Unemployment in a Production Network

Finn Schüle and Haoyu Sheng

August 9, 2023

Abstract

Modern economy features rich production linkages and frictional, segmented labor markets. We develop a new theoretical framework to study how they interact. Productivity shocks in relatively upstream sectors affect production in other sectors directly through the production of intermediate goods and indirectly through labor markets. Because different sectors hire similar workers, changes in labor demand in one sector affect the number of available workers in other sectors. This changes hiring costs and, therefore, output across the network. We find that, under a wide range of wage assumptions, labor market frictions amplify the response of aggregate output to sector-specific productivity shocks. We apply our model to analyze the impact of the Russia-Ukraine war during the "Great Resignation." Our model generates a modest decline in output, a pronounced increase in tightness, and relative price increases in energy-intensive sectors and their downstream sectors.

JEL Codes: E1, J3, J6

Schüle: finn_schuele@brown.edu, Brown University. Sheng: haoyu_sheng@brown.edu, Brown University. We thank Gabriel Chodorow-Reich, Eduardo Dávila, Fernando Duarte, Gauti Eggertsson, Amy Handlan, Yann Koby, and Pascal Michaillat for helpful comments and advice. We thank Rosemary Kaiser for her insightful discussion. We thank seminar speakers at Brown University for their invaluable feedback. All errors are our own. The code for this paper can be found here.

1. Introduction

Modern economies feature rich production networks. As illustrated by the simple example in Figure 1, car manufacturers buy steel and computer chips and sell trucks to transportation firms. The transportation firms, on the other hand, transport goods for the steel and chip producers. As the recent chip shortage [add citation] demonstrates, a shock to one sector can have wide-reaching consequences for firms in other sectors and consumers. A decline in the production of computer chips affects car manufacturers. As cars and trucks become more expensive, transportation firms suffer. Because chips and steel manufactures need transportation for their products, the original shock feeds back to those sectors as well.

Indeed, a burgeoning literature highlights that these production linkages between firms in different sectors matter for the propagation of sector level shocks (Baqaee and Rubbo 2022), how we measure productivity and the social cost of distortions (Baqaee and Farhi 2019, 2020), the optimal conduct of monetary policy (Rubbo 2020; La'O and Tahbaz-Salehi 2022), and that sector level shocks may even be a source of aggregate fluctuations and growth (Acemoglu et al. 2012; Acemoglu and Azar 2020).

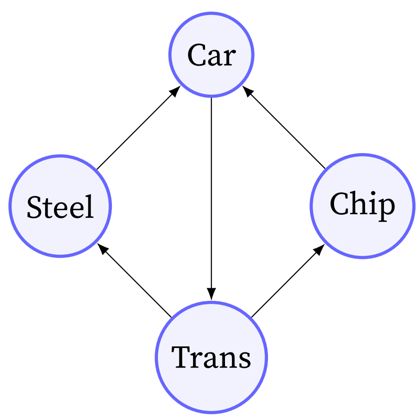


FIGURE 1. An example of a production network. "Car" refers to car manufacturers, "Steel" to steel manufacturer, "Chip" to computer chip manufacturers, and "Trans" to transportation firms. Modern economies are full of networks like this one, with consequences for the aggregate effects of sector level productivities and distortions. The arrows go from suppliers to customers.

We extend this existing literature by showing that frictional labor markets matter for shock propagation in production networks. Figure 2 augments the example from Figure 1 with simple labor markets. Now, a decline in chip production affects the number of engineers employed by chip producers. Car producers adjust their production because they not only use chips but also hire engineers. Meanwhile, car producers also decrease their labor demand for factory workers as a part of their production adjustment process. Steel producers now hire factory workers in a slacker labor market, and the decrease in recruiting costs allows steel producers to allocate more workers to steel production instead of recruiting.

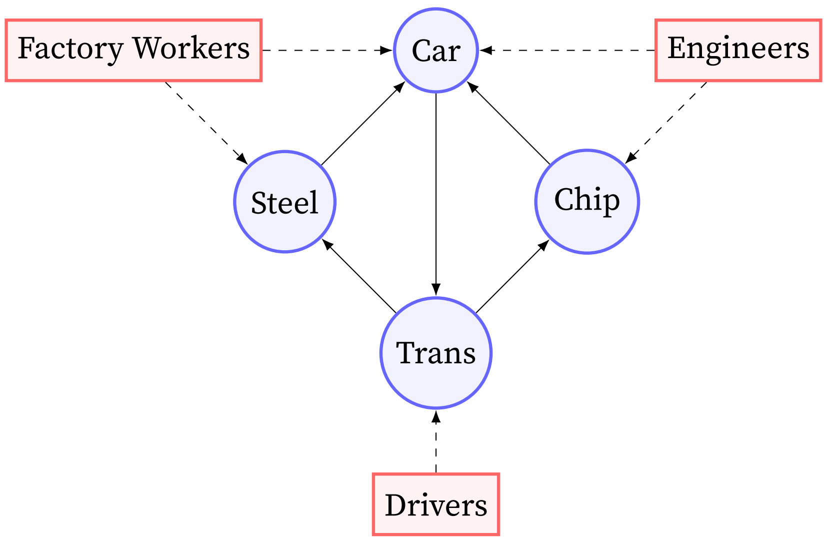


FIGURE 2. A production network with frictional and segmented labor markets.

As demonstrated by the simple example, the interplay between labor market frictions and production linkages introduces a novel shock propagation channel and impacts both aggregate and sectoral economic variables of interest such as output and unemployment. Understanding such interaction is key to painting an accurate and granular picture of how microeconomic shocks generate macroeconomic fluctuations and helps policymakers craft and evaluate policies. For example, we can dissect how energy price spikes caused by the Russia-Ukraine war impact output and unemployment for different sectors, or how the CHIPS and Science Act, which boosts domestic research and manufacturing of semiconductors in the United States, changes aggregate output and unemployment. More generally, we can answer the following questions: What are the impacts of idiosyncratic technology or labor supply shocks on sector and aggregate output and unemployment? How do idiosyncratic shocks generate comovements in tightness across labor markets? How do labor market inefficiencies and characteristics, such as matching frictions, mobility costs, and wage schedules, affect the network propagation of shocks?

In this paper, we extend the production network framework by incorporating matching frictions. We build a static, multisector production network model that features a representative household, many production sectors, perfect competition in product markets, and segmented, frictional labor markets. The form of labor market segmentation is flexible, and our model allows for geographic, sectoral, or occupational labor market segmentation.

With this setup, we develop a general theory of how technology shocks propagate and aggregate across the production network under search frictions. In particular, we show that the aggregate impact on output and unemployment can be decomposed into two components. The first term resembles the foundational aggregation theorem of Hulten (1978), in which the aggregate output response is a sales-share-weighted sum of sectoral technology shocks. The second term, which we call the search-and-matching channel, involves complex interaction between tightness, production structure, and labor market structure. We show that we can recover Hulten's Theorem when relative wages adjust exactly according to the occupational-labor-share-weighted marginal product of labor, which we call the network-adjusted marginal product of labor. Intuitively, when wages change by the network adjusted marginal product of labor, tightness remains unchanged, eliminating all additional propagation coming from labor market frictions.

Whether the search-and-matching channel dampens or amplifies technology shocks depends largely on how wages adjust. Since the search and matching setup admits a wide range of assumptions about wages with very different quantitative consequences, as demonstrated by Shimer (2005) and Hall (2005), we view this as an empirical question. Nevertheless, we use two toy examples to illustrate the determinants of the direction of amplification. First, we shut off production linkages in the model. We show that the direction of amplification depends on how wages respond relative to the marginal product of labor, as well as whether posting additional vacancies changes the number of productive workers. Since hiring an additional worker requires more vacancies as labor markets get tighter, and posting vacancies requires firms to allocate workers towards recruiting, there is an inflection point where an additional vacancy results in no change in the productive workforce. We then solve a simple two-sector vertical production networks model with one labor market, to show that the direction of amplification now depends on how wages respond to the network-adjusted marginal product of labor. We further illustrate that our frictional labor market setup allows for upstream propagation of shocks, which is absent in efficient production networks with Cobb-Douglas utility and production functions.

In addition to technology shocks, our setup allows us to explore another set of shocks—shocks to the size of the labor force. We can think of these shocks as coming from an underlying model of occupational choice, as in Humlum (2021), or an underlying model of migration, as in Fernandez-Villaverde (2020). Unlike in the standard production networks setup, where a positive shock to the size of the labor force mechanically increases output because firms are forced to hire all available workers, our framework allows the hiring decision of firms to vary depending on wages. For instance, if wages do not change by much in response to positive labor force shocks, firms do not increase their hiring by much, dampening the overall effect on output but generating large changes to unemployment.

To test the empirical relevance of the theoretical channels outlined above, we calibrate our model to US data. We use survey-based vacancy and hiring data from the Job Openings and Labor Turnover Survey (JOLTS), unemployment data from the Current Population Survey (CPS), occupation data from the Occupational Employment and Wage Statistics (OEWS), and industry sales shares from the U.S Bureau of Economic Analysis (BEA). We find that, assuming network price adjusted wages do not change, a one-percent positive technology shock to the durables manufacturing sector increases aggregate output by 0.56 percent, and decreases aggregate unemployment by 0.44 percent. Without search and matching frictions, the same shock would increase output by only 0.25 percent and would not affect unemployment. Importantly, we observe how unemployment changes across occupations, with unemployment decreases by 0.99 percent for production workers, by 0.77 percent for engineers, and by 0.70 percent for construction workers.

In section 5 we demonstrate how to use our calibrated model for macroeconomic applications. In particular, we examine the employment and output consequences of a simultaneous negative shock to energy supply and labor force participation to explore the consequences of the Russia-Ukraine War related reduction in global oil supply at a time when many workers still had not returned to the labor force after Covid. In a model without search frictions, this generates large changes in output and no changes in labor market tightness. In our model, on the other hand, this combination of shocks generates a decline in output that is half as large, sizeable increases in labor market tightness, and larger increases in relative prices in energy intensive sectors relative to the education and health services sector, a sector where prices stayed relatively constant. We view all three as more realistic given 2022 U.S. data. For simplicity and to keep the focus on labor market frictions, our current model is static, features no nominal rigidities, and is entirely real. As a result, we can only speak to relative prices across sectors. However, in future work we hope to extend the model to incorporate dynamics and nominal rigidities. This would allow us explore the consequences of a combination of factor and labor supply shocks for price dynamics, extending Minton and Wheaton (2022) to incorporate labor market frictions and Benigno and Eggertsson (2023) to incorporate shocks to supply chain inputs. We believe a combination of a non-linear Phillips curve in labor market tightness, as in Benigno and Eggertsson (2023), and the gradual propagation of factor prices through a production network, as in Minton and Wheaton (2022), would paint a realistic picture of the current inflation episode.

This paper contributes to the production network literature that started with Long and Plosser (1983) and Acemoglu et al. (2012). Since an early contribution by Jones (2011), which demonstrates that missallocation accross a production network can generate more realistic transition dynamics in a neoclassical growth model, a recent literature has incorporated inefficiencies into production network models, including markups and financial frictions (Liu 2019; Baqaee and Farhi 2020; Bigio and La'O 2020) and nominal rigidities (Rubbo 2020; La'O and Tahbaz-Salehi 2022; Minton and Wheaton 2022). While these models offer a more realistic depiction of product and financial market inefficiencies, their treatment of labor markets is simple, considering labor as either perfectly inelastically supplied, or supplied according to the disutility of work in household's utility function. Howard (2019) builds a two-sector, two-country, endogenous production network model with matching frictions and finds that labor market efficiency has modest impact on production network density. Huneeus et al. (2023) builds a production network model with firm labor market power to study the impact of production linkages on earnings inequality. Like La'O and Tahbaz-Salehi (2022) and Rubbo (2020), who consider New Keynesian production network models, our model features endogenous wedges, in our case generated by search and matching frictions. To our knowledge, we are the first to build a tractable and flexible model that allows for arbitrary network structure, labor market structure, and wage assumptions to study how the endogenous network matching wedge impact the propagation and aggregation of microeconomic shocks.

This paper also contributes to the literature on factor reallocation and the aggregate impact of differential regional responses to shocks. Like Adão et al. (2019), who study the differential and aggregate impact of trade shocks on U.S. labor markets, our model allows us to explore both the aggregate and regional affects of shocks in a unified framework. Our model differs in that we incorporate labor market frictions, allowing us to speak to involuntary unemployment. Labor market frictions also naturally generate the high sensitivity of employment to wages documented empirically by Adão et al. (2019). Like Chodorow-Reich and Wieland (2020), who study the impact of labor reallocation across industries over the business cycle, we build a multi-sector search-and-matching model of the labor market. We demonstrate that incorporating production linkages to the multi-sector model qualitatively and quantitatively changes the propagation of shocks to technology. Therefore, although this is not the focus of our paper, production linkages would presumably alter labor reallocation patterns as well. To the best of our knowledge, our paper is the first to shed light on how production linkages impact reallocation and co-movements in labor markets under search-and-matching frictions.

The remainder of the paper is organized as follows. Section 2 outlines our model and defines the equilibrium. Section 3 derives expressions for first order changes in output and employment in response to changes in technology and the size of the labor force. Section 4 describes the data we use to calibrate the model and presents illustrative examples to demonstrate the quantitative importance of labor markets. Section 5 works through an application of our model to energy supply shocks. Section 6 concludes.

2. Model

Our model is a static multi-sector production networks model (Baqaee and Farhi 2020; Bigio and La'O 2020). The model features J production sectors, indexed by i, and $\mathcal{O}$ occupations index by o. Production requires intermediate inputs, labor, and fixed factors. There is a separate, *frictional* labor market for each occupation. Because we are interested in energy cost shocks our model features $\mathcal{K}$ additional factors of production. In our empirical application, one of these will be energy. In this section, we present the model and characterize its equilibrium. For expositional clarity, the model we present features Cobb-Douglas production functions and preferences. We derive general results for constant return-to-scale technology in Appendix B.

2.1. Environment

2.1.1. Households

Preferences. The economy features a representative household with homothetic preferences over the goods from each sector

$$\mathcal{U}(\{c_i\}_{i=1}^{J}) = \sum_{i=1}^{J} \sigma_i \log c_i,$$

where

$$\sum_i \sigma_i = 1.$$

Budget constraint. The representative household inealastically supplies a labor force of size H_o to occupation o. In addition, the household owns the fixed factors required for production. As a result, the household's budget constraint is:

$$\sum_{i=1}^{J} p_i c_i = \sum_{o=1}^{\mathcal{O}} w_o L_o + \sum_{k=1}^{\mathcal{K}} r_k K_k,$$

where c_i is the final consumption of sector i's output, p_i is the price of the sector i good, w_o is the wage in occupation o, and L_o is the labor used in sector o. r_k are the prices of the additional factors K_k of production. We can think of these $\mathcal{K}$ additional inputs into production as capital or energy.

Optimization. The household faces the following optimization problem:

$$\max_{\{c_i\}_{i=1}^{J}} \mathcal{U}(\{c_i\}_{i=1}^{J}),$$

subject to

$$\sum_{i=1}^{J} p_i c_i = \sum_{o=1}^{\mathcal{O}} w_o L_o + \sum_{k=1}^{\mathcal{K}} r_k K_k.$$

The consumption choices satisfy the first order condition

$$\sigma_i = \frac{p_i c_i}{\sum_{j=1}^{J} p_j c_j}. \tag{1}$$

2.1.2. Sectors

Production. A representative firm in sector i uses workers in occupation o, N_{io}, intermediate inputs from sector j, x_{ij}, and $\mathcal{K}$ additional factors K_{ik} to produce output y_i using Cobb-Douglas technology. We refer to the representative firm directly as a sector.

$$y_i = A_i \Pi_{j=1}^{J} \Pi_{o=1}^{\mathcal{O}} \Pi_{k=1}^{\mathcal{K}} x_{ij}^{\alpha_{ij}} N_{io}^{\beta_{io}} K_{ik}^{\kappa_{ik}}, \tag{2}$$

where

$$\sum_{j=1}^{J} \alpha_{ij} + \sum_{o=1}^{\mathcal{O}} \beta_{io} + \sum_{k=1}^{\mathcal{K}} \kappa_{ik} = 1.$$

The goods produced by a sector can be either an intermediate good for other sectors, or a consumption good for the household.

Profits. A sector's profit is given by the difference between its revenue and costs. The production costs include labor cost, intermediate input cost, and fixed factor cost. The profit π_i for sector i is:

$$\pi_i = p_i y_i - \sum_{o=1}^{\mathcal{O}} w_o L_{io} - \sum_{j=1}^{J} p_j x_{ij} - \sum_{k=1}^{\mathcal{K}} r_k K_{ik},$$

where

$$L_{io} = (1 + \tau_o(\theta_o)) N_{io}.$$

Here L_{io} denotes the total number of workers from occupation o hired by sector i. $\tau_o = \frac{L_{io} - N_{io}}{N_{io}}$ is the recruiter-producer ratio. We explain the distinction between recruiters and producers in subsubsection 2.1.3.

