* Modelagem Cinemática

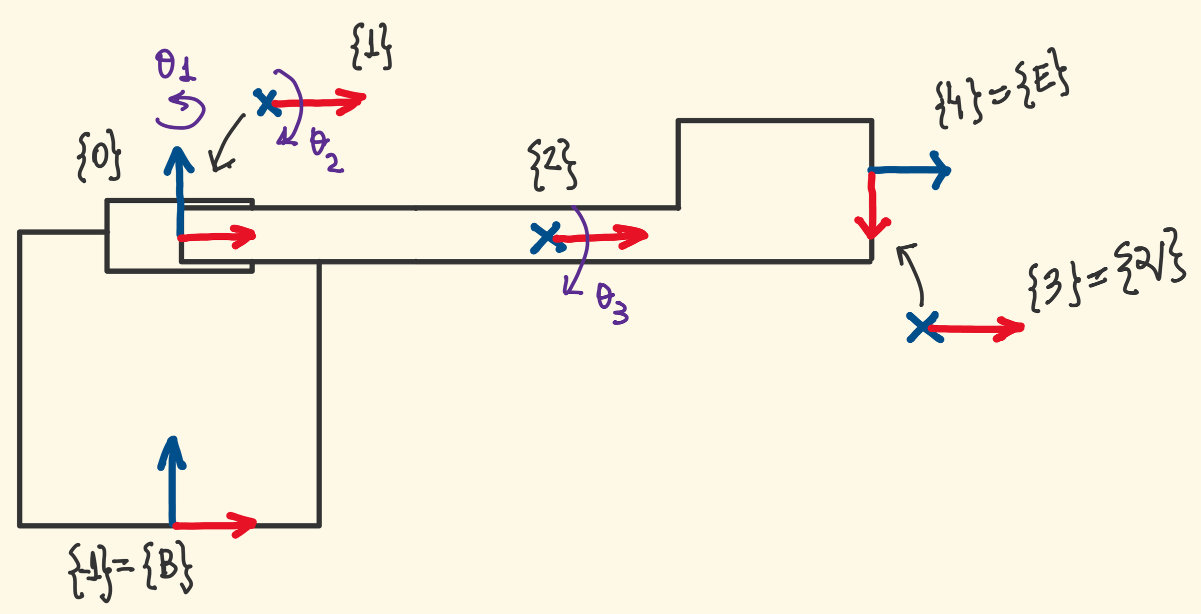

Tabela DH

i	θ	d	a	α
1	θ_1	0	0	$-\pi/2$
2	θ_2	0	a_2	0
3	θ_3	0	a_3	0
4	$\pi/2$	0	$-a_4$	$\pi/2$

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$$ {}^{E}_{B}\xi = {}^{0}_{B}\xi(0,d_0,0,0)\cdot{}^{1}_{0}\xi(\theta_1,0,0,-\pi/2)\cdot{}^{2}_{1}\xi(\theta_2,0,a_2,0)\cdot{}^{3}_{2}\xi(\theta_3,0,a_3,0)\cdot{}^{4}_{3}\xi(\pi/2,0,-a_4,\pi/2) $$

onde $$ \xi^{i}_{i-1}(\theta_i,d_i,a_i,\alpha_i) = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Calculando as transformações:

$$ {}^{0}_{B}\xi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{1}_{0}\xi = \begin{bmatrix} c\theta_1 & 0 & -s\theta_1 & 0 \\ s\theta_1 & 0 & -c\theta_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{2}_{1}\xi = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & a_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & a_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{3}_{2}\xi = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_3 c\theta_3 \\ s\theta_3 & c\theta_3 & 0 & a_3 s\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{4}_{3}\xi = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -a_4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Usar programação simbólica

