$$
\begin{aligned}
d\log p_i &= \sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} \left[d\log w_o + d\log(1+\tau_o(\theta_o))\right] + \sum_{j=1}^{N} \varepsilon^{f_i}_{x_{ij}} \left[d\log p_j\right] + \sum_{k=1}^{\mathcal{K}} \varepsilon^{f_i}_{K_{ik}} d\log r_k - d\log A_i \\
&= \sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} \left[d\log w_o + \varepsilon^{1+\tau_o}_{\theta_o} d\log\theta_o\right] + \sum_{j=1}^{J} \varepsilon^{f_i}_{x_{ij}} \left[d\log p_j\right] + \sum_{k=1}^{\mathcal{K}} \varepsilon^{f_i}_{K_{ik}} d\log r_k - d\log A_i \\
&= \sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} \left[d\log w_o - \tau_o(\theta_o)\varepsilon^{\Omega_o}_{\theta_o} d\log\theta_o\right] + \sum_{j=1}^{J} \varepsilon^{f_i}_{x_{ij}} \left[d\log p_j\right] + \sum_{k=1}^{\mathcal{K}} \varepsilon^{f_i}_{K_{ik}} d\log r_k - d\log A_i
\end{aligned}
$$

Stacking equations over sectors, we can write

$$
d\log \boldsymbol{p} = \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} \left[d\log \boldsymbol{w} - \boldsymbol{\Omega}\boldsymbol{\mathcal{T}} d\log\theta\right] + \boldsymbol{\Omega} d\log \boldsymbol{p} + \varepsilon^{\boldsymbol{f}}_{\boldsymbol{K}} d\log \boldsymbol{r} - d\log \boldsymbol{A}
$$

Which implies

$$
d\log \boldsymbol{p} = \boldsymbol{\Psi}\left[\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} \left[d\log \boldsymbol{w} - \boldsymbol{\Omega}\boldsymbol{\mathcal{T}} d\log\theta\right] + \varepsilon^{\boldsymbol{f}}_{\boldsymbol{K}} d\log \boldsymbol{r} - d\log \boldsymbol{A}\right]
$$

Or equivalently

$$
\left(\boldsymbol{I} - \boldsymbol{\Psi}\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}\boldsymbol{\mathcal{L}}\right) d\log \boldsymbol{p} = \boldsymbol{\Psi}\left[\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} \left[d\log \boldsymbol{w} - \boldsymbol{\mathcal{L}} d\log \boldsymbol{p} - \boldsymbol{\Omega}\boldsymbol{\mathcal{T}} d\log\theta\right] + \varepsilon^{\boldsymbol{f}}_{\boldsymbol{K}} d\log \boldsymbol{r} - d\log \boldsymbol{A}\right]
$$

Where $d\log \boldsymbol{r}$ is pinned down by market clearing in the additional factor market for K_k. Assuming Cobb-Douglas production, for any i

$$
d\log r_k = d\log y_i + d\log p_i - d\log K_{ik}
$$

Which means that $d\log k_{ik} = d\log K_{jk}$ for all i, j. Furthermore, this means that $d\log K_{ik} = d\log K^s_k$ for all i. So we can write,

$$
d\log r_k = d\log y_i + d\log p_i - d\log K^s_k
$$

Which pins down $d\log r_k$ given a numeraire sector.

A.1.2. Output Propagation

Since the log-linearized expression for the Domar weight must hold for every sector, we can write

$$\begin{aligned} d\log\lambda_i - d\log\lambda_j &= d\log p_i - d\log p_j + d\log y_i - d\log y_j \\ &= d\log x_{ij} - d\log \varepsilon^{f_i}_{x_{ij}} - d\log y_j \\ \Rightarrow d\log x_{ij} &= d\log\lambda_i - d\log\lambda_j + d\log y_j + d\log \varepsilon^{f_i}_{x_{ij}} \end{aligned}$$

Plugging back into the production function,

$$\begin{aligned} d\log y_i &= d\log A_i + \sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} d\log N_{io} + \sum_{j=1}^{J} \varepsilon^{f_i}_{x_{ij}} + d\log y_j + \sum_{k=1}^{\mathcal{K}} \varepsilon^{f_i}_{K_{ik}} d\log K_{ik} \\ &= d\log A_i + \sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} d\log N_{io} + \sum_{j=1}^{J} \varepsilon^{f_i}_{x_{ij}} d\log y_j + \sum_{k=1}^{\mathcal{K}} \varepsilon^{f_i}_{K_{ik}} d\log K^s_K \end{aligned}$$

Using the definition of labor demand,

$$\begin{aligned} \sum_i \frac{l_{io}}{L_o} d\log N_{io} &= d\log L^d_o - d\log(1+\tau_o(\theta_o)) \\ &= d\log L^d_o + \tau_o(\theta_o)\varepsilon^{Q_o}_{\theta_o} d\log\theta_o \end{aligned}$$

From output labor usage, we have that labor usage ratio for an occupation by two different sectors as:

$$\frac{l_{io}}{l_{jo}} = \frac{\varepsilon^{f}_{N_{io}}\lambda_i}{\varepsilon^{f}_{N_{jo}}\lambda_j}$$

for any $l_{io}, l_{jo} > 0$

Log-linearizing it, assuming Cobb-Douglas preferences, yields:

$$d\log l_{io} = d\log l_{jo}$$

Also since, $d\log l_{io} = d\log N_{io} + d\log(1+\tau_o(\theta_o)) = d\log l_{jo} = d\log N_{jo} + d\log(1+\tau_o(\theta_o))$, we have that $d\log N_{io} = d\log N_{jo}$

Using the labor market clearing condition, and the definition of labor supply,

$$\sum_k \frac{l_{ko}}{L_o} d\log N_{ko} = \left(\varepsilon^{\mathcal{F}_o}_{\theta_o} + \tau_o(\theta_o)\varepsilon^{\mathcal{Q}_o}_{\theta_o}\right) d\log\theta_o + d\log H_o$$
$$\Rightarrow d\log N_{io} \underbrace{\sum_k \frac{l_{ko}}{L_o}}_{=1} = \left(\varepsilon^{\mathcal{F}_o}_{\theta_o} + \tau_o(\theta_o)\varepsilon^{\mathcal{Q}_o}_{\theta_o}\right) d\log\theta_o + d\log H_o$$

