PEARSON
Education

Electric Machines

Theory, Operation, Applications, Adjustment, and Control

Second Edition

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Magnetics, Electromagnetic Forces, Generated Voltage, and Energy Conversion

1.1 INTRODUCTION

This chapter starts with a brief review of electromagnetism and magnetic circuits, which are normally included in a basic circuits or physics course. This review is followed by a discussion of the development of the mechanical forces that are caused by the interaction of magnetic fields and that form the basis for all motor action. Faraday's law provides the basis from which all magnetically induced voltages are derived. The relationship between applied torque and countertorque is developed and visualized through the application of Lenz's law and the "flux bunching" rule.

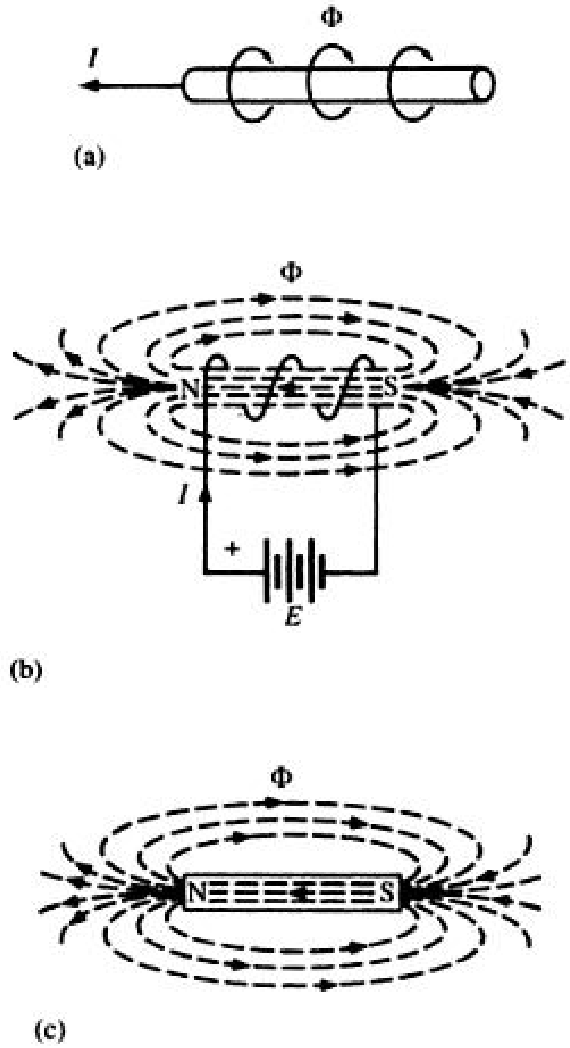
1.2 MAGNETIC FIELD

A magnetic field is a condition resulting from electric charges in motion. The magnetic field of a permanent magnet is attributed to the uncompensated spinning of electrons about their own axis within the atomic structure of the material and to the parallel alignment of these electrons with similar uncompensated electron spins in the adjacent atoms. Groups of adjacent atoms with parallel magnetic spins are called domains. The magnetic field surrounding a current-carrying conductor is caused by the movement of electric charges in the form of an electric current.

For convenience in visualization and analysis, magnetic fields are represented on diagrams by closed loops. These loops, called magnetic flux lines, have been assigned a specific direction that is related to the polarity of a magnet, or the direction of current in a coil or a conductor.

The direction of the magnetic field around a current can be determined by the *right-hand rule:* Grasp the conductor with the right hand, with the thumb pointing in the direction of conventional current, and the fingers will curl in the direction of the magnetic field. This can be visualized in Figure 1.1(a).

FIGURE 1.1
Direction of magnetic flux: (a) around a current-carrying conductor; (b) in a coil; (c) about a magnet.



In a similar manner, to determine the direction of the magnetic field generated by a current through a coil of wire, grasp the coil with the right hand, with the fingers curled in the direction of the current, and the thumb will point in the direction of the magnetic field. This can be visualized in Figure 1.1(b).

The direction of the magnetic field supplied by a magnet is out from the north pole and into the south pole, but is south-to-north within the magnet, as shown in Figure 1.1(c).

1.3 MAGNETIC CIRCUIT DEFINED

Each magnetic circuit shown in Figure 1.2 is an arrangement of ferromagnetic materials called a *core* that forms a path to contain and guide the magnetic flux in a specific direction. The core shape shown in Figure 1.2(a) is used in transformers. Figure 1.2(b)

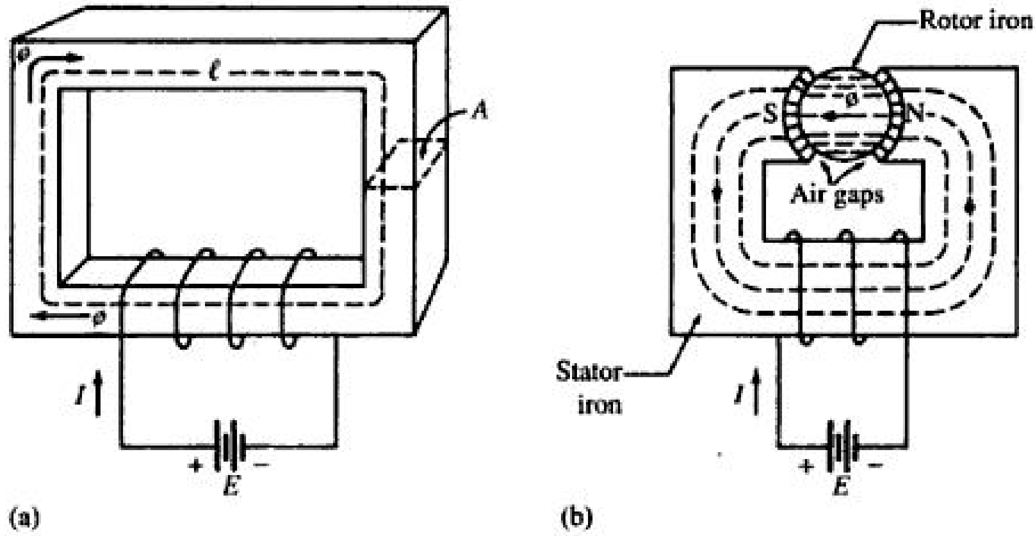


FIGURE 1.2
Magnetic circuit: (a) for a transformer; (b) for a simple two-pole motor.

shows the magnetic circuit of a simple two-pole motor; it includes a stator core, a rotor core, and two air gaps. Note that the flux always takes the shortest path across an air gap.

Magnetomotive Force

The ampere-turns (A-t) of the respective coils in Figure 1.2 represent the driving force, called *magnetomotive force* or *mmf*, that causes a magnetic field to appear in the corresponding magnetic circuits. Expressed in equation form,

$$\mathcal{F} = N \cdot I \qquad (1\text{–}1)$$

where: $\mathcal{F}$ = magnetomotive force (mmf) in ampere-turns (A-t)
N = number of turns in coil
I = current in coil (A)

Magnetic Field Intensity

Magnetic field intensity, also called mmf gradient, is defined as the magnetomotive force per unit length of magnetic circuit, and it may vary from point to point throughout the magnetic circuit. The average magnitude of the field intensity in a *homogeneous* section of a magnetic circuit is numerically equal to the mmf across the section divided by the effective length of the magnetic section. That is,

$$H = \frac{\mathcal{F}}{\ell} = \frac{N \cdot I}{\ell} \qquad (1\text{–}2)$$

where: H = magnetic field intensity (A-t/m)
ℓ = mean length of the magnetic circuit, or section (m)
$\mathcal{F}$ = mmf (A-t)

Note that in a homogeneous magnetic circuit of uniform cross section, the field intensity is the same at all points in the magnetic circuit. In composite magnetic circuits, consisting of sections of different materials and/or different cross-sectional areas, however, the magnetic field intensity differs from section to section.

Magnetic field intensity has many useful applications in magnetic circuit calculations. One specific application is calculating the *magnetic-potential difference,* also called *magnetic drop* or *mmf drop,* across a section of a magnetic circuit. The magnetic drop in ampere-turns per meter of magnetic core length in a magnetic circuit is analogous to the voltage drop in volts per meter of conductor length in an electric circuit.

Flux Density

The flux density is a measure of the concentration of lines of flux in a particular section of a magnetic circuit. Expressed mathematically, and referring to the homogeneous core in Figure 1.2(a),

$$B = \frac{\Phi}{A} \tag{1–3}$$

where: Φ = flux, webers (Wb)
A = cross-sectional area (m^2)
B = flux density (Wb/m^2), or teslas (T)

1.4 RELUCTANCE AND THE MAGNETIC CIRCUIT EQUATION

A very useful equation that expresses the relationship between magnetic flux, mmf, and the reluctance of the magnetic circuit is

$$\Phi = \frac{\mathcal{F}}{\mathcal{R}} = \frac{N \cdot I}{\mathcal{R}} \tag{1–4}$$

where: Φ = magnetic flux (Wb)
$\mathcal{F}$ = magnetomotive force (A-t)
$\mathcal{R}$ = reluctance of magnetic circuit (A-t/Wb)

Reluctance $\mathcal{R}$ is a measure of the opposition the magnetic circuit offers to the flux and is analogous to resistance in an electric circuit. The reluctance of a magnetic circuit, or section of a magnetic circuit, is related to its length, cross-sectional area, and permeability. Solving Eq. (1–4) for $\mathcal{R}$, dividing numerator and denominator by ℓ, and rearranging terms,

$$\mathcal{R} = \frac{N \cdot I}{\Phi} = \frac{N \cdot I/\ell}{\Phi/\ell} = \frac{H}{B \cdot A/\ell} = \frac{\ell}{(B/H) \cdot A}$$

Defining

$$\mu = \frac{B}{H} \tag{1–5}$$

$$\mathfrak{R} = \frac{\ell}{\mu A} \tag{1–6}$$

where: B = flux density (Wb/m^2), or teslas (T)
H = magnetic field intensity (A-t/m)
ℓ = mean length of magnetic circuit (m)
A = cross-sectional area (m^2)
μ = permeability of material (Wb/A-t · m)

Equation (1–6) applies to a homogeneous section of a magnetic circuit of uniform cross section.

Magnetic Permeability

The ratio $\mu = B/H$ is called magnetic permeability and has different values for different degrees of magnetization of a specific magnetic core material.

1.5 RELATIVE PERMEABILITY AND MAGNETIZATION CURVES

Relative permeability is the ratio of the permeability of a material to the permeability of free space; it is, in effect, a figure of merit that is very useful for comparing the magnetizability of different magnetic materials whose relative permeabilities are known. Expressed in equation form,

$$\mu_r = \frac{\mu}{\mu_0} \tag{1–7}$$

where: μ_0 = permeability of free space = $4\pi 10^{-7}$ (Wb/A-t · m)
μ_r = relative permeability, a dimensionless constant
μ = permeability of material (Wb/A-t · m)

Representative graphs of Eq. (1–5) for some commonly used ferromagnetic materials are shown in Figure 1.3. The graphs, called *B–H curves, magnetization curves,* or *saturation curves,* are very useful in design, and in the analysis of machine and transformer behavior.

The four principal sections of a typical magnetization curve are illustrated in Figure 1.4. The curve is concave up for "low" values of magnetic field intensity, exhibits a somewhat (but not always) linear characteristic for "medium" field intensities, and then is concave down for "high" field intensities, eventually flattening to an almost horizontal line for "very high" intensities. The part of the curve that is concave down is known as the *knee* of the curve, and the "flattened" section is the *saturation region.*

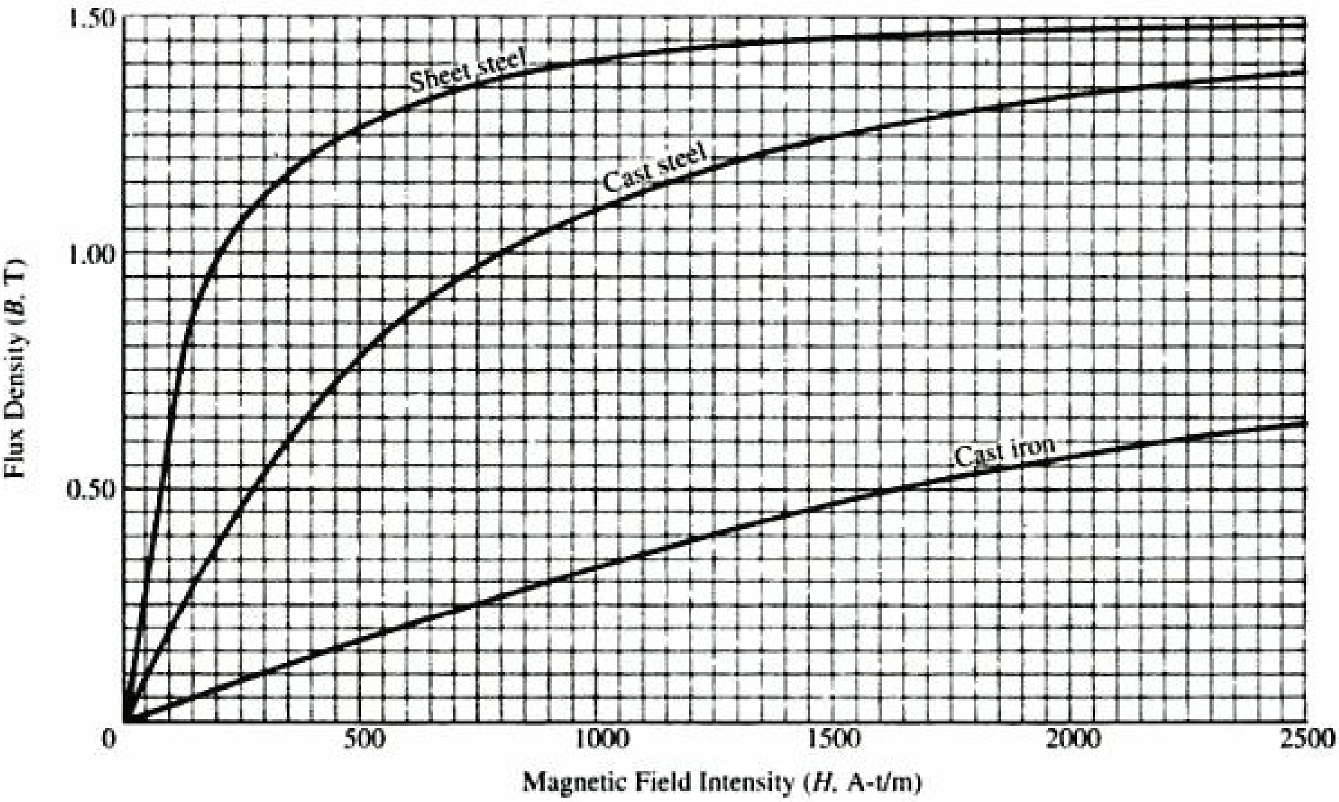


FIGURE 1.3
Representative B–H curves for some commonly used ferromagnetic materials.

Magnetic saturation is complete when all of the magnetic domains of the material are oriented in the direction of the applied magnetomotive force. Saturation begins at the start of the knee region and is essentially complete when the curve starts to flatten.

Depending on the specific application, the magnetic core of an apparatus may be operated in the linear region, and/or the saturation region. For example, transformers and AC machines are operated in the linear region and lower end of the knee; self-excited DC generators and DC motors are operated in the upper end of the knee region, extending into the saturation region; separately excited DC generators are operated in the linear and lower end of the knee region.

Magnetization curves supplied by manufacturers for specific electrical steel sheets or casting are usually plotted on semilog paper, and often include a curve of relative permeability vs. field intensity, as shown in Figure 1.5.[1]

[1] Figure 1.5, as furnished by the manufacturer, has the magnetic field intensity expressed in oersteds, a cgs unit. To convert to A-t/m multiply by 79.577. See Appendix K for other conversion factors. Although not shown, the minimum value of $\mu_r = 1.0$, and it occurs when saturation is complete, resulting in $\mu = \mu_0$.

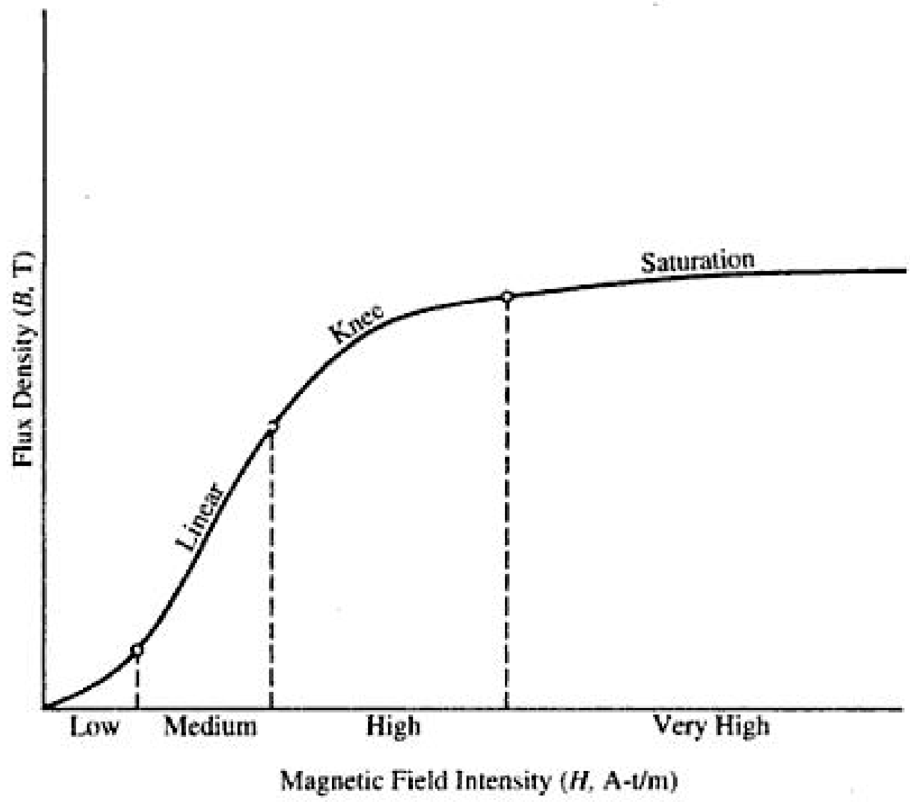


FIGURE 1.4
Exaggerated magnetization curve illustrating the four principal sections.

The relationship between the relative permeability and the reluctance of a magnetic core is obtained by solving Eq. (1–7) for μ, and then substituting into Eq. (1–6). The result is

$$\mathcal{R} = \frac{\ell}{\mu A} = \frac{\ell}{\mu_r \mu_0 A} \tag{1–8}$$

Equation (1–8) indicates that the reluctance of a magnetic circuit is affected by the relative permeability of the material, which, as shown in Figure 1.5, is dependent on the magnetization, and hence is not constant.

EXAMPLE 1.1 (a) Determine the voltage that must be applied to the magnetizing coil in Figure 1.6(a) in order to produce a flux density of 0.200 T in the air gap. *Flux fringing,* which always occurs along the sides of an air gap, as shown in Figure 1.6(b), will be assumed negligible. Assume the magnetization curve for the core material (which is homogeneous) is that given in Figure 1.5. The coil has 80 turns and a resistance of 0.05 Ω. The cross section of the core material is 0.0400 m^2.

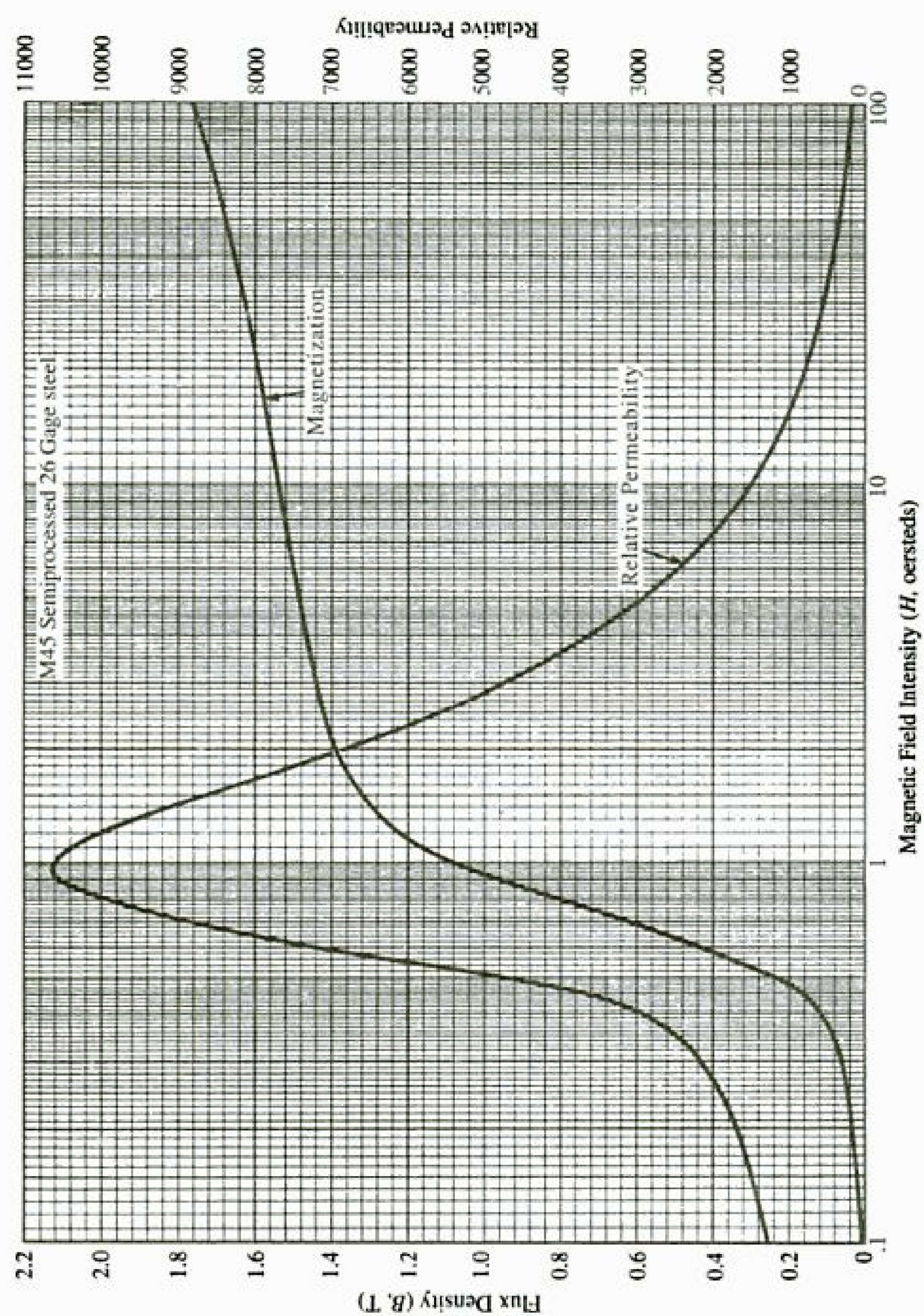


FIGURE 1.5

Magnetization and permeability curves for electrical sheet steel used in magnetic applications. (Courtesy USX Corp.)

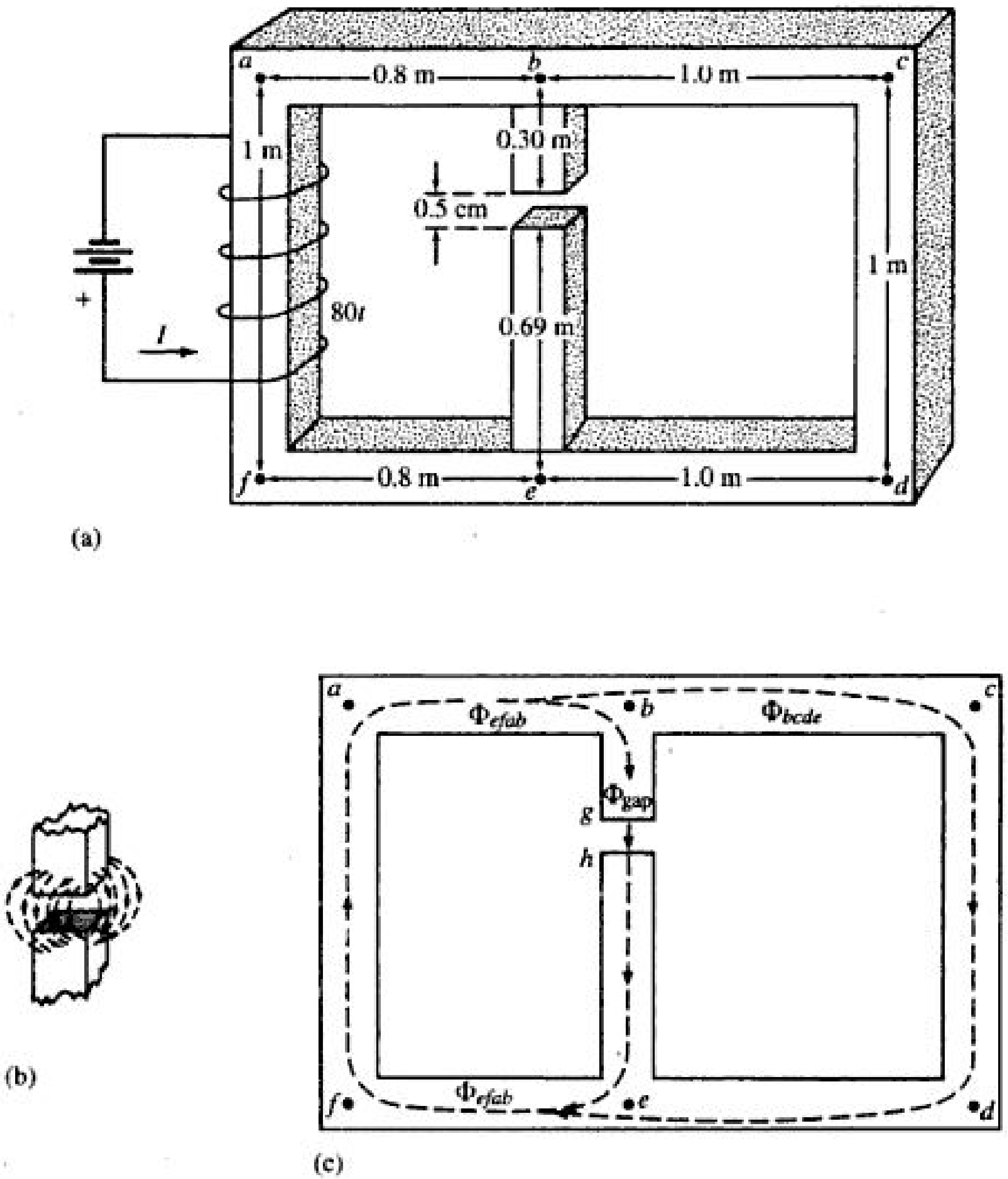


FIGURE 1.6
Magnetic circuit for Example 1.1: (a) physical layout and dimensions; (b) flux fringing; (c) flux distribution.

(b) Using Eqs. (1–5) and (1–7), determine the relative permeability of each of the three legs of the core, and compare the calculated values with the corresponding values obtained from the permeability curve in Figure 1.5.

Solution

(a) The physical layout and dimensions of the magnetic circuit shown in Figure 1.6(a) are used in conjunction with the B–H curve to determine the magnetic field intensity in the component parts. The flux distribution is shown in Figure 1.6(c). The procedure for solving the problem is as follows:

Step 1: Determine Φ_{gap}, and $\mathcal{F}_{bghe}$.

Step 2: Determine: H_{bcde}, B_{bcde}, and Φ_{bcde}.

Step 3: Determine Φ_{efab}, B_{efab}, H_{efab}, and $\mathcal{F}_{efab}$.

Step 4: Determine $\mathcal{F}_T$, and, knowing the number of turns in the coil, determine the required current.

Step 5: Using Ohm's law, determine the required voltage.

The flux in the center section is

$$\Phi_{gap} = B_{gap}A_{gap} = 0.2 \times 0.04 = 0.008 \text{ Wb}$$

The flux density throughout the two cores of the center leg is 0.2 T. The field intensity required to provide a flux density of 0.2 T in each of the two cores in the center leg is obtained from the magnetization curve in Figure 1.5. The corresponding field intensity, obtained from the curve is

$$H_{0.30} = H_{0.69} \approx 0.47 \times 79.577 = 37.4 \text{ A-t/m}$$

The resultant magnetic-potential difference across each core of the center leg is determined from Eq. (1–2):

$$\mathcal{F}_{0.30} = H \cdot \ell = 37.4 \times 0.30 = 11.22 \text{ A-t}$$

$$\mathcal{F}_{0.69} = H \cdot \ell = 37.4 \times 0.69 = 25.81 \text{ A-t}$$

The magnetic-potential difference required across the air gap to obtain a flux density of 0.20 T is obtained from Eq. (1–5), where $\mu_{gap} = \mu_0$.

$$\mu_{gap} = \frac{B_{gap}}{H_{gap}} \quad \Rightarrow \quad 4\pi 10^{-7} = \frac{0.2}{H_{gap}}$$

$$H_{gap} = 159{,}155 \text{ A-t/m}$$

The resultant magnetic-potential difference across the air gap is

$$\mathcal{F}_{gap} = H_{gap}\ell_{gap} = 159{,}155(0.005) = 795.77 \text{ A-t}$$

Thus, the total magnetic-potential difference across the center leg is

$$\mathcal{F}_{bghe} = \mathcal{F}_{0.30} + \mathcal{F}_{0.69} + \mathcal{F}_{gap} = 11.22 + 25.81 + 795.77 = 833 \text{ A-t}$$

Note that the magnetic-potential drop across the 0.005-m air gap is 795.77 A-t, whereas the combined magnetic drop across the 0.30-m and 0.69-m cores total only $11.22 + 25.81 = 37.03$ A-t. *The greatest magnetic-potential drop occurs across an* *air gap.* Thus, to reduce the amount of ampere-turns required to obtain a desired flux density, air gaps in electrical machinery are kept small.

Since $\mathcal{F}_{bghe}$ is also the magnetic-potential difference across section *bcde*, the magnetic field intensity in that region is

$$H_{bcde} = \frac{\mathcal{F}_{bcde}}{\ell_{bcde}} = \frac{833}{1 + 1 + 1} = 277.67 \text{ A-t/m}$$

Converting to oersteds,

$$277.67 \div 79.577 = 3.49 \text{ oersteds}$$

The corresponding flux density, as obtained from the magnetization curve in Figure 1.5 is

$$B_{bcde} \approx 1.45 \text{ T}$$

Thus, the flux in section *bcde* is

$$\Phi_{bcde} = BA = 1.45 \times 0.04 = 0.058 \text{ Wb}$$

The total magnetic flux supplied by the coil is

$$\Phi_{efab} = \Phi_{\text{gap}} + \Phi_{bcde} = 0.008 + 0.058 = 0.066 \text{ Wb}$$

$$B_{efab} = \frac{\Phi}{A} = \frac{0.066}{0.04} = 1.65 \text{ T}$$

The field intensity required to provide a flux density of 1.65 T in the left leg, as obtained from the magnetization curve in Figure 1.5, is $\approx$ 37 oersteds. Thus,

$$H_{efab} = 37 \times 79.577 = 2944.35 \text{ A-t/m}$$

The mmf drop in section *efab* is

$$\mathcal{F}_{efab} = H \cdot \ell = 2944.35(1 + 0.8 + 0.8) = 7655.31 \text{ A-t}$$

The total mmf that must be supplied by the magnetizing coil is

$$\begin{aligned}
\mathcal{F}_T &= \mathcal{F}_{bghe} + \mathcal{F}_{efab} = 7655.31 + 833 = 8488.31 \text{ A-t} \\
\mathcal{F}_{\text{coil}} &= NI \quad \Rightarrow \quad 8488.31 = 80 \times I \\
I &= 106.1 \text{ A} \\
V &= IR = 106.1 \times 0.05 = \underline{5.30 \text{ V}}
\end{aligned}$$

(b) Combining Eqs. (1–5) and (1–7),

$$\mu_r = \frac{\mu}{\mu_0} = \frac{B/H}{4\pi \times 10^{-7}} = \frac{B}{4\pi \times 10^{-7} \cdot H}$$

$$\mu_{\text{left}} = \frac{1.65}{4\pi \times 10^{-7} \times 2944} = 446$$

$$\mu_{\text{center}} = \frac{0.20}{4\pi \times 10^{-7} \times 37.4} = 4256$$

$$\mu_{\text{right}} = \frac{1.45}{4\pi \times 10^{-7} \times 277.67} = 4156.1$$

Note that even though the core is homogeneous throughout, the permeability is not the same in all parts of the core. The left leg, with the greater magnetization, is approaching saturation, and thus has a much lower permeability than the other legs.

The following table compares the relative permeability of the core legs, as obtained from the curve in Figure 1.5, with the calculated values.

Core	H (A-t/m)	B (T)	μ_r (calc)	μ_r (curve)
Left leg	2944	1.65	446	450
Center leg	37.4	0.20	4256	4000
Right leg	277.67	1.45	4156	4100

1.6 ANALOGIES BETWEEN ELECTRIC AND MAGNETIC CIRCUITS

The relationship between mmf, flux, and reluctance in a magnetic circuit is an analog of the relationship between emf, current, and resistance, respectively, in an electric circuit.

$$\Phi = \frac{\mathcal{F}}{\mathcal{R}} \qquad I = \frac{E}{R}$$

where: Φ corresponds to I
$\mathcal{F}$ corresponds to E
$\mathcal{R}$ corresponds to R

Continuing the analogy, the equivalent reluctance of n reluctances in series is

$$\mathcal{R}_{\text{ser}} = \mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \cdots + \mathcal{R}_n \tag{1–9}$$

The equivalent reluctance of n reluctances in parallel is

$$\frac{1}{\mathcal{R}_{\text{par}}} = \frac{1}{\mathcal{R}_1} + \frac{1}{\mathcal{R}_2} + \frac{1}{\mathcal{R}_3} + \cdots + \frac{1}{\mathcal{R}_n}$$

or

$$\mathcal{R}_{\text{par}} = \frac{1}{1/\mathcal{R}_1 + 1/\mathcal{R}_2 + 1/\mathcal{R}_3 + \cdots + 1/\mathcal{R}_n} \tag{1–10}$$

An equivalent magnetic circuit that shows the analogous relationship to an electric circuit is often used to solve magnetic circuit problems that may otherwise be more difficult to visualize. For example, the components of the series–parallel circuit shown in Figure 1.7(a) are represented as lumped reluctances in the equivalent magnetic circuit shown in Figure 1.7(b). Using the methods developed for electric circuits, the total reluctance of the series–parallel magnetic circuit is

$$\mathcal{R}_T = \mathcal{R}_1 + \frac{\mathcal{R}_2 \cdot \mathcal{R}_3}{\mathcal{R}_2 + \mathcal{R}_3}$$

EXAMPLE 1.2

Assume that flux Φ_1 in Figure 1.7(a) is 0.250 Wb, and that the magnetic circuit parameters for this condition are

$$\mathcal{R}_1 = 10{,}500 \text{ A-t/Wb}$$
$$\mathcal{R}_2 = 40{,}000 \text{ A-t/Wb}$$
$$\mathcal{R}_3 = 30{,}000 \text{ A-t/Wb}$$

The magnetizing coil is wound with 140 turns of copper wire. Determine (a) the current in the coil; (b) the magnetic-potential difference across $\mathcal{R}_3$; (c) the flux in $\mathcal{R}_2$.

Solution

(a) Applying basic circuit concepts to the equivalent magnetic circuit in Figure 1.7(b),

$$\mathcal{R}_{\text{par}} = \frac{\mathcal{R}_2 \mathcal{R}_3}{\mathcal{R}_2 + \mathcal{R}_3} = \frac{40{,}000 \times 30{,}000}{40{,}000 + 30{,}000} = 17{,}142.8571 \text{ A-t/Wb}$$

$$\mathcal{R}_{\text{circ}} = \mathcal{R}_1 + \mathcal{R}_{\text{par}} = 10{,}500 + 17{,}142.8571 = 27{,}642.8571 \text{ A-t/Wb}$$

$$\Phi = \frac{NI}{\mathcal{R}} \quad \Rightarrow \quad 0.250 = \frac{140 \times I}{27{,}642.8571}$$

$$I = 49.3622 \quad \Rightarrow \quad \underline{49.36 \text{ A}}$$

(b) The magnetic drop across $\mathcal{R}_1$ is

$$\mathcal{F}_1 = \Phi_T \cdot \mathcal{R}_1 = 0.25 \times 10{,}500 = 2625 \text{ A-t}$$

Referring to Figure 1.7(b),

$$\mathcal{F}_T = \mathcal{F}_1 + \mathcal{F}_{\text{par}} \quad \Rightarrow \quad 49.3622 \times 140 = 2625 + \mathcal{F}_{\text{par}}$$

$$\mathcal{F}_3 = \mathcal{F}_{\text{par}} = 4285.7143 \quad \Rightarrow \quad \underline{4285.71 \text{ A-t}}$$

(c)

$$\Phi_2 = \frac{\mathcal{F}_{\text{par}}}{\mathcal{R}_2} = \frac{4285.7143}{40{,}000} = \underline{0.1071 \text{ Wb}}$$

Or, using the magnetic analog of the current divider rule,

$$\Phi_2 = \Phi_T \times \frac{\mathcal{R}_3}{\mathcal{R}_2 + \mathcal{R}_3} = 0.25 \times \frac{30{,}000}{40{,}000 + 30{,}000} = \underline{0.1071 \text{ Wb}}$$

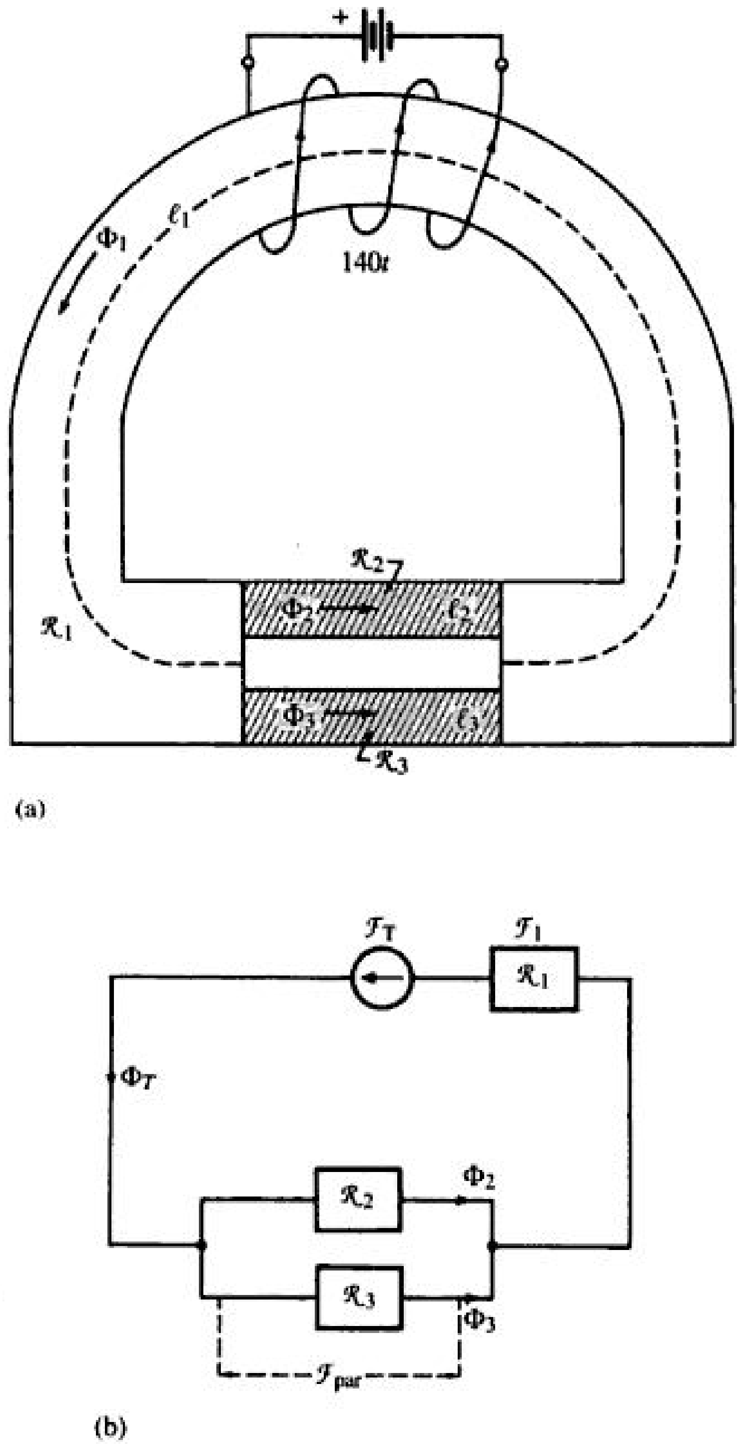


FIGURE 1.7
Magnetic circuit for Example 1.2: (a) physical layout; (b) equivalent magnetic circuit.

1.7 MAGNETIC HYSTERESIS AND HYSTERESIS LOSS

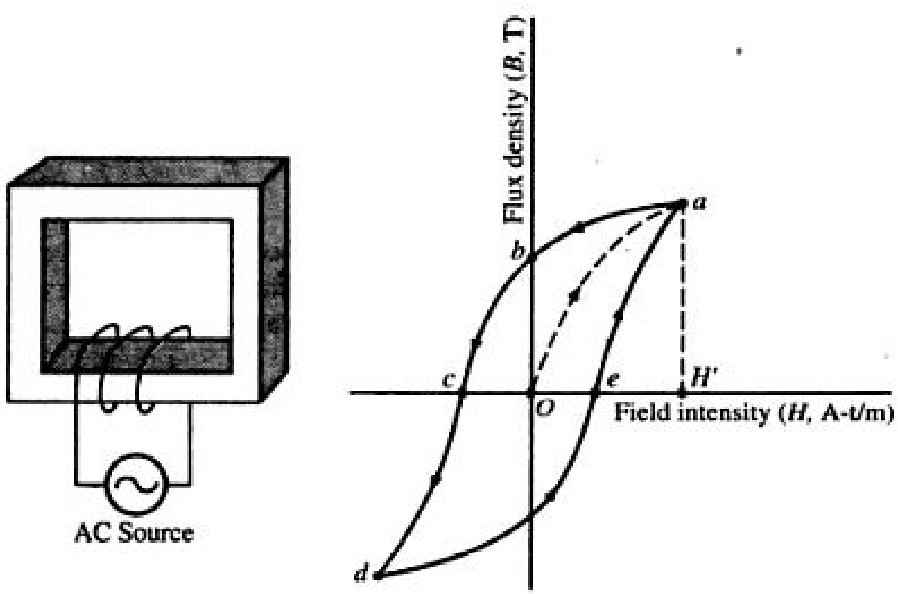
If an alternating magnetomotive force is applied to a magnetic material, as shown in Figure 1.8(a), and the flux density B plotted against the magnetic field intensity H, the resultant curve will indicate a lack of retraceability. This phenomenon, shown in Figure 1.8(b), is called *hysteresis*, and the resultant curve is called an *hysteresis loop*.

Starting with an unmagnetized ferromagnetic core, point O on the curve, $H = 0$ and $B = 0$. Increasing the coil current in the positive direction increases the ampere-turns, and hence the magnetic field intensity. From Eqs. (1–1) and (1–2),

$$H = \frac{NI}{\ell}$$

When the current reaches its maximum value, the flux density and magnetic field intensity have their respective maximum values, and the curve is at point a; this initial trace of the curve, drawn with a broken line, is called the *virgin section* of the curve. As the current decreases, the curve follows a different path, and when the current is reduced to zero, H is reduced to zero, but the flux density in the core lags behind, holding at point b on the curve. The flux density at point b is the *residual magnetism*. This lagging of flux behind the magnetizing force is the *hysteresis effect*.

As the alternating current and associated magnetic field intensity increase in the negative direction, the residual magnetism decreases but remains positive until point c is reached, at which time the flux density in the core is zero. The negative field intensity required to force the residual magnetism to zero is called the *coercive force*, and is



(a) (b)

FIGURE 1.8
(a) Magnetic circuit with an alternating mmf; (b) representative hysteresis loop.

represented by line O–c on the H axis. As the current continues its alternations, the plot of B vs. H follows points c–d–e–a–b–c on the hysteresis loop.

Magnetic hysteresis affects the rate of response of magnetic flux to a magnetizing force. In electrical apparatus such as transformers, in which the desired characteristic necessitates a quick and proportional response of flux to a change in mmf, with little residual magnetism, a high-grade silicon steel is used. Machines such as self-excited generators require steel that retains sufficient residual magnetism to permit the buildup of voltage. Stepper motors and some DC motors require permanent magnets with a very high magnetic retentivity (high hysteresis). Thus, the choice of magnetic materials is dictated by the application.

Magnetic Hysteresis Loss

If an alternating voltage is connected to the magnetizing coil, as shown in Figure 1.8(a), the alternating magnetomotive force causes the magnetic domains to be constantly reoriented along the magnetizing axis. This molecular motion produces heat, and the harder the steel the greater the heat. The power loss due to hysteresis for a given type and volume of core material varies directly with the frequency and the nth power of the maximum value of the flux density wave. Expressed mathematically,

$$P_h = k_h \cdot f \cdot B_{max}^n \qquad (1\text{–}11)$$

where:
P_h = hysteresis loss (W/unit mass of core)
f = frequency of flux wave (Hz)
B_{max} = maximum value of flux density wave (T)
k_h = constant
n = Steinmetz exponent[2]

The constant k_h is dependent on the magnetic characteristics of the material, its density, and the units used. The area enclosed by the hysteresis loop is equal to the hysteresis energy in joules/cycle/cubic-meter of material.

EXAMPLE 1.3 The hysteresis loss in a certain electrical apparatus operating at its rated voltage and rated frequency of 240 V and 25 Hz is 846 W. Determine the hysteresis loss if the apparatus is connected to a 60-Hz source whose voltage is such as to cause the flux density to be 62 percent of its rated value. Assume the Steinmetz exponent is 1.4.

Solution

From Eq. (1–11),

$$\frac{P_{h1}}{P_{h2}} = \frac{[k_h \cdot f \cdot B_{max}^n]_1}{[k_h \cdot f \cdot B_{max}^n]_2} \qquad \Rightarrow \qquad P_{h2} = P_{h1} \times \frac{[k_h \cdot f \cdot B_{max}^n]_2}{[k_h \cdot f \cdot B_{max}^n]_1}$$

$$P_{h2} = 846 \times \frac{60}{25} \times \left[\frac{0.62}{1.0}\right]^{1.4} = \underline{1.04 \text{ kW}}$$

[2] The Steinmetz exponent varies with the core material and has an average value of 1.6 for silicon steel sheets.

1.8 INTERACTION OF MAGNETIC FIELDS (MOTOR ACTION)

When two or more sources of magnetic fields are arranged so that their fluxes, or a component of their fluxes, are parallel within a common region, a mechanical force will be produced that tends to either force the sources of flux together or force them apart. A force of repulsion will occur if the two magnetic sources have components of flux that are parallel and in the same direction; this will be indicated by a net increase in flux called "flux bunching" in the common region. A force of attraction will occur if the respective fluxes have components that are parallel and in opposite directions; this will be indicated by a net subtraction of flux in the common region.

Forces on Adjacent Conductors

The interaction of magnetic fields of adjacent current-carrying conductors produces mechanical forces that tend to bring together or separate the two conductors. If the currents in adjacent conductors are in opposite directions, as shown in Figure 1.9(a), the respective components of flux in the common region will be in the same direction,

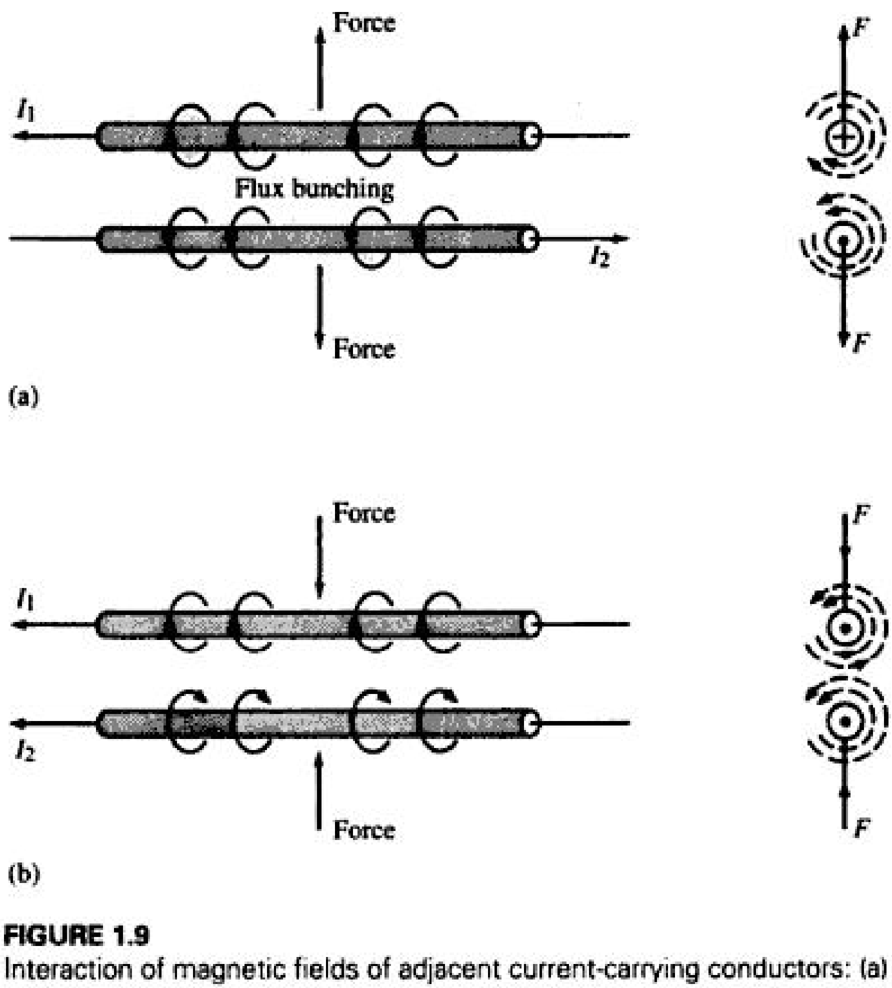


FIGURE 1.9
Interaction of magnetic fields of adjacent current-carrying conductors: (a) currents in opposite direction; (b) currents in same direction.

and as indicated by flux bunching, a separating force will be produced on the conductors. If the currents in adjacent conductors are in the same direction, as shown in Figure 1.9(b), the respective components of flux in the common region will be in opposite directions, and the net reduction in flux indicates a force of attraction.

