# Unemployment in a Production Network

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# Motivation

# Motivation

Economies feature production linkages and frictional labor markets

Important to understand how production linkages interact with matching frictions and unemployment

This allows us to answer an array of important questions, such as:

- What is the impact of sector-specific productivity shocks on unemployment?
- How do labor market frictions affect shock propagation?
- What is the efficient allocation of unemployment across sectors?

# Related Literature

Two points of departure

1. Mismatch literature, e.g. [Şahin et al., 2014]
   - Linear production independent sectors, doesn't capture spillovers from moving people
2. Production networks literature
   - [Acemoglu et al., 2012, Acemoglu and Azar, 2020, Baqaee, 2018, Baqaee and Farhi, 2019, Baqaee and Rubbo, 2022, La'O and Tahbaz-Salehi, 2021]

# Variables of Interests

1. How does sector level output $d\log \boldsymbol{y}$ and unemployment $d\log \boldsymbol{u}$ respond to shocks to productivity $d\log \boldsymbol{A}$ and to the size of labor force $d\log \boldsymbol{H}$?
2. How do sector level responses aggregate to real GDP $d\log Y$ and unemployment $d\log U$?

This requires:

- $d\log \boldsymbol{p}$: How sector level prices change.
- $d\log \boldsymbol{w}$: How sector level wages change.
- $d\log \boldsymbol{\theta}$: How sector level tightnesses change.

# Our Model

# Sectors

$J$ sectors, indexed by $i$, produce with CRTS technology:

$$y_i = A_i f_i \left(N_i, \{x_{ij}\}_{j=1}^{J}\right)$$

Firms maximize profits:

$$\max_{N_i, \{x_{ij}\}_{j=1}^{J}} p_i f_i \left(N_i, \{x_{ij}\}_{j=1}^{J}\right) - w_i(1 + \tau_i(\theta_i))N_i - \sum_{j=1}^{J} p_j x_{ij}$$

This requires:

$$p_i f_{i,x_{ij}} = p_j \qquad \text{(FOC } x_{ij}\text{)}$$

$$p_i f_{i,N_i} = w_i \left(1 + \tau_i(\theta_i)\right) \qquad \text{(FOC } N_i\text{)}$$

# Labor Markets

Each sector has separate labor market, with new hires governed by matching technology $m$:

$$h_i = \phi_i m\left(u_i, v_i\right)$$

We assume balanced flows:

$$L_i^s(\theta_i) = \frac{\mathcal{F}_i(\theta_i)}{s_i + \mathcal{F}_i(\theta_i)} H_i$$
$$\tau_i(\theta_i) = \frac{r_i s_i}{\mathcal{Q}_i(\theta_i) - r_i s_i}$$

# Households

A final goods producer aggregates output by CRTS technology:

$$Y = \max_{\{c_i\}_{i=1}^{J}} \mathcal{D}\left(\{c_i\}_{i=1}^{J}\right),$$

subject to

$$\sum_{i=1}^{J} p_i c_i = \sum_{i=1}^{J} w_i L_i.$$

# Equilibrium Conditions (Goods market)

Firms:

$$\varepsilon^{f_i}_{x_{ij}} = \frac{p_j x_{ij}}{p_i y_i} \qquad \text{(Intermediate input decision)}$$

$$\varepsilon^{f_i}_{N_i} = (1 + \tau_i(\theta_i)) \frac{w_i N_i}{p_i y_i} \qquad \text{(Labor input decision)}$$

Households:

$$\varepsilon^{\mathcal{D}}_{c_i} = \frac{p_i c_i}{\sum_{k=1}^{J} p_k c_k} \qquad \text{(Consumption decision)}$$

Market clearing:

$$y_i = c_i + \sum_{j=1}^{J} x_{ji} \qquad \text{(Goods market clearing)}$$

# Equilibrium conditions (Labor market)

Labor market:

$$L_i^d = \varepsilon_{N_i}^{f_i} \frac{p_i y_i}{w_i} \qquad \text{(Labor demand)}$$

$$L_i^s = \frac{\mathcal{F}_i(\theta_i)}{s_i + \mathcal{F}_i(\theta_i)} H_i \qquad \text{(Labor supply)}$$