Plugging this back into the linearized production function gives:

$$d\log y_i = d\log A_i + \sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} \left[\left(\varepsilon^{\mathcal{F}_o}_{\theta_o} + \tau_o(\theta_o)\varepsilon^{\mathcal{Q}_o}_{\theta_o}\right) d\log\theta_o + d\log H_o\right]$$
$$+ \sum_{j=1}^{J} \varepsilon^{f_i}_{x_{ij}} d\log y_j + \sum_{k=1}^{\mathcal{K}} \varepsilon^{f_i}_{K_{ik}} d\log K^s_k$$

Stacking over sectors gives,

$$d\log \boldsymbol{y} = d\log \boldsymbol{A} + \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} (\mathfrak{F} + \mathfrak{Q}\mathfrak{T})\, d\log\theta + \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} d\log \boldsymbol{H} + \boldsymbol{\Omega} d\log \boldsymbol{y} + \varepsilon^{\boldsymbol{f}}_{\boldsymbol{K}} d\log \boldsymbol{K}^s$$

Which implies

$$d\log \boldsymbol{y} = \boldsymbol{\Psi}\left(d\log \boldsymbol{A} + \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} (\mathfrak{F} + \mathfrak{Q}\mathfrak{T})\, d\log\theta + \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} d\log \boldsymbol{H} + \varepsilon^{\boldsymbol{f}}_{\boldsymbol{K}} d\log \boldsymbol{K}^s\right)$$

A.1.3. Tightness Propagation

Labor market clearing implies that changes in labor demand have to equal changes in labor supply:

$$d\log L^s_o(\theta, \boldsymbol{H}) = d\log L^d_o(\theta, \boldsymbol{A}).$$
$$\varepsilon^{\mathcal{F}_o}_{\theta_o} d\log\theta_o + d\log H_o = \sum_{i=1}^{J} \frac{l_{io}}{L^d_o} d\log l_{io}(\theta_o)$$

Where $\frac{l_{io}}{L^d_o} = \frac{\varepsilon^{f_i}_{N_{io}} p_i y_i}{\sum_{j=1}^{J} \varepsilon^{f_j}_{N_{jo}} p_j y_j}$[7]. For every sector i we have

$$d\log l_{io}(\theta_o) = d\log \varepsilon^{f_i}_{N_{io}} - d\log w_o + d\log p_i + d\log y_i$$

Which implies that

$$d\log \boldsymbol{L^d}(\theta) = \text{diag}\left(\mathcal{L} d\log \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}\right) - \left[d\log \boldsymbol{w} - \mathcal{L} d\log \boldsymbol{p}\right] + \mathcal{L} d\log \boldsymbol{y}$$

since $\sum_{i=1}^{J} \frac{l_{io}}{L^d_o} = 1$ for all o. Plugging in for $d\log \boldsymbol{y}$ gives

$$\begin{aligned} d\log \boldsymbol{L^d}(\theta) = {} & \text{diag}\left(\mathcal{L} d\log \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}\right) - \left[d\log \boldsymbol{w} - \mathcal{L} d\log \boldsymbol{p}\right] \\ & + \mathcal{L}\Psi\left[d\log \boldsymbol{A} + \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}\left(\mathcal{F} + \mathfrak{Q}\mathfrak{T}\right) d\log\theta + \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} d\log \boldsymbol{H} + \varepsilon^{\boldsymbol{f}}_{\boldsymbol{K}} d\log \boldsymbol{K}^s\right] \end{aligned}$$

Labor market clearing implies

$$\mathcal{F} d\log\theta + d\log \boldsymbol{H} = \mathcal{L}\Psi\left[d\log \boldsymbol{A} + \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}\left(\mathcal{F} + \mathfrak{Q}\mathfrak{T}\right) d\log\theta + \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} d\log \boldsymbol{H} + \varepsilon^{\boldsymbol{f}}_{\boldsymbol{K}} d\log \boldsymbol{K}^s\right] - \left[d\log \boldsymbol{w} - \mathcal{L} d\log \boldsymbol{p}\right]$$

Which pins down first order changes in log tightness as

$$d\log\theta = \left[\mathcal{F} - \Xi_\theta\right]^{-1}\left[\mathcal{L}\Psi d\log \boldsymbol{A} - \left[d\log \boldsymbol{w} - \mathcal{L} d\log \boldsymbol{p}\right] + \left[\mathcal{L}\Psi\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} - \boldsymbol{I}\right] d\log \boldsymbol{H} + \mathcal{L}\Psi\varepsilon^{\boldsymbol{f}}_{\boldsymbol{K}} d\log \boldsymbol{K}^s\right]$$

Where $\Xi_\theta = \mathcal{L}\Psi\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}\left[\mathcal{F} + \mathfrak{Q}\mathfrak{T}\right]$.

Therefore, combining these propagation expressions, we are able to expres $d\log\theta$, $d\log \boldsymbol{y}$, and $d\log \boldsymbol{p}$ in terms of exogenous shocks.

A.2. Proof for Theorem 1 and corrollaries

Let $\lambda_i = \frac{p_i y_i}{G}$, where $G = \sum_j p_j c_j = GDP = \sum_{o=1}^{\mathcal{O}} w_o L_o$, denote the final sales share of GDP for sector i. We have that:

$$p_j x_{ij} = \varepsilon^{f_i}{}_{x_{ij}} \lambda_i G.$$

[7] One implication of this formula is that I think we should be able to check whether the elasticities $\left\{\left\{\varepsilon^{f}_{N_{io}}\right\}_{o=1}^{\mathcal{O}}\right\}_{i=1}^{J}$ are consistent with the Domar weights.