Under severe short-circuit conditions, the forces between adjacent conductors can be high enough to physically crush the insulation of transformers, motors, and generators, bend bus bars, tear switchboards apart, and cause switches and circuit breakers to come apart with explosive violence. Thus, in those applications where the available short-circuit current is of a magnitude that would cause destruction of apparatus if a fault occurred, special current-limiting devices, as well as mechanical bracing and conductor support must be installed **[1], [2].**

1.9 ELEMENTARY TWO-POLE MOTOR

Figure 1.10 shows a rotor core, containing two insulated conductors in rotor slots, and the rotor centered between the poles of a stationary magnet (called the stator). The + mark on the end of conductor *A* is the tail end of an arrow that represents the direction of current in conductor *A*. The dot in the center of conductor *B* is the point of an arrow indicating the direction of current in conductor *B*. The direction of flux around each conductor is determined by the right-hand rule.

The broken lines show the paths of component fluxes, assuming the rotor and stator were energized at different times. The dotted line indicates the direction and path of the resultant flux with both rotor and stator energized at the same time. Note that the net flux on top of conductor *A*, due to the magnet and due to the current in the conductor, is additive (bunching), indicating a downward mechanical force *F*, as shown in Figure 1.10. A similar action occurs at the bottom of conductor *B*, causing an

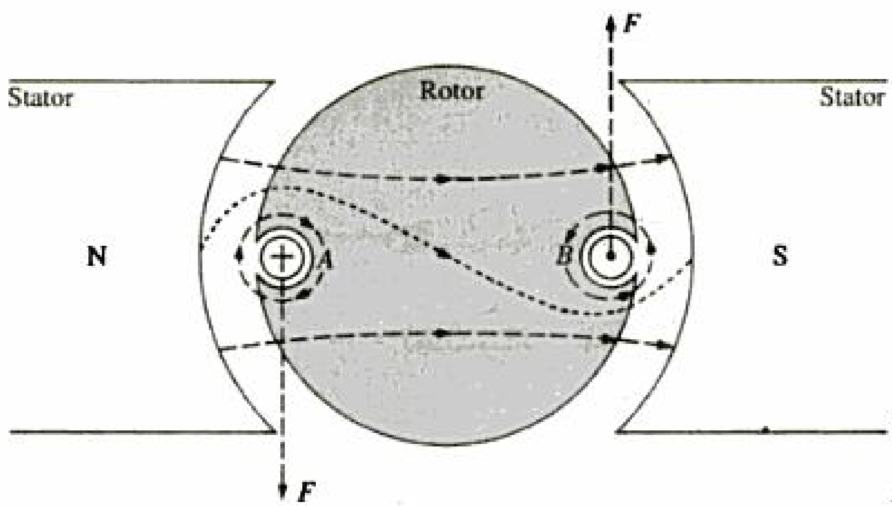


FIGURE 1.10
Motor action.

upward mechanical force. The net result is a counterclockwise (CCW)-turning moment or torque, called *motor action.*

1.10 MAGNITUDE OF THE MECHANICAL FORCE EXERTED ON A CURRENT-CARRYING CONDUCTOR SITUATED IN A MAGNETIC FIELD (BLI RULE)

The magnitude of the mechanical force exerted on a straight conductor that is carrying an electric current and situated within and perpendicular to a magnetic field, as shown in Figure 1.11(a), is expressed by

$$F = B \cdot \ell_{eff} \cdot I \tag{1–12}$$

where: F = mechanical force (N)
B = flux density of stator field (T)
I = current in rotor conductor (A)
ℓ_{eff} = effective length of rotor conductor (m)

The effective length of a conductor is that component of its length that is immersed in and normal to the magnetic field. Thus, if the conductor is not perpendicular to the magnetic field as shown in Figure 1.11(b), the effective length of the conductor is

$$\ell_{eff} = \ell \sin \alpha$$

Angle β is called the skewing angle, which may range from 0 to 30 degrees in electrical machines.

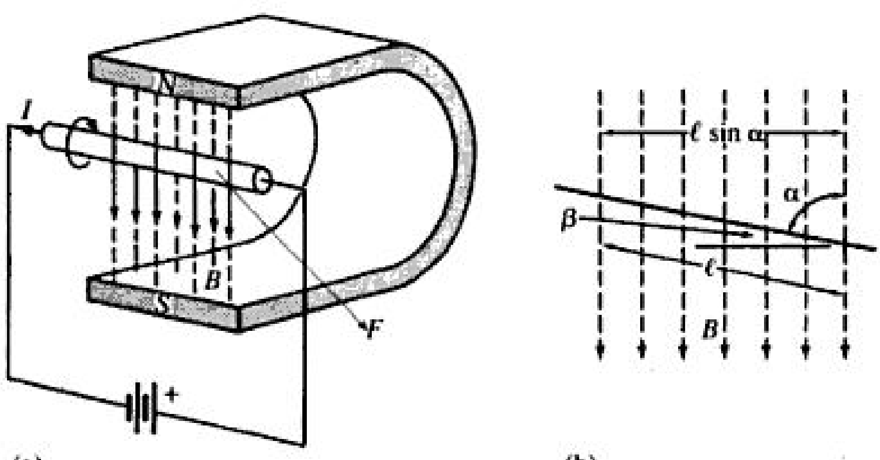


FIGURE 1.11
(a) Conductor carrying current, situated within and perpendicular to the B-field of a permanent magnet; (b) conductor skewed $\beta°$.

The direction of the mechanical force exerted on the conductor in Figure 1.11(a) is determined by flux bunching.

Developed Torque

Figure 1.12(a) shows a rotor coil made up of a single loop, situated in a two-pole stator field of uniform flux density. The effective length of each conductor (coil side) does not include the *end connections*. The end connections, also called *end turns*, are used to connect the conductors in series, but because they are not immersed in the field, they do not develop torque. The distance d between the center of the shaft and the center of a conductor is the moment arm.

The direction of developed torque may be determined from an end view of the conductors and magnet poles, as seen from the battery end in Figure 1.12(b). The direction of flux due to the known direction of current was determined by the right-hand

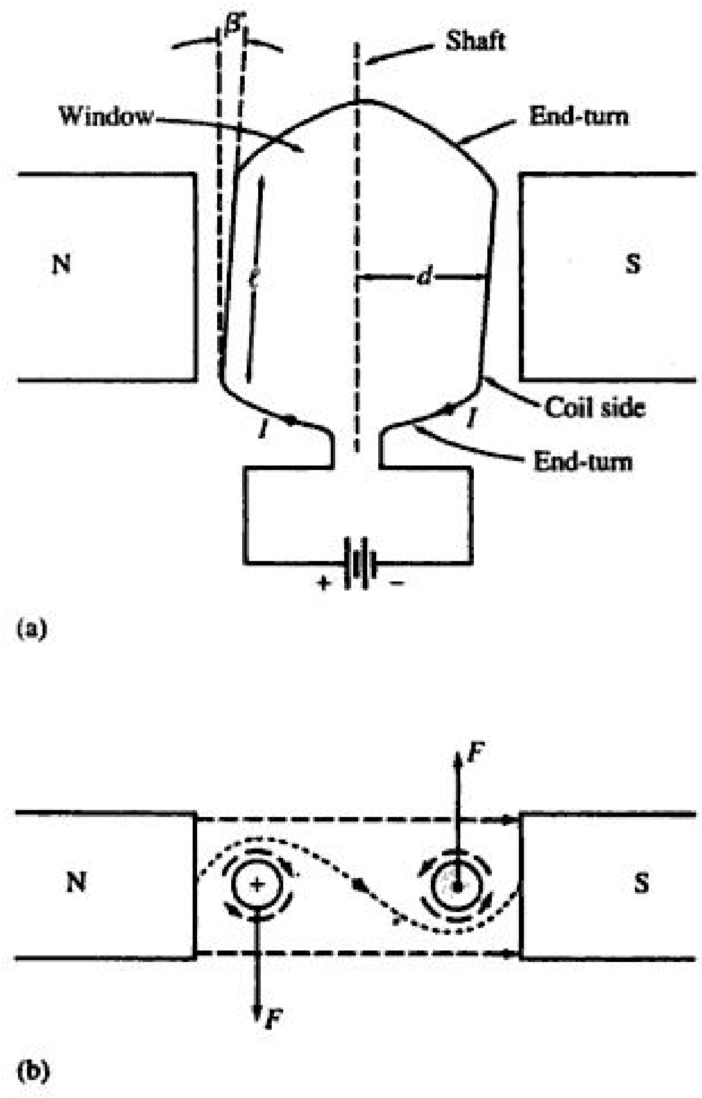


FIGURE 1.12
(a) Single-loop rotor coil carrying an electric current and situated in a two-pole field; (b) end view of coil, showing direction of developed force.

rule, and the direction of the mechanical force on each conductor, due to the interaction of the magnetic fields, was determined by the flux bunching effect. The resultant torque, produced by the two-conductor couple, is CCW and has a magnitude equal to

$$T_D = 2 \cdot F \cdot d \qquad \text{N} \cdot \text{m} \tag{1–13}$$

Substituting Eq. (1–12) into Eq. (1–13),

$$T_D = 2 \cdot B \cdot \ell_{eff} \cdot I \cdot d \qquad \text{N} \cdot \text{m} \tag{1–14}$$

EXAMPLE 1.4 Assume each coil side in Figure 1.12(a) has a length of 0.30 m and a skew angle of 15°. The distance between the center of each conductor and the center of the shaft is 0.60 m. The combined resistance of the coil and its connections to a 36-V battery is 4.0 Ω. If the stator field has a uniform flux density of 0.23 T between the poles, determine the magnitude and direction of the developed torque.

Solution

From Figure 1.11(b),

$$\alpha = 90° - \beta° = 90° - 15° = 75°$$

$$I = \frac{E_{bat}}{R} = \frac{36}{4.0} = 9.0 \text{ A}$$

$$T = 2 \cdot B \cdot I(\ell \sin \alpha) \cdot d = 2 \times 0.23 \times 9(0.3 \sin 75°) \times 0.60 = \underline{0.72 \text{ N} \cdot \text{m}}$$

The direction of the developed torque is counterclockwise, as indicated in Figure 1.12(b).

1.11 ELECTROMAGNETICALLY INDUCED VOLTAGES (GENERATOR ACTION)

The magnitude of the voltage induced in a coil by electromagnetic induction is directly proportional to the number of series-connected turns in the coil, and to the rate of change of flux through its window. This relationship, known as *Faraday's law*, is expressed mathematically as

$$e = N\frac{d\phi}{dt} \tag{1–15}$$

where: e = induced voltage (electromotive force, emf) (V)
N = number of series-connected turns
$d\phi/dt$ = rate of change of flux through window (Wb/s)

The basic Faraday relationship expressed in Eq. (1–15) is often converted by mathematical manipulation to other forms for solution of specific groups of problems.

Electromagnetically induced voltages are generated by relative motion or transformer action. Voltages generated by transformer action are due to flux varying with time through the window of a stationary coil. Voltages generated by relative motion involve a moving coil and a stationary magnet, or a moving magnet and a stationary coil. Voltages caused by relative motion are called *speed voltages* or "flux cutting" voltages.

In accordance with Lenz's law, the voltage, current, and associated flux, generated by transformer action, or relative motion between a conductor and a magnetic field, will always be induced in a direction to oppose the action that caused it.[3] In a transformer, the flux due to current generated in a transformer coil will be in a direction to oppose the change in flux that caused it.

In the case of a conductor driven by an applied force, the flux due to current generated in the conductor will set up a counterforce in opposition to the applied force. In a rotating machine, the flux due to generated current in the conductors will set up a countertorque (motor action) in opposition to the driving torque of the prime mover. In fact, as will be shown in subsequent chapters, all generators may be operated as motors and all motors may be operated as generators.

Speed Voltages and the BLV Rule

A closed loop consisting of two conductors X and Y, and a set of conducting rails, is situated within a uniform magnetic field, as shown in Figure 1.13(a); conductor Y is clamped and conductor X is moving to the right at velocity v meters per second. The window in Figure 1.13(a) is the area enclosed by conductor X, conductor Y, and the conducting rails. As conductor X moves to the right, the window area increases, causing the flux through the window to increase with time, inducing a voltage in the loop.

Expressing the flux in terms of the flux density and the area of the window,

$$\phi = B \cdot A$$

Taking the derivative with respect to time,

$$\frac{d\phi}{dt} = B \cdot \frac{dA}{dt}$$

Substituting into Eq. (1–15)

$$e = N \cdot B \frac{dA}{dt} \tag{1–16}$$

From Figure 1.13(a), the increment increase in window area, as conductor X moves to the right, may be expressed in terms of length ℓ and an increment increase in distance (ds) along the rails. That is,

$$dA = \ell ds \tag{1–17}$$

[3] When applying Lenz's law, it will be assumed that the circuit is complete, resulting in a current and its associated flux.

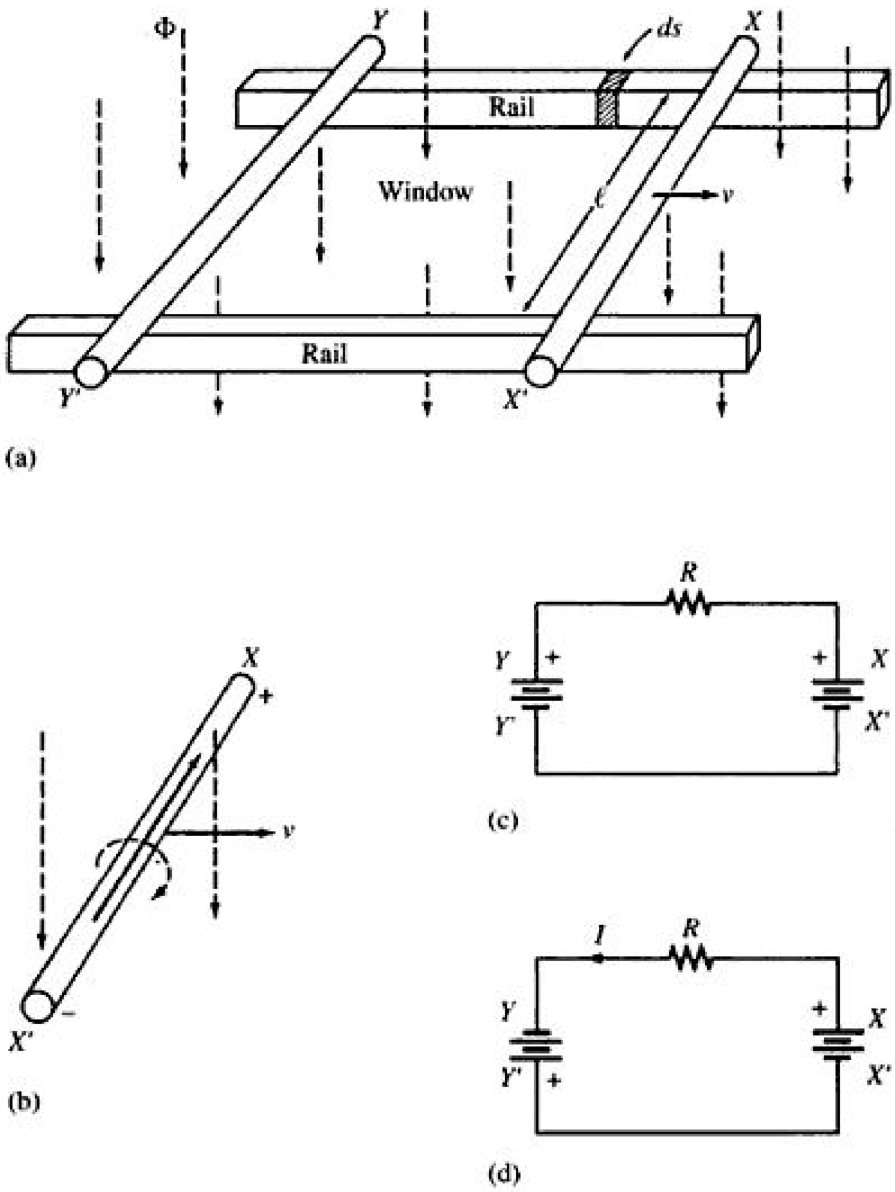


FIGURE 1.13
(a) Closed loop consisting of two conductors and a set of conducting rails; (b) direction of emf and current caused by conductor X moving to the right; (c) equivalent circuit, both conductors moving in the same direction; (d) equivalent circuit, conductors moving in opposite directions.

Substituting into Eq. (1–16), and noting that $N = 1$ for a single loop,

$$e = B \cdot \ell \cdot \frac{ds}{dt} \tag{1–18}$$

Since ds/dt represents the velocity of the conductor, Eq. (1–18) may be rewritten as

$$e = B \cdot \ell \cdot v \tag{1–19}$$

where: e = induced voltage (V)
B = flux density of field (T)
ℓ = effective length of conductor (m)
v = velocity of conductor (m/s)

Note: For the loop formed by the rod and rails in Figure 1.13(a), conductor X is the only moving conductor.

Since the emf was generated by an applied force driving conductor X to the right, the induced voltage and associated current will be in a direction to develop a counterforce. For this to happen, flux bunching must occur on the right side of conductor X, as shown in Figure 1.13(b). This establishes the direction of conductor flux, and the right-hand rule may then be used to determine the direction of the associated current, and hence the direction of the induced emf. Thus, the direction of induced emf *within the conductor* is away from the reader, as shown in Figure 1.13(b), causing terminal X to be positive with respect to terminal X'.

Equation (1–19) defines a speed-voltage generated by a conductor of length ℓ, *cutting flux lines* while moving at velocity v through (and normal to) a magnetic field of density B, and is called the *$B\ell v$ rule.*

The equivalence of the $B\ell v$ rule and the $d\phi/dt$ through the window method for determining the generation of an emf is further demonstrated in the following two examples.

1. If both conductor X and conductor Y in Figure 1.13(a) are moved to the right by an applied force and at the same speed, they would each cut the same number of flux lines, at the same speed and in the same direction, and thus generate the same voltage. The respective voltage directions *within the conductors* would be Y' to Y and X' to X. As a result, the net voltage around the loop (and thus the current in the loop) would be zero. The corresponding equivalent circuit is shown in Figure 1.13(c), resistor R is the equivalent total resistance of conductors and rails.

   Analyzing the same conditions, on the basis of $d\phi/dt$ through the window, indicates that with both coil sides moving at the same speed and in the same direction, $d\phi/dt$ through the window will be zero, resulting in zero voltage generated in the loop.
2. If conductor Y is moved to the left while conductor X is moved to the right, both at the same speed, they would each cut the same number of flux lines, at the same speed, *but in opposite directions.* Thus, the voltage *within* conductor Y would be from Y to Y', while the voltage *within* conductor X would be from X' to X. The net voltage in the loop formed by the conductors and the rails would be doubled. This is the case for almost all rotating machines that use coils[4]; the two coil sides always move in opposite directions with respect to the flux from the field poles. The corresponding equivalent circuit is shown in Figure 1.13(d).

[4]Acyclic machines, also called homopolar or unipolar machines, use conducting cylinders instead of coils [3].

> Analyzing the same conditions on the basis of $d\phi/dt$ through the window indicates that with the coil sides moving in opposite directions, the $d\phi/dt$ through the window will double, generating twice the voltage that would otherwise occur if only one coil side moved.

Thus, in the case of rotating machines, it is not necessary to look for a rate of change of flux through a window in order to determine whether or not a voltage is generated. *If a conductor "cuts flux," a voltage is generated.*

EXAMPLE 1.5 Determine the length of conductor required to generate 2.5 V when passing through and normal to a magnetic field of 1.2 T at a speed of 8.0 m/s.

Solution

$$e = B\ell v \quad \Rightarrow \quad 2.5 = 1.2 \times \ell \times 8.0$$

$$\ell = 0.26 \text{ m}$$

1.12 ELEMENTARY TWO-POLE GENERATOR

Figure 1.14(a) shows a closed coil situated within a magnetic field and driven in a clockwise direction by the prime mover. To satisfy Lenz's law, the induced voltage, current, and associated flux must be in a direction that will develop a *countertorque* to oppose the driving torque of the prime mover. For this to happen, flux bunching must occur on the top of coil side B and the bottom of coil side A, as shown in Figure 1.14(b). With the direction of conductor flux known, the direction of the respective emfs may be determined by applying the right-hand rule; the emf and current are toward the reader in A, and away from the reader in B. Thus, as viewed from the south pole in Figure 1.14(a) the current in the coil is in a CCW direction.

Sinusoidal Emfs

Referring to the elementary generator in Figure 1.14(a), if the coil rotates at a constant angular velocity in a uniform magnetic field, the variation of flux through the coil window will be sinusoidal.

$$\phi = \phi_{max} \sin(\omega t) \tag{1–20}$$

where: ωt = instantaneous angle that the plane of the coil makes with the flux lines (rad)

Φ_{max} = maximum flux through coil window (Wb)

Referring to Figure 1.14(a), the maximum flux through the coil window occurs when the window of the coil is parallel to the pole face.

The rate-of-change of flux through the window as the coil rotates within the magnetic field is

$$\frac{d\phi}{dt} = \omega\Phi_{max}\cos(\omega t) \tag{1–21}$$

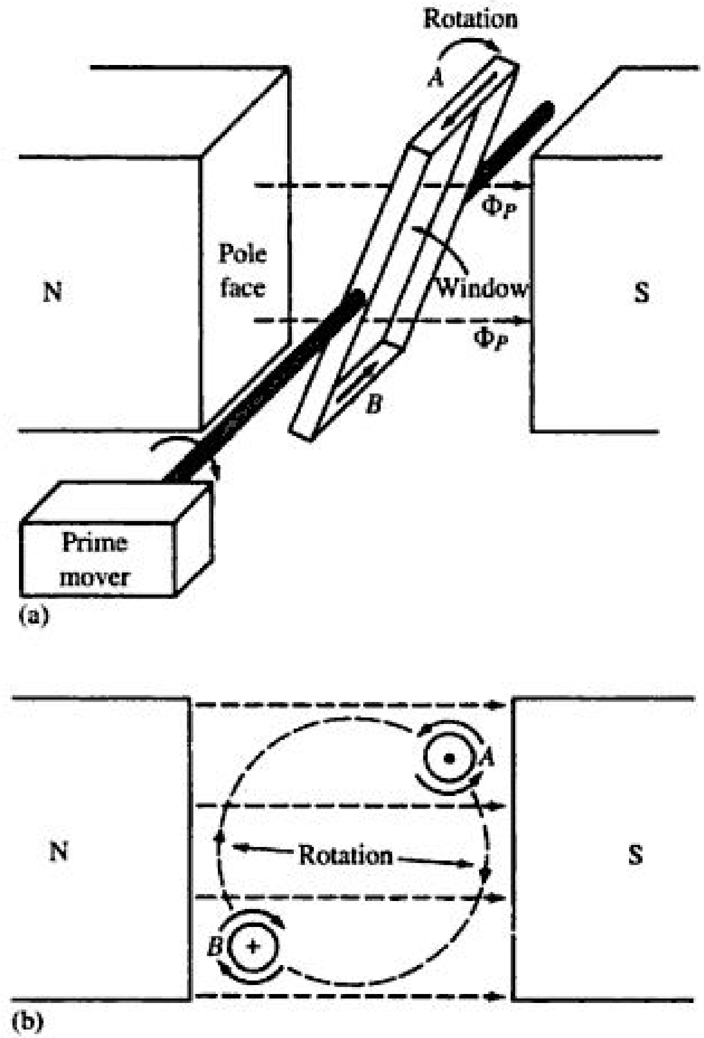


FIGURE 1.14
(a) Closed coil rotating CW within a magnetic field; (b) direction of emf and current for the instant shown in (a).

Substituting Eq. (1–21) into Eq. (1–15),

$$e = N\frac{d\phi}{dt} = N \cdot \omega\Phi_{max}\cos(\omega t) \tag{1–22}$$

The maximum value of the voltage wave in Eq. (1–22) is

$$E_{max} = \omega N\Phi_{max} = 2\pi f N\Phi_{max} \tag{1–23}$$

Dividing both sides by $\sqrt{2}$

$$\frac{E_{max}}{\sqrt{2}} = \frac{2\pi f N\Phi_{max}}{\sqrt{2}} \tag{1–24}$$

$$E_{rms} = 4.44 fN\Phi_{max} \tag{1–25}$$

where: f = frequency of the sinusoidal flux through the window, and hence the frequency of the generated emf (Hz)
N = number of series-connected turns in coil

Note: Equation (1–25) may also be expressed in terms of rotational or angular velocity:

$$E_{rms} = n \cdot \Phi_{max} \cdot k_n \tag{1–26}$$

or

$$E_{rms} = \omega \cdot \Phi_{max} \cdot k_\omega \tag{1–27}$$

where: ω = angular velocity (rad/s)
n = rotational speed (r/s or r/min)
k_n, k_ω = constants[5]

Frequencies currently used in electrical power applications are 25 Hz, 50 Hz, 60 Hz, and 400 Hz. The 60-Hz system is used primarily in North America; the 50-Hz system is used throughout Europe and most other countries; the 400-Hz system is the preferred system for aircraft and spacecraft because of its light weight; and the 25-Hz system is used extensively for traction motors in railroads **[4], [5], [6].**

EXAMPLE 1.6 An elementary four-pole generator with a six-turn rotor coil generates the following voltage wave:

$$e = 24.2 \sin(36 \cdot t)$$

Determine (a) the frequency; (b) the pole flux.

Solution

(a) $\omega = 2\pi f \quad \Rightarrow \quad 36 = 2\pi f \quad \Rightarrow \quad f = 5.7296 \text{ Hz}$

(b) $E_{rms} = 4.44 f N \Phi_{max} \quad \Rightarrow \quad \dfrac{24.2}{\sqrt{2}} = 4.44 \times 5.7296 \times 6 \times \Phi_{max}$

$\Phi_{max} = \underline{0.112 \text{ Wb}}$

1.13 ENERGY CONVERSION IN ROTATING ELECTRICAL MACHINES

All rotating electrical machines may be operated as either motors or as generators. If mechanical energy is supplied to the shaft, the machine converts mechanical energy to electrical energy. If electrical energy is applied to the machine windings, the machine converts electrical energy to mechanical energy. *Regardless of the direction of energy flow, however, all electrical machines (when operating) generate voltage and develop torque at the same time.* If operating as a motor, it develops torque and a counter-emf; if operating as a generator, it develops an emf; and if supplying a load, it develops a countertorque.

[5] The constant depends on the units used and the number of series-connected turns in the coil.

1.14 EDDY CURRENTS AND EDDY-CURRENT LOSSES

Eddy currents are circulating currents produced by transformer action in the iron cores of electrical apparatus. Figure 1.15(a) shows a block of iron that may be viewed as an infinite number of concentric shells or loops. The eddy voltages generated in these shells by a changing magnetic field are proportional to the rate of change of flux through the window of the respective shells. Thus,

$$e_e \propto \frac{d\phi}{dt}$$

Expressed in terms of frequency and flux density, as obtained from Eq. (1–25),

$$E_e \propto f \cdot B_{\max} \qquad \textbf{(1–28)}$$

Slicing the core into many laminations and insulating one from the other will reduce the magnitude of the eddy currents by providing smaller paths, and hence lower eddy voltages. This is shown in Figure 1.15(b). Laminated cores are made by stacking insulated steel stampings to the desired thickness or depth. Each lamination is insulated by a coating of insulating varnish or oxide on one or both sides. Laminating the core results in much smaller shells, significantly reducing the heat losses in the iron.

The eddy-current loss, expended as heat power in the resistance of each shell, is proportional to the square of the eddy voltage.

$$P_e \propto E_e^2 \qquad \textbf{(1–29)}$$

Substituting Eq. (1–28) into Eq. (1–29) and applying a proportionality factor results in

$$P_e = k_e f^2 B_{\max}^2 \qquad \textbf{(1–30)}$$

where: P_e = eddy-current loss (W/unit mass)
f = frequency of flux wave (Hz)

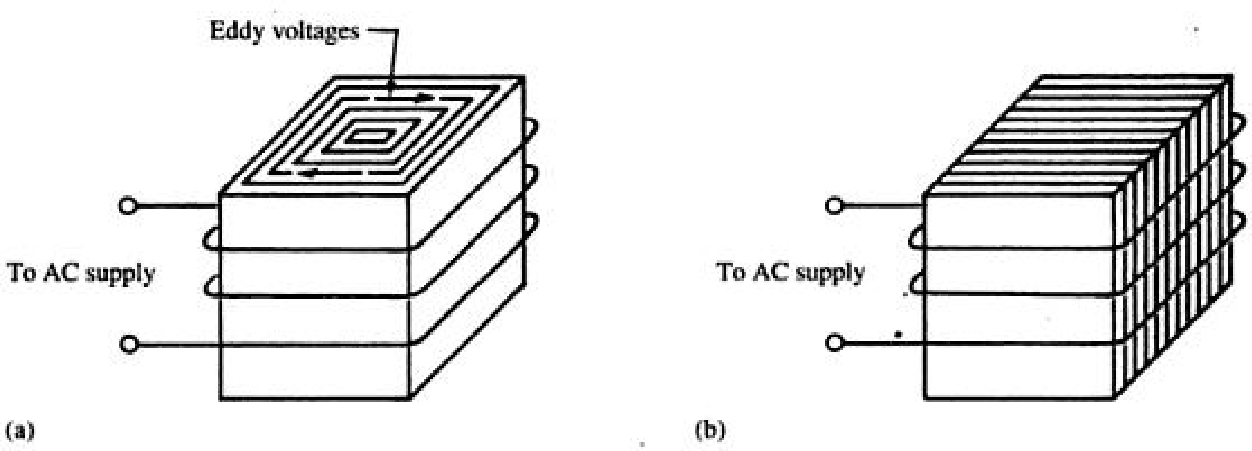


FIGURE 1.15
(a) Eddy currents in solid iron core; (b) laminated core.

B_{max} = maximum value of flux density wave (T)
k_e = constant

The constant k_e is dependent on the lamination thickness, electrical resistivity, density and mass of the core material, and the units used.

EXAMPLE 1–7 The eddy-current loss in a certain electrical apparatus operating at its rated voltage and rated frequency of 240 V and 25 Hz is 642 W. Determine the eddy-current loss if the apparatus is connected to a 60-Hz source whose voltage is such as to cause the flux density to be 62 percent of its rated value.

Solution
From Eq. (1–30),

$$\frac{P_{e1}}{P_{e2}} = \frac{[k_e f^2 B_{max}^2]_1}{[k_e f^2 B_{max}^2]_2} \quad \Rightarrow \quad P_{e2} = P_{e1} \times \left[\frac{f_2}{f_1}\right]^2 \times \left[\frac{B_{max,2}}{B_{max,1}}\right]^2$$

$$P_{e2} = 642 \times \left[\frac{60}{25}\right]^2 \times \left[\frac{0.62}{1.0}\right]^2 = \underline{1.42 \text{ kW}}$$

1.15 MULTIPOLAR MACHINES, FREQUENCY, AND ELECTRICAL DEGREES

The magnetic circuit for an elementary four-pole generator is shown in Figure 1.16(a). The four poles of the stator core are alternately north and south, and an *armature coil* wound on the rotor core spans one-quarter of the rotor circumference. The stator is marked off in *space degrees,* also called *mechanical degrees.* If the rotor coil is positioned at the 0° reference, as shown in Figure 1.16(a), maximum flux from the north pole will enter the outside face of the coil window. At the 45° position, shown in Figure 1.16(b), the net flux passing through the window is zero; the number of lines entering the window is equal to the number of lines leaving the same side of the window. At 90°, the flux through the window reaches its maximum value in the opposite direction, etc.

A plot of the variation of flux through the coil window for one revolution of the rotor is shown in Figure 1.16(c); the variation of flux is assumed to be essentially sinusoidal.

Note that for a four-pole machine, such as that shown in Figure 1.16, one revolution of the rotor causes two complete cycles of flux to pass through the coil window, *one cycle per pair of poles.* Similarly, a six-pole machine would produce three cycles per revolution, etc. Expressed as an equation,

$$f = \frac{Pn}{2} \qquad (1\text{–}31)$$

where: f = frequency (Hz)
P = number of poles
n = rotational speed (r/s)

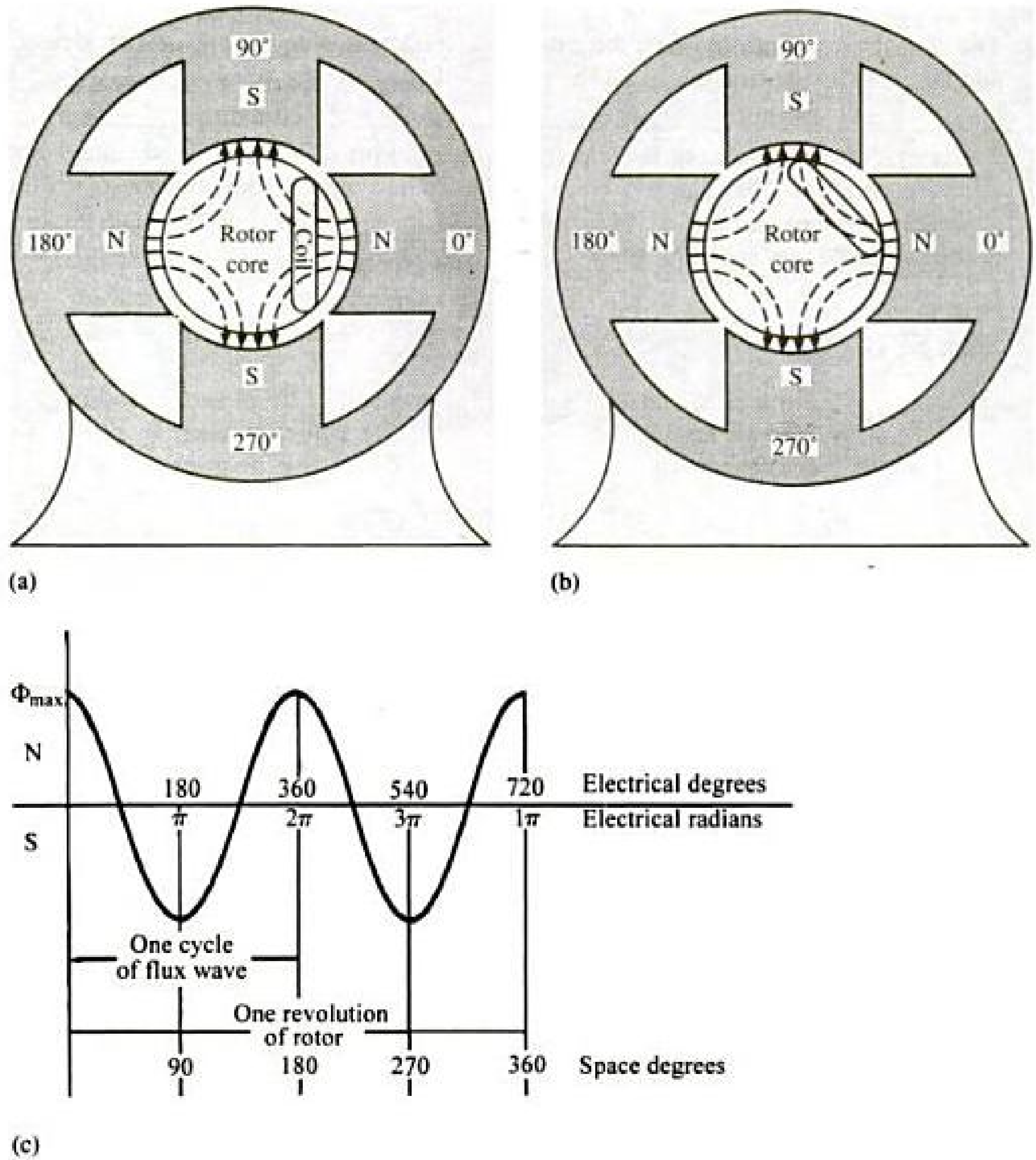


FIGURE 1.16

Four-pole generator: (a) flux through coil window is at maximum value; (b) net flux through coil window is zero; (c) variation of flux through coil window as rotor turns in CCW direction.

Note also that, for a four-pole machine, 720° of the periodic wave corresponds to 360° of angular displacement of the rotor. Hence, to differentiate between the degrees of an electrical quantity and the degrees of space displacement, the former are known as electrical degrees or time degrees, and the latter as space degrees. This distinction is also used in radian measure, namely, electrical radians and space radians.

As indicated in Figure 1.16(c), the relationship between electrical degrees and space degrees is

$$\text{Elec. deg.} = \text{space deg.} \times \frac{P}{2} \tag{1–32}$$

where: P = the number of poles

Unless otherwise specified, angular measurements used in electrical transactions in this text, and in other electrical texts, are expressed in electrical degrees or electrical radians. Adjacent poles are always 180 electrical degrees (π electrical radians) apart.

EXAMPLE 1.8 A special-purpose 80-pole, 100-kVA generator is operating at 20 r/s. Determine (a) the number of cycles per revolution; (b) the number of electrical degrees per revolution; (c) the frequency in Hz.

Solution

(a) Two poles per cycle, or 40 cycles.

(b) $\text{Elec. deg.} = 360 \times \dfrac{80}{2} = 14{,}400$

(c) $f = \dfrac{Pn}{2} = \dfrac{80 \times 20}{2} = \underline{800\text{ Hz}}$

EXAMPLE 1.9 The voltage generated in a 15-turn armature coil by a four-pole rotating field is 100 V. If the flux per pole is 0.012 Wb, determine (a) frequency of the generated emf; (b) speed of the rotor.

Solution

(a) From Eq. (1–25),

$$f = \frac{E_{\text{rms}}}{4.44 \times N \times \phi_{\max}} = \frac{100}{4.44 \times 15 \times 0.012} = 125.13 \quad \Rightarrow \quad 125\text{ Hz}$$

(b) From Eq. (1–30),

$$n = \frac{2f}{P} = \frac{2 \times 125.13}{4} = 62.57\text{ r/s} \quad \text{or} \quad 60 \times 62.57 = 3754\text{ r/min}$$

SUMMARY OF EQUATIONS FOR PROBLEM SOLVING

$$\mathcal{F} = N \cdot I \qquad N \tag{1–1}$$

$$H = \frac{\mathcal{F}}{\ell} = \frac{N \cdot I}{\ell} \qquad \text{A-t/m} \tag{1–2}$$

$$B = \frac{\Phi}{A} \qquad \text{T} \tag{1–3}$$

$$\Phi = \frac{\mathcal{F}}{\mathcal{R}} = \frac{N \cdot I}{\mathcal{R}} \qquad \text{Wb} \tag{1–4}$$

$$\mu = \frac{B}{H} \qquad \text{Wb/A-t} \cdot \text{m} \tag{1–5}$$

$$\mathcal{R} = \frac{\ell}{\mu A} \qquad \text{A-t/Wb} \tag{1–6}$$

$$\mu_r = \frac{\mu}{\mu_0} \tag{1–7}$$

$$\mathcal{R} = \frac{\ell}{\mu_r \mu_0 A} \qquad \text{A-t/Wb} \tag{1–8}$$

$$\mathcal{R}_{ser} = \mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \cdots + \mathcal{R}_n \tag{1–9}$$

$$\frac{1}{\mathcal{R}_{par}} = \frac{1}{\mathcal{R}_1} + \frac{1}{\mathcal{R}_2} + \frac{1}{\mathcal{R}_3} + \cdots + \frac{1}{\mathcal{R}_n}$$

or

$$\mathcal{R}_{par} = \frac{1}{1/\mathcal{R}_1 + 1/\mathcal{R}_2 + 1/\mathcal{R}_3 + \cdots + 1/\mathcal{R}_n} \tag{1–10}$$

$$P_h = k_h \cdot f \cdot B^n_{max} \qquad \text{W} \tag{1–11}$$

$$F = B \cdot \ell_{eff} \cdot I \tag{1–12}$$

$$\alpha = 90^\circ - \beta$$

$$T_D = 2 \cdot B \cdot \ell_{eff} \cdot I \cdot d \qquad \text{N} \cdot \text{m} \tag{1–14}$$

$$e = N\frac{d\phi}{dt} \qquad \text{V} \tag{1–15}$$

$$e = B \cdot \ell \cdot v \qquad \text{V} \tag{1–19}$$

$$E_{rms} = 4.44 fN\Phi_{max} \tag{1–25}$$

$$E_{rms} = n \cdot \Phi_{max} \cdot k_n \qquad \text{V} \tag{1–26}$$

$$E_{rms} = \omega \cdot \Phi_{max} \cdot k_\omega \qquad \text{V} \tag{1–27}$$

$$P_e = k_e f^2 B^2_{max} \qquad \text{W} \tag{1–30}$$

$$f = \frac{Pn}{2} \qquad \text{Hz} \tag{1–31}$$

$$\text{Elec. deg.} = \text{space deg.} \times \frac{P}{2} \tag{1–32}$$

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REVIEW QUESTIONS

1. Sketch a coil connected to a DC source, and indicate the direction of current in the coil and the direction of magnetic flux around the connecting wires, coil, and battery.
2. Differentiate between magnetic field intensity and magnetomotive force, and state the units for each.
3. (a) How is the reluctance of a section of magnetic material related to the material and its dimensions? (b) Is the reluctance of a magnetic material dependent on the degree of magnetization? Explain.
4. Is the permeability of a given block of magnetic material constant? Explain.
5. Differentiate between permeability and relative permeability.
6. What is flux fringing and where does it occur?
7. Explain why, in any series magnetic circuit (or series branch of a magnetic circuit) containing an air gap, the greatest magnetic-potential drop occurs across the air gap.
8. List the analogous relationships that exist between an electric circuit and a magnetic circuit.
9. What is magnetic hysteresis, and what effect does it have on the rate of response of magnetic circuits to a magnetizing force?
10. Sketch an hysteresis loop and discuss the behavior of the loop as the magnetizing current goes through the first 1.5 cycles. Assume the magnetic core was initially in an unmagnetized state.
11. (a) What causes magnetic hysteresis loss, and how is it affected by the frequency and density of the flux wave? (b) How is the hysteresis loss related to the hysteresis loop?
12. Sketch two parallel conductors in the vertical plane with currents in opposite directions. Show the directions of current, component magnetic fields, resultant field, and direction of mechanical force exerted on each conductor.
13. Repeat Question 12, assuming the currents are in the same direction.

14. Sketch a conductor carrying direct current, situated in and normal to the magnetic field of a permanent magnet. Show the directions of current, component magnetic fields, resultant field, and direction of mechanical force exerted on the conductor and on the poles.
15. Make a sketch of a one-turn coil situated in the field of a permanent magnet, and explain how current in the coil produces torque. Indicate on the sketch the direction of current, respective directions of the component magnetic fields, and the direction of the resultant two-conductor couple.
16. Using appropriate sketches and Lenz's law, explain (a) how speed voltages are generated and indicate their direction; (b) how transformer voltages are generated and indicate their direction.
17. Explain why all electrical machines (when operating) develop torque and generate voltage at the same time.
18. Explain how eddy currents are generated in magnetic cores and how they can be minimized.
19. How are eddy-current losses affected by the frequency and density of the flux wave?

PROBLEMS

1–1/4 The magnetic circuit of an inductance coil has a reluctance of 1500 A-t/Wb. The coil is wound with 200 turns of aluminum wire, and draws 3 A when connected to a 24-V battery. Determine (a) the core flux; (b) the resistance of the coil.

1–2/4 A magnetic circuit constructed of sheet steel has an average length of 1.3 m and a cross-sectional area of 0.024 m^2. A 50-turn coil wound on the ring has a resistance of 0.82 Ω, and draws 2 A from a DC supply. The reluctance of the core for this condition is 7425 A-t/Wb. Determine (a) flux density; (b) voltage applied.

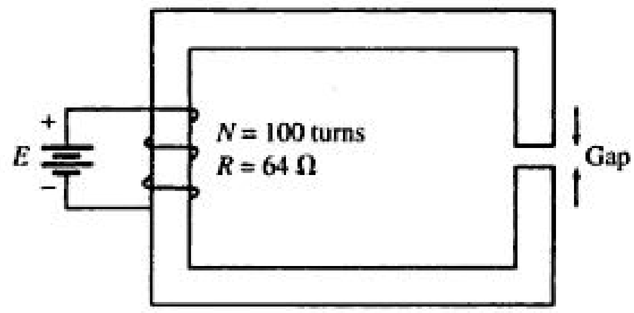
1–3/4 A magnetic circuit has an average length of 1.4 m and a cross-sectional area of 0.25 m^2. Excitation is provided by a 140-turn, 30-Ω coil. Determine the voltage required to establish a flux density of 1.56 T. The reluctance of the magnetic circuit, when operating at this flux density is 768 A-t/Wb.

1–4/5 A ferromagnetic core in the shape of a doughnut has a cross-sectional area of 0.11 m^2 and an average length of 1.4 m. The permeability of the core is 1.206×10^{-3} Wb/A-t · m. Determine the reluctance of the magnetic circuit.

1–5/5 A magnetic circuit has a mean length of 0.80 m, a cross-sectional area of 0.06 m^2, and a relative permeability of 2167. Connecting its 340-turn 64-Ω magnetizing coil to a DC circuit causes a 56-V drop across the coil. Determine the flux density in the core.

1–6/5 The magnetic circuit shown in Figure 1.17 has a mean core length of 52 cm and a cross-sectional area of 18 cm^2. The length of the air gap is 0.14 cm. Determine the battery voltage required to obtain a flux density of 1.2 T in the air gap. Use the magnetization curve shown in Figure 1.5.

FIGURE 1.17
Magnetic circuit for Problem 1–6/5.



1–7/5 The mean length and cross-sectional area of the core shown in Figure 1.18 are 1.5 m and 0.08 m^2, respectively. The core is made of cast steel and the magnetization curve for the material is shown in Figure 1.3. The 260-turn magnetizing coil has a resistance of 27.75 Ω, and is connected to a 240-V DC supply. Determine (a) magnetic field intensity; (b) core flux density and core flux; (c) relative permeability of the core; (d) reluctance of the magnetic circuit.

1–8/5 Repeat Problem 1–7, assuming a sheet steel core.

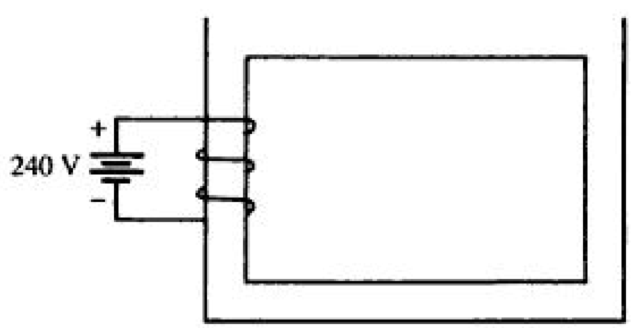
1–9/5 Repeat Problem 1–7, assuming a cast-iron core.

1–10/6 A magnetic circuit, composed of two half-rings of different core materials, is joined at the ends to form a doughnut. The cross-sectional area of the core is 0.14 m^2, and the reluctances of the two halves are 650 A-t/Wb and 244 A-t/Wb, respectively. A coil of 268 turns and 5.2-Ω resistance is wound around the doughnut and connected to a 45-V battery. Determine (a) the core flux; (b) repeat (a), assuming the half-rings are separated 0.12 cm at each end (assume no fringing), and the reluctance of the half-rings does not change; (c) the magnetic drop across each air gap in (b).

1–11/7 A coil wound around a ferromagnetic core is supplied from a 25-Hz source. Determine the percent change in hysteresis loss if the coil is connected to a 60-Hz sources and the resultant flux density is reduced by 60%. Assume the Steinmetz coefficient is 1.65, and voltage is constant.

1–12/7 A certain electrical apparatus operating at rated voltage and rated frequency has an hysteresis loss of 250 W. Determine the hysteresis loss if the fre-

FIGURE 1.18
Magnetic circuit for Problem 1–7/5.



quency is reduced to 60.0 percent rated frequency, and the applied voltage is adjusted to provide 80.0 percent rated flux density. Assume the Steinmetz exponent is 1.6.

1–13/10 A conductor 0.32 m long with 0.25-Ω resistance is situated within and normal to a uniform magnetic field of 1.3 T. Determine (a) the voltage drop across the conductor that would cause a force of 120 N to be exerted on the conductor; (b) repeat part (a), assuming a skew angle of 25°.

1–14/10 A rotor coil consisting of 30 series-connected turns, with a total resistance of 1.56 Ω, is situated within a uniform magnetic field of 1.34 T. Each coil side has a length of 54 cm, is displaced 22 cm from the center of the rotor shaft, and has a skew angle of 8.0°. Sketch the system and determine the coil current required to obtain a shaft torque of 84 N · m.

1–15/11 Determine the required linear velocity of a 0.54-m conductor that will generate 30.6 V when cutting flux in a 0.86-T magnetic field.

1–16/11 A conductor 1.2 m long is moving at a constant velocity of 5.2 m/s through and normal to a uniform magnetic field of 0.18 T. Determine the generated voltage.

1–17/12 Determine the frequency and rms voltage generated by a three-turn coil rotating at 12 r/s within a four-pole field that has a pole flux of 0.28 Wb/pole.

1–18/12 Determine the rotational speed required to generate a sinusoidal voltage of 24 V in a 25-turn coil that rotates within a two-pole field of 0.012 Wb/pole.