Optimization. Sectors choose $\left\{N_{io}\right\}_{o=1}^{\mathcal{O}}$, $\left\{x_{ij}\right\}_{j=1}^{J}$, and $\left\{K_{ik}\right\}_{ik}^{\mathcal{K}}$ to maximize profits, or equivalently to minimize costs, taking all prices and wages as given

$$\max_{\left\{N_{io}\right\}_{o=1}^{\mathcal{O}},\left\{x_{ij}\right\}_{j=1}^{J},\left\{K_{ik}\right\}_{k=1}^{\mathcal{K}}} \pi_i\left(\left\{N_{io}\right\}_{o=1}^{\mathcal{O}},\left\{x_{ij}\right\}_{j=1}^{J},\left\{K_{ik}\right\}_{k=1}^{\mathcal{K}}\right).$$

The profit maximization problem gives the first order conditions

$$\alpha_{ij} = \frac{p_j x_{ij}}{p_i y_i}, \tag{3}$$

$$\beta_{io} = (1+\tau_o(\theta_o))\,\frac{w_o N_{io}}{p_i y_i}, \tag{4}$$

$$\kappa_{ik} = \frac{r_k K_{ik}}{p_i y_i}. \tag{5}$$

2.1.3. Labor Markets

We assume there are $\mathcal{O}$ occupations with separate labor markets. Labor market o has a labor force of H_o possible workers, who all start out unemployed at the beginning of the single period. Firms must hire recruiters in order to post vacancies. We assume that recruiters in labor force o are themselves type o workers. The exogenous recruiting cost, r_o, measures the units of type o labor required for a firm to maintain each posted vacancy in occupation o.

Matching Functions. Hires are generated by a Cobb-Douglas matching function in occupation-level unemployment H_o and vacancies V_o, which measures all vacancy postings for occupation o,

$$h_o = \phi_o H_o^{\eta_o} V_o^{1-\eta_o}.$$

The number of aggregate vacancy postings V_o is the sum of sectoral vacancy postings $\sum_{i=1}^{J} v_{io}$. The occupational labor market tightness is $\theta_o = \frac{V_o}{H_o}$[1].

[1]Since our model is static and the entire labor force starts out unemployed, beginning of period unemployment is H_o. This is the relevant level of unemployment for determining the number of hires generated by the matching function. We use the more conventional U_o to denote equilibrium or end-of-period unemployment.

The vacancy-filling rate $\mathcal{Q}_o$ and the job-finding rate $\mathcal{F}_o$ are thus:

$$\mathcal{Q}_o(\theta_o) = \frac{h_o}{V_o} = \phi_o\theta_o^{-\eta_o}, \quad \mathcal{F}_o(\theta_o) = \frac{h_o}{H_o} = \phi_o\theta_o^{1-\eta_o}.$$

These are, respectively, the probability of a posted vacancy being filled and the probability of an unemployed worker finding a job.

Labor Supply. A fraction $\mathcal{F}_o(\theta_o)$ of the labor force finds a job and is employed at the end of the period. Labor supply satisfies

$$L_o^s(\theta_o) = \mathcal{F}_o(\theta_o)H_o. \tag{6}$$

And end-of-period unemployment is

$$U_o = H_o - L_o^s(\theta_o).$$

Labor Demand. We assume firms take the occupation level tightness as given.[2] We denote sector i's demand for occupation o labor by L_{io}^d. This labor demand is determined by the firm's profit maximization problem. Due to the matching frictions outline above, $L_{io}^d = \mathcal{Q}(\theta_o)v_{io}$: The total number of type o hires in sector i is equal to the type o vacancy filling rate times the number of vacancies posted by sector i firms.

Because firms must employ recruiters to post vacancies, $r_o v_i o$ of the total labor hired by sector i does not engage in production. The number of productive occupation o workers is $N_{io} = \mathcal{Q}(\theta_o)v_{io} - r_o v_{io}$. Rearranging yields the following expression for the recruiter-producer ratio for occupation o,

$$\tau_o(\theta_o) \equiv \frac{r_o v_{io}}{N_{io}} = \frac{r_o}{\mathcal{Q}_o(\theta_o) - r_o}.$$

In the language of the production networks literature, τ_o acts as an endogenous wedge on sectors' labor costs. This endogenous wedge plays an important role in how shocks propagate through the production network through labor demand. In particular, τ_o drives a wedge between the total number of employed occupation o workers and the number of occupation o workers engaged in production, $L_{io}^d(\theta_{io}) = (1+\tau_o(\theta_o))N_{io}$.

Finally, we define aggregate occupation o labor demand as the sum of sectoral labor

[2] We can think of each sector as being populated by many identical competitive firms so that each firm only has an infinitesimal impact on aggregate vacancies, and therefore on aggregate tightness.

demands.

$$L_o^d(\theta_o) = \sum_{i=1}^{J} L_{io}^d(\theta_o) = \sum_{i=1}^{J}(1+\tau_o(\theta_o))N_{io} \tag{7}$$

Wages. In matching models, workers and firms meet in a situation of bilateral monopoly. The resulting mutual gains from trade mean that wages are not determined by equilibrium conditions of the model[3], and must instead be pinned down by some wage setting norm chosen by the researcher. For now, we assume a general functional form for wages:

$$\boldsymbol{w} = g(\boldsymbol{p}, \boldsymbol{A}, \boldsymbol{H}, \boldsymbol{K^s}). \tag{8}$$

Where $\boldsymbol{p}$ is a $J \times 1$ vector of sectoral prices, $\boldsymbol{A}$ is a $J \times 1$ vector of sectoral productivities, $\boldsymbol{H}$ is an $\mathcal{O} \times 1$ vector of occupational labor force sizes, and $\boldsymbol{K}^s$ is a $\mathcal{K} \times 1$ vector of the additional factor supplies. This wage assumption nests both nominally rigid and real rigid wages, revenue sharing between workers and firms, and fully flexible wages. For example, nominal wages can be a function of productivity as in Blanchard and Galí (2010) and Michaillat (2012), or constant. Because prices are themselves a function of $\boldsymbol{A}, \boldsymbol{H}$, and $\boldsymbol{K^s}$, including prices in $\boldsymbol{p}$ is simply a matter of convenience. It allows us to express real wage rigidity more compactly in section 3.

2.1.4. Equilibrium

Given exogenous variables $\left\{\{A_i\}_{i=1}^J, \{H_o\}_{o=1}^{\mathcal{O}}, \left\{K_k^s\right\}_{k=1}^K\right\}$ and a wage function g, the equilibrium is a collection of allocations $\left\{\left\{y_i, \{x_{ij}\}_{j=1}^J, c_i, \{N_{io}\}_{o=1}^{\mathcal{O}}, \{K_{ik}\}_{k=1}^K\right\}_{i=1}^J, \{\theta_o\}_{o=1}^{\mathcal{O}}\right\}$ and prices $\left\{\{p_i\}_{i=1}^J, \{w_o\}_{o=1}^{\mathcal{O}}, \{r_k\}_{k=1}^{\mathcal{K}}\right\}$ such that

(i) The allocations solve the household's problem (Equation 1) at the prevailing prices.

(ii) The allocations solve the firm's problem (Equations 2 - 5) at the prevailing prices.

[3] Wages are only constrained to fall within a range where both workers and firms benefit from the match. However, this range can be wide since workers usually strongly prefer employment to unemployment and finding a new match is costly for firms.

(iii) Goods, factor, and labor markets clear

$$y_i = c_i + \sum_{j=1}^{J} x_{ij} \qquad \forall i \in \{1, 2, \ldots, J\} \tag{9}$$

$$K_k^s = \sum_{i=1}^{J} K_{ik} \qquad \forall k \in \{1, 2, \ldots, \mathcal{K}\} \tag{10}$$

$$L_o^d = L_o^s \qquad \forall o \in \{1, 2, \ldots, \mathcal{O}\}. \tag{11}$$

(iv) Wages are set according to Equation 8.

3. Theoretical Results: The Propagation, Aggregation, and Amplification of Shocks

In this section, we describe the main theoretical results of our paper. We define shocks to be small proportional changes in the exogenous variables. We first derive the propagation of technology, labor supply, and factor supply shocks at the dis-aggregated occupation and sector level. We compute the first-order proportional responses of output and unemployment, the main endogenous variables of interest in the model. We then present our aggregation theorem for idiosyncratic shocks. We show how to generalize the results to any constant returns production function in the case of one occupation per sector in appendix B.

This section also highlights the theoretical importance of modeling labor market frictions and production linkages in conjunction. Labor demand for a particular occupation depends not only on that occupations importance to each sector, but also on the production linkages between sectors. Labor demand, in turn, affects tightness in a frictional labor market. This creates an endogenous matching wedge which affects the propagation of shocks. In the following section, we show that the effects of this wedge are salient for output and unemployment.

3.1. The Propagation of Shocks

We are interested in how three sets of endogenous variables—sector level output, occupation level unemployment, and sector level relative prices—change in response to changes in technology and the size of the labor force. With these variables, we can compute the aggregate variables of interest: output and unemployment.

Before deriving our results, we introduce the following reduced-form relationship between relative wages and the shocks:

$$d\log \boldsymbol{w} - \mathcal{L} d\log \boldsymbol{p} = \boldsymbol{\Lambda_A} d\log \boldsymbol{A} + \boldsymbol{\Lambda_H} d\log \boldsymbol{H} + \boldsymbol{\Lambda}_K d\log \boldsymbol{K}^s,$$

where $d\log \boldsymbol{w}$ and $d\log \boldsymbol{H}$ are $\mathcal{O} \times 1$ dimensional vectors capturing first order changes in wages and the size of the labor force, $d\log \boldsymbol{p}$ and $d\log \boldsymbol{A}$ are $J \times 1$ dimensional vectors capturing first order changes in prices and productivities, and $d\log \boldsymbol{K}^s$ is a $\mathcal{K} \times 1$ dimensional vector of first order changes in factor supply. $\mathcal{L}$ is the occupational-share matrix, a $\mathcal{O} \times J$ matrix with the share of occupation o workers employed in each sector along the rows. The (o, j)-th entry is $\frac{l_{jo}}{L_o}$. This matrix captures each sector's importance as an employer of occupation o workers. $\boldsymbol{\Lambda}_A$, $\boldsymbol{\Lambda}_H$, and $\boldsymbol{\Lambda}_K$ are $\mathcal{O} \times J$, $\mathcal{O} \times \mathcal{O}$, and $\mathcal{O} \times \mathcal{K}$ coefficient matrices that capture how wages respond to technology, labor force, and factor supply shocks in each other sector, occupation, and factor market. For instance, the (o, i)-th entry of $\boldsymbol{\Lambda}_A$ captures how wages in occupation o respond to technology shocks in sector i.

This relationship is a first-order approximation of the wage function in Equation 8. We choose to represent the relative wages as the difference between change in nominal occupational wages and change in occupation-share weighted prices, because this is the effective wage that determines labor demand for workers in each occupation. As we show below, relative prices are themselves determined by changes in exogenous variables such as technology shocks, labor force shocks, and factor supply shocks. Working with price-adjusted wages is therefore an algebraically convenient but innocuous choice. Working with nominal wages would lead to more complicate but ultimately equivalent expressions.

We now derive how shocks propagate to labor market tightness, output, and unemployment. These first order effects are summarized in proposition 1. For a detailed derivation see appendix A.1.

Proposition 1. *Let* $\boldsymbol{\Psi} = (\boldsymbol{I} - \boldsymbol{\Omega})^{-1}$ *denote the Leontief inverse, where* $\boldsymbol{\Omega}$ *is the* $J \times J$ *input-output matrix. Let* $\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}$ *and* $\varepsilon^{\boldsymbol{f}}_{\boldsymbol{K}}$ *denote* $J \times \mathcal{O}$ *and* $J \times \mathcal{K}$ *matrices of production elasticities to labor and factor inputs respectively. Let* $\mathcal{M}$ *be a diagonal matrix with the* $\mathcal{O}$ *matching elasticities along the main diagonal. Finally, let* $\mathcal{T}$ *be a diagonal matrix with the* $\mathcal{O}$ *recruiter-producer ratios along the main diagonal.*

Given occupational labor supply shocks $d\log \boldsymbol{H} = \left[d\log H_1, \cdots, d\log H_{\mathcal{O}}\right]'$, *sectoral productivity shocks* $d\log \boldsymbol{A} = \left[d\log A_1, \cdots, d\log A_J\right]'$, *and factor supply shocks* $d\log \boldsymbol{K}^s =$

$\left[d\log K_1^s, \cdots, d\log K_{\mathcal{K}}^s\right]'$, *the first-order responses of labor market tightness* $d\log\theta = \left[d\log\theta_1, \cdots, d\log\theta_{\mathcal{O}}\right]$ *follows*

$$d\log\theta = \Pi_{\theta,A} d\log \boldsymbol{A} + \Pi_{\theta,H} d\log \boldsymbol{H} + \Pi_{\theta,K} d\log \boldsymbol{K}^s$$

where

$$\begin{aligned}
\Pi_{\theta,A} &= \left[\boldsymbol{I} - \mathcal{M} - \Xi_\theta\right]^{-1}(\mathcal{L}\Psi - \Lambda_A),\\
\Pi_{\theta,H} &= \left[\boldsymbol{I} - \mathcal{M} - \Xi_\theta\right]^{-1}\left(\mathcal{L}\Psi\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I} - \Lambda_H\right),\\
\Pi_{\theta,K} &= \left[\boldsymbol{I} - \mathcal{M} - \Xi_\theta\right]^{-1}\left(\mathcal{L}\Psi\varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}} - \Lambda_K\right),\\
\Xi_\theta &= \mathcal{L}\Psi\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left[\boldsymbol{I} - \mathcal{M}(\boldsymbol{I} + \mathcal{T})\right].
\end{aligned}$$

The first order response of sectoral output $d\log \boldsymbol{y} = \left[d\log y_1, \cdots, d\log y_J\right]'$ *follows*

$$d\log \boldsymbol{y} = \Pi_{y,A} d\log \boldsymbol{A} + \Pi_{y,H} d\log \boldsymbol{H} + \Pi_{y,K} d\log \boldsymbol{K}^s,$$

where

$$\begin{aligned}
\Pi_{y,A} &= \Psi\left[\boldsymbol{I} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left(\boldsymbol{I} - \underbrace{\mathcal{M}(\boldsymbol{I} + \mathcal{T})}_{\text{search cost}}\right)\underbrace{\left[\boldsymbol{I} - \mathcal{M} - \Xi_\theta\right]^{-1}\left[\mathcal{L}\Psi - \Lambda_A\right]}_{\Pi_{\theta,\boldsymbol{A}}}\right],\\
\Pi_{y,H} &= \Psi\left[\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left(\boldsymbol{I} - \underbrace{\mathcal{M}(\boldsymbol{I} + \mathcal{T})}_{\text{search cost}}\right)\underbrace{\left[\boldsymbol{I} - \mathcal{M} - \Xi_\theta\right]^{-1}\left[\mathcal{L}\Psi\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I} - \Lambda_H\right]}_{\Pi_{\theta,\boldsymbol{H}}}\right],\\
\Pi_{y,K} &= \Psi\left[\varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left(\boldsymbol{I} - \underbrace{\mathcal{M}(\boldsymbol{I} + \mathcal{T})}_{\text{search cost}}\right)\underbrace{\left[\boldsymbol{I} - \mathcal{M} - \Xi_\theta\right]^{-1}\left[\mathcal{L}\Psi\varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}} - \Lambda_K\right]}_{\Pi_{\theta,\boldsymbol{K}}}\right].
\end{aligned}$$

Finally, the expression for labor supply implies that first order changes in the end of period occupational unemployment level $d\log \boldsymbol{U} = \left[d\log U_1, d\log U_2, \cdots, d\log U_{\mathcal{O}}\right]$ *follow*

$$d\log \boldsymbol{U} = -(\boldsymbol{I} - \mathcal{M})\, d\log\theta + d\log \boldsymbol{H}.$$

PROOF. See appendix A.1. □

We now unpack the intuition behind these expressions. First, we consider the response in tightness. The equilibrium response in tightness is jointly determined by changes in labor supply and labor demand. Labor supply increases when the size of the labor force increases or when the job-finding rate rises. Labor demand increases when the vacancy-filling rate rises, wages fall relative to prices, or productivity rises.

Intuitively, a productivity shock affects labor demand by directly altering a sector's productive capability. A productivity shock also impacts prices and output, and therefore indirectly changes labor usage in other sectors through production linkages. Algebraically, the difference between the occupation-share adjusted Leontief inverse $\mathcal{L}\boldsymbol{\Psi}$ and the relative wage coefficients $\boldsymbol{\Lambda}$ captures the positive net effect an exogenous shock has on labor demand. The multiplicative constant $\left[\boldsymbol{I} - \mathcal{M} - \Xi_\theta\right]^{-1}$ captures how the changes in labor demand cascade through the network, accounting for the effects of labor market frictions.

