Unemployment in a Production Network

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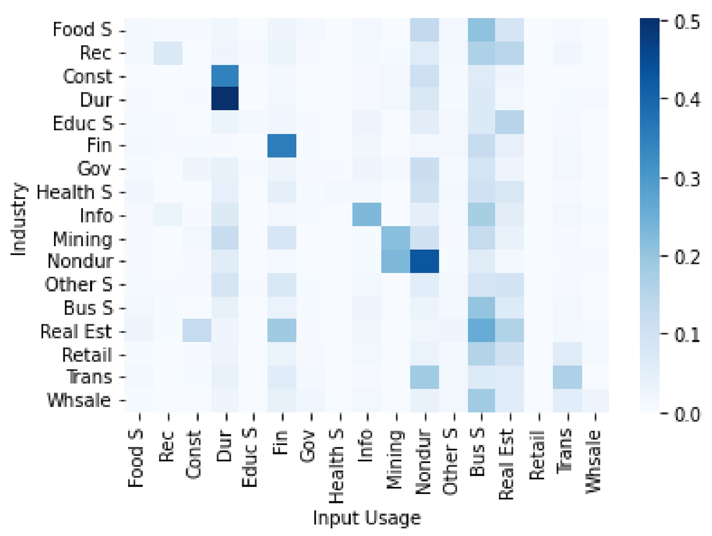
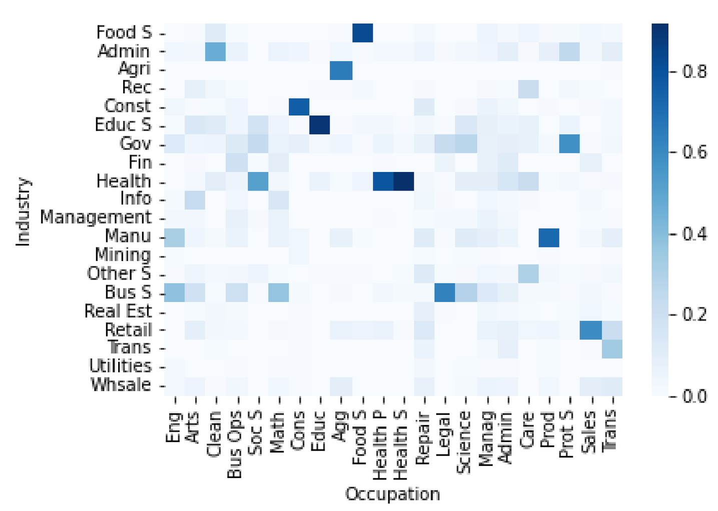
Motivation

Economies feature production linkages.

Labor markets are frictional and segmented.

Labor markets and industries are inter-connected.





Research Questions:

- How do labor market frictions and segmentation impact network propagation of shocks?
- How do we design optimal labor market policies in a network economy?

Model Setup

There are $\mathcal{O}$ occupations and J sectors. Occupation o has labor force H_o.

- Sector i uses inputs from other sectors x_{ij} and labor N_o:

$$y_i = A_i f_i\left(\{N_{io}\}_{o=1}^{\mathcal{O}}, \{x_{ij}\}_{j=1}^{J}\right)$$

- Household preferences represented by final goods producer:

$$Y = \max_{\{c_i\}_{i=1}^J} \mathcal{D}\left(\{c_i\}_{i=1}^J\right) \quad \text{s.t} \quad \sum_{i=1}^{J} p_i c_i = \sum_{o=1}^{\mathcal{O}} w_o L_o(\theta_o)$$

- Labor markets are subject to search frictions:

$$\underbrace{h_o = m_o(H_o, V_o)}_{\text{Matching function}}, \qquad \underbrace{V_o = \sum_{i=1}^{J} v_{io}}_{\text{Vacancies}}, \qquad \underbrace{\theta_o \equiv \frac{V_o}{H_o}}_{\text{Tightness}},$$

$$\underbrace{\mathcal{Q}_o(\theta_o) \equiv \frac{m_o(H_o, V_o)}{V_o}}_{\text{Vacancy-filling rate}}, \qquad \underbrace{\mathcal{F}_o(\theta_o) \equiv \frac{m_o(H_o, V_o)}{H_o}}_{\text{Job-finding rate}}, \qquad \underbrace{\tau_o(\theta_o) \equiv \frac{r_o}{\mathcal{Q}_o(\theta_o) - r_o}}_{\text{Recruiter-producer ratio}}.$$

- Consider Cobb-Douglas case here, but results generalizable to CRTS technology.

