PHYS 517 Project: Gravitational Collapse of Dust Star

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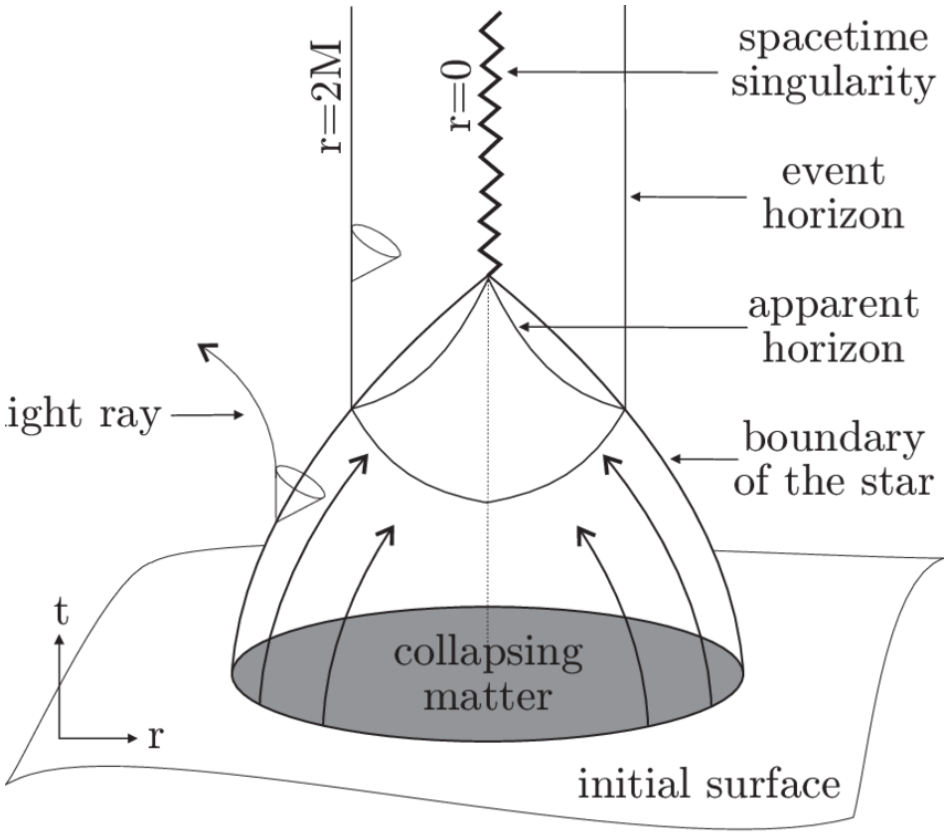
Schwarzschild Black Hole

- Prediction of Einstein's field equations.
- Spherically symmetric, no angular momentum or charge.
- Metric:

$$\mathrm{d}s^2 = -(1 - \frac{2M}{r})\,\mathrm{d}t^2 + (1 - \frac{2M}{r})^{-1}\,\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega^2\,.$$

- All sphericall symmetric spacetime must look like this in the vacuum exterior.

Gravitational Collapse



Joshi, Pankaj. (2013). Spacetime Singularities. Springer Handbook of Spacetime. 10.1007/978-3-642-41992-8_20.

Simulating Gravitational Collapse of a Dust Star

- Metric evolution: Use ADM formulation. (Alternate form of Einstein's field equations $G_{ab} = 8\pi T_{ab}$)
- Dust: Matter with no self-interaction. Equation of motion for each dust particle is the geodesic equation.
- Alternate between solving for the metric and solving for dust motion, repeat 40000 time steps with $\Delta t = 0.0004$.

ADM Formulation

- Metric in isotropic coordinates:

$$\mathrm{d}s^2 = -\left(\alpha^2 - A^2\beta^2\right)\mathrm{d}t^2 + 2A^2\beta\,\mathrm{d}r\,\mathrm{d}t + A^2(\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega^2)$$

- Hamiltonian constraint:

$$\frac{1}{r^2}\partial_r(r^2\partial_r A^{1/2}) = -\frac{1}{4}A^{5/2}(8\pi\rho)$$

- Discretization in r: Roughly 10000 points on $[0, 150]$.
- Solve the nonlinear equation using finite difference method.