Market clearing:

$$L_i^s = L_i^d \qquad \text{(Labor market equilibrium)}$$

# Production network notation

$$\lambda_i = \frac{p_i y_i}{\sum_j p_j c_j} \qquad \text{(Sales share)}$$

$$\boldsymbol{\Omega} = \begin{bmatrix} \varepsilon^{f_1}_{x_{11}} & \varepsilon^{f_1}_{x_{12}} & \cdots & \varepsilon^{f_1}_{x_{1J}} \\ \varepsilon^{f_2}_{x_{21}} & \varepsilon^{f_2}_{x_{22}} & \cdots & \varepsilon^{f_2}_{x_{2J}} \\ \vdots & \vdots & \ddots & \vdots \\ \varepsilon^{f_J}_{x_{J1}} & \varepsilon^{f_J}_{x_{J2}} & \cdots & \varepsilon^{f_J}_{x_{JJ}} \end{bmatrix} \qquad \text{(Input-output matrix)}$$

$$\boldsymbol{\Psi} = (\boldsymbol{I} - \boldsymbol{\Omega})^{-1} \qquad \text{(Leontief inverse)}$$

$$\boldsymbol{\varepsilon}^{\boldsymbol{f}}_{\boldsymbol{N}} = \begin{bmatrix} \varepsilon^{f_1}_{N_1} & \cdots & \varepsilon^{f_J}_{N_J} \end{bmatrix}', \ \boldsymbol{\varepsilon}^{\boldsymbol{\Omega}}_{\boldsymbol{\theta}} = \begin{bmatrix} \varepsilon^{\Omega_1}_{\theta_1} & \cdots & \varepsilon^{\Omega_J}_{\theta_J} \end{bmatrix}'$$

$$\boldsymbol{\tau}(\boldsymbol{\theta}) = \begin{bmatrix} \tau_1(\theta_1) & \cdots & \tau_J(\theta_J) \end{bmatrix}', \ \boldsymbol{\mathfrak{F}} = \begin{bmatrix} \frac{s_1}{s_1 + \mathcal{F}_1(\theta_1)} \varepsilon^{\mathcal{F}_1}_{\theta_1} & \cdots & \frac{s_J}{s_J + \mathcal{F}_J(\theta_J)} \varepsilon^{\mathcal{F}_J}_{\theta_J} \end{bmatrix}'$$

# Theoretical Results

# Our Goal

Express $d \log Y$, where

$$d \log Y = \varepsilon_{\boldsymbol{c}}^{\mathcal{D}\prime} d \log \boldsymbol{c}$$

in terms of shocks $d \log \boldsymbol{A}$ and $d \log \boldsymbol{H}$ and primitives

This requires:

- $d \log \boldsymbol{p}$
- $d \log \boldsymbol{w}$
- $d \log \boldsymbol{\theta}$
- $d \log \boldsymbol{y}$

# Price Propagation

From the production function and the input decisions, sector level log price changes satisfy

$$
\begin{aligned}
d\log \boldsymbol{p} = \underbrace{\boldsymbol{\Psi}}_{\text{production linkages}} \Big( & \underbrace{\operatorname{diag}\left(\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}\right) d\log \boldsymbol{w}}_{\text{factor prices}} \\
& - \underbrace{\left(\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}\right) \operatorname{diag}\left(\boldsymbol{\tau}\right) \operatorname{diag}\left(\varepsilon^{\Omega}_{\boldsymbol{\theta}}\right) d\log \boldsymbol{\theta}}_{\text{searching and matching}} \\
& - \underbrace{d\log \boldsymbol{A}}_{\text{productivity}} \Big) \qquad \text{(Pricing Equation)}
\end{aligned}
$$

# Sales shares

From goods market clearing, changes in sales shares satisfy

$$d\boldsymbol{\lambda} = \left[ d\varepsilon_{\boldsymbol{c}}^{\mathcal{D}\prime} + \boldsymbol{\lambda}' d\boldsymbol{\Omega} \right] \boldsymbol{\Psi} \qquad \text{(Sales Shares)}$$