From household's maximization problem, I have that $p_i c_i = \varepsilon^{\mathcal{D}}_{c_i} G$. Combining the two gives:

$$\varepsilon^{\mathcal{D}}_{c_j} G = p_j y_j = p_j (c_j + \sum_i x_{ij}) = \left(\varepsilon^{\mathcal{D}}_{c_j} + \sum_i \varepsilon^{f_i} x_{ij} \lambda_i \right) G$$
$$\Rightarrow \boldsymbol{\lambda} = \boldsymbol{\Psi}' \varepsilon^{\boldsymbol{D}}_{\boldsymbol{c}}.$$

The aggregate labor force, employment, and unemployment are $H^{agg} = \sum_{o=1}^{\mathcal{O}} H_o$, $L^{agg} = \sum_{o=1}^{\mathcal{O}} L_o$, and $U^{agg} = \sum_{o=1}^{\mathcal{O}} U_o$. Changes in aggregates are therefore given by

$$dH^{agg} = \sum_{o=1}^{\mathcal{O}} dH_o$$
$$dL^{agg} = \sum_{o=1}^{\mathcal{O}} dL_o$$
$$dU^{agg} = \sum_{o=1}^{\mathcal{O}} dU_o$$

Or in terms of log changes

$$d \log H^{agg} = \frac{1}{H^{agg}} \sum_{o=1}^{\mathcal{O}} H_o d \log H_o$$
$$d \log L^{agg} = \frac{1}{L^{agg}} \sum_{o=1}^{\mathcal{O}} L_o d \log L_o$$
$$d \log U^{agg} = \frac{1}{U^{agg}} \sum_{o=1}^{\mathcal{O}} U_o d \log U_o$$

In matrix notation

$$d \log H^{agg} = \frac{1}{H^{agg}} \boldsymbol{H}' d \log \boldsymbol{H}$$
$$d \log L^{agg} = \frac{1}{L^{agg}} \boldsymbol{L}' d \log \boldsymbol{L}$$
$$d \log U^{agg} = \frac{1}{U^{agg}} \boldsymbol{U}' d \log \boldsymbol{U}$$

Substituting in for $d\log \boldsymbol{L}$

$$
\begin{aligned}
d\log L^{agg} &= \frac{1}{L^{agg}}\boldsymbol{L'}\left[\mathcal{L}d\log\ \boldsymbol{y} - \left[d\log\boldsymbol{w} - \mathcal{L}d\log\ \boldsymbol{p}\right]\right] \\
&= \frac{1}{L^{agg}}\boldsymbol{L'}\left[\mathcal{L}\left[\boldsymbol{\Pi}_{y,A}d\log\boldsymbol{A} + \boldsymbol{\Pi}_{y,H}d\log\boldsymbol{H} - \boldsymbol{\Lambda}_A d\log\boldsymbol{A} - \boldsymbol{\Lambda}_H d\log\boldsymbol{H}\right]\right] \\
&= \boldsymbol{\Pi}_{L^{agg},A}d\log\boldsymbol{A} + \boldsymbol{\Pi}_{L^{agg},H}d\log\boldsymbol{H}
\end{aligned}
$$

Where

$$
\begin{aligned}
\boldsymbol{\Pi}_{L^{agg},A} &= \frac{1}{L^{agg}}\boldsymbol{L}'\left[\mathcal{L}\boldsymbol{\Pi}_{y,A} - \boldsymbol{\Lambda}_A\right] \\
\boldsymbol{\Pi}_{L^{agg},H} &= \frac{1}{L^{agg}}\boldsymbol{L}'\left[\mathcal{L}\boldsymbol{\Pi}_{y,H} - \boldsymbol{\Lambda}_H\right]
\end{aligned}
$$

And

$$
\begin{aligned}
d\log U^{agg} &= \frac{1}{U^{agg}}\boldsymbol{U}'\left[d\log\boldsymbol{H} - d\log\boldsymbol{L}\right] \\
&= \frac{1}{U^{agg}}\boldsymbol{U}'\left[d\log\boldsymbol{H} - \left[\mathcal{L}\left[\boldsymbol{\Pi}_{y,A}d\log\boldsymbol{A} + \boldsymbol{\Pi}_{y,H}d\log\boldsymbol{H} - \boldsymbol{\Lambda}_A d\log\boldsymbol{A} - \boldsymbol{\Lambda}_H d\log\boldsymbol{H}\right]\right]\right] \\
&= \boldsymbol{\Pi}_{U^{agg},A}d\log\boldsymbol{A} + \boldsymbol{\Pi}_{U^{agg},H}d\log\boldsymbol{H} \qquad \blacksquare
\end{aligned}
$$

Where

$$
\begin{aligned}
\boldsymbol{\Pi}_{U^{agg},A} &= \frac{1}{U^{agg}}\boldsymbol{U}'\left[\boldsymbol{\Lambda}_A - \mathcal{L}\boldsymbol{\Pi}_{y,A}\right] \\
\boldsymbol{\Pi}_{U^{agg},H} &= \frac{1}{U^{agg}}\boldsymbol{U}'\left[\boldsymbol{I} + \boldsymbol{\Lambda}_H - \mathcal{L}\boldsymbol{\Pi}_{y,H}\right]
\end{aligned}
$$

Appendix B. Results for general CRTS production functions and one occupation per sector

In this section we generalize our results to any constant returns to scale production function, under the assumption that there is one type of labor per sector. This generalization results in additional terms that capture how the production elasticities change when shocks hit the economy. The expressions are otherwise similar to above. The model setup is identical, we just do not impose Cobb-Douglas technology and instead impose $\mathcal{O} = J$.

B.1. Price changes

First order changes in prices remain largely unchanged and satisfy

$$\left(\boldsymbol{I} - \boldsymbol{\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\right) d\log \boldsymbol{p} = \boldsymbol{\Psi}\left[\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left[d\log \boldsymbol{w} - d\log \boldsymbol{p} - \mathfrak{Q}\mathfrak{T} d\log\theta\right] - d\log \boldsymbol{A}\right]$$

B.2. Sales Share Propagation

We can rewrite the goods market clearing condition in terms of Domar weights:

$$y_i = c_i + \sum_{j=1}^{J} x_{ji}$$

$$\Rightarrow \frac{p_i y_i}{\sum_{k=1}^{J} p_k c_k} = \frac{p_i c_i}{\sum_{k=1}^{J} p_k c_k} + \sum_{j=1}^{J} \frac{p_i x_{ji}}{p_j x_j} \frac{p_j x_j}{\sum_{k=1}^{J} p_k c_k}$$

$$\Rightarrow \lambda_i = \varepsilon_{c_i}^{\mathcal{D}} + \sum_{j=1}^{J} \varepsilon_{x_{ji}}^{f_j} \lambda_j, \tag{A1}$$

where $\lambda_i = \frac{p_i y_i}{\sum_{k=1}^{J} p_k c_k}$ is the Domar weight of sector i.