Equilibrium

Equilibrium is a collection of prices $\{p_i\}_{i=1}^J$, outputs $\{y_i\}_{i=1}^J$, intermediate inputs $\left\{\{x_{ij}\}_{j=1}^J\right\}_{i=1}^J$, labor inputs $\left\{\{N_{io}\}_{o=1}^{\mathcal{O}}\right\}_{i=1}^J$, consumption choices $\{c_i\}_{i=1}^J$, tightnesses $\{\theta_o\}_{o=1}^{\mathcal{O}}$, wages, $\{w_o\}_{o=1}^{\mathcal{O}}$, labor demands $\left\{L_o^d\right\}_{o=1}^{\mathcal{O}}$, and labor supplies $\{L_o^s\}_{o=1}^{\mathcal{O}}$, such that

$$\varepsilon_{x_{ij}}^{f_i} = \frac{p_j x_{ij}}{p_i y_i},$$

$$\varepsilon_{N_{io}}^{f_i} = \frac{w_o(1+\tau_o(\theta_o))N_o}{p_i y_i} \Rightarrow l_{io}^d = \varepsilon_{N_{io}}^{f_i} \frac{p_i y_i}{w_o} \qquad \text{(Sector FOCs)}$$

$$\varepsilon_{c_i}^{\mathcal{D}} = \frac{p_i c_i}{\sum_{j=1}^J p_j c_j} \qquad \text{(Consumption maximization)}$$

$$y_i = c_i + \sum_{j=1}^{J} x_{ij} \qquad \text{(Goods market clearing)}$$

$$L_o^d = \sum_{i=1}^{J} l_{io}^d = \mathcal{F}_o(\theta_o) H_o = L_o^s \qquad \text{(Labor market clearing)}$$

Network Notation

Use bold letters to denote vectors and matrices.

$$\boldsymbol{\mathcal{Q}} = \text{diag}\left(\begin{bmatrix}\varepsilon_{\theta_1}^{\mathcal{Q}_1} & \cdots & \varepsilon_{\theta_{\mathcal{O}}}^{\mathcal{Q}_{\mathcal{O}}}\end{bmatrix}\right), \boldsymbol{\mathcal{F}} = \text{diag}\left(\begin{bmatrix}\varepsilon_{\theta_1}^{\mathcal{F}_1} & \cdots & \varepsilon_{\theta_{\mathcal{O}}}^{\mathcal{F}_{\mathcal{O}}}\end{bmatrix}\right), \boldsymbol{\mathcal{T}} = \text{diag}\left(\begin{bmatrix}\varepsilon_{\theta_1}^{\mathcal{T}_1} & \cdots & \varepsilon_{\theta_{\mathcal{O}}}^{\mathcal{T}_{\mathcal{O}}}\end{bmatrix}\right)$$

$$\boldsymbol{\Omega} = \begin{bmatrix}\varepsilon_{x_{11}}^{f_1} & \cdots & \varepsilon_{x_{1J}}^{f_1} \\ \vdots & \ddots & \vdots \\ \varepsilon_{x_{J1}}^{f_J} & \cdots & \varepsilon_{x_{JJ}}^{f_J}\end{bmatrix}, \boldsymbol{\varepsilon}_{\boldsymbol{N}}^{\boldsymbol{f}} = \begin{bmatrix}\varepsilon_{N_1 1}^{f_1} & \cdots & \varepsilon_{N_{1\mathcal{O}}}^{f_1} \\ \vdots & \ddots & \vdots \\ \varepsilon_{N_{J1}}^{f_J} & \cdots & \varepsilon_{N_{J\mathcal{O}}}^{f_J}\end{bmatrix}, \boldsymbol{\mathcal{L}} = \begin{bmatrix}\frac{l_{11}}{L_1} & \cdots & \frac{l_{J1}}{L_1} \\ \vdots & \ddots & \vdots \\ \frac{l_{1\mathcal{O}}}{L_{\mathcal{O}}} & \cdots & \frac{l_{J\mathcal{O}}}{L_{\mathcal{O}}}\end{bmatrix}, \boldsymbol{\Psi} = (\boldsymbol{I} - \boldsymbol{\Omega})^{-1}$$