1–19/12 The flux through the window of a 20-turn coil varies with time in the following manner:

$$\phi = 1.2 \sin(28 \cdot t) \qquad \text{Wb}$$

Determine (a) the frequency and rms value of voltage generated in the coil; (b) the equation representing the voltage wave.

1–20/14 A coil wound around a ferromagnetic core is supplied by a 120-V, 25-Hz source. Determine the percent change in eddy-current loss if the coil is connected to a 120-V, 60-Hz source.

1–21/14 A certain electrical apparatus operating at rated voltage and rated frequency has an eddy-current loss of 212.6 W. Determine the eddy-current loss if the frequency is reduced to 60.0 percent rated frequency and the applied voltage is adjusted to provide 80.0 percent rated flux density.

2

Transformer Principles

2.1 INTRODUCTION

The principle of transformer action is based on the work of Michael Faraday (1791–1867), whose discoveries in electromagnetic induction showed that, given two magnetically coupled coils, a changing current in one coil will induce an electromotive force in the other coil. Such electromagnetically induced emfs are called *transformer voltages,* and coils specifically arranged for such purposes are called *transformers.*

Transformers are very versatile. They are used to raise or lower voltage in AC distribution and transmission systems; to provide reduced-voltage starting of AC motors; to isolate one electric circuit from another; to superimpose an alternating voltage on a DC circuit; and to provide low voltage for solid-state control, for battery charging, door bells, etc.

The principle of transformer action is also applicable in many ways to motors, generators, and control apparatus. A specific example is the application of the equivalent-circuit model of the transformer, developed in this chapter, to the analysis of induction-motor performance in Chapter 4.

2.2 CONSTRUCTION OF POWER AND DISTRIBUTION TRANSFORMERS

The two basic types of transformer construction used for power and distribution applications are shown in Figure 2.1. Note that the high-voltage coils are wound with a greater number of turns of smaller cross-section conductor than the low-voltage coils. The core type, shown in Figure 2.1(a), has primary and secondary coils wound on different legs, and the shell type, shown in Figure 2.1(b), has both coils wound on the same leg. The wider spacing between primary and secondary in the core-type transformer gives it an advantage in high-voltage applications. The shell type, however, has the advantage of less leakage flux.

Transformer core material is made of nonaging, cold-rolled, high-permeability silicon steel laminations, and each lamination is insulated with a varnish or oxide coating

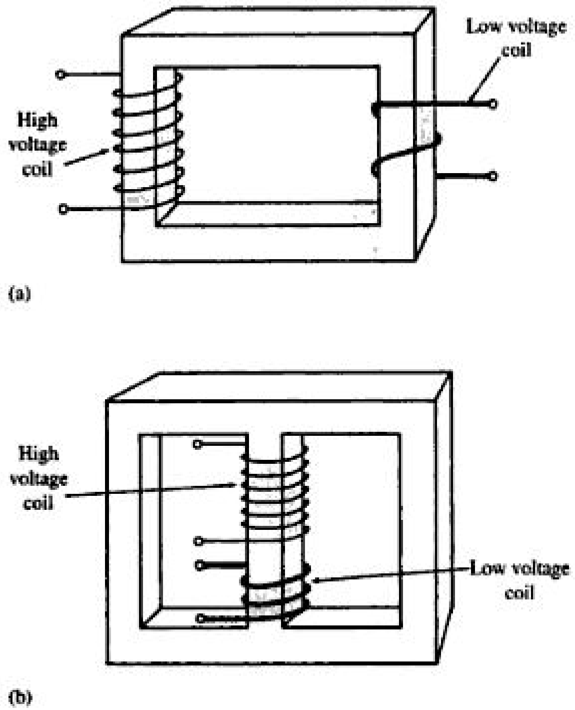


FIGURE 2.1
Transformer construction: (a) core type; (b) shell type.

to reduce eddy currents. The coils are wound with insulated aluminum conductor or insulated copper conductor, depending on design considerations. Cooling is provided by air convection, forced air, insulating liquids, or gas.

Ventilated Dry-Type Transformers

Ventilated dry-type transformers are cooled by natural air convection. The principal application for this type of transformer is in schools, hospitals, and shopping areas, where large groups of people are present and potential hazards to personnel from burning oil or toxic gases must be avoided. The ventilated dry-type transformer, however, requires periodic maintenance, such as removal of dust or dirt from the windings by light brushing, vacuuming, and/or blowing with dry air.

Gas-Filled Dry-Type Transformers

Gas-filled dry-type transformers are cooled with nitrogen or other dielectric gases, such as fluorocarbon C_2F_6 and sulfurhexafluoride SF_6. These transformers can be installed indoors, outdoors, or in underground environments. Gas-filled transformers are hermetically sealed and require only periodic checks of gas pressure and temperature.

Liquid-Immersed Transformers

Liquid-immersed transformers, such as that shown in Figure 2.2, have hermetically sealed tanks filled with insulating liquid to provide both insulation and cooling. Cooling fins on the tank provide for convection cooling of the insulating liquid. Forced cooling with pumps and/or fans is also provided on larger power transformers. The

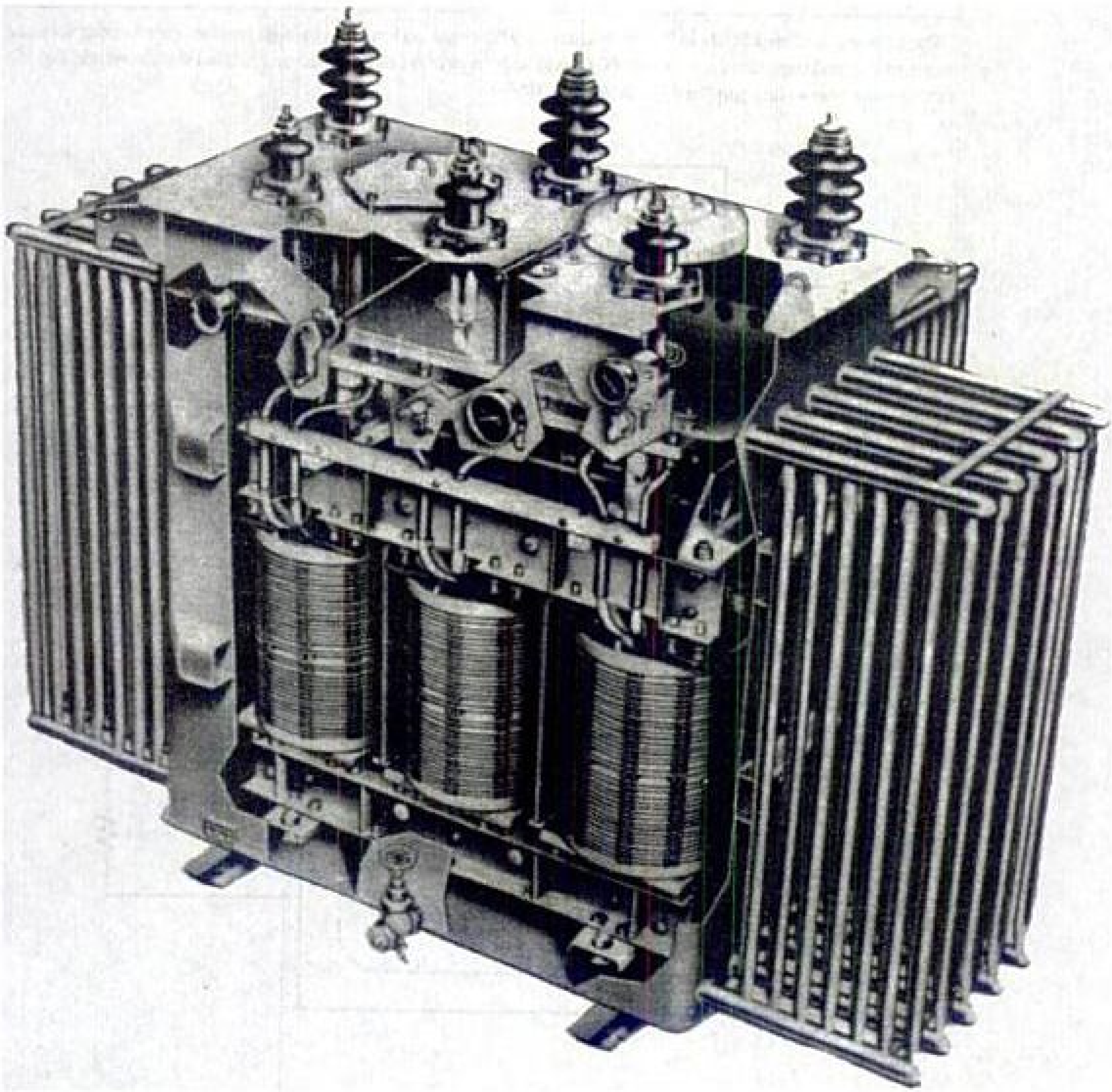


FIGURE 2.2
Cutaway view of a large three-phase oil-cooled power transformer. (Courtesy, TECO Westinghouse)

insulating liquids used are mineral oil and silicone oil. Polychlorinated biphenyls (PCBs) called askarels[1] were used in earlier construction but are no longer permitted.

2.3 PRINCIPLE OF TRANSFORMER ACTION

The principle of transformer action is explained with the aid of Figure 2.3(a), which shows coil 1 connected to a battery through a switch, and coil 2 connected to a resistor. Closing the switch causes a clockwise (CW) buildup of flux in the iron core, generating

[1] The EPA has declared PCBs to be toxic liquids, and they are no longer permitted in new construction. Existing transformers and capacitors containing PCBs had to be replaced or detoxified and refilled with nontoxic liquids, and all work was to be completed by October 1, 1990.

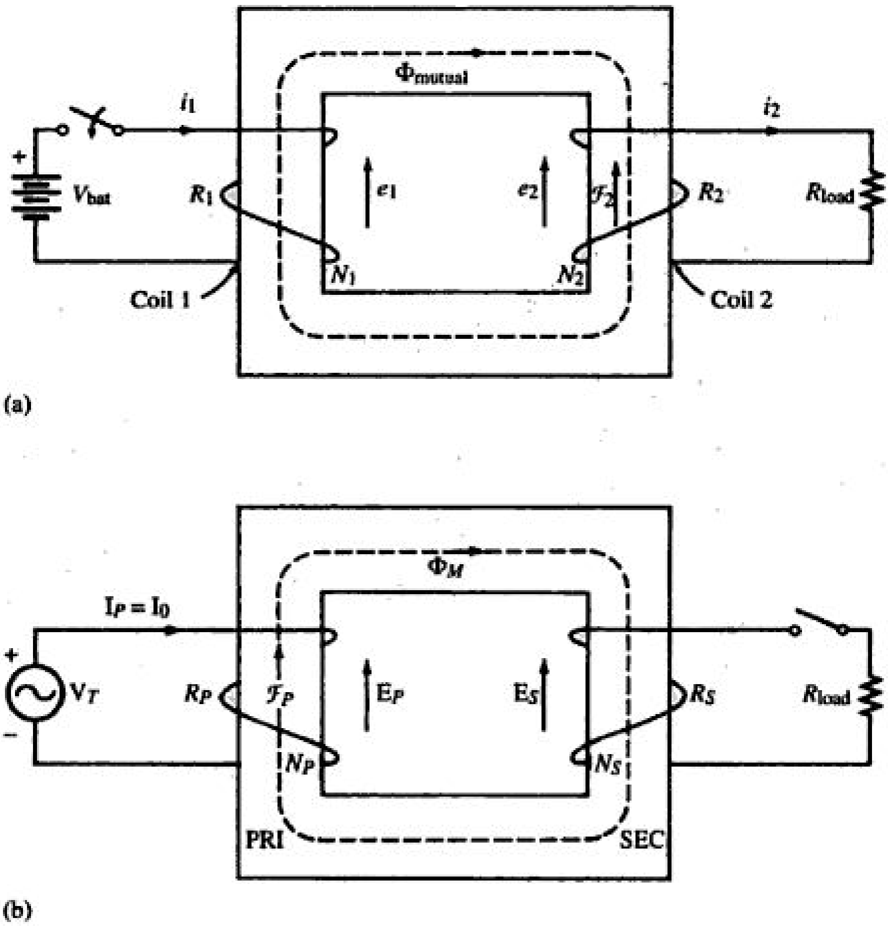


FIGURE 2.3
(a) Transformer with battery in primary circuit to aid in the explanation of transformer action; (b) transformer with sinusoidal source and no load on secondary.

a voltage in each coil that is proportional to the number of turns in the coil and the rate of change of flux through the respective coils. Assuming no leakage, the same flux (called the *mutual flux*) exists in both coils. Thus,

$$e_1 = N_1\frac{d\phi}{dt} \qquad e_2 = N_2\frac{d\phi}{dt}$$

where: N_1 = turns in coil 1
N_2 = turns in coil 2

In accordance with Lenz's law, the voltage generated in each coil will be induced in a direction that opposes the action that caused it. Thus, the induced emf in coil 1 must be opposite in direction to the battery voltage, as shown in Figure 2.3(a). This opposing voltage, shown as e_1 in Figure 2.3(a), is called a counter-emf (cemf).

In the case of coil 2, the induced emf and associated current must be in a direction that will develop a counterclockwise (CCW) mmf to oppose the buildup of flux in its window. Thus, with the direction of mmf known, the direction of induced emf and associated current may be determined by applying the right-hand rule to coil 2. *Note:* The induced emfs and secondary current in Figure 2.3(a) are transients. When Φ_{mutual} reaches steady state, $d\phi/dt = 0$, the induced emfs $= 0$, and $i_2 = 0$.

2.4 TRANSFORMERS WITH SINUSOIDAL VOLTAGES

Figure 2.3(b) shows a transformer with one winding (called the primary) connected to a sinusoidal source, and the other winding (called the secondary) connected to a switch and a resistor load. The currents and voltages are expressed as phasors. The directions of the induced voltages are the same as in Figure 2.3(a) and are determined in the same manner.

For this *preliminary discussion,* the following simplifying assumptions will be made: (1) The permeability of the core is constant over the range of transformer operation, and thus the reluctance of the core is constant; and (2) there is no leakage flux, hence the same flux links both primary and secondary windings.

The voltages induced in the primary and secondary windings by the sinusoidal variation of flux in the respective coil windows, expressed in terms of rms values, are[2]

$$E_P = 4.44N_P f\Phi_{max} \tag{2-1}$$

$$E_S = 4.44N_S f\Phi_{max} \tag{2-2}$$

Dividing Eq. (2–1) by Eq. (2–2),

$$\frac{E_P}{E_S} = \frac{N_P}{N_S} \tag{2-3}$$

[2] See Section 1.12, Chapter 1.

where: E_P = voltage induced in primary (V)
E_S = voltage induced in secondary (V)
N_P = turns in primary coil
N_S = turns in secondary coil

Thus, assuming no leakage flux, the ratio of induced voltages is equal to the ratio of turns.

EXAMPLE 2.1 Determine the peak value of sinusoidal flux in a transformer core that has a primary of 200 turns and is connected to a 240-V, 60-Hz, 50-kVA source.

Solution

$$E_P = 4.44 N_P f \Phi_{max}$$

$$\Phi_{max} = \frac{E_P}{4.44 N_P f} = \frac{240}{4.44 \times 200 \times 60} = 4.5 \times 10^{-3}\,\text{Wb}$$

EXAMPLE 2.2 A 15-kVA, 2400—240-V, 60-Hz transformer[3] has a magnetic core of 50-cm^2 cross section and a mean length of 66.7 cm. The application of 2400 V causes a magnetic field intensity of 450 A-t/m rms, and a maximum flux density of 1.5 T. Determine (a) the turns ratio; (b) the number of turns in each winding; (c) the magnetizing current.[4]

Solution

(a) The turns ratio is equal to the ratio of induced emfs, and is approximately equal to the nameplate voltage ratio. Thus,

$$\frac{N_P}{N_S} = \frac{E_P}{E_S} \approx \frac{V_P}{V_S} = \frac{2400}{240} = 10$$

(b)

$$\Phi_{max} = B_{max} \times A = 1.5 \times \frac{50}{10^4} = 7.5 \times 10^{-3}\,\text{Wb}$$

$$E_P = 4.44 N_P f \Phi_{max} \quad \Rightarrow \quad N_P = \frac{E_P}{4.44 f \Phi_{max}}$$

$$N_P = \frac{2400}{4.44 \times 60 \times 7.5 \times 10^{-3}} = \underline{1201\ \text{turns}}$$

$$\frac{N_P}{N_S} = 10 \quad \Rightarrow \quad \frac{1201}{N_S} = 10$$

$$N_S = \underline{120\ \text{turns}}$$

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[3] The long dash (called an em dash) indicates that the voltages are from different windings. See Section 3.3 on transformer nameplates.

[4] See Section 1.3, Chapter 1, for relationship between magnetic field intensity and magnetomotive force.

(c) From Eq. (1–2),

$$H = \frac{N_P I_M}{\ell}$$

where: H = magnetic field intensity (A-t/m, rms)
N_P = turns in primary winding
I_M = magnetizing current (A, rms)
ℓ = mean length of core (m)

Solving for I_M and substituting known values,

$$I_M = \frac{H\ell}{N_P} = \frac{450 \times 0.667}{1201} = \underline{0.25\ \text{A}}$$

2.5 NO-LOAD CONDITIONS

With no load connected to the secondary, the current in the primary is just enough to establish the magnetic flux needed for transformer action, and to supply the hysteresis and eddy-current losses in the iron.[5] This no-load current, called the *exciting current,* varies between 1 and 2 percent of rated current in large power transformers, and may be as high as 6 percent of rated current in very small distribution transformers.

The exciting current can be divided into two right-angle components: a core-loss component that supplies the hysteresis and eddy-current losses in the iron, and a magnetizing component that establishes the *mutual flux* (Φ_M) that links both primary and secondary windings. These components are shown in Figure 2.4(a), and form the equivalent-circuit model of a transformer operating at *no load.* The corresponding phasor diagram for the exciting current and its right-angle components is shown in Figure 2.4(b). Note that the exciting current lags the applied voltage by a large angle;

[5] In effect, at no load, the transformer is nothing more than an impedance coil.

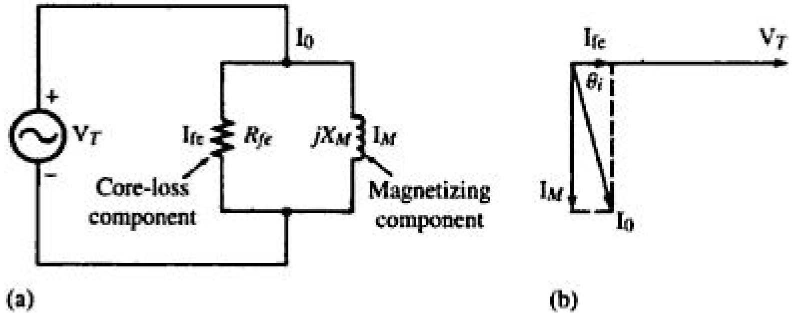


FIGURE 2.4
(a) Equivalent-circuit model of transformer with no load on secondary; (b) phasor diagram showing no-load conditions.

this may be as much as 85° in high-efficiency transformers. Expressing the exciting current in terms of its quadrature components,

$$\mathbf{I}_{fe} = \frac{\mathbf{V}_T}{R_{fe}}$$

$$\mathbf{I}_M = \frac{\mathbf{V}_T}{jX_M}$$

$$\mathbf{I}_0 = \mathbf{I}_{fe} + \mathbf{I}_M \qquad \textbf{(2–4)}$$

where: $\mathbf{I}_0$ = exciting current
$\mathbf{I}_{fe}$ = core-loss component
$\mathbf{I}_M$ = magnetizing component
X_M = fictitious magnetizing reactance that accounts for the magnetizing current
R_{fe} = fictitious resistance that accounts for the core loss
$\mathbf{V}_T$ = voltage applied to primary

EXAMPLE 2.3 A 25-kVA, 2400—240-V, 60-Hz single-phase distribution transformer, operating at no load in the step-down mode, draws 138 W at a power factor (F_P) of 0.210 lagging. Using the equivalent circuit shown in Figure 2.4, determine (a) the exciting current and its quadrature components; (b) the equivalent magnetizing reactance and equivalent core-loss resistance; (c) and (d) repeat parts (a) and (b) for the transformer in the step-up mode.

Solution

(a) The phase angle of the exciting current is determined from the power-factor angle using[6]

$$\theta = (\theta_v - \theta_i)$$

where: θ = power-factor angle
θ_i = angle of current phasor
θ_v = angle of applied voltage phasor

Unless otherwise specified, the phase angle of applied voltage is assumed to be zero.

$$\theta = \cos^{-1}(F_P) = \cos^{-1}0.210 = 77.8776 \quad \Rightarrow \quad 77.88°$$

$$77.88° = (0 - \theta_i)$$

$$\theta_i = -77.88°$$

Referring to Figure 2.4(a), only the core-loss component draws active power. Hence,

$$P_{core} = V_T I_{fe} \quad \Rightarrow \quad 138 = 2400 \times I_{fe}$$

$$I_{fe} = \underline{0.0575\ \text{A}}$$

---

[6] See Appendix A.5, power relationships in a single-phase system.

The magnitudes $|\mathbf{I}_0|$ and $|\mathbf{I}_M|$ are determined from geometry of the corresponding phasor diagram in Figure 2.4(b),

$$\cos\theta_i = \frac{I_{fe}}{I_0} \quad \Rightarrow \quad 0.21 = \frac{0.0575}{I_0}$$

$$I_0 = \underline{0.2738\ \text{A}}$$

$$\tan\theta_i = \frac{I_M}{I_{fe}} \quad \Rightarrow \quad \tan(-77.8776°) = \frac{-I_M}{0.0575}$$

$$I_M = \underline{0.268\ \text{A}}$$

Or, applying the Pythagorean theorem to Figure 2.4(b),

$$I_0 = \sqrt{I_{fe}^2 + I_M^2} \quad \Rightarrow \quad I_M = \sqrt{I_0^2 - I_{fe}^2}$$

$$I_M = \sqrt{(0.2738)^2 + (0.0575)^2} = 0.268\ \text{A}$$

(b) Using high-side data,

$$I_M = \frac{V_T}{X_M} \quad \Rightarrow \quad 0.2677 = \frac{2400}{X_M}$$

$$X_M = 8965 \quad \Rightarrow \quad \underline{8.97\ \text{k}\Omega}$$

$$I_{fe} = \frac{V_T}{R_{fe}} \quad \Rightarrow \quad 0.0575 = \frac{2400}{R_{fe}}$$

$$R_{fe} = 41{,}739 \quad \Rightarrow \quad \underline{41.7\ \text{k}\Omega}$$

(c) The core loss and power factor are the same whether operating in the step-down or step-up mode. Hence, using the 240-V side as the primary,

$$P_{core} = V_T I_{fe} \quad \Rightarrow \quad 138 = 240 \times I_{fe}$$

$$I_{fe} = \underline{0.575\ \text{A}}$$

$$\cos\theta_i = \frac{I_{fe}}{I_0} \quad \Rightarrow \quad 0.21 = \frac{0.575}{I_0}$$

$$I_0 = 2.738 \quad \Rightarrow \quad \underline{2.74\ \text{A}}$$

$$\tan\theta_i = \frac{I_M}{I_{fe}} \quad \Rightarrow \quad \tan(-77.8776) = \frac{-I_M}{0.575}$$

$$I_M = 2.677 \quad \Rightarrow \quad \underline{2.68\ \text{A}}$$

(d) Using low-side data,

$$I_M = \frac{V_T}{X_M} \quad \Rightarrow \quad 2.677 = \frac{240}{X_M}$$

$$X_M = \underline{89.7\ \Omega}$$

$$I_{fe} = \frac{V_T}{R_{fe}} \quad \Rightarrow \quad 0.575 = \frac{240}{R_{fe}}$$

$$R_{fe} = \underline{417.4\ \Omega}$$

No-Load Ampere-Turns and Its Components

Multiplying Eq. (2–4) by the primary turns expresses the no-load mmf in terms of its quadrature components:

$$N_P\mathbf{I}_0 = N_P\mathbf{I}_{fe} + N_P\mathbf{I}_M \tag{2–5}$$

Component $N_P\mathbf{I}_{fe}$ does not contribute to the development of mutual flux, but serves only to oscillate the magnetic domains and to generate eddy currents in the core. If there were no core losses, component $N_P\mathbf{I}_{fe}$ would not exist, and the exciting ampere-turns would be reduced to only that required to establish the mutual flux.

Component $N_P\mathbf{I}_M$, called the magnetizing ampere-turns, produces the mutual flux and hence transformer action. The mutual flux expressed in terms of the rms magnetizing current is

$$\Phi_M = \frac{N_P I_M}{\mathcal{R}_{\text{core}}} \tag{2–6}$$

where: Φ_M = mutual flux produced by the magnetizing component of exciting current
I_M = magnetizing current
$\mathcal{R}_{\text{core}}$ = reluctance of transformer core

Applying Kirchhoff's voltage law to the primary circuit in Figure 2.3(b), and noting that $\mathbf{I}_P = \mathbf{I}_0$ at no load,

$$\mathbf{V}_T = \mathbf{I}_P R_P + \mathbf{E}_P \tag{2–7}$$

Solving for $\mathbf{I}_P$,

$$\mathbf{I}_P = \frac{\mathbf{V}_T - \mathbf{E}_P}{R_P} \tag{2–8}$$

where: $\mathbf{V}_T$ = applied voltage
$\mathbf{I}_P$ = primary current
$\mathbf{E}_P$ = voltage induced in the primary
R_P = resistance of primary winding

Voltage $\mathbf{E}_P$ is the cemf in the primary coil caused by the sinusoidal variation of flux in its window.

2.6 TRANSIENT BEHAVIOR WHEN LOADING AND UNLOADING

In accordance with Lenz's law, the emf induced in the secondary will be in a direction that opposes the change in flux that caused it. Hence, when a load is placed on the secondary winding, the instantaneous direction of the secondary current will set up an mmf of its own in opposition to the primary mmf. This is shown in Figure 2.5. Thus, for a very brief instant of time the core flux will decrease to

$$\phi_M = \frac{N_P i_M - N_S i_S}{\mathcal{R}_{\text{core}}} \tag{2–9}$$

The decrease in flux causes a decrease in cemf, which, in accordance with Eq. (2–8), causes an increase in primary current. The additional primary current ($\mathbf{I}_{P,\text{load}}$), called the *load component of primary current*, adds its mmf to the magnetizing component, causing the flux to increase. Thus,

$$\phi_M = \frac{N_P i_M + N_P i_{P,\text{load}} - N_S i_S}{\mathcal{R}_{\text{core}}} \tag{2–10}$$

The primary current increases until $N_P \mathbf{I}_{P,\text{load}} = N_S \mathbf{I}_S$, at which point both $\mathbf{\Phi}_M$ and $\mathbf{E}_P$ will have returned to essentially the same values they had before the switch was closed; any difference between $\mathbf{E}_P$ at no load and $\mathbf{E}_P$ under load conditions is due to the additional (but small) increase in voltage drop due to the resistance of the primary winding. Thus, the final steady-state primary current *under load conditions* will be

$$\mathbf{I}_P = \mathbf{I}_{\text{fe}} + \mathbf{I}_M + \mathbf{I}_{P,\text{load}}$$
$$\mathbf{I}_P = \mathbf{I}_0 + \mathbf{I}_{P,\text{load}} \tag{2–11}$$

Removing load from the secondary causes the opposite effect to take place. Opening the switch in Figure 2.5 causes $\mathbf{I}_S$ and hence $N_s\mathbf{I}_s$ to drop to zero. The resultant transient increase in mutual flux produces a transient increase in cemf, causing the primary current to drop back to its initial no-load value.

Although described as a step-by-step process, the actual behavior, when loading or unloading, is essentially simultaneous and takes place in a fraction of a second. *Note.* This entire discussion assumed constant permeability and no leakage flux.

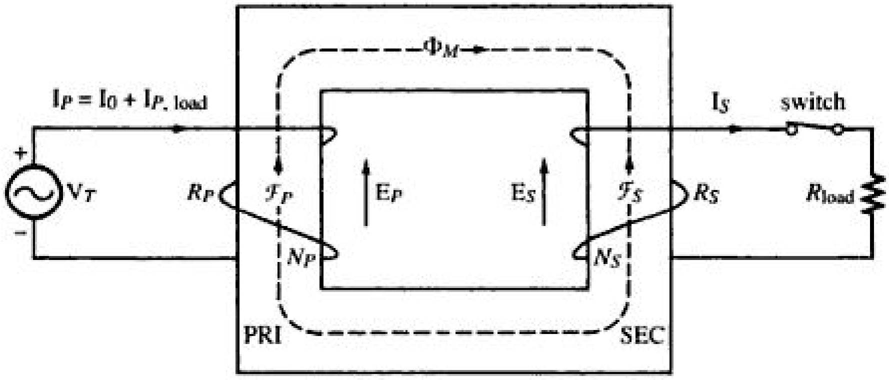


FIGURE 2.5
Relative directions of secondary current and secondary mmf for one-half cycle when load switch is closed.

2.7 EFFECT OF LEAKAGE FLUX ON THE OUTPUT VOLTAGE OF A REAL TRANSFORMER

All of the flux in a real transformer is not common to both primary and secondary windings. The flux in a real transformer has three components: mutual flux, primary leakage flux, and secondary leakage flux. This is shown in Figure 2.6, where, in order to simplify visualization and analysis, only a few representative leakage paths are shown. For the transformer shown in Figure 2.6, the primary leakage flux (caused by primary current) links only the primary turns, the secondary leakage flux (caused by secondary current) links only the secondary turns, and the mutual flux (due to the magnetizing component of the exciting current) links both windings.

The relationship between coil flux, leakage flux, and mutual flux, for the respective primary and secondary coils shown in Figure 2.6, are

$$\Phi_P = \Phi_M + \Phi_{\ell p} \tag{2–12}$$

$$\Phi_S = \Phi_M - \Phi_{\ell s} \tag{2–13}$$

where:
- Φ_P = net flux in window of primary coil
- Φ_S = net flux in window of secondary coil
- Φ_M = mutual flux
- $\Phi_{\ell p}$ = leakage flux associated with the primary coil
- $\Phi_{\ell s}$ = leakage flux associated with the secondary coil

Equations (2–12) and (2–13) illustrate how the leakage flux in both windings serves to reduce the output voltage of the secondary; the mutual flux is less than the available primary flux because of primary leakage, and the net flux in the secondary is the mutual flux less the secondary leakage. Less flux in the secondary coil results in a lower secondary voltage than if no leakage were present.

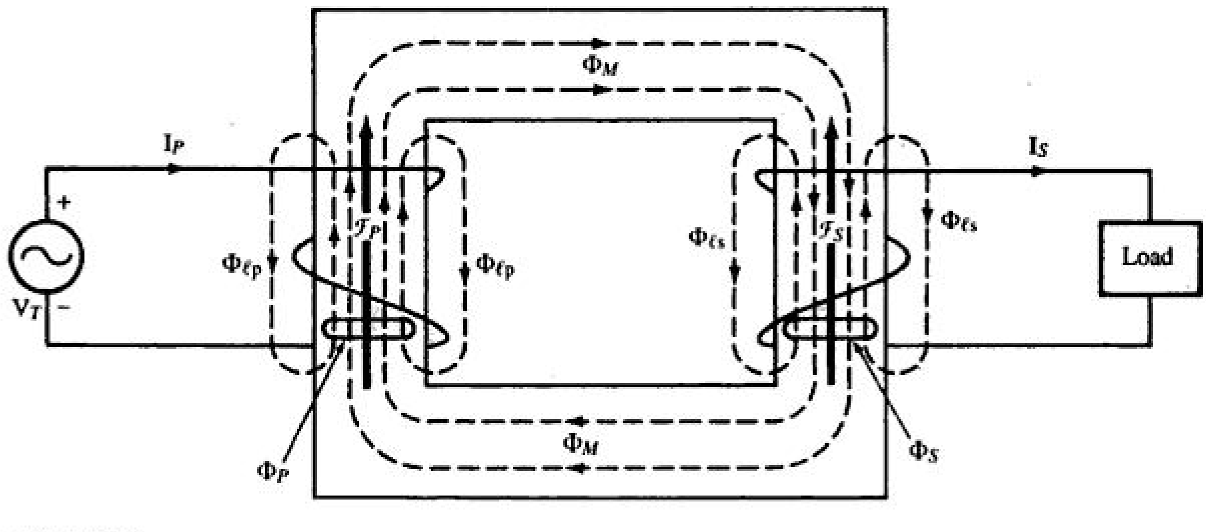


FIGURE 2.6
Component fluxes in the core of a loaded transformer.

The voltage drop caused by leakage flux is proportional to the load current. The greater the load current, the greater the magnitudes of both the primary and secondary ampere turns, and hence the greater the respective leakage fluxes in both primary and secondary windings. Although leakage flux has an adverse effect on the transformer output voltage, it proves an asset under severe short-circuit conditions; the large voltage drop caused by the intense leakage flux limits the current to a lower value than would otherwise occur if no leakage were present and thus helps to avoid damage to the transformer.

2.8 IDEAL TRANSFORMER

An ideal transformer is a hypothetical transformer that has no leakage flux and no core losses; the permeability of its core is infinite, it requires no exciting current to maintain the flux, and its windings have zero resistance. Although an ideal transformer does not exist, its mathematical relationships have practical applications in the development of equivalent circuits for real transformers, for the development of equivalent circuits for induction motors, and for impedance transformation applications.

The basic relationships for the ideal transformer are developed with the aid of Figure 2.7, which shows a load connected across the secondary terminals of an ideal transformer. The primed symbols are used to designate the induced voltages and input impedance of an ideal transformer.

Turns Ratio

The turns ratio a *is the ratio of the number of turns in the high-voltage winding to the number of turns in the low-voltage winding* [3]. It is equal to the ratio of voltages in the ideal transformer and is approximately equal to the voltage ratio of the real transformer (*high side to low side*), with no load connected to the secondary; the effects of leakage flux and winding resistance are insignificant at no load. Hence, when information

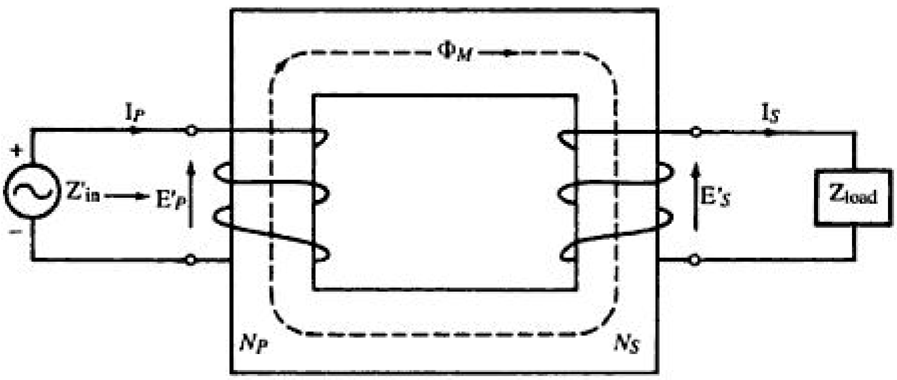


FIGURE 2.7
Ideal transformer.

about the turns ratio is not available, and voltage measurements at no load cannot be made, the nameplate voltage ratio can be used as an approximation of the turns ratio.

Thus, in terms of high-side (HS) and low-side (LS) values,

$$a = \frac{N_{HS}}{N_{LS}} = \frac{E'_{HS}}{E'_{LS}} \approx \frac{V_{HS}}{V_{LS}} \tag{2–14}$$

where: a = turns ratio
V_{HS}/V_{LS} = nameplate voltage ratio
E'_{HS}/E'_{LS} = ratio of induced voltages

Thus, referring to the ideal transformer in Figure 2.7, and *assuming* the primary is the high-voltage winding,

$$\frac{\mathbf{E}'_P}{\mathbf{E}'_S} = \frac{N_P}{N_S} = a$$

$$\mathbf{E}'_P = a\mathbf{E}'_S$$

Input Impedance of an Ideal Transformer

The input impedance looking into the primary terminals of the ideal transformer shown in Figure 2.7 is

$$\mathbf{Z}'_{in} = \frac{\mathbf{E}'_P}{\mathbf{I}_P} \tag{2–15}$$

Voltages $\mathbf{E}'_P$ and $\mathbf{E}'_S$ are induced by the same flux, and thus must have the same phase angle. Hence,

$$\frac{\mathbf{E}'_P}{\mathbf{E}'_S} = \frac{E'_P\angle\alpha}{E'_S\angle\alpha} = a$$

$$\mathbf{E}'_P = a\mathbf{E}'_S \tag{2–16a}$$

The apparent power input to the ideal transformer must equal the apparent power output. Expressed as phasor power (see Appendix A–5),

$$\mathbf{E}'_P\mathbf{I}^*_P = \mathbf{E}'_S\mathbf{I}^*_S$$

$$\mathbf{I}^*_P = \frac{\mathbf{E}'_S}{\mathbf{E}'_P}\cdot\mathbf{I}^*_S \quad \Rightarrow \quad \mathbf{I}^*_P = \frac{1}{a}\mathbf{I}^*_S$$

Therefore,

$$\mathbf{I}_P = \frac{1}{a}\mathbf{I}_S \tag{2–16b}$$

Substituting Eqs. (2–16a) and (2–16b) into Eq. (2–15),

$$\mathbf{Z}'_{in} = \frac{a\mathbf{E}'_S}{\mathbf{I}_S/a} = a^2\frac{\mathbf{E}'_S}{\mathbf{I}_S} \tag{2–17}$$

Applying Ohm's law to the secondary circuit in Figure 2.7,

$$\mathbf{Z}_{\text{load}} = \frac{\mathbf{E}'_S}{\mathbf{I}_S} \tag{2–18}$$

Substituting Eq. (2–18) into Eq. (2–17),

$$\mathbf{Z}'_{\text{in}} = a^2\mathbf{Z}_{\text{load}} \tag{2–19}$$

Equation (2–19) indicates that a well-designed transformer, with very low leakage flux, can be used as an *impedance multiplier.* The multiplication factor is equal to the square of the turns ratio. Transformers specifically designed for this purpose are called *impedance-matching transformers,* and have applications in audio systems **[2]**.

EXAMPLE 2.4 An ideal transformer with a primary of 200 turns and a secondary of 20 turns has its primary connected to a 120-V, 60-Hz supply, and its secondary connected to a $100\angle 30°$-Ω load. Determine (a) the secondary voltage; (b) the load current; (c) the input current to the primary; (d) the input impedance looking into the primary terminals. *Note:* For purposes of simplification in problem solving, unless otherwise specified it will be assumed (throughout the text) that the phase angle of the input voltage is zero degrees.

Solution

(a) Using Figure 2.7 as a guide,

$$a = \frac{N_{\text{HS}}}{N_{\text{LS}}} = \frac{200}{20} = 10$$

$$\frac{E'_P}{E'_S} = a \quad \Rightarrow \quad E'_{\text{LS}} = \frac{120}{10} = \underline{12 \text{ V}}$$

(b)
$$\mathbf{I}_S = \frac{\mathbf{E}'_S}{\mathbf{Z}_{\text{load}}} = \frac{12\angle 0°}{100\angle 30°} = \underline{0.12\angle -30° \text{ A}}$$

(c)
$$\mathbf{I}_P = \frac{1}{a}\cdot \mathbf{I}_S = \frac{0.12\angle -30°}{10} = \underline{0.012\angle -30 \text{ A}}$$

(d)
$$\mathbf{Z}'_{\text{in}} = a^2\mathbf{Z}_{\text{load}} = 10^2 \times 100\angle 30° = \underline{10\angle 30° \text{ k}\Omega}$$

2.9 LEAKAGE REACTANCE AND THE EQUIVALENT CIRCUIT OF A REAL TRANSFORMER

Calculations to determine the overall voltage drop in a transformer, for different magnitudes and different power factors of loads, must take into consideration the effect of leakage flux. To facilitate such calculations, voltage drops caused by leakage flux are expressed in terms of fictitious *leakage reactances:* these derived mathematical quantities, when multiplied by the current in them, will result in voltage drops equal to those brought about by the respective leakage fluxes.

The voltage generated by the sinusoidal variation of flux through the window of any coil is expressed as[7]

$$e = 2\pi f N\Phi_{max}\cos(2\pi f t)$$
$$E_{max} = 2\pi f N\Phi_{max} \qquad (2\text{–}30)$$

Expressing equation set (2–29) in terms of maximum flux (I_{max} causes Φ_{max}), substituting each (in turn) in Eq. (2–30), and using appropriate subscripts,

$$E_{\ell p,max} = 2\pi f N_P\left(\frac{N_P I_{P,max}}{\mathcal{R}_{\ell p}}\right) \qquad E_{\ell s,max} = 2\pi f N_S\left(\frac{N_S I_{S,max}}{\mathcal{R}_{\ell s}}\right)$$

$$E_{\ell p,max} = 2\pi f\left(\frac{N_P^2}{\mathcal{R}_{\ell p}}\right)I_{P,max} \qquad E_{\ell s,max} = 2\pi f\left(\frac{N_S^2}{\mathcal{R}_{\ell s}}\right)I_{S,max}$$

Dividing both sides of each equation by $\sqrt{2}$ to obtain rms values,

$$E_{\ell p} = 2\pi f\left(\frac{N_P^2}{\mathcal{R}_{\ell p}}\right)I_P \qquad E_{\ell s} = 2\pi f\left(\frac{N_S^2}{\mathcal{R}_{\ell s}}\right)I_S \qquad (2\text{–}31)$$

The inductance of a coil is related to the number of turns in the coil and the reluctance of its magnetic circuit in the following manner [2]:

$$L = \frac{N^2}{\mathcal{R}} \qquad (2\text{–}32)$$

Substituting Eq. (2–32) into equation set (2–31),

$$E_{\ell p} = (2\pi f L_{\ell p})I_P \qquad E_{\ell s} = (2\pi f L_{\ell s})I_S \qquad (2\text{–}33)$$

Thus,

$$E_{\ell p} = I_P X_{\ell p} \qquad E_{\ell s} = I_S X_{\ell s} \qquad (2\text{–}34)$$

where: $E_{\ell p}$ = leakage voltage of primary (rms)
$E_{\ell s}$ = leakage voltage of secondary (rms)
$X_{\ell p} = 2\pi f L_{\ell p}$ = leakage reactance of primary (Ω)
$X_{\ell s} = 2\pi f L_{\ell s}$ = leakage reactance of secondary (Ω)
$L_{\ell p}$ = leakage inductance of primary (H)
$L_{\ell s}$ = leakage inductance of secondary (H)

As indicated in equation set (2–34), the voltage drops due to leakage flux may be expressed in terms of the respective leakage reactances and the associated primary and secondary currents.

The final two-winding equivalent circuit of the transformer, expressing the voltage drops due to leakage flux in terms of leakage reactance drops, is shown in Figure 2.9. The parallel branch, representing the path for exciting current $\mathbf{I}_0$, contains

[7] See Section 1.12, Chapter 1.

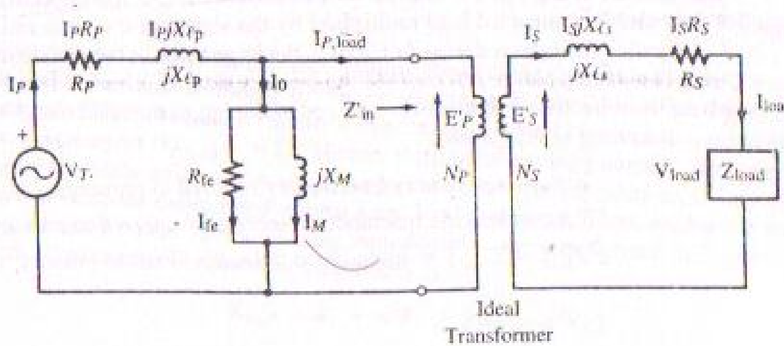


FIGURE 2.9
Leakage reactance in equivalent-circuit model.

a fictitious resistance R_{fe} that dissipates heat energy at the same rate as does the actual hysteresis and eddy-current losses in the core, and a fictitious magnetizing reactance X_M that draws the same magnetizing current as does the actual transformer.

2.10 EQUIVALENT IMPEDANCE OF A TRANSFORMER

The equivalent-circuit models of the transformer, shown in Figures 2.8(b) and 2.9, are very useful for analyzing the individual effects of winding resistance and leakage effects in primary and secondary windings. For purposes of simplified calculations of engineering problems, however, the actual transformer is replaced by an equivalent impedance in series with the source voltage and the load.

Applying the relationships developed for the ideal transformer in Section 2.8 to the ideal transformer in Figure 2.9,

$$\mathbf{Z}'_{in} = a^2\frac{\mathbf{E}'_S}{\mathbf{I}_S} \qquad (2\text{–}17)$$

Applying Ohm's law to the secondary circuit in Figure 2.9,

$$\mathbf{I}_S = \frac{\mathbf{E}'_S}{R_S + jX_{\ell s} + \mathbf{Z}_{load}} \qquad (2\text{–}35)$$

$$\frac{\mathbf{E}'_S}{\mathbf{I}_S} = R_S + jX_{\ell s} + \mathbf{Z}_{load} \qquad (2\text{–}36)$$

Substituting Eq. (2–36) into Eq. (2–17) and multiplying through,

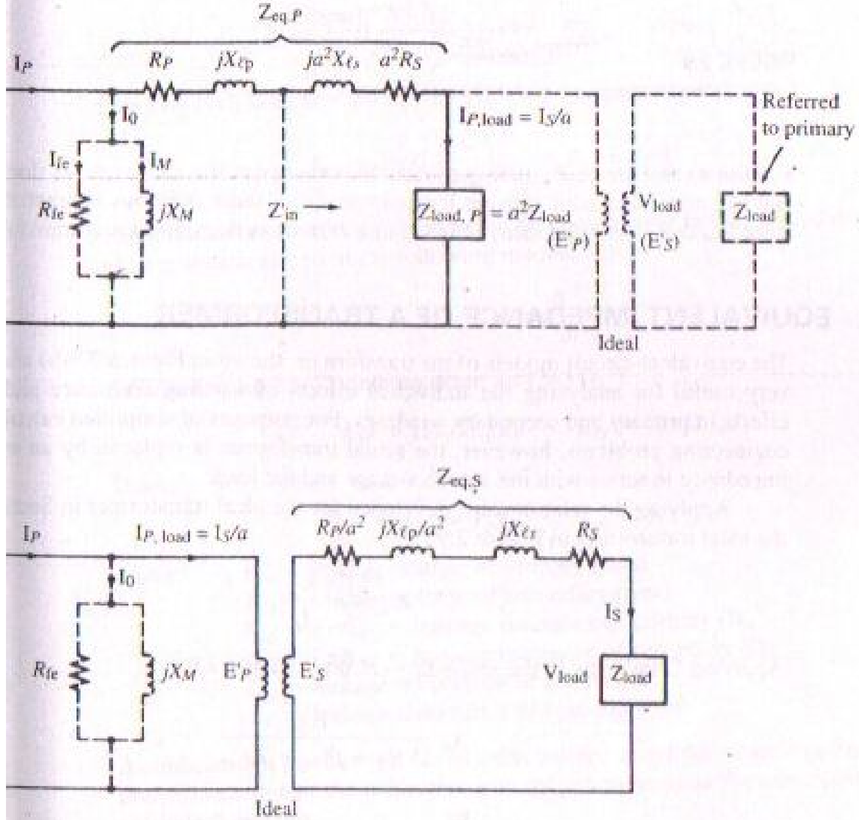
$$\mathbf{Z}'_{in} = a^2(R_S + jX_{\ell s} + \mathbf{Z}_{load}) \qquad (2\text{–}37)$$

$$\mathbf{Z}'_{in} = a^2R_S + ja^2X_{\ell s} + a^2\mathbf{Z}_{load} \qquad (2\text{–}38)$$

As indicated in Eqs. (2–37) and (2–38), impedance $\mathbf{Z}'_{in}$ is the impedance of the secondary and its connected load multiplied by the square of the turns ratio. Impedance $\mathbf{Z}'_{in}$ is called the *reflected impedance*; it is the impedance of the secondary and its connected load reflected (or *referred*) to the primary side. This is shown in Figure 2.10(a), where the reflected impedances are placed to the left of the ideal transformer.

Referring to Eq. (2–38),

$$a^2R_S = \text{resistance of secondary } referred \text{ to primary}$$
$$a^2X_{\ell_S} = \text{leakage reactance of secondary } referred \text{ to primary}$$
$$a^2Z_{load} = Z_{load,P} = \text{impedance of load } referred \text{ to primary}$$



E 2.10
nt circuits: (a) parameters referred to primary; (b) parameters referred to secondary.