If production technology and utility are Cobb-Douglas, sales shares satisfy

$$d\boldsymbol{\lambda} = \mathbf{0}$$

# Sector level output

With sales shares and prices, we can back out sector level log output changes:

$$d \log \boldsymbol{y} = -d \log \boldsymbol{p} + d \log \boldsymbol{\lambda} + \boldsymbol{\Xi} d \log \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}, \qquad \text{(Sector Level Output)}$$

where $\boldsymbol{\Xi} = \left(\boldsymbol{I} - \operatorname{diag}\left(\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}\right) \boldsymbol{\Psi}\right)^{-1} \operatorname{diag}\left(\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}\right)$.

If production technology is Cobb-Douglas

$$d \log \boldsymbol{y} = -d \log \boldsymbol{p}$$

## Sector level tightness

Given shocks $d\log \boldsymbol{A}$ and $d\log \boldsymbol{H}$, and a general wage schedule $\boldsymbol{w}(\boldsymbol{\theta}, \boldsymbol{A})$, labor market clearing implies

$$\overbrace{\operatorname{diag}(\mathfrak{F})\, d\log \boldsymbol{\theta} + d\log \boldsymbol{H}}^{d\log \boldsymbol{L}^s} = \overbrace{-d\log \boldsymbol{w}(\boldsymbol{\theta}, \boldsymbol{A}) + d\log \boldsymbol{\lambda}(\boldsymbol{\theta}, \boldsymbol{A})}^{d\log L^d}$$
$$+ (\boldsymbol{I} + \Xi)\, d\log \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}(\boldsymbol{\theta}, \boldsymbol{A})$$

If the labor schedule $\boldsymbol{w}$ is differentiable in $\boldsymbol{A}$ and $\boldsymbol{\theta}$

$$d\log \boldsymbol{\theta} = \left(\operatorname{diag}(\mathfrak{F}) - \varepsilon^{\boldsymbol{L}^d}_{\boldsymbol{\theta}}\right)^{-1} \varepsilon^{\boldsymbol{L}^d}_{\boldsymbol{A}}\, d\log \boldsymbol{A} - \left(\operatorname{diag}(\mathfrak{F}) - \varepsilon^{\boldsymbol{L}^d}_{\boldsymbol{\theta}}\right)^{-1} d\log \boldsymbol{H}$$

# Sector level tightness - Cobb-Douglas

If production technology is Cobb-Douglas $\boldsymbol{\varepsilon}_{\boldsymbol{\theta}}^{\boldsymbol{L}^d} = \boldsymbol{\varepsilon}_{\boldsymbol{\theta}}^{\boldsymbol{w}}$ and $\boldsymbol{\varepsilon}_{\boldsymbol{A}}^{\boldsymbol{L}^d} = \boldsymbol{\varepsilon}_{\boldsymbol{A}}^{\boldsymbol{w}}$

$$d\log\boldsymbol{\theta} = \left(\operatorname{diag}\left(\mathcal{F}\right) - \boldsymbol{\varepsilon}_{\boldsymbol{\theta}}^{\boldsymbol{w}}\right)^{-1}\boldsymbol{\varepsilon}_{\boldsymbol{A}}^{\boldsymbol{w}} d\log\boldsymbol{A} - \left(\operatorname{diag}\left(\mathcal{F}\right) - \boldsymbol{\varepsilon}_{\boldsymbol{\theta}}^{\boldsymbol{w}}\right)^{-1} d\log\boldsymbol{H}$$

Assuming Cobb-Douglas production matrices of wage elasticities $\boldsymbol{\varepsilon}_{\boldsymbol{\theta}}^{\boldsymbol{w}}$, $\boldsymbol{\varepsilon}_{\boldsymbol{A}}^{\boldsymbol{w}}$ are sufficient statistics for changes in tightness.

In this case, if wages are rigid, changes in tightness are not related to production linkages. This is a special property for Cobb-Douglas production functions, since no expenditure switching occurs.