Shock Propagation

We derive first order response of sectoral output and occupational employment to shocks to technology and the size of the labor force. Mutual gains from trade means wages are not pinned down by model, convenient to assume:

$$d \log \boldsymbol{w} - \boldsymbol{\mathcal{L}} d \log \boldsymbol{p} = \boldsymbol{\Lambda_A} d \log \boldsymbol{A} + \boldsymbol{\Lambda_H} d \log \boldsymbol{H}.$$

First order changes in output can be decomposed into supply and demand-side forces:

$$d \log \boldsymbol{y} = \boldsymbol{\Pi}_{y,A} d \log \boldsymbol{A} + \boldsymbol{\Pi}_{y,H} d \log \boldsymbol{H},$$

where

$$\boldsymbol{\Pi}_{y,A} = \boldsymbol{\Psi}\left[\boldsymbol{I} + \underbrace{\boldsymbol{\varepsilon}_{\boldsymbol{N}}^{\boldsymbol{f}}(\boldsymbol{\mathcal{F}} + \boldsymbol{\mathcal{Q}}\boldsymbol{\mathcal{T}})\left[\boldsymbol{\mathcal{F}} - \boldsymbol{\Xi_\theta}\right]^{-1}\left[\boldsymbol{\mathcal{L}}\boldsymbol{\Psi} - \boldsymbol{\Lambda_A}\right]}_{\text{Search frictions}}\right],$$

$$\boldsymbol{\Pi}_{y,H} = \boldsymbol{\Psi}\boldsymbol{\varepsilon}_{\boldsymbol{N}}^{\boldsymbol{f}}\left[\boldsymbol{I} + \underbrace{(\boldsymbol{\mathcal{F}} + \boldsymbol{\mathcal{Q}}\boldsymbol{\mathcal{T}})\left[\boldsymbol{\mathcal{F}} - \boldsymbol{\Xi_\theta}\right]^{-1}\left[\boldsymbol{\mathcal{L}}\boldsymbol{\Psi}\boldsymbol{\varepsilon}_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I} - \boldsymbol{\Lambda_H}\right]}_{\text{Search frictions}}\right],$$

$$\boldsymbol{\Xi_\theta} = \boldsymbol{\mathcal{L}}\boldsymbol{\Psi}\boldsymbol{\varepsilon}_{\boldsymbol{N}}^{\boldsymbol{f}}\left[\boldsymbol{\mathcal{F}} + \boldsymbol{\mathcal{Q}}\boldsymbol{\mathcal{T}}\right].$$

First order changes in employment are

$$d \log \boldsymbol{L} = \boldsymbol{\Pi}_{L,A} d \log \boldsymbol{A} + \boldsymbol{\Pi}_{L,H} d \log \boldsymbol{H}$$

Where

$$\boldsymbol{\Pi}_{L,A} = \boldsymbol{\mathcal{F}}\left[\boldsymbol{\mathcal{F}} - \boldsymbol{\Xi_\theta}\right]^{-1}\left[\boldsymbol{\mathcal{L}}\boldsymbol{\Psi} - \boldsymbol{\Lambda_A}\right]$$

$$\boldsymbol{\Pi}_{L,H} = \boldsymbol{I} + \boldsymbol{\mathcal{F}}\left[\boldsymbol{\mathcal{F}} - \boldsymbol{\Xi_\theta}\right]^{-1}\left[\boldsymbol{\mathcal{L}}\boldsymbol{\Psi}\boldsymbol{\varepsilon}_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I} - \boldsymbol{\Lambda_H}\right]$$

Aggregate Effects

We use demand elasticities to compute aggregate output change:

$$\begin{aligned} d \log Y &= \boldsymbol{\varepsilon}_{\boldsymbol{c}}^{\mathcal{D}'} d \log \boldsymbol{y} \\ &= \boldsymbol{\Pi}_{Y,A} d \log \boldsymbol{A} + \boldsymbol{\Pi}_{Y,H} d \log \boldsymbol{H} \end{aligned}$$

Benchmark 1 - No Search Friction:

$$\boldsymbol{\Pi}_{Y,A} = \boldsymbol{\varepsilon}_{\boldsymbol{c}}^{\mathcal{D}'}\boldsymbol{\Psi}, \qquad \text{(Hulten's Theorem)}$$

$$\boldsymbol{\Pi}_{Y,H} = \boldsymbol{\varepsilon}_{\boldsymbol{c}}^{\mathcal{D}'}\boldsymbol{\Psi}\boldsymbol{\varepsilon}_{\boldsymbol{N}}^{\boldsymbol{f}}.$$