Note that the parallel branch representing the exciting current in Figure 2.9 is shown shifted to the input terminals of the transformer in Figure 2.10. When operating at or near rated load, the load component of primary current is significantly greater than the exciting current ($\mathbf{I}_{P,load} >> \mathbf{I}_0$). Hence, shifting the exciting current component to the input terminals will not cause any appreciable error in calculations involving transformer behavior under rated or near-rated load conditions. Thus, neglecting the exciting current branch, the *equivalent impedance* of the transformer in Figure 2.10(a), with all parameters referred to the primary is

$$\mathbf{Z}_{eq,P} = R_P + a^2R_S + j(X_{\ell_P} + a^2X_{\ell_S}) \tag{2–39}$$

$$\mathbf{Z}_{eq,P} = R_{eq,P} + jX_{eq,P} \tag{2–40}$$

Figure 2.10(b) shows an equivalent circuit with all parameters *referred to the secondary*, where

$$R_P/a^2 = \text{resistance of primary referred to secondary}$$
$$X_{\ell_P}/a^2 = \text{leakage reactance of primary referred to secondary}$$

Thus, the equivalent impedance of the transformer shown in Figure 2.10(b), with the primary parameters referred to the secondary, is

$$\mathbf{Z}_{eq,S} = R_S + R_P/a^2 + j(X_{\ell_S} + X_{\ell_P}/a^2) \tag{2–41}$$

$$\mathbf{Z}_{eq,S} = R_{eq,S} + jX_{eq,S} \tag{2–42}$$

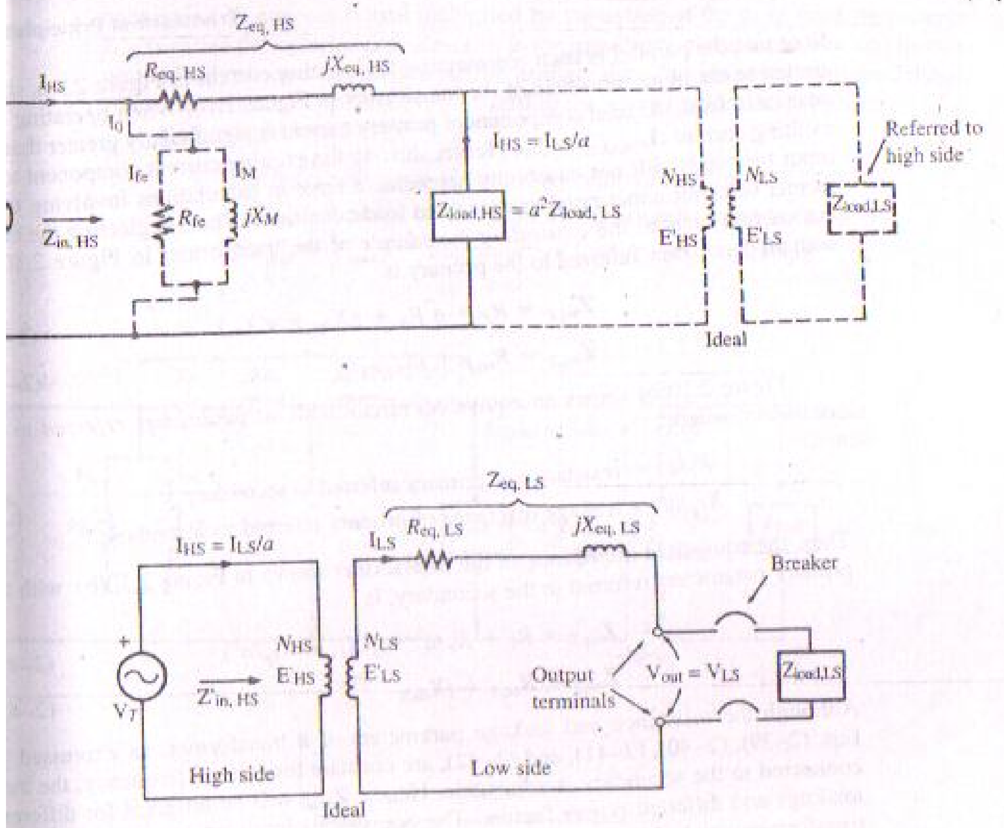
Although the resistance and leakage parameters of a transformer, as expressed in Eqs. (2–39), (2–40), (2–41), and (2–42), are constant for a given frequency, the load connected to the secondary is adjustable. Hence, $\mathbf{Z}_{load}$ will be different for different loadings and different power factors. The equivalent-circuit parameters for a given transformer may be obtained from the transformer nameplate, from the manufacturer, or from a test procedure outlined in Section 2.14.

High-Side, Low-Side

Power and distribution transformers may be used to either step up or step down voltage. Hence, it is convenient to refer to the two windings as the high-voltage side (HS) and the low-voltage side (LS). This is shown in Figure 2.11 for *step-down* operation and is a modification of Figure 2.10.

The exciting current branch, shown with broken lines in Figure 2.11(a), may be omitted when making calculations involving operations at or near rated load; for such loadings, the load component of primary current is so much greater than the exciting current that the exciting current may be neglected. However, *when making calculations for loadings less than 25 percent rated load, the no-load components must be considered if significant errors in current calculations are to be avoided.*

The same circuits shown in Figure 2.11 for step-down operation may also be used for step-up operations by referring the transformer parameters to the low side, as



URE 2.11
valent circuits for step-down operation: (a) in terms of high-side values; (b) in terms of low-side values.

shown in Figure 2.12, and converting the high-side load impedance to the low side using

$$Z_{load,LS} = \frac{1}{a^2} Z_{load,HS} \tag{2–43}$$

Note that the lower voltage rating and higher current rating of the low side (compared to the high side) requires the low side to have fewer turns of larger cross-sectional area conductor. Hence, the equivalent impedance of the transformer referred to the low side will always be less than the equivalent impedance referred to the high side.

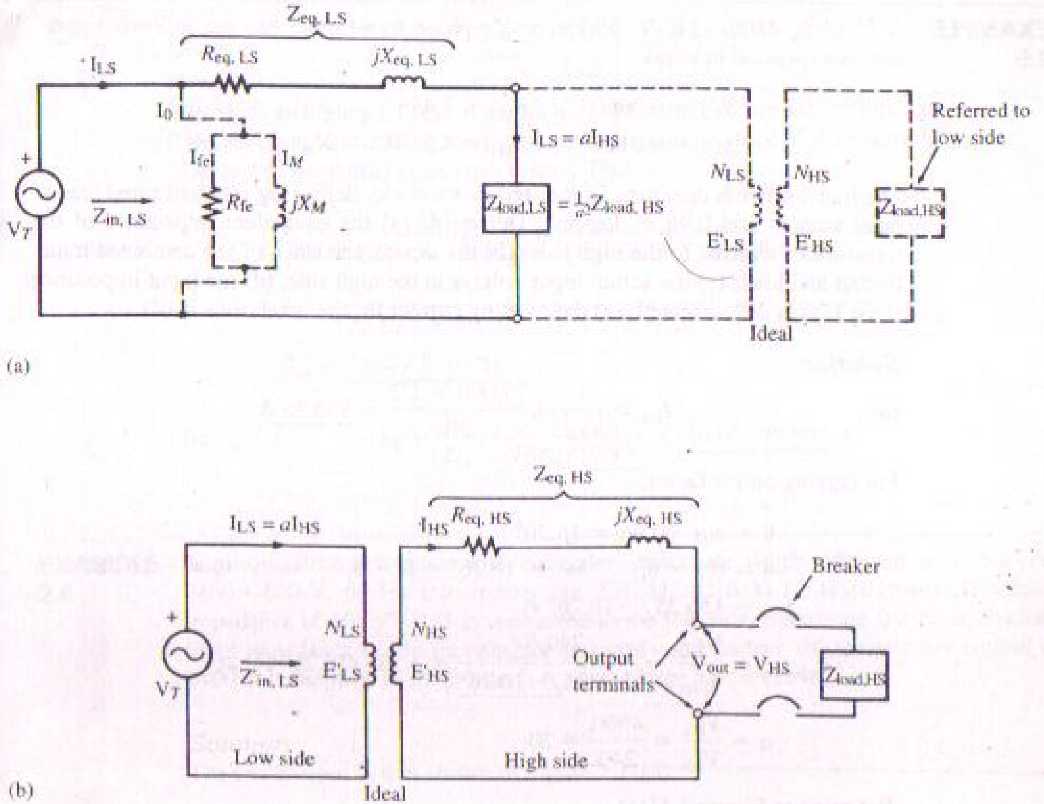


FIGURE 2.12
Equivalent circuits for step-up operation: (a) in terms of low-side values; (b) in terms of high-side values.

$$Z_{eq,LS} = \frac{Z_{eq,HS}}{a^2} \tag{2–43a}$$

The circuit models shown in Figures 2.11(a) and 2.12(a) are used to determine the input impedance of the combined transformer and load. The circuit models shown in Figures 2.11(b) and 2.12(b) are used to determine the no-load voltage and the voltage regulation.[8]

[8] See Section 2.11 for a discussion of voltage regulation.

EXAMPLE 2.5 A 75-kVA, 4800—240-V, 60-Hz, single-phase transformer has the following parameters expressed in ohms:

$$R_{LS} = 0.00600 \quad R_{HS} = 2.488 \quad R_{fe,HS} = 44{,}202$$
$$X_{LS} = 0.0121 \quad X_{HS} = 4.8384 \quad X_{M,HS} = 7798.6$$

The transformer is operating in the step-down mode, delivering one-half rated load at rated voltage and 0.96 F_p lagging. Determine (a) the equivalent impedance of the transformer referred to the high side; (b) the input impedance of the combined transformer and load; (c) the actual input voltage at the high side; (d) the input impedance if the load is disconnected; (e) the exciting current for the conditions in (d).

Solution

(a)
$$I_{LS} = \frac{S}{V_{LS}} = \frac{75{,}000 \times 1/2}{240} = \underline{156.25\text{ A}}$$

For lagging power factor,

$$\theta = \cos^{-1}0.96 = 16.26°$$
$$\theta = (\theta_v - \theta_i) \quad \Rightarrow \quad 16.26° = 0 - \theta_i \quad \Rightarrow \quad \theta_i = -16.26°$$
$$\mathbf{I}_{LS} = 156.25\underline{/-16.26°}\text{ A}$$
$$\mathbf{Z}_{load,LS} = \frac{\mathbf{V}_{load}}{\mathbf{I}_{load}} = \frac{240\underline{/0°}}{156.25\underline{/-16.26°}} = 1.536\underline{/16.26°}\ \Omega$$
$$a \approx \frac{V_{HS}}{V_{LS}} = \frac{4800}{240} = 20$$

Referring to Figure 2.11(a),

$$\mathbf{Z}_{eq,HS} = R_{eq,HS} + jX_{eq,HS} = R_{HS} + a^2R_{LS} + j(X_{HS} + a^2X_{LS})$$
$$\mathbf{Z}_{eq,HS} = 2.488 + 20^2(0.00600) + j(4.8384 + 20^2(0.0121))$$
$$\mathbf{Z}_{eq,HS} = 4.888 + j9.678 = \underline{10.84\underline{/63.2°}\ \Omega}$$

(b) Referring to Figure 2.11(a), and neglecting the exciting current branch,

$$\mathbf{Z}_{load,HS} = a^2\mathbf{Z}_{load,LS} = 20^2 \times 1.536\underline{/16.26°} = 614.40\ \underline{/16.26°}\ \Omega$$
$$\mathbf{Z}_{load,HS} = 589.82 + j172.03\ \Omega$$
$$\mathbf{Z}_{in} = \mathbf{Z}_{load,HS} + \mathbf{Z}_{eq,HS} = (589.82 + j172.03) + (4.888 + j9.678)$$
$$\mathbf{Z}_{in} = 594.71 + j181.71 = \underline{621.85\underline{/16.99°}\ \Omega}$$

(c)
$$I_{HS} = \frac{I_{LS}}{a} = \frac{156.25}{20} = 7.81\text{ A}$$
$$V_T = I_{HS}Z_{in} = 7.81 \times 621.85 = \underline{4857\text{ V}}$$

(d) With the load disconnected, the output load and current are zero. Thus,

$$Z_{load} = \frac{V_{load}}{I_{load}} = \frac{V_{load}}{0} = \infty$$

Referring to Figure 2.11(a), with the load disconnected, the net impedance looking into the input terminals of the transformer is the impedance of the exciting branch. The rest of the model is an open circuit. Thus,

$$\mathbf{Z}_{in} = \frac{1}{(1/R_{fe}) + (1/jX_M)} = \frac{1}{(1/44{,}202) + (1/j7798.6)}$$
$$\mathbf{Z}_{in} = \frac{1}{22.623 \times 10^{-6} - j128.228 \times 10^{-6}} = \frac{10^6}{130.208\underline{/-79.99°}}$$
$$\mathbf{Z}_{in} = \underline{7680\underline{/79.99°}\ \Omega}$$

(e)
$$\mathbf{I}_0 = \frac{\mathbf{V}_T}{\mathbf{Z}_{in}} = \frac{4857\underline{/0°}}{7680\underline{/79.99°}} = \underline{0.63\underline{/-79.99°}\text{ A}}$$

EXAMPLE 2.6 The equivalent resistance and equivalent reactance (high side) for a 37.5-kVA, 2400—600-V, 60-Hz transformer are 2.80 Ω and 6.00 Ω, respectively. If a load impedance of $10.0\underline{/20°}$ Ω is connected to the low side, determine (a) the equivalent input impedance of the transformer and load combination; (b) the primary current if 2400 V is supplied to the primary; (c) the voltage across the load.

Solution

The circuit used is that shown in Figure 2.11(a).

(a)
$$a \approx \frac{V_{HS}}{V_{LS}} = \frac{2400}{600} = 4.0$$
$$\mathbf{Z}_{load,HS} = a^2\mathbf{Z}_{load,LS} = 4^2 \times 10\underline{/20°} = 160\underline{/20°} = (150.351 + j54.723)\ \Omega$$
$$\mathbf{Z}_{in} = 2.8 + j6.0 + 150.351 + j54.723 = \underline{164.75\underline{/21.63°}\ \Omega}$$

(b)
$$\mathbf{I}_{HS} = \frac{\mathbf{V}_T}{\mathbf{Z}_{in}} = \frac{2400\underline{/0°}}{164.75\underline{/21.63°}} = \underline{14.57\underline{/-21.63°}\text{ A}}$$

(c) Referring to Figure 2.11(a), the voltage *across the reflected load* is

$$\mathbf{E}'_{HS} = \mathbf{I}_{HS}a^2\mathbf{Z}_{load,LS} = 14.57\underline{/-21.63°} \times 4^2 \times 10\underline{/20°} = \underline{2330.8\underline{/-1.63°}\text{ V}}$$

The actual voltage across the *real load* is the voltage at the secondary of the ideal transformer in Figure 2.11(a). Thus,

$$\mathbf{E}'_{LS} = \frac{\mathbf{E}'_{HS}}{a} = \frac{2330.8\underline{/-1.63°}}{4} = \underline{582.7\underline{/-1.63°}\text{ V}}$$

.11 VOLTAGE REGULATION

The effects of leakage flux and winding resistance in a transformer cause internal voltage drops that result in different output voltages for different loads. *The difference between the output voltage at no load and the output voltage at rated load, divided by the output voltage at rated load, is called the voltage regulation* of the transformer, and is commonly used as a figure of merit when comparing transformers [2]. Expressed mathematically,

$$\text{reg} = \frac{E_{\text{nl}} - V_{\text{rated}}}{V_{\text{rated}}} \tag{2–44}$$

where: E_{nl} = voltmeter reading at the output terminals when no load is connected to the transformer

V_{rated} = voltmeter reading at the output terminals when the transformer is supplying rated apparent power

Although Eq. (2–44) expresses the voltage regulation in decimal form, called *per-unit regulation*, it may also be expressed in percent.

The no-load and full-load voltages in Eq. (2–44) must be *all* high-side values or *all* low-side values. The voltage regulation will be the same, however, whether all high-side values or all low-side values are used.

The voltage regulation of a transformer, along with voltage, current, frequency, and apparent power ratings, are required data when specifying replacement transformers, when selecting transformers for parallel operation, when selecting transformers for polyphase arrangements, or when selecting transformers that will be used in distribution systems that feed large induction motors.

Although the regulation of a transformer may be determined from a set of no-load and full-load voltage measurements, as expressed in Eq. (2–44), this requires loading the transformer to its rated value at the desired power factor. Since this is seldom easy to accomplish, and in most cases is impractical, a mathematical determination using the equivalent circuit in Figure 2.11(b) or Figure 2.12(b) is preferred.

Referring to Figure 2.11(b), $\mathbf{E}'_{\text{LS}}$ is the no-load voltage. It is the voltage that appears across the output terminals when the load is removed (circuit breaker open); removing the load causes $\mathbf{I}_{\text{LS}} = 0$, which causes $\mathbf{I}_{\text{LS}}\mathbf{Z}_{\text{eq,LS}} = 0$, resulting in an output voltage equal to $\mathbf{E}'_{\text{LS}}$. Thus, the no-load voltage for *rated load conditions at a specified power factor* is determined by applying Kirchhoff's voltage law to the secondary and solving for $\mathbf{E}'_{\text{LS}}$. Referring to Figure 2.11(b), and assuming *rated load* on the secondary,

$$\mathbf{E}'_{\text{LS}} = \mathbf{I}_{\text{LS}}\mathbf{Z}_{\text{eq,LS}} + \mathbf{V}_{\text{LS}} \tag{2–45}$$

where: $\mathbf{I}_{\text{LS}}$ = rated low-side current at specified power factor

$\mathbf{V}_{\text{LS}}$ = rated low-side voltage (output V, breaker closed)

$\mathbf{E}'_{\text{LS}}$ = no-load low-side voltage (output V, breaker open)

$\mathbf{Z}_{\text{eq,LS}}$ = equivalent impedance of transformer referred to low side

EXAMPLE 2.7

The equivalent low-side parameters of a 250-kVA, 4160—480-V, 60-Hz transformer are $R_{\text{eq,LS}} = 0.00920\ \Omega$, and $X_{\text{eq,LS}} = 0.0433\ \Omega$. The transformer is operating in the step-down mode and is delivering rated current at rated voltage to a 0.840 power-factor lagging load. Determine (a) the no-load voltage; (b) the actual input voltage at the high side; (c) the high-side current; (d) the input impedance; (e) the voltage regulation; (f) the voltage regulation if the power factor of the load is 0.840 leading; (g) sketch the tip-to-tail phasor diagram of the secondary circuit for the 0.840 power-factor lagging load. Show all voltage drops.

Solution

(a)
$$I_{\text{LS}} = \frac{250{,}000}{480} = 520.83\text{ A} \qquad \theta = \cos^{-1}0.840 = 32.86^\circ$$

For a lagging power-factor load, the load current lags the load voltage as shown in Figure 2.13(a). Thus, from Figure 2.13(a),

$$\mathbf{V}_{\text{LS}} = 480\underline{/0^\circ}\text{ V} \qquad \mathbf{I}_{\text{LS}} = 520.83\underline{/-32.86^\circ}\text{ A}$$

Using Figure 2.11(b) as a guide,

$$\mathbf{E}'_{\text{LS}} = \mathbf{I}_{\text{LS}}R_{\text{eq,LS}} + \mathbf{I}_{\text{LS}}jX_{\text{eq,LS}} + \mathbf{V}_{\text{LS}}$$
$$\mathbf{E}'_{\text{LS}} = 520.83\underline{/-32.86^\circ} \times 0.0092 + 520.83\underline{/-32.86^\circ} \times j0.0433 + 480\underline{/0^\circ}$$
$$\mathbf{E}'_{\text{LS}} = 4.79\underline{/-32.86^\circ} + 22.55\underline{/57.14^\circ} + 480\underline{/0^\circ}$$
$$\mathbf{E}'_{\text{LS}} = 4.024 - j2.599 + 12.235 + j18.94 + 480 + j0 = 496.53\underline{/1.886^\circ}\text{ V}$$

(b)
$$a = \frac{E'_{\text{HS}}}{E'_{\text{LS}}} = \frac{V_{\text{HS}}}{V_{\text{LS}}} = \frac{4160}{480} = 8.667$$

From Figure 2.11(b),

$$\mathbf{V}_T = \mathbf{E}'_{\text{HS}} = a\mathbf{E}'_{\text{LS}} = 8.667 \times 496.53\underline{/1.886^\circ} = 4303.4\underline{/1.886^\circ}\text{ V}$$

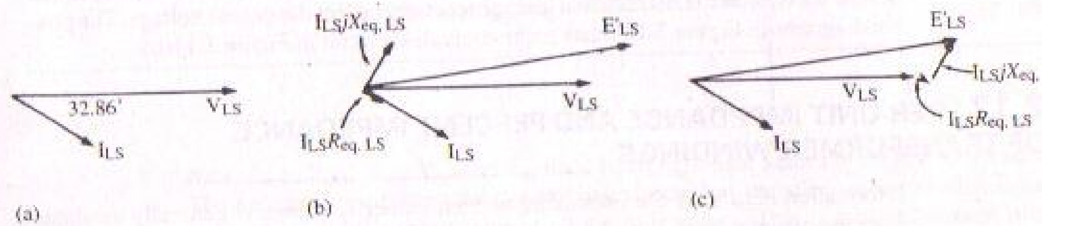


FIGURE 2.13
Phasor diagrams for Example 2.7: (a) low-side output; (b) component phasors; (c) tip-to-tail addition.

(c) $$\mathbf{I}_{HS} = \frac{\mathbf{I}_{LS}}{a} = \frac{520.83\angle -32.86^\circ}{8.667} = 60.09\angle -32.86^\circ \text{ A}$$

(d) From Figure 2.11(b),

$$\mathbf{Z}'_{in} = \frac{\mathbf{V}_T}{\mathbf{I}_{HS}} = \frac{4303.4\angle 1.886^\circ}{60.09\angle -32.86^\circ} = 71.62\angle 34.74^\circ\ \Omega$$

(e) $$\text{reg} = \frac{E_{nl} - V_{rated}}{V_{rated}} = \frac{496.53 - 480}{480} = 0.0344 \quad \text{or} \quad 3.44\%$$

(f) For a 0.840 leading power factor, $\mathbf{I}_{LS} = 520.83\angle +32.86^\circ$. Thus,

$$\mathbf{E}'_{LS} = 520.83\angle +32.86^\circ \times 0.0092 + 520.83\angle +32.86^\circ \times j0.0433 + 480\angle 0^\circ$$

$$\mathbf{E}'_{LS} = 4.79\angle 32.86^\circ + 22.55\angle 122.86^\circ + 480\angle 0^\circ$$

$$\mathbf{E}'_{LS} = 4.024 + j2.599 + (-12.235 + j18.94) + 480 + j0 = 472.28\angle 2.61^\circ$$

$$\text{reg} = \frac{E_{nl} - V_{rated}}{V_{rated}} = \frac{472.28 - 480}{480} = -0.0161 \quad \text{or} \quad -1.61\%$$

Note that the effect of a leading power-factor load is to cause a voltage rise in the transformer, resulting in a negative regulation. The voltage rise is a resonance effect caused by the combined leakage reactance of the transformer and the capacitance characteristic of the load.

(g) The voltage drops due to the equivalent resistance and equivalent reactance are

$$\mathbf{I}_{LS}R_{eq,LS} = 520.83\angle -32.86^\circ \times 0.0092 = 4.79\angle -32.86^\circ \text{ V}$$

$$\mathbf{I}_{LS}X_{eq,LS} = 520.83\angle -32.86^\circ \times j0.0433 = 22.6\angle 57.14^\circ \text{ V}$$

A phasor diagram showing the component voltages for the 0.840 power-factor lagging load is shown in Figure 2.13(b) and the corresponding tip-to-tail diagram is shown in Figure 2.13(c). Although the diagrams are not drawn to scale, because of the large difference in magnitudes between V_{LS} and the voltage drops, it does provide a perspective of how transformer resistance and leakage reactance affect the output voltage. The phasor diagrams in Figure 2.13 relate to the equivalent circuit in Figure 2.11(b).

.12 PER-UNIT IMPEDANCE AND PERCENT IMPEDANCE F TRANSFORMER WINDINGS

Information regarding the impedance of transformer windings is generally available from the manufacturer, or from the transformer nameplate as per-unit (PU) impedance or percent impedance.[9] Per-unit impedance (Z_{PU}) also called per-unit impedance

[9] Percent values are per-unit values times 100.

voltage, is the ratio of the voltage drop within the transformer caused by transformer impedance, to the rated voltage of the transformer, when operating at rated current. Thus,

$$\left.\begin{aligned} Z_{PU} &= \frac{I_{rated}Z_{eq}}{V_{rated}} \\ R_{PU} &= \frac{I_{rated}R_{eq}}{V_{rated}} \\ X_{PU} &= \frac{I_{rated}X_{eq}}{V_{rated}} \end{aligned}\right\} \qquad (2\text{–}46)$$

where: Z_{PU} = per-unit impedance
R_{PU} = per-unit resistance
X_{PU} = per-unit reactance

Note: V_{rated} and I_{rated} are also called *base voltage* and *base current*, respectively.

Per-unit impedance of a transformer is often expressed in terms of a *base impedance* obtained from the transformer rating:

$$Z_{base} = \frac{V_{rated}}{I_{rated}} \qquad (2\text{–}47)$$

To express Z_{base} in terms of transformer apparent power, multiply the numerator and denominator of Eq. (2–47) by V_{rated}. Thus,

$$Z_{base} = \frac{V^2_{rated}}{V_{rated} \cdot I_{rated}} = \frac{V^2_{rated}}{S_{rated}} \qquad (2\text{–}48)$$

Solving Eq. (2–47) for V_{rated}, and substituting into equation set (2–46), expresses the per-unit values of Z, R, and X in terms of the base impedance:

$$\left.\begin{aligned} Z_{PU} &= \frac{I_{rated}Z_{eq}}{I_{rated}Z_{base}} = \frac{Z_{eq}}{Z_{base}} \\ R_{PU} &= \frac{I_{rated}R_{eq}}{I_{rated}Z_{base}} = \frac{R_{eq}}{Z_{base}} \\ X_{PU} &= \frac{I_{rated}X_{eq}}{I_{rated}Z_{base}} = \frac{X_{eq}}{Z_{base}} \end{aligned}\right\} \qquad (2\text{–}49)$$

Note: I_{rated}, V_{rated}, R_{eq}, X_{eq}, and Z_{eq} must be all high-side values or all low-side values. The per-unit impedance (and percent impedance) has the same value whether calculated using all high-side values or all low-side values. This is a big advantage when making calculations involving systems that have more than one transformer, each at a different voltage level.

The per-unit system has its greatest application in the solution of network problems involving several voltage levels and it is used extensively in power system analysis [4]. All per-unit values are dimensionless.

The per-unit impedance, in terms of its components, is

$$Z_{PU} = R_{PU} + jX_{PU} \tag{2–50}$$

$$Z_{PU} = \sqrt{R_{PU}^2 + X_{PU}^2} \tag{2–51}$$

$$\alpha = \tan^{-1}\left(\frac{X_{PU}}{R_{PU}}\right) \tag{2–52}$$

Angle α is the phase angle of the per-unit impedance, and is the same for percent impedance and equivalent impedance.[10]

Transformers rated above 100 kVA have conductors of such large cross-sectional area that $X_{PU} >> R_{PU}$. Thus, for very large transformers,

$$\underset{kVA>100}{Z_{PU}} \approx X_{PU} \tag{2–53}$$

Although Eq. (2–53) is a close approximation for large transformers, it is often applied to calculations involving smaller transformers if no other data are available.

EXAMPLE

The percent resistance and percent reactance of a 75-kVA, 2400—240-V, 60-Hz transformer are 0.90 and 1.30, respectively. Determine (a) percent impedance; (b) rated high-side current; (c) equivalent resistance and equivalent reactance referred to the high side; (d) high-side fault current if an accidental short circuit of 0.016 Ω (resistive) occurs at the secondary when 2300 V is impressed across the primary.

Solution

(a) $$Z = \sqrt{(R)^2 + (X)^2} = \sqrt{0.90^2 + 1.30^2} = \underline{1.58\%}$$

(b) $$I_{HS} = \frac{75{,}000}{2400} = \underline{31.25\text{ A}}$$

(c) $$R_{PU} = \frac{I_{HS}R_{eq,HS}}{V_{HS}} \qquad X_{PU} = \frac{I_{HS}X_{eq,HS}}{V_{HS}}$$

$$0.009 = \frac{31.25R_{eq,HS}}{2400} \qquad 0.013 = \frac{31.25X_{eq,HS}}{2400}$$

$$\underline{R_{eq,HS} = 0.691\ \Omega} \qquad \underline{X_{eq,HS} = 0.998\ \Omega}$$

[10] Impedance phase angle α may be approximated at 76° to 80° for single-phase transformers with apparent power ratings ≥500 kVA; 70° to 76° for apparent power ratings between 100 and 500 kVA. A table of representative impedances is given in Appendix J.

(d) The equivalent circuit is shown in Figure 2.14.

$$\mathbf{Z}_{in} = \mathbf{Z}_{eq,HS} + a^2\mathbf{Z}_{short} \qquad a = \frac{2400}{240} = 10$$

$$\mathbf{Z}_{in} = 0.691 + j0.998 + 10^2(0.016) = 2.499\underline{/23.54°}\ \Omega$$

$$\mathbf{I}_{HS} = \frac{\mathbf{V}_{HS}}{\mathbf{Z}_{HS}} = \frac{2300\underline{/0°}}{2.499\underline{/23.54°}} = \underline{920\underline{/-23.54°}\text{ A}}$$

Calculating Voltage Regulation From Per-Unit Values

The voltage regulation of a transformer (operating at rated voltage and rated current) may be determined from the power factor of the load and the known per-unit values of transformer reactance and resistance, without having to calculate load currents and voltage drops. Referring to Figure 2.15(a),

$$\mathbf{E}'_{LS} = \mathbf{I}_{LS}R_{eq,LS} + \mathbf{I}_{LS}jX_{eq,LS} + \mathbf{V}_{LS} \tag{2–54}$$

where: $\mathbf{V}_{LS}$ = output voltage, breaker closed
$\mathbf{E}'_{LS}$ = output voltage, breaker open

The component phasors in Eq. (2–54) are shown on the phasor diagram in Figure 2.15(b) for a lagging power-factor load, with the current phasor drawn as the reference phasor at 0°. The diagram is not drawn to scale.

The *magnitude* of the no-load low-side voltage is obtained by resolving $\mathbf{V}_{LS}$ into vertical and horizontal components, and applying the Pythagorean theorem. Thus, referring to Figure 2.15(b),

$$E'_{LS} = \sqrt{(I_{LS}R_{eq,LS} + V_{LS}\cos\theta)^2 + (I_{LS}X_{eq,LS} + V_{LS}\sin\theta)^2} \tag{2–55}$$

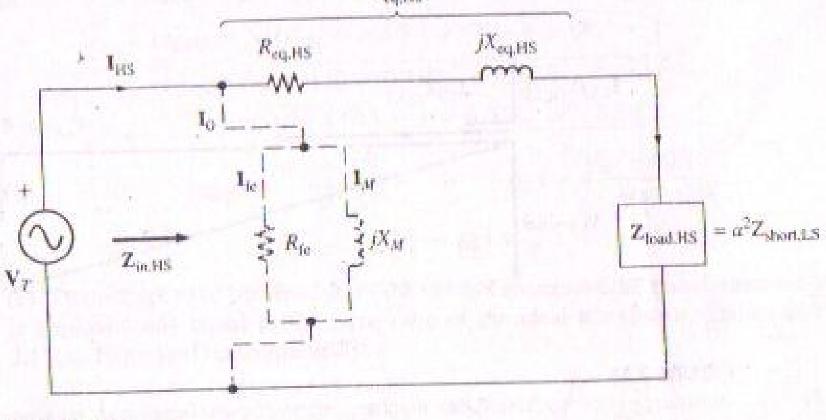


FIGURE 2.14
Equivalent circuit for Example 2.8.

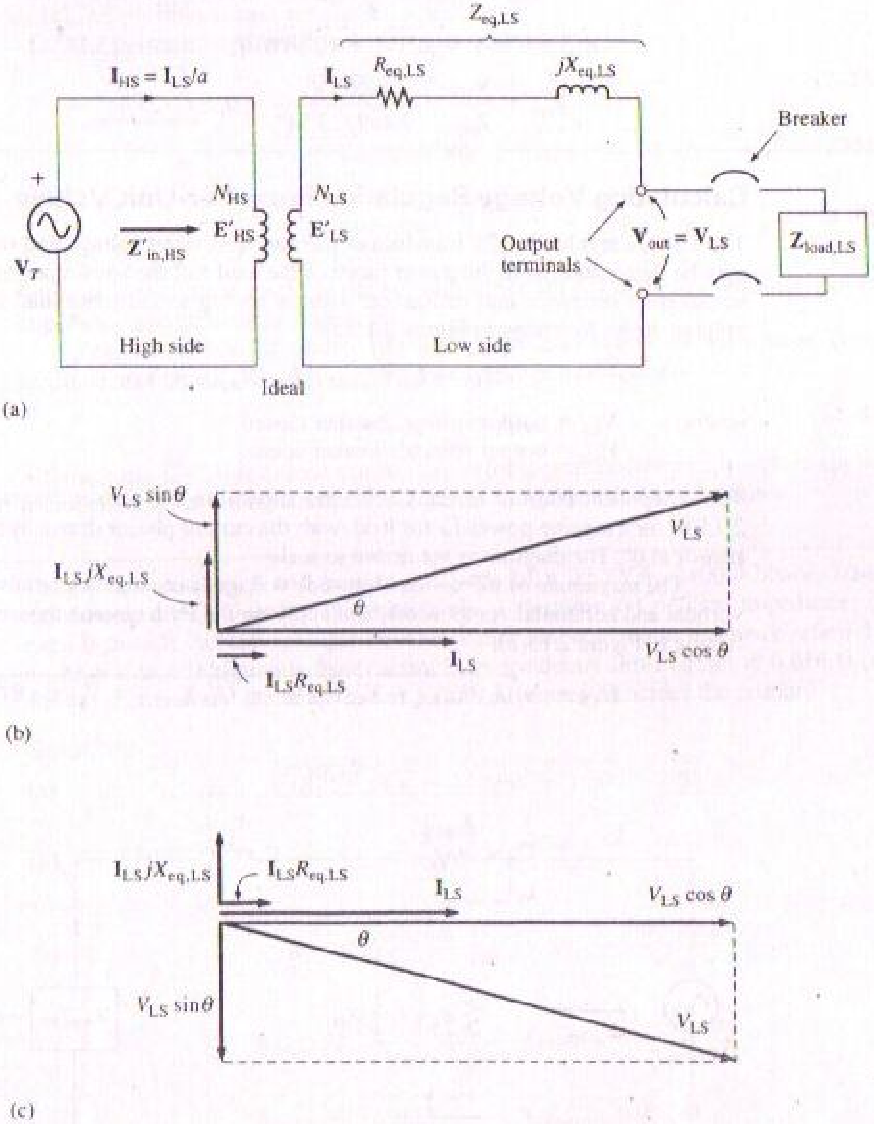


FIGURE 2.15
(a) Equivalent circuit; (b) phasor diagram, lagging power-factor load; (c) phasor diagram, leading power-factor load.

Substituting Eq. (2–55) into Eq. (2–44) and simplifying,

$$\text{reg}_{PU} = \frac{\sqrt{(I_{LS}R_{eq,LS} + V_{LS}\cos\theta)^2 + (I_{LS}X_{eq,LS} + V_{LS}\sin\theta)^2} - V_{LS}}{V_{LS}} \tag{2–56}$$

Dividing numerator and denominator by V_{LS},

$$\text{reg}_{PU} = \sqrt{\left(\frac{I_{LS}R_{eq,LS}}{V_{LS}} + \cos\theta\right)^2 + \left(\frac{I_{LS}X_{eq,LS}}{V_{LS}} + \sin\theta\right)^2} - 1 \tag{2–57}$$

Substituting appropriate equations from equation set (2–46) into Eq. (2–57),

$$\text{reg}_{PU} = \sqrt{(R_{PU} + \cos\theta)^2 + (X_{PU} + \sin\theta)^2} - 1 \tag{2–58}$$

Note: Angle θ, shown in Figure 2.15, is called the *power-factor angle;* it is positive for lagging power-factor loads, and negative for leading power-factor loads.[11] The cosine of the power-factor angle is the power factor. Expressed mathematically,

$$\theta = \cos^{-1}F_P \qquad \text{for lagging power-factor loads}$$
$$\theta = -\cos^{-1}F_P \qquad \text{for leading power-factor loads}$$

[11] See Appendix A.5 for the relationship between power, power factor, and power-factor angle.

EXAMPLE 2.9

A single-phase distribution transformer, rated at 50 kVA, 7200—600 V, is supplying rated kVA at 600 V and 0.75 power-factor lagging. The percent resistance and percent reactance are 1.3 and 3.8, respectively. Determine (a) transformer regulation; (b) secondary voltage when the load is disconnected; (c) input voltage that must be applied to the primary in order to obtain rated secondary voltage when carrying rated load at 0.75 power-factor lagging.

Solution

(a) $\theta = \cos^{-1}F_P = \cos^{-1}0.750 = 41.41°$ $\qquad \sin 41.41° = 0.661$

$$\text{reg}_{PU} = \sqrt{(R_{PU} + \cos\theta)^2 + (X_{PU} + \sin\theta)^2} - 1$$

$$\text{reg}_{PU} = \sqrt{0.0130 + 0.750)^2 + (0.038 + 0.661)^2} - 1$$

$$\text{reg}_{PU} = 1.035 - 1 = \underline{0.035} \text{ or } \underline{3.5\%}$$

(b) $$\text{reg}_{PU} = \frac{V_{nl} - V_{rated}}{V_{rated}} \Rightarrow 0.035 = \frac{V_{nl} - 600}{600}$$

$$V_{nl} = \underline{621 \text{ V}}$$

(c) The voltage ratio obtained from the voltage ratings on the transformer nameplate is approximately equal to the turns ratio of the ideal transformer shown in Figure 2.15(a). Expressed mathematically,

$$\frac{E'_{HS}}{E'_{LS}} = \frac{7200}{600} \Rightarrow E'_{HS} = E'_{LS} \times \frac{7200}{600} \tag{2–56a}$$

Referring to Figure 2.15(a), with no load connected to the secondary (load breaker open), $I_l = 0$. Thus, there are no voltage drops in the secondary circuit, and

$$E'_{LS} = V_{nl} = 621 \text{ V}$$

Substituting into Eq. (2–56a),

$$E'_{HS} = 621 \times \frac{7200}{600} = \underline{7452.5 \text{ V}}$$

XAMPLE 10 Assume the transformer in Example 2.9 is operating at rated kVA and 600 V, but the power factor of the load is 0.75 *leading*. Determine (a) transformer regulation; (b) secondary voltage when the load is disconnected; (c) input voltage that must be applied to the primary in order to obtain rated secondary voltage when carrying rated load at 0.75 power factor leading.

Solution

(a) $\theta = -\cos^{-1} F_P = -\cos^{-1} 0.750 = -41.41° \qquad \sin(-41.41°) = -0.661$

$$\text{reg}_{PU} = \sqrt{(R_{PU} + \cos\theta)^2 + (X_{PU} + \sin\theta)^2} - 1$$

$$\text{reg}_{PU} = \sqrt{(0.0130 + 0.750)^2 + (0.038 - 0.661)^2} - 1$$

$$\text{reg}_{PU} = 0.9853 - 1 = \underline{-0.0147 \text{ or } -1.5\%}$$

(b) $\text{reg}_{PU} = \dfrac{V_{nl} - V_{rated}}{V_{rated}} \quad \Rightarrow \quad -0.0147 = \dfrac{V_{nl} - 600}{600}$

$$V_{nl} = \underline{591.2 \text{ V}}$$

(c) $E'_{HS} = 591.2 \times \dfrac{7200}{600} = \underline{7094 \text{ V}}$

Note: As shown in part (b) of Example 2.9, for lagging power-factor loads the transformer regulation is positive; this is also true for unity power-factor loads. However, for loads that have sufficiently leading power factors, as shown in Example 2.10, the voltage regulation will be negative.

Voltage Regulation at Other Than Rated Load

Equation (2–58) is applicable to transformers operating at rated load. If operating at other than rated load, Eq. (2–58) must be modified to reflect the actual per-unit load connected to the secondary. Making the modification,

$$\text{reg}_{PU} = \sqrt{(S_{PU} \times R_{PU} + \cos\theta)^2 + (S_{PU} \times X_{PU} + \sin\theta)^2} - 1 \qquad (2\text{–}58a)$$

$$I_{PU} = \frac{I}{I_{rated}} = S_{PU} = \frac{S}{S_{rated}}$$

where: S_{PU} = per-unit apparent power of load (PU)
S = apparent power of load (VA)
S_{rated} = rated apparent power of transformer (VA)
I_{PU} = per-unit load current (PU)
I = load current (A)
I_{rated} = rated current of secondary (A)

EXAMPLE 2.11 A 25-kVA, 7620—480-V distribution transformer is supplying a 10-kVA load at 0.65 power-factor lagging. The percent *IR* drop and the percent *IX* drop are 1.2 and 1.4, respectively. Determine the transformer regulation for the specific load.

Solution

$S_{PU} = \dfrac{S}{S_{rated}} = \dfrac{10}{25} = 0.0394 \qquad \theta = \cos^{-1} 0.65 = 49.49° \qquad \sin 49.49° = 0.76$

Substituting into Eq. (2–58a),

$$\text{reg}_{PU} = \sqrt{(0.0394 \times 0.0124 + 0.65)^2 + (0.0394 \times 0.014 + 0.76)^2} - 1$$

$$\text{reg}_{PU} = \underline{0.738 \text{ or } 73.8\%}$$

2.13 TRANSFORMER LOSSES AND EFFICIENCY

Transformer losses include the I^2R losses in the primary and secondary windings, and the hysteresis and eddy-current losses (core losses) in the iron. These losses are the same whether operating in the step-up or step-down mode.

The efficiency of a transformer is the ratio of the power out to the power in, and may be expressed in decimal form, called *per-unit efficiency*, or expressed as *percent efficiency* by multiplying by 100.

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{core} + I_{HS}^2 R_{HS} + I_{LS}^2 R_{LS}} \qquad (2\text{–}59)$$

where: η = efficiency

$$P_{core} = P_h + P_e \qquad (2\text{–}60)$$

$$P_e = k_e f^2 B_{max}^2 \qquad (2\text{–}61)$$

$$P_h = k_h f B_{max}^{1.6} \qquad (2\text{–}62)$$

From Eq. (2–1),

$$\Phi_{max} \propto \frac{V_T}{f}$$

Thus,

$$B_{max} \propto \frac{V_T}{f}$$

Obtaining proportionalities by substituting into Eqs. (2–61) and (2–62), respectively,

$$P_e \propto f^2\left(\frac{V_T}{f}\right)^2 \propto V_T^2 \tag{2–63}$$

$$P_h \propto f\left(\frac{V_T}{f}\right)^{1.6} \tag{2–64}$$

The hysteresis component of the total core losses is generally greater than the eddy-current component, $P_h > P_e$.

As indicated in Eq. (2–63), the eddy-current losses are proportional to the square of the applied voltage. However, as shown in Eq. (2–64), the hysteresis losses are affected by both the frequency and the applied voltage. Hence, assuming the frequency and magnitude of applied voltage are constant, the core loss will be essentially constant for all load conditions up to the transformer rating; slight changes in leakage flux from no load to full load will have an insignificant effect on the core loss.

The combined conductor losses of both primary and secondary windings may be expressed in terms of the *equivalent resistance referred to the high side or referred to the low side.* That is,

$$(I_{HS}^2 R_{HS} + I_{LS}^2 R_{LS}) = I_{HS}^2 R_{eq,HS} = I_{LS}^2 R_{eq,LS} \tag{2–65}$$

Substituting Eq. (2–65) into Eq. (2–59),

$$\eta = \frac{P_{out}}{P_{out} + P_{core} + I^2 R_{eq}} \tag{2–66}$$

where I and R_{eq} *are both high-side values or are both low-side values.*

Depending on its apparent power rating, the efficiency of distribution transformers and power transformers varies from 96 to more than 99 percent; the larger transformers have the higher efficiencies.

EXAMPLE 2–12 A 50-kVA, 450—230-V, 60-Hz transformer has percent resistance and percent leakage reactance of 1.25 and 2.24, respectively. Its efficiency at rated voltage, rated frequency, and rated apparent power at 0.860 power-factor lagging is 96.5 percent. Determine (a) the core loss; (b) the core loss if operating at rated load current and 0.860 power factor from a 375-V, 50-Hz supply (assume the hysteresis loss is 71.0 percent of the total core loss); (c) the efficiency for the conditions in (b); (d) the efficiency if the load is disconnected.

Solution

(a)
$$I_{HS} = \frac{50{,}000}{450} = 111.11\ \text{A}$$

$$R_{PU} = \frac{I_{rated}R_{eq}}{V_{rated}} \quad\Rightarrow\quad R_{eq} = R_{PU}\cdot\frac{V_{rated}}{I_{rated}}$$

Using high-side values,

$$R_{eq,HS} = 0.0125 \times \frac{450}{111.11} = 0.0506\ \Omega$$

$$P_{out} = S_{rated}\cdot F_P = 50{,}000 \times 0.860 = 43{,}000\ \text{W}$$

$$P_{in} = \frac{P_{out}}{\eta} = \frac{43{,}000}{0.965} = 44{,}559.59\ \text{W}$$

$$P_{out} = P_{in} - P_{core} - I_{HS}^2 R_{eq,HS} \quad\Rightarrow\quad P_{core} = P_{in} - P_{out} - I_{HS}^2 R_{eq,HS}$$

$$P_{core} = 44{,}559.59 - 43{,}000 - (111.11)^2 \times 0.0506 = \underline{934.9\ \text{W}}$$

(b)
$$P_{h,60} = 0.71 \times 934.9 = 663.78\ \text{W}$$

$$P_{e,60} = 934.9 - 663.78 = 271.12\ \text{W}$$

From Eq. (2–63),

$$\frac{P_{e,60}}{P_{e,50}} = \frac{(V_T)_{60}^2}{(V_T)_{50}^2} \quad\Rightarrow\quad P_{e,50} = P_{e,60} \times \left[\frac{V_{T,50}}{V_{T,60}}\right]^2$$

$$P_{e,50} = 271.12 \times \left[\frac{375}{450}\right]^2 = 188.3\ \text{W}$$

From Eq. (2–64),

$$\frac{P_{h,60}}{P_{h,50}} = \frac{f_{60}(V_T/f)_{60}^{1.6}}{f_{50}(V_T/f)_{50}^{1.6}} \quad\Rightarrow\quad P_{h,50} = P_{h,60} \times \frac{50}{60} \times \left[\frac{V_{T,50}}{V_{T,60}} \times \frac{60}{50}\right]^{1.6}$$

$$P_{h,50} = 663.78 \times \frac{50}{60} \times \left[\frac{375}{450} \times \frac{60}{50}\right]^{1.6} = 553.15\ \text{W}$$

$$P_{core,50} = 188.3 + 553.15 = \underline{741.45\ \text{W}}$$

(c)
$$P_{out} = 375 \times 111.11 \times 0.860 = 35{,}832.98\ \text{W}$$

Discounting small changes in skin effect due to changes in frequency, the equivalent resistance is essentially the same. Thus,

$$I_{HS}^2 R_{eq,HS} = 111.11^2 \times 0.0506 = 624.68\ \text{W}$$

Substituting into Eq. (2–66),

$$\eta = \frac{35{,}832.98}{35{,}832.98 + 741.45 + 624.68} = \underline{0.963 \quad\text{or}\quad 96.3\%}$$

(d) With the load disconnected, $P_{out} = 0$. Thus, the efficiency is zero.

Calculating Efficiency From Per-Unit Values

Quick-and-easy calculations of efficiency may be accomplished if the transformer parameters and the core loss are given in per-unit values or percent values. The appropriate equation is derived by first expressing P_{out} in Eq. (2–66) in terms of apparent power. Thus,

$$\eta = \frac{S \times F_P}{S \times F_P + P_{core} + I^2 R_{eq}} \tag{2-67}$$

where: F_P = per-unit power factor
S = apparent power of connected load (VA)
I = load current

Dividing both numerator and denominator in Eq. (2–67) by the *rated apparent power* of the transformer,

$$\eta = \frac{(S/S_{rated}) \times F_P}{(S/S_{rated}) \times F_P + (P_{core}/S_{rated}) + (I^2 R_{eq}/S_{rated})} \tag{2-68}$$

where: η = per-unit efficiency
$S_{rated} = V_{rated} I_{rated}$ = rated apparent power, also called *base apparent power*

Defining:

$$\frac{S}{S_{rated}} = S_{PU} = \text{per-unit apparent power of load} \tag{2-69}$$

$$\frac{P_{core}}{S_{rated}} = P_{core,PU} = \text{per-unit core loss} \tag{2-70}$$

Multiplying the numerator and denominator of the $(I^2 R_{eq}/S_{rated})$ term in Eq. (2–68) by I_{rated}, and then rearranging its components,

$$\frac{I^2 R_{eq}}{S_{rated}} = \frac{I^2 R_{eq}}{V_{rated} I_{rated}} \cdot \frac{I_{rated}}{I_{rated}} = \left[\frac{I^2}{I^2_{rated}}\right]\left[\frac{I_{rated} R_{eq}}{V_{rated}}\right]$$

Defining $I_{PU} = (I/I_{rated})$ ·

$$\frac{I^2 R_{eq}}{S_{rated}} = I^2_{PU} \cdot R_{PU} \tag{2-71}$$

Substituting Eqs. (2–69), (2–70), and (2–71) into Eq. (2–68),

$$\eta = \frac{S_{PU} \times F_P}{S_{PU} \times F_P + P_{core,PU} + I^2_{PU} \times R_{PU}} \tag{2-72}$$

Equation (2–72) is applicable to all loads. When operating at *rated conditions*, $S_{PU} = 1$, $I_{PU} = 1$, and Eq. (2–72) reduces to

$$\eta_{rated} = \frac{F_P}{F_P + P_{core\,PU} + R_{PU}} \tag{2-73}$$

The components in Eqs. (2–72) and (2–73) may be expressed *all in per-unit* or *all in percent*. However, the calculated efficiency will be in per-unit.

EXAMPLE 2.13 A 100-kVA, 4800—240-V, 60-Hz transformer is operating at *rated conditions* and 80.0 percent power factor. The core loss, resistance, and leakage reactance, *expressed in percent*, are 0.450, 1.46, and 3.38, respectively. Determine the efficiency at (a) rated load and 80% power factor; (b) 70% load and 80% power factor.

Solution

(a) Substituting into Eq. (2–73),

$$\eta_{rated} = \frac{0.800}{0.800 + 0.0045 + 0.0146} = \underline{0.977 \text{ or } 97.7\%}$$

(b)
$$S_{PU} = \frac{S_{load}}{S_{rated}} = \frac{70}{100} = 0.70$$

and since I_{load} is proportional to S_{load}

$$I_{PU} = S_{PU} = 0.70$$

Substituting into Eq. (2–72),

$$\eta = \frac{0.70 \times 0.80}{0.70 \times 0.80 + 0.0045 + 0.70^2 \times 0.0146} = \underline{0.979 \text{ or } 97.9\% \text{ efficient}}$$

Note: There is very little change in efficiency.