An increase in labor demand results, all else equal, in an increase in labor market tightness. This increase in labor market tightess raises the labor supply, bringing the labor market into equilibrium.

Second, we analyze the response in sectoral output. The first-order impact of a shock on sectoral output can be split into two terms: a term that captures what the response would be in a frictionless economy, and a term that captures the effects of labor market frictions through adjustments in tightness. How much output changes when tightness changes depends on the search costs, $\mathcal{M}\left(\boldsymbol{I} + \mathcal{T}\right)$. Search costs determine how much more of the workforce needs to be allocated to recruiting when tightness rises. In other words, $\boldsymbol{I} - \mathcal{M}\left(\boldsymbol{I} + \mathcal{T}\right)$ captures how much of an increase in labor demand results in an increase in productive workers, rather than an increase in recruiters.

Last, we examine the response in equilibrium occupational unemployment levels: the number workers who search for jobs and cannot find one. Algebraically, the fraction of workers who do not find a job by the end of the period is equal to $U_o = (1 - f_o(\theta_o))H_o$. Therefore, a change in exogenous variables impacts the unemployment level through changes in tightness. If a labor market becomes tighter, then the unemployment level goes down. In addition, a change in labor force participation for an occupation impacts its unemployment level directly, as more workers look for jobs in that labor market.

Proposition 2 describes how relative prices respond to shocks the productivity, the size of the labor force, or factor supplies. While the relative price changes are not necessary for determining how real output or employment respond to shocks, they are interesting in their own right. They provide an additional set of restrictions to test the realism of our model and as we discuss briefly in section 5.

PROPOSITION 2. *The first-order responses of relative sectoral and factor prices are pinned down by labor supply, technology, and factor supply shocks up to a numeraire. The first-order responses in sectoral prices satisfy*

$$\left(\boldsymbol{I}-\Psi\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\mathcal{L}\right)d\log\boldsymbol{p}=\Pi_{p,A}d\log\boldsymbol{A}+\Pi_{p,H}d\log\boldsymbol{H}+\Pi_{p,K}d\log\boldsymbol{K}^{s}+\Psi\varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}}d\log\boldsymbol{r},$$

and

$$\Pi_{p,A}=\Psi\left[\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left(\Lambda_{A}+\mathcal{M}\mathcal{T}\underbrace{\left[\boldsymbol{I}-\mathcal{M}-\Xi_{\theta}\right]^{-1}\left[\mathcal{L}\Psi-\Lambda_{A}\right]}_{\Pi_{\theta,\boldsymbol{A}}}\right)-\boldsymbol{I}\right],$$

$$\Pi_{p,H}=\Psi\left[\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left(\Lambda_{H}+\mathcal{M}\mathcal{T}\underbrace{\left[\boldsymbol{I}-\mathcal{M}-\Xi_{\theta}\right]^{-1}\left(\mathcal{L}\Psi\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}-\boldsymbol{I}-\Lambda_{\boldsymbol{H}}\right)}_{\Pi_{\theta,\boldsymbol{H}}}\right)\right],$$

$$\Pi_{p,K}=\Psi\left[\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\left(\Lambda_{K}+\mathcal{M}\mathcal{T}\underbrace{\left[\boldsymbol{I}-\mathcal{M}-\Xi_{\theta}\right]^{-1}\left(\mathcal{L}\Psi\varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}}-\Lambda_{K}\right)}_{\Pi_{\theta,\boldsymbol{K}}}\right)\right].$$

The first-order responses in fixed factor prices satisfy

$$d\log\boldsymbol{r}=\mathbf{1}d\log y_{num}-d\log\boldsymbol{K}^{\boldsymbol{s}},$$

where $d\log y_{num}$ denotes the change in output in the numeraire sector, which is determined as per proposition 1.

PROOF. See appendix A.1. □

In our framework, relative prices are determined by market clearing in a perfectly competitive market. The price of a good produced by a particular sector responds to changes in prices in all other sectors, and Ψ captures the co-movement and interaction of prices throughout the production network. Prices also depend on the effective cost of employing workers. In models without search frictions, prices respond directly to change in wages, which corresponds to the Λ terms in the expressions above.

In our model, wage adjustments impact prices through an additional tightness channel, which corresponds to the product of the matching elasticity matrix, the recruiter-producer matrix, and the Π_θ matrices. This is because wage adjustments impact labor market tightness, and changes in labor market tightness changes the number of recruiters needed to hire a certain number of workers. In addition, the price is also directly linked to the level of productivity in that sector. Thus, a productivity shock impacts the system of prices directly through production and indirectly through adjustments in tightness, and a labor supply shock impacts prices solely through adjustments in labor market tightness. Additionally, the first-order responses in factor prices follow from the numeraire sector's optimal factor usage (Equation 5).

At the dis-aggregate level, labor market structure, frictions, and production linkages interact to contribute meaningfully to how prices and allocations respond. In particular, changes in labor demand depend on how sectors connect to each other through the use of intermediate goods, and what common occupations they hire from. The changes in labor demand lead to changes in labor market tightness, which acts as an endogenous wedge that impacts output and price responses.

3.2. The Aggregate Impact of Shocks

We are interested in how the impact of idiosyncratic shocks aggregate to impact output. With a Cobb-Douglas preference, the first-order response of aggregate output is given by $d\log Y = \boldsymbol{\sigma}' d\log \boldsymbol{y}$. Using Proposition 1, we arrive at the following result for first-order changes in aggregate output.

THEOREM 1. *Given idiosyncratic labor supply shocks $d\log \boldsymbol{H}$ and productivity shocks $d\log \boldsymbol{A}$, the log change in real GDP is:*

$$d\log Y = \Pi_{\boldsymbol{A}} d\log \boldsymbol{A} + \Pi_{\boldsymbol{H}} d\log \boldsymbol{H} + \Pi_{\boldsymbol{K}} d\log \boldsymbol{K}^s,$$

where

$$\begin{aligned}
\Pi_{\boldsymbol{A}} &= \lambda' \left(\boldsymbol{I} + \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} \left(\boldsymbol{I} - \mathcal{M}\left(\boldsymbol{I} + \mathcal{T}\right)\right) \left[\boldsymbol{I} - \mathcal{M} - \Xi_\theta\right]^{-1} \left(\mathcal{L}\Psi - \Lambda_A\right) \right), \\
\Pi_{\boldsymbol{H}} &= \lambda' \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} \left(\boldsymbol{I} + \left(\boldsymbol{I} - \mathcal{M}\left(\boldsymbol{I} + \mathcal{T}\right)\right) \left[\boldsymbol{I} - \mathcal{M} - \Xi_\theta\right]^{-1} \left(\left[\mathcal{L}\Psi \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} - \boldsymbol{I}\right] - \Lambda_H \right) \right), \\
\Pi_{\boldsymbol{K}} &= \lambda' \left(\varepsilon^{\boldsymbol{f}}_{\boldsymbol{K}} + \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} \left(\boldsymbol{I} - \mathcal{M}\left(\boldsymbol{I} + \mathcal{T}\right)\right) \left[\boldsymbol{I} - \mathcal{M} - \Xi_\theta\right]^{-1} \left(\mathcal{L}\Psi \varepsilon^{\boldsymbol{f}}_{\boldsymbol{K}} - \Lambda_K \right) \right)
\end{aligned}$$

and $\boldsymbol{\lambda} = \boldsymbol{\Psi}'\boldsymbol{\sigma}$ denotes the sectors' sales shares.

PROOF. The result follows from $d\log Y = \sigma' d\log \boldsymbol{y}$ and proposition 1. □

Theorem 1 follows from Proposition 1, as we can compute the change in real GDP by weighing the change in real output for all sectors by the household's demand elasticities. Note that the matrix product between the Leontief inverse and the demand elasticities is equal to the sales shares. This property results from household's maximization problem, the firms' profit maximization decision, goods market clearing, and Cobb-Douglas production functions (see appendix A.2). Therefore, the aggregate impact of productivity and labor supply shocks can be summarized as the sales-share-weighted impact of these shocks on sectoral output directly through production and indirectly through labor markets.

Finally, we are interested in the response of aggregate unemployment to sector specific shocks. Using the results in Theorem 1, we can derive changes in aggregate unemployment and relative prices.

COROLLARY 1. *Given idiosyncratic labor supply shocks $d\log \boldsymbol{H}$ and productivity shocks $d\log \boldsymbol{A}$, the first-order response in aggregate unemployment is:*

$$d\log U^{agg} = \Pi_{U^{agg},A} d\log \boldsymbol{A} + \Pi_{U^{agg},H} d\log \boldsymbol{H},$$

where

$$\Pi_{U^{agg},A} = \frac{1}{U^{agg}} \boldsymbol{U}' \left[\Lambda_A - \mathcal{L}\Pi_{y,A}\right],$$
$$\Pi_{U^{agg},H} = \frac{1}{U^{agg}} \boldsymbol{U}' \left[\boldsymbol{I} + \Lambda_H - \mathcal{L}\Pi_{y,H}\right].$$

The first-order change in the relative price of the aggregate consumption good is:

$$d\log P = \sigma' d\log \boldsymbol{p},$$

where $d\log \boldsymbol{p}$ follows proposition 2.

PROOF. See appendix A.2 □

The change in unemployment is a weighted average of the first-order changes in the size of the labor force and employment. The employment level change for each labor market in equilibrium equates the change in labor demand in that labor market to labor supply. The change in labor demand in that labor market depends on how relative wages respond to shocks, as well as how sectoral output changes for sectors that use that particular type of labor.

The first-order response in aggregate price level is the changes in sectoral prices weighted by their consumption share, which is equivalent to the demand elasticities of consumption when preferences are Cobb-Douglas. Note that one can solve out the price changes explicitly by picking a numeraire, for example, by assuming $d\log p_1 = 0$. However, we don't explicitly do it here in order to maintain a simpler expression for the relationship between the price system and productivity, fixed factor, and labor force shocks.

3.3. Labor Market Frictions and Amplification

Hulten (1978)'s theorem states that in an efficient economy, the first-order effect of an idiosyncratic productivity shock to an industry on aggregate output is equal to that industry's sales share. We compare our results to Hulten's foundational theorem and examine how the interaction between production linkages and labor market frictions impacts aggregate output. In particular, we analyze the conditions under which Hulten's theorem holds, as well as when labor market frictions interact with production linkages to amplify the aggregation of shocks.

In the discussion following Proposition 1, labor market inefficiencies impact output propagation through adjustments in tightness. Tightness generates an additional wedge between wages and the marginal product of labor. This additional wedge exists because firms have to dedicate more resources to recruit in tighter labor markets, increasing the marginal cost of labor. When network price adjusted wages change exactly proportionally to the marginal product of labor, tightness remains unchanged, the first-order impact of search costs is eliminated, and Hulten's theorem holds.

COROLLARY 2. *Hulten's theorem holds for technology shocks whenever network price adjusted wages change exactly proportionally to the network adjusted marginal product of labor. That is when*

$$d\log \boldsymbol{w} - \mathcal{L} d\log \boldsymbol{p} = \mathcal{L} d\log MP,$$

where $d\log MP$ is a matrix of changes to the marginal product of each type of labor in each sector. Furthermore, when this condition holds, aggregate changes in response to labor force and factor supply shocks are independent of matching frictions.

PROOF. The derivative of production with respect to labor inputs, along with the firms first order conditions, imply that the network adjusted marginal product of labor satisfies

$$\mathcal{L} d \log \mathrm{MP} = \mathfrak{T}\mathcal{M} d \log \theta + d \log \boldsymbol{w} - \mathcal{L} d \log \boldsymbol{p}.$$

Imposing that network price adjusted wage changes are exactly proportional to changes in the network adjusted marginal product of labor implies

$$\mathfrak{T}\mathcal{M} d \log \theta = 0.$$

$\mathfrak{T}\mathcal{M}$ is a diagonal matrix with non-zero diagonal elements, therefore

$$d \log \theta = 0.$$

Since search-and-matching operates through changes in $d \log \theta$, this implies that search-and-matching has no impact on the propagation of shocks. In particular, in this case

$$d \log Y = \lambda' \left[d \log \boldsymbol{A} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} d \log \boldsymbol{H} + \varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}} d \log \boldsymbol{K}^{s} \right]$$

The aggregate output response to technology shocks is $\lambda' d \log \boldsymbol{A}$, this is exactly Hulten's theorem. The aggregate output response to the other shocks depends only on production parameters, matching frictions play no role. □

When wages do not respond to shut off changes in tightness, labor market frictions impact the aggregation of idiosyncratic shocks. We can formally characterize the search channel of idiosyncratic shocks as the following:

COROLLARY 3. *When wages do not respond exactly proportionally to the network adjusted marginal product of labor, matching frictions generate deviations from Hulten's theorem captured by*

$$\begin{aligned}
\Pi_{search,A} &= \lambda' \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left(\boldsymbol{I} - \mathcal{M}\left(\boldsymbol{I} + \mathfrak{T}\right)\right) \left[\boldsymbol{I} - \mathcal{M} - \Xi_{\theta}\right]^{-1} \left(\mathcal{L}\Psi - \Lambda_{A}\right), \\
\Pi_{search,H} &= \lambda' \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left(\boldsymbol{I} - \mathcal{M}\left(\boldsymbol{I} + \mathfrak{T}\right)\right) \left[\boldsymbol{I} - \mathcal{M} - \Xi_{\theta}\right]^{-1} \left(\left[\mathcal{L}\Psi \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I}\right] - \Lambda_{H}\right), \\
\Pi_{search,K} &= \lambda' \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left(\boldsymbol{I} - \mathcal{M}\left(\boldsymbol{I} + \mathfrak{T}\right)\right) \left[\boldsymbol{I} - \mathcal{M} - \Xi_{\theta}\right]^{-1} \left(\mathcal{L}\Psi \varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}} - \Lambda_{K}\right).
\end{aligned}$$

PROOF. Follows from Theorem 1 and 2. □

The search channel is a product of sales shares $\boldsymbol{\lambda}$, the labor elasticity matrix $\boldsymbol{\varepsilon}_{\boldsymbol{N}}^{\boldsymbol{f}}$, the search cost term $(\boldsymbol{I} - \mathcal{M}(\boldsymbol{I} + \mathcal{T}))$, and the first-order response in labor market tightness. We want to examine whether the search channel amplifies the aggregate impact of shocks. Before we proceed, we first define amplification:

DEFINITION 1. *For a shock $x \in \{A, H, K\}$, the search channel* ***amplifies*** *the impact of the shock if $\boldsymbol{\Pi}_{search,x} > 0$ element-wise.*

The amplification definition we have here is strict. When this definition holds, a shock to any sector, occupation, or factor has an additional impact on output through the search channel. This impact, as the name amplification suggests, has the same sign as the shock.

In theory, whether amplification occurs depends on the magnitudes of the matching elasticities matrix $\mathcal{M}$, the recruiter-producer ratio matrix $\mathcal{T}$, production structures, occupational structure, as well as the wage schedules. We show that under common and nonrestrictive parametrization, network-adjusted rigid wages are enough to guarantee amplification of shocks through the search channel.

ASSUMPTION 1. *For occupation $o \in \{1, 2, \ldots, \mathcal{O}\}$, the matching elasticity η_o and recruiter-producer ratio satisfy $\frac{1}{(1+\tau_o)} > \eta_o$.*

This assumption states that ratio between productive workers and total workers is greater than the matching elasticity. Petrongolo and Pissarides (2001) claims that a matching elasticity between 0.5 and 0.7 is plausible and Landais et al. (2018) finds that the share of recruiters in the workforce is around 2.3%. For this inequality to break, the share of recruiters would have to be at least 30%, which is unrealistic.

Before we dive into the amplification result, we define what it means for wages to be rigid:

DEFINITION 2. *Wages are* ***rigid*** *in response to:*

- *technology shocks if $\mathcal{L}\boldsymbol{\Psi} - \boldsymbol{\Gamma}_{\boldsymbol{A}}$ is non-negative, with one strictly positive element in each column*
- *labor force shocks if $\mathcal{L}\boldsymbol{\Psi}\boldsymbol{\varepsilon}_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I} - \boldsymbol{\Gamma}_{\boldsymbol{H}}$ is non-negative, with one strictly positive element in each column*
- *factor supply shocks if $\mathcal{L}\boldsymbol{\Psi}\boldsymbol{\varepsilon}_{\boldsymbol{K}}^{\boldsymbol{f}} - \boldsymbol{\Gamma}_{\boldsymbol{K}}$ is non-negative, with one strictly positive element in each column*

Essentially, we define wages to be rigid when they adjust less than the occupation and network adjusted total/labor/factor productivity. Such definition of rigidity is much less restrictive than wage rigidity without production networks. Take the wage response to total factor productivity as an example, Hall (2005), Blanchard and Galí (2010), and Michaillat (2012) define rigid wages as adjusting less than proportionally to changes in productivity, and Haefke et al. (2013) estimates wage elasticity to be around 0.8 for new hires and 0.24 for all workers. In a production network, however, take $\mathcal{L} = \boldsymbol{I}$ for example, the diagonal elements of the Leontief inverse $\boldsymbol{\Psi}$ can often be greater than 1. Therefore, a wage schedule that is viewed as "flexible" in the conventional sense can be seemed as rigid here.