Particle method

- Store the radial coordinate, radial velocity, angular velocity and mass of each particle.
- Update at each time step using the geodesic equation.
- Density calculation (mean field scheme):

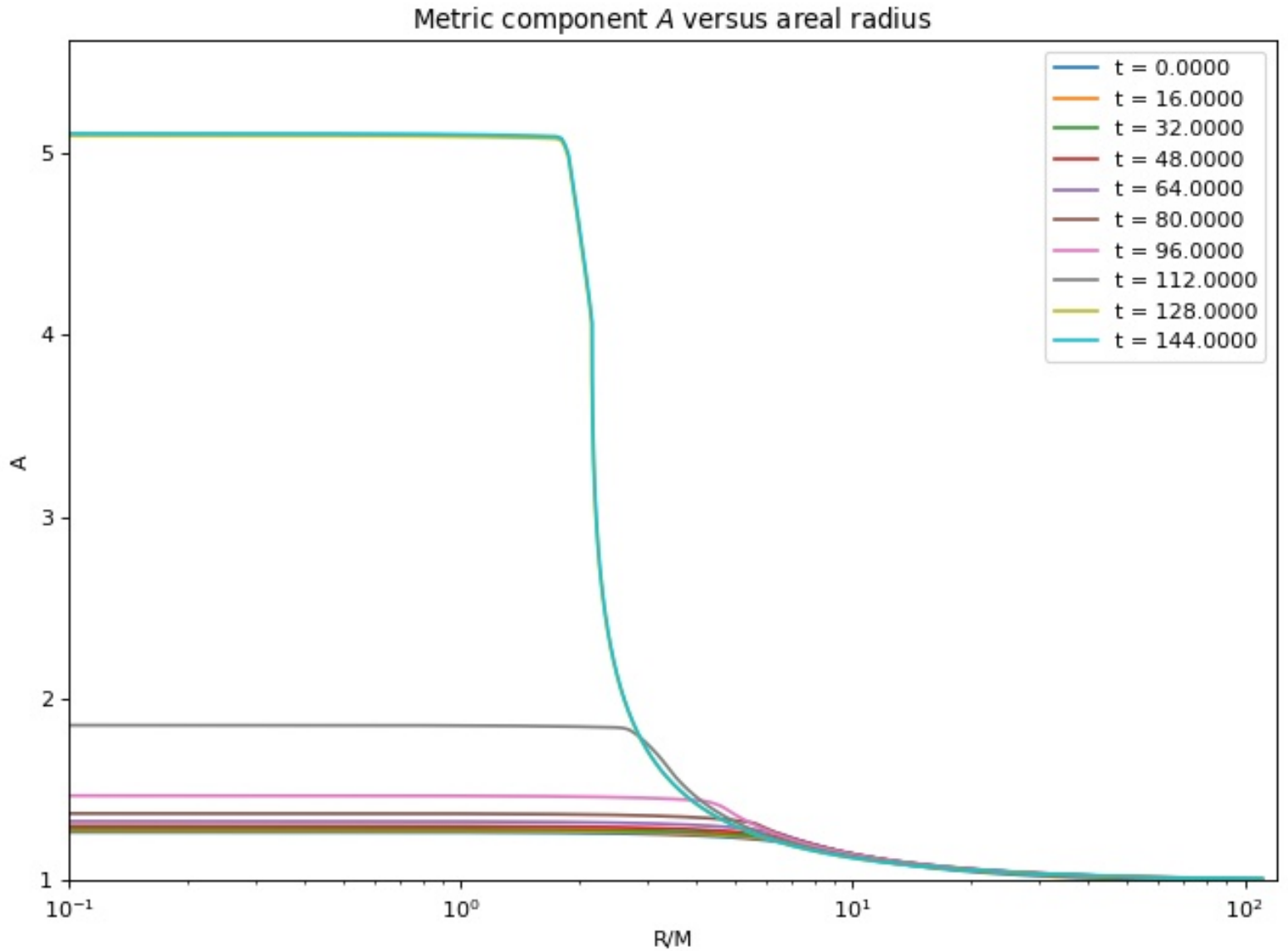
$$\rho = \sum_i m_i n_i W^2$$

$$W \equiv \alpha u^0 = (1 + \frac{u_r^2}{A^2} + \frac{u_\phi^2}{r^2 A^2})^{1/2}$$