2.14 DETERMINATION OF TRANSFORMER PARAMETERS

If transformer parameters are not readily available from the nameplate or from the manufacturer, they can be approximated from an open-circuit test (also called a no-load test) and a short-circuit test.

Open-Circuit Test

The purpose of the open-circuit test is to determine the magnetizing reactance X_M and the equivalent core-loss resistance R_{fe}. The connections and instrumentation required for this test are shown in Figure 2.16(a).

For safety in testing and instrumentation, the open-circuit test is generally made on the low-voltage side. The test is performed at rated frequency and rated low-side

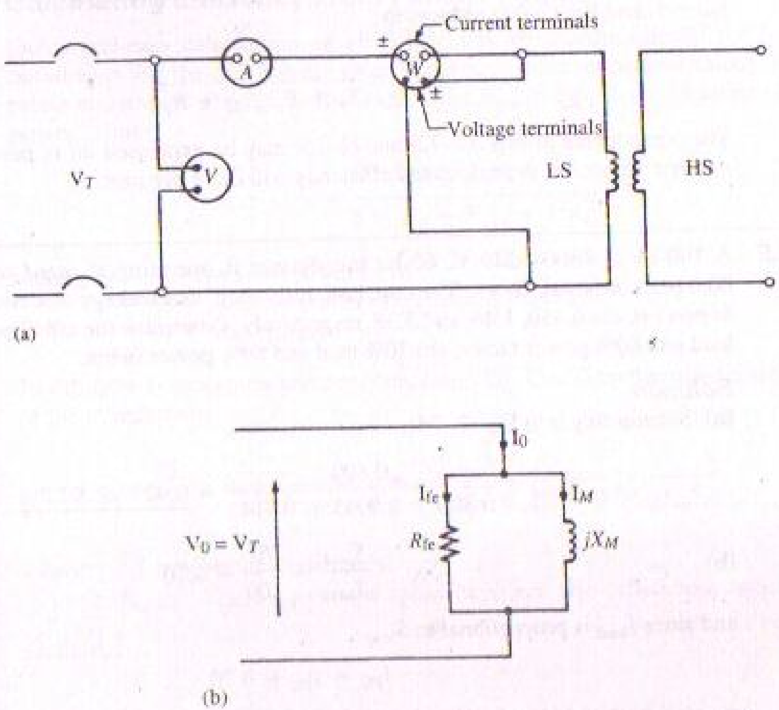


FIGURE 2.16
Open-circuit test: (a) connection diagram; (b) equivalent circuit.

voltage; the high-side terminals should be covered with insulating material to prevent accidental contact. Since no load is connected to the secondary, the copper losses in the secondary are zero, and the copper losses in the primary are negligible. Thus, the wattmeter reading (for the open-circuit test) is essentially core losses. The equivalent open-circuit model of the transformer is shown in Figure 2.16(b).

Assuming the wattmeter, voltmeter, and ammeter readings taken during the open-circuit test are P_{OC}, V_{OC}, and I_{OC}, respectively, and that the test was made on the low side, the open-circuit parameters referred to the low side may be obtained by substituting into the following equations:[12]

$$\left.\begin{aligned} P_{OC} &= V_{OC} I_{fe} & I_{OC} &= \sqrt{I_{fe}^2 + I_M^2} \\ R_{fe,LS} &= \frac{V_{OC}}{I_{fe}} & X_{M,LS} &= \frac{V_{OC}}{I_M} \end{aligned}\right\} \tag{2–74}$$

[12] If an analog wattmeter is used for the no-load test, wattmeter losses may be an appreciable part of the wattmeter reading, and must be subtracted before calculating the open-circuit parameters [2].

Short-Circuit Test

The purpose of the short-circuit test is to determine the equivalent resistance, equivalent leakage reactance, and equivalent impedance of the transformer windings. The connections and instrumentation required for the test are shown in Figure 2.17(a). The high side of the transformer under test is connected to the supply line through an adjustable-voltage autotransformer, and the low-voltage side is jumped by connecting a short piece of large cross-sectional area copper across its terminals; in effect, the secondary is shorted. The jumper represents a load impedance of almost zero ohms. That is, $Z_{load} \approx 0\ \Omega$. Thus, by jumping the secondary, the test provides data that include the effects of primary and secondary resistance, and primary and secondary leakage, but excludes the load impedance. Furthermore, short circuiting the secondary causes the flux density to be reduced to a very low value, making the core losses insignificant. Thus, the wattmeter reading (for the short-circuit test) is essentially copper losses. The equivalent series circuit model is shown in Figure 2.17(b).

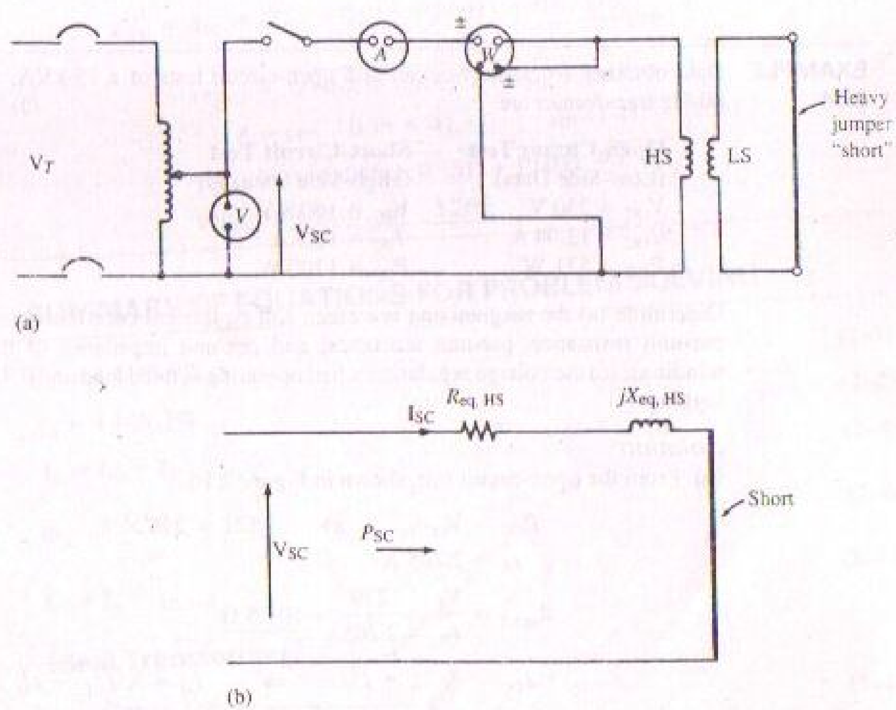


FIGURE 2.17
Short-circuit test: (a) connection diagram; (b) equivalent circuit.

Test Procedure With the variable voltage set to zero, the breaker is closed, the voltage is gradually raised until the ammeter indicates approximately rated high-side current, and the instruments are then read. Assuming the wattmeter, voltmeter, and ammeter readings taken during the short-circuit test are P_{SC}, V_{SC}, and I_{SC}, respectively, and the test is made on the high side, the equivalent resistance, equivalent reactance, and equivalent impedance may be obtained by substituting values into the following set of equations:

$$\left.\begin{aligned} I_{SC} &= \frac{V_{SC}}{Z_{eq,HS}} \qquad P_{SC} = I_{SC}^2 R_{eq,HS} \\ Z_{eq,HS} &= \sqrt{R_{eq,HS}^2 + X_{eq,HS}^2} \end{aligned}\right\} \tag{2–75}$$

The short-circuit test may be made using either winding. For reasons of lower current input and meter sizing, however, the high-voltage winding is preferred. If the measurements are made on the low side, the resultant values determined from the test would be the equivalent resistance, equivalent reactance, and equivalent impedance referred to the low side.

EXAMPLE 2.14 Data obtained from short-circuit and open-circuit tests of a 75-kVA, 4600—230-V, 60-Hz transformer are

Open-Circuit Test (Low-Side Data)	Short-Circuit Test (High-Side Data)
$V_{OC} = 230$ V	$V_{SC} = 160.8$ V
$I_{OC} = 13.04$ A	$I_{SC} = 16.3$ A
$P_{OC} = 521$ W	$P_{SC} = 1200$ W

Determine (a) the magnetizing reactance and equivalent core-loss resistance; (b) the per-unit resistance, per-unit reactance, and per-unit impedance of the transformer windings; (c) the voltage regulation when operating at rated load and 0.75 power-factor lagging.

Solution

(a) From the open-circuit test, shown in Figure 2.16,

$$P_{OC} = V_{OC} I_{fe} \quad\Rightarrow\quad 521 = 230 \times I_{fe}$$

$$I_{fe} = 2.265 \text{ A}$$

$$R_{fe,LS} = \frac{V_0}{I_{fe}} = \frac{230}{2.265} = \underline{101.5\ \Omega}$$

$$I_{OC} = \sqrt{I_{fe}^2 + I_M^2} \quad\Rightarrow\quad I_M = \sqrt{I_{OC}^2 - I_{fe}^2}$$

$$I_M = \sqrt{(13.04)^2 - (2.265)^2} = 12.842 \text{ A}$$

$$X_{M,LS} = \frac{V_{OC}}{I_M} = \frac{230}{12.842} = \underline{17.91\ \Omega}$$

(b) From the short-circuit test, shown in Figure 2.17,

$$I_{SC} = \frac{V_{SC}}{Z_{eq,HS}} \quad\Rightarrow\quad 16.3 = \frac{160.8}{Z_{eq,HS}}$$

$$Z_{eq,HS} = 9.865\ \Omega$$

$$P_{SC} = I_{SC}^2 R_{eq,HS} \quad\Rightarrow\quad 1200 = 16.3^2 R_{eq,HS}$$

$$R_{eq,HS} = 4.517\ \Omega$$

$$Z_{eq,HS} = \sqrt{R_{eq,HS}^2 + X_{eq,HS}^2} \quad\Rightarrow\quad X_{eq,HS} = \sqrt{Z_{eq,HS}^2 - R_{eq,HS}^2}$$

$$X_{eq,HS} = \sqrt{(9.865)^2 - (4.517)^2} = 8.77\ \Omega$$

$$I_{HS} = \frac{S}{V_{HS}} = \frac{75{,}000}{4600} = 16.3 \text{ A}$$

$$R_{PU} = \frac{I_{HS} R_{eq,HS}}{V_{HS}} = \frac{16.3 \times 4.517}{4600} = \underline{0.016}$$

$$X_{PU} = \frac{I_{HS} X_{eq,HS}}{V_{HS}} = \frac{16.3 \times 8.77}{4600} = \underline{0.031}$$

$$Z_{PU} = R_{PU} + jX_{PU} = 0.016 + j0.031 = \underline{0.035\angle 62.7^\circ}$$

(c)

$$\text{reg}_{PU} = \sqrt{(R_{PU} + \cos\theta)^2 + (X_{PU} + \sin\theta)^2} - 1$$

$$\theta = \cos^{-1} 0.75 = 41.41^\circ \qquad \sin 41.41^\circ = 0.661$$

$$\text{reg}_{PU} = \sqrt{(0.016 + 0.75)^2 + (0.031 + 0.661)^2} - 1$$

$$\text{reg}_{PU} = \underline{0.0326} \text{ or } \underline{3.26\%}$$

SUMMARY OF EQUATIONS FOR PROBLEM SOLVING

$$E_P = 4.44 N_P f \Phi_{max} \tag{2–1}$$

$$E_S = 4.44 N_S f \Phi_{max} \tag{2–2}$$

$$\mathbf{I}_0 = \mathbf{I}_{fe} + \mathbf{I}_M \tag{2–4}$$

$$\Phi_M = \frac{N_P I_M}{\mathcal{R}_{core}} \tag{2–6}$$

$$\mathbf{I}_P = \mathbf{I}_0 + \mathbf{I}_{P,load} \tag{2–11}$$

Ideal Transformer

$$a = \frac{N_{HS}}{N_{LS}} \approx \frac{V_{HS}}{V_{LS}} \tag{2–14}$$

$$\mathbf{E}'_P = a\mathbf{E}'_S \tag{2-16a}$$

$$\mathbf{I}_P = \frac{1}{a}\mathbf{I}_S \tag{2-16b}$$

$$\mathbf{Z}'_{\text{in}} = a^2\mathbf{Z}_{\text{load}} \tag{2-19}$$

Real Transformer

$$\mathbf{Z}_{\text{eq,HS}} = R_{\text{HS}} + a^2R_{\text{LS}} + j(X_{\text{HS}} + a^2X_{\text{LS}}) \tag{2-39}$$

$$\mathbf{Z}_{\text{eq,HS}} = R_{\text{eq,HS}} + jX_{\text{eq,HS}} \tag{2-40}$$

$$\mathbf{Z}_{\text{eq,LS}} = R_{\text{LS}} + R_{\text{HS}}/a^2 + j(X_{\text{LS}} + X_{\text{HS}}/a^2) \tag{2-41}$$

$$\mathbf{Z}_{\text{eq,LS}} = R_{\text{eq,LS}} + jX_{\text{eq,LS}} \tag{2-42}$$

$$\mathbf{Z}_{\text{load,LS}} = \frac{1}{a^2}\mathbf{Z}_{\text{load,HS}} \tag{2-43}$$

$$\mathbf{Z}_{\text{eq,LS}} = \frac{\mathbf{Z}_{\text{eq,HS}}}{a^2} \tag{2-43a}$$

$$\text{reg} = \frac{E - V_{\text{rated}}}{V_{\text{rated}}} \tag{2-44}$$

$$\text{reg}_{\text{PU}} = \sqrt{(R_{\text{PU}} + \cos\theta)^2 + (X_{\text{PU}} + \sin\theta)^2} - 1 \tag{2-58}$$

$$\text{reg}_{\text{PU}} = \sqrt{(S_{\text{PU}} \times R_{\text{PU}} + \cos\theta)^2 + (S_{\text{PU}} \times X_{\text{PU}} + \sin\theta)^2} - 1 \tag{2-58a}$$

$$\left.\begin{aligned} Z_{\text{PU}} &= \frac{I_{\text{rated}}Z_{\text{eq}}}{V_{\text{rated}}} \\ R_{\text{PU}} &= \frac{I_{\text{rated}}R_{\text{eq}}}{V_{\text{rated}}} \\ X_{\text{PU}} &= \frac{I_{\text{rated}}X_{\text{eq}}}{V_{\text{rated}}} \end{aligned}\right\} \tag{2-46}$$

$$Z_{\text{base}} = \frac{V_{\text{rated}}}{I_{\text{rated}}} = \frac{V_{\text{rated}}^2}{S_{\text{rated}}} \tag{2-48}$$

$$\left.\begin{aligned} Z_{\text{PU}} &= \frac{Z_{\text{eq}}}{Z_{\text{base}}} \\ R_{\text{PU}} &= \frac{R_{\text{eq}}}{Z_{\text{base}}} \\ X_{\text{PU}} &= \frac{X_{\text{eq}}}{Z_{\text{base}}} \end{aligned}\right\} \tag{2-49}$$

$$Z_{\text{PU}} = R_{\text{PU}} + jX_{\text{PU}} \tag{2-50}$$

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{core}} + I_{\text{HS}}^2R_{\text{HS}} + I_{\text{LS}}^2R_{\text{LS}}} \tag{2-59}$$

$$P_{\text{core}} = P_h + P_e \tag{2-60}$$

$$P_e = k_e f^2 B_{\max}^2 \tag{2-61}$$

$$P_h = k_h f B_{\max}^{1.6} \tag{2-62}$$

$$P_e \propto f^2\left(\frac{V_T}{f}\right)^2 \propto V_T^2 \tag{2-63}$$

$$P_h \propto f\left(\frac{V_T}{f}\right)^{1.6} \tag{2-64}$$

$$\eta = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{core}} + I^2R_{\text{eq}}} \tag{2-66}$$

$$\eta = \frac{S_{\text{PU}} \times F_P}{S_{\text{PU}} \times F_P + P_{\text{core,PU}} + I_{\text{PU}}^2 \times R_{\text{PU}}} \tag{2-72}$$

$$\eta_{\text{rated}} = \frac{F_P}{F_P + P_{\text{core,PU}} + R_{\text{PU}}} \tag{2-73}$$

Open-Circuit Test

$$\left.\begin{aligned} P_{\text{OC}} &= V_{\text{OC}}I_{\text{fe}} & I_{\text{OC}} &= \sqrt{I_{\text{fe}}^2 + I_M^2} \\ R_{\text{fe,LS}} &= \frac{V_{\text{OC}}}{I_{\text{fe}}} & X_{M,\text{LS}} &= \frac{V_{\text{OC}}}{I_M} \end{aligned}\right\} \tag{2-74}$$

Short-Circuit Test

$$\left.\begin{aligned} I_{\text{SC}} &= \frac{V_{\text{SC}}}{Z_{\text{eq,HS}}} & P_{\text{SC}} &= I_{\text{SC}}^2R_{\text{eq,HS}} \\ Z_{\text{eq,HS}} &= \sqrt{R_{\text{eq,HS}}^2 + X_{\text{eq,HS}}^2} \end{aligned}\right\} \tag{2-75}$$

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REVIEW QUESTIONS

1. Describe the difference in construction between core-type and shell-type transformers and state the advantages of each.
2. Describe the different methods used for cooling power and distribution transformers.
3. Why is the iron core of a transformer laminated?
4. Explain why the core loss of a transformer does not change with changes in load.
5. Explain why the current in the primary of a transformer increases when a load is placed across the secondary. What causes the primary current to level off to a value that is just sufficient to carry the load plus losses?
6. What is leakage flux and how does it affect the output of a transformer?
7. Explain how leakage flux in the primary and leakage flux in the secondary affect the secondary voltage.
8. If it were possible to design a transformer with no leakage flux, would this be desirable? Explain.
9. What is meant by the voltage regulation of a transformer and how is this information useful to an applications engineer?
10. Explain why a leading power-factor load tends to cause a rise in voltage above the no-load value?
11. Distinguish between equivalent impedance, per-unit impedance, and percent impedance as they apply to a transformer, and indicate the unique advantages offered by the per-unit system.
12. (a) Explain the nature of hysteresis and eddy-current losses. (b) How are these losses affected by the magnitude and frequency of the applied voltage? (c) How are the core losses minimized during transformer design?

13. Explain why core loss remains essentially constant over the kilovoltampere load range of a transformer.
14. Explain the effect powering a transformer at higher than its rated primary voltage will have on (a) its output voltage; (b) its efficiency.
15. What transformer parameters are determined by the short-circuit test? Sketch the appropriate circuit and indicate how the parameters are determined.
16. What transformer parameters are determined by the open-circuit test? Sketch the appropriate circuit and indicate how the parameters are determined.
17. What precautions should be observed when making the open-circuit test?
18. Explain why the short-circuit test minimizes the core losses.
19. Explain why the open-circuit test minimizes the copper losses.

PROBLEMS

2–1/4 A 22,000-V, 60-Hz generator is connected to the high-voltage side of a 22,000—2200-V, 500-kVA step-down transformer. If the resultant core flux is 0.0683 Wb (max), determine (a) the number of turns of wire in the secondary coil; (b) the new core flux if the driving voltage is increased by 20 percent and the frequency is decreased by 5 percent.

2–2/4 A 2400—115-V transformer has a sinusoidal flux expressed by $\phi = 0.113 \sin 188.5t$. Determine the primary and secondary turns.

2–3/4 A core-type transformer rated at 37.5 kVA, 2400—480 V, and 60 Hz has a core whose mean length is 1.07 m and whose cross-sectional area is 95 cm^2. The application of rated voltage causes a magnetic field intensity of 352 A-t/m (rms), and a maximum flux density of 1.505 T. Determine (a) the number of turns in the primary and the secondary; (b) the magnetizing current when operating as a step-up transformer.

2–4/4 A 2000-kVA, 4800—600-V, 60-Hz, core-type transformer operating at no load in the step-down mode draws a magnetizing current equal to 2 percent rated current. The core has a mean length of 3.15 m, and is operated at a flux density of 1.55 T. The magnetic field intensity is 360 A-t/m. Determine (a) the magnetizing current; (b) the number of turns in the two coils; (c) the core flux; (d) the cross-sectional area of the core.

2–5/5 The exciting current for a certain 50-kVA, 480—240-V, 60-Hz transformer is 2.5 percent of rated current at a phase angle of 79.8°. Sketch the equivalent circuit and phasor diagram for the no-load conditions and, assuming operation is in the step-down mode, determine (a) the exciting current; (b) the core-loss component of the exciting current; (c) the magnetizing current; (d) the core loss.

2–6/5 A single-phase oil-cooled distribution transformer rated at 200 kVA, 7200—460 V, and 60 Hz has a core loss of 1100 W, of which 74 percent is due to hysteresis. The magnetizing current is 1.5 percent of rated current.

Sketch the appropriate equivalent circuit and phasor diagram and, assuming step-down operation, determine (a) the magnetizing current and the core-loss component of exciting current; (b) the exciting current; (c) the no-load power factor; (d) the eddy-current losses.

2–7/5 The hysteresis and eddy-current losses for a 75-kVA, 480—120 V, 60-Hz transformer are 215 W and 115 W, respectively. The magnetizing current is 2.5 percent rated current, and the transformer is operating in the step-up mode. Sketch the appropriate equivalent circuit and phasor diagram and determine (a) the exciting current; (b) the no-load power factor; (c) the reactive power input at no load.

2–8/8 A 480—120-V, 60-Hz transformer has its high-voltage winding connected to a 460-V system, and its low-voltage winding connected to a $24\angle 32.8^\circ$-Ω load. Assume the transformer is ideal. Determine (a) the secondary voltage; (b) secondary current; (c) primary current; (d) input impedance at the primary terminals; (e) active, reactive, and apparent power drawn by the load.

2–9/8 A 7200—240-V, 60-Hz transformer is connected for step-up operation, and a $144\angle 46^\circ$-Ω load is connected to the secondary. Assume the transformer is ideal and the input voltage is 220 V at 60 Hz. Determine (a) secondary voltage; (b) secondary current; (c) primary current; (d) input impedance at primary terminals of transformer; (e) active, reactive, and apparent power input to the transformer.

2–10/8 A 200-kVA, 2300—230-V, 60-Hz transformer operating at rated voltage in the step-down mode is supplying 150 kVA at 0.654 power-factor lagging. Assume the transformer is ideal. Determine (a) secondary current; (b) impedance of the load; (c) primary current.

2–11/8 A 50-Hz ideal transformer with a 5-to-1 turns ratio has a low-side current of $15.6\angle -32^\circ$ A when operating in the step-down mode and feeding a load impedance of $8\angle 32^\circ$ Ω. Sketch the circuit and determine (a) low-side voltage; (b) high-side voltage; (c) high-side current; (d) active, reactive, and apparent power input to the transformer.

2–12/10 A 100-kVA, 60-Hz, 7200—480-V, single-phase transformer has the following parameters.

$$R_{HS} = 2.98\ \Omega \qquad X_{HS} = 6.52\ \Omega$$
$$R_{LS} = 0.021\ \Omega \qquad X_{LS} = 0.031\ \Omega$$

Determine the equivalent impedance of the transformer (a) referred to the high side; (b) referred to the low side.

2–13/10 A 30-kVA, 60-Hz, 2400—600-V transformer has the following parameters in ohms:

$$R_{HS} = 1.86 \qquad X_{HS} = 3.41 \qquad X_{M,HS} = 4962$$
$$R_{LS} = 0.15 \qquad X_{LS} = 0.28 \qquad R_{fe,HS} = 19{,}501$$

Determine the equivalent impedance of the transformer (a) referred to the high side; (b) referred to the low side.

2–14/10 A single-phase, 25-kVA, 2200—600-V, 60-Hz transformer used for step-down operation has the following parameters expressed in ohms:

$$R_{HS} = 1.40 \qquad X_{HS} = 3.20 \qquad X_{M,HS} = 5011$$
$$R_{LS} = 0.11 \qquad X_{LS} = 0.25 \qquad R_{fe,HS} = 18{,}694$$

Sketch the appropriate equivalent circuit and determine (a) the input voltage required to obtain an output of 25 kVA at 600 V and 0.8 power-factor lagging; (b) the load component of primary current; (c) the exciting current.

2–15/10 A 100-kVA, 60-Hz, 7200—480-V, single-phase transformer has the following parameters expressed in ohms:

$$R_{HS} = 3.06 \qquad X_{HS} = 6.05 \qquad X_{M,HS} = 17{,}809$$
$$R_{LS} = 0.014 \qquad X_{LS} = 0.027 \qquad R_{fe,HS} = 71{,}400$$

The transformer is supplying a load that draws rated current at 480 V and 75 percent power-factor lagging. Sketch the appropriate equivalent circuit and determine (a) the equivalent resistance and equivalent reactance referred to the high side; (b) the input impedance of the combined transformer and load; (c) the load component of high-side current; (d) the input voltage to the transformer; (e) the exciting current and its components; (f) the input impedance at no load.

2–16/10 A 75-kVA, 60-Hz, 4160—240-V, single-phase transformer operating in the step-down mode is feeding a $1.45\angle -38.74^\circ$-Ω load at 270 V. The transformer parameters expressed in ohms are:

$$R_{LS} = 0.0072 \qquad X_{LS} = 0.0128$$
$$R_{HS} = 2.16 \qquad X_{HS} = 3.84$$

Sketch the appropriate equivalent circuit and determine (a) the equivalent impedance of the transformer referred to the high side; (b) the input impedance; (c) the voltage impressed at the high-side terminals that results in a load voltage of 270 V. (d) Sketch the phasor diagram for the low-side voltage and current, and determine the power factor at the high side of the transformer.

2–17/11 The parameters for a 250-kVA, 2400—480-V, single-phase transformer operating at rated voltage, rated kVA, and 0.82 power-factor lagging, are $X_{eq,HS} = 1.08\ \Omega$ and $R_{eq,HS} = 0.123\ \Omega$. The transformer is operating in the step-down mode. Sketch the appropriate equivalent circuit and determine (a) the equivalent low-side parameters; (b) the no-load voltage; (c) the voltage regulation at 0.82 power-factor lagging.

2–18/11 Re-solve Problem 2–17/11(b) and (c) assuming operation in the step-up mode and 0.70 power-factor leading.

2–19/11 A 333-kVA, 60-Hz, 4160—2400-V transformer operating in the step-down mode has an equivalent resistance and equivalent reactance referred to the high side of 0.5196 Ω and 2.65 Ω, respectively. Assume operation is at rated voltage, rated load, and 0.95 power-factor leading. Sketch the appropriate equivalent circuit and determine (a) the no-load voltage, (b) the voltage regulation; (c) the combined input impedance of the transformer and load.

2–20/11 A 100-kVA, 4800—480-V, 60-Hz, single-phase distribution transformer has 6 V/turn and an equivalent impedance referred to the high side of $8.48\angle 71°$ Ω. The transformer is operating in the step-down mode supplying a 50-kVA, unity power-factor load at 480 V. Determine (a) the output voltage when the load is removed; (b) the inherent voltage regulation of the transformer when operating at 78 percent power-factor lagging. *Note:* By definition, inherent voltage regulation infers rated kVA.

2–21/11 A 37.5-kVA, 6900—230-V, 60-Hz, single-phase transformer is operating in the step-down mode at rated load, rated voltage, and 0.68 power-factor lagging. The equivalent resistance and reactance referred to the low side are 0.0224 Ω and 0.0876 Ω, respectively. The magnetizing reactance and equivalent core-loss resistance (high side) are 43,617 Ω and 174,864 Ω, respectively. Determine (a) the output voltage when the load is removed; (b) the voltage regulation; (c) the combined input impedance of transformer and load; (d) the exciting current and input impedance at no load.

2–22/11 A 500-kVA, 7200—600-V, 60-Hz transformer is operating in the step-down mode at rated kVA and 0.83 power-factor lagging. The output voltage when the load is removed is 625 V. Determine the equivalent impedance of the transformer referred to the high side (assume the equivalent resistance is negligible). *Hint:* Draw a phasor diagram showing **I**, **E**, **V**, and the impedance drop. Use trigonometry to solve for IX_{eq} and then determine X_{eq}.

2–23/12 A 25-kVA, 480—120-V, 60-Hz transformer has a 2.1 percent impedance. Determine (a) the equivalent impedance referred to the high side; (b) the equivalent impedance referred to the low side.

2–24/12 The percent impedance and the percent resistance of a 25-kVA, 7200—600-V, 60-Hz transformer are 2.3 and 1.6 percent, respectively. Determine (a) the percent reactance; (b) the equivalent resistance, equivalent reactance, and equivalent impedance referred to the high side; (c) repeat (b) for the equivalent low-side values.

2–25/12 A 500-kVA, 7200—240-V, 60-Hz transformer with a 2.2 percent impedance was severely damaged as a result of a dead short across the secondary terminals. Determine (a) the short-circuit current; (b) the required percent impedance of a replacement transformer that will limit the low-side short-circuit current to 60,000 A.

2–26/12 A 167-kVA, 60-Hz, 600—240-V, 60-Hz, 4.1 percent impedance distribution transformer with 46 turns on the high side is operating at rated load and 0.82 power-factor lagging. Determine (a) the voltage regulation; (b) the no-load voltage; (c) the core flux; (d) the cross-sectional area of the core if the transformer is operating at a maximum flux density of 1.4 T.

2–27/12 A 150-kVA, 2300—240-V, 60-Hz transformer is operating at rated load and 90 percent power-factor lagging. The resistance and reactance of the transformer, expressed in per-unit values, are 0.0127 and 0.0380, respectively. Determine the inherent voltage regulation.

2–28/12 A 75-kVA, 4160—460-V, 60-Hz transformer is operating at 76 percent rated load and 85 percent power-factor leading. The resistance and reactance of the transformer, expressed in per-unit values, are 0.0160 and 0.0311, respectively. Determine the inherent voltage regulation.

2–29/12 A 50-kVA, 4370—600-V, 60-Hz transformer is operating at 80 percent rated load and 75 percent power-factor lagging. The resistance and reactance of the transformer, expressed in per-unit values, are 0.0156 and 0.0316, respectively. Determine the inherent voltage regulation.

2–30/12 A single-phase distribution transformer, rated at 50 kVA, 450—120 volt, is supplying rated kVA at 120 V and 0.80 power-factor lagging. The percent resistance and percent reactance are 1.0 and 4.4, respectively. Determine (a) transformer regulation; (b) secondary voltage when the load is disconnected; (c) input voltage that must be applied to the primary in order to obtain rated secondary voltage when carrying rated load at 0.80 power-factor lagging.

2–31/12 A single-phase distribution transformer, rated at 75 kVA and 450—230 volt, is supplying rated kVA at 230 V and 0.9 power-factor lagging. The percent resistance and percent reactance are 1.8 and 3.7, respectively. Determine (a) transformer regulation; (b) secondary voltage when the load is disconnected; (c) input voltage that must be applied to the primary in order to obtain rated secondary voltage when carrying rated load at 0.9 power-factor lagging.

2–32/12 A single-phase distribution transformer, rated at 50 kVA and 480—240 volt is supplying rated kVA at 240 V and 0.85 power-factor leading. The percent resistance and percent reactance are 1.1 and 4.6, respectively. Determine (a) transformer regulation; (b) secondary voltage when the load is disconnected; (c) input voltage that must be applied to the primary in orde to obtain rated secondary voltage when carrying rated load at 0.85 power factor leading.

2–33/12* A 200-kVA, 2300—230-V, 60-Hz transformer has percent resistance an percent leakage reactance of 1.24 and 4.03 respectively. For 10° increment of power-factor angle, tabulate and plot percent regulation vs. power facto for operation between 0.5 lagging and 0.5 leading.

2–34/13 A 150-kVA, 7200—600-V, 60-Hz, single-phase transformer operating rated conditions has an hysteresis loss of 527 W, an eddy-current loss c 373 W, and a conductor loss of 2000 W. The transformer is to be used on

*Solution by computer is recommended.

50-Hz system, with the restriction that it maintain the same maximum core flux and the same total losses. Determine (a) the new voltage rating; (b) the new kVA rating.

2–35/13 A 75-kVA, 450—120-V, 60-Hz, single-phase transformer has percent resistance and percent reactance of 1.75 and 3.92, respectively. Its efficiency when operating at rated voltage, rated frequency, and rated load at 0.74 power-factor lagging is 97.1 percent. Determine (a) the core loss; (b) the core loss and efficiency if the transformer is powered at the same voltage, load, and power factor, but at 50 Hz. Assume the core-loss ratio $P_h : P_e$ is 2.5.

2–36/13 A 200-kVA, 7200—600-V, 60-Hz transformer is operating at rated load and 90 percent power-factor lagging. The core loss, resistance, and reactance of the transformer, expressed in per-unit values, are 0.0056, 0.0133, and 0.0557, respectively. Determine (a) the efficiency; (b) the inherent voltage regulation; (c) the efficiency and regulation at 30 percent load and 80 percent power-factor lagging.

2–37/13* A 50-kVA, 2300—230-V, 60-Hz transformer supplies a 0.8 lagging power-factor load whose kVA is adjustable from no load to 120 percent rated kVA. The percent resistance, percent reactance, and percent core loss are 1.56, 3.16, and 0.42, respectively. For 2-kVA increments of load, tabulate and plot the efficiency of the transformer from no load to 120 percent rated load.

2–38/14 A short-circuit test performed on a 150-kVA, 4600—230-V, 60-Hz transformer provided the following data:

$$V_{SC} = 182 \text{ V} \qquad I_{SC} = 32.8 \text{ A} \qquad P_{SC} = 1902 \text{ W}$$

Determine (a) per-unit resistance and per-unit reactance; (b) regulation when operated at 0.6 power-factor lagging.

2–39/14 The following test data were obtained from short-circuit and open-circuit tests of a 50-kVA, 2400—600-V, 60-Hz transformer.

$$\begin{aligned} V_{OC} &= 600 \text{ V} & V_{SC} &= 76.4 \text{ V} \\ I_{OC} &= 3.34 \text{ A} & I_{SC} &= 20.8 \text{ A} \\ P_{OC} &= 484 \text{ W} & P_{SC} &= 754 \text{ W} \end{aligned}$$

Determine (a) the equivalent high-side parameters; (b) regulation; (c) efficiency at rated load and 0.92 power-factor lagging.

2–40/14 Data from short-circuit and open-circuit tests of a 25-kVA, 6900—230-V, 60-Hz transformer are:

$$\begin{aligned} V_{OC} &= 230 \text{ V} & V_{SC} &= 513 \text{ V} \\ I_{OC} &= 5.4 \text{ A} & I_{SC} &= 3.6 \text{ A} \\ P_{OC} &= 260 \text{ W} & P_{SC} &= 465 \text{ W} \end{aligned}$$

Determine (a) the magnetizing reactance referred to the high side; (b) the per-unit parameters; (c) efficiency; (d) voltage regulation at 0.65 per-unit load and 84 percent power-factor leading; (e) low-side voltage when the load is removed; (f) voltage that must be applied to the primary in order to obtain the low-side voltage in (e).

2–41/14 Data from short-circuit and open-circuit tests of a 60-Hz, 100-kVA, 4600—230-V transformer are:

$$\begin{aligned} V_{OC} &= 230 & V_{SC} &= 172.3 \\ I_{OC} &= 14 & I_{SC} &= 20.2 \\ P_{OC} &= 60 & P_{SC} &= 1046 \end{aligned}$$

Determine (a) the magnetizing reactance referred to the high-side; (b) the per-unit parameters; (c) efficiency; (d) voltage regulation at 0.85 per-unit load and 89 percent power-factor lagging; (e) low-side voltage when the load is removed; (f) voltage that must be applied to the primary in order to obtain the low-side voltage in (e).

3-30/10 Three single-phase transformers are used to supply a total of 750-kVA at 450-V to a balanced three-phase load. The three-phase input to the bank is 2400-V. Determine (a) the bank ratio and the required transformer ratio if delta–wye connected; (b) the bank ratio and the required transformer ratio if delta–delta connected. (c) If each transformer is rated at 400-kVA, and the bank is delta–delta connected, will the bank be able to safely carry the load if one transformer is disconnected?

3-31/11 A bank of three single-phase 500-kVA transformers, connected delta–delta, was accidentally paralleled with a 400-kVA delta–wye bank. Although there was no load connected to the paralleled transformers, the resultant high circulating current between the transformer banks caused the tie breaker to trip. Both banks have a bank ratio of 7200—240-V. The impedance of transformers in the delta–delta bank is 2.2 percent, and is 3.1 percent in the delta–wye bank, both calculated on their respective bases. Determine the magnitude of the current that circulated between banks until the breaker tripped.

3-32/11 A 200-kVA, 60-Hz, 4600—460 Y/266-V, three-phase transformer (Δ–Y) was accidentally paralleled with a 200-kVA, 4600—460-V, 60-Hz, delta–delta, three-phase transformer. The per-unit impedance per phase is $0.0448\angle 72.33^\circ$ for the delta–delta bank, and is $0.0420\angle 68.42^\circ$ for the delta–wye bank, both calculated on their respective bases. Determine the magnitude of the line current circulating between banks.

4

Principles of Three-Phase Induction Machines

4.1 INTRODUCTION

Induction machines represent a class of rotating apparatus that includes induction motors, induction generators, induction frequency converters, induction phase converters, and electromagnetic slip couplings. Induction motors can be used effectively in all motor applications, except where very high torque or very fine adjustable speed control is required. Induction motors can range in size from fractional horsepower to more than 100,000 horsepower. They are more rugged, require less maintenance, and are less expensive than direct-current motors of equal power and speed ratings.

The induction motor was invented by Nikola Tesla (1856–1943) in 1888. It requires no electrical connections to the rotating member; the transfer of energy from the stationary member to the rotating member is by means of electromagnetic induction. A rotating magnetic field, produced by a stationary winding (called the stator), induces an alternating emf and current in the rotor. The resultant interaction of the induced rotor current with the rotating field of the stationary winding produces motor torque.

The torque-speed characteristic of an induction motor is directly related to the resistance and reactance of the rotor. Hence, different torque-speed characteristics may be obtained by designing rotor circuits with different ratios of rotor resistance to rotor reactance.

4.2 INDUCTION-MOTOR ACTION

An elementary three-phase two-pole induction motor is shown in Figure 4.1(a). The stator (stationary member) consists of three "blocks of iron" spaced 120° apart. The three coils wound around the iron blocks are connected in wye and energized from a three-phase system. The rotor consists of a laminated steel core containing conductors that are joined at the ends to form a cage similar to that used for exercising squirrels; hence, the name squirrel-cage rotor. When the stator windings are energized

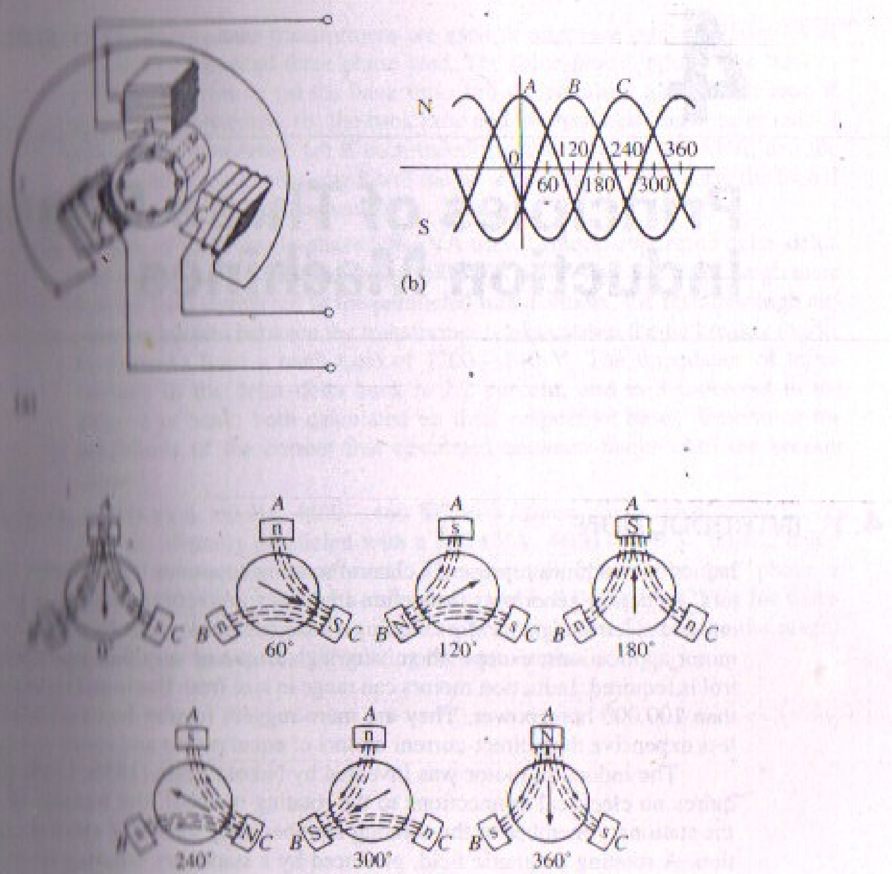


FIGURE 4.1
(a) Elementary three-phase induction motor; (b) three-phase flux waves; (c) instantaneous direction of resultant stator flux.

from a three-phase system, the currents in the three coils reach their maximum values at different instants. Since the three currents are displaced from each other by 120 electrical degrees, their respective flux contributions will also be displaced by 120°, as shown in Figure 4.1(b). Figure 4.1(c) is keyed to Figure 4.1(b), and shows the instantaneous direction of stator flux as it passes through the rotor at different instants of time. For example, at zero degrees, phase *A* is a maximum north pole, while phases *B* and *C* are weak south poles; at 60° phase *C* is a strong south pole, while phases *A* and *B* are weak north poles; at 120° phase *B* is a strong north pole, while phases *A* and *C* are weak south poles, and so forth. The large arrows indicate the instantaneous di-

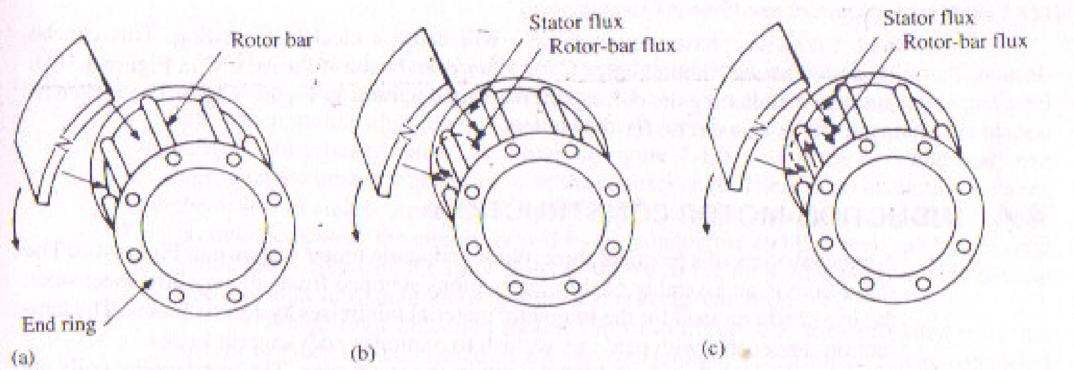


FIGURE 4.2
(a) Rotating field sweeping a rotor bar; (b) direction of flux generated around rotor bar; (c) direction of rotor-bar current.

rection of the resultant flux. The different angular positions assumed by the resultant flux vector show the plane of the flux to be revolving in a counterclockwise (CCW) direction. Although the flux generated by each coil is only an alternating flux, the combined flux contributions of the three staggered coils, carrying currents at appropriate sequential phase angles, produce a two-pole rotating flux. It is the rotating flux, not the alternating flux, that produces induction-motor action.

The rotating flux (also called *rotating field*) produced by the three-phase currents in the stationary coils, may be likened to the rotating field produced by a magnet sweeping around the rotor, as shown in Figure 4.2(a). The rotating magnetic field "cuts" the rotor bars (conductors) in its CCW sweep around the rotor. The speed of the rotating field is called the *synchronous speed.*

In accordance with Lenz's law, the voltage, current, and flux generated by the relative motion between a conductor and a magnetic field will be in a direction to oppose the relative motion. Hence, to satisfy Lenz's law, the conductors must develop a mechanical force or thrust in the same direction as the rotating flux (CCW). For this to happen, "flux bunching" must occur on the right side of the conductor; thus, the generated flux due to rotor-bar current must be clockwise (CW) as shown in Figure 4.2(b). The direction of rotor-bar current that produces this CW flux is determined by the right-hand rule and is shown in Figure 4.2(c).

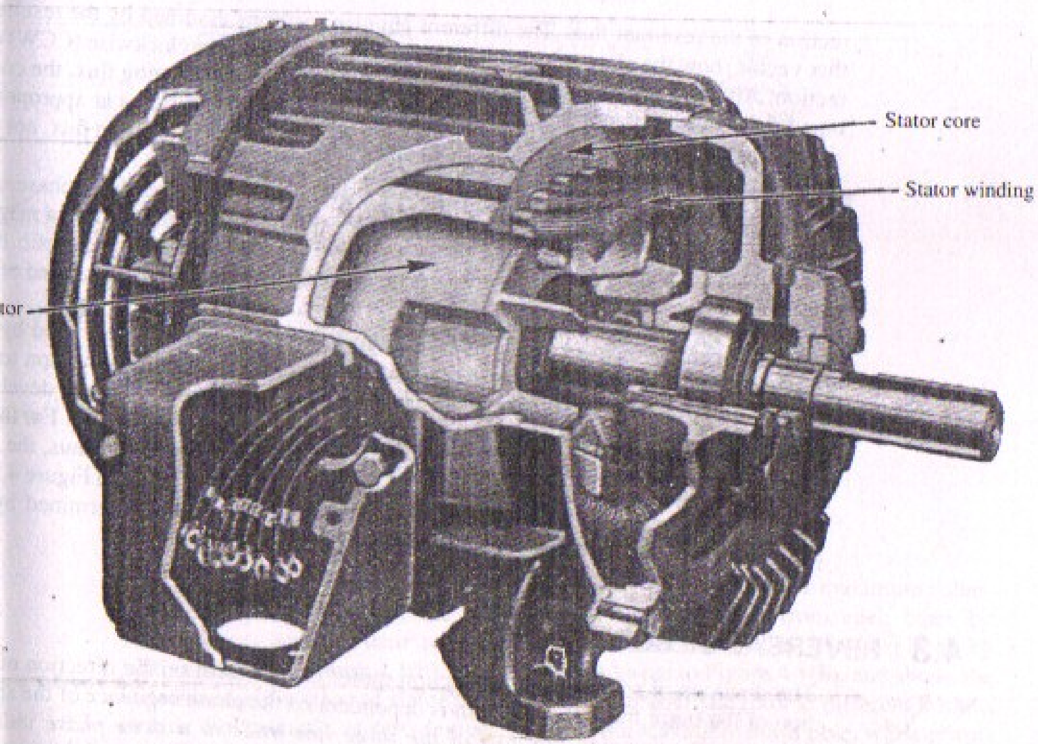
4.3 REVERSAL OF ROTATION

The direction of rotation of an induction motor is dependent on the direction of rotation of the stator flux, which in turn is dependent on the phase sequence of the applied voltage. *Interchanging any two of the three line-leads to a three-phase induction motor will reverse the phase sequence, thus reversing the rotation of the motor.* As shown in Figure 4.1, the phase sequence *ABC* causes a CCW rotation of the magnetic field. Likewise, phase sequence *CBA* will cause a clockwise rotation. This can be illustrated by substituting letter *C* for letter *A* and letter *A* for letter *C* in Figure 4.1(b), and then resketching the corresponding flux diagrams in Figure 4.1(c). The resketching is left as an exercise for the student.

.4 INDUCTION-MOTOR CONSTRUCTION

A cutaway view of a practical three-phase induction motor is shown in Figure 4.3. The stator core is an assembly of thin laminations stamped from silicon-alloy sheet steel; the use of silicon steel for the magnetic material minimizes hysteresis losses. The laminations are coated with oxide or varnish to minimize eddy-current losses.

Insulated coils are set in slots within the stator core. The overlapping coils are connected in series or parallel arrangements to form phase groups, and the phase groups are connected wye or delta. The connections, wye or delta, series or parallel, are dictated by voltage and current requirements.



GURE 4.3
itaway view of a three-phase induction motor (Courtesy Siemens Energy and Automation)

The rotors are of two basic types: squirrel cage and wound rotor. Small squi cage rotors, such as that shown in Figure 4.4(a), use a slotted core of laminated into which molten aluminum is cast to form the conductors, end rings, and fan bla Larger squirrel-cage rotors, as shown in Figure 4.4(b), use brass bars and brass rings that are brazed together to form the squirrel cage. There is no insulation betw the iron core and the conductors, and none is needed; the current induced in the r is contained within the circuit formed by the conductors and end rings, also called connections. Skewing the rotor slots, as shown in Figure 4.4(a), helps avoid craw (locking in at subsynchronous speeds) and reduces vibration.

A wound-rotor induction motor, shown in Figure 4.5, uses insulated coils are set in slots and connected in a wye arrangement. The rotor circuit is compl through a set of slip rings, carbon brushes, and a wye-connected rheostat. The th phase rheostat is composed of three rheostats connected in wye; a common leve used to simultaneously adjust all three rheostat arms. Moving the rheostat to the "z resistance" position, extreme left in Figure 4.5, shorts the resistors and simulat squirrel-cage motor. The rheostat is used to adjust starting torque and running spe

The transfer of energy from the stator to the rotor, whether squirrel cage wound rotor, is by means of electromagnetic induction and occurs in a manner sim to that in a transformer. For that reason, the stator is often referred to as the prim and the rotor as the secondary. Since the energy to do work is transferred electrom netically across the air gap between the stator and the rotor, the air gap is made q small so as to offer minimum reluctance.