Now, we characterize when the search channel amplifies different types of shocks:

PROPOSITION 3. *Under Assumption 1, for a shock $x \in \{A, H, K\}$, the search channel amplifies the shock when wages are rigid in response to x*

PROOF. When $\frac{1}{(1+\tau_o)} > \eta_o$, $(\boldsymbol{I} - \mathcal{M}(\boldsymbol{I} + \mathfrak{T}))$ is greater than 0 on the diagonals, and the equilibrium adjustment coefficients $\left[\boldsymbol{I} - \mathcal{M} - \Xi_\theta\right]^{-1}$ is non-negative element-wise and positive on the diagonals(shown in Appendix C). □

In other words, proposition 3 shows that, when wages adjust less than the occupation and network-adjusted total/labor/factor productivity, the search channel amplifies the aggregate impact of idiosyncratic shocks on output. In other words, network wage rigidity is sufficient to guarantee amplification of shocks through the search and matching channel. Here, we do not argue the wage rigidity assumptions are necessary for search channel amplification. For example, wages can be more flexible for sectors that are not as important, and the aggregate impact of these flexible adjustments can be out-weighted by rigid adjustments in more important sectors.

4. Calibration

So far, we have established qualitatively that incorporating search frictions in a production network economy amplifies the dis-aggregate and aggregate economic impact of microeconomic shocks. In this section, we test the quantitative importance of the aforementioned amplification mechanism by calibrating our model to the U.S. economy. Subsection 4.1 documents the data sources we use. Subsection 4.2 discusses our assumptions about labor market structure and wage schedules. Subsection 4.4 reports our calibrated parameters. Subsection 4.5 presents the calibrated model's quantitative response to a 1% shock in productivity to the durable manufacturing sector.

4.1. Data Sources

Vacancies and Hires. We use vacancy and hiring data from The Job Openings and Labor Turnover Survey (JOLTs), which provides survey-based measures of job openings and hires at a monthly frequency. This survey data is available for 13 industries that roughly correspond to the two-digit NAICS classification. The data is available from Dec 2000 to Feb 2023 at a monthly frequency and from 2001 to 2022 at a yearly frequency.

Unemployment. We use sector level unemployment from the Current Population Survey (CPS). The data cover 13 sectors at a monthly frequency for the same range as the JOLTs data.

Major Occupations. We use occupation level data from the BLS's 2021 Occupational Employment and Wage Statistics(OEWS) for sectors at the two and three-digit NAICS classification level to construct sector-by-sector major occupational employment and wages. We define a major occupation to be an occupation at the 2-digit Standard Occupational Classification (SOC) level.

Input-output Linkages. The BEA Make and Use tables at the three-digit NAICS level allow us to calculate the intermediate input intensity of each sector, the labor intensity of each sector, and the elasticity of final consumption demand to each sectors output. We use employee compensation, recorded in the Use table to calculate labor elasticities.

Factor Shares. In our calibration, we consider two factors: capital and energy. Capital and energy shares for the two and three-digit NAICS classification are available in the BEA-BLS Integrated Industry-level Production Accounts (KLEMS).

Although JOLTs and the CPS include labor market data at a level that corresponds roughly to the two-digit NAICS classification, this correspondence is not exact. Whenever this correspondence is not exact, we use data at the two or three-digit NAICS classification level and aggregate back up to match the 13 CPS industries (Horowitz and Planting 2009).

4.2. Labor Market Assumptions

4.2.1. Labor Market Structure

In this calibration exercise, we split the labor force into major occupation categories and allow firms in each sector to use a mix of the different major occupations in production. Workers are constrained to remain in one occupation, but because firms in different sectors use multiple occupations as an input, workers are no longer constrained to remain in just one sector. This is a realistic feature of the labor market, as switching into an industry where the same occupational skills are demanded is much easier than acquiring a new set of skills to switch into a new occupation. This feature allows for sector-to-sector labor transitions, a potentially important feature of the labor market in many countries. For instance, in the U.S. about 12% to 20% of jobs switchers also change industries at the 1-digit level (Kambourov and Manovskii 2008; Parrado et al. 2007). Neffke et al. (2017) find that nearly 59% of German job switchers change industry at the most aggregate German industry grouping.

Our major occupations calibration allows for realistic industry-to-industry flows of workers, but at the cost of higher data requirements at the major occupational level. In this calibration, we try our best to address the high data requirements by imputing certain occupational parameters from sector-level data. In general, our model is flexible and one can modify the labor market specification and readily apply our model to other scenarios. For example, we can assume labor markets to be fully rigid geographically to study the localized impact of industry shocks. Also, the quantitative importance of our search channel is robust across labor market specification. We include results for one unique occupation per production sector in the appendix.

4.3. Wage Schedule

Wages play an important role in how shocks propagate in our model economy. How much wage adjust determines the response in tightness after an economic shock. In fact, as Theorem 2 shows, for the right assumption about wages, search frictions can have no effect whatsoever on shock propagation. For other generally rigid wage schedules, we also want to quantify the size of amplification. To avoid taking a strong stance on exactly how wages change, we test the quantitative consequences of shocks in our model under a set of several different wage assumptions commonly used in the literature.

Throughout this section, we report results for the following wage assumptions

(i) The network price adjusted wage changes half as much as the network adjusted marginal product of labor[4]:

(0.5MP) $$d \log \boldsymbol{w} - \mathcal{L} d \log \boldsymbol{p} = 0.5 \mathcal{L} d \log \mathrm{MP}.$$

(ii) The network price adjusted wage does not change in response to shocks:

(Rigid Real) $$d \log \boldsymbol{w} - \mathcal{L} \delta \log \boldsymbol{p} = 0.$$

(iii) The nominal wage does not change in response to shocks:

(Rigid Nominal) $$d \log \boldsymbol{w} = 0.$$

4.4. Calibration Procedure and Parameter Values

We provide an overview of our calibration procedure and some key parameter values we acquire. Due to large number of parameters resulting from the interaction between the production network and labor markets, we leave the bulk of our calibration procedures and the exact calibrated parameter values to Appendix D.

Figure 3 reports the calibrated input-output matrix $\boldsymbol{\Omega}$. It shows that a sector's production usually relies on intermediate goods produced by other sectors. The parameters are calibrated by computing each sector's expenditure share on commodities produced by other sectors. The detailed calibration process can be found in subsection D.1.

[4] In our setup, this is similar to assuming real wage Nash bargaining with equal bargaining weights for firms and workers.

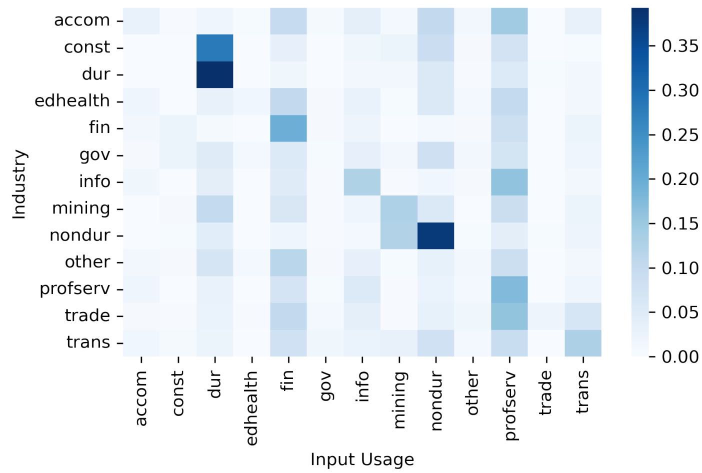
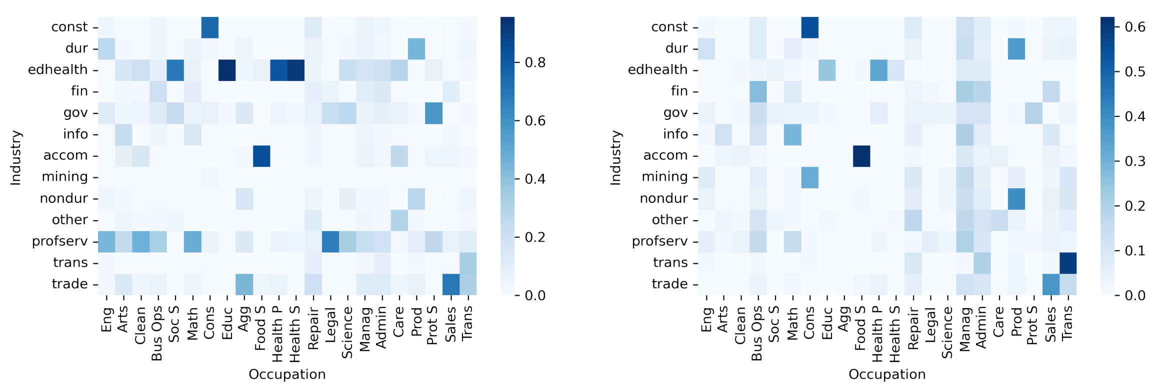


FIGURE 3. Input-output table for major sectors, roughly corresponding to the NAICS two-digit sectors. The (i, j)-th element on the heat map denotes the share of sector i's revenue spent on intermediate goods produced by sector j.

Figure 4A and Figure 4B report the calibrated values for the occupation share matrix $\mathcal{L}$ and occupation elasticity matrix $\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}$. These two figures show that many of the major occupation, such as administrative workers, are important in production across many sectors. $\mathcal{L}$ is directly computed from the employment data in OEWS, and $\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}$ is computed using occupational wage and employment data for each sector. The detailed calibration process for the labor elasticities can be found in subsection D.3.



A. Occupation share

B. Occupation elasticity of production

FIGURE 4. *Left:* the share of given occupation used by different industries. The (i, j)-th element denotes the share of occupation j that is employed by sector i. Each column sums to 1. *Right:* the occupation elasticity of production. The (i, j)-th element denotes the share of labor expenditure sector i spends on workers in occupation j. Each row sums to 1.

We currently don't have access to occupational labor market characteristics, so we impute unemployment, vacancy, the number of recruiters, and tightness for each occupation from sector level data in JOLTS and CPS. We assume the total number of unemployment, vacancy, and recruiters for an occupation is the sum of unemployment, vacancy, and recruiters across sectors, weighted by the sectors' labor expenditure shares of that particular occupation. We proxy recruiting efforts in each sector by the number of human resources workers they employ. This likely underestimates the amount of recruiting efforts, since non-HR workers may also partake in recruiting new workers. The details of the calibration process can be found in subsection D.4.

We then estimate occupation-specific matching efficiency and elasticity from imputed occupation hires, vacancies, and unemployment at the monthly frequency. In particular, we estimate the following equation:

$$\log H_{o,t} = \log \phi_o + \eta_o \log U_{o,t} + (1 - \eta_o) \log V_{o,t} + \epsilon_{o,t},$$

where ϕ_o is the matching efficiency and η_o the matching elasticity with respect to unemployment in occupation o. We report the estimated parameters in subsection D.4.

4.5. Effects of a 1% shock to productivity in the durable manufacturing sector

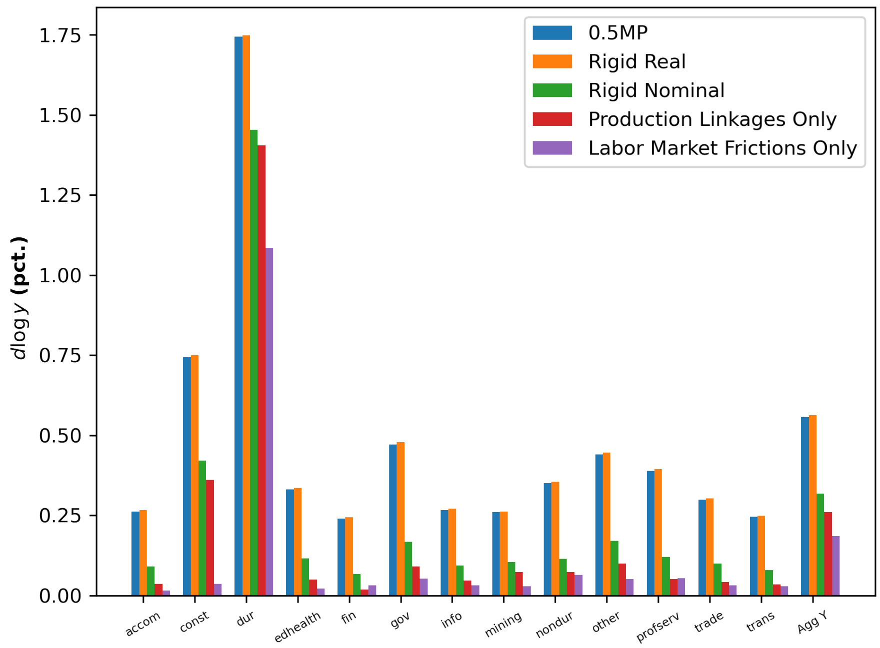
Now, we quantify the amplification effects of labor market frictions in the calibrated production network using results from Proposition 1. Specifically, we compare our model with two benchmark cases. First, we consider the case where labor markets are friction-less (i.e. $\mathcal{M} = \boldsymbol{I}$). Such comparison has been examined theoretically in subsection 3.3. Additionally, we consider the case with no production linkages and only labor market frictions (i.e. $\boldsymbol{\Psi} = \boldsymbol{I}$). We assume relative wages in this benchmark case to be rigid.

Figures 5A shows the response of output to a 1% shock to productivity in the durable manufacturing sector for our model economy and the two benchmarks. The red bars represent the benchmark with only production linkages, and the purple bars represent the benchmark with only labor market frictions. In the network model without labor market frictions, the 1% shock to productivity in durable manufacturing boosts output in the durable manufacturing sector by 1.40% and boosts aggregate output by 0.26%. In the labor frictions only benchmark in which wages follow "Rigid Real", the shock results in a 1.08% boost in durable manufacturing output, and a 0.18% aggregate output boost. Thus, production linkages alone are quantitatively more important than labor market frictions for the amplification of productivity shocks to the durable manufacturing sector.

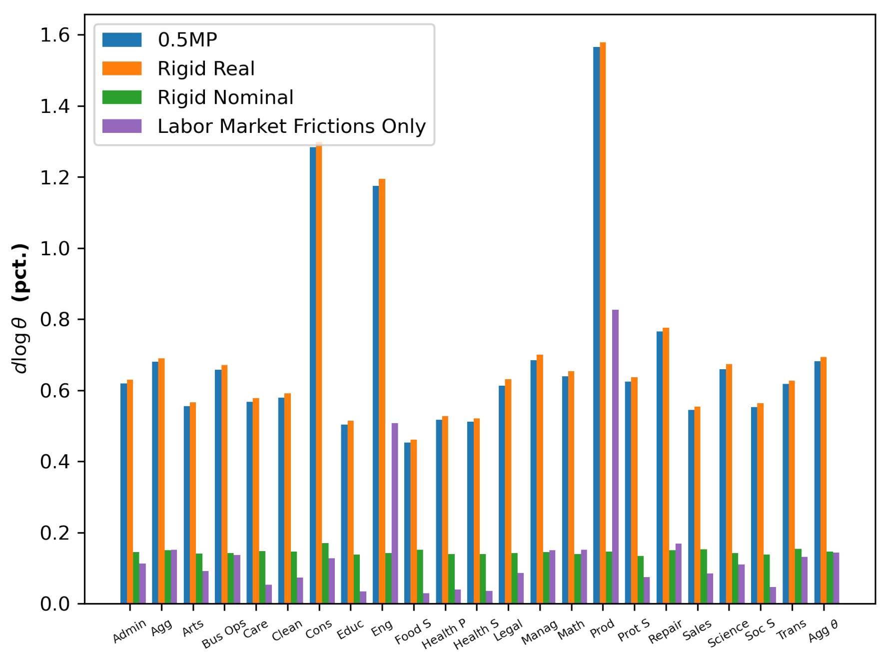
Nonetheless, the interaction between these two can significantly boost the effects of the productivity shock on output. For instance, under the same wage specification as in the labor frictions only benchmark (“Rigid Real"), output in the durable manufacturing sector increases by 1.75%. Aggregate output rises by 0.56%, which is more than the combined aggregate output response for the two benchmark models. This illustrates the quantitative importance to model production linkages and search frictions in conjunction. As theory predicts, the amplification patterns are general across wage specifications.