With CES, however, network linkages will still come into play through $d\log\boldsymbol{\lambda}$ and $d\log\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}$

# Summary

We now have a system of sector-level prices, quantities, and tightness:

$$d\log \boldsymbol{p} = \boldsymbol{\Psi}\left(\operatorname{diag}\left(\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}\right)\left(d\log \boldsymbol{w} - \operatorname{diag}\left(\boldsymbol{\tau}\right)\operatorname{diag}\left(\varepsilon^{\Omega}_{\boldsymbol{\theta}}\right) d\log \boldsymbol{\theta}\right) - d\log \boldsymbol{A}\right) \qquad \text{(pricing)}$$

$$d\boldsymbol{\lambda} = \left[d\varepsilon^{\mathcal{D}\prime}_{\boldsymbol{c}} + \boldsymbol{\lambda}' d\boldsymbol{\Omega}\right]\boldsymbol{\Psi} \qquad \text{(sales shares)}$$

$$d\log \boldsymbol{\theta} = \left(\operatorname{diag}\left(\mathfrak{F}\right) - \varepsilon^{\boldsymbol{L^d}}_{\boldsymbol{\theta}}\right)^{-1}\varepsilon^{\boldsymbol{L^d}}_{\boldsymbol{A}}\, d\log \boldsymbol{A} - \left(\operatorname{diag}\left(\mathfrak{F}\right) - \varepsilon^{\boldsymbol{L^d}}_{\boldsymbol{\theta}}\right)^{-1} d\log \boldsymbol{H} \qquad \text{(tightness)}$$

$$d\log \boldsymbol{y} = -d\log \boldsymbol{p} + d\log \boldsymbol{\lambda} + \boldsymbol{\Xi}\, d\log \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}, \qquad \text{(sector-level output)}$$

with assumptions on wage schedules determining $d\log \boldsymbol{w}$

# General aggregation formula

$$
\begin{aligned}
d\log Y &= d\log \sum_i p_i c_i - \sum_i \frac{p_i c_i}{\sum_j p_j c_j} d\log p_i \\
&= {\varepsilon_{\boldsymbol{c}}^{\mathcal{D}}}' d\log \boldsymbol{c} \\
&= {\boldsymbol{\varepsilon}_{\boldsymbol{c}}^{\mathcal{D}}}' \left( d\log \varepsilon_{\boldsymbol{c}}^{\mathcal{D}} + d\log \boldsymbol{y} - d\log \boldsymbol{\lambda} \right)
\end{aligned}
$$

# Aggregation under Cobb-Douglas

With Cobb-Douglas utility and production function,

$$\begin{aligned} d\log Y &= \varepsilon_{\boldsymbol{c}}^{\mathcal{D}'}(d\log \boldsymbol{y}) \\ &= -\varepsilon_{\boldsymbol{c}}^{\mathcal{D}'}\left[\boldsymbol{\Psi}(\operatorname{diag}\left(\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\right)(d\log \boldsymbol{w} - \operatorname{diag}(\boldsymbol{\tau})\operatorname{diag}\left(\varepsilon_{\boldsymbol{\theta}}^{\Omega}\right)d\log\boldsymbol{\theta}) - d\log\boldsymbol{A})\right] \end{aligned}$$

where

$$d\log\boldsymbol{\theta} = \left(\operatorname{diag}(\mathcal{F}) - \varepsilon_{\boldsymbol{\theta}}^{\boldsymbol{w}}\right)^{-1}\varepsilon_{\boldsymbol{A}}^{\boldsymbol{w}}d\log\boldsymbol{A} - \left(\operatorname{diag}(\mathcal{F}) - \varepsilon_{\boldsymbol{\theta}}^{\boldsymbol{w}}\right)^{-1}d\log\boldsymbol{H}$$

With fixed wages and labor force participation, this reduces to Hulten's theorem:

$$d\log Y = \varepsilon_{\boldsymbol{c}}^{\mathcal{D}'}\boldsymbol{\Psi}d\log\boldsymbol{A}$$