Each coil of an induction motor stator spans a portion of the stator circumfere equal to or slightly less than the pole pitch; the pole pitch is equal to the stator circu ference divided by the number of stator poles, and it may be expressed in terms of tor slots or stator arc. For example, each coil of a four-pole stator spans one-quarte less of the circumference. If the coil span (also called coil pitch) is equal to the p pitch it is called a full-pitch winding; if the span is less than full pitch, it is calle fractional-pitch winding. Figure 4.6 shows the coil span for full-pitch four-pole eight-pole stator windings.[2] The three black arcs represent the end view of three sta coils, each representing one phase. The angles in Figure 4.6 are in mechanical degre

4.5 SYNCHRONOUS SPEED

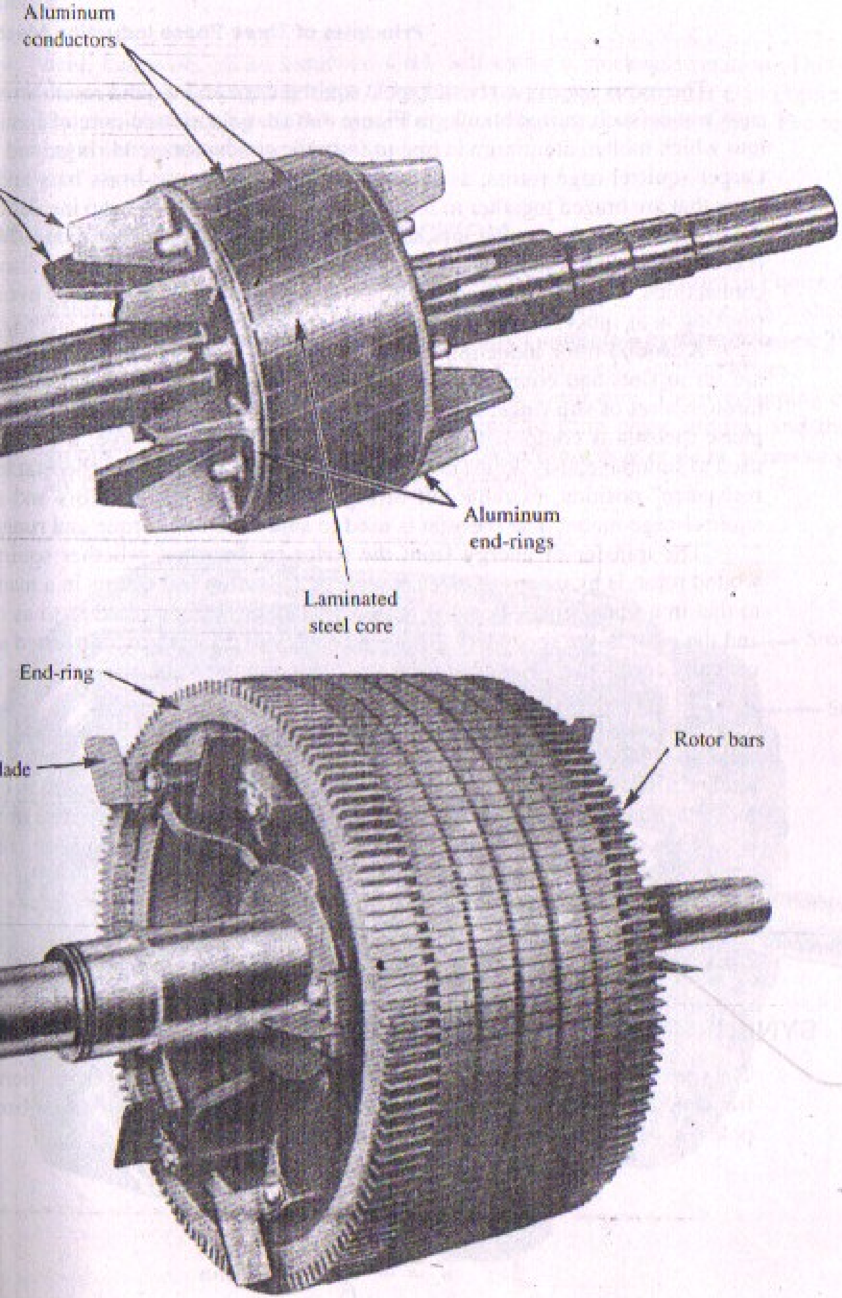
The speed of the rotating flux, called *synchronous speed*, is directly proportional to frequency of the supply voltage and inversely proportional to the number of pairs poles: poles only occur in pairs. Expressed mathematically,

$$n_s = \frac{f_s}{P/2} = \frac{2 \times f_s}{P} \quad \text{r/s}$$

$$n_s = \frac{120 \times f_s}{P} \quad \text{r/min} \qquad (4-$$

[1] See Section 5.9, Chapter 5, for application of rheostats to wound-rotor motors.

[2] See Appendix B for more details on stator windings.



.4
ge rotor: (a) cast-aluminum conductors; (b) brazed conductors and end rings. (Courtesy
nergy and Automation)

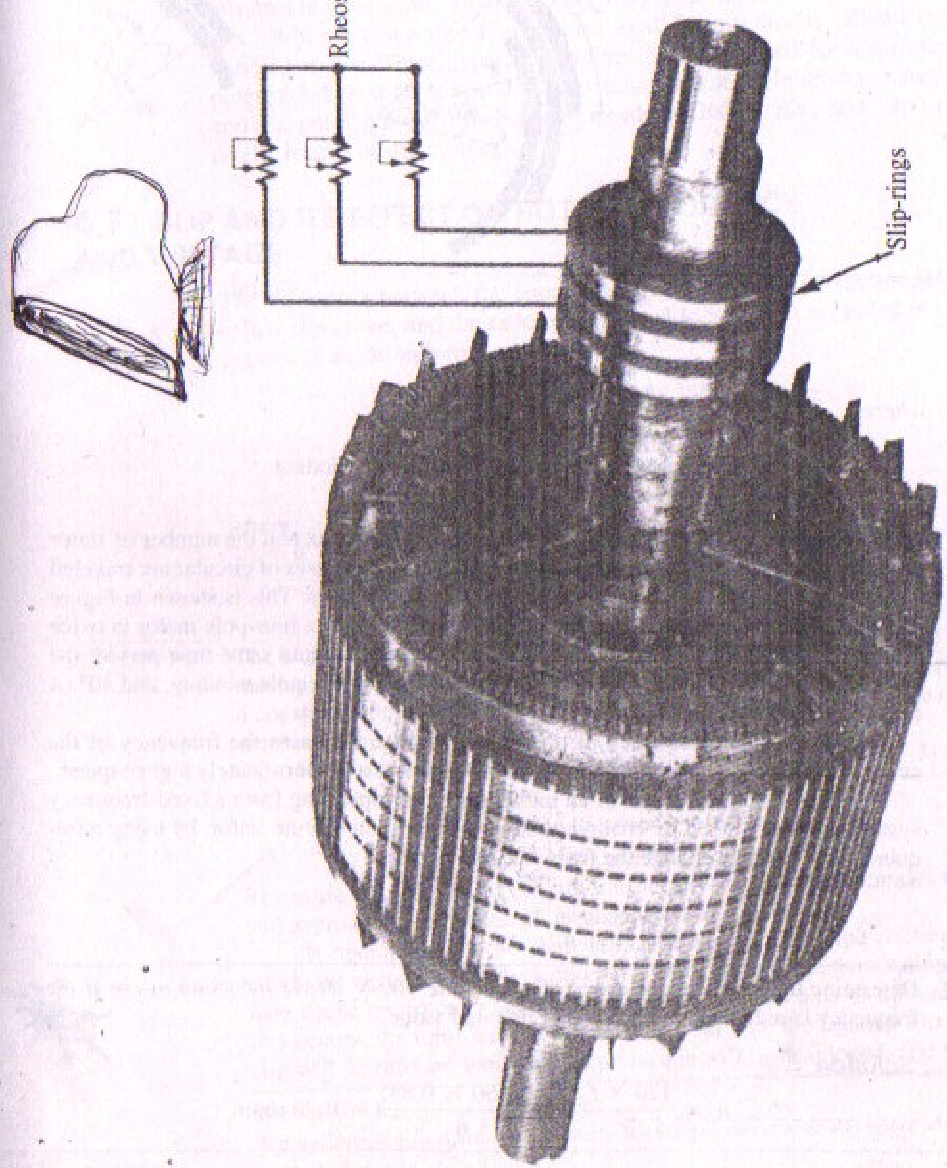
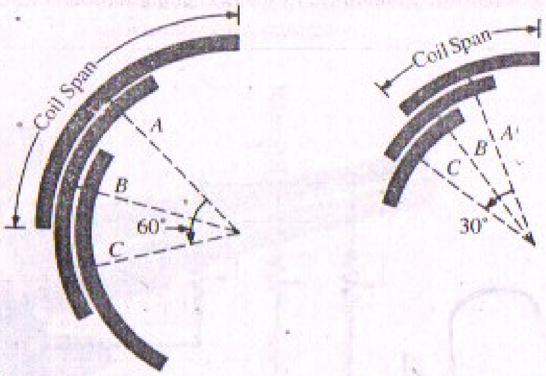


FIGURE 4.5
Wound-rotor induction motor showing rheostat connections. (Courtesy Dresser-Rand, Electric Machinery)

IGURE 4.6
oil span for (a) four-pole winding; (b) eight-
ole winding.



(a) (b)

where: f_s = frequency of the three-phase supply
n_s = synchronous speed
P = number of poles formed by the stator winding

The relationship between the speed of the rotating flux and the number of stator poles may be visualized by comparing the *mechanical degrees* of circular arc traveled by the flux in motors with different numbers of stator poles. This is shown in Figure 4.6, where the circular arc traveled by the rotating field of a four-pole motor is twice that of an eight-pole motor, assuming the same frequency and same time period; the centerline of flux rotates 60° (phase A to phase C) for a four-pole winding, and 30° (A to C) for an eight-pole winding.

Increasing the frequency of the supply voltage increases the frequency of the current in the stator coils, causing the flux to rotate at a proportionately higher speed.

The synchronous speed of an induction motor operating from a fixed-frequency system can be changed by changing the number of poles in the stator, by using a frequency converter to change the frequency, or both.

XAMPLE .1 Determine the synchronous speed of a six-pole 460-V 60-Hz induction motor if the frequency is reduced to 85 percent of its rated value.

Solution

$$n_s = \frac{120 \times f_s}{P} = \frac{120(60 \times 0.85)}{6} = 1020 \text{ r/min}$$

4.6 MULTISPEED FIXED-FREQUENCY POLE-CHANGING MOTORS

Pole changing may be accomplished by using separate windings for each speed, or by reconnecting the windings of specially designed machines called *consequent-pole* motors.[3] When two separate windings are used, the machine is called a two-speed two-winding motor. Three separate windings, each arranged for a different number of poles, result in a three-speed three-winding motor. Pole arrangements of two, four, and six poles provide synchronous speeds of 3600, 1800, and 1200 r/min, respectively, from a 60-Hz system.

4.7 SLIP AND ITS EFFECT ON ROTOR FREQUENCY AND VOLTAGE

The difference between the speed of the rotating flux and the speed of the rotor is called *slip speed*, and the ratio of slip speed to synchronous speed is called *slip*. Expressed in equation form:

$$n = n_s - n_r \tag{4–2}$$

$$s = \frac{n_s - n_r}{n_s} \tag{4–3}$$

where: n = slip speed (r/min)
n_s = synchronous speed (r/min)
n_r = rotor speed (r/min)
s = slip (pu)

The slip, as expressed in Eq. (4–3), is called *per-unit slip*.[4] The slip depends on the mechanical load connected to the rotor shaft (assuming a constant supply voltage and a constant supply frequency). Increasing the shaft load decreases the rotor speed, thu increasing the slip.

If the rotor is blocked to prevent turning, $n_r = 0$, and Eq. (4–3) reduces to

$$s = \frac{n_s - 0}{n_s} = 1$$

Releasing the brake allows the rotor to accelerate. The slip decreases with acceleratio and approaches zero when all mechanical load is removed.

If operating with no shaft load, and the windage and friction are sufficient small, the very low relative motion between the rotor and the rotating flux of the stat may cause the rotor to become magnetized along an axis of minimum reluctance. this occurs, the rotor will lock in synchronism with the rotating flux of the stator; t slip will be zero, no induction motor torque will be developed, and the motor will a

[3]See Appendix B.
[4]If slip is given in percent, it must be divided by 100 to obtain the per-unit value before substituting into equati

as a reluctance-synchronous motor.[5] The application of a small shaft load will cause it to pull out of synchronism, however, and induction-motor action will again occur. Solving Eq. (4–3) for n_r expresses the rotor speed in terms of slip:

$$n_r = n_s(1 - s) \tag{4–4}$$

Effect of Slip on Rotor Frequency

The frequency of the voltage induced in a rotor loop by a rotating magnetic field is given by[6]

$$f_r = \frac{P \times n}{120} \tag{4–5}$$

where: f_r = rotor frequency (Hz)
P = number of stator poles
n = slip speed (r/min)

Substituting Eq. (4–2) into Eq. (4–5)

$$f_r = \frac{P(n_s - n_r)}{120} \tag{4–6}$$

From Eq. (4–3)

$$n_s - n_r = sn_s$$

Substituting into Eq. (4–6)

$$f_r = \frac{sPn_s}{120} \tag{4–7}$$

If the rotor is blocked so that it cannot turn, $s = 1$, and Eq. (4–7) becomes

$$f_{BR} = \frac{Pn_s}{120} \tag{4–8}$$

where f_{BR} = frequency of voltage generated in the blocked rotor. Substituting Eq. (4–8) into Eq. (4–7) results in the general expression for rotor frequency in terms of slip and blocked-rotor frequency. Thus,

$$f_r = sf_{BR} \tag{4–9}$$

At blocked rotor, also called locked rotor, there is no relative motion between rotor and stator, the slip is 1.0, and the frequency of the voltage generated in the rotor is identical to the frequency of the applied stator voltage. That is,

$$f_{BR} = f_{stator}$$

[5] See Section 7.2, Chapter 7.

[6] See Section 1.15, Chapter 1.

Effect of Slip on Rotor Voltage

Referring to Figure 4.2, the voltage generated in a rotor loop (formed by two rotor bars and the end connections) as it is swept by the rotating stator flux is given by[7]

$$E_r = 4.44Nf_r\Phi_{max}$$

Substituting Eq. (4–9) into Eq. (1–25),

$$E_r = 4.44Nsf_{BR}\Phi_{max} \tag{4–10}$$

At blocked rotor, $s = 1$ and Eq. (4–10) becomes

$$E_{BR} = 4.44Nf_{BR}\Phi_{max} \tag{4–11}$$

Substituting Eq. (4–11) into Eq. (4–10),

$$E_r = sE_{BR} \tag{4–12}$$

Equation (4–12) is the general expression for the voltage induced in a rotor loop at any rotor speed, in terms of blocked-rotor voltage and slip.

[7] See Section 1.12, Chapter 1.

EXAMPLE 4.2

The frequency and induced voltage in the rotor of a certain six-pole wound-rotor induction motor, whose shaft is blocked, are 60 Hz and 100 V, respectively. Determine the corresponding values when the rotor is running at 1100 r/min.

Solution

$$n_s = \frac{120f_s}{P} = \frac{120 \times 60}{6} = \underline{1200 \text{ r/min}}$$

$$s = \frac{n_s - n_r}{n_s} = \frac{1200 - 1100}{1200} = \underline{0.0833}$$

$$f_r = sf_{BR} = 0.0833 \times 60 = \underline{5.0 \text{ Hz}}$$

$$E_r = sE_{BR} = 0.0833 \times 100 = \underline{8.33 \text{ V}}$$

4.8 EQUIVALENT CIRCUIT OF AN INDUCTION-MOTOR ROTOR

A feeling for the characteristic behavior of a three-phase induction motor may be ob- tained by manipulation and analysis of an equivalent-circuit model representing one phase of an induction-motor rotor, such as that shown in Figure 4.7(a). Note, however, that the power and torque developed by a three-phase motor is three times that devel- oped by one of its phases. For purposes of simplicity in analysis, it will be assumed that the stator is an ideal stator, in that it produces a rotating magnetic field of constant amplitude and constant speed, and that it has no core losses, no copper losses, and

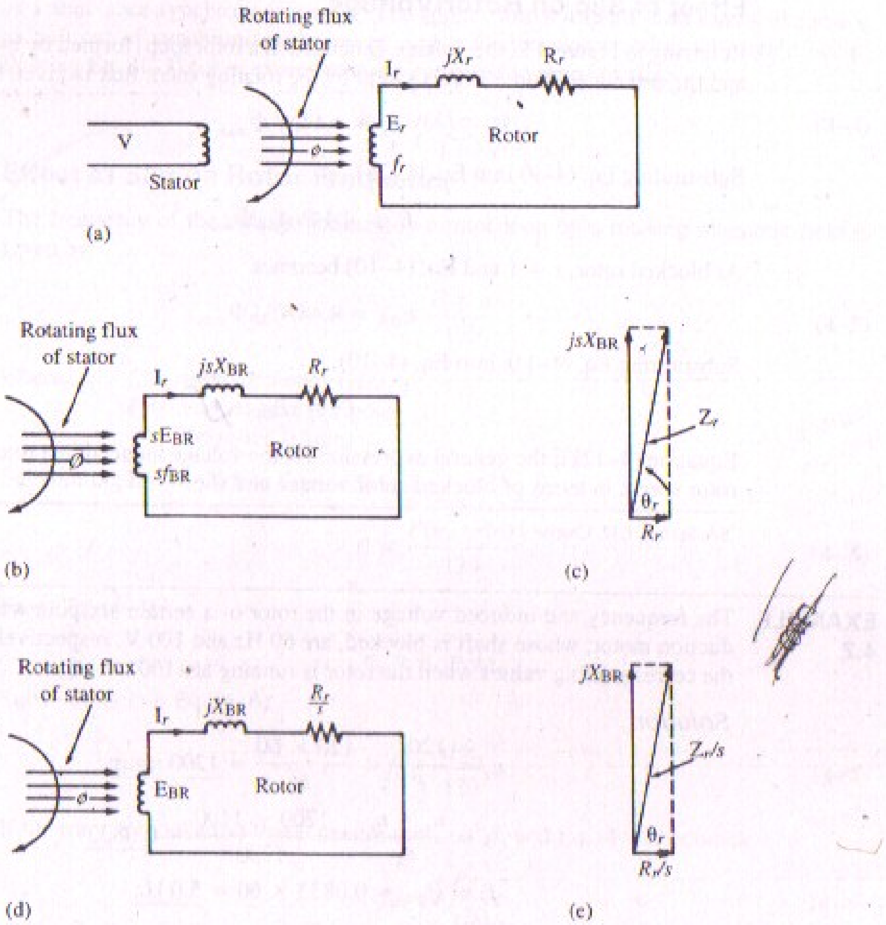


FIGURE 4.7
Equivalent circuits and corresponding impedance diagrams for an induction motor with an imaginary (ideal) stator and a real rotor.

voltage drops.[8] The rotor is represented by an electrically isolated closed circuit containing resistance and reactance acted on by an induced rotor voltage E_r. The rotor voltage is generated at frequency f_r by the rotating flux of the stator. The model, shown in Figure 4.7(a), represents one phase of a wound rotor, or one phase of an equivalent wound rotor if the rotor is squirrel cage.

[8]The effect of stator parameters (resistance and reactance) on induction-motor performance is discussed in Section 5.4, Chapter 5.

The rotor resistance is dependent on the length, cross-sectional area, resistivit and skin effect of the rotor conductors, as well as external rheostat resistance if it is wound rotor like that shown in Figure 4.5. The inductive reactance X_r of the roto called *leakage reactance*, is caused by leakage flux and is dependent on the shape the rotor conductors, its depth in the iron core, the frequency of the rotor voltage, ar the length of the air gap between the rotor iron and the stator iron.[9]

The leakage reactance of the rotor, expressed in terms of rotor frequency ar rotor inductance, is

$$X_r = 2\pi f_r L_r \qquad \text{(4–1}$$

Substituting Eq. (4–9) into Eq. (4–13) and simplifying,

$$X_r = 2\pi(sf_{BR})L_r = s(2\pi f_{BR} L_r)$$

$$X_r = sX_{BR} \qquad \text{(4–1}$$

Replacing X_r, E_r, and f_r in Figure 4.7(a) with their equivalent values in terms slip results in Figure 4.7(b). The rotor impedance, as determined from the associat impedance diagram in Figure 4.7(c), is

$$Z_r = R_r + jsX_{BR} \qquad \text{(4–1}$$

Applying Ohm's law to the rotor circuit in Figure 4.7(b),

$$\mathbf{I}_r = \frac{s\mathbf{E}_{BR}}{\mathbf{Z}_r} = \frac{s\mathbf{E}_{BR}}{R_r + jsX_{BR}} \qquad \text{(4–1}$$

Dividing both numerator and denominator by s,

$$\mathbf{I}_r = \frac{\mathbf{E}_{BR}}{\mathbf{Z}_r/s} = \frac{\mathbf{E}_{BR}}{R_r/s + jX_{BR}} \qquad \text{(4–}$$

A modified equivalent series circuit and associated impedance diagram, correspond to Eq. (4–17), are shown in Figures 4–7(d) and (e), respectively. The constant block rotor voltage in Figure 4.7(d), combined with an equivalent rotor resistance that va with the slip, provides a convenient tool for analysis of induction-motor behavior.

Expressing the rotor current in terms of magnitude and phase angle,[10]

$$\mathbf{I}_r = \frac{E_{BR}\angle 0^\circ}{(Z_r/s)\angle\theta_r} = \frac{E_{BR}}{Z_r/s}\angle -\theta_r$$

The magnitude of the rotor current is

$$I_r = \frac{E_{BR}}{Z_r/s} \qquad \text{(4–}$$

[9]Leakage reactance, which also occurs in transformers, is explained in Section 2.9, Chapter 2.
[10]For convenience, the phase angle of $\mathbf{E}_{BR}$ is assumed to be zero degrees.

Expressing Z_r/s and θ_r in terms of their components, as shown in Figure 4.7(e),

$$I_r = \frac{E_{BR}}{\sqrt{(R_r/s)^2 + X_{BR}^2}} \tag{4-19}$$

$$\theta_r = \tan^{-1}\left(\frac{X_{BR}}{R_r/s}\right) \tag{4-20}$$

E The rotor of a certain 25-hp, six-pole, 60-Hz induction motor has equivalent resistance and equivalent reactance per phase of 0.10 Ω and 0.54 Ω, respectively. The blocked-rotor voltage/phase (E_{BR}) is 150 V. If the rotor is turning at 1164 r/min, determine (a) synchronous speed; (b) slip; (c) rotor impedance; (d) rotor current; (e) rotor current if changing the shaft load resulted in 1.24 percent slip; (f) speed for the conditions in (e).

Solution

(a) $n_s = \dfrac{120f}{P} = \dfrac{120 \times 60}{6} = \underline{1200 \text{ r/min}}$

(b) $s = \dfrac{n_s - n_r}{n_s} = \dfrac{1200 - 1164}{1200} = \underline{0.030}$

(c) $\mathbf{Z}_r = \dfrac{R_r}{s} + jX_{BR} = \dfrac{0.010}{0.03} + j0.54 = 3.3768\underline{/9.20^\circ} \Rightarrow \underline{3.38/9.20^\circ\ \Omega}$

(d) $\mathbf{I}_r = \dfrac{\mathbf{E}_{BR}}{\mathbf{Z}_r} = \dfrac{150\underline{/0^\circ}}{3.3768\underline{/9.20^\circ}} = 44.421\underline{/-9.2^\circ} \Rightarrow \underline{44.4/-9.2^\circ \text{ A}}$

(e) $\mathbf{Z}_r = \dfrac{R_r}{s} + jX_{BR} = \dfrac{0.10}{0.0124} + j0.54 = 8.08257\underline{/3.83} \Rightarrow \underline{8.08/3.83^\circ\ \Omega}$

$\mathbf{I}_r = \dfrac{\mathbf{E}_{BR}}{\mathbf{Z}_r} = \dfrac{150\underline{/0^\circ}}{8.08257\underline{/3.83^\circ}} = 18.558\underline{/-3.83^\circ} \Rightarrow \underline{18.6/-3.83^\circ \text{ A}}$

(f) $n_r = n_s(1-s) = 1200(1-0.0124) = \underline{1185 \text{ r/min}}$

LOCUS OF THE ROTOR CURRENT

The changes that take place in rotor-impedance angle θ_r, and rotor-current magnitude I_r as an unloaded induction motor accelerates from standstill (blocked rotor) to synchronous speed, are shown in Figure 4.8(a). The curves are plots of Eqs. (4–20) and (4–19), respectively. *Note that the rotor current and the rotor impedance angle have their greatest values at blocked rotor, both decrease in value as the rotor accelerates, and both approach zero as the rotor approaches synchronous speed.* Note also that, *for low values of slip* ($s < 0.05$) *the rotor current is proportional to the slip.*

A phasor diagram representing the magnitude and phase angle of the rotor current for values of slip from $s = 1$ to $s = 0$ is shown in Figure 4.8(b). As indicated, the

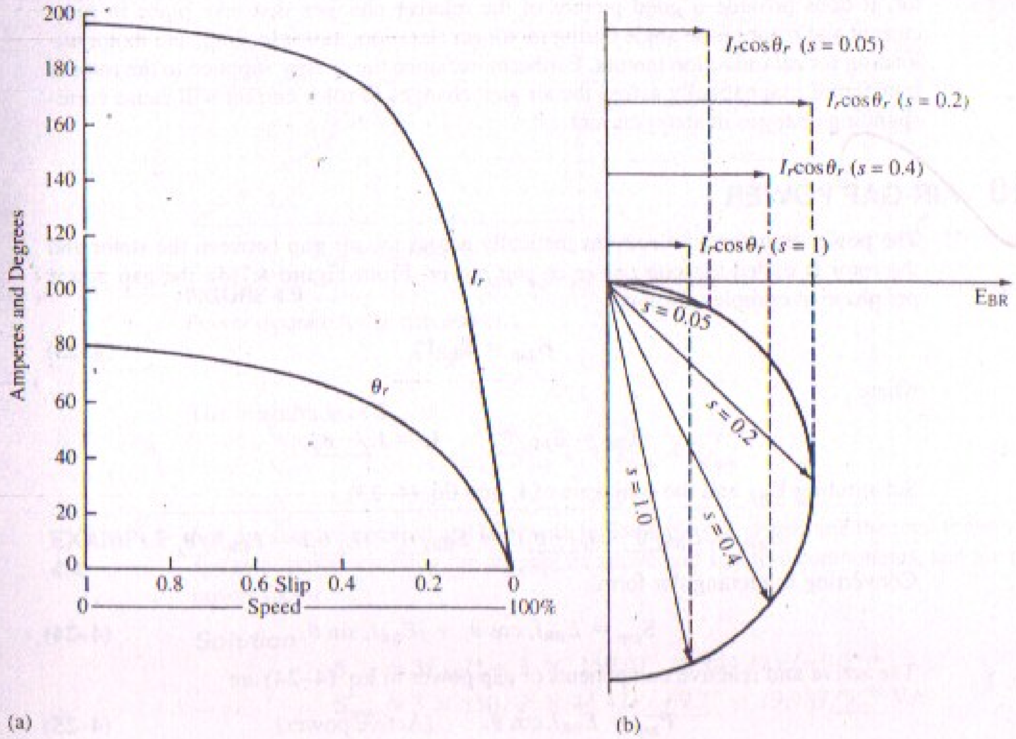


FIGURE 4.8
(a) Rotor current and rotor-impedance angle vs. speed for a representative induction motor; (b) locus of rotor-current phasor.

current phasor changes in both magnitude and phase angle as the machine accelerates from blocked rotor ($s = 1$) to synchronous speed ($s = 0$). *Note that the locus of the current phasor is a semicircle.* Proof of its semicircle character is obtained by expressing, Z_r/s in terms of $\sin\theta_r$, and then substituting into Eq. (4–18). Thus, from Figure 4.7(e),

$$\frac{Z_r}{s} = \frac{X_{BR}}{\sin\theta_r} \tag{4-21}$$

Substituting Eq. (4–21) into Eq. (4–18) and simplifying,

$$I_r = \frac{E_{BR}}{X_{BR}}\sin\theta_r \tag{4-22}$$

Equation (4–22) is the polar equation for a circle that is tangent to the horizontal axis at the origin and whose diameter is E_{BR}/X_{BR}.

Although the "circle diagram" in Figure 4.8(b) is for a machine with an ideal stator, it does provide a good picture of the relative changes that take place in rotor current and rotor phase angle during motor acceleration, motor loading, and motor unloading for *all* induction motors. Furthermore, since the energy supplied to the rotor is transferred magnetically across the air gap, changes in rotor current will cause corresponding changes in stator current.

AIR-GAP POWER

The power transferred electromagnetically across the air gap between the stator and the rotor is called *air-gap power* or *gap power*. From Figure 4.7(d), the gap power per phase in complex form is[11]

$$\mathbf{S}_{gap} = \mathbf{E}_{BR}\mathbf{I}_r^* \qquad (4\text{–}23)$$

where

$$\mathbf{E}_{BR} = E_{BR}\underline{/0^\circ} \qquad \mathbf{I}_r = I_r\underline{/-\theta_r}$$

Substituting $\mathbf{E}_{BR}$ and the conjugate of $\mathbf{I}_r$ into Eq. (4–23),

$$\mathbf{S}_{gap} = E_{BR}\underline{/0^\circ} \cdot (I_r\underline{/-\theta_r})^* = E_{BR}\underline{/0^\circ} \cdot (I_r\underline{/\theta_r}) = E_{BR}I_r\underline{/\theta_r}$$

Converting to rectangular form,

$$\mathbf{S}_{gap} = E_{BR}I_r\cos\theta_r + jE_{BR}I_r\sin\theta_r \qquad (4\text{–}24)$$

The active and reactive components of gap power in Eq. (4–24) are

$$P_{gap} = E_{BR}I_r\cos\theta_r \quad \text{(Active power)} \qquad (4\text{–}25)$$

$$Q_{gap} = E_{BR}I_r\sin\theta_r \quad \text{(Reactive power)} \qquad (4\text{–}26)$$

where: E_{BR} = blocked rotor voltage
I_r = magnitude of rotor current
θ_r = rotor impedance angle
$\cos\theta_r$ = power factor of rotor

Active component P_{gap} supplies the shaft power output as well as friction, windage, and heat losses in the rotor.

Reactive component Q_{gap} supplies the reactive power for the alternating magnetic field about the rotor current. Component Q_{gap} is not dissipated; it follows a sinusoidal pattern as it "see-saws" across the gap between the stator and the rotor.

Components P_{gap} and Q_{gap} may be represented in a power diagram as two sides of a right triangle whose diagonal is S_{gap} as shown in Figure 4.9. From the geometry of the power diagram,

$$S_{gap} = P_{gap} + jQ_{gap} \qquad (4\text{–}27)$$

[11] See Appendix A.5 for a review of complex power, also called phasor power.

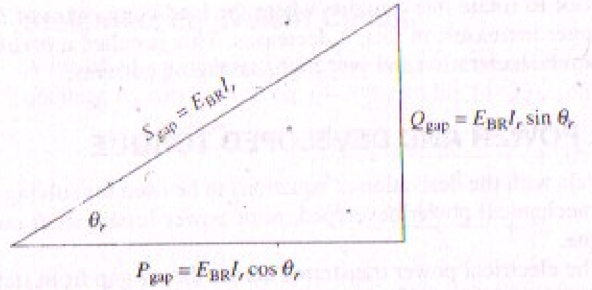


FIGURE 4.9
Power diagram for air-gap power.

The magnitude of S_{gap} is

$$S_{gap} = \sqrt{P_{gap}^2 + Q_{gap}^2} \qquad (4\text{–}28)$$

EXAMPLE 4.4 For the motor operating at 1164 r/min in Example 4.3, determine the total three-phase apparent power crossing the air gap, its active and reactive components, and the rotor power factor.

Solution

$$\mathbf{S}_{gap} = 3E_{BR}I_r^* = 3 \times 150\underline{/0^\circ} \times (44.421\underline{/-9.2^\circ})^*$$

$$\mathbf{S}_{gap} = 3 \times 150\underline{/0^\circ} \times 44.421\underline{/+9.2^\circ} = \underline{19{,}989\underline{/9.2^\circ}\text{ VA}}$$

Converting to rectangular form,

$$\mathbf{S}_{gap} = (19{,}732 + j3197)\text{ VA}$$

$$P_{gap} = \underline{19{,}732\text{ W}} \qquad Q_{gap} = \underline{3196\text{ var}}$$

$$F_P = \cos 9.2^\circ = \underline{0.99}$$

Interpreting the Circle Diagram

Blocked-rotor voltage E_{BR} is assumed constant because it is proportional to an assumed constant flux density. Thus, referring to Eq. (4–25), P_{gap} is proportional to $I_r\cos\theta_r$. This is shown in Figure 4.8(b) as the projection of the current phasor on the horizontal axis.

Assume the motor is partly loaded and operating at slip $s = 0.05$. Increasing the shaft load on the motor causes it to slow down and the slip to increase. The increased slip increases the magnitude and phase angle of the rotor current. This causes the current phasor in Figure 4.8(b) to elongate and rotate clockwise to a position that results in an $I_r\cos\theta_r$ sufficient to carry the load. Severely overloading the motor can cause the current phasor to rotate into a region where the load component of motor current $I_r \cos\theta_r$ no longer increases; in fact, it decreases. This is called a *breakdown condition*, causing rapid deceleration and very high, damaging currents.

MECHANICAL POWER AND DEVELOPED TORQUE

This section deals with the derivation of equations to be used for solving problems relating to slip, mechanical power developed, rotor power losses, shaft power out, and developed torque.

Most of the electrical power transferred across the air gap from stator to rotor is converted to mechanical power; the remainder is expended as I^2R heat-power losses in the rotor conductors. Expressed as an equation,

$$P_{gap} = P_{mech} + P_{rcl} \quad \text{W} \tag{4–29}$$

where P_{rcl} = rotor conductor losses.

Examination of the *equivalent circuit* for one phase of the rotor in Figure 4.7(d), however, indicates that there is no provision for mechanical power. In fact, with respect to Figure 4.7(d), all of the air-gap power is dissipated as heat losses in the *equivalent resistance R_r/s*; the reactance X_{BR} draws no active power. Thus, the total air-gap power delivered to the rotor for all three phases expressed in terms of R_r/s is

$$P_{gap} = \frac{3I_r^2R_r}{s} \quad \text{W} \tag{4–30}$$

$$P_{gap} = \frac{P_{rcl}}{s} \quad \text{W} \tag{4–31}$$

Equation (4–30) expresses the heat power expended in *equivalent rotor resistance R_r/s*. The *real rotor resistance*, as shown in Figure 4.7(a), however, is R_r. Thus, the actual heat power expended in the *real rotor conductors* for all three phases is

$$P_{rcl} = 3I_r^2R_r \quad \text{W} \tag{4–32}$$

Substituting Eqs. (4–30) and (4–32) into Eq. (4–29),

$$\frac{3I_r^2R_r}{s} = P_{mech} + 3I_r^2R_r \tag{4–33}$$

Solving Eq. (4–33) for P_{mech}, and rearranging terms,

$$P_{mech} = \frac{3I_r^2R_r(1-s)}{s} \quad \text{W} \tag{4–34}$$

Substituting Eq. (4–30) into Eq. (4–34),

$$P_{mech} = P_{gap}(1-s) \quad \text{W} \tag{4–35}$$

Equation (4–35) represents the total mechanical power developed at slip s.

Steinmetz Equivalent Circuit

A modified equivalent circuit that is used extensively in induction-motor analysis is obtained by substituting Eq. (4–34) into Eq. (4–33), and dividing through by $3I_r^2$:

$$\frac{3I_r^2R_r}{s} = \frac{3I_r^2R_r(1-s)}{s} + 3I_r^2R_r$$

$$\frac{R_r}{s} = \frac{R_r(1-s)}{s} + R_r \tag{4–36}$$

Equation (4–36) indicates that the equivalent resistance R_r/s of Figure 4.7(d) can be split into two series-connected components as shown in Figure 4.10,[12]

where: R_r = actual resistance per phase of the rotor windings (Ω)
$R_r(1-s)/s$ = equivalent resistance per phase that expends energy at a rate equal to the mechanical power produced (Ω)

Developed Torque

From Eq. (4–4),

$$\frac{n_r}{n_s} = (1-s)$$

Substituting into Eq. (4–34) and simplifying results in the following equation for mechanical power developed in the rotor of a three-phase motor in terms of rotor speed:

$$P_{mech} = \frac{3I_r^2R_rn_r}{sn_s} \quad \text{W} \tag{4–37}$$

Motor nameplates and motor data, supplied by manufacturers and the National Electrical Manufacturers Association (NEMA), are expressed in hp, r/min, and lb-ft

[12] The equivalent-circuit model of the rotor shown in Figure 4.10 was developed by Charles Proteus Steinmetz.

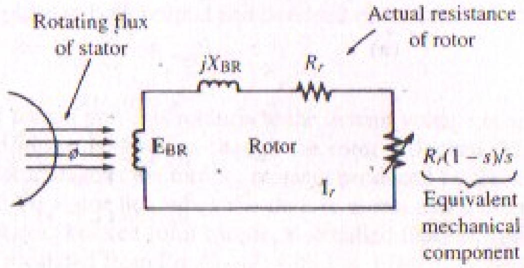


FIGURE 4.10
Equivalent circuit of rotor, with R_r/s split into actual rotor resistance and an equivalent mechanical component.

torque. Hence, these units will be used exclusively in all electric-motor problems.[13] Thus, converting Eq. (4–37) to horse-power,

$$P_{mech} = \frac{3I_r^2 R_r n_r}{746 s n_s} \quad \text{hp} \qquad \textbf{(4–38)}$$

The basic equation that relates horsepower to developed torque and rotor speed is

$$P_{mech} = \frac{T_D n_r}{5252} \quad \text{hp} \qquad \textbf{(4–39)}$$

where: T_D = developed torque (lb-ft)
n_r = shaft speed (r/min)
P_{mech} = mechanical power developed in rotor (hp)

Substituting Eq. (4–39) into Eq. (4–38) and solving for T_D,

$$\frac{T_D n_r}{5252} = \frac{3I_r^2 R_r n_r}{746 s n_s}$$

$$T_D = \frac{7.04 \times 3I_r^2 R_r}{s n_s} = \frac{21.12 I_r^2 R_r}{s n_s} \quad \text{lb-ft} \qquad \textbf{(4–40)}$$

Substituting Eq. (4–30) into Eq. (4–40),

$$T_D = \frac{7.04 P_{gap}}{n_s} \quad \text{lb-ft} \qquad \textbf{(4–41)}$$

Equations (4–40) and (4–41) represent the pound-foot torque developed in the rotor of a three-phase induction motor.

[13] Nameplates on machines used in the United States indicate r/min as RPM. Data on nameplates of foreign motors are expressed in SI units: rad/s, watts, and newton · meters (N · m).

LE A three-phase, 460-V, 25-hp, 60-Hz, four-pole induction motor operating at reduced load requires 14.58-kW input to the rotor. The rotor copper losses are 263 W, and the combined friction, windage, and stray power losses are 197 W. Determine (a) shaft speed; (b) mechanical power developed; (c) developed torque.

Solution

(a)

$$P_{gap} = \frac{P_{rcl}}{s} \Rightarrow \qquad 14{,}580 = \frac{263}{s}$$

$$s = 0.018$$

$$n_s = \frac{120 f_s}{P} = \frac{120 \times 60}{4} = 1800 \text{ r/min}$$

$$n_r = n_s(1 - s) = 1800(1 - 0.018) = \underline{1767.6 \text{ r/min}}$$

(b)

$$P_{mech} = P_{gap} - P_{rcl} = 14{,}580 - 263 = \underline{14{,}317 \text{ W}}$$

Expressed in terms of horsepower,

$$P_{mech} = \frac{14{,}317}{746} = \underline{19.19 \text{ hp}}$$

(c)

$$P_{mech} = \frac{T_D n_r}{5252} \Rightarrow \quad T_D = \frac{5252 P_{mech}}{n_r}$$

$$T_D = \frac{5252 \times 19.19}{1767.6} = \underline{57.0 \text{ lb-ft}}$$

Or using Eq. (4–41),

$$T_D = \frac{7.04 P_{gap}}{n_s} = \frac{7.04 \times 14{,}580}{1800} = \underline{57.0 \text{ lb-ft}}$$

4.12 TORQUE-SPEED CHARACTERISTIC

The developed torque, as expressed in Eq. (4–40), is a function of two variables: rotor current and slip. Substituting current equation (4–19) into torque equation (4–40) results in the expression for torque as a function of only one variable, slip. Making this substitution,

$$T_D = \frac{21.12 R_r}{s n_s} \cdot \left[\frac{E_{BR}}{\sqrt{(R_r/s)^2 + X_{BR}^2}} \right]^2$$

$$T_D = \frac{21.12 R_r E_{BR}^2}{s n_s [(R_r/s)^2 + X_{BR}^2]} \quad \text{lb-ft} \qquad \textbf{(4–42)}$$

A plot of Eq. (4–42), called the torque-speed characteristic of an induction motor, is shown in Figure 4.11. The inset is the locus of the rotor-current phasor extracted from Figure 4.8(b); it is used to show the correlation between rotor current and developed torque for an induction motor with an ideal stator. Although derived for a machine with an ideal stator, the torque-speed curve and the discussions associated with it are representative of the general behavior of real machines and, as such, provides the reader with a feel for what takes place in both normal and overload operation.

Locked-Rotor Torque

If the rotor is at rest, mechanical inertia prevents rotation at the instant voltage is applied to the stator; in effect, the motor behaves as though the rotor is locked. The torque developed at the locked-rotor stage is the turning moment produced by the interaction of the rotor current and the stator flux when the shaft is at rest and a three-phase voltage is applied to the stator. Locked-rotor torque, also called blocked-rotor torque, or static torque, may be calculated from Eq. (4–42) with $s = 1.0$.

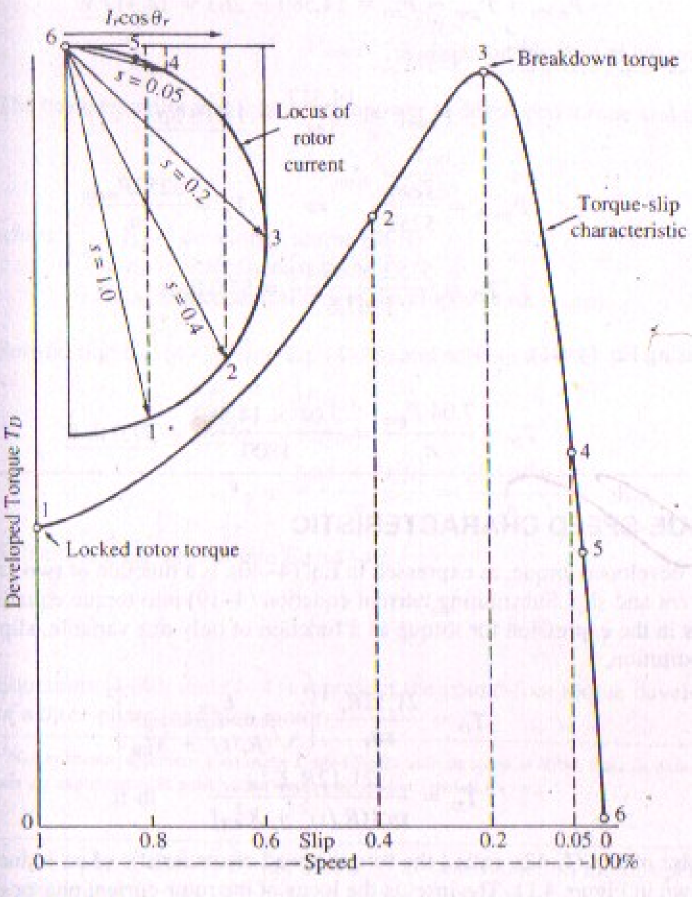


FIGURE 4.11
Representative torque-speed characteristic of an induction motor and corresponding locus of rotor current phasor.

The locked-rotor torque may vary somewhat with different standstill positions of the rotor with respect to the stator. Since this can affect starting, the locked-rotor torque data supplied by a manufacturer is the smallest of the locked rotor values obtained for different rotor positions. Poorly designed motors, or motors rewound for different speeds, or repaired by cutting out defective coils may have significantly lower values of locked-rotor torque at certain angular positions.

If the load torque on the shaft is equal to or exceeds the locked-rotor torque, the motor will not start. Should this occur, and protective devices do not clear the machine from the line, the motor will burn out! Locked-rotor conditions are indicated by point 1 in Figure 4.11.

Acceleration

Assuming the load torque on the shaft is less than the locked-rotor torque, the rotor will accelerate. As the machine accelerates from its standstill position, the slip decreases, causing the magnitude and phase angle of rotor current to decrease. This is shown in Figures 4.11 and 4.8, where $I_r \cos\theta_r$ increases from its low value at blocked rotor to some maximum value, called the breakdown value, and then decreases again with further acceleration. Note the correlation between developed torque (T_D) and $I_r \cos\theta_r$ in Figure 4.11.

Approaching Synchronous Speed

If the shaft is lightly loaded, the rotor speed approaches that of the rotating flux. The current becomes quite small, and even though it is almost in phase with the induced emf, the very low value of $I_r \cos\theta_r$ results in a very low T_D. If there is no load on the shaft, the rotor may sometimes lock in synchronism with the rotating flux. This becomes possible when the relative motion between the two is so small that the rotor iron becomes magnetized along some axis of minimum reluctance, and locks in synchronism with the rotating flux:[14] Under such conditions, the machine no longer develops induction-motor torque; the slip is zero and no current appears in the rotor. The reluctance torque is very small, however, and a light load on the shaft will pull it out of synchronism.

Behavior During Loading and Breakdown

Assume the induction motor portrayed in Figure 4.11 is operating at no load (point 6 on the curve and on the phasor diagram). For this condition, the load torque is essentially windage and friction. As shaft load is applied, the load torque becomes greater than the developed torque, and the motor slows down; the resultant increase in slip causes an increase in $I_r \cos\theta_r$, which in turn causes an increase in the developed torque. If rated load torque is applied to the shaft, the motor will decelerate until the increase in developed torque caused by the increase in slip equals the load torque on the shaft plus windage, friction, and stray load. The motor will then operate at the steady-state speed indicated by point 5.

Further increases in shaft load (overload) cause additional deceleration, accompanied by increases in $I_r \cos\theta_r$, and thus increases in developed torque. If, however, the load torque on the shaft is increased to a value greater than the maximum torque that the machine can develop (point 3), the machine will "break down"; increases in slip, due to increases in shaft load above the breakdown value, cause a rapid decrease in $I_r \cos\theta_r$, and hence a rapid decrease in developed torque. The machine will suffer a sharp drop in speed and may stop. The very high current associated with a high slip will burn out the motor windings unless protective devices remove the machine from

[14] This is called reluctance-motor action, and is discussed in Section 7.2, Chapter 7.

the line. *The breakdown torque is defined as the maximum torque that a motor can develop while being loaded (at rated voltage and rated frequency) without suffering an abrupt drop in speed.*

Although an induction motor can be operated momentarily at overloads up to the breakdown point, it cannot do so continuously without overheating and causing severe damage to both stator and rotor. To prevent damage if a sustained overload occurs, motor control circuits use overload relays and/or solid-state devices to trip the machine from the line.

No-Load Conditions

If there is no load on the shaft, the rotor will run at or near synchronous speed and the rotor current will be at or near zero. Under such conditions, the line current drawn by the stator will be only enough to produce the rotating magnetic field and supply the friction, windage, and iron losses. Thus, in a way, the no-load current drawn by the stator of an induction motor is similar to the exciting current of a transformer that supplies only the transformer flux and iron losses.

Neglecting the induction motor "exciting current," the stator (pri) current will be directly proportional to the rotor (sec) current.[15] Increasing the shaft load increases the rotor current, causing a proportional increase in stator current:

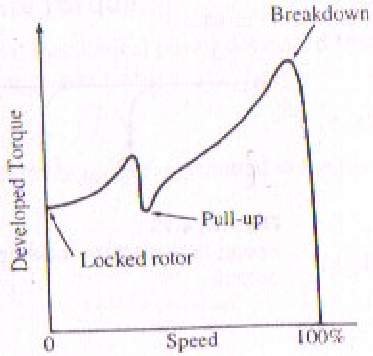
$$I_{stator} \propto I_{rotor} \qquad (4\text{-}43)$$

PARASITIC TORQUES

The periodic variation of magnetic-circuit reluctance, caused by rotor and stator slots, results in a nonsinusoidal space distribution of the rotating flux. Analysis of this rotating flux pattern shows it to consist of a number of rotating fields of different speeds called *space harmonics*. The first harmonic, called the *fundamental*, runs at a speed corresponding to the number of poles in the actual winding. The fifth space-harmonic rotates backward at one-fifth the speed of the fundamental, the seventh space-harmonic rotates forward at one-seventh the speed of the fundamental, and so forth. There are no even space harmonics and no third harmonics or its multiples. Although the fundamental dominates, the component torques produced by the fifth and seventh harmonics, called *parasitic torques* or *harmonic torques*, can cause undesirable bumps and dips in the motor torque-speed characteristic during acceleration, and may even cause the rotor to lock in at some subsynchronous speed and "crawl." Figure 4.12 shows the effect of parasitic torques on the torque-speed characteristic. The presence of significant dips in the torque-speed characteristic of an induction motor may indicate a defective design, a damaged rotor, or improper repair of a damaged stator [4], [5].

[15] The complete equivalent circuit of an induction motor, including stator and rotor windings, is similar to the equivalent circuit of a transformer. See Section 5.4, Chapter 5, for more details.

FIGURE 4.12
Effect of parasitic torques on the torque-speed characteristic of an induction motor.