Benchmark 2 - No Production Network:

$$\boldsymbol{\Pi}_{Y,A} = \boldsymbol{\varepsilon}_{\boldsymbol{c}}^{\mathcal{D}'}\left(\boldsymbol{\Lambda_A} - \boldsymbol{\mathcal{F}}\boldsymbol{\mathcal{T}}^{-1}\boldsymbol{\mathcal{Q}}^{-1}(\boldsymbol{I} - \boldsymbol{\Lambda_A})\right),$$

$$\boldsymbol{\Pi}_{Y,H} = \boldsymbol{\varepsilon}_{\boldsymbol{c}}^{\mathcal{D}'}\left((\boldsymbol{I} + \boldsymbol{\Lambda_H}) + \boldsymbol{\mathcal{F}}\boldsymbol{\mathcal{T}}^{-1}\boldsymbol{\mathcal{Q}}^{-1}\boldsymbol{\Lambda_H}\right).$$

Our Model:

$$\boldsymbol{\Pi}_{Y,A} = \boldsymbol{\varepsilon}_{\boldsymbol{c}}^{\mathcal{D}'}\boldsymbol{\Psi}\left(\boldsymbol{I} + \boldsymbol{\varepsilon}_{\boldsymbol{N}}^{\boldsymbol{f}}(\boldsymbol{\mathcal{F}} + \boldsymbol{\mathcal{Q}}\boldsymbol{\mathcal{T}})\left[\boldsymbol{\mathcal{F}} - \boldsymbol{\Xi_\theta}\right]^{-1}(\boldsymbol{\mathcal{L}}\boldsymbol{\Psi} - \boldsymbol{\Lambda_A})\right),$$

$$\boldsymbol{\Pi}_{Y,H} = \boldsymbol{\varepsilon}_{\boldsymbol{c}}^{\mathcal{D}'}\boldsymbol{\Psi}\boldsymbol{\varepsilon}_{\boldsymbol{N}}^{\boldsymbol{f}}\left(\boldsymbol{I} + (\boldsymbol{\mathcal{F}} + \boldsymbol{\mathcal{Q}}\boldsymbol{\mathcal{T}})\left[\boldsymbol{\mathcal{F}} - \boldsymbol{\Xi_\theta}\right]^{-1}\left(\left[\boldsymbol{\mathcal{L}}\boldsymbol{\Psi}\boldsymbol{\varepsilon}_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I}\right] - \boldsymbol{\Lambda_H}\right)\right).$$

Takeaways:

- We are able to obtain efficient aggregation whenever $\boldsymbol{\Lambda_A} = \boldsymbol{\mathcal{L}}\boldsymbol{\Psi}$ and $\boldsymbol{\Lambda_H} = \boldsymbol{\mathcal{L}}\boldsymbol{\Psi}\boldsymbol{\varepsilon}_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I}$,
- Production structure and search frictions interact in meaningful ways.

Data and Calibration

Labor markets: Cobb-Douglas matching technology

$$h_o = \phi_o U_o^{\eta} V_o^{1-\eta},$$

with

- $\eta = 0.5$, r_o so that mean $(\tau_o(\theta_o)) \approx 2.3\%$,
- U_o, V_o from JOLTS.

Product markets: input output tables and sector gross output from BEA.

Households: preference parameters are computed from consumption shares.

In this calibration, we assume the occupations are sector-specific ($\boldsymbol{\mathcal{L}} = \boldsymbol{I}$).

Quantitative Results

For our relative wage assumption, we assume:

$$\boldsymbol{\Lambda_A} = \gamma_A \boldsymbol{\mathcal{L}}\boldsymbol{\Psi},$$

$$\boldsymbol{\Lambda_H} = \gamma_H\left[\boldsymbol{\mathcal{L}}\boldsymbol{\Psi}\boldsymbol{\varepsilon}_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I}\right].$$