# Summary

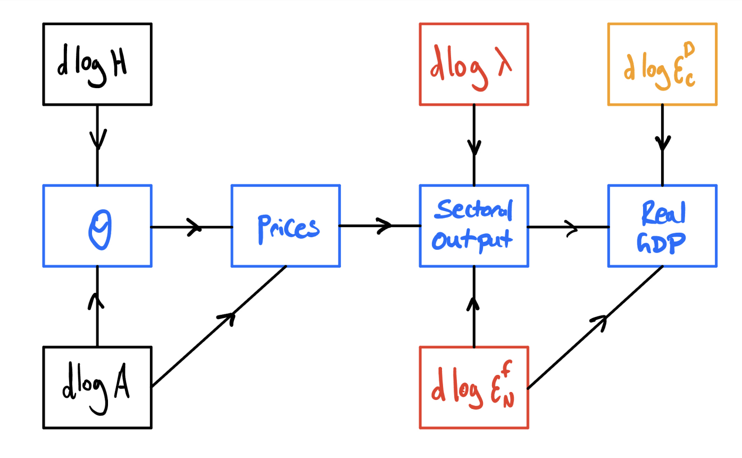


**Figure 1:** Summary of Solution Method

# Applications: Cobb-Douglas

# Calibration: Labor Market

Cobb-Douglas matching technology

$$h_i = \phi_i U_i^{\eta} V_i^{1-\eta}$$

set

- $\eta = 0.5$
- $r_i = 0.96$ for all $i$
- $s_i = 0.03$ for all $i$
- $\phi_i$ from [Şahin et al., 2014]
- $U_i$, $V_i$ from JOLTS

# Calibration: Product Market

Input output tables from BEA

- We use the 2007 6-digit NAICS classification level table, and aggregate back up to match the JOLTs industries

Sector gross output from BEA

# Key Parameters

| Sector | $\lambda$ | $\varepsilon_N^f$ | $\phi$ |
|---|---|---|---|
| Accommodation and food services | 0.05 | 0.46 | 1.53 |
| Arts, entertainment, and recreation | 0.02 | 0.43 | 1.68 |
| Construction | 0.09 | 0.42 | 1.66 |
| Durable goods | 0.19 | 0.28 | 0.84 |
| Educational services | 0.02 | 0.60 | 0.94 |
| Finance and insurance | 0.14 | 0.39 | 0.85 |
| Government | 0.20 | 0.60 | 0.87 |
| Health care and social assistance | 0.11 | 0.56 | 0.93 |
| Information | 0.08 | 0.34 | 0.76 |
| Mining | 0.03 | 0.29 | 1.71 |
| Nondurable goods | 0.18 | 0.17 | 0.96 |
| Other services, except government | 0.04 | 0.54 | 1.14 |
| Professional and business services | 0.18 | 0.55 | 1.43 |
| Real estate and rental and leasing | 0.18 | 0.13 | 1.41 |
| Retail trade | 0.09 | 0.54 | 1.47 |
| Transportation and warehousing | 0.06 | 0.38 | 1.14 |
| Wholesale trade | 0.09 | 0.52 | 1.21 |

**Table 1:** Key parameter values

# What is the effect of a $1\%$ productivity shock in all sectors on output and unemployment?

**Case 1: Fixed nominal wages**

Set $d\log \boldsymbol{H} = \mathbf{0}$, $d\log \boldsymbol{w} = \mathbf{0}$ (constant nominal wage)

- $\varepsilon_{\boldsymbol{\theta}}^{\boldsymbol{w}} = \varepsilon_{\boldsymbol{A}}^{\boldsymbol{w}} = \mathbf{0} \Rightarrow$
  - $d\log \boldsymbol{L} = d\log \boldsymbol{U} = 0$
  - $d\log Y = \underbrace{\boldsymbol{\varepsilon}_{\boldsymbol{c}}^{\mathcal{D}'}\boldsymbol{\Psi}}_{\boldsymbol{\lambda}'} d\log \boldsymbol{A}$ (Hulten's Theorem)

This gives the same output response to technology shocks as a standard production networks model.