Figures 5B shows the response of tightness to a 1% shock to productivity in the durable manufacturing sector for our model economy and the two benchmarks. Without labor market frictions, productivity shocks have no effect on tightness or unemployment, so we omit the production linkages only benchmark in this graph. In the labor market frictions benchmark, the productivity shock results in a 0.14% increase in aggregate tightness, as well as sizable increases in engineers and production workers, the two labor markets durable manufacturing hires heavily from. In our model with rigid relative wage, the same shock results in a 0.69% increase in aggregate tightness, as well as tightness increases across most occupations, since durable manufacturing is connected to many sectors through production linkages, and these sectors hire from many labor markets. Therefore, combining production linkages with labor market frictions create significant amplification and comovement in the tightness response of occupational and aggregate labor markets.

This example demonstrates the production implication for incorporating labor market frictions into a production network, as well as the labor market implication for incorporating production linkages into a model with labor market frictions. It is therefore key to model these two economic mechanisms in tandem, as modeling them separately will miss important quantitative amplification.



A. Sectoral and aggregate output response



B. Occupational tightness response

FIGURE 5. Response of output, tightness, and unemployment to a 1% shock to technology in the durable manufacturing sector.

5. Application: Adverse Energy and Labor Supply Shocks to the U.S Economy

Our model features two types of non-intermediate production inputs: those that are subject to search and matching, such as labor, and those that are not, such as energy. We apply our model to evaluate the effect of these two types of factor shocks. Specifically, we analyze the aggregate and granular impact of energy supply and labor supply shocks, which the US experienced in recent years due to the Russian-Ukraine war and the decrease in labor force participation during COVID. We provide a qualitative illustration of how these adverse factor supply shocks would impact sectoral production and disaggregated labor market tightness. While our model focuses on real variables, we demonstrate that relative prices move in a realistic way.

5.1. Effects of an adverse energy shock

On March 8, 2022, the White House issued a ban on imports of Russian oil, liquefied natural gas, and coal. Such ban constitutes a large adverse energy shock, as the U.S imported nearly 700, 000 barrels of crude oil and refined petroleum products from Russia per day in 2021. In this section, we examine the impact of an adverse energy shock, using the major occupation setup. Figure 6A shows the response of output to the adverse energy shock. The rigid wage specifications predicts an aggregate output decline of around 1% in response to an adverse 10% energy shock, twice as large as when search and matching frictions are absent. The transportation sector is most negatively impacted, followed by the accommodation sector, the construction sector, and the trade sector, as they have high energy elasticities. Search and matching frictions amplify the impact of energy shocks significantly through the search and matching channel. These amplifications create large responses in other downstream sectors, such as government and professional services, despite their low energy elasticities.

figures/energy_shock/energy_KshockY.png

A. Sectoral and aggregate output response to an adverse energy shock

figures/energy_shock/energy_KshockT.png

B. Occupational tightness response to an adverse energy shock

figures/energy_shock/energy_KshockTsec.png

C. Sectoral tightness response to an adverse energy shock

figures/energy_shock/energy_KshockP.png

D. Relative price response to an adverse energy shock

FIGURE 6. First-order responses to an adverse energy shock under sector-specific occupations specification under major occupations specification.

Figure 6B plots the response in occupational labor market tightness to the adverse energy shock. The labor market tightness for management and transportation service workers decreases the most, at around 2%. Similarly, the construction occupation also experience a large decline in tightness. The decline in labor tightness for transportation service workers and construction workers is straightforward, as the construction and transportation sectors are energy-intensive. Most other occupations experience a similar and sizable decline in labor market tightness. This is because sectors, who are affected directly through having higher energy elasticities, or indirectly through their production linkages with energy-intensive sectors, lower their labor demand for the occupations they employ, thus changing labor market tightness across occupations.

Figure 6C plots the response in sectoral labor market tightness reconstructed from occupational tightness movements. Note that this reconstruction is only valid under our calibration assumptions, and should provide a rough estimate of how sectoral tightness changes. We see the transportation sector experiences the largest decrease in tightness of around 2%. The decline in tightness for other sectors appears to be similar.

Figure 6D documents the first-order price changes, using education and health as the numeraire sector. We choose the education and health as the numeraire for one simple reason, which is that inflation for medical care service was 0.4% from 2022 to 2023, much more stable than most other goods and services. We find that, first-order price responses differ across sectors and across wage specifications. We observe the largest price increases in sectors that suffer the largest output declines. In particular, we see a large increase in transportation prices. This is qualitatively consistent with large observed increases in transportation prices between 2022 and 2023[5]. We also see price increases in the accommodation sector and the construction sector.

5.2. Effects of adverse labor supply shocks

Another prominent feature of the U.S labor market after the COVID-19 pandemic is the occurrence of the “Great Reshuffle" and the “Great Resignation." These labor market phenomena signify structural labor market changes, such as more people quitting their jobs, more people retiring early, and people switching jobs for better pay and work-life balance. We focus on two aspects of this phenomenon. First, we assume that there is a reduction in labor supply for the service occupations, as suggested by the high quit rates in the accommodation sector. According to the Bureau of Labor Statistics(BLS), in September 2021, the quit rate for the accommodation sector was 6.6%, whereas the quit rate for private industries overall was 3.4%. Second, we assume there is a reduction in the overall labor participation rate, as evidenced by Hobijn and Şahin (2022). In both cases we assume the magnitude of the shock for each impacted occupation is 2%. This is consistent with Hobijn and Şahin (2022)’s finding that the labor force participation rate declined by roughly 1.2 percentage points. We examine the macroeconomic impact of these two specifications in the context of our model separately.

To start off, we examine the impact of adverse labor supply shocks to the service occupations. Figures 7A,7C, 7E and7G show the output, tightness, and relative price responses to a 2% adverse labor supply shock to the following ONET major occupation classification: building and grounds cleaning and maintenance occupations, food preparation and serving related occupations, personal care and service occupations,

[5]According to the Bureau of Transportation Statistics, transportation prices rose 15.5% overall between 2022 and 2023

and transportation and material moving occupations. Overall, while the efficient benchmark exhibits large movements in output and prices, our specifications show very small changes. However, when it comes to labor market tightness, we can see clear responses in the occupations we shock. From Figure 7E can also see that sectoral tightness increases for all sectors, but increases the most for sectors such as accommodation, transportation, retail, and other services.

figures/covid_labor_shocks/energy_HservshockY.png

A. Output response to service occupation shocks

figures/covid_labor_shocks/energy_HallshockY.png

B. Output response to labor force participation shocks

figures/covid_labor_shocks/energy_HservshockT.png

C. Occupational tightness response to service occupation shocks

figures/covid_labor_shocks/energy_HallshockT.png

D. Occupational tightness response to labor force participation shocks

figures/covid_labor_shocks/energy_HservshockTsec.png

E. Sectoral tightness response to service occupation shocks

figures/covid_labor_shocks/energy_HallshockTsec.p

F. Sectoral tightness response to labor force participation shocks

figures/covid_labor_shocks/energy_HservshockP.png

G. Relative price response to an adverse labor supply shock in service occupations

figures/covid_labor_shocks/energy_HallshockP.png

H. Relative price response to an adverse labor supply shock in all occupations

FIGURE 7. Left: First-order responses to an adverse labor supply shock in service occupations. Right: First-order responses to an adverse labor supply shock in all occupations.

Why is the output response to labor supply shock so muted? The effect of output depends on how much the cost of productive workers changes. Since the recruiter producer ratio is the US economy tends to be fairly small, when wages do not change directly an adverse labor supply shock to service occupations does not alter the cost of productive workers substantially. Therefore, the shock has negligible effects on output. The small effects of occupational labor supply shocks on output and prices means that changes in labor demand for other types of occupation are small as well, which is why we see very little movement in tightness for other occupations.

Next, we analyze the impact of adverse labor supply shocks to all occupations. Figures 7B,7D, 7F, and 7H document the output, tightness, and relative price responses to an adverse shock to all occupations. Overall, the patterns of responses are similar to what we observe for service occupation shocks. The only difference is that, the output responses, while still small compared to the efficient benchmark, are visible on the graphs. This suggests that the change in labor demand induced by negative output responses can dampen the increase in tightness we observe, although the effect are quantitatively negligible.

5.3. Combination Shocks

Now, we combine the energy shock and the labor participation shock. Although the first-order impact of these shocks are linearly additive, it is helpful to see how they affect the economy together. Figures 8A, 8B, 8C, and 8D show the combined output, tightness and relative price responses to the shocks that we believe are relevant for the U.S economy in the past few years. Overall, we see a moderate drop in real output, an increase in tightness across all occupations and sectors, and increases in relative prices for sectors such as transportation, accommodation, and construction.

figures/combo_shocks/energy_HKshockY.png

A. Output response to energy and labor force participation shocks

figures/combo_shocks/energy_HKshockT.png

B. Occupational tightness response to energy and labor force participation shocks

figures/combo_shocks/energy_HKshockTsec.png

C. Sectoral tightness response to energy and labor force participation shocks

figures/combo_shocks/energy_HKshockP.png

D. Relative price response to energy and labor force participation shocks

FIGURE 8. First-order responses to a shock in labor force participation rate combined with an adverse energy shock.

Compared to the efficient production networks model, does our model do a better job at capturing reality? We find that combined energy and labor supply shocks have an impact on aggregate output that is half of what the conventional model would have predicted. This is mainly due to the smaller effect of labor supply shocks on output in our model. While conventional production network models don't feature unemployment, we find that our model predicts an increase in labor market tightness across sectors. In addition, our model predicts a larger increase in relative prices for sectors that have large energy elasticities, as well as sectors that are linked to them through production and occupations. While our results should only be viewed as qualitative as we have only chosen approximate shock values,our models predictions of a small output contraction and tighter labor markets seem consistent with the U.S. data. U.S. real GDP did decline from Q4 2021 to Q2 2022, but the decline was muted, while labor markets have been exceptionally tight. The standard production networks model would instead predict a much larger decline in output and no movement in tightness.

6. Conclusion

Modern economies feature production networks and frictional, segmented labor markets. We show that accounting for both can alter our understanding of how shocks propagate in a quantitative and qualitatively meaningful way. The impact of matching frictions on network propagation depends on the assumption about wages. We show that under a wide range of assumptions about wages, labor market frictions amplify shocks to productivity. But it is nevertheless possible for the market frictions to dampen shock propagation. This highlights that determining the empirical response of wages to shocks across the network is an import avenue for future work. In addition, a combination of negative energy supply shocks and negative labor force shocks generates simultaneously tight labor markets and depressed output in our framework. While our current model cannot speak to the effect on the price level, in future work we hope to extend the model to incorporate dynamics and nominal rigidities to help paint a realistic picture of the current inflation episode.

References

Acemoglu, Daron and Pablo D. Azar, "Endogenous Production Networks," *Econometrica*, 2020, *88* (1), 33–82.

___ **, Vasco M. Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi**, "The Network Origins of Aggregate Fluctuations," *Econometrica*, 2012, *80* (5), 1977–2016.

Adão, Rodrigo, Costas Arkolakis, and Federico Esposito, "General Equilibrium Effects in Space: Theory and Measurement," 2019.

Baqaee, David and Elisa Rubbo, "Micro Propagation and Macro Aggregation," 2022.

Baqaee, David Rezza and Emmanuel Farhi, "The Macroeconomic Impact of Microeconomic Shocks: Beyond Hulten's Theorem," *Econometrica*, 2019, *87* (4), 1155–1203.

___ **and** ___ , "Productivity and Misallocation in General Equilibrium," *The Quarterly Journal of Economics*, 2020, *135* (1), 105–163.

Benigno, Pierpaolo and Gauti Eggertsson, "It's Baaack: The Surge in Inflation in the 2020s and the Return of the Non-Linear Phillips Curve," 2023.

Bigio, Saki and Jennifer La'O, "Distortions in Production Networks," *The Quarterly Journal of Economics*, 2020, *135* (4), 2187–2253.

Blanchard, Olivier and Jordi Galí, "Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment," *American Economic Journal: Macroeconomics*, April 2010, *2* (2), 1–30.

Chodorow-Reich, Gabriel and Johannes Wieland, "Secular Labor Reallocation," *Journal of Political Economy*, 2020, *128* (6), 2245–2287.

Fernandez-Villaverde, Jesus, "Solving High-Dimensional Dynamic Programming Problems using Deep Learning," 2020.

Haefke, Christian, Marcus Sonntag, and Thijs van Rens, "Wage rigidity and job creation," *Journal of Monetary Economics*, November 2013, *60* (8), 887–899.

Hall, Robert E., "Employment Fluctuations with Equilibrium Wage Stickiness," *The American Economic Review*, 2005, *95* (1), 50–65.

Hobijn, Bart and Ayşegül Şahin, "Missing Workers and Missing Jobs Since the Pandemic," 2022.

Horowitz, Karen J. and Mark A. Planting, "Concepts and Methods of the U.S. Input-Output Accounts," *Bureau of Economic Analysis*, 2009.

Howard, Jacob, "Labor-Market Frictions and Endogenous Production-Network Formation," November 2019.

Hulten, C. R., "Growth Accounting with Intermediate Inputs," *The Review of Economic*

Studies, 1978, *45* (3), 511–518.

Humlum, Anders, "Robot Adoption and Labor Market Dynamics," 2021, p. 69.

Huneeus, Federico, Kory Kroft, and Kevin Lim, "Earnings Inequality in Production Networks," 2023.

Jones, Charles I., "Misallocation, Economic Growth, and Input-Output Economics," 2011.

Kambourov, Gueorgui and Iourii Manovskii, "Rising Occupational and Industry Mobility in the United States: 1968–97," *International Economic Review*, 2008, *49* (1), 41–79.

Landais, Camille, Pascal Michaillat, and Emmanuel Saez, "A Macroeconomic Approach to Optimal Unemployment Insurance: Theory," *American Economic Journal: Economic Policy*, May 2018, *10* (2), 152–181.

La'O, Jennifer and Alireza Tahbaz-Salehi, "Optimal Monetary Policy in Production Networks," *Econometrica*, 2022, *90* (3), 1295–1336.

Liu, Ernest, "Industrial Policies in Production Networks," *The Quarterly Journal of Economics*, 2019, *134* (4), 1883–1948.

Long, John B. and Charles I. Plosser, "Real Business Cycles," *Journal of Political Economy*, 1983, *91* (1), 39–69.

Michaillat, Pascal, "Do Matching Frictions Explain Unemployment? Not in Bad Times," *The American Economic Review*, June 2012, *102* (4), 1721–1750. Num Pages: 30 Place: Nashville, United States Publisher: American Economic Association Section: Shorter Papers.

Minton, Robert and Brian Wheaton, "Hidden Inflation in Supply Chains: Theory and Evidence," 2022.

Neffke, Frank M.H., Anne Otto, and Antje Weyh, "Inter-industry labor flows," *Journal of Economic Behavior & Organization*, 2017, *142*, 275–292.

Parrado, Eric, Asena Caner, and Edward N. Wolff, "Occupational and industrial mobility in the United States," *Labour Economics*, 2007, *14* (3), 435–455.

Petrongolo, Barbara and Christopher A Pissarides, "Looking into the Black Box: A Survey of the Matching Function," *Journal of Economic Literature*, June 2001, *39* (2), 390–431.

Rubbo, Elisa, "Networks, Phillips Curves, and Monetary Policy," 2020.

Shimer, Robert, "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *The American Economic Review*, 2005, *95* (1), 25–49.

Appendix A. Proofs

A.1. Proof for Propositions 1 and 2

We first show how to prove Proposition 2 by assuming Cobb-Douglas preferences and production.

A.1.1. Price Propagation

Log-linearizing the production function, for each sector i, we have:

$$d\log y_i = \underbrace{\varepsilon^{f_i}_{A_i}}_{=1} d\log A_i + \sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} d\log N_{io} + \sum_{j=1}^{N} \varepsilon^{f_i}_{x_{ij}} d\log x_{ij} + \sum_{k=1}^{\mathcal{K}} \varepsilon^{f_i}_{K_{ik}} d\log K_{ik}$$

Plugging in Equation 4 and Equation 3, the first order conditions for optimal input usage, into the log-linearized production function gives

$$\begin{aligned}
d\log y_i = &\sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} \left[d\log \varepsilon^{f_i}_{N_{io}} + d\log y_i + d\log p_i - d\log w_o - d\log\left(1+\tau_o(\theta_o)\right)\right] \\
&+ \sum_{j=1}^{N} \varepsilon^{f_i}_{x_{ij}} \left[d\log \varepsilon^{f_i}_{x_{ij}} + d\log y_i + d\log p_i - d\log p_j\right] \\
&+ \sum_{k=1}^{\mathcal{K}} \varepsilon^{f_i}_{K_{ik}} \left[d\log \varepsilon^{f_i}_{K_{ik}} + d\log y_i + d\log p_i - d\log r_k\right] + d\log A_i \\
&= \left[d\log y_i + d\log p_i\right] \underbrace{\left[\sum_{k=1}^{\mathcal{K}} \varepsilon^{f_i}_{K_{ik}} + \sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} + \sum_{j=1}^{N} \varepsilon^{f_i}_{x_{ij}}\right]}_{=1 \text{ by crts}} + \underbrace{\left[\sum_{k=1}^{\mathcal{K}} d\varepsilon^{f_i}_{k_{ik}} + \sum_{o=1}^{\mathcal{O}} d\varepsilon^{f_i}_{N_{io}} + \sum_{j=1}^{N} d\varepsilon^{f_i}_{x_{ij}}\right]}_{=0 \text{ by crts}} \\
&- \sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} \left[d\log w_o + d\log(1+\tau_o(\theta_o))\right] - \sum_{j=1}^{N} \varepsilon^{f_i}_{x_{ij}} \left[d\log p_j\right] - \sum_{k=1}^{\mathcal{K}} \varepsilon^{f_i}_{K_{ik}} d\log r_k + d\log A_i,
\end{aligned}$$

where the second equality holds because the sum of elasticities equals one for constant returns to scale technology and $\varepsilon^{f_i}_{x_{ij}} d\log \varepsilon^{f_i}_{x_{ij}} = d\varepsilon^{f_i}_{x_{ij}}$.

