Mismatch Unemployment in a Production Network

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Motivation

- Economies feature production linkages and frictional labor markets
- Important to understand how production linkages interact with matching frictions and unemployment
- **Research Questions:**
  - How does the optimal sectoral allocation of unemployment change when production linkages are present?
  - How much of aggregate unemployment change is attributed to mismatch frictions in a production network?

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Literature Review

Related Literature

- **Production Network:** Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi (2012)
- **Mismatch Unemployment:** Şahin, Song, Topa and Violante (2014)

Baseline Economy

Baseline: setup

- N sectors indexed by i.
- Unit mass of workers (employed or unemployed and searching in sector i):

$$\sum_{i=1}^{N}(e_i + u_i) = 1.$$

- Linear production technology in labor:

$$y_i = L_i = e_i + h_i.$$

- Labor markets are frictional:

$$h_i = \phi_i m(u_i, v_i).$$

Baseline: efficiency without production linkages

- **The social planner's problem**: reallocate unemployed workers to maximize total output

$$\max_{\{u_i\}_{i=1}^N} \sum_{i=1}^N \left[e_i + \phi_i m(u_i, v_i)\right] \text{ s.t.} \sum_{i=1}^N (e_i + u_i) = 1.$$

- Optimal unemployment allocation satisfies

$$\phi_i m_u \left(u_i^*, v_i\right) = \phi_j m_u \left(u_j^*, v_j\right).$$

Intuition: want to assign more unemployed workers to sectors with more efficient matching technology.

Baseline: mismatch index

- Let h denote aggregate new hires across sectors:

$$h = \Phi u^{1-\eta} v^{\eta} \sum_{i=1}^{j} \phi_i \left(\frac{u_i}{u}\right)^{1-\eta} \left(\frac{v_i}{v}\right)^{\eta}.$$

- Mismatch index measures employment lost due to inefficient allocation of unemployed workers across sectors:

$$\mathcal{M}_B = 1 - \frac{h}{h^*} = 1 - \sum_{i=1}^{N} \frac{\phi_i}{\overline{\phi}} \left(\frac{u_i}{u}\right)^{1-\eta} \left(\frac{v_i}{v}\right)^{\eta},$$

where $\overline{\phi} = \sum_{i=1}^{N} \phi_i^{\frac{1}{\eta}} \frac{v_i}{v}$.

Vertical Economy

Two-sector vertical economy: setup

- Suppose $N = 2$ with upstream and downstream sectors:

$$y_1 = L_1^{\alpha_1},$$
$$y_2 = L_2^{\alpha_2} y_1^{a_{21}}.$$

- The social planner maximizes final goods production:

$$\max_{\{u_i\}_{i=1}^N} \left(e_2 + \phi_2 m(u_2, v_2)\right)^{\alpha_2} \left(e_1 + \phi_1 m(u_2, v_2)\right)^{\alpha_1 a_{21}},$$

$$\text{s.t.} \sum_{i=1}^{2} (e_i + u_i) = 1.$$

Two-sector vertical economy: efficiency

- Optimal unemployment satisfies:

$$\frac{\alpha_1 a_{21}}{L_1^*}\phi_1 m_u(u_1^*, v_1) = \frac{\alpha_2}{L_2^*}\phi_2 m_u(u_2^*, v_2).$$

Intuition: New channel $\rightarrow$ optimal to have more workers searching in the upstream sector if the upstream sector is more important in downstream production.

- Extending to N industry vertical economy yields:

$$\frac{\alpha_i \prod_{k=i+1}^{K} a_{k,k-1}}{L_i^*}\phi_i m_u\left(u_i^*, v_i\right) = \frac{\alpha_j \prod_{k=j+1}^{K} a_{k,k-1}}{L_j^*}\phi_j m_u\left(u_j^*, v_j\right).$$

General Production Network

General production network: firms

- Firms in sector i produce output using Cobb-Douglas production technology:

$$y_i = L_i^{\alpha_i} \prod_{j=1}^{N} x_{ij}^{a_{ij}},$$

where x_{ij} is amount of good j used in production of good i.

- This gives profit maximization problem:

$$\max_{\{x_{ij}\}_{j=1}^{N}} p_i L_i^{\alpha_i} \prod_{j=1}^{N} x_{ij}^{a_{ij}} - \sum_{j=1}^{N} p_j x_{ij} - wL_i.$$

FOCs imply

$$p_j x_{ij} = a_{ij} p_i y_i.$$

General production network: households

- Representative household solves:

$$\max_{\{C_i\}_{i=1}^N} U\left(\{C_i\}_{i=1}^N\right) = \max_{\{C_i\}_{i=1}^N} \prod_{i=1}^N C_i^{\theta_i},$$

$$\text{s.t. } \sum_{i=1}^N p_i C_i = wL.$$

Let $G = wL = \sum_{i=1}^N p_i C_i = GDP$, household FOCs imply:

$$p_i C_i = \theta_i G \ \forall i.$$

General production network: efficiency

- The optimal allocation equalizes the marginal contribution of changing unemployment to the utility of the representative household across sectors.