# What is the effect of a $1\%$ productivity shock in all sectors on output and unemployment?

**Case 2: Partially adjusted wages**

Suppose $d\log \boldsymbol{H} = \mathbf{0}$, $d\log \boldsymbol{w} = \gamma d\log \boldsymbol{p}$ for $\gamma \neq 1$

- $\varepsilon_{\boldsymbol{\theta}}^{\boldsymbol{w}} = -\left(\boldsymbol{I} - \gamma\Psi\mathrm{diag}\left(\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\right)\right)^{-1}\gamma\Psi\mathrm{diag}\left(\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\right)\mathrm{diag}\left(\boldsymbol{\tau}\right)\mathrm{diag}\left(\varepsilon_{\boldsymbol{\theta}}^{\Omega}\right)$
- $\varepsilon_{\boldsymbol{A}}^{\boldsymbol{\theta}} = -\left(\boldsymbol{I} - \gamma\Psi\mathrm{diag}\left(\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\right)\right)^{-1}\gamma\Psi$

# Response of output to technology shocks

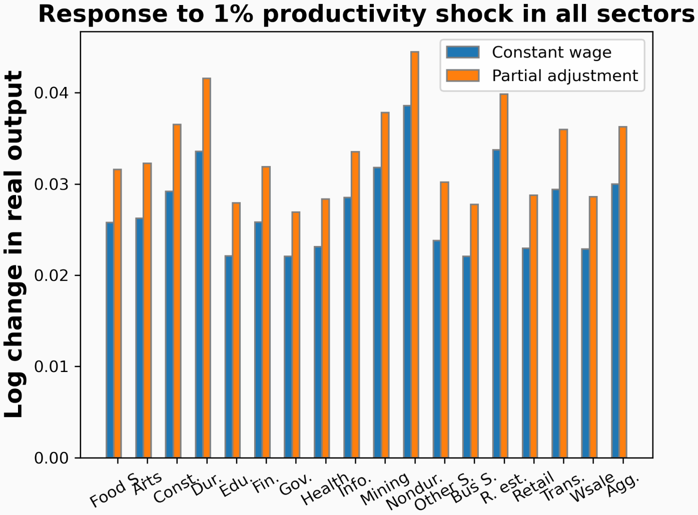


**Figure 2:** Response of output

# Response of unemployment to technology shocks

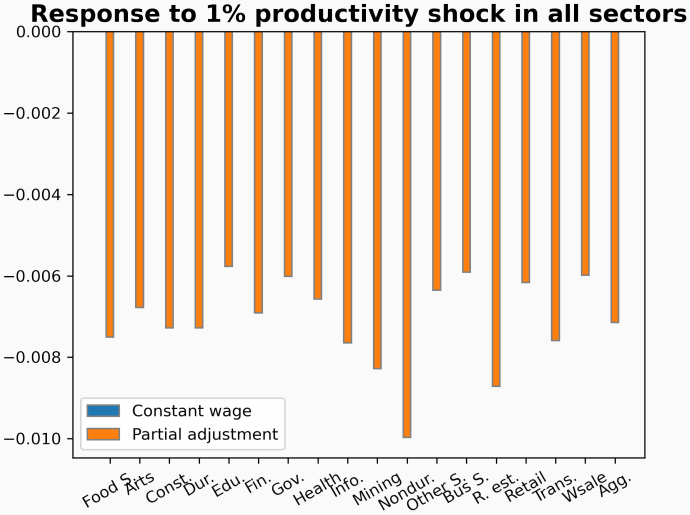


**Figure 3:** Response of unemployment rate

# Response of output and unemployment to sector-specific technology shocks

Can also ask what is response to shock in any given sector?