By stacking (A1) for each sector, we get the following expression for Domar weights across the production network.

$$\boldsymbol{\lambda}' = \varepsilon_{\boldsymbol{c}}^{\mathcal{D}'} + \boldsymbol{\lambda}'\boldsymbol{\Omega}$$

We can see how Domar weights change across the production network by totally differentiating

$$d\boldsymbol{\lambda}' = d\varepsilon_{\boldsymbol{c}}^{\mathcal{D}'} + d\boldsymbol{\lambda}'\boldsymbol{\Omega} + \boldsymbol{\lambda}' d\boldsymbol{\Omega}$$

$$\Rightarrow d\boldsymbol{\lambda}' = \left[d\varepsilon_{\boldsymbol{c}}^{\mathcal{D}'} + \boldsymbol{\lambda}' d\boldsymbol{\Omega}\right]\boldsymbol{\Psi} \tag{A2}$$

The Domar weights will help us express how shocks propagate to output.

B.3. Output changes

We can now write changes in output in terms of changes in tightness, technology, the size of the labor force, and changes in production elasticities, including changes in

Domar weights, as

$$
\begin{aligned}
d\log \boldsymbol{y} = \boldsymbol{\Psi}\left(d\log \boldsymbol{A} + \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}\left(\mathcal{F} + \mathcal{Q}\mathcal{T}\right) d\log \theta + \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} d\log \boldsymbol{H}\right) \\
- \boldsymbol{\Psi} d\log \mathcal{E} + \boldsymbol{\Psi}\left(\operatorname{diag}\left(\boldsymbol{\Omega}\mathbf{1}\right) - \boldsymbol{\Omega}\right) d\log \boldsymbol{\lambda}
\end{aligned}
$$

Where $\mathbf{1}$ is a $J \times 1$ vector of ones and $d\log \mathcal{E}$ is the $J \times 1$ vector of diagonal elements of $\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} d\log \varepsilon^{\boldsymbol{f}\prime}_{\boldsymbol{N}}$.

B.4. Tightness changes

Much like output, changes in tightness now also depends on changes in the elasticities of the production functions. The expressiong for changes in tightness is

$$
\begin{aligned}
d\log \theta = \left[\mathcal{F} - \Xi_{\theta}\right]^{-1}\left[\boldsymbol{\Psi} d\log \boldsymbol{A} - \left[d\log \boldsymbol{w} - d\log \boldsymbol{p}\right] + \left[\boldsymbol{\Psi}\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} - \boldsymbol{I}\right] d\log \boldsymbol{H}\right] \\
+ \left[\mathcal{F} - \Xi_{\theta}\right]^{-1}\left[\operatorname{diag}\left(\mathcal{L} d\log \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}\right) + \boldsymbol{\Psi}\left[\left(\operatorname{diag}\left(\boldsymbol{\Omega}\mathbf{1}\right) - \boldsymbol{\Omega}\right) d\log \boldsymbol{\lambda} - d\log \mathcal{E}\right]\right]
\end{aligned}
$$

Where $\Xi_{\theta} = \boldsymbol{\Psi}\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}\left[\mathcal{F} + \mathcal{Q}\mathcal{T}\right]$. Notice, all terms in the second line are zero assuming Cobb-Douglas production technology.

B.5. Aggregation

Aggregate output now satisfies

$$
\begin{aligned}
d\log Y &= \varepsilon^{\mathcal{D}\prime}_{\boldsymbol{c}} d\log \boldsymbol{c} \\
&= \varepsilon^{\mathcal{D}\prime}_{\boldsymbol{c}}\left(d\log \varepsilon^{\mathcal{D}}_{\boldsymbol{c}} d\log \boldsymbol{y} - d\log \boldsymbol{\lambda}\right)
\end{aligned}
$$

Appendix C. Amplification Proofs

We want to show that $\left(\boldsymbol{I} - \mathcal{M} - \Xi_{\theta}\right)^{-1}$, where $\Xi_{\theta} = \mathcal{L}\boldsymbol{\Psi}\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}\left[\boldsymbol{I} - \mathcal{M}\left(\boldsymbol{I} + \mathcal{T}\right)\right]$, is non-negative element-wise.

We have

$$
\left(\boldsymbol{I} - \mathcal{M} - \Xi_{\theta}\right)^{-1} = \left(\boldsymbol{I} - \Xi_{\theta}\left(\boldsymbol{I} - \mathcal{M}\right)^{-1}\right)^{-1}\left(\boldsymbol{I} - \mathcal{M}\right)^{-1}
$$

Since $\left(\boldsymbol{I} - \mathcal{M}\right)^{-1}$ is a diagonal matrix with positive diagonal elements, it suffices to show that $\boldsymbol{I} - \Xi_{\theta}\left(\boldsymbol{I} - \mathcal{M}\right)^{-1}$ is an M-matrix, since M-matrices are inverse non-negative.

$\boldsymbol{I} - \Xi_\theta\,(\boldsymbol{I} - \mathcal{M})^{-1}$ is a Z-matrix since its off-diagonals are negative. If $\boldsymbol{I} - \Xi_\theta\,(\boldsymbol{I} - \mathcal{M})^{-1}$ is diagonally dominant then it is also and M-matrix. Let a_{ij} denote the (i, j)-th element of $\boldsymbol{I} - \Xi_\theta\,(\boldsymbol{I} - \mathcal{M})^{-1}$. Row diagonal dominance requires:

$$|a_{ii}| \geq \sum_{j \neq i} |a_{ij}| \quad \forall i$$

For simplicity, we consider the case with one occupation per sector $\boldsymbol{L} = \boldsymbol{I}$, but the logic behind this proof follows for general $\boldsymbol{L}$.

To start with, we show that the row sums of $\boldsymbol{\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}$ is less than or equal to 1.

Let B_{ij} denote the $(i, j) - th$ element of $(\boldsymbol{I} - \boldsymbol{\Omega})$, and Ψ_{ij} the $(i, j) - th$ element of Ψ, and

$$\sum_k \Psi_{ik} B_{kj} = \delta_{ij}$$

We have:

$$\sum_k \Psi_{ik} \sum_j B_{kj} = \sum_k \sum_j \Psi_{ik} B_{kj} = \sum_j \delta_{ij} = 1,$$

since $\boldsymbol{\Psi}$ is the inverse of $(\boldsymbol{I} - \boldsymbol{\Omega})$.