Rearranging terms gives

$$
\begin{aligned}
d\log p_i &= \sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} \left[d\log w_o + d\log(1+\tau_o(\theta_o))\right] + \sum_{j=1}^{N} \varepsilon^{f_i}_{x_{ij}} \left[d\log p_j\right] + \sum_{k=1}^{\mathcal{K}} \varepsilon^{f_i}_{K_{ik}} d\log r_k - d\log A_i \\
&= \sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} \left[d\log w_o + \varepsilon^{1+\tau_o}_{\theta_o} d\log \theta_o\right] + \sum_{j=1}^{J} \varepsilon^{f_i}_{x_{ij}} \left[d\log p_j\right] + \sum_{k=1}^{\mathcal{K}} \varepsilon^{f_i}_{K_{ik}} d\log r_k - d\log A_i \\
&= \sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} \left[d\log w_o - \tau_o(\theta_o)\varepsilon^{\Omega_o}_{\theta_o} d\log \theta_o\right] + \sum_{j=1}^{J} \varepsilon^{f_i}_{x_{ij}} \left[d\log p_j\right] + \sum_{k=1}^{\mathcal{K}} \varepsilon^{f_i}_{K_{ik}} d\log r_k - d\log A_i
\end{aligned}
$$

Stacking equations over sectors, we can write

$$
d\log \boldsymbol{p} = \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} \left[d\log \boldsymbol{w} - \boldsymbol{\Omega}\boldsymbol{\mathcal{T}} d\log \theta\right] + \boldsymbol{\Omega} d\log \boldsymbol{p} + \varepsilon^{\boldsymbol{f}}_{\boldsymbol{K}} d\log \boldsymbol{r} - d\log \boldsymbol{A}
$$

Which implies

$$
d\log \boldsymbol{p} = \boldsymbol{\Psi} \left[\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} \left[d\log \boldsymbol{w} - \boldsymbol{\Omega}\boldsymbol{\mathcal{T}} d\log \theta\right] + \varepsilon^{\boldsymbol{f}}_{\boldsymbol{K}} d\log \boldsymbol{r} - d\log \boldsymbol{A}\right]
$$

Or equivalently

$$
\left(\boldsymbol{I} - \boldsymbol{\Psi}\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}}\boldsymbol{\mathcal{L}}\right) d\log \boldsymbol{p} = \boldsymbol{\Psi} \left[\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} \left[d\log \boldsymbol{w} - \boldsymbol{\mathcal{L}} d\log \boldsymbol{p} - \boldsymbol{\Omega}\boldsymbol{\mathcal{T}} d\log \theta\right] + \varepsilon^{\boldsymbol{f}}_{\boldsymbol{K}} d\log \boldsymbol{r} - d\log \boldsymbol{A}\right]
$$

Where $d\log \boldsymbol{r}$ is pinned down by market clearing in the additional factor market for K_k. Assuming Cobb-Douglas production, for any i

$$
d\log r_k = d\log y_i + d\log p_i - d\log K_{ik}
$$

Which means that $d\log k_{ik} = d\log K_{jk}$ for all i, j. Furthermore, this means that $d\log K_{ik} = d\log K^s_k$ for all i. So we can write,

$$
d\log r_k = d\log y_i + d\log p_i - d\log K^s_k
$$

Which pins down $d\log r_k$ given a numeraire sector.

A.1.2. Output Propagation

Since the log-linearized expression for the Domar weight must hold for every sector, we can write

$$\begin{aligned} d\log\lambda_i - d\log\lambda_j &= d\log p_i - d\log p_j + d\log y_i - d\log y_j \\ &= d\log x_{ij} - d\log \varepsilon^{f_i}_{x_{ij}} - d\log y_j \\ \Rightarrow d\log x_{ij} &= d\log\lambda_i - d\log\lambda_j + d\log y_j + d\log \varepsilon^{f_i}_{x_{ij}} \end{aligned}$$

Plugging back into the production function,

$$\begin{aligned} d\log y_i &= d\log A_i + \sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} d\log N_{io} + \sum_{j=1}^{J} \varepsilon^{f_i}_{x_{ij}} + d\log y_j + \sum_{k=1}^{\mathcal{K}} \varepsilon^{f_i}_{K_{ik}} d\log K_{ik} \\ &= d\log A_i + \sum_{o=1}^{\mathcal{O}} \varepsilon^{f_i}_{N_{io}} d\log N_{io} + \sum_{j=1}^{J} \varepsilon^{f_i}_{x_{ij}} d\log y_j + \sum_{k=1}^{\mathcal{K}} \varepsilon^{f_i}_{K_{ik}} d\log K^s_K \end{aligned}$$

Using the definition of labor demand,

$$\begin{aligned} \sum_i \frac{l_{io}}{L_o} d\log N_{io} &= d\log L^d_o - d\log(1+\tau_o(\theta_o)) \\ &= d\log L^d_o + \tau_o(\theta_o)\varepsilon^{Q_o}_{\theta_o} d\log\theta_o \end{aligned}$$

From output labor usage, we have that labor usage ratio for an occupation by two different sectors as:

$$\frac{l_{io}}{l_{jo}} = \frac{\varepsilon^{f}_{N_{io}}\lambda_i}{\varepsilon^{f}_{N_{jo}}\lambda_j}$$

for any $l_{io}, l_{jo} > 0$

Log-linearizing it, assuming Cobb-Douglas preferences, yields:

$$d\log l_{io} = d\log l_{jo}$$

Also since, $d\log l_{io} = d\log N_{io} + d\log(1+\tau_o(\theta_o)) = d\log l_{jo} = d\log N_{jo} + d\log(1+\tau_o(\theta_o))$, we have that $d\log N_{io} = d\log N_{jo}$

Using the labor market clearing condition, and the definition of labor supply,

$$\sum_k \frac{l_{ko}}{L_o} d\log N_{ko} = \left(\varepsilon_{\theta_o}^{\mathcal{F}_o} + \tau_o(\theta_o)\varepsilon_{\theta_o}^{\mathcal{Q}_o}\right) d\log\theta_o + d\log H_o$$

$$\Rightarrow d\log N_{io} \underbrace{\sum_k \frac{l_{ko}}{L_o}}_{=1} = \left(\varepsilon_{\theta_o}^{\mathcal{F}_o} + \tau_o(\theta_o)\varepsilon_{\theta_o}^{\mathcal{Q}_o}\right) d\log\theta_o + d\log H_o$$

Plugging this back into the linearized production function gives:

$$d\log y_i = d\log A_i + \sum_{o=1}^{\mathcal{O}} \varepsilon_{N_{io}}^{f_i} \left[\left(\varepsilon_{\theta_o}^{\mathcal{F}_o} + \tau_o(\theta_o)\varepsilon_{\theta_o}^{\mathcal{Q}_o}\right) d\log\theta_o + d\log H_o\right]$$
$$+ \sum_{j=1}^{J} \varepsilon_{x_{ij}}^{f_i} d\log y_j + \sum_{k=1}^{\mathcal{K}} \varepsilon_{K_{ik}}^{f_i} d\log K_k^s$$

Stacking over sectors gives,

$$d\log \boldsymbol{y} = d\log\boldsymbol{A} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} (\mathfrak{F} + \mathfrak{Q}\mathfrak{T})\, d\log\theta + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} d\log\boldsymbol{H} + \boldsymbol{\Omega} d\log \boldsymbol{y} + \varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}} d\log\boldsymbol{K}^s$$

Which implies

$$d\log \boldsymbol{y} = \boldsymbol{\Psi}\left(d\log\boldsymbol{A} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} (\mathfrak{F} + \mathfrak{Q}\mathfrak{T})\, d\log\theta + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} d\log\boldsymbol{H} + \varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}} d\log\boldsymbol{K}^s\right)$$

A.1.3. Tightness Propagation

Labor market clearing implies that changes in labor demand have to equal changes in labor supply:

$$d\log L_o^s(\theta, \boldsymbol{H}) = d\log L_o^d(\theta, \boldsymbol{A}).$$

$$\varepsilon_{\theta_o}^{\mathcal{F}_o} d\log\theta_o + d\log H_o = \sum_{i=1}^{J} \frac{l_{io}}{L_o^d} d\log l_{io}(\theta_o)$$

Where $\frac{l_i o}{L_o^d} = \frac{\varepsilon_{N_{io}}^{f_i} p_i y_i}{\sum_{j=1}^J \varepsilon_{N_{jo}}^{f_j} p_j y_j}$[6]. For every sector i we have

$$d \log l_{io}(\theta_o) = d \log \varepsilon_{N_{io}}^{f_i} - d \log w_o + d \log p_i + d \log y_i$$

Which implies that

$$d \log \boldsymbol{L^d}(\boldsymbol{\theta}) = \text{diag}\left(\mathcal{L} d \log \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\right) - \left[d \log \boldsymbol{w} - \mathcal{L} d \log \boldsymbol{p}\right] + \mathcal{L} d \log \boldsymbol{y}$$

since $\sum_{i=1}^J \frac{l_{io}}{L_o^d} = 1$ for all o. Plugging in for $d \log \boldsymbol{y}$ gives

$$\begin{aligned} d \log \boldsymbol{L^d}(\boldsymbol{\theta}) = \text{diag}\left(\mathcal{L} d \log \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\right) - \left[d \log \boldsymbol{w} - \mathcal{L} d \log \boldsymbol{p}\right] \\ + \mathcal{L}\boldsymbol{\Psi}\left[d \log \boldsymbol{A} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}(\mathcal{F} + \mathcal{Q}\mathcal{T})\, d \log \theta + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} d \log \boldsymbol{H} + \varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}} d \log \boldsymbol{K}^s\right] \end{aligned}$$

Labor market clearing implies

$$\mathcal{F} d \log \theta + d \log \boldsymbol{H} = \mathcal{L}\boldsymbol{\Psi}\left[d \log \boldsymbol{A} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}(\mathcal{F} + \mathcal{Q}\mathcal{T})\, d \log \theta + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} d \log \boldsymbol{H} + \varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}} d \log \boldsymbol{K}^s\right] - \left[d \log \boldsymbol{w} - \mathcal{L} d \log \boldsymbol{p}\right]$$

Which pins down first order changes in log tightness as

$$d \log \theta = \left[\mathcal{F} - \Xi_\theta\right]^{-1}\left[\mathcal{L}\boldsymbol{\Psi} d \log \boldsymbol{A} - \left[d \log \boldsymbol{w} - \mathcal{L} d \log \boldsymbol{p}\right] + \left[\mathcal{L}\boldsymbol{\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I}\right] d \log \boldsymbol{H} + \mathcal{L}\boldsymbol{\Psi}\varepsilon_{\boldsymbol{K}}^{\boldsymbol{f}} d \log \boldsymbol{K}^s\right]$$

Where $\Xi_\theta = \mathcal{L}\boldsymbol{\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}[\mathcal{F} + \mathcal{Q}\mathcal{T}]$.

Therefore, combining these propagation expressions, we are able to expres $d \log \theta$, $d \log \boldsymbol{y}$, and $d \log \boldsymbol{p}$ in terms of exogenous shocks.

A.1.4. Unemployment Propagation

Occupational unemployment is given by:

$$U_o = H_o - L_o,$$

[6]One implication of this formula is that I think we should be able to check whether the elasticities $\left\{\left\{\varepsilon_{N_{io}}^{f}\right\}_{o=1}^{\mathcal{O}}\right\}_{i=1}^{J}$ are consistent with the Domar weights.

which implies that:

$$d \log U_o = \frac{H_o}{U_o} d \log H_o - \frac{L_o}{U_o} d \log L_o.$$

Using the definition for labor supply, we have that:

$$\begin{aligned} d \log U_o &= \frac{H_o}{U_o} d \log H_o - \frac{L_o}{U_o} \left((1-\eta_o)\, d \log \theta_o + d \log H_o\right) \\ &= d \log H_o - \frac{L_o}{U_o} (1-\eta_o)\, d \log \theta_o \end{aligned}$$

Alternatively, we can rewrite the expression above in terms of job-finding rates:

$$d \log U_o = d \log H_o - \frac{f_o(\theta_o)}{1 - f_o(\theta_o)} (1-\eta_o)\, d \log \theta_o.$$

A.2. Proof for Theorem 1 and corrollaries

Let $\lambda_i = \frac{p_i y_i}{G}$, where $G = \sum_j p_j c_j = GDP = \sum_{o=1}^{\mathcal{O}} w_o L_o$, denote the final sales share of GDP for sector i. We have that:

$$p_j x_{ij} = \varepsilon^{f_i}{}_{x_{ij}} \lambda_i G.$$

From household's maximization problem, I have that $p_i c_i = \varepsilon^{\mathcal{D}}_{c_i} G$. Combining the two gives:

$$\begin{aligned} &\varepsilon^{\mathcal{D}}_{c_j} G = p_j y_j = p_j (c_j + \sum_i x_{ij}) = \left(\varepsilon^{\mathcal{D}}_{c_j} + \sum_i \varepsilon^{f_i}{}_{x_{ij}} \lambda_i \right) G \\ &\Rightarrow \boldsymbol{\lambda} = \boldsymbol{\Psi}' \varepsilon^{\boldsymbol{D}}_{\boldsymbol{c}}. \end{aligned}$$

The aggregate labor force, employment, and unemployment are $H^{agg} = \sum_{o=1}^{\mathcal{O}} H_o$, $L^{agg} = \sum_{o=1}^{\mathcal{O}} L_o$, and $U^{agg} = \sum_{o=1}^{\mathcal{O}} U_o$. Changes in aggregates are therefore given by

$$\begin{aligned} dH^{agg} &= \sum_{o=1}^{\mathcal{O}} dH_o \\ dL^{agg} &= \sum_{o=1}^{\mathcal{O}} dL_o \end{aligned}$$

$$dU^{agg} = \sum_{o=1}^{\mathcal{O}} dU_o$$

Or in terms of log changes

$$\begin{aligned}
d\log H^{agg} &= \frac{1}{H^{agg}} \sum_{o=1}^{\mathcal{O}} H_o d\log H_o \\
d\log L^{agg} &= \frac{1}{L^{agg}} \sum_{o=1}^{\mathcal{O}} L_o d\log L_o \\
d\log U^{agg} &= \frac{1}{U^{agg}} \sum_{o=1}^{\mathcal{O}} U_o d\log U_o
\end{aligned}$$

In matrix notation

$$\begin{aligned}
d\log H^{agg} &= \frac{1}{H^{agg}} \boldsymbol{H}' d\log \boldsymbol{H} \\
d\log L^{agg} &= \frac{1}{L^{agg}} \boldsymbol{L}' d\log \boldsymbol{L} \\
d\log U^{agg} &= \frac{1}{U^{agg}} \boldsymbol{U}' d\log \boldsymbol{U}
\end{aligned}$$

Substituting in for $d\log \boldsymbol{L}$

$$\begin{aligned}
d\log L^{agg} &= \frac{1}{L^{agg}} \boldsymbol{L'} \left[\mathcal{L} d\log \boldsymbol{y} - \left[d\log \boldsymbol{w} - \mathcal{L} d\log \boldsymbol{p}\right]\right] \\
&= \frac{1}{L^{agg}} \boldsymbol{L'} \left[\mathcal{L} \left[\Pi_{y,A} d\log \boldsymbol{A} + \Pi_{y,H} d\log \boldsymbol{H} - \Lambda_A d\log \boldsymbol{A} - \Lambda_H d\log \boldsymbol{H}\right]\right] \\
&= \Pi_{L^{agg},A} d\log \boldsymbol{A} + \Pi_{L^{agg},H} d\log \boldsymbol{H}
\end{aligned}$$

Where

$$\begin{aligned}
\Pi_{L^{agg},A} &= \frac{1}{L^{agg}} \boldsymbol{L}' \left[\mathcal{L} \Pi_{y,A} - \Lambda_A\right] \\
\Pi_{L^{agg},H} &= \frac{1}{L^{agg}} \boldsymbol{L}' \left[\mathcal{L} \Pi_{y,H} - \Lambda_H\right]
\end{aligned}$$

And

$$\begin{aligned}
d\log U^{agg} &= \frac{1}{U^{agg}} \boldsymbol{U}' \left[d\log \boldsymbol{H} - d\log \boldsymbol{L}\right] \\
&= \frac{1}{U^{agg}} \boldsymbol{U}' \left[d\log \boldsymbol{H} - \left[\mathcal{L} \left[\Pi_{y,A} d\log \boldsymbol{A} + \Pi_{y,H} d\log \boldsymbol{H} - \Lambda_A d\log \boldsymbol{A} - \Lambda_H d\log \boldsymbol{H}\right]\right]\right]
\end{aligned}$$

$$= \Pi_{U^{agg},A} d\log \boldsymbol{A} + \Pi_{U^{agg},H} d\log \boldsymbol{H}$$

Where

$$\Pi_{U^{agg},A} = \frac{1}{U^{agg}} \boldsymbol{U}' \left[\Lambda_A - \mathcal{L}\Pi_{y,A} \right]$$
$$\Pi_{U^{agg},H} = \frac{1}{U^{agg}} \boldsymbol{U}' \left[\boldsymbol{I} + \Lambda_H - \mathcal{L}\Pi_{y,H} \right]$$

Appendix B. Results for general CRTS production functions and one occupation per sector

In this section we generalize our results to any constant returns to scale production function, under the assumption that there is one type of labor per sector. This generalization results in additional terms that capture how the production elasticities change when shocks hit the economy. The expressions are otherwise similar to above. The model setup is identical, we just do not impose Cobb-Douglas technology and instead impose $\mathcal{O} = J$.

