A–3–9. Consider the servo system shown in Figure 3–29(a). The motor shown is a servomotor, a dc motor designed specifically to be used in a control system. The operation of this system is as follows: A pair of potentiometers acts as an error-measuring device. They convert the input and output positions into proportional electric signals. The command input signal determines the angular position r of the wiper arm of the input potentiometer. The angular position r is the reference input to the system, and the electric potential of the arm is proportional to the angular position of the arm. The output shaft position determines the angular position c of the wiper arm of the output potentiometer. The difference between the input angular position r and the output angular position c is the error signal e, or

$$e = r - c$$

The potential difference $e_r - e_c = e_v$ is the error voltage, where e_r is proportional to r and e_c is proportional to c; that is, $e_r = K_0 r$ and $e_c = K_0 c$, where K_0 is a proportionality constant. The error voltage that appears at the potentiometer terminals is amplified by the amplifier whose gain constant is K_1. The output voltage of this amplifier is applied to the armature circuit of the dc motor. A fixed voltage is applied to the field winding. If an error exists, the motor develops a torque to rotate the output load in such a way as to reduce the error to zero. For constant field current, the torque developed by the motor is

$$T = K_2 i_a$$

where K_2 is the motor torque constant and i_a is the armature current.

When the armature is rotating, a voltage proportional to the product of the flux and angular velocity is induced in the armature. For a constant flux, the induced voltage e_b is directly proportional to the angular velocity $d\theta/dt$, or

$$e_b = K_3 \frac{d\theta}{dt}$$

where e_b is the back emf, K_3 is the back emf constant of the motor, and θ is the angular displacement of the motor shaft.

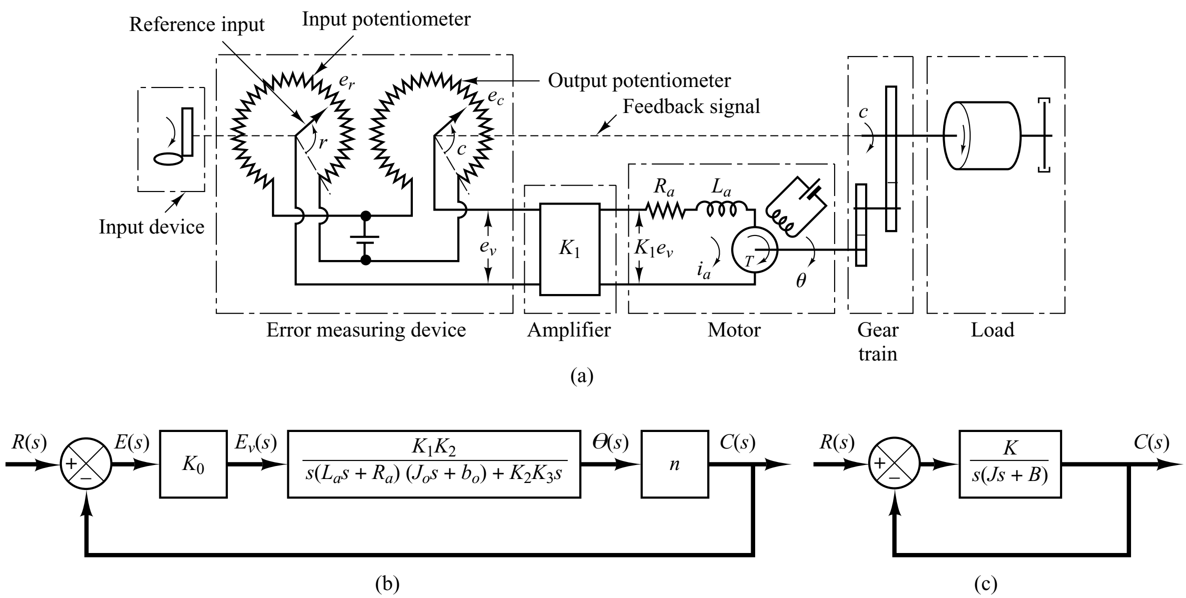

Figure 3–29
(a) Schematic diagram of servo system; (b) block diagram for the system; (c) simplified block diagram.

Obtain the transfer function between the motor shaft angular displacement θ and the error voltage e_v. Obtain also a block diagram for this system and a simplified block diagram when L_a is negligible.

Solution. The speed of an armature-controlled dc servomotor is controlled by the armature voltage e_a. (The armature voltage $e_a = K_1 e_v$ is the output of the amplifier.) The differential equation for the armature circuit is

$$L_a \frac{di_a}{dt} + R_a i_a + e_b = e_a$$

or

$$L_a \frac{di_a}{dt} + R_a i_a + K_3 \frac{d\theta}{dt} = K_1 e_v \tag{3–46}$$

The equation for torque equilibrium is

$$J_0 \frac{d^2\theta}{dt^2} + b_0 \frac{d\theta}{dt} = T = K_2 i_a \tag{3–47}$$

where J_0 is the inertia of the combination of the motor, load, and gear train referred to the motor shaft and b_0 is the viscous-friction coefficient of the combination of the motor, load, and gear train referred to the motor shaft.