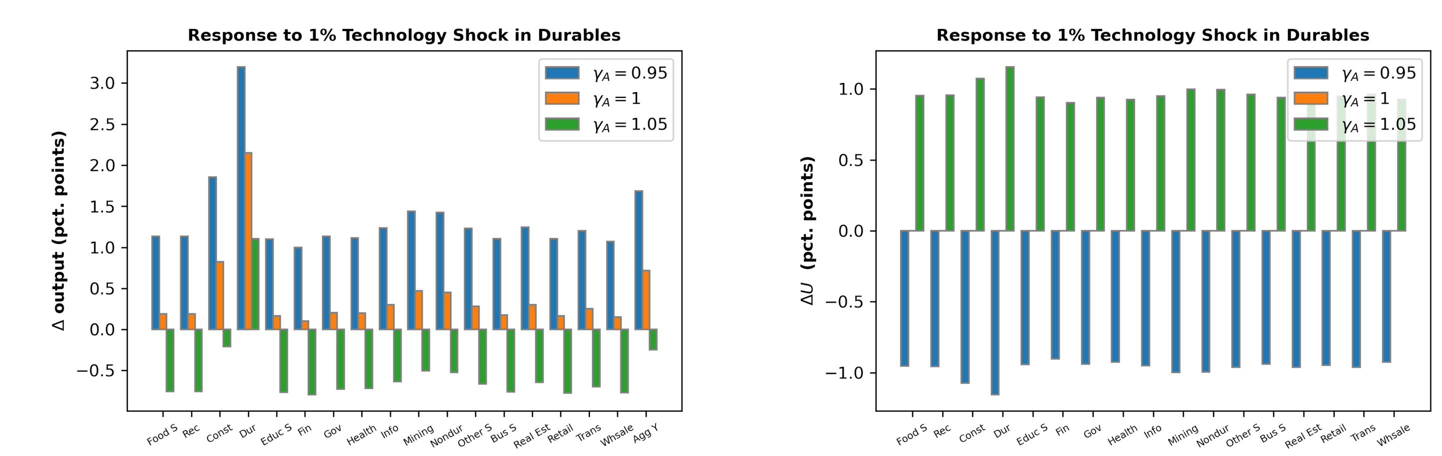


Figure 1. Response of sector level output and unemployment to a 1% productivity shock in the durables sector

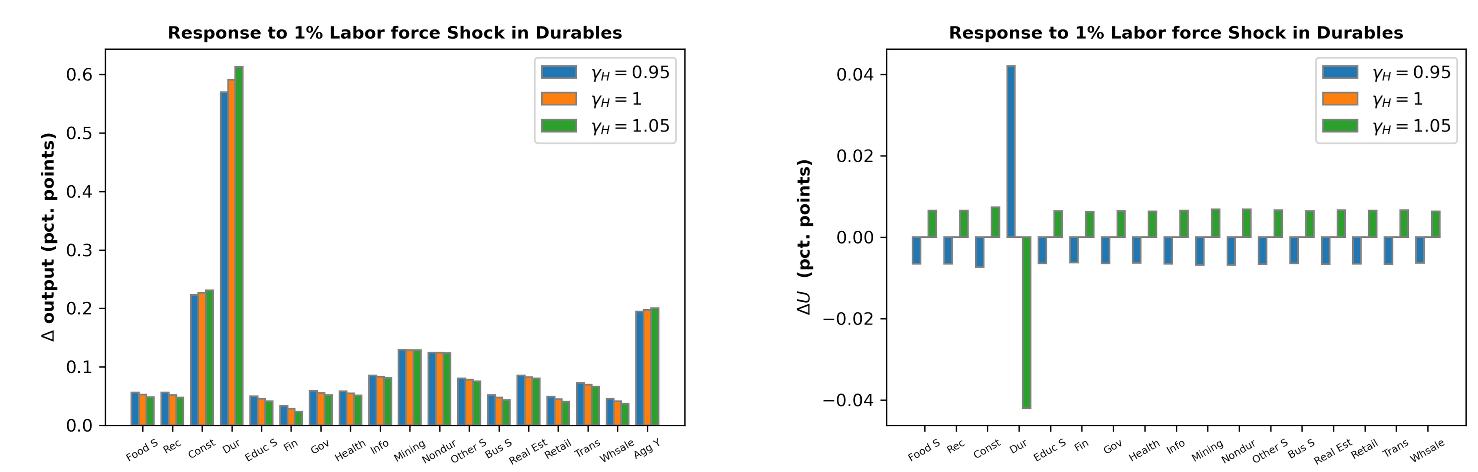


Figure 2. Response of sector level output and unemployment to a 1% shock to the size of the labor force in the durables sector

With varying relative wage assumptions, we observe:

- significant amplification / dampening of productivity shocks,
- small differences in responses to labor force shocks compared to efficient benchmark,
  - mostly due to our sector-specific occupation assumption.

Next Steps

- Calibrate realistic occupation model with Occupational Employment and Wage Statistics.
- Motivate our approach with empirical irregularities on comovements in sectoral factor reallocation.