For each k, the (k, k)-th element of $\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}$, β_{kk}, is smaller than or equal to $\sum_j B_{kj}$ by the constant returns of the production functions and non-negative factor shares.

Thus, for each row i in the matrix $\boldsymbol{\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}$, the row sum is given by:

$$\sum_k \Psi_{ik} \beta_{kk} \leq 1$$

Let x_{ij} denote the (i, j)-th element of the matrix $\boldsymbol{\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}$, row diagonal dominance requires that for each row i,

$$1 - x_{ii} \frac{(1 - \eta_i\,(1 + \tau_i))}{1 - \eta_i} \geq \sum_{j \neq i} x_{ij} \frac{\left(1 - \eta_j\left(1 + \tau_j\right)\right)}{1 - \eta_j}$$

Rewriting it yields:

$$1 \geq \sum_j x_{ij} \frac{\left(1 - \eta_j\left(1 + \tau_j\right)\right)}{1 - \eta_j},$$

which holds because $\tau_j \geq 0$.

Appendix D. Horizontal Two-Sector Model

In this section we present a horizontal two sector model. This model is slightly more complicated than the simple example presented in the main text. It, however, allows us to highlight a few additional features of production networks with labor market frictions relative to similarly structured models featuring only one of the two.

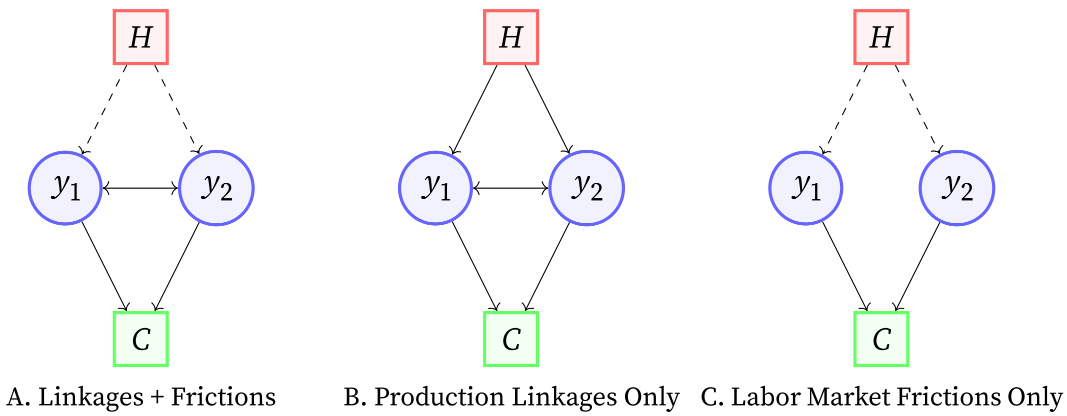

FIGURE A1. Structure of two-sector horizontal economies. The two sectors are denoted by y_1 and y_2. Final consumption is denoted by C. H is the labor force, which both sectors use in production. Arrows denote links in the production process, who sells what to whom. Solid arrows denote frictionless trade. Dashed arrows denote frictional trade.

Appendix E. Data and Calibration Details

This appendix describes our data in greater detail.

E.1. Input-output matrix

We use the 3-digit 2021 BEA Make and Use tables accessible at https://www.bea.gov/industry/input-output-accounts-data to calculate the relevant production elasticities[8]. The 3-digit Make and Use tables record the nominal amount of each 71 commodities made by and used by each of 71 industries. The commodities are denoted using the same codes as the industries, but they are conceptually distinct as each industry can produce more than one commodity.

For consistency with the industry classifications in JOLTs and the CPS unemployment by sector series, we collapse the 3-digit tables to a 13 sector table. The table below outlines the mapping from the NAICS 2-digit classification codes to our industry classifications. The mapping from 2-digit codes to 3-digit codes is readily available online.

Industry Name	Short Name	2-digit codes
Leisure and Hospitality	accom	71, 72
Construction	const	33
Durable goods	dur	33DG
Education and Health Services	edhealth	61, 62
Financial Activities	fin	52, 53
Government	gov	G
Information	info	51
Mining	mining	21
Nondurable good	nondur	11, 31ND
Other services, except government	other	81
Professional and business services	profserv	54, 55, 56
Wholesale and Retail trade	trade	42, 44RT
Transportation and Utilities	trans	22, 48TW

TABLE A1. Mapping from NAICS classification to our industries.

[8]See https://www.bea.gov/sites/default/files/methodologies/IOmanual_092906.pdf for a detailed description of how these tables are generated.

With the 13-sector make and use tables in hand, we can construct production elasticities in intermediate inputs and to labor, and demand elasticities. Let M_{ij} denote the nominal value of commodity i made by industry j. Let U_{ij} denote the nominal amount of commodity i used by industry j. The two tables below demonstrate the elements of the Make and Use tables.

	Sector 1	Sector 2	$\cdots$	Sector J	Total Industry Output
Sector 1	M_{11}	M_{21}	$\cdots$	M_{J1}	$\sum_{i=1}^{J} M_{i1}$
Sector 2	M_{12}	M_{22}	$\cdots$	M_{J2}	$\sum_{i=1}^{J} M_{i2}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
Sector J	M_{1J}	M_{2J}	$\cdots$	M_{JJ}	$\sum_{i=1}^{J} M_{iJ}$
Total Commodity Output	$\sum_{j=1}^{J} M_{1j}$	$\sum_{j=1}^{J} M_{2j}$	$\cdots$	$\sum_{j=1}^{J} M_{Jj}$	—