B.1. Price changes

First order changes in prices remain largely unchanged and satisfy

$$\left(\boldsymbol{I} - \Psi \varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} \right) d\log \, \boldsymbol{p} = \Psi \left[\varepsilon^{\boldsymbol{f}}_{\boldsymbol{N}} \left[d\log \boldsymbol{w} - d\log \, \boldsymbol{p} - \mathfrak{Q}\mathfrak{T} d\log \theta \right] - d\log \boldsymbol{A} \right]$$

B.2. Sales Share Propagation

We can rewrite the goods market clearing condition in terms of Domar weights:

$$y_i = c_i + \sum_{j=1}^{J} x_{ji}$$
$$\Rightarrow \frac{p_i y_i}{\sum_{k=1}^{J} p_k c_k} = \frac{p_i c_i}{\sum_{k=1}^{J} p_k c_k} + \sum_{j=1}^{J} \frac{p_i x_{ji}}{p_j x_j} \frac{p_j x_j}{\sum_{k=1}^{J} p_k c_k}$$

(A1)
$$\Rightarrow \lambda_i = \varepsilon^{\mathcal{D}}_{c_i} + \sum_{j=1}^{J} \varepsilon^{f_j}_{x_{ji}} \lambda_j,$$

where $\lambda_i = \frac{p_i y_i}{\sum_{k=1}^{J} p_k c_k}$ is the Domar weight of sector i.

By stacking (A1) for each sector, we get the following expression for Domar weights across the production network.

$$\lambda' = \varepsilon_{\boldsymbol{c}}^{\mathcal{D}'} + \lambda' \boldsymbol{\Omega}$$

We can see how Domar weights change across the production network by totally differentiating

$$\begin{aligned} d\boldsymbol{\lambda}' &= d\varepsilon_{\boldsymbol{c}}^{\mathcal{D}'} + d\boldsymbol{\lambda}'\boldsymbol{\Omega} + \boldsymbol{\lambda}' d\boldsymbol{\Omega} \\ \Rightarrow d\boldsymbol{\lambda}' &= \left[d\varepsilon_{\boldsymbol{c}}^{\mathcal{D}'} + \boldsymbol{\lambda}' d\boldsymbol{\Omega}\right] \boldsymbol{\Psi} \end{aligned} \tag{A2}$$

The Domar weights will help us express how shocks propagate to output.

B.3. Output changes

We can now write changes in output in terms of changes in tightness, technology, the size of the labor force, and changes in production elasticities, including changes in Domar weights, as

$$\begin{aligned} d\log \boldsymbol{y} = \boldsymbol{\Psi} &\left(d\log \boldsymbol{A} + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left(\mathcal{F} + \mathcal{Q}\mathcal{T}\right) d\log \theta + \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} d\log \boldsymbol{H}\right) \\ &- \boldsymbol{\Psi} d\log \mathcal{E} + \boldsymbol{\Psi} \left(\operatorname{diag}\left(\boldsymbol{\Omega}\mathbf{1}\right) - \boldsymbol{\Omega}\right) d\log \boldsymbol{\lambda} \end{aligned}$$

Where $\mathbf{1}$ is a $J \times 1$ vector of ones and $d\log \mathcal{E}$ is the $J \times 1$ vector of diagonal elements of $\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} d\log \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}'}$.

B.4. Tightness changes

Much like output, changes in tightness now also depends on changes in the elasticities of the production functions. The expressiong for changes in tightness is

$$\begin{aligned} d\log \theta = &\left[\mathcal{F} - \Xi_\theta\right]^{-1} \left[\boldsymbol{\Psi} d\log \boldsymbol{A} - \left[d\log \boldsymbol{w} - d\log \boldsymbol{p}\right] + \left[\boldsymbol{\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} - \boldsymbol{I}\right] d\log \boldsymbol{H}\right] \\ &+ \left[\mathcal{F} - \Xi_\theta\right]^{-1} \left[\operatorname{diag}\left(\mathcal{L} d\log \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}\right) + \boldsymbol{\Psi}\left[\left(\operatorname{diag}\left(\boldsymbol{\Omega}\mathbf{1}\right) - \boldsymbol{\Omega}\right) d\log \boldsymbol{\lambda} - d\log \mathcal{E}\right]\right] \end{aligned}$$

Where $\Xi_\theta = \boldsymbol{\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} \left[\mathcal{F} + \mathcal{Q}\mathcal{T}\right]$. Notice, all terms in the second line are zero assuming Cobb-Douglas production technology.

B.5. Aggregation

Aggregate output now satisfies

$$
\begin{aligned}
d \log Y &= \varepsilon_{\boldsymbol{c}}^{\mathcal{D}\prime} d \log \boldsymbol{c} \\
&= \varepsilon_{\boldsymbol{c}}^{\mathcal{D}\prime} \left(d \log \varepsilon_{\boldsymbol{c}}^{\mathcal{D}} d \log \boldsymbol{y} - d \log \boldsymbol{\lambda} \right)
\end{aligned}
$$

Appendix C. Amplification Proofs

We want to show that $(\boldsymbol{I} - \mathcal{M} - \Xi_\theta)^{-1}$, where $\Xi_\theta = \mathcal{L} \boldsymbol{\Psi} \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}} [\boldsymbol{I} - \mathcal{M}(\boldsymbol{I} + \mathcal{T})]$, is non-negative element-wise.

We have

$$
(\boldsymbol{I} - \mathcal{M} - \Xi_\theta)^{-1} = \left(\boldsymbol{I} - \Xi_\theta (\boldsymbol{I} - \mathcal{M})^{-1} \right)^{-1} (\boldsymbol{I} - \mathcal{M})^{-1}
$$

Since $(\boldsymbol{I} - \mathcal{M})^{-1}$ is a diagonal matrix with positive diagonal elements, it suffices to show that $\boldsymbol{I} - \Xi_\theta (\boldsymbol{I} - \mathcal{M})^{-1}$ is an M-matrix, since M-matrices are inverse non-negative.

$\boldsymbol{I} - \Xi_\theta (\boldsymbol{I} - \mathcal{M})^{-1}$ is a Z-matrix since its off-diagonals are negative. If $\boldsymbol{I} - \Xi_\theta (\boldsymbol{I} - \mathcal{M})^{-1}$ is diagonally dominant then it is also and M-matrix. Let a_{ij} denote the (i, j)-th element of $\boldsymbol{I} - \Xi_\theta (\boldsymbol{I} - \mathcal{M})^{-1}$. Row diagonal dominance requires:

$$
|a_{ii}| \geq \sum_{j \neq i} |a_{ij}| \quad \forall i
$$

For simplicity, we consider the case with one occupation per sector $\mathcal{L} = \boldsymbol{I}$, but the logic behind this proof follows for general $\mathcal{L}$.

To start with, we show that the row sums of $\boldsymbol{\Psi} \varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}$ is less than or equal to 1.

Let B_{ij} denote the $(i, j) - th$ element of $(\boldsymbol{I} - \boldsymbol{\Omega})$, and Ψ_{ij} the $(i, j) - th$ element of Ψ, and

$$
\sum_k \Psi_{ik} B_{kj} = \delta_{ij}
$$

We have:

$$
\sum_k \Psi_{ik} \sum_j B_{kj} = \sum_k \sum_j \Psi_{ik} B_{kj} = \sum_j \delta_{ij} = 1,
$$

since $\boldsymbol{\Psi}$ is the inverse of $(\boldsymbol{I} - \boldsymbol{\Omega})$.

For each k, the (k, k)-th element of $\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}$, β_{kk}, is smaller than or equal to $\sum_j B_{kj}$ by the constant returns of the production functions and non-negative factor shares.

Thus, for each row i in the matrix $\boldsymbol{\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}$, the row sum is given by:

$$\sum_k \Psi_{ik}\beta_{kk} \leq 1$$

Let x_{ij} denote the (i, j)-th element of the matrix $\boldsymbol{\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}$, row diagonal dominance requires that for each row i,

$$1 - x_{ii}\frac{(1 - \eta_i (1 + \tau_i))}{1 - \eta_i} \geq \sum_{j \neq i} x_{ij}\frac{\left(1 - \eta_j \left(1 + \tau_j\right)\right)}{1 - \eta_j}$$

Rewriting it yields:

$$1 \geq \sum_j x_{ij}\frac{\left(1 - \eta_j \left(1 + \tau_j\right)\right)}{1 - \eta_j},$$

which holds because $\tau_j \geq 0$.

Since $\tau_j > 0$, we actually have a strict inequality, where

$$1 > \sum_j x_{ij}\frac{\left(1 - \eta_j \left(1 + \tau_j\right)\right)}{1 - \eta_j}.$$

Now we can look at the case for general $\mathcal{L}$. We only need to show that the row sum of $\mathcal{L}\boldsymbol{\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}$ is no greater than 1.

First, we have the (i, j)-th element of $\mathcal{L}\boldsymbol{\Psi}$ is:

$$\sum_{k=1}^{J} \mathcal{L}_{ik}\Psi_{kj}$$

Thus, the (i, q)-th element of $\mathcal{L}\boldsymbol{\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}$ is:

$$\sum_{j=1}^{J} \beta_{jq}\left(\sum_{k=1}^{J} \mathcal{L}_{ik}\Psi_{kj}\right)$$

The row sum for the i-th row of $\boldsymbol{\mathcal{L}\Psi}\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}$ is thus:

$$\sum_{q=1}^{\mathcal{O}} \sum_{j=1}^{J} \beta_{jq} \left(\sum_{k=1}^{J} \mathcal{L}_{ik} \Psi_{kj} \right) = \sum_{k=1}^{J} \mathcal{L}_{ik} \sum_{j=1}^{J} \Psi_{kj} \sum_{q=1}^{\mathcal{O}} \beta_{jq}$$

By definition, for any j, $\sum_{q=1}^{\mathcal{O}} \beta_{jq} \leq \sum_j B_{kj}$, which implies that

$$\sum_{k=1}^{J} \mathcal{L}_{ik} \sum_{j=1}^{J} \Psi_{kj} \sum_{q=1}^{\mathcal{O}} \beta_{jq} \leq \sum_{k=1}^{J} \mathcal{L}_{ik} \leq 1$$

In fact, we can also show that $\boldsymbol{I} - \Xi_{\theta} \left(\boldsymbol{I} - \mathcal{M} \right)^{-1}$ has value at least 1 on the diagonals. This can be proven by rewriting $\boldsymbol{I} - \Xi_{\theta} \left(\boldsymbol{I} - \mathcal{M} \right)^{-1}$ as a Neumann series, which converges because

$$1 > \sum_{j} x_{ij} \frac{\left(1 - \eta_j \left(1 + \tau_j\right)\right)}{1 - \eta_j}.$$

Appendix D. Data and Calibration Details

This appendix describes our data in greater detail.

D.1. Input-output matrix

We use the 3-digit 2021 BEA Make and Use tables accessible at https://www.bea.gov/industry/input-output-accounts-data to calculate the relevant production elasticities[7]. The 3-digit Make and Use tables record the nominal amount of each 71 commodities made by and used by each of 71 industries. The commodities are denoted using the same codes as the industries, but they are conceptually distinct as each industry can produce more than one commodity.

For consistency with the industry classifications in JOLTs and the CPS unemployment by sector series, we collapse the 3-digit tables to a 13 sector table. The table below outlines the mapping from the NAICS 2-digit classification codes to our industry classifications. The mapping from 2-digit codes to 3-digit codes is readily available online.

Industry Name	Short Name	2-digit codes
Leisure and Hospitality	accom	71, 72
Construction	const	33
Durable goods	dur	33DG
Education and Health Services	edhealth	61, 62
Financial Activities	fin	52, 53
Government	gov	G
Information	info	51
Mining	mining	21
Nondurable good	nondur	11, 31ND
Other services, except government	other	81
Professional and business services	profserv	54, 55, 56
Wholesale and Retail trade	trade	42, 44RT
Transportation and Utilities	trans	22, 48TW

TABLE A1. Mapping from NAICS classification to our industries.

[7] See https://www.bea.gov/sites/default/files/methodologies/IOmanual_092906.pdf for a detailed description of how these tables are generated.

With the 13-sector make and use tables in hand, we can construct production elasticities in intermediate inputs and to labor, and demand elasticities. Let M_{ij} denote the nominal value of commodity i made by industry j. Let U_{ij} denote the nominal amount of commodity i used by industry j. The two tables below demonstrate the elements of the Make and Use tables.

	Sector 1	Sector 2	$\cdots$	Sector J	Total Industry Output
Sector 1	M_{11}	M_{21}	$\cdots$	M_{J1}	$\sum_{i=1}^{J} M_{i1}$
Sector 2	M_{12}	M_{22}	$\cdots$	M_{J2}	$\sum_{i=1}^{J} M_{i2}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
Sector J	M_{1J}	M_{2J}	$\cdots$	M_{JJ}	$\sum_{i=1}^{J} M_{iJ}$
Total Commodity Output	$\sum_{j=1}^{J} M_{1j}$	$\sum_{j=1}^{J} M_{2j}$	$\cdots$	$\sum_{j=1}^{J} M_{Jj}$	—

TABLE A2. Make table

	Sector 1	Sector 2	$\cdots$	Sector J	Total Intermediate Uses	Total Final Uses
Sector 1	U_{11}	U_{12}	$\cdots$	U_{1J}	$\sum_{j=1}^{J} U_{1j}$	$\sum_{j=1}^{J} U_{1j} + p_1 c_1$
Sector 2	U_{21}	U_{22}	$\cdots$	U_{2J}	$\sum_{j=1}^{J} U_{2j}$	$\sum_{j=1}^{J} U_{2j} + p_2 c_2$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
Sector J	U_{J1}	U_{J2}	$\cdots$	U_{JJ}	$\sum_{j=1}^{J} U_{Jj}$	$\sum_{j=1}^{J} U_{Jj} + p_J c_J$
Total Intermediate Inputs	$\sum_{i=1}^{J} U_{i1}$	$\sum_{i=1}^{J} U_{i2}$	$\cdots$	$\sum_{i=1}^{J} U_{iJ}$	—	—
Total industry output	$\sum_{i=1}^{J} U_{i1} + w_1(1+\tau_1)N_1$	$\sum_{i=1}^{J} U_{i2} + w_2(1+\tau_2)N_2$	$\cdots$	$\sum_{i=1}^{J} U_{iJ} + w_J(1+\tau_J)N_J$	—	—

TABLE A3. Use table

First, we calculate the fraction of commodity i produced by industry j by dividing the entry in along each row by the corresponding "total industry output"

$$m_{ij} = \frac{M_{ij}}{\sum_{j=1}^{J} M_{ji}}$$

Second, we calculate the share of commodity i in industry j's total uses as by dividing each entry in the column corresponding to industry j by the corresponding "Total industry output"

$$u_{ij} = \frac{U_{ij}}{\sum_{i=1}^{J} U_{ij} + w_j(1+\tau_j)N_j}$$

We form the two matrices

$$\boldsymbol{M} = \begin{bmatrix} m_{11} & m_{21} & \cdots & m_{J1} \\ m_{12} & m_{22} & \cdots & m_{J2} \\ \vdots & \vdots & \ddots & \vdots \\ m_{1J} & m_{2J} & \cdots & m_{JJ} \end{bmatrix}, \boldsymbol{U} = \begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1J} \\ u_{21} & u_{22} & \cdots & u_{2J} \\ \vdots & \vdots & \ddots & \vdots \\ u_{J1} & u_{J2} & \cdots & u_{JJ} \end{bmatrix}$$

Then, we can calculate our input output matrix by

$$\boldsymbol{\Omega} = [\boldsymbol{MU}]'$$

Given our assumption of constant returns to scale and zero profits, the difference between total intermediate inputs and total industry output is the nominal income paid to workers in each sector. We abstract from the other components of total industry output in the IO accounts, taxes and gross operating surplus, as they have no model counterpart in our setup. We can therefore calculate the labor elasticities from the Use table as "Total industry output" – "Total intermediate inputs" ÷ "Total industry output."

$$\varepsilon^{f_j}_{N_j} = \frac{w_j(1+\tau_j)N_j}{\sum_{i=1}^{J} U_{ij} + w_j(1+\tau_j)N_j}$$

Finally, we can back out the demand elasticities from "Total intermediate uses" and "Total final uses" columns of the Uses table.

$$p_i c_i = \sum_{j=1}^{J} U_{ij} + p_i c_i - \sum_{j=1}^{J} U_{ij}$$

We can the work out the elasticities by

$$\varepsilon^{\mathcal{D}}_{c_i} = \frac{p_i c_i}{\sum_{i=1}^{J} p_i c_i}$$

Finally, to ensure that constant returns to scale holds we rescale our elasticities proportionally to ensure they sum to one. This adjustment is minor and is only needed because we drop the small "Used" and "rest of world adjustment" categories. It does not change any elasticity by more than 3 percent.