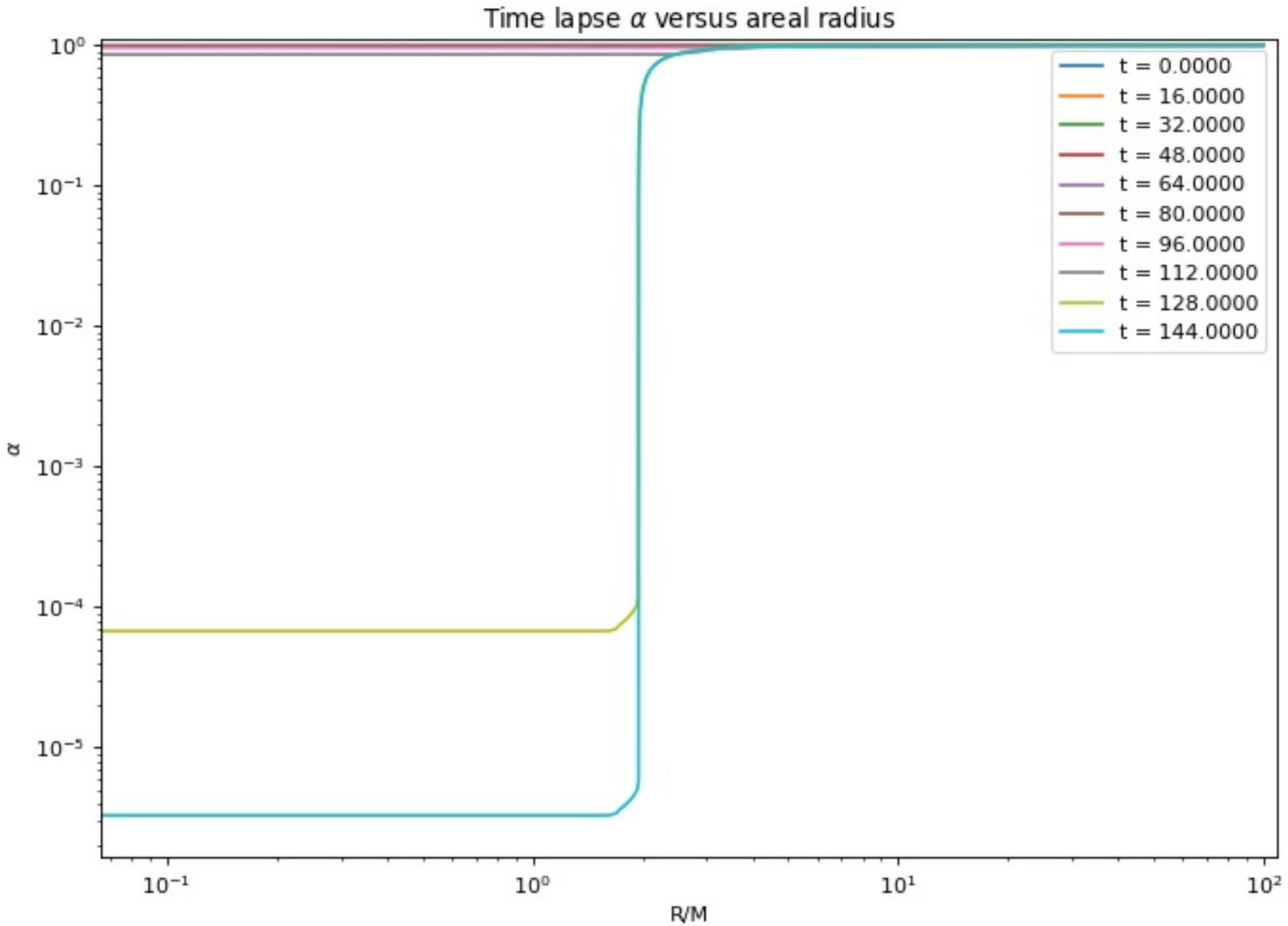
$$n_i = \frac{1}{4\pi W A^3 r^2}\,\delta(r - r_i)$$

- Simulated $N = 200000$ particles with mass $m = 0.00001$.