TABLE A2. Make table

	Sector 1	Sector 2	$\cdots$	Sector J	Total Intermediate Uses	Total Final Uses
Sector 1	U_{11}	U_{12}	$\cdots$	U_{1J}	$\sum_{j=1}^{J} U_{1j}$	$\sum_{j=1}^{J} U_{1j} + p_1 c_1$
Sector 2	U_{21}	U_{22}	$\cdots$	U_{2J}	$\sum_{j=1}^{J} U_{2j}$	$\sum_{j=1}^{J} U_{2j} + p_2 c_2$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
Sector J	U_{J1}	U_{J2}	$\cdots$	U_{JJ}	$\sum_{j=1}^{J} U_{Jj}$	$\sum_{j=1}^{J} U_{Jj} + p_J c_J$
Total Intermediate Inputs	$\sum_{i=1}^{J} U_{i1}$	$\sum_{i=1}^{J} U_{i2}$	$\cdots$	$\sum_{i=1}^{J} U_{iJ}$	—	—
Total industry output	$\sum_{i=1}^{J} U_{i1} + w_1(1+\tau_1)N_1$	$\sum_{i=1}^{J} U_{i2} + w_2(1+\tau_2)N_2$	$\cdots$	$\sum_{i=1}^{J} U_{iJ} + w_J(1+\tau_J)N_J$	—	—

TABLE A3. Use table

First, we calculate the fraction of commodity i produced by industry j by dividing the entry in along each row by the corresponding "total industry output"

$$m_{ij} = \frac{M_{ij}}{\sum_{j=1}^{J} M_{ji}}$$

Second, we calculate the share of commodity i in industry j's total uses as by dividing each entry in the column corresponding to industry j by the corresponding "Total industry output"

$$u_{ij} = \frac{U_{ij}}{\sum_{i=1}^{J} U_{ij} + w_j(1+\tau_j)N_j}$$

We form the two matrices

$$\boldsymbol{M} = \begin{bmatrix} m_{11} & m_{21} & \cdots & m_{J1} \\ m_{12} & m_{22} & \cdots & m_{J2} \\ \vdots & \vdots & \ddots & \vdots \\ m_{1J} & m_{2J} & \cdots & m_{JJ} \end{bmatrix}, \boldsymbol{U} = \begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1J} \\ u_{21} & u_{22} & \cdots & u_{2J} \\ \vdots & \vdots & \ddots & \vdots \\ u_{J1} & u_{J2} & \cdots & u_{JJ} \end{bmatrix}$$

Then, we can calculate our input output matrix by

$$\boldsymbol{\Omega} = [\boldsymbol{MU}]'$$

Given our assumption of constant returns to scale and zero profits, the difference between total intermediate inputs and total industry output is the nominal income paid to workers in each sector. We abstract from the other components of total industry output in the IO accounts, taxes and gross operating surplus, as they have no model counterpart in our setup. We can therefore calculate the labor elasticities from the Use table as "Total industry output" – "Total intermediate inputs" ÷ "Total industry output."

$$\varepsilon_{N_j}^{f_j} = \frac{w_j(1+\tau_j)N_j}{\sum_{i=1}^{J} U_{ij} + w_j(1+\tau_j)N_j}$$

Finally, we can back out the demand elasticities from "Total intermediate uses" and "Total final uses" columns of the Uses table.

$$p_i c_i = \sum_{j=1}^{J} U_{ij} + p_i c_i - \sum_{j=1}^{J} U_{ij}$$

We can the work out the elasticities by

$$\varepsilon_{c_i}^{\mathcal{D}} = \frac{p_i c_i}{\sum_{i=1}^{J} p_i c_i}$$

Finally, to ensure that constant returns to scale holds we rescale our elasticities proportionally to ensure they sum to one. This adjustment is minor and is only needed because we drop the small "Used" and "rest of world adjustment" categories. It does not change any elasticity by more than 3 percent.

We report the resulting estimates of the production elasticities, labor elasticities, and demand elasticities in the tables below. In tables A4 and A5 we assume all non-intermediate, non-energy, and non-capital, spending goes to labor income, which automatically imposes constant returns but leads to large labor shares.

Sector	Labor Elasticity $\left(\varepsilon_N^f\right)$	Demand Elasticity $\left(\varepsilon_c^{\mathcal{D}}\right)$
accom	0.510	0.051
const	0.474	0.065
dur	0.434	0.138
edhealth	0.616	0.129
fin	0.617	0.165
gov	0.626	0.132
info	0.571	0.043
mining	0.518	0.008
nondur	0.352	0.151
other	0.608	0.024
profserv	0.591	0.071
trade	0.523	0.000
trans	0.492	0.022

TABLE A4. Labor elasticities and demand elasticities according the BEA make use tables for 13-industry classification, rounded to 3 decimal places. We assume all non-intermediate spending goes to labor income.

	accom	const	dur	edhealth	fin	gov	info	mining	nondur	other	profserv	trade	trans
accom	0.029	0.002	0.017	0.003	0.099	0.007	0.036	0.008	0.103	0.014	0.143	0.000	0.030
const	0.001	0.000	0.282	0.000	0.033	0.001	0.016	0.022	0.089	0.006	0.072	0.000	0.004
dur	0.001	0.001	0.393	0.000	0.016	0.001	0.012	0.013	0.056	0.003	0.055	0.004	0.012
edhealth	0.019	0.000	0.030	0.015	0.103	0.006	0.028	0.004	0.056	0.009	0.101	0.000	0.012
fin	0.011	0.023	0.007	0.000	0.195	0.005	0.020	0.001	0.008	0.005	0.083	0.001	0.022
gov	0.006	0.023	0.048	0.011	0.055	0.004	0.032	0.013	0.081	0.012	0.072	0.000	0.017
info	0.017	0.001	0.040	0.000	0.047	0.003	0.125	0.002	0.015	0.005	0.160	0.001	0.013
mining	0.001	0.005	0.010	0.000	0.060	0.002	0.017	0.128	0.056	0.001	0.090	0.000	0.022
nondur	0.001	0.003	0.044	0.000	0.020	0.002	0.008	0.124	0.377	0.003	0.040	0.004	0.021
other	0.013	0.005	0.066	0.010	0.113	0.005	0.033	0.003	0.032	0.013	0.086	0.000	0.012
nprofserv	0.020	0.000	0.029	0.001	0.070	0.004	0.054	0.002	0.026	0.008	0.176	0.000	0.017
trade	0.006	0.002	0.025	0.002	0.104	0.009	0.038	0.002	0.031	0.015	0.159	0.021	0.063
trans	0.014	0.007	0.022	0.000	0.080	0.015	0.025	0.030	0.080	0.011	0.093	0.001	0.129

TABLE A5. Production elasticities to intermediate inputs at 13-sector level ($\mathbf{\Omega}$), rounded to 3 decimal places. We assume all non-intermediate spending goes to labor income.