Suppose there is 1% shock to productivity in the durable goods sector with

1. Case 1: Constant wages
2. Case 2: Partial adjustment of wages

# Response of output to sector-specific technology shocks

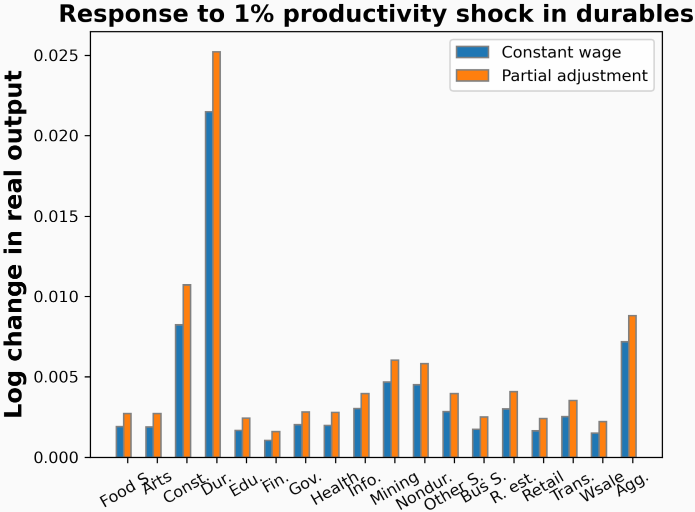


**Figure 4:** Response of output

# Response of unemployment to sector-specific technology shocks

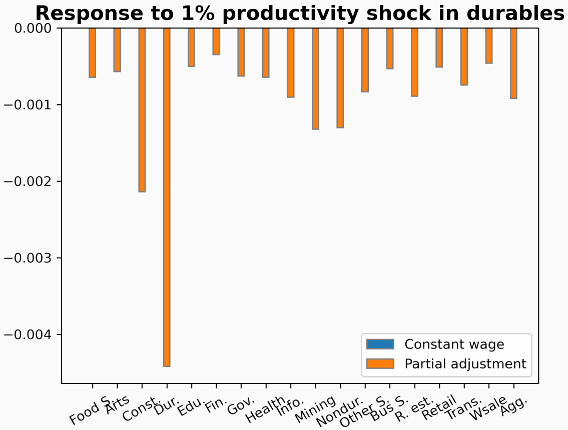


**Figure 5:** Response of unemployment rate

# Response to technology shocks - takeaways

- Labor market mechanisms introduce an additional channel of productivity shock propagation. General equilibrium response of tightness can amplify or dampen shocks.
- This framework allows us to quantify the impacts of shocks on sectoral co-movement of unemployment

## What is the effect of a $1\%$ negative shock to the size of the labor force in every sector?

**Case 1:** Set $d \log \boldsymbol{A} = d \log \boldsymbol{w} = 0$ (fixed nominal wages)

- $d \log \boldsymbol{\theta} = -\mathrm{diag}\left(\mathcal{F}\right)^{-1} d \log \boldsymbol{H}$

**Case 2:** Set $d \log \boldsymbol{A} = 0$, $d \log \boldsymbol{w} = \gamma d \log \boldsymbol{p}$ (partially adjusted wages)

- $d \log \boldsymbol{\theta} = -\left(\mathrm{diag}\left(\mathcal{F}\right) + \varepsilon_{\boldsymbol{\theta}}^{\boldsymbol{w}}\right)^{-1} d \log \boldsymbol{H}$

Where $\varepsilon_{\boldsymbol{\theta}}^{\boldsymbol{w}}$ is same as above.