4.14 PULL-UP TORQUE

The *pull-up torque* of an induction motor is the minimum torque developed by the motor during the period of acceleration from rest to the speed at which breakdown torque occurs. For the torque-speed characteristic in Figure 4.12, the pull-up torque is the value of torque at the bottom of the dip caused by a parasitic torque. If the pull-up torque is less than the load torque on the shaft, the motor will not accelerate past the pull-up point.

4.15 LOSSES, EFFICIENCY, AND POWER FACTOR

Calculations involving overall motor efficiency must take into account the losses that occur in both the stator and the rotor. The stator losses include all hysteresis losses and eddy-current losses in stator and rotor (called core losses), and I^2R losses in the stator winding (called stator conductor losses or stator copper losses). Thus, given the input power to the stator, and the stator losses, the net power crossing the air gap is

$$P_{gap} = P_{in} - P_{core} - P_{scl} \quad \text{W} \qquad (4\text{-}44)$$

where:
P_{in} = total 3-phase power input to stator
P_{core} = core loss
P_{scl} = stator conductor loss

Figure 4.13 shows the flow of power from stator input to shaft output, and accounts for the losses in both stator and rotor. The *power-flow diagram* is a useful adjunct to problem solving in that it often suggests a convenient method of solution. As indicated in the power-flow diagram, the total power loss for the motor is

$$P_{loss} = P_{scl} + P_{core} + P_{rcl} + P_{f,w} + P_{stray} \quad \text{W} \qquad (4\text{-}45)$$

The friction losses (f) are due to bearing friction plus friction between carbon brushes and slip rings if a wound rotor motor, and the windage losses (w) are due to the shaft-mounted cooling fan plus other air disturbances caused by rotation; the friction and

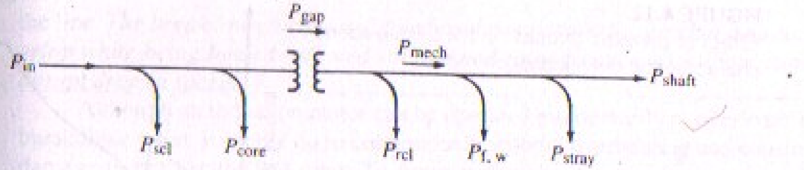


FIGURE 4.13
Power-flow diagram showing flow of power from stator input to shaft output.

windage losses are essentially constant for the normal load range of machines that conform to NEMA designs *A, B,* and *C*. Such machines change speed only slightly from no load to 115 percent rated load.[16]

The stray load loss is a collection of small losses not otherwise accounted for that vary with the load and, for calculation purposes, are assumed to be proportional to the square of the rotor current [1]. Expressed mathematically,

$$P_{stray} \propto I_r^2 \tag{4-46}$$

The stray losses include eddy-current losses in the stator conductors due to stator slot-leakage flux, losses in the end turns, end shields, and other parts in the end region due to stray flux, losses in the rotor due to harmonics produced by the stator load current, etc. [2].

Typical magnitudes of induction-motor losses, expressed as a percentage of the total loss, and the factors affecting these losses, are given in Table 4.1 for three-phase four-pole NEMA design *B* motors between 1 and 125 hp [3]. NEMA design motors are discussed in Chapter 5.

[16] NEMA design classifications and their industrial applications are discussed in Chapter 5.

TABLE 4.1
Typical induction motor losses for four-pole motors

Losses	Percent of Total Losses	Factors Affecting These Losses
P_{scl}	35–40	Stator conductor size
P_{rcl}	15–25	Rotor conductor size
P_{core}	15–25	Type and quantity of magnetic material
P_{stray}	10–15	Primarily manufacturing and design methods
$P_{f,w}$	5–10	Selection and design of fans and bearings

Source: J. Keinz, and R. Houlton, NEMA Nominal Efficiency—What Is It and Why? *IEEE Trans. Industry and Applications*, Vol. IA-17, No. 5, Sept./Oct., 1981. © 1981 IEEE. Reprinted by permission.

Useful Shaft-Power Output and Shaft Torque

The useful shaft-power output is equal to the total mechanical power developed from all three phases minus friction, windage, and stray power losses.

$$P_{shaft} = P_{mech} - P_{f,w} - P_{stray} \quad \text{W} \tag{4-47}$$

Shaft torque is the output torque of the motor. It is the torque transmitted to the load, and may be determined from

$$P_{shaft} = \frac{T_{shaft} \cdot n_r}{5252} \tag{4-48}$$

Efficiency

The efficiency of an induction motor is equal to the ratio of the useful power out to the total power in. Expressed as an equation

$$\eta = \frac{P_{shaft}}{P_{in}} \tag{4-49}$$

The efficiency in Eq. (4–49) is expressed in decimal form and generally called *per-unit efficiency*.

Power Factor

Power factor is the ratio of active power to apparent power. Thus, for the induction motor, the power factor is

$$F_P = \frac{P_{in}}{S_{in}} \tag{4-50}$$

$$S_{in} = \sqrt{3}\, V_{line} I_{line}$$

where: F_P = power factor (pu)
P_{in} = active power (W)
S_{in} = apparent power (VA)

Note the difference between power factor and efficiency.

EXAMPLE 4.6

A three-phase, 230-V, 60-Hz, 100-hp, six-pole induction motor operating at rated conditions has an efficiency of 91.0 percent and draws a line current of 248 A. The core loss, stator copper loss, and rotor conductor loss are 1697 W, 2803 W, and 1549 W, respectively. Determine (a) power input; (b) total losses; (c) air-gap power; (d) shaft speed; (e) power factor; (f) combined windage, friction, and stray load loss; (g) shaft torque.

Solution

The power-flow diagram is shown in Figure 4.14.

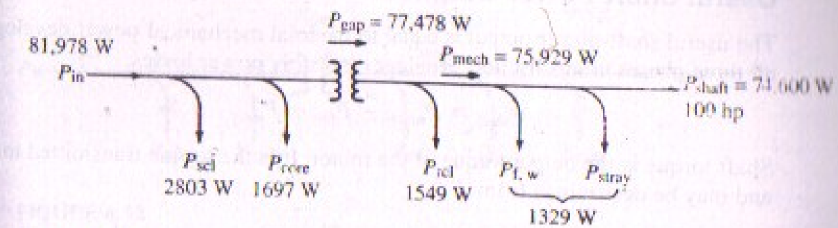


FIGURE 4.14
Power-flow diagram for Example 4.6.

(a)
$$\eta = \frac{P_{shaft}}{P_{in}} \quad \Rightarrow \quad 0.910 = \frac{100 \times 746}{P_{in}}$$
$$P_{in} = \underline{81{,}978\ \text{W}}$$

(b)
$$P_{loss} = P_{in} - P_{shaft} = 81{,}978 - 100 \times 746 = \underline{7378\ \text{W}}$$

(c) From Figure 4.14,
$$P_{gap} = P_{in} - P_{core} - P_{scl} = 81{,}978 - 1697 - 2803 = \underline{77{,}478\ \text{W}}$$

(d)
$$P_{gap} = \frac{P_{rcl}}{s} \quad \Rightarrow \quad 77{,}478 = \frac{1549}{s}$$
$$s = 0.0200$$
$$n_s = \frac{120f}{P} = \frac{120 \times 60}{6} = 1200\ \text{r/min}$$
$$n_r = n_s(1-s) = 1200(1-0.0200) = \underline{1176\ \text{r/min}}$$

(e)
$$S = \sqrt{3}\,V_{line}I_{line} = \sqrt{3} \times 230 \times 248 = 98{,}796\ \text{VA}$$
$$F_P = \frac{P_{in}}{S_{in}} = \frac{81{,}978}{98{,}796} = \underline{0.83}$$

(f)
$$P_{loss} = P_{core} + P_{scl} + P_{rcl} + P_{w,f} + P_{stray}$$
$$7378 = 1697 + 2803 + 1549 + P_{w,f} + P_{stray}$$
$$P_{w,f} + P_{stray} = \underline{1329\ \text{W}}$$

(g) From Eq. (4–39)
$$T_{shaft} = \frac{5252 \times P_{shaft}}{n_r} = \frac{5252 \times 100}{1176} = \underline{446.6\ \text{lb-ft}}$$

SUMMARY OF EQUATIONS FOR PROBLEM SOLVING

$$n_s = \frac{f_s}{P/2} = \frac{2 \times f_s}{P} \quad \text{r/s} \qquad n_s = \frac{120 \times f_s}{P} \quad \text{r/min} \tag{4–1}$$

$$n = n_s - n_r \tag{4–2}$$

$$s = \frac{n_s - n_r}{n_s} \tag{4–3}$$

$$n_r = n_s(1-s) \tag{4–4}$$

$$f_r = \frac{P \times n}{120} \tag{4–5}$$

$$f_r = \frac{P(n_s - n_r)}{120} \tag{4–6}$$

$$f_r = \frac{sPn_s}{120} \tag{4–7}$$

$$f_{BR} = \frac{Pn_s}{120} \tag{4–8}$$

$$f_r = sf_{BR} \tag{4–9}$$

$$E_r = sE_{BR} \tag{4–12}$$

$$\mathbf{I}_r = \frac{s\mathbf{E}_{BR}}{\mathbf{Z}_r} = \frac{s\mathbf{E}_{BR}}{R_r + jsX_{BR}} \tag{4–16}$$

$$I_r = \frac{E_{BR}}{\sqrt{(R_r/s)^2 + X_{BR}^2}} \tag{4–19}$$

$$\theta_r = \tan^{-1}\left(\frac{X_{BR}}{R_r/s}\right) \tag{4–20}$$

$$P_{gap} = E_{BR}I_r\cos\theta_r \tag{4–25}$$

$$P_{gap} = P_{mech} + P_{rcl} \quad \text{W} \tag{4–29}$$

$$P_{gap} = \frac{3I_r^2R_r}{s} \qquad P_{gap} = \frac{P_{rcl}}{s} \quad \text{W} \tag{4–30, 4–31}$$

$$P_{rcl} = 3I_r^2R_r \quad \text{W} \tag{4–32}$$

$$P_{mech} = P_{gap}(1-s) \quad \text{W} \tag{4–35}$$

$$P_{mech} = \frac{3I_r^2R_rn_r}{sn_s} \quad W \qquad P_{mech} = \frac{T_Dn_r}{5252} \quad hp \qquad (4–37, 4–39)$$

$$T_D = \frac{21.12I_r^2R_r}{sn_s} \quad lb\text{-}ft \qquad T_D = \frac{7.04P_{gap}}{n_s} \quad lb\text{-}ft \qquad (4–40, 4–41)$$

$$P_{loss} = P_{scl} + P_{core} + P_{rcl} + P_{f,w} + P_{stray} \quad W \qquad (4–45)$$

$$P_{shaft} = P_{mech} - P_{f,w} - P_{stray} \quad W \qquad (4–47)$$

$$\eta = \frac{P_{shaft}}{P_{in}} \qquad F_P = \frac{P_{in}}{S_{in}} \qquad S_{in} = \sqrt{3}\, V_{line}I_{line} \qquad (4–49, 4–50)$$

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REVIEW QUESTIONS

1. Explain how a rotating flux is produced in the stator of a three-phase induction motor.
2. With the aid of suitable sketches, explain how a rotating flux causes a squirrel-cage rotor to rotate.
3. What is phase sequence and how does it affect the operation of an induction motor?
4. Make two separate sketches showing the line connections to a three-phase induction motor for different directions of rotation.
5. State two reasons for skewing rotor slots.
6. (a) Differentiate between a squirrel-cage motor and a wound-rotor motor. (b) How is the speed of a wound-rotor motor adjusted?
7. Differentiate between synchronous speed, rotor speed, slip speed, and slip.
8. What two methods are used to change the synchronous speed of a three-phase induction motor?

9. Explain how slip affects rotor frequency and rotor voltage.
10. (a) Draw the circle diagram for the rotor of an induction motor. (b) Using the circle diagram as an aid to your analysis, explain the changes that take place in air-gap power as the rotor accelerates from standstill to near synchronous speed.
11. Differentiate between air-gap power, mechanical power developed, and shaft power out.
12. (a) Sketch the equivalent circuit for an induction-motor rotor and the related impedance diagram. (b) Determine from the impedance diagram the magnitude and phase angle of the rotor impedance in terms of its components.
13. (a) Draw the circle diagram for the rotor of an induction motor. (b) Using the circle diagram as an aid to your analysis, explain the changes that take place in air-gap power as the rotor accelerates from standstill to near synchronous speed.
14. (a) Sketch a representative torque-slip characteristic of a squirrel-cage induction motor and circle the points corresponding to locked rotor, breakdown torque, and rated torque. (b) Sketch the circle diagram for the rotor and draw the current phasors corresponding to the points circled in (a). (c) Using the sketches as an aid to your analysis, explain in detail the behavior of an induction motor as the machine is loaded from no load, to full load, to breakdown; assume that the machine had accelerated to rated speed before loading. Include in your analysis the reasons for changes in motor torque with increased shaft load.
15. What causes parasitic torques and what adverse effect can they have on induction-motor operation?
16. Differentiate between locked-rotor torque, pull-up torque, and breakdown torque.
17. Differentiate between efficiency and power factor.
18. List the types of losses in an induction motor and state the factors affecting these losses.
19. Sketch the power-flow diagram for an induction motor and show the relationship between power in, air-gap power, shaft power out.

PROBLEMS

4–1/7 A four-pole, 60-Hz, 10-hp, 460-V, three-phase induction motor operates at 1750 r/min when fully loaded and at its rated frequency and rated voltage. Determine (a) synchronous speed; (b) slip speed; (c) per-unit slip.

4–2/7 A 100-hp, 16-pole, 460-V, three-phase, 60-Hz induction motor has a slip of 2.4 percent when running at rated conditions. Determine (a) synchronous speed; (b) rotor speed; (c) rotor frequency.

4–3/7 A 60-Hz, four-pole, 450-V, three-phase induction motor operating at rated conditions has a speed of 1775 r/min. Determine (a) synchronous speed; (b) slip; (c) slip speed; (d) rotor frequency.

4–4/7 A 200-hp, 2300-V, three-phase, 60-Hz, wound-rotor induction motor has a blocked-rotor voltage of 104-V. The shaft speed and slip speed, when operating at rated load, are 1775 r/min and 25 r/min, respectively. Determine (a) number of poles; (b) slip; (c) rotor frequency; and (d) rotor voltage at slip speed.

4–5/7 A six-pole three-phase induction motor is operating at 480 r/min from a 25-Hz, 230-V supply. The voltage induced in the rotor when blocked is 90 V. Determine (a) slip speed; (b) rotor frequency and rotor voltage at 480 r/min.

4–6/7 A 100-hp, three-phase induction motor, operating at rated load, runs at 423 r/min when connected across a 450-V, 60-Hz supply. The slip at this load is 0.06. Determine (a) synchronous speed; (b) number of stator poles; (c) rotor frequency.

4–7/7 A four-pole/eight-pole, multispeed, 60-Hz, 10-hp, 240-V, three-phase induction motor operating with four poles runs at 1750 r/min when fully loaded and at its rated voltage and frequency. Determine (a) slip speed; (b) percent slip; (c) the synchronous speed if operating in the eight-pole mode and at 20 percent rated frequency.

4–8/11 A 20-hp, 230-V, 60-Hz, four-pole, three-phase induction motor operating at rated load has a rotor copper loss of 331 W, and a combined friction, windage, and stray power loss of 249 W. Determine (a) mechanical power developed; (b) air-gap power; (c) shaft speed; (d) shaft torque.

4–9/11 A 12-pole, 50-Hz, 20-hp, 220-V, squirrel-cage motor operating at rated conditions runs at 480 r/min, is 85 percent efficient, and has a power factor of 0.73 lagging. Determine (a) synchronous speed; (b) slip; (c) line current; (d) rated torque; (e) rotor frequency.

4–10/11 A three-phase, 230-V, 30-hp, 50-Hz, six-pole induction motor is operating with a shaft load that requires 21.3kW of input to the rotor. The rotor copper losses are 1.05 kW, and the combined friction, windage, and stray power losses for this load are 300 W. Determine (a) shaft speed; (b) mechanical power developed; (c) developed torque; (d) shaft torque; (e) percent of rated horsepower load that the machine is required to deliver.

4–11/15 A 30-hp, three-phase, 12-pole, 460-V, 60-Hz induction motor operating at reduced load draws a line current of 35 A, and has an efficiency and power factor of 90 and 79 percent, respectively. The stator conductor loss, rotor conductor loss, and core loss are 837 W, 485 W, and 375 W, respectively. Sketch the power-flow diagram, enter known values, and determine (a) input power; (b) shaft horsepower; (c) total losses; (d) rotor speed; (e) shaft torque; (f) combined windage, friction, and stray load loss.

4–12/15 A three-phase 5000-hp, 4000-V, 60-Hz, four-pole induction motor is operating at 4130 V, 60 Hz, and 67 percent rated load. The breakdown of losses for this load are as follows: stator conductors, 12.4 kW; rotor conductors, 9.92 kW; core, 12.44 kW; stray power, 10.2 kW; friction and windage, 18.2 kW. Sketch the power-flow diagram, enter known values, and determine (a) shaft speed; (b) shaft torque; (c) developed torque; (d) input power to the stator; (e) overall efficiency.

4–13/15 A 10-pole, 125-hp, 575-V, 60-Hz, three-phase induction motor operating at rated conditions draws a line current of 125 A and has an overall efficiency of 93 percent. The core loss, stator conductor loss, and rotor conductor loss are 1053 W, 2527 W, and 1755 W, respectively. Sketch the power-flow diagram, substitute values, and determine (a) shaft speed; (b) developed torque; (c) shaft torque; (d) power factor; (e) combined windage, friction, and stray power loss.

4–14/15 A 40-hp, 50-Hz, 2300-V, eight-pole induction motor is operating at 80 percent rated load and 6 percent reduced voltage. The efficiency and power factor for these conditions are 85 and 90 percent, respectively. The combined windage, friction, and stray power losses are 1011 W, the rotor conductor losses are 969 W, and the stator conductor losses are 1559 W. Sketch the power-flow diagram, enter values, and determine (a) mechanical power developed; (b) shaft speed; (c) shaft torque; (d) slip speed; (e) line current; (f) core loss.

4–15/15 A three-phase, 5-hp, 60-Hz, 115-V, four-pole induction motor operating at rated voltage, rated frequency, and 125 percent rated load has an efficiency of 85.4 percent. The stator conductor loss, rotor conductor loss, and core loss are 223.2 W, 153 W, and 114.8 W, respectively. Sketch the power-flow diagram, enter the given data, and determine (a) shaft speed; (b) shaft torque; (c) loss in torque due to the combined friction, windage, and stray power.

4–16/15 A three-phase, 50-hp, 230-V, 60-Hz, four-pole induction motor is operating at rated load, rated voltage, and rated frequency. Assume a system overload results in a 5 percent drop in frequency, and a 7 percent drop in voltage. To help reduce the system load, the shaft load is reduced to 70 percent rated horsepower, resulting in a line current of 100 A. Assume the losses for the new operating conditions are as follows: stator conductor loss, 1015 W; rotor conductor loss, 696 W; core loss, 522 W; and the combined windage, friction, and stray power loss is 667 W. Sketch the power-flow diagram, enter given data, and determine (a) percent efficiency; (b) speed; (c) shaft torque; (d) power factor.

4–17/15 A three-phase, 25-hp, 230-V, 60-Hz, two-pole induction motor drives a load that demands a constant torque regardless of speed (constant load torque). The machine is operating at rated voltage, rated frequency, and its rated speed of 3575 r/min. Determine the shaft horsepower, speed, and efficiency if the frequency drops to 54 Hz. The power factor and line current for the new conditions are 89 percent and 55 A, respectively, and the respective stator conductor loss, rotor conductor loss, and core loss, are 992.7 W, 496 W, and 546 W, respectively.

5

Classification, Performance, Applications, and Operation of Three-Phase Induction Machines

5.1 INTRODUCTION

Selecting the best induction motor for a specific application requires consideration of many factors and often presents a complex problem that requires sound judgment and considerable experience. To extract the optimum performance from a driven machine, the motor must be selected to match as closely as possible the operating characteristics of the load. To do this, a host of questions must be answered. What are the power, torque, and speed characteristics of the driven load? Must the speed be constant, adjustable, or inherently variable? Is the machine to be operated on continuous, short-time, or intermittent duty? What are the external conditions under which the motor will be required to operate? What about the ambient temperature in which the machine is to operate? Perhaps special insulation is required. What type of control is needed—manual, magnetic, or solid-state; full voltage or reduced voltage? What are the voltage and frequency constraints?

In an effort to assist the purchaser in selecting and obtaining the proper motor for the particular application, the National Electrical Manufacturers Association (NEMA) developed product standards for motors that include frame dimensions, voltage and frequency, power ratings, service factors, temperature rises, and performance characteristics. The benefits derived from these standards are greater availability of motors, a sounder basis for accurate comparison of machines, prompter repair service, and shorter delivery time.

NEMA data stamped on motor nameplates provide a wealth of information on motor operation, characteristics, and applications. Properly operated, within the bounds of its nameplate ratings, the machine will provide many years of efficient and reliable service. However, when operating at off-rated frequency, off-rated voltage, overloaded, in wrong ambient, etc., the performance of the machine will be different, with the amount of deviation from the expected normal operation depending on the percent variation in voltage, frequency, temperature, and so forth.

Sustained operation with unbalanced line voltages can cause a decrease in locked-rotor torque and breakdown torque, as well as severe overheating with a high probability of shortened life, unless the motor is derated as specified by NEMA for the particular unbalanced conditions.

High in-rush current associated with every full-voltage start (or attempted full-voltage start) causes severe thermal and mechanical stresses on rotor and stator components, as well as causing large voltage drops in the distribution system. Reduction of high in-rush current may be accomplished through various starting methods that use current-limiting impedances, autotransformers, reconnection of windings, solid-state starting, and the like.

The expected motor current, developed torque, and speed may be calculated for specific sets of conditions using the resistance and reactance of the motor windings. Simplifying approximations for "normal running" and blocked-rotor conditions make calculations relatively easy. Induction-motor parameters are available from the manufacturer or may be approximated through appropriate electrical tests.

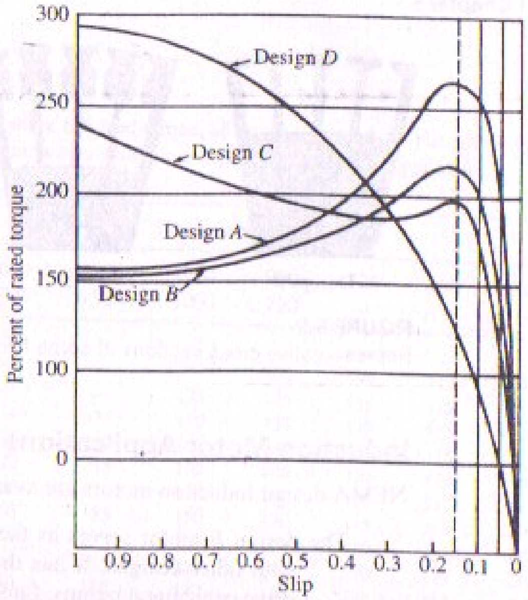
A very interesting aspect of induction motors is their application as an induction generator. Driven by wind turbines, gas turbines, and the like, they range in size from a few kilowatts to more than 10 MW and are used in the sequential production of two forms of energy: process steam and electrical energy.

CLASSIFICATION AND PERFORMANCE ACTERISTICS OF NEMA-DESIGN SQUIRREL-CAGE CTION MOTORS

The National Electrical Manufacturers Association standardized five basic design categories of induction motors to match the torque-speed requirements of the most common types of mechanical loads. Representative torque-speed characteristics of four of these basic designs are shown in Figure 5.1. Note that the design *C* motor has its maximum torque occur at blocked rotor ($s = 1$), the design *D* motor has its maximum torque occur at (or near) blocked rotor, and the design *A* and design *B* motors have their respective maximum torques occur at a slip of approximately 0.15. The design *E* (energy efficient)[1] motor, not shown, has a torque-speed characteristic somewhat similar to that of the design *B* motor shown in Figure 5.1.

[1]The Energy Policy Act of 1992 (EPACT92) requires that general-purpose, foot-mounted, T-frame, continuous duty, single-speed, NEMA design A and B induction motors of two, four, and six poles, manufactured after October 4, 1997, have a NEMA nominal efficiency stamped on the motor nameplate. Motor ratings between 1 and 200 hp, 230/460 V, 60-Hz, are affected. Tables of nominal efficiencies for different horsepower and pole arrangements are provided in Reference [9].

FIGURE 5.1
Torque-speed characteristics of basic NEMA-design squirrel-cage induction motors.



The characteristic curves in Figure 5.1 are "ideal" curves in that they do not include the effect of parasitic torques.[2] Parasitic torques cause dips in the torque-speed characteristic and are always present to some extent. The magnitude and location of these dips cannot be determined from the motor parameters. Hence, if this information is critical to a particular application, the manufacturer should be contacted for the actual test characteristics of the specific motor.

The different torque-speed characteristics shown in Figure 5.1 are obtained by selecting the proper combination of rotor and stator resistance and rotor and stator leakage reactance, with the rotor parameters playing the dominant role.

Representative cross sections of the three most commonly used NEMA-design squirrel-cage rotors are shown in Figure 5.2. The design *D* rotor has relatively high-resistance, low-reactance rotor bars close to the surface. Design *B* rotors and design *A* (not shown) have low-resistance rotor bars that extend deeper into the iron, resulting in low R_r and high X_{BR}. The design *C* rotor combines the features of both design *B* and design *D* rotors; it has high resistance with low reactance at the surface bars, and low resistance with high reactance at the deeper bars. Design *E* motors utilize thinner laminations of low-loss steel to minimize eddy-current losses; longer cores for low flux density to improve power factor and minimize rated current; larger cross-section conductors in the rotor and stator to reduce I^2R losses; special design low-loss cooling fans and bearings to reduce windage and friction loss; and special design winding configurations to minimize stray load losses.

[2]See parasitic torques in Section 4.12, Chapter 4.

Design *B* Design *C* Design *D*

FIGURE 5.2
Representative cross sections of some NEMA-design rotors.

Induction Motor Applications

NEMA design induction motors are available for almost all applications:

The design *B* motor serves as the basis for comparison of motor performance with other designs. It has the broadest field of application and is used to drive centrifugal pumps, fans, blowers, and machine tools.

The design *A* motor has essentially the same characteristics as the design *B*, except for a somewhat higher breakdown torque. Since its starting current is higher, however, its field of application is limited.

The design *C* motor has a higher locked-rotor torque, but a lower breakdown torque than the design *B*. The higher starting torque makes it suitable for driving plunger pumps, vibrating screens, and compressors without unloading devices. The starting current and slip at rated torque are essentially the same as for the design *B*.

The design *D* motor has a very high locked-rotor torque and a high slip. Its principal field of application is in high-inertia loads such as flywheel-equipped punch presses, elevators, and hoists.

The design *E* motor is a high-efficiency motor that is used to drive centrifugal pumps, fans, blowers, and machine tools. However, except for isolated cases, the locked-rotor torque, breakdown torque, and pull-up torque of design *E* motors are somewhat lower than that of design *B* motors for the same power and synchronous speed ratings. Furthermore, the locked-rotor current (starting current) of design *E* motors is significantly higher than that of design *B* motors for the same power and synchronous speed ratings.

NEMA TABLES [9]

The *minimum* values of locked-rotor torque, breakdown torque, and pull-up torque, as specified for NEMA-design squirrel-cage medium-size induction motors with continuous ratings, are given in Table 5.1 through 5.7, respectively, for specific horsepower, frequency, and synchronous speed ratings. These minimum torque values are expressed

TABLE 5.1
Minimum locked-rotor torque, in percent of full-load torque, of single-speed, 60–50-Hz, polyphase, squirrel-cage, continuous-rated, medium motors with rated voltage and frequency applied for NEMA designs *A, B, C,* and *D.*

hp	Synchronous Speed (rpm) 60 Hz / 50 Hz: 3600 / 3000	1800 / 1500	1200 / 1000	900 / 750	720 / —	600 / —	514 / —
Designs A and B							
$\frac{1}{2}$	—	—	—	140	140	115	110
$\frac{3}{4}$	—	—	175	135	135	115	110
1	—	275	170	135	135	115	110
$1\frac{1}{2}$	175	250	165	130	130	115	110
2	170	235	160	130	125	115	110
3	160	215	155	130	125	115	110
5	150	185	150	130	125	115	110
$7\frac{1}{2}$	140	175	150	125	120	115	110
10	135	165	150	125	120	115	110
15	130	160	140	125	120	115	110
20	130	150	135	125	120	115	110
25	130	150	135	125	120	115	110
30	130	150	135	125	120	115	110
40	125	140	135	125	120	115	110
50	120	140	135	125	120	115	110
60	120	140	135	125	120	115	110
75	105	140	135	125	120	115	110
100	105	125	125	125	120	115	110
125	100	110	125	120	115	115	110
150	100	110	120	120	115	115	110
200	100	100	120	120	115	115	—
250	70	80	100	100	—	—	—
300	70	80	100	—	—	—	—
350	70	80	100	—	—	—	—
400	70	80	—	—	—	—	—
450	70	80	—	—	—	—	—
500	70	80	—	—	—	—	—
Design C							
1		285	255	225			
1.5		285	250	225			
2		285	250	225			
3		270	250	225			
5		255	250	225			
$7\frac{1}{2}$		250	225	200			
10		250	225	200			
15		225	210	200			
20–200, inclusive		200	200	200			

Design D: 150 hp and smaller with 4, 6, and 8 poles, 275 percent full-load torque.

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TABLE 5.2
Minimum locked-rotor torque, in percent of full-load torque, of single-speed, 60–50-Hz, polyphase, squirrel-cage, continuous-rated, medium motors with rated voltage and frequency applied for NEMA design *E*.

hp	60 Hz / 50 Hz: 3600 / 3000	1800 / 1500	1200 / 1000	900 / 750
	Synchronous Speed (rpm)			
$\frac{1}{2}$	190	200	170	150
$\frac{3}{4}$	190	200	170	150
1	180	190	170	150
$1\frac{1}{2}$	180	190	160	140
2	180	190	160	140
3	170	180	160	140
5	160	170	150	130
$7\frac{1}{2}$	150	160	150	130
10	150	160	150	130
15	140	150	140	120
20	140	150	140	120
25	130	140	140	120
30	130	140	140	120
40	120	130	130	120
50	120	130	130	120
60	110	120	120	110
75	110	120	120	110
100	100	110	110	100
125	100	110	110	100
150	90	100	100	90
200	90	100	100	90
250	80	90	90	90
300	80	90	90	
350	75	75	75	
400	75	75		
450	75	75		
500	75	75		

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as a *percent of rated torque,* and assume that rated voltage and rated frequency are applied to the stator.

To determine the minimum values of locked-rotor torque, breakdown torque, and pull-up torque for a specific machine, calculate rated torque from the nameplate data, and then multiply it by the respective percentages in the NEMA tables. Although the torque values obtained from the tables are minimum values, *for application considerations, it is best to assume that the minimum values are the actual values.*

TABLE 5.3
Minimum breakdown torque, in percent of full-load torque, of single-speed, 60–50-Hz, polyphase, squirrel-cage, continuous-rated, medium motors with rated voltage and frequency applied for NEMA designs *A,B,* and *C.*

hp	60 Hz / 50 Hz: 3600 / 3000	1800 / 1500	1200 / 1000	900 / 750	720 / —	600 / —	514 / —
	Synchronous Speed (rpm)						
Designs A and B							
$\frac{1}{2}$	—	—	—	225	200	200	200
$\frac{3}{4}$	—	—	275	220	200	200	200
1	—	300	265	215	200	200	200
$1\frac{1}{2}$	250	280	250	210	200	200	200
2	240	270	240	210	200	200	200
3	230	250	230	205	200	200	200
5	215	225	215	205	200	200	200
$7\frac{1}{2}$	200	215	205	200	200	200	200
10–125, inclusive	200	200	200	200	200	200	200
150	200	200	200	200	200	200	200
200	200	200	200	200	200	200	—
250	175	175	175	175	—	—	—
300–350	175	175	175	—	—	—	—
400–500, inclusive	175	175	—	—	—	—	—
Design C							
3		200	225	200			
5		200	200	200			
$7\frac{1}{2}$–20		200	190	190			
25–200, inclusive		190	190	190			

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EXAMPLE 5.1 Determine the minimum values of locked-rotor torque, breakdown torque, and pull-up torque that can be expected from a three-phase, 10-hp, 460-V, 60-Hz, six-pole, NEMA design *C* motor whose rated speed is 1150 r/min.

Solution

$$n_s = \frac{120f}{P} = \frac{120(60)}{6} = 1200 \text{ r/min}$$

$$\text{hp} = \frac{Tn}{5252}$$

$$10 = \frac{T(1150)}{5252}$$

$$T_{\text{rated}} = 45.67 \text{ lb-ft}$$

TABLE 5.4
Minimum breakdown torque, in percent of full-load torque, of single-speed, 60–50-Hz, polyphase, squirrel-cage, continuous-rated, medium motors with rated voltage and frequency applied for NEMA design *E*.

	Synchronous Speed (rpm)			
60 Hz	3600	1800	1200	900
hp / 50 Hz	3000	1500	1000	750
$\frac{1}{2}$	200	200	170	160
$\frac{3}{4}$	200	200	170	160
1	200	200	180	170
$1\frac{1}{2}$	200	200	190	180
2	200	200	190	180
3	200	200	190	180
5	200	200	190	180
$7\frac{1}{2}$	200	200	190	180
10	200	200	180	170
15	200	200	180	170
20	200	200	180	170
25	190	190	180	170
30	190	190	180	170
40	190	190	180	170
50	190	190	180	170
60	180	180	170	170
75	180	180	170	170
100	180	180	170	160
125	180	180	170	160
150	170	170	170	160
200	170	170	170	160
250	170	170	160	160
300	170	170	160	—
350	160	160	160	—
400	160	160	—	—
450	160	160	—	—
500	160	160	—	—

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TABLE 5.5
Minimum pull-up torque, in percent of full-load torque of single-speed, 60–50-Hz, polyphase, squirrel-cage, continuous-rated, medium motors with rated voltage and frequency applied for NEMA designs *A* and *B*.

	Synchronous Speed (rpm)						
60 Hz	3600	1800	1200	900	720	600	514
hp / 50 Hz	3000	1500	1000	750	—	—	—
$\frac{1}{2}$	—	—	—	100	100	100	100
$\frac{3}{4}$	—	—	120	100	100	100	100
1	—	190	120	100	100	100	100
$1\frac{1}{2}$	120	175	115	100	100	100	100
2	120	165	110	100	100	100	100
3	110	150	110	100	100	100	100
5	105	130	105	100	100	100	100
$7\frac{1}{2}$	100	120	105	100	100	100	100
10	100	115	105	100	100	100	100
15	100	110	100	100	100	100	100
20	100	105	100	100	100	100	100
25	100	105	100	100	100	100	100
30	100	105	100	100	100	100	100
40	100	100	100	100	100	100	100
50	100	100	100	100	100	100	100
60	100	100	100	100	100	100	100
75	95	100	100	100	100	100	100
100	95	100	100	100	100	100	100
125	90	100	100	100	100	100	100
150	90	100	100	100	100	100	—
200	90	90	100	100	100	—	—
250	65	75	90	90	—	—	—
300	65	75	90	—	—	—	—
350	65	75	90	—	—	—	—
400	65	75	—	—	—	—	—
450	65	75	—	—	—	—	—
500	65	75	—	—	—	—	—

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From Table 5.1, a 10-hp, design *C* motor with a synchronous speed of 1200 r/min should have a minimum locked-rotor torque equal to 225 percent full-load torque. Thus,

$$T_{\text{locked rotor}} = 2.25(45.67) = \underline{102.8 \text{ lb-ft}}$$

From Table 5.3, the minimum breakdown torque is 190%.

$$T_{\text{breakdown}} = 1.90(45.67) = \underline{86.8 \text{ lb-ft}}$$

From Table 5.6, the minimum pull-up torque is 165%.

$$T_{\text{pull-up}} = 1.65(45.67) = \underline{75.4 \text{ lb-ft}}$$

TABLE 5.6
Minimum pull-up torque, in percent of full-load torque of single speed, 60–50-Hz, polyphase, squirrel-cage, continuous-rated, medium motors with rated voltage and frequency applied for NEMA design *C*.

	Synchronous Speed (rpm)		
	60 Hz 1800	1200	900
hp	50 Hz 1500	1000	750
1	195	180	165
1½	195	175	160
2	195	175	160
3	180	175	160
5	180	175	160
7½	175	165	150
10	175	165	150
15	165	150	140
20	165	150	140
25	150	150	140
30	150	150	140
40	150	150	140
50	150	150	140
60	140	140	140
75	140	140	140
100	140	140	140
125	140	140	140
150	140	140	140
200	140	140	140

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The Upgrading Problem [12]

Before replacing a design *B* motor with a design *E* motor of the same horsepower and synchronous speed ratings, be sure to check the NEMA tables to see if the design *E* motor has sufficient torque to start and accelerate the load. The following is a comparison of the significant points on the motor speed-torque curves for design *B* and design *E* motors, both rated 60 hp with a synchronous speed of 1800 rpm:

Minimum Torque in Percent of Rated Torque from NEMA Tables

NEMA Design	Locked Rotor	Breakdown	Pull-Up
B	140	200	100
E	120	180	90

TABLE 5.7
Minimum pull-up torque, in percent of full-load torque of single-speed, 60–50-Hz, polyphase, squirrel-cage, continuous-rated, medium motors with rated voltage and frequency applied for NEMA design *E*.

	Synchronous Speed (rpm)			
	60 Hz 3600	1800	1200	900
hp	50 Hz 3000	1500	1000	750
$\frac{1}{2}$	130	140	120	110
$\frac{3}{4}$	130	140	120	110
1	120	130	120	110
1½	120	130	110	100
2	120	130	110	100
3	110	120	110	100
5	110	120	110	100
7½	100	110	110	100
10	100	110	110	100
15	100	110	100	90
20	100	110	100	90
25	90	100	100	90
30	90	100	100	90
40	90	100	100	90
50	90	100	100	90
60	80	90	90	80
75	80	90	90	80
100	70	80	80	70
125	70	80	80	70
150	70	80	80	70
200	70	80	80	70
250	60	70	70	70
300	60	70	70	—
350	60	60	60	—
400	60	60	—	—
450	60	60	—	—
500	60	60	—	—

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Note that, for the same horsepower and speed ratings (60 hp, 1800 rpm), the design *E* motor has lower minimum critical torques than the design *B* motor. This may cause problems. For the given load, the motor must be able to develop sufficient locked-rotor torque to start, sufficient pull-up torque to accelerate, and sufficient breakdown torque to handle any peak loads. It would also be wise to check with the manufacturer of the motor for their recommendations.

4 MOTOR PERFORMANCE AS A FUNCTION OF MACHINE RAMETERS, SLIP, AND STATOR VOLTAGE

Motor performance as a function of machine parameters, slip, and applied stator voltage requires analysis of the complete equivalent-circuit model of an induction motor, including both rotor and stator circuits, as shown in Figure 5.3. The rotor circuit is identical to that previously shown in Figure 4.7(d) in Chapter 4. The stator circuit includes stator resistance R_s, stator leakage reactance X_s, resistance R_{fe}, which accounts for hysteresis and eddy-current losses in the iron, and magnetizing reactance X_M, which accounts for the magnetizing component of the exciting current.

The circuit model shown in Figure 5.3 is similar to that of a transformer (see Figure 2.9 in Chapter 2), where the resistance and leakage reactance are separated from the respective primary and secondary windings, leaving an ideal transformer between the two. Because of its similarity, the equivalent-circuit reductions previously developed for transformers may be adapted to induction motors. Thus, Figure 5.3 may be reduced to the simple series–parallel circuit shown in Figure 5.4, with all parameters referred to the stator. The relationship between the actual parameters shown in Figure 5.3 and the parameters referred to the stator (all per phase), as shown in Figure 5.4, are:

$R_2 = a^2R_r = R_r$ referred to the stator

$X_2 = a^2X_{BR} = X_{BR}$ referred to the stator

$\mathbf{I}_2 = \mathbf{I}_r/a = \mathbf{I}_r$ referred to the stator

$\mathbf{E}_2 = \mathbf{E}_s = a\mathbf{E}_{BR} = \mathbf{E}_{BR}$ referred to the stator

$a = N_s/N_r$ = ratio of stator turns per phase to rotor turns per phase[3]

R_{fe} = equivalent resistance per phase that accounts for the core loss

X_M = equivalent reactance per phase that accounts for the magnetizing current

[3] For squirrel-cage rotors, the ratio is stator turns per phase to equivalent wound-rotor turns per phase.

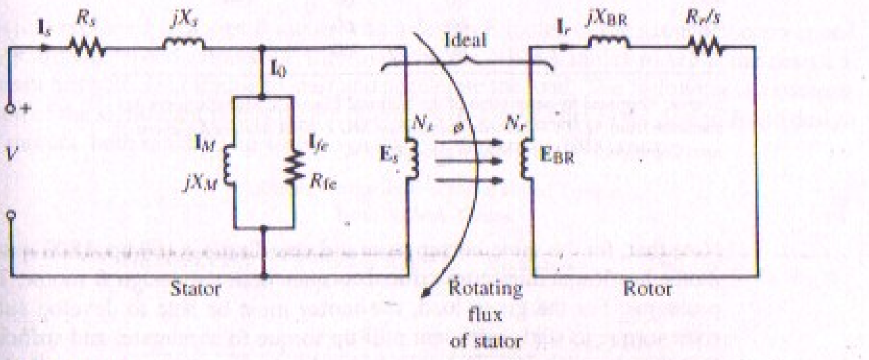


FIGURE 5.3
Equivalent-circuit model of an induction motor showing rotor and stator as separate circuits.

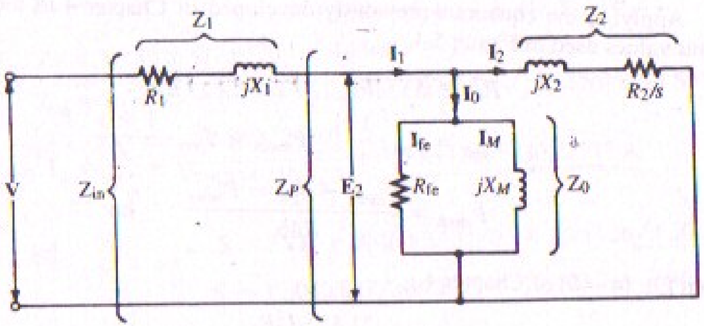


FIGURE 5.4
Equivalent series–parallel model of an induction motor with all parameters referred to the stator.

$\mathbf{I}_0$ = exciting current (no-load current) per phase

R_r = *actual* rotor resistance per phase

$\mathbf{I}_{fe}$ = core loss component of exciting current

$\mathbf{I}_M$ = magnetizing component of exciting current

X_{BR} = *actual* blocked-rotor reactance per phase

$\mathbf{I}_r$ = *actual* rotor current per phase

$\mathbf{V}$ = *actual* voltage per phase applied to the stator

$\mathbf{I}_1$ = *actual* stator current per phase

NOTHING ELSE MATTERS

Power, Torque, Speed, Losses, and Efficiency

Stator–conductor losses, rotor–conductor losses, core losses, air-gap power, mechanical power developed, developed torque, shaft horsepower, shaft torque, speed, and efficiency may be readily determined by first solving the equivalent-circuit shown in Figure 5.4 for $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{E}_2$, and then substituting into the appropriate equations previously developed in Chapter 4. Thus, from Figure 5.4,

$$\mathbf{Z}_2 = \frac{R_2}{s} + jX_2 \qquad \mathbf{Z}_0 = \frac{R_{fe} \cdot jX_M}{R_{fe} + jX_M}$$

$$\mathbf{Z}_P = \frac{\mathbf{Z}_2 \cdot \mathbf{Z}_0}{\mathbf{Z}_2 + \mathbf{Z}_0} \qquad \mathbf{Z}_{in} = \mathbf{Z}_1 + \mathbf{Z}_P$$

$$\mathbf{I}_1 = \frac{\mathbf{V}}{\mathbf{Z}_{in}} \qquad \mathbf{E}_2 = \mathbf{I}_1 \cdot \mathbf{Z}_P \qquad \mathbf{I}_2 = \frac{\mathbf{E}_2}{\mathbf{Z}_2}$$

Note: Parameter data supplied by manufacturers, or in technical papers, or as given in professional licensing examinations sometimes omit R_{fe}. To solve problems when R_{fe} is not given, simply equate $\mathbf{Z}_0 = jX_M$. The resultant error will be relatively small.

Applying the equations previously developed in Chapter 4 to the equivalent-circuit values used in Figure 5.4,

$$P_{scl} = 3 \cdot I_1^2 R_1 \qquad P_{rcl} = 3 \cdot I_2^2 R_2$$

$$P_{gap} = P_{rcl} \cdot \frac{1}{s} \qquad P_{mech} = P_{rcl} \cdot \frac{1-s}{s}$$

$$P_{shaft} = \frac{P_{mech} - P_{f,w} - P_{stray}}{746} \quad \text{hp}$$

From Eq. (4–40) of Chapter 4,

$$T_D = \frac{21.12 \cdot I_2^2 R_2}{s \cdot n_s} \quad \text{lb-ft} \tag{5–1}$$

The core loss expressed in terms of R_{fe} in Figure 5.4 is

$$P_{core} = \frac{3E_2^2}{R_{fe}} \tag{5–2}$$

EXAMPLE 5.2 A 60-Hz, 15-hp, 460-V, three-phase, six-pole, wye-connected induction motor is driving a centrifugal pump at 1185 r/min. The combined friction, windage, and stray power losses are 166 W, and the motor parameters (in ohms per phase) referred to the stator are:

$$R_1 = 0.200 \quad R_2 = 0.250 \quad X_M = 42.0$$
$$X_1 = 1.20 \quad X_2 = 1.29 \quad R_{fe} = 317$$

Determine (a) slip; (b) line current; (c) apparent power, active power, reactive power, and power factor of the motor; (d) equivalent rotor current; (e) stator copper loss; (f) rotor copper loss; (g) core loss; (h) air-gap power; (i) mechanical power developed; (j) developed torque; (k) shaft horsepower; (l) shaft torque; (m) efficiency. (n) Sketch the power-flow diagram.

Solution

(a)

$$n_s = \frac{120f}{P} = \frac{120 \times 60}{6} = 1200 \text{ r/min}$$

$$s = \frac{n_s - n_r}{n_s} = \frac{1200 - 1185}{1200} = \underline{0.0125}$$

(b) Referring to Figure 5.4,

$$\mathbf{Z}_2 = \frac{R_2}{s} + jX_2 = \frac{0.250}{0.0125} + j1.29 = 20 + j1.29 = 20.0416\underline{/3.6905^\circ}\ \Omega$$

$$\mathbf{Z}_0 = \frac{R_{fe} \cdot jX_M}{R_{fe} + jX_M} = \frac{317(42.0\underline{/90^\circ})}{317 + j42.0} = 41.6361\underline{/82.4527^\circ} = 5.4687 + j41.2754\ \Omega$$

$$\mathbf{Z}_P = \frac{\mathbf{Z}_2 \cdot \mathbf{Z}_0}{\mathbf{Z}_2 + \mathbf{Z}_0} = \frac{(20.0416\underline{/3.6905^\circ})(41.6361\underline{/82.4527^\circ})}{(20 + j1.29) + (5.4687 + j41.2754)}$$

$$\mathbf{Z}_P = 16.8226\underline{/27.0370^\circ} = 14.9841 + j7.6470\ \Omega$$

$$\mathbf{Z}_{in} = \mathbf{Z}_1 + \mathbf{Z}_P = (0.200 + j1.20) + (14.9841 + j7.6470) = 17.5735\underline{/30.2271^\circ}\ \Omega$$

$$\mathbf{I}_1 = \frac{\mathbf{V}}{\mathbf{Z}_{in}} = \frac{(460/\sqrt{3})\underline{/0^\circ}}{17.5735\underline{/30.2271^\circ}} = \underline{15.1126\underline{/-30.2271^\circ}\ \text{A}}$$

(c)

$$\mathbf{S} = 3\mathbf{V}\mathbf{I}_1^* = 3(460/\sqrt{3})\underline{/0^\circ} \times 15.1126\underline{/+30.2271^\circ}$$

$$\mathbf{S} = 12{,}040.857\underline{/30.2271^\circ} = 10{,}403.7 + j6061.7 \text{ VA}$$

Thus,

$$P_{in} = 10{,}404 \text{ W} \Rightarrow \underline{10.4 \text{ kW}}$$
$$Q_{in} = 6062 \text{ var} \Rightarrow \underline{6.06 \text{ kvar}}$$
$$S_{in} = 12{,}041 \text{ VA} \Rightarrow \underline{12.0 \text{ kVA}}$$
$$F_P = \cos(30.23^\circ) = 0.864 \quad \text{or} \quad \underline{86.4\%}$$

(d) From Figure 5.4,

$$\mathbf{E}_2 = \mathbf{I}_1\mathbf{Z}_P = (15.1126\underline{/-30.2271^\circ})(16.8226\underline{/27.037^\circ}) = 254.2332\underline{/-3.1901^\circ}\text{ V}$$

$$\mathbf{I}_2 = \frac{\mathbf{E}_2}{\mathbf{Z}_2} = \frac{254.2332\underline{/-3.1901^\circ}}{20.0416\underline{/3.6905^\circ}} = \underline{12.6853\underline{/-6.8806^\circ}\ \text{A}}$$

(e)

$$P_{scl} = 3I_1^2R_1 = 3(15.1126)^2(0.20) = 137.03 \Rightarrow \underline{137 \text{ W}}$$

(f)

$$P_{rcl} = 3I_2^2R_2 = 3(12.6853)^2(0.25) = 120.69 \Rightarrow \underline{121 \text{ W}}$$

(g)

$$P_{core} = 3\left(\frac{E_2^2}{R_{fe}}\right) = 3\frac{(254.2332)^2}{317} = 611.68 \Rightarrow \underline{612 \text{ W}}$$

(h)

$$P_{gap} = \frac{P_{rcl}}{s} = \frac{120.6876}{0.0125} = 9655.20 \Rightarrow \underline{9655 \text{ W}}$$

(i)

$$P_{mech} = \frac{P_{rcl}(1-s)}{s} = \frac{120.6876(1 - 0.0125)}{0.0125} = 9534.3 \Rightarrow \underline{9534 \text{ W}}$$

(j)

$$T_D = \frac{21.12 \cdot I_2^2R_2}{s \cdot n_s} = \frac{21.12(12.6853)^2(0.25)}{0.0125 \times 1200} = \underline{56.64 \text{ lb-ft}}$$

(k)

$$\text{Loss} = P_{scl} + P_{rcl} + P_{core} + P_{f,w} + P_{stray}$$

$$\text{Loss} = 137.03 + 120.69 + 611.68 + 166 = 1035 \text{ W}$$

$$P_{shaft} = \frac{P_{in} - \text{loss}}{746} = \frac{10{,}404 - 1035}{746} = \underline{12.56 \text{ hp}}$$

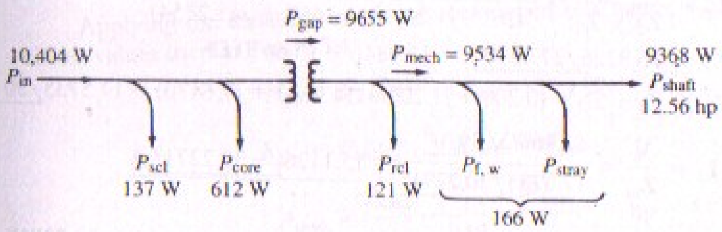


FIGURE 5.5
Power-flow diagram for Example 5.2.