By eliminating i_a from Equations (3–46) and (3–47), we obtain

$$\frac{\Theta(s)}{E_v(s)} = \frac{K_1 K_2}{s(L_a s + R_a)(J_0 s + b_0) + K_2 K_3 s} \tag{3–48}$$

We assume that the gear ratio of the gear train is such that the output shaft rotates n times for each revolution of the motor shaft. Thus,

$$C(s) = n\Theta(s) \tag{3–49}$$

The relationship among $E_v(s)$, $R(s)$, and $C(s)$ is

$$E_v(s) = K_0[R(s) - C(s)] = K_0 E(s) \tag{3–50}$$

The block diagram of this system can be constructed from Equations (3–48), (3–49), and (3–50), as shown in Figure 3–29(b). The transfer function in the feedforward path of this system is

$$G(s) = \frac{C(s)}{\Theta(s)} \frac{\Theta(s)}{E_v(s)} \frac{E_v(s)}{E(s)} = \frac{K_0 K_1 K_2 n}{s[(L_a s + R_a)(J_0 s + b_0) + K_2 K_3]}$$

When L_a is small, it can be neglected, and the transfer function $G(s)$ in the feedforward path becomes

$$G(s) = \frac{K_0 K_1 K_2 n}{s[R_a(J_0 s + b_0) + K_2 K_3]}$$

$$= \frac{K_0 K_1 K_2 n / R_a}{J_0 s^2 + \left(b_0 + \dfrac{K_2 K_3}{R_a}\right)s} \tag{3–51}$$

The term $[b_0 + (K_2 K_3 / R_a)]s$ indicates that the back emf of the motor effectively increases the viscous friction of the system. The inertia J_0 and viscous friction coefficient $b_0 + (K_2 K_3 / R_a)$ are referred to the motor shaft. When J_0 and $b_0 + (K_2K_3/R_a)$ are multiplied by $1/n^2$, the inertia and viscous-friction coefficient are expressed in terms of the output shaft. Introducing new parameters defined by

$$J = J_0/n^2 = \text{moment of inertia referred to the output shaft}$$

$$B = [b_0 + (K_2K_3/R_a)]/n^2 = \text{viscous-friction coefficient referred to the output shaft}$$

$$K = K_0K_1K_2/nR_a$$

the transfer function $G(s)$ given by Equation (3–51) can be simplified, yielding

$$G(s) = \frac{K}{Js^2 + Bs}$$

or

$$G(s) = \frac{K_m}{s(T_m s + 1)}$$

where

$$K_m = \frac{K}{B}, \qquad T_m = \frac{J}{B} = \frac{R_a J_0}{R_a b_0 + K_2 K_3}$$

The block diagram of the system shown in Figure 3–29(b) can thus be simplified as shown in Figure 3–29(c).

PROBLEMS

B–3–1. Obtain the equivalent viscous-friction coefficient b_{eq} of the system shown in Figure 3–30.

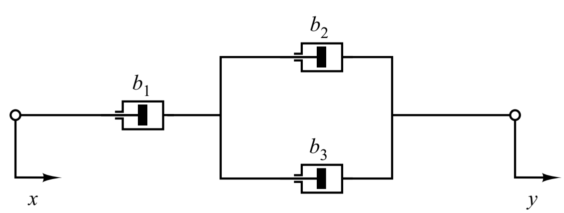

Figure 3–30
Damper system.

B–3–2. Obtain mathematical models of the mechanical systems shown in Figures 3–31(a) and (b).

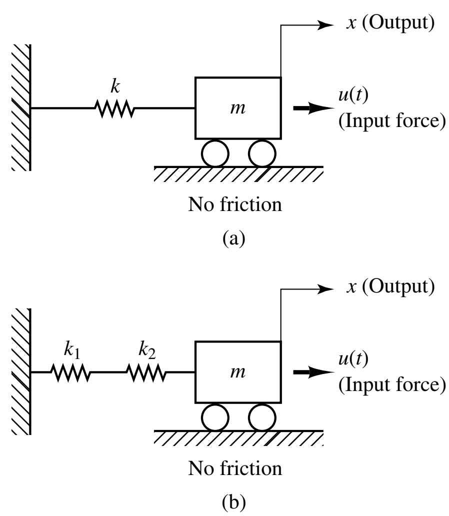

Figure 3–31
Mechanical systems.