E.2. Matching Parameters

We estimate the parameters of the sector specific matching function from monthly data on hires and vacancies from JOLTs and unemployment from the CPS. In particular, we estimate

$$\log H_{i,t} = \log \phi_i + \eta_i \log U_{i,t} + (1 - \eta_i) \log V_{i,t} + \epsilon_{i,t}$$

by least squares. ϕ_i is the matching efficiency in sector i and η_i is the matching elasticity with respect to unemployment in sector i. We report the resulting estimates in the table below.

	Matching Efficiency $\left(\widehat{\phi}_i\right)$	Unemployment Elasticity $(\widehat{\eta}_i)$
accom	1.185	0.401
const	1.106	0.507
dur	0.688	0.364
edhealth	0.703	0.336
fin	0.705	0.329
gov	0.640	0.291
info	0.703	0.275
mining	1.236	0.262
nondur	0.779	0.391
other	0.848	0.441
profserv	1.077	0.372
trade	1.009	0.430
trans	0.862	0.439

TABLE A6. Matching function parameter estimates. Based on monthly hiring, unemployment, and vacancy data from Jan 2000 to Feb 2023.

Finally, we use the sector level proportion of HR workers as a proxy for the recruiter producer ratio. The resulting recruiter producer ratios are reported below

	τ_i
accom	0.002
const	0.002
dur	0.007
edhealth	0.005
fin	0.008
gov	0.011
info	0.013
mining	0.005
nondur	0.007
other	0.018
profserv	0.020
trade	0.003
trans	0.001

TABLE A7. Estimated recruiter producer ratios based on the number of HR workers in industry i over total employment in industry i.

E.3. Computing Occupational Worker Share

For our occupational labor market calibration, we need to compute $\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}$, which is the occupational worker elasticity of production. To do this, we obtain wage and employment data for ONET major occupations at 3-digit sector level from the Occupational Employment and Wage Statistics (OES). For each sector i, we compute $\varepsilon_{N_i o}^{f}$ as:

$$\varepsilon_{N_{io}}^{f} = \varepsilon_{Ni}^{f} \frac{w_{io} L_{io}}{\sum_o w_{io} L_{io}},$$

where ε_N^f is the labor share we obtained earlier from the input-output table.

Table A8 contains our calibration estimates.

	Admin	Agg	Arts	Bus Ops	Care	Clean	Cons	Educ	Eng	Food S	Health P	Health S	Legal	Manag	Math	Prod	Prot S	Repair	Sales	Science	Soc S	Trans
accom	1.7	0.0	0.9	0.8	1.9	1.6	0.0	0.2	0.0	23.9	0.1	0.0	0.0	3.5	0.1	0.3	0.4	0.7	1.4	0.0	0.0	0.8
const	2.8	0.0	0.1	3.0	0.0	0.1	22.9	0.0	0.7	0.0	0.0	0.0	0.0	5.6	0.1	0.6	0.0	3.4	1.0	0.1	0.0	1.1
dur	1.9	0.0	0.2	2.3	0.0	0.1	0.6	0.0	4.0	0.0	0.0	0.0	0.1	4.6	1.9	11.5	0.0	1.3	1.2	0.2	0.0	1.4
edhealth	4.1	0.0	0.4	1.4	0.6	0.8	0.1	12.9	0.0	0.7	17.8	5.3	0.0	4.3	0.7	0.1	0.3	0.4	0.1	0.6	2.1	0.4
fin	7.2	0.0	0.2	10.6	0.1	0.3	0.1	0.0	0.1	0.0	0.3	0.0	0.6	8.4	3.2	0.0	0.1	1.3	6.3	0.0	0.1	0.3
gov	6.8	0.1	0.5	8.6	0.6	0.8	2.3	1.0	2.6	0.2	4.0	0.5	2.7	6.6	2.4	0.9	12.0	2.3	0.2	2.5	2.9	2.1
info	2.4	0.0	4.4	4.0	0.2	0.0	0.1	0.1	0.6	0.1	0.0	0.0	0.3	7.2	10.3	0.1	0.0	2.0	3.2	0.0	0.0	0.2
mining	1.3	0.0	0.0	1.4	0.0	0.0	7.3	0.0	1.8	0.0	0.0	0.0	0.1	3.7	0.4	1.4	0.0	1.9	0.6	0.7	0.0	2.4
nondur	1.8	0.1	0.2	1.2	0.0	0.2	0.1	0.0	1.0	0.3	0.0	0.0	0.0	3.3	0.3	9.8	0.0	1.7	1.2	0.8	0.0	2.3
other	6.4	0.0	1.7	6.1	8.0	0.8	0.2	0.8	0.3	0.5	0.3	0.7	0.3	9.4	1.0	2.4	0.3	9.5	1.9	0.3	1.6	3.4
profserv	5.4	0.0	1.1	8.6	0.1	2.3	0.7	0.1	3.4	0.1	1.2	0.3	3.1	10.5	7.9	1.0	1.1	0.7	2.2	1.2	0.1	1.7
trade	3.9	0.1	0.5	1.7	0.1	0.2	0.1	0.0	0.2	0.7	1.6	0.1	0.0	5.0	0.8	1.1	0.1	2.3	15.2	0.1	0.0	6.0
trans	6.2	0.0	0.0	0.4	0.0	0.2	0.4	0.0	0.4	0.0	0.0	0.0	0.0	0.5	0.2	0.9	0.0	2.9	0.5	0.1	0.0	18.2

TABLE A8. Occupational worker elasticity of output, in percentage terms, rounded to 1 decimal place.

E.4. Imputing Occupation Labor Market Parameters

For the occupational labor market specification, we need to calibrate unemployment, vacancy, and tightness of each occupation. We currently don't have access to occupational labor market characteristics, so we instead impute these parameters. For simplicity, we assume the total number of unemployment and vacancy for an occupation is the sum of unemployment and vacancy across sectors, weighted by the sectors' wage shares of that particular occupation:

$$
\begin{aligned}
V_o &= \sum_i \frac{\varepsilon^f_{N_{io}}}{\varepsilon^f_{N_i}} V_i, \\
H_o &= \sum_i \frac{\varepsilon^f_{N_{io}}}{\varepsilon^f_{N_i}} H_i,
\end{aligned}
$$

where V denotes vacancy and H denotes labor supply, which is equivalent to unemployment in our static setup. The intuition behind this is that each sector's contribution to vacancy postings and the number of people looking for jobs in that sector for an occupation is proportional to how much the sector relies on that occupation.