# Response of output to labor force shocks

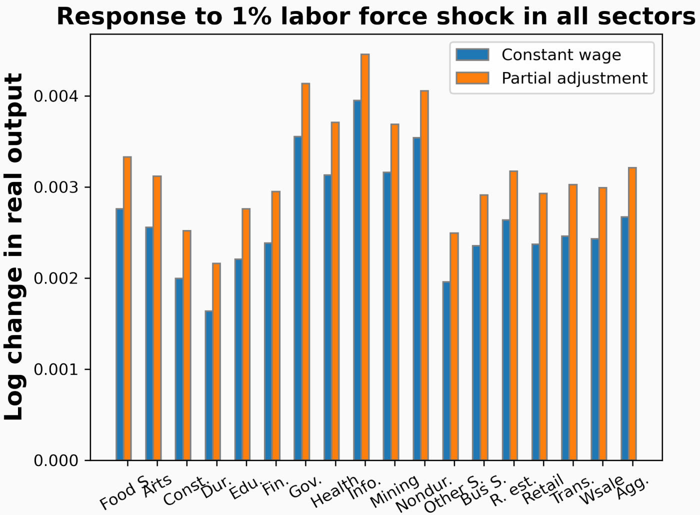


**Figure 6:** Response of output

# Response of unemployment to labor force shocks

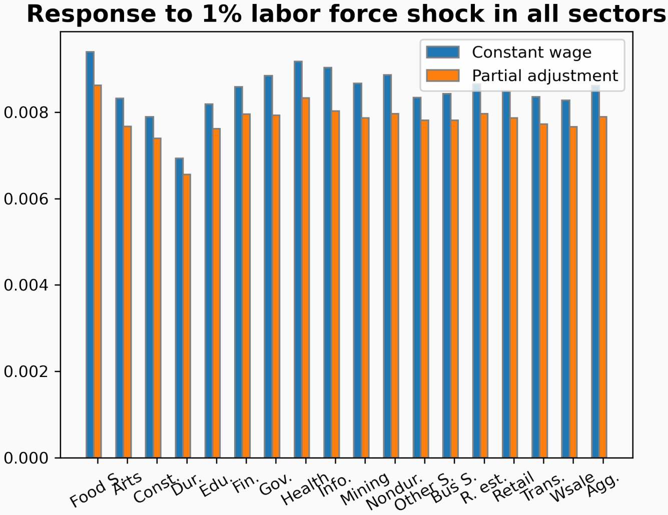


**Figure 7:** Response of unemployment rate

# Response of output to sector-specific labor force shocks

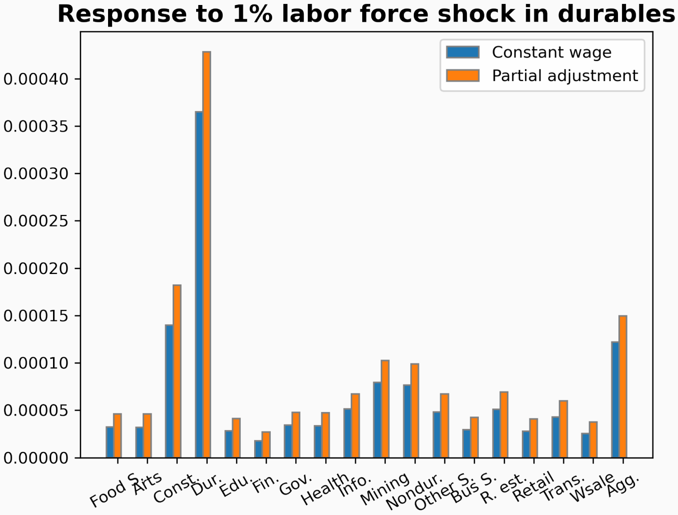


**Figure 8:** Response of output

# Response of unemployment to sector-specific labor force shocks

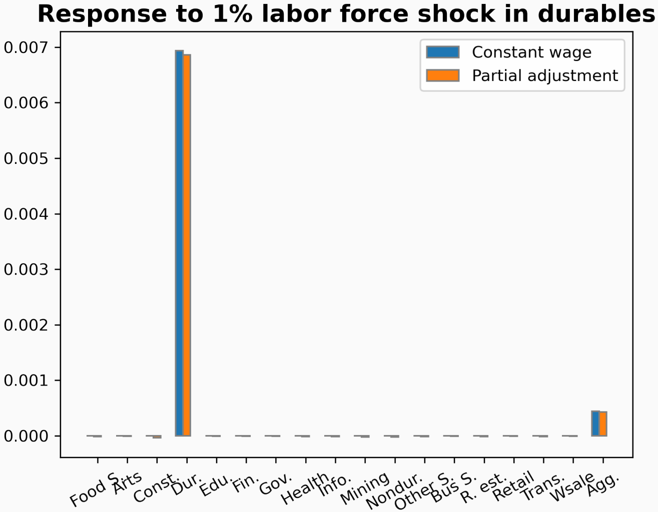


**Figure 9:** Response of unemployment rate

# Response to labor-force shocks - takeaways

- Labor force shocks have small impacts on output - limited impact of $d \log H$ on $d \log(1 + \tau(\theta))$
- Labor force shocks have small spillovers on unemployment - this is because sector labor markets are linked through goods markets.

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