Multiplicando tudo:

$$ {}^{E}_{B}\xi = \begin{bmatrix} & R(\theta) & & f_x(\theta) \\ & & & f_y(\theta) \\ & & & f_z(\theta) \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad J(\theta) = \begin{bmatrix} \frac{\partial f_x(\theta)}{\partial\theta_1} & \frac{\partial f_x(\theta)}{\partial\theta_2} & \frac{\partial f_x(\theta)}{\partial\theta_3} \\ \frac{\partial f_y(\theta)}{\partial\theta_1} & \frac{\partial f_y(\theta)}{\partial\theta_2} & \frac{\partial f_y(\theta)}{\partial\theta_3} \\ \frac{\partial f_z(\theta)}{\partial\theta_1} & \frac{\partial f_z(\theta)}{\partial\theta_2} & \frac{\partial f_z(\theta)}{\partial\theta_3} \end{bmatrix} $$

* Modelagem Dinâmica

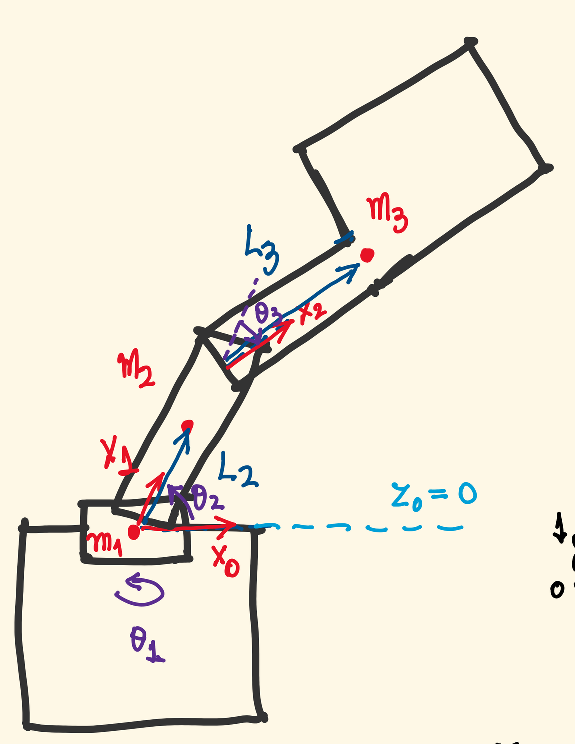

Tabela DH – Centros de Massa

i	θ	d	a	α
1	θ_1	0	0	$-\pi/2$
2	θ_2	0	L_2	0
2	θ_2	0	$2L_2$	0
3	θ_3	0	L_3	0

Calculando as posições dos C.Ms:

$$ {}^{1}_{0}\xi = \begin{bmatrix} c\theta_1 & 0 & -s\theta_1 & 0 \\ s\theta_1 & 0 & -c\theta_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow P_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} $$

$$ {}^{2}_{0}\xi = {}^{1}_{0}\xi\,{}^{2}_{1}\xi = \begin{bmatrix} c\theta_1 & 0 & -s\theta_1 & 0 \\ s\theta_1 & 0 & -c\theta_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & L_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & L_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c\theta_{12} & -c\theta_1 s\theta_2 & -s\theta_1 & L_2 c\theta_{12} \\ s\theta_1 c\theta_2 & -s\theta_{12} & -c\theta_1 & L_2 s\theta_1 c\theta_2 \\ -s\theta_2 & -c\theta_2 & 0 & -L_2 s\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ \rightarrow P_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} L_2 c\theta_1 c\theta_2 \\ L_2 s\theta_1 c\theta_2 \\ -L_2 s\theta_2 \end{bmatrix} $$