We report the resulting estimates of the production elasticities, labor elasticities, and demand elasticities in the tables below. In tables A4 and A5 we assume all non-intermediate, non-energy, and non-capital, spending goes to labor income, which automatically imposes constant returns but leads to large labor shares.

Sector	Labor Elasticity $\left(\varepsilon_N^f\right)$	Demand Elasticity $\left(\varepsilon_c^{\mathcal{D}}\right)$
accom	0.510	0.051
const	0.474	0.065
dur	0.434	0.138
edhealth	0.616	0.129
fin	0.617	0.165
gov	0.626	0.132
info	0.571	0.043
mining	0.518	0.008
nondur	0.352	0.151
other	0.608	0.024
profserv	0.591	0.071
trade	0.523	0.000
trans	0.492	0.022

TABLE A4. Labor elasticities and demand elasticities according the BEA make use tables for 13-industry classification, rounded to 3 decimal places. We assume all non-intermediate spending goes to labor income.

	accom	const	dur	edhealth	fin	gov	info	mining	nondur	other	profserv	trade	trans
accom	0.029	0.002	0.017	0.003	0.099	0.007	0.036	0.008	0.103	0.014	0.143	0.000	0.030
const	0.001	0.000	0.282	0.000	0.033	0.001	0.016	0.022	0.089	0.006	0.072	0.000	0.004
dur	0.001	0.001	0.393	0.000	0.016	0.001	0.012	0.013	0.056	0.003	0.055	0.004	0.012
edhealth	0.019	0.000	0.030	0.015	0.103	0.006	0.028	0.004	0.056	0.009	0.101	0.000	0.012
fin	0.011	0.023	0.007	0.000	0.195	0.005	0.020	0.001	0.008	0.005	0.083	0.001	0.022
gov	0.006	0.023	0.048	0.011	0.055	0.004	0.032	0.013	0.081	0.012	0.072	0.000	0.017
info	0.017	0.001	0.040	0.000	0.047	0.003	0.125	0.002	0.015	0.005	0.160	0.001	0.013
mining	0.001	0.005	0.010	0.000	0.060	0.002	0.017	0.128	0.056	0.001	0.090	0.000	0.022
nondur	0.001	0.003	0.044	0.000	0.020	0.002	0.008	0.124	0.377	0.003	0.040	0.004	0.021
other	0.013	0.005	0.066	0.010	0.113	0.005	0.033	0.003	0.032	0.013	0.086	0.000	0.012
nprofserv	0.020	0.000	0.029	0.001	0.070	0.004	0.054	0.002	0.026	0.008	0.176	0.000	0.017
trade	0.006	0.002	0.025	0.002	0.104	0.009	0.038	0.002	0.031	0.015	0.159	0.021	0.063
trans	0.014	0.007	0.022	0.000	0.080	0.015	0.025	0.030	0.080	0.011	0.093	0.001	0.129

TABLE A5. Production elasticities to intermediate inputs at 13-sector level ($\mathbf{\Omega}$), rounded to 3 decimal places. We assume all non-intermediate spending goes to labor income.

D.2. Matching Parameters

We estimate the parameters of the sector specific matching function from monthly data on hires and vacancies from JOLTs and unemployment from the CPS. In particular, we estimate

$$\log H_{i,t} = \log \phi_i + \eta_i \log U_{i,t} + (1 - \eta_i) \log V_{i,t} + \epsilon_{i,t}$$

by least squares. ϕ_i is the matching efficiency in sector i and η_i is the matching elasticity with respect to unemployment in sector i. We report the resulting estimates in the table below.

	Matching Efficiency $\left(\widehat{\phi}_i\right)$	Unemployment Elasticity $(\widehat{\eta}_i)$
accom	1.185	0.401
const	1.106	0.507
dur	0.688	0.364
edhealth	0.703	0.336
fin	0.705	0.329
gov	0.640	0.291
info	0.703	0.275
mining	1.236	0.262
nondur	0.779	0.391
other	0.848	0.441
profserv	1.077	0.372
trade	1.009	0.430
trans	0.862	0.439

TABLE A6. Matching function parameter estimates. Based on monthly hiring, unemployment, and vacancy data from Jan 2000 to Feb 2023.

Finally, we use the sector level proportion of HR workers as a proxy for the recruiter producer ratio. The resulting recruiter producer ratios are reported below

	τ_i
accom	0.002
const	0.002
dur	0.007
edhealth	0.005
fin	0.008
gov	0.011
info	0.013
mining	0.005
nondur	0.007
other	0.018
profserv	0.020
trade	0.003
trans	0.001

TABLE A7. Estimated recruiter producer ratios based on the number of HR workers in industry i over total employment in industry i.

D.3. Computing Occupational Worker Share

For our occupational labor market calibration, we need to compute $\varepsilon_{\boldsymbol{N}}^{\boldsymbol{f}}$, which is the occupational worker elasticity of production. To do this, we obtain wage and employment data for ONET major occupations at 3-digit sector level from the Occupational Employment and Wage Statistics (OES). For each sector i, we compute $\varepsilon_{N_i o}^{f}$ as:

$$\varepsilon_{N_{io}}^{f} = \varepsilon_{Ni}^{f} \frac{w_{io} L_{io}}{\sum_o w_{io} L_{io}},$$

where ε_{N}^{f} is the labor share we obtained earlier from the input-output table.

Table A8 contains our calibration estimates.

	Admin	Agg	Arts	Bus Ops	Care	Clean	Cons	Educ	Eng	Food S	Health P	Health S	Legal	Manag	Math	Prod	Prot S	Repair	Sales	Science	Soc S	Trans
accom	1.7	0.0	0.9	0.8	1.9	1.6	0.0	0.2	0.0	23.9	0.1	0.0	0.0	3.5	0.1	0.3	0.4	0.7	1.4	0.0	0.0	0.8
const	2.8	0.0	0.1	3.0	0.0	0.1	22.9	0.0	0.7	0.0	0.0	0.0	0.0	5.6	0.1	0.6	0.0	3.4	1.0	0.1	0.0	1.1
dur	1.9	0.0	0.2	2.3	0.0	0.1	0.6	0.0	4.0	0.0	0.0	0.0	0.1	4.6	1.9	11.5	0.0	1.3	1.2	0.2	0.0	1.4
edhealth	4.1	0.0	0.4	1.4	0.6	0.8	0.1	12.9	0.0	0.7	17.8	5.3	0.0	4.3	0.7	0.1	0.3	0.4	0.1	0.6	2.1	0.4
fin	7.2	0.0	0.2	10.6	0.1	0.3	0.1	0.0	0.1	0.0	0.3	0.0	0.6	8.4	3.2	0.0	0.1	1.3	6.3	0.0	0.1	0.3
gov	6.8	0.1	0.5	8.6	0.6	0.8	2.3	1.0	2.6	0.2	4.0	0.5	2.7	6.6	2.4	0.9	12.0	2.3	0.2	2.5	2.9	2.1
info	2.4	0.0	4.4	4.0	0.2	0.0	0.1	0.1	0.6	0.1	0.0	0.0	0.3	7.2	10.3	0.1	0.0	2.0	3.2	0.0	0.0	0.2
mining	1.3	0.0	0.0	1.4	0.0	0.0	7.3	0.0	1.8	0.0	0.0	0.0	0.1	3.7	0.4	1.4	0.0	1.9	0.6	0.7	0.0	2.4
nondur	1.8	0.1	0.2	1.2	0.0	0.2	0.1	0.0	1.0	0.3	0.0	0.0	0.0	3.3	0.3	9.8	0.0	1.7	1.2	0.8	0.0	2.3
other	6.4	0.0	1.7	6.1	8.0	0.8	0.2	0.8	0.3	0.5	0.3	0.7	0.3	9.4	1.0	2.4	0.3	9.5	1.9	0.3	1.6	3.4
profserv	5.4	0.0	1.1	8.6	0.1	2.3	0.7	0.1	3.4	0.1	1.2	0.3	3.1	10.5	7.9	1.0	1.1	0.7	2.2	1.2	0.1	1.7
trade	3.9	0.1	0.5	1.7	0.1	0.2	0.1	0.0	0.2	0.7	1.6	0.1	0.0	5.0	0.8	1.1	0.1	2.3	15.2	0.1	0.0	6.0
trans	6.2	0.0	0.0	0.4	0.0	0.2	0.4	0.0	0.4	0.0	0.0	0.0	0.0	0.5	0.2	0.9	0.0	2.9	0.5	0.1	0.0	18.2

TABLE A8. Occupational worker elasticity of output, in percentage terms, rounded to 1 decimal place.

D.4. Imputing Occupation Labor Market Parameters

For the occupational labor market specification, we need to calibrate unemployment, vacancy, and tightness of each occupation. We currently don't have access to occupational labor market characteristics, so we instead impute these parameters. For simplicity, we assume the total number of unemployment and vacancy for an occupation is the sum of unemployment and vacancy across sectors, weighted by the sectors' wage shares of that particular occupation:

$$V_o = \sum_i \frac{\varepsilon^f_{N_{io}}}{\varepsilon^f_{N_i}} V_i,$$

$$H_o = \sum_i \frac{\varepsilon^f_{N_{io}}}{\varepsilon^f_{N_i}} H_i,$$

where V denotes vacancy and H denotes labor supply, which is equivalent to unemployment in our static setup. The intuition behind this is that each sector's contribution to vacancy postings and the number of people looking for jobs in that sector for an occupation is proportional to how much the sector relies on that occupation.

Note that, with this imperfect simplifying assumption, we can back out changes in sectoral tightness. For sector j, the first-order response in tightness is: Tightness for sector j is:

$$\begin{aligned}
\theta_j = \frac{V_j}{H_j} &= \frac{\sum_o \frac{V_{jo}}{V_o} V_o}{\sum_o \frac{H_{jo}}{H_o} H_o} \\
\Rightarrow d\log\theta_j &= d\log V_j - d\log H_j \\
&= \sum_o \frac{V_o}{V_j}\frac{V_{jo}}{V_o} d\log V_o - \frac{H_o}{H_j}\frac{H_{jo}}{H_o} d\log H_o \\
&= \sum_o \frac{V_{jo}}{V_j} d\log V_o - \frac{H_{jo}}{H_j} d\log H_o \\
&= \sum_o \frac{\varepsilon^f_{N_{jo}}}{\varepsilon^f_{N_j}} d\log\theta_o.
\end{aligned}$$

We estimate the matching elasticities using the same methodology from appendix section D.2. Table A9 reports the estimated coefficients.

Occupation	Matching Efficiency ($\hat{\phi}_o$)	Unemployment Elasticity ($\hat{\eta}_o$)
Admin	0.883	0.368
Agg	0.907	0.390
Arts	0.939	0.352
Bus Ops	0.879	0.356
Care	0.978	0.378
Clean	1.009	0.372
Cons	1.054	0.461
Educ	0.713	0.338
Eng	0.871	0.357
Food S	1.163	0.398
Health P	0.749	0.346
Health S	0.731	0.343
Legal	0.923	0.354
Manag	0.915	0.368
Math	0.905	0.342
Prod	0.797	0.374
Prot S	0.744	0.319
Repair	0.909	0.390
Sales	0.975	0.398
Science	0.829	0.355
Soc S	0.701	0.334
Trans	0.924	0.406

TABLE A9. Matching parameters for major occupations

Additionally, following our vacancy assumption, we assume the number of recruiters each sector dedicates to recruiting a particular occupation is proportional to the occupation elasticity of production. In other words:

$$R_o = \sum_i \frac{\varepsilon^f_{N_{io}}}{\varepsilon^f_{N_i}} R_i,$$

Occupation	τ_o
Admin	0.006
Agg	0.002
Arts	0.011
Bus Ops	0.014
Care	0.006
Clean	0.006
Cons	0.004
Educ	0.005
Eng	0.017
Food S	0.002
Health P	0.009
Health S	0.003
Legal	0.027
Manag	0.019
Math	0.020
Prod	0.006
Prot S	0.009
Repair	0.007
Sales	0.005
Science	0.014
Soc S	0.006
Trans	0.003

TABLE A10. Recruiter producer ratios based on the number of estimated recruiters in occupation o.

where R_i is the number of recruiters in sector i. This is also implicitly assuming that the recruiting cost for the occupations are the same.

Since we have the total employment for each occupation from the OES, we can therefore compute the recruiter-producer ratios. Table A10 reports the estimated recruiter-producer ratio for different occupations.

Appendix E. Additional Results on Calibrated Shock Propagation.

Below, we report the responses of output and unemployment to technology shocks in each of the 13 sectors in our calibration.

E.1. Responses to technology shocks across sectors: One Occupation Per Sector

figures/calib2/accom_AshockY.png	figures/calib2/const_AshockY.png
figures/calib2/edhealth_AshockY.png	figures/calib2/fin_AshockY.png
figures/calib2/gov_AshockY.png	figures/calib2/info_AshockY.png

FIGURE A1. Responses of output to 1% technology shocks other sectors.

figures/calib2/mining_AshockY.png

figures/calib2/nondur_AshockY.png

figures/calib2/other_AshockY.png

figures/calib2/profserv_AshockY.png

figures/calib2/trade_AshockY.png

figures/calib2/trans_AshockY.png

FIGURE A2. Responses of output to 1% technology shocks other sectors.

figures/calib2/accom_AshockU.png figures/calib2/const_AshockU.png

figures/calib2/edhealth_AshockU.png figures/calib2/fin_AshockU.png

figures/calib2/gov_AshockU.png figures/calib2/info_AshockU.png

FIGURE A3. Responses of unemployment to 1% labor force shocks other sectors.

figures/calib2/mining_AshockU.png figures/calib2/nondur_AshockU.png

figures/calib2/other_AshockU.png figures/calib2/profserv_AshockU.png

figures/calib2/trade_AshockU.png figures/calib2/trans_AshockU.png

FIGURE A4. Responses of unemployment to 1% labor force shocks other sectors.

E.2. Responses to technology shocks across sectors: Major Occupations

figures/calib3/accom_AshockY.png

figures/calib3/const_AshockY.png

figures/calib3/edhealth_AshockY.png

figures/calib3/fin_AshockY.png

figures/calib3/gov_AshockY.png

figures/calib3/info_AshockY.png

FIGURE A5. Responses of output to 1% technology shocks other sectors.

figures/calib3/mining_AshockY.png figures/calib3/nondur_AshockY.png

figures/calib3/other_AshockY.png figures/calib3/profserv_AshockY.png

figures/calib3/trade_AshockY.png figures/calib3/trans_AshockY.png

FIGURE A6. Responses of output to 1% technology shocks other sectors.

figures/calib3/accom_AshockU.png

figures/calib3/const_AshockU.png

figures/calib3/edhealth_AshockU.png

figures/calib3/fin_AshockU.png

figures/calib3/gov_AshockU.png

figures/calib3/info_AshockU.png

FIGURE A7. Responses of unemployment to 1% labor force shocks other sectors.

figures/calib3/mining_AshockU.png figures/calib3/nondur_AshockU.png

figures/calib3/other_AshockU.png figures/calib3/profserv_AshockU.png

figures/calib3/trade_AshockU.png figures/calib3/trans_AshockU.png

FIGURE A8. Responses of unemployment to 1% labor force shocks other sectors.