Note that, with this imperfect simplifying assumption, we can back out changes in sectoral tightness. For sector j, the first-order response in tightness is: Tightness for sector j is:

$$
\begin{aligned}
\theta_j = \frac{V_j}{H_j} &= \frac{\sum_o \frac{V_{jo}}{V_o} V_o}{\sum_o \frac{H_{jo}}{H_o} H_o} \\
\Rightarrow d\log\theta_j &= d\log V_j - d\log H_j \\
&= \sum_o \frac{V_o}{V_j}\frac{V_{jo}}{V_o} d\log V_o - \frac{H_o}{H_j}\frac{H_{jo}}{H_o} d\log H_o \\
&= \sum_o \frac{V_{jo}}{V_j} d\log V_o - \frac{H_{jo}}{H_j} d\log H_o \\
&= \sum_o \frac{\varepsilon^f_{N_{jo}}}{\varepsilon^f_{N_j}} d\log\theta_o.
\end{aligned}
$$

We estimate the matching elasticities using the same methodology from appendix section E.2. Table A9 reports the estimated coefficients.

Occupation	Matching Efficiency ($\hat{\phi}_o$)	Unemployment Elasticity ($\hat{\eta}_o$)
Admin	0.883	0.368
Agg	0.907	0.390
Arts	0.939	0.352
Bus Ops	0.879	0.356
Care	0.978	0.378
Clean	1.009	0.372
Cons	1.054	0.461
Educ	0.713	0.338
Eng	0.871	0.357
Food S	1.163	0.398
Health P	0.749	0.346
Health S	0.731	0.343
Legal	0.923	0.354
Manag	0.915	0.368
Math	0.905	0.342
Prod	0.797	0.374
Prot S	0.744	0.319
Repair	0.909	0.390
Sales	0.975	0.398
Science	0.829	0.355
Soc S	0.701	0.334
Trans	0.924	0.406

TABLE A9. Matching parameters for major occupations

Additionally, following our vacancy assumption, we assume the number of recruiters each sector dedicates to recruiting a particular occupation is proportional to the occupation elasticity of production. In other words:

$$R_o = \sum_i \frac{\varepsilon^f_{N_{io}}}{\varepsilon^f_{N_i}} R_i,$$

Occupation	τ_o
Admin	0.006
Agg	0.002
Arts	0.011
Bus Ops	0.014
Care	0.006
Clean	0.006
Cons	0.004
Educ	0.005
Eng	0.017
Food S	0.002
Health P	0.009
Health S	0.003
Legal	0.027
Manag	0.019
Math	0.020
Prod	0.006
Prot S	0.009
Repair	0.007
Sales	0.005
Science	0.014
Soc S	0.006
Trans	0.003

TABLE A10. Recruiter producer ratios based on the number of estimated recruiters in occupation o.

where R_i is the number of recruiters in sector i. This is also implicitly assuming that the recruiting cost for the occupations are the same.

Since we have the total employment for each occupation from the OES, we can therefore compute the recruiter-producer ratios. Table A10 reports the estimated recruiter-producer ratio for different occupations.

Appendix F. Additional Results on Calibrated Shock Propagation.

Below, we report the responses of output and unemployment to technology shocks in each of the 13 sectors in our calibration.

F.1. Responses to technology shocks across sectors: One Occupation Per Sector

figures/calib2/accom_AshockY.png

figures/calib2/const_AshockY.png

figures/calib2/edhealth_AshockY.png

figures/calib2/fin_AshockY.png

figures/calib2/gov_AshockY.png

figures/calib2/info_AshockY.png

FIGURE A2. Responses of output to 1% technology shocks other sectors.

figures/calib2/mining_AshockY.png

figures/calib2/nondur_AshockY.png

figures/calib2/other_AshockY.png

figures/calib2/profserv_AshockY.png

figures/calib2/trade_AshockY.png

figures/calib2/trans_AshockY.png

FIGURE A3. Responses of output to 1% technology shocks other sectors.

figures/calib2/accom_AshockU.png figures/calib2/const_AshockU.png

figures/calib2/edhealth_AshockU.png figures/calib2/fin_AshockU.png

figures/calib2/gov_AshockU.png figures/calib2/info_AshockU.png

FIGURE A4. Responses of unemployment to 1% labor force shocks other sectors.

figures/calib2/mining_AshockU.png figures/calib2/nondur_AshockU.png

figures/calib2/other_AshockU.png figures/calib2/profserv_AshockU.png

figures/calib2/trade_AshockU.png figures/calib2/trans_AshockU.png

FIGURE A5. Responses of unemployment to 1% labor force shocks other sectors.

F.2. Responses to technology shocks across sectors: Major Occupations

figures/calib3/accom_AshockY.png figures/calib3/const_AshockY.png

figures/calib3/edhealth_AshockY.png figures/calib3/fin_AshockY.png

figures/calib3/gov_AshockY.png figures/calib3/info_AshockY.png

FIGURE A6. Responses of output to 1% technology shocks other sectors.

figures/calib3/mining_AshockY.png figures/calib3/nondur_AshockY.png

figures/calib3/other_AshockY.png figures/calib3/profserv_AshockY.png

figures/calib3/trade_AshockY.png figures/calib3/trans_AshockY.png

FIGURE A7. Responses of output to 1% technology shocks other sectors.

figures/calib3/accom_AshockU.png	figures/calib3/const_AshockU.png
figures/calib3/edhealth_AshockU.png	figures/calib3/fin_AshockU.png
figures/calib3/gov_AshockU.png	figures/calib3/info_AshockU.png

FIGURE A8. Responses of unemployment to 1% labor force shocks other sectors.

figures/calib3/mining_AshockU.png

figures/calib3/nondur_AshockU.png

figures/calib3/other_AshockU.png

figures/calib3/profserv_AshockU.png

figures/calib3/trade_AshockU.png

figures/calib3/trans_AshockU.png

FIGURE A9. Responses of unemployment to 1% labor force shocks other sectors.