$$\frac{dU}{du_i} = \frac{dU}{du_j}$$

- Optimal allocation satisfies

$$\frac{\lambda_i \alpha_i \phi_i m_u \left(u_i^*, v_i\right)}{L_i^*} = \frac{\lambda_j \alpha_j \phi_j m_u \left(u_j^*, v_j\right)}{L_j^*}$$

Where $\lambda_i = \sum_{j=1}^{N} \theta_j \rho_{ji}$.

General production network: some new notation

- Can summarize the production linkages in the matrix A where the ijth element of A is a_{ij}.
- Let ρ_{ij} be the ijth element of the matrix $(I - A)^{-1}$, where structure of A guarantees invertibility and

$$(I - A)^{-1} = \sum_{k=0}^{\infty} A^k$$

$$\Rightarrow \rho_{ij} = a_{ij} + \sum_{r=1}^{N} a_{ir} a_{rj} + \cdots$$

Intuition: ρ_{ij} measures importance of industry j as direct and indirect input into industry i.

General production network: efficiency

- Optimal allocation satisfies

$$\frac{\lambda_i \alpha_i \phi_i m_u \left(u_i^*, v_i\right)}{L_i^*} = \frac{\lambda_j \alpha_j \phi_j m_u \left(u_j^*, v_j\right)}{L_j^*}$$

where $\lambda_i = \sum_{j=1}^{N} \theta_j \rho_{ji}$.
Intuition: Now, the optimal number of searchers in each sector depends in complex way (summarized by λ_i) on the structure of the production network.

Special cases of general result

- **Horizontal economy:** $\rho_{ii} = 1$, $\rho_{ji} = 0 \forall j \neq i$. When $\theta_i = \theta_j = \theta$ and $\alpha_i = \alpha_j = \alpha$, we have:

$$\frac{\phi_i m_u (u_i^*, v_i)}{L_i^*} = \frac{\phi_j m_u \left(u_j^*, v_j\right)}{L_j^*}.$$

- **Vertical economy:** $\theta_N = 1$, $\theta_i = 0 \forall i \neq N$, $\lambda_i = \rho_{Ni} = \prod_{k=i+1}^{K} a_{k,k-1}$. We have:

$$\frac{\alpha_i \prod_{k=i+1}^{N} a_{k,k-1}}{L_i^*} \phi_i m_u (u_i^*, v_i) = \frac{\alpha_j \prod_{k=j+1}^{K} a_{k,k-1}}{L_j^*} \phi_j m_u \left(u_j^*, v_j\right).$$

General production network: mismatch index

- Can apply similar mismatch index to above for correctly specified $\overline{\phi}$:

$$\mathcal{M}_G = 1 - \frac{h}{h^*} = 1 - \sum_{i=1}^{N} \frac{\phi_i}{\overline{\phi}} \left(\frac{u_i}{u}\right)^{1-\eta} \left(\frac{v_i}{v}\right)^{\eta}.$$

Measuring Mismatch in the Data

Measuring mismatch

- **Goal:** Estimate mismatch unemployment at the industry level based on formula derived above.
- Requires:
  - Data on sector level unemployment, vacancies, and hires
  - Estimation of sector level matching functions
  - Estimates or data on production linkages between sectors
  - Data on size of each sector relative to GDP

Possible Data sources

- **JOLTS:** Vacancies and hires at industry level.
  - **BurningGlass:** Alternative to JOLTS for vacancy postings.
- **CPS:** Unemployment at industry level.
- **BEA input-output tables:** Estimated production linkages between sectors.

Estimating sector level matching functions

- **Endogeneity:** Vacancies chosen by profit maximizing firms are a function of matching efficiency.
- Borowczyk-Martins, Jolivet, Postel-Vinay (2012) suggest using lags of vacancies and unemployment as instruments, argue these are valid under certain assumptions about process for shocks to matching efficiency.
- Şahin et al. (2014) argue sources of endogeneity are low-frequency movements in matching efficiency and directly control for these with a quartic time trend. Run

$$\log(h_{it}/u_{it}) = \log(\phi_i) + \gamma' quartic_t + \eta \log(v_{it}/u_{it}) + \epsilon_{it}$$

References

Acemoglu, D., Carvalho, V.M., Ozdaglar, A., Tahbaz-Salehi, A., 2012. The Network Origins of Aggregate Fluctuations. Econometrica 80, 1977–2016. URL: https://onlinelibrary.wiley.com/doi/abs/10.3982/ECTA9623, doi:10.3982/ECTA9623. _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.3982/ECTA9623.

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