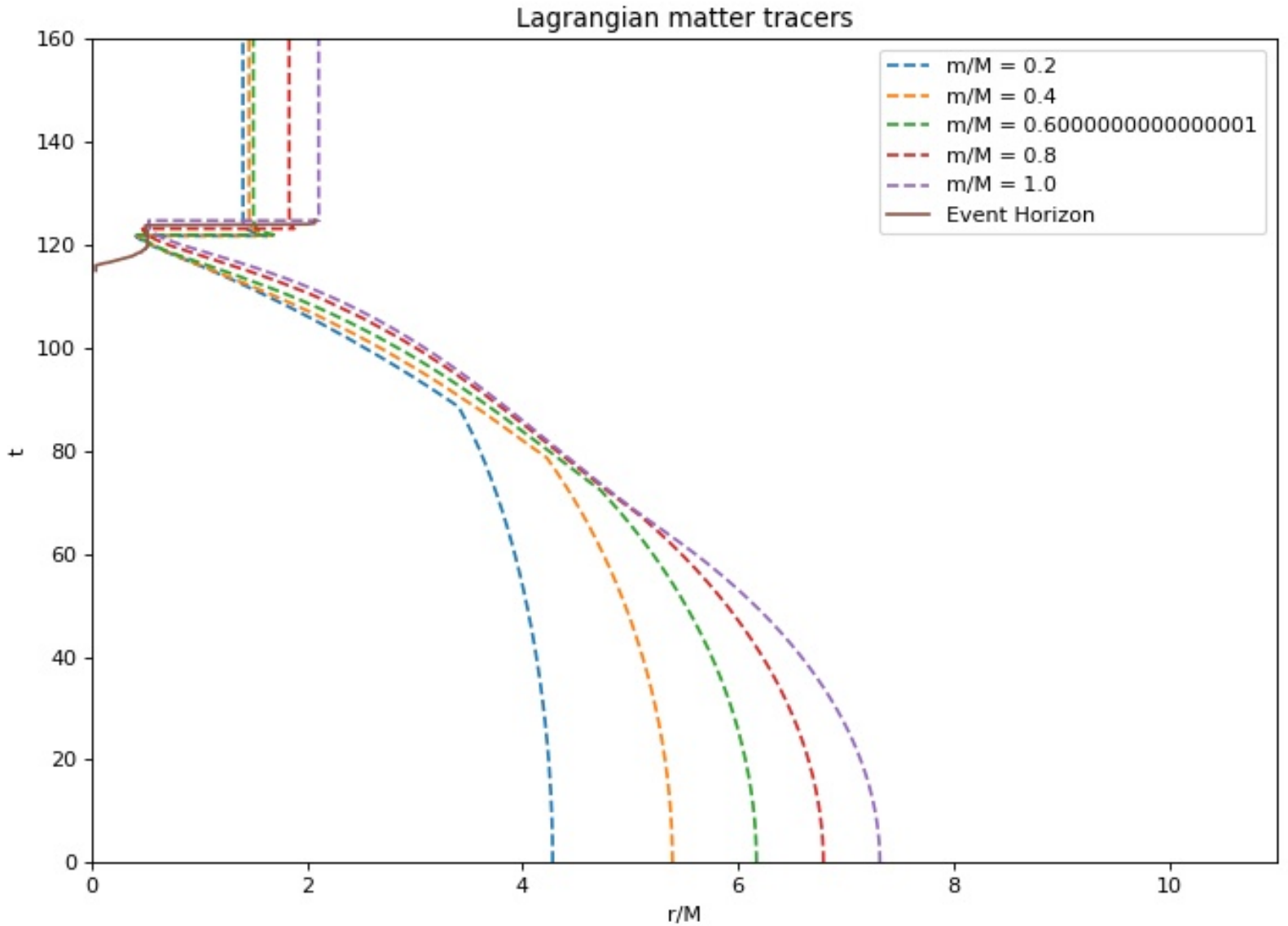
Results: Metric



Results: Time Lapse



Results: Matter Tracer



The End