$$ {}^{3}_{0}\xi = {}^{2}_{0}\xi\,{}^{3}_{2}\xi = \begin{bmatrix} c\theta_{12} & -c\theta_1 s\theta_2 & -s\theta_1 & 2L_2 c\theta_{12} \\ s\theta_1 c\theta_2 & -s\theta_{12} & -c\theta_1 & 2L_2 s\theta_1 c\theta_2 \\ -s\theta_2 & -c\theta_2 & 0 & -2L_2 s\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & L_3 c\theta_3 \\ s\theta_3 & c\theta_3 & 0 & L_3 s\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{3}_{0}\xi = \begin{bmatrix} & R & & L_3 c\theta_{123} - L_3 c\theta_1 s\theta_{23} + 2L_2 c\theta_{12} \\ & & & L_3 s\theta_1 c\theta_{23} - L_3 s\theta_{123} + 2L_2 s\theta_1 c\theta_2 \\ & & & -L_3 s\theta_2 c\theta_3 - L_3 c\theta_2 s\theta_3 - 2L_2 s\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ \rightarrow P_3 = \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \begin{bmatrix} L_3 c\theta_1 (c\theta_{23} - s\theta_{23}) + 2L_2 c\theta_1 c\theta_2 \\ L_3 s\theta_1 (c\theta_{23} - s\theta_{23}) + 2L_2 s\theta_1 c\theta_2 \\ -L_3 (s\theta_2 c\theta_3 + c\theta_2 s\theta_3) - 2L_2 s\theta_2 \end{bmatrix} $$

$$ \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \begin{bmatrix} L_3 c\theta_1 c(\theta_2+\theta_3) + 2L_2 c\theta_1 c\theta_2 \\ L_3 s\theta_1 c(\theta_2+\theta_3) + 2L_2 s\theta_1 c\theta_2 \\ -L_3 s(\theta_2+\theta_3) - 2L_2 s\theta_2 \end{bmatrix} $$

* Calculando as equações do movimento dinâmico:

$$ \tau_i - b_i\dot\theta_i = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot\theta_i}\right) + \frac{\partial \mathcal{L}}{\partial \theta_i} $$

* Calculo lagrangiano

$$ \mathcal{L} = T - V $$

A energia potencial é dada por (tomando referência em $z_0 = 0$):

$$ V(\theta) = \sum_{i=1}^{3} m_i g z_i = m_2 g z_2 + m_3 g z_3 $$

$$ V(\theta) = -m_2 g L_2 s\theta_2 - m_3 g \left[L_3 s(\theta_2+\theta_3) + 2L_2 s\theta_2 \right] $$

A energia cinética é dada por:

$$ T = \frac{1}{2}\sum_{i=1}^{3} m_i v_i^2 \text{, onde } v_i^2 = \dot x_i^2 + \dot y_i^2 + \dot z_i^2 . $$

Calculando as componentes das velocidades dos CMs:

$$ \begin{bmatrix} \dot x_2 \\ \dot y_2 \\ \dot z_2 \end{bmatrix} = \begin{bmatrix} -L_2 (s\theta_1 c\theta_2 \dot\theta_1 + c\theta_1 s\theta_2 \dot\theta_2) \\ L_2 (c\theta_1 c\theta_2 \dot\theta_1 - s\theta_1 s\theta_2 \dot\theta_2) \\ -L_2 c\theta_2 \dot\theta_2 \end{bmatrix} $$

$$ \begin{bmatrix} \dot x_3 \\ \dot y_3 \\ \dot z_3 \end{bmatrix} = 2\begin{bmatrix} \dot x_2 \\ \dot y_2 \\ \dot z_2 \end{bmatrix} + \begin{bmatrix} -L_3 (s\theta_1 c(\theta_2+\theta_3)\dot\theta_1 + c\theta_1 s(\theta_2+\theta_3)(\dot\theta_2+\dot\theta_3)) \\ L_3 (c\theta_1 c(\theta_2+\theta_3)\dot\theta_1 - s\theta_1 s(\theta_2+\theta_3)(\dot\theta_2+\dot\theta_3)) \\ -L_3 c(\theta_2+\theta_3)(\dot\theta_2+\dot\theta_3) \end{bmatrix} $$

O lagrangiano do sistema fica dado por:

$$ \boxed{\mathcal{L} = \frac{1}{2}m_2(\dot x_2^2 + \dot y_2^2 + \dot z_2^2) + \frac{1}{2}m_3(\dot x_3^2 + \dot y_3^2 + \dot z_3^2) + m_2 g L_2 s\theta_2 + m_3 g\left[L_3 s(\theta_2+\theta_3) + 2L_2 s\theta_2\right]} $$