(l)

$$\text{hp} = \frac{Tn}{5252}$$

$$12.56 = \frac{T(1185)}{5252}$$

$$T = \underline{55.7 \text{ lb-ft}}$$

(m)

$$\eta = \frac{P_{out}}{P_{in}} = \frac{12.56 \times 746}{10{,}404} = 0.900 \quad \text{or} \quad \underline{90.0\%}$$

(n) The power-flow diagram is shown in Figure 5.5.

SHAPING THE TORQUE-SPEED CHARACTERISTIC

The maximum torque that an induction motor can develop, for a given applied voltage and frequency, is dependent on the relative magnitudes of R_1, X_1, and X_2, and is independent of rotor resistance R_2. The slip at which this maximum torque occurs, however, is directly proportional to R_2. These two very significant relationships are not readily apparent from Eq. (5–1).

To show how T_D is related to the motor parameters requires solving for $\mathbf{I}_2$ in Figure 5.4, and then substituting the result into Eq. (5–1). Unfortunately, the resultant messy mathematical expression would completely obscure the basic relationships being sought. A simplified mathematical expression that is easier to interpret may be obtained by shifting the exciting current components shown in Figure 5.4 to the input terminals, as shown in Figure 5.6(a). Although the *approximate equivalent circuit* shown in Figure 5.6(a) is very useful for developing a reasonably accurate expression that clearly shows how rotor and stator parameters affect the value of $T_{D,\max}$, and the value of slip at which $T_{D,\max}$ occurs, it should not be used in place of Figure 5.4 for precise current, power, and efficiency calculations.

Solving the circuit in Figure 5.6(a) for $\mathbf{I}_2$,

$$\mathbf{I}_2 \cong \frac{\mathbf{V}}{R_1 + jX_1 + R_2/s + jX_2} \tag{5–3}$$

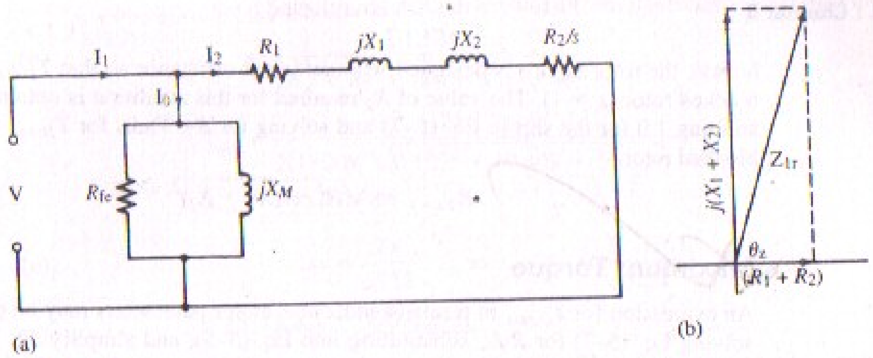


FIGURE 5.6
(a) Approximate equivalent circuit of an induction motor; (b) phasor diagram for locked-rotor conditions ($s = 1$).

$$|\mathbf{I}_2| \cong \frac{|\mathbf{V}|}{\sqrt{[(R_1 + R_2/s)^2 + (X_1 + X_2)^2]}} \tag{5–4}$$

Substituting Eq. (5–4) into Eq. (5–1),

$$T_D \cong \frac{21.12V^2R_2/s}{[(R_1 + R_2/s)^2 + (X_1 + X_2)^2]n_s} \tag{5–5}$$

As indicated in Eq. (5–5), for a given slip and given machine parameters, the developed torque is proportional to the square of the applied stator voltage. That is,

$$T_D \propto V^2 \tag{5–6}$$

This very useful relationship has significant applications in motor starting and motor breakdown problems.

Slip at Which Maximum Torque Occurs

Inspection of the torque-slip curves in Figure 5.1 shows that at breakdown the slope of the curve is zero. Hence, the slip at which maximum torque occurs may be determined by taking the derivative of Eq. (5–5) with respect to s, and then solving for the value of s that makes the slope (derivative) equal to zero. The resulting mathematical expression is

$$s_{T_{D,\max}} \cong \frac{R_2}{\sqrt{R_1^2 + (X_1 + X_2)^2}} \tag{5–7}$$

Note that the slip at which $T_{D,\max}$ occurs is directly proportional to rotor resistance R_2. Thus, in those applications that require a very high starting torque (e.g., elevators or hoists), the rotor circuit is designed with sufficient resistance so that $T_{D,\max}$ occurs at blocked rotor ($s = 1$). The value of R_2 required for this condition is obtained by substituting 1.0 for the slip in Eq. (5–7) and solving for R_2. Thus, for $T_{D,\max}$ to occur at blocked rotor,

$$R_{2,s=1} = \sqrt{R_1^2 + (X_1 + X_2)^2} \tag{5–8}$$

Maximum Torque

An expression for $T_{D,\max}$ in terms of induction-motor parameters may be obtained by solving Eq. (5–7) for R_2/s, substituting into Eq. (5–5), and simplifying. Thus, from Eq. (5–7),

$$\frac{R_2}{s} = \sqrt{R_1^2 + (X_1 + X_2)^2} \tag{5–9}$$

Substituting Eq. (5–9) into Eq. (5–5) and simplifying,

$$T_{D,\max} \cong \frac{21.12V^2\sqrt{R_1^2 + (X_1 + X_2)^2}}{n_s[(R_1 + \sqrt{R_1^2 + (X_1 + X_2)^2})^2 + (X_1 + X_2)^2]}$$

$$T_{D,\max} \cong \frac{21.12V^2}{2n_s[\sqrt{R_1^2 + (X_1 + X_2)^2} + R_1]} \tag{5–10}$$

As indicated in Eq. (5–10), the maximum torque (breakdown torque) that a given induction motor can develop is independent of rotor resistance. *Changing the value of rotor resistance will change the slip at which* $T_{D,\max}$ *occurs, but will not change the value of* $T_{D,\max}$.

PLE A three-phase, 40-hp, 460-V, four-pole, 60-Hz squirrel-cage induction motor has a rated speed of 1751 r/min, and the following parameters expressed in ohms:

$$R_1 = 0.102 \qquad R_2 = 0.153 \qquad R_{fe} = 102.2$$
$$X_1 = 0.409 \qquad X_2 = 0.613 \qquad X_M = 7.665$$

Determine (a) the speed at which maximum torque is developed; (b) the maximum torque that the machine can develop; (c) rated shaft torque; (d) which NEMA design fits this motor.

Solution

(a)
$$s_{T_{D,\max}} = \frac{R_2}{\sqrt{R_1^2 + (X_1 + X_2)^2}} = \frac{0.153}{\sqrt{(0.102)^2 + (0.409 + 0.613)^2}}$$

$$s_{T_{D,\max}} = 0.1490$$

$$n_s = \frac{120f}{P} = \frac{120 \cdot 60}{4} = 1800 \text{ r/min}$$

$$n_r = n_s(1 - s) = 1800(1 - 0.1490) = \underline{1532 \text{ r/min}}$$

(b)
$$T_{D,\max} = \frac{21.12V^2}{2n_s[\sqrt{R_1^2 + (X_1 + X_2)^2} + R_1]}$$

$$T_{D,\max} = \frac{21.12(460/\sqrt{3})^2}{2(1800)[\sqrt{0.102^2 + (0.409 + 0.613)^2} + 0.102]}$$

$$T_{D,\max} = \underline{366.5 \text{ lb-ft}}$$

(c)
$$\text{hp} = \frac{Tn}{5252} \Rightarrow 40 = \frac{T \cdot 1751}{5252}$$

$$T_{D,\text{shaft}} = \underline{120.0 \text{ lb-ft}}$$

(d) Maximum torque is developed at a slip of 0.1490. This places the machine in the design *A* category, as seen from the curves in Figure 5.1.

EXAMPLE 5.4 A three-phase, 50-hp, 460-V, 60-Hz, four-pole, design *B* induction motor operating at rated load, rated voltage, and rated frequency has an operating speed of 1760 r/min. If to reduce a system overload, the utility drops the line voltage to 90 percent rated voltage, determine (a) the amount of torque that must be removed from the motor shaft in order to maintain 1760 r/min; (b) the expected minimum starting torque for the lower voltage; (c) the percent change in developed torque caused by the 10 percent drop in system voltage.

Solution

(a) At rated conditions,

$$\text{hp} = \frac{Tn}{5252} \Rightarrow 50 = T \times \frac{1760}{5252}$$

$$T_{\text{rated}} = 149.2 \text{ lb-ft}$$

Using proportionality (5–6), the developed torque at 1760 r/min and 90 percent rated voltage is

$$T_{D2} = T_{D1}\left(\frac{V_2}{V_1}\right)^2 = 149.2\left(\frac{460 \times 0.90}{460}\right)^2 = \underline{120.9 \text{ lb-ft}}$$

The required reduction in torque is

$$149.2 - 120.9 = \underline{28.3 \text{ lb-ft}}$$

(b) The expected minimum locked-rotor torque at rated voltage and rated frequency, as obtained from Table 5.1, is 140 percent rated torque.

$$T_{lr} = 1.40 \times 149.2 = 208.88 \text{ lb-ft}$$

The expected minimum starting torque at 90 percent rated voltage is

$$T_{lr2} = T_{lr1}\left(\frac{V_2}{V_1}\right)^2 = 208.88 \times \left(\frac{460 \times 0.90}{460}\right)^2 = \underline{169.2 \text{ lb-ft}}$$

(c) The percent change in torque caused by a 10 percent drop in system voltage is as follows. At 1760 r/min,

$$\frac{120.9 - 149.2}{149.2} = -0.19 \quad \text{or} \quad \underline{-19\%}$$

At locked rotor,

$$\frac{169.2 - 208.88}{208.88} = -0.19 \quad \text{or} \quad \underline{-19\%}$$

Note that a 10 percent drop in applied stator voltage results in a 19 percent drop in developed torque.

SOME USEFUL APPROXIMATIONS FOR …AL RUNNING AND OVERLOAD …ITIONS OF SQUIRREL-CAGE MOTORS

Normal running conditions are defined as operating between no load and 15 percent overload with rated voltage and rated frequency. The 15 percent overload represents the permissible continuous overload for motors with a 1.15 service factor on their respective nameplates. When operating under these conditions, the slip is very small, usually <0.03, permitting very simplified mathematical approximations that are useful for solving many induction-motor problems.

Recall equations for rotor current and developed torque, Eqs. (5–4) and (5–5), respectively:

$$|\mathbf{I}_2| \cong \frac{|\mathbf{V}|}{\sqrt{[(R_1 + R_2/s)^2 + (X_1 + X_2)^2]}} \tag{5–4}$$

$$T_D \cong \frac{21.12V^2R_2/s}{[(R_1 + R_2/s)^2 + (X_1 + X_2)^2]n_s} \tag{5–5}$$

In Eqs. (5–4) and (5–5), for very low values of slip,

$$R_1 \ll \frac{R_2}{s} \gg (X_1 + X_2)$$

Thus, for values of $s \le 0.03$, the bracketed expression in the denominators of Eqs. (5–4) and (5–5) may be replaced by R_2/s without introducing any significant error.[4] That is,

$$\left[\left(R_1 + \frac{R_2}{s}\right)^2 + (X_1 + X_2)^2\right]_{s \le 0.03} \Rightarrow \left(\frac{R_2}{s}\right)^2$$

[4]A slip of $s \le 0.03$ is an arbitrary constraint that provides good approximations for most machines. Exact determinations, however, require the procedure outlined in Section 5.4, when calculating rotor current and developed torque.

Substituting into Eqs. (5–4) and (5–5) results in the following approximations:

$$\underset{s \le 0.03}{I_2} \cong \frac{V}{R_2/s} = \frac{V \cdot s}{R_2} \tag{5–11}$$

$$\underset{s \le 0.03}{T_D} \cong \frac{21.12V^2R_2/s}{(R_2/s)^2 n_s} = \frac{21.12V^2 \cdot s}{R_2 n_s} \tag{5–12}$$

Examination of Eqs. (5–11) and (5–12) shows that for $s \le 0.03$, both I_2 and T_D are directly proportional to the slip. Thus, expressed as a proportion, and assuming rated voltage and rated frequency,

$$\underset{s \le 0.03}{I_2} \propto s \tag{5–13}$$

$$\underset{s \le 0.03}{T_D} \propto s \tag{5–14}$$

Graphical justification for proportionalities (5–13) and (5–14) is illustrated in Figure 5.7. The curves show the behavior of rotor current and rotor torque *as the motor is loaded from its running no-load condition.*[5]

The approximations developed in this section may also be applied to problems involving overloads of up to 150 percent or more of rated torque, providing the initial and final conditions of rotor current and developed torque are known to lie on (or close to) the linear section of the respective curve. It is important, however, to note that when operating under high overload conditions, rapid and severe heating of the motor will occur; sustained overload operation at high overloads will cause severe damage to both rotor and stator.

[5]The curves in Figure 5.7 were plotted using the motor parameters in Example 5.2.

EXAMPLE 5.5 A 575-V, 100-hp, 60-Hz, 12-pole, wye-connected squirrel-cage motor operating at rated torque load and 591.1 r/min draws a line current of 89.2 A at rated voltage and rated frequency. The motor parameters expressed in ohms are

$$R_1 = 0.060 \quad R_2 = 0.055 \quad R_{fe} = 67.0$$
$$X_1 = 0.034 \quad X_2 = 0.034 \quad X_M = 11.22$$

If an increase in shaft load causes T_D to increase by 25 percent (an obvious overload), determine for the new conditions (a) shaft speed; (b) rotor current referred to the stator. Assume the overload operation is known to lie on the linear sections of the respective torque-slip and current-slip curves.

Solution

(a) $$n_s = \frac{120f}{P} = 120 \times \frac{60}{12} = 600 \text{ r/min}$$

At rated load,

$$s = \frac{n_s - n_r}{n_s} = \frac{600 - 591.1}{600} = 0.01483$$

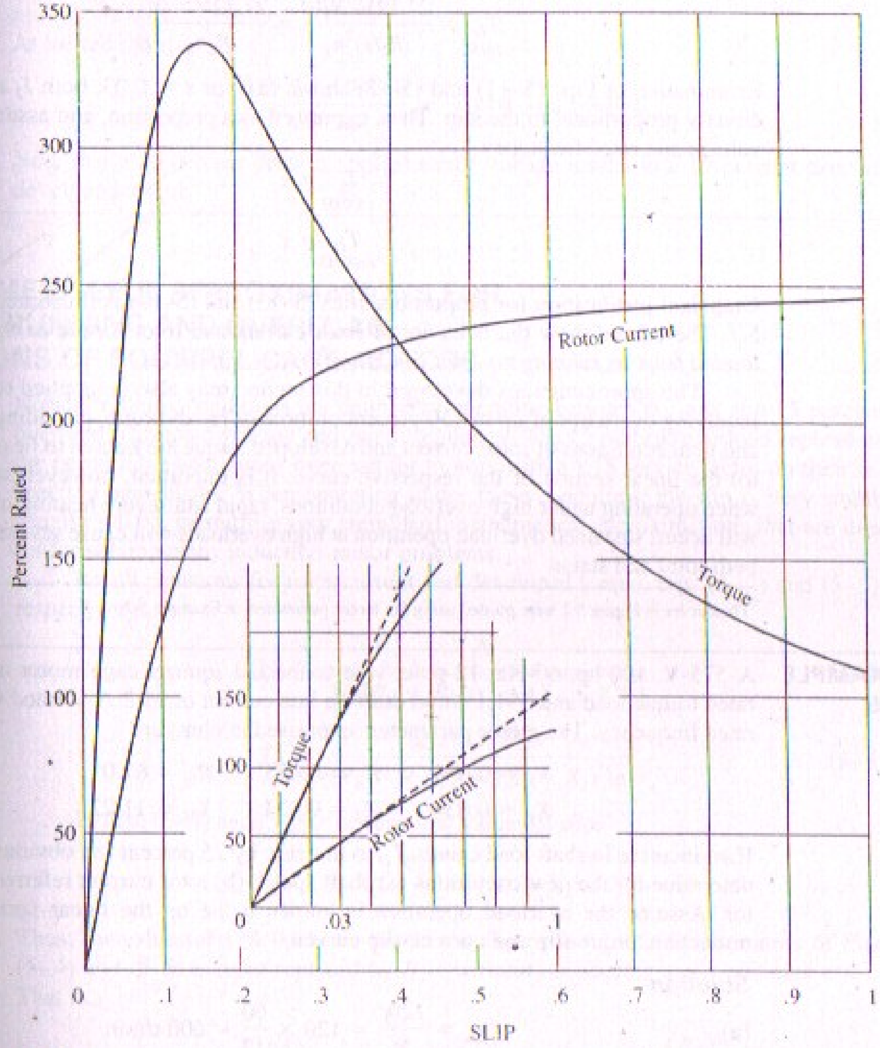


FIGURE 5.7
Rotor current and developed torque vs. slip.

From proportionality (5–14),

$$s_2 = s_1 \cdot \frac{T_{D2}}{T_{D1}} = 0.01483 \times \frac{1.25 \times T_{D1}}{T_{D1}} = 0.01854$$

$$n_r = n_s(1 - s) = 600(1 - 0.01854) = \underline{589 \text{ r/min}}$$

(b)

$$I_2\Big|_{s \le 0.03} = \frac{V \cdot s}{R_2} = \frac{(575/\sqrt{3}) \cdot (0.01854)}{0.055} = \underline{112 \text{ A}}$$

5.7 NEMA CONSTRAINTS ON VOLTAGE AND FREQUENCY

NEMA-rated induction motors are expected to operate successfully at rated load, as long as variations in applied voltage and applied frequency do not exceed the following constraints [9]:

1. A voltage variation of up to ±10 percent rated voltage while operating at rated frequency.
2. A frequency variation of ±5 percent rated frequency, while operating at rated voltage.
3. A combined variation in voltage and frequency, with the sum of the *absolute* values of the respective variations not exceeding 10 percent providing the frequency does not exceed 5 percent of rated frequency.

Note: In accordance with constraint 3, a 9 percent *rise* in system voltage, accompanied by a 4 percent *drop* in system frequency would cause a combined variation of 9% + 4% = 13%. Adding algebraically 9% + (−4%) = 5% is *incorrect!* The calculation must be an *addition of the absolute values.* Off-standard frequency and off-standard voltage each have an adverse effect on motor performance, and *the adverse effects of a decrease in frequency do not offset the adverse effects of an increase in voltage*, and vice versa.

5.8 EFFECT OF OFF-RATED VOLTAGE AND OFF-RATED FREQUENCY ON INDUCTION-MOTOR PERFORMANCE

Induction-motor speed, current, and developed torque are a function of the source frequency and voltage. Thus, a significant deviation from rated motor frequency can have serious adverse effects on motor operation. Large interconnected utilities have a relatively stable frequency and stable voltage. However, isolated power plants, such as those found on ships, offshore drilling rigs, and in certain rural areas, may at times experience both off-rated voltage and off-rated frequency.

Effect on Running Torque

The effect of different frequencies and different voltages on the developed torque may be determined by expressing the synchronous speed in Eq. (5–12) in terms of frequency. Making the substitution,

$$\underset{s \le 0.03}{T_D} \cong \frac{21.12V^2 \cdot s}{R_2(120f/P)} \qquad (5\text{–}15)$$

Equation (5–15) indicates that for *all values of slip ≤ 0.03 the developed torque is proportional to the slip, to the square of the applied voltage, and inversely proportional to the frequency*. That is,

$$\underset{s \le 0.03}{T_D} \propto \frac{V^2 \cdot s}{f} \qquad (5\text{–}16)$$

Note: Changes in friction, windage, and stray power losses are generally very small for ranges in slip ≤ 0.03. Hence, if these losses are not known, and $s \le 0.03$, T_{shaft} may be substituted for T_D in proportionality (5–16) without introducing a significant error.

A 230-V, 20-hp, 60-Hz, six pole, three-phase induction motor driving a constant torque load at rated frequency, rated voltage, and rated horsepower has a speed of 1175 r/min, and an efficiency of 92.1 percent. Determine (a) the new operating speed if a system disturbance causes a 10 percent drop in voltage and a 6 percent drop in frequency; (b) the new shaft horsepower. Assume that windage, friction, and stray power losses are essentially constant.

Solution

(a)

$$V_2 = 0.90(230) = 207 \text{ V}$$

$$f_2 = 0.94(60) = 56.4 \text{ Hz}$$

$$n_{s1} = 120f_1/P = 120(60)/6 = 1200 \text{ r/min}$$

$$n_{s2} = 120f_2/P = 120(56.4)/6 = 1128 \text{ r/min}$$

$$s_1 = (n_{s1} - n_{r1})/n_{s1} = (1200 - 1175)/1200 = 0.02083$$

With a constant torque load and $s_1 \le 0.03$,

$$\left[\frac{V^2 \cdot s}{f}\right]_1 = \left[\frac{V^2 \cdot s}{f}\right]_2$$

$$s_2 = s_1 \cdot \left(\frac{V_1}{V_2}\right)^2 \cdot \frac{f_2}{f_1} = 0.02083 \times \left(\frac{230}{0.90 \times 230}\right)^2 \times \frac{60 \times 0.94}{60}$$

$$s_2 = 0.02417$$

$$n_{r2} = n_{s2}(1 - s_2) = 1128(1 - 0.02417) = \underline{1101 \text{ r/min}}$$

(b)

$$P = \frac{T \cdot n}{5252} \quad \Rightarrow \quad \frac{P_2}{P_1} = \frac{T_2 \cdot n_{r2}}{T_1 \cdot n_{r1}}$$

Thus, with a constant torque load, $T_2 = T_1$,

$$P_2 = P_1 \times \frac{T_2 \cdot n_{r2}}{T_1 \cdot n_{r1}} = 20 \times \frac{T_1 \cdot 1101}{T_1 \cdot 1175} = \underline{18.7 \text{ hp}}$$

Effect on Locked-Rotor Current

Calculations to determine locked-rotor current are based solely on the applied voltage and the locked-rotor input impedance. Thus, referring to the approximate equivalent circuit in Figure 5.6(a), and noting that $s = 1.0$, at blocked rotor,

$$\underset{s=1.0}{I_2} = \frac{V}{Z_{lr}} = \frac{V}{\sqrt{(R_1 + R_2)^2 + (X_1 + X_2)^2}} \qquad (5\text{–}17)$$

where: Z_{lr} = locked-rotor impedance (Ω)

Except for high-slip machines such as the design *D* motor, or other special high-slip designs, *most squirrel-cage induction motors are low-resistance, high-reactance machines, whose impedance angle at locked rotor is ≥ 75°*. This is shown in Figure 5.6(b), where

$$\theta_z = \arctan\left(\frac{X_1 + X_2}{R_1 + R_2}\right) \qquad (5\text{–}18)$$

Note that, with angle $\theta_z \ge 75°$,

$$|\mathbf{Z}_{lr}| \cong X_1 + X_2$$

Hence, with $s = 1.0$, and $\theta_z \ge 75°$, Eq. (5–17) may be approximated as

$$\underset{s=1.0,\theta_z \ge 75°}{I_2} \cong \frac{V}{X_1 + X_2} \qquad (5\text{–}19)$$

Expressing the leakage reactance in terms of frequency, and factoring,

$$\underset{s=1.0,\theta_z \ge 75°}{I_2} \cong \frac{V}{f(2\pi L_1 + 2\pi L_2)} \qquad (5\text{–}20)$$

Equation (5–20) indicates that the locked-rotor current is directly proportional to the applied voltage and inversely proportional to the applied frequency. That is,

$$\underset{s=1.0,\theta_z \ge 75°}{I_2} \propto \frac{V}{f} \qquad (5\text{–}21)$$

Furthermore, from Figure 5.6(a),

$$\mathbf{I}_1 = \mathbf{I}_2 + \mathbf{I}_0$$

And, at locked rotor,

$$|\mathbf{I}_2| >> |\mathbf{I}_0|$$

Hence, at locked rotor, the stator current (I_1, defined as I_{lr}) is approximately equal to the rotor current (I_2),

$$I_{lr} = I_1 \cong I_2 \tag{5–22}$$

Thus rewriting Eq. (5–21),

$$I_{lr}\Big|_{s=1.0,\ \theta_2 \cong 75^\circ} \propto \frac{V}{f} \tag{5–23}$$

E A 20-hp, four-pole, three-phase, 230-V, 60-Hz, design *B*, wye-connected motor draws 151 A when started at rated voltage and rated frequency. Determine the expected locked-rotor line current if the motor is started from a 220-V, 50-Hz system.

Solution

Using proportionality (5–23),

$$\frac{I_{lr1}}{I_{lr2}} = \frac{V_1/f_1}{V_2/f_2} \quad \Rightarrow \quad I_{lr2} = I_{lr1} \cdot \frac{V_2/f_2}{V_1/f_1}$$

$$I_{lr2} = 151 \times \frac{220/50}{230/60} = \underline{173 \text{ A}}$$

Note: In those industries where there is a mix of 25-Hz and 60-Hz power systems, extreme care must be exercised to prevent the accidental connection of a 25-Hz motor to a 60-Hz system, and vice versa. For example, connecting a motor rated at 25 Hz, 425 r/min, to a 60-Hz system would cause a dangerous overspeed approximating 425(60/25) = 1020 r/min.[6] Connecting a 60-Hz motor, whose locked-rotor current is 1085 A, to a 25-Hz system (with the same line voltage) will cause the locked-rotor current to be approximately 1085(60/25) = 2604 A, resulting in a burned-out motor.

Effect on Locked-Rotor Torque

From the BLI rule as expressed in Eq. (1–12), Chapter 1, the force on a rotor conductor (and hence the torque) is proportional to the current in the conductor and the stator flux density in which it is immersed. Expressed mathematically,

$$T_D \propto B_{stator} I_{rotor}$$

Furthermore, the flux density is proportional to the stator current.

$$B_{stator} \propto I_{stator}$$

[6] See Table 5.11, Section 5.18, for allowable overspeeds of induction motors.

Thus,

$$T_D \propto I_{stator} I_{rotor}$$

Using the terminology in Figure 5.6(a), at locked rotor,

$$T_{lr} \propto I_1 I_2$$

However, from Eq. (5–22), at locked rotor,

$$I_{lr} = I_1 \cong I_2$$

Thus, at locked rotor,

$$T_{lr} \propto I_{lr}^2 \tag{5–23a}$$

Substituting proportionality (5–23) into proportionality (5–23a),

$$I_{lr} = \left[\frac{V}{f}\right]^2 \tag{5–23b}$$

Proportionality (5–23b) shows that the locked-rotor torque is directly proportional to the square of the applied voltage, and inversely proportional to the square of the frequency. Although the effects of magnetic saturation and conductor skin effect were not considered, proportionality (5–23b) is accurate enough for practical applications.

EXAMPLE 5.8 (a) From the NEMA tables, determine the expected minimum locked-rotor torque for a 75-hp, four-pole, 60-Hz, 240-V, 1750-rpm, design *E* motor. (b) Repeat (a) assuming system overloading made it mandatory to drop the voltage and frequency to 230 V and 58 Hz, respectively.

Solution

(a) From Table 5.2, the minimum locked-rotor torque is 120% rated.

$$\text{hp}_{rated} = \frac{T_{rated} \times n_{rated}}{5252} \quad \Rightarrow \quad T_{rated} = \frac{5252 \times \text{hp}_{rated}}{n_{rated}}$$

$$T_{rated} = \frac{5252 \times 75}{1750} = 225 \text{ lb-ft}$$

$$T_{lr} = 225 \times 1.20 = \underline{270 \text{ lb-ft}}$$

(b) Using proportionality (5–23b)

$$\frac{T_{lr2}}{T_{lr1}} = \frac{\left[\dfrac{V_2}{f_2}\right]^2}{\left[\dfrac{V_1}{f_1}\right]^2} \quad \Rightarrow \quad T_{lr2} = T_{lr1} \times \left[\frac{V_2}{f_2}\right]^2 \times \left[\frac{f_1}{V_1}\right]^2$$

$$T_{lr2} = 270 \times \left[\frac{230}{58}\right]^2 \times \left[\frac{60}{240}\right]^2 = \underline{265 \text{ ft-lb}}$$

Operating 60-Hz Motors on a 50-Hz System

Operating an induction motor at significantly below rated frequency, such as operating a 60-Hz motor at 50 Hz, causes a significant decrease in magnetizing reactance and, because of magnetic saturation effects, an out-of-proportion increase in magnetizing current. The net result is severe overheating of the motor windings. To prevent overheating, a reduction in applied frequency must be accompanied by a reduction in applied voltage. Simply stated, *the ratio of volts per hertz must be kept constant.* General-purpose, three-phase, 60-Hz, NEMA-design induction motors with two, four, six, or eight poles are capable of operating satisfactorily from 50-Hz systems, *provided the horsepower and voltage ratings at 50 Hz are 5/6 of the corresponding ratings at 60 Hz.* When operating in this manner, overheating will not occur, and the locked-rotor torque and breakdown torque at 50 Hz will be essentially the same as for 60-Hz operation [9].

These relationships, expressed mathematically are:

$$V_{50} = \frac{5}{6}V_{60} \tag{5–23c}$$

$$\text{hp}_{50} = \frac{5}{6}\text{hp}_{60} \tag{5–23d}$$

$$\left[\frac{T \cdot n_r}{5252}\right]_{50} = \frac{5}{6}\left[\frac{T \cdot n_r}{5252}\right]_{60} \tag{5–23e}$$

Substituting proportionality (5–16) and simplifying,

$$\left[\frac{V^2 \cdot s \cdot n_r}{f}\right]_{50} = \frac{5}{6}\left[\frac{V^2 \cdot s \cdot n_r}{f}\right]_{60}\Bigg\} s \le 0.03 \tag{5–23f}$$

A 40-hp, 460-V, 60-Hz, four-pole, three-phase, design *B* induction motor operating at rated voltage, rated frequency, and rated horsepower runs at 1770 r/min and draws a line current of 52.0 A. If the machine is operated at 5/6 rated horsepower from a 385-V, 50-Hz supply, determine (a) shaft r/min; (b) slip.

Solution

(a) $$n_{s,60} = \frac{120(60)}{4} = 1800 \text{ r/min}$$

$$n_{s,50} = \frac{120(50)}{4} = 1500 \text{ r/min}$$

$$s_{60} = \frac{n_s - n_r}{n_s} = \frac{1800 - 1770}{1800} = 0.01667$$

$$\text{hp}_{50} = \frac{5}{6}\text{hp}_{60}$$

$$\left[\frac{T \cdot n_r}{5252}\right]_{50} = \frac{5}{6}\left[\frac{T \cdot n_r}{5252}\right]_{60}$$

Substituting proportionality (5–16) and simplifying,

$$\left[\frac{V^2 \cdot s \cdot n_r}{f}\right]_{50} = \frac{5}{6}\left[\frac{V^2 \cdot s \cdot n_r}{f}\right]_{60}\Bigg\} s \le 0.03$$

Substituting given and calculated values and solving for s_{50},

$$\left[\frac{(385)^2 \cdot s_{50} \cdot n_{r,50}}{50}\right] = \frac{5}{6}\left[\frac{(460)^2(0.01667)(1770)}{60}\right]$$

$$s_{50} = \frac{29.251}{n_{r,50}}$$

From Eq. (4–3),

$$s_{50} = \frac{1500 - n_{r,50}}{1500}$$

Equating and solving for $n_{r,50}$

$$\frac{29.251}{n_{r,50}} = \left[\frac{1500 - n_{r,50}}{1500}\right]$$

$$n_{r,50}^2 - 1500n_{r,50} + 43{,}876.5 = 0$$

Using the quadratic formula,

$$n_{r,50} = \frac{1500 \pm \sqrt{(-1500)^2 - 4(43{,}876.5)}}{2}$$

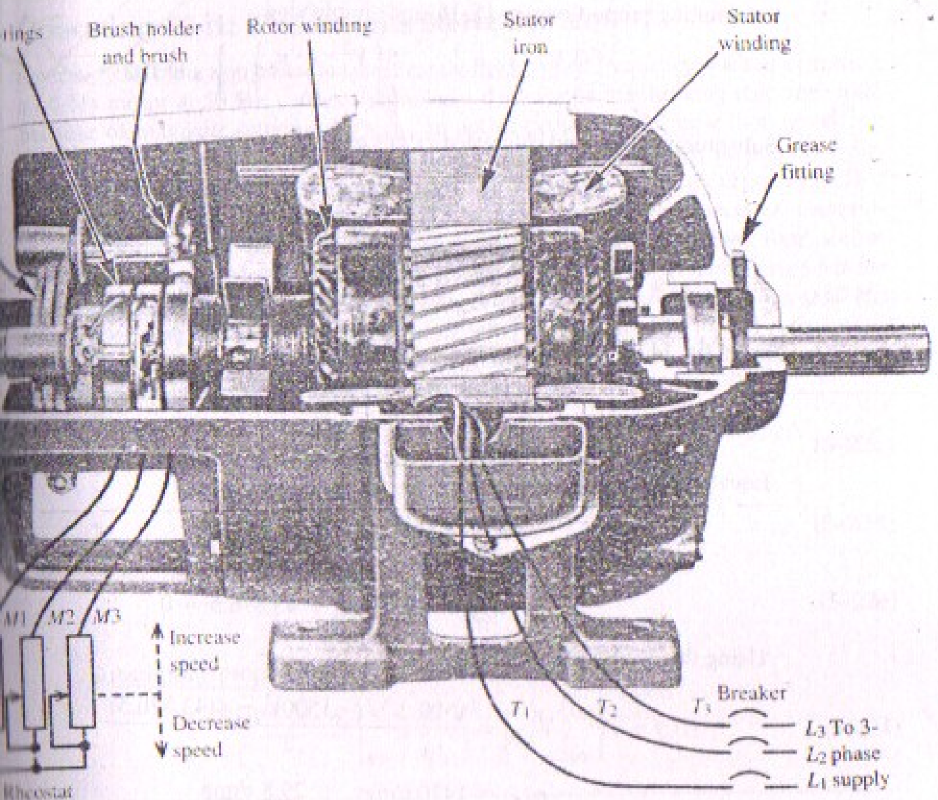
$$n_{r,50} = \underline{1470 \text{ r/min}} \qquad \underbrace{\underline{29.8 \text{ r/min}}}_{\text{not valid}}$$

Proportionality (5–16) is valid for $s \le 0.03$. The slip at 29.8 r/min, however, is $(1500 - 29.8)/1500 = 0.980$. Hence, the 29.8 r/min is not valid.

(b) $$s_{50} = \frac{1500 - 1470}{1500} = \underline{0.020}$$

5.9 WOUND-ROTOR INDUCTION MOTOR

A wound-rotor induction motor, shown in Figure 5.8 and Figure 4.5, uses a wound rotor in place of a squirrel-cage rotor. The wound rotor has a regular three-phase winding similar to that of the stator and is wound with the same number of poles. The phases are usually wye connected and terminate at the slip rings. A wye-connected rheostat with a common lever is used to adjust the resistance of the rotor circuit. The rheostat provides speed control, torque adjustment at locked rotor, and current limiting during starting and acceleration.

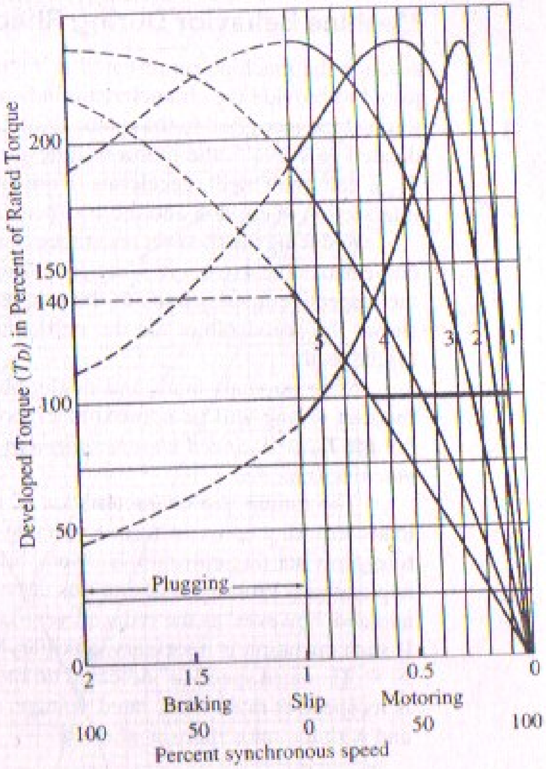


5.8
view of a wound-rotor motor. (Courtesy Magnetek Louis Allis Company)

Figure 5.9 illustrates a typical family of torque-slip curves that may be obtained through rotor-rheostat adjustment. Curve 5 is obtained with maximum rheostat resistance in the rotor circuit, and curve 1 is obtained with the rheostat shorted ($R_{rheo} = 0\ \Omega$). Changing the rheostat setting changes the slip at which $T_{D\text{-max}}$ occurs, but does not change the value of $T_{D\text{-max}}$. Note that increasing the resistance of the rheostat causes $T_{D\text{-max}}$ to shift to the left. A rheostat setting that results in curve 3 causes $T_{D\text{-max}}$ to occur at locked rotor ($s = 1$). Higher values of rotor circuit resistance, such as that represented by curve 4 and curve 5, cause $T_{D\text{-max}}$ to occur at values of slip greater than 1.0.

A slip greater than 1.0 can only occur during *plugging* operations. Plugging is the electrical reversal of a motor before it comes to rest and is accomplished by interchanging any two of the three line leads going to the stator. The characteristics when plugging are shown with broken lines in Figure 5.9. When in the plugging mode, n_r is negative with respect to synchronous speed. Hence, the slip when plugging is greater than 1.0.

FIGURE 5.9
Family of torque-slip (speed) curves for a representative wound-rotor motor.



Although plugging is sometimes used to provide a fast stop or a fast reversal of squirrel-cage or wound-rotor motors, damage due to overheating may occur unless the motor and/or control circuit are specifically designed to prevent excessive current during such operations. The high transient torques produced by plugging may also damage the driven equipment.

The slip at which $T_{D,\max}$ occurs may be determined from the machine parameters by modifying Eq. (5–7) to include the rheostat resistance. The resultant equation is:

$$s_{T_{D,\max}} = \frac{R_2 + R'_{rheo}}{\sqrt{R_1^2 + (X_1 + X_2)^2}} \tag{5–24}$$

where R'_{rheo} = rotor-rheostat resistance referred to the stator. That is,

$$R'_{rheo} = a^2 R_{rheo} \tag{5–24a}$$

where: a = stator-to-rotor turns-ratio

As indicated in Eq. (5–24), the slip at which $T_{D,\max}$ occurs is directly proportional to the per phase value of the rotor circuit resistance. Expressed as a proportion,

$$s_{T_{D,\max}} \propto (R_2 + R'_{rheo}) \tag{5–25}$$

Machine Behavior During Rheostat Adjustment

Assume the machine represented in Figure 5.9 is at rest ($s = 1$), the rheostat is adjusted to provide the characteristic indicated by curve 5, a constant 100-percent-rated torque load is applied to the motor shaft, and full voltage is applied to the stator. As indicated on curve 5, the motor will develop 140-percent-rated torque, and since $T_D > T_{load}$, the rotor will accelerate from standstill to a speed corresponding to the intersection of curve 5 and the 100-percent-rated torque line.

Reducing the rheostat resistance causes the torque-slip characteristic to shift from curve 5 to curve 4 to curve 3, etc., resulting in higher speeds. The heavy line connecting the respective intersections of the torque-slip curves with the 100 percent torque line shows the speed range for the particular motor, rheostat settings, and 100 percent torque load.

With no shaft load, and negligible windage and friction, the speed for every rheostat setting will be approximately synchronous speed; this is indicated in Figure 5.9 for $T_D \cong 0$. *Speed changes through rheostat control can be obtained only if the machine is loaded.*

The torque-slip characteristics of wound-rotor motors make them adaptable to loads requiring constant-torque variable-speed drives, high starting torques, and relatively low starting currents. Blowers, hoists, compressors, and stokers are some of its applications. Prolonged operations at below 50 percent synchronous speed should be avoided, however, as the reduced ventilation at slow speed may overheat the machine. If such operation is necessary, auxiliary cooling is required.

The rated speed as indicated on the nameplate of a wound-rotor induction motor is its speed at rated load, rated voltage, rated frequency, rated operating temperature, and with the rotor rheostat shorted.

E A family of torque-slip curves for a wye-connected, 400-hp, 2300-V, 14-pole, 60-Hz, wound-rotor induction motor is shown in Figure 5.10. Curves A and D indicate the extremes of rheostat adjustment. Determine (a) the range of rotor speeds available by rheostat adjustment, assuming 100 percent rated torque load on the shaft; (b) the rheostat resistance required to obtain 260 percent rated torque when starting. The ratio of stator turns per phase to rotor turns per phase is 3.8, and the motor parameters, in ohms/phase, are

$$R_1 = 0.403 \qquad R_2 = 0.317$$
$$X_1 = 1.32 \qquad X_2 = 1.32 \qquad X_M = 35.46$$

Solution

(a) $$n_s = \frac{120f}{P} = 120 \times \frac{60}{14} = 514.29 \text{ r/min}$$

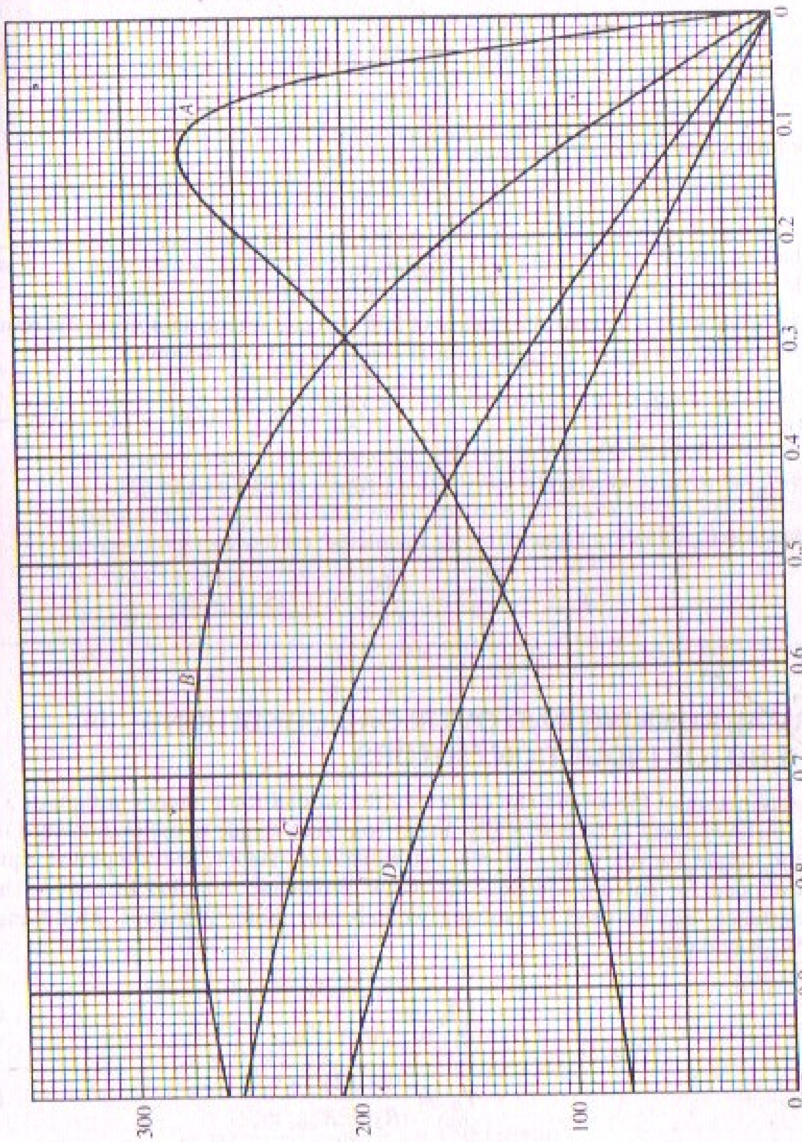


FIGURE 5.10
Torque-speed curves for Example 5.10.

From the intersection of the curves with the 100 percent torque line, low speed occurs at $s \cong 0.395$, and high speed occurs at $s \cong 0.02$. Thus, for low speed (curve D).

$$n_r = n_s(1 - s)$$
$$n_r = 514.29(1 - 0.395)$$
$$n_r = \underline{311 \text{ r/min}}$$

For high speed (curve A),

$$n_r = n_s(1 - s)$$
$$n_r = 514.29(1 - 0.02)$$
$$n_r = \underline{504 \text{ r/min}}$$

(b) Curve B has a locked-rotor torque of 260 percent. A rheostat setting that results in curve B will cause $T_{D,\max}$ to occur at $s = 0.74$ and is as follows:

$$s_{T_{D,\max}} = \frac{R_2 + R'_{\text{rheo}}}{\sqrt{R_1^2 + (X_1 + X_2)^2}} \quad\Rightarrow\quad R'_{\text{rheo}} = s_{T_{D,\max}} \cdot \sqrt{R_1^2 + (X_1 + X_2)^2} - R_2$$

$$R'_{\text{rheo}} = 0.74 \times \sqrt{(0.403)^2 + (1.32 + 1.32)^2} - 0.317 = 1.66\ \Omega$$

From Eq. (5–24a),

$$R_{\text{rheo}} = \frac{R'_{\text{rheo}}}{a^2} = \frac{1.66}{(3.8)^2} = \underline{0.115\ \Omega/\text{phase}}$$

0 NORMAL RUNNING AND OVERLOAD CONDITIONS WOUND-ROTOR INDUCTION MOTORS

Examination of the torque-slip curves for the wound-rotor motors in Figures 5.9 and 5.10 shows them to be essentially linear from zero torque to approximately 110 percent torque for all values of rheostat resistance. Hence, the simplified equations developed for solving squirrel-cage motor problems, modified to include rheostat resistance, can be used for solving wound-rotor motor problems. Modifying Eqs. (5–11) and (5–12),

$$I_{2\,(s \le 0.03)} \cong \frac{V \cdot s}{R_2 + R'_{\text{rheo}}} \tag{5–26}$$

$$T_{D\,(s \le 0.03)} \cong \frac{21.12V^2 \cdot s}{(R_2 + R'_{\text{rheo}})n_s} \tag{5–27}$$

Examination of Eqs. (5–26) and (5–27) shows that for $s \le 0.03$, both I_2 and T_D are directly proportional to the slip and inversely proportional to $(R_2 + R'_{\text{rheo}})$. Because of the added resistance in the rotor circuit, however, the constraint for $s \le 0.03$ for

squirrel-cage motors does not necessarily apply for all rheostat settings of a wound-rotor motor. For example, curve 5 in Figure 5.9 is essentially linear from $s = 0$ to $s = 0.50$. Thus, expressed as a proportion, and assuming rated voltage, rated frequency, and operation in the linear region,

$$I_{2\,\text{linear}} \propto \frac{s}{R_2 + R'_{\text{rheo}}} \tag{5–28}$$

$$T_{D\,\text{linear}} \propto \frac{s}{R_2 + R'_{\text{rheo}}} \tag{5–29}$$

EXAMPLE 5.11 A three-phase, wye-connected, 400-hp, four-pole, 380-V, 50-Hz, wound-rotor induction motor operating at rated conditions with the rheostat shorted has a slip of 0.0159. The machine parameters expressed in ohms are

$$R_1 = 0.00536 \qquad R_2 = 0.00613 \qquad R_{fe} = 7.66$$
$$X_1 = 0.0383 \qquad X_2 = 0.0383 \qquad X_M = 0.5743$$

Determine (a) the rotor frequency; (b) the slip at which $T_{D,\max}$ occurs; (c) the rotor speed at one-half rated torque load; (d) the rheostat resistance per phase required to operate the machine at 1000 r/min and one-half rated torque load (assume the motor is operating on the linear section of the curve and the stator to rotor turns ratio is 2.0); (e) rated torque.

Solution

(a)

$$f_r = sf_{BR} = 0.0159 \times 50 = \underline{0.795 \text{ Hz}}$$

(b)

$$s_{T_{D,\max}} = \frac{R_2 + R'_{\text{rheo}}}{\sqrt{R_1^2 + (X_1 + X_2)^2}} = \frac{0.00613 + 0}{\sqrt{0.00536^2 + (0.0383 + 0.0383)^2}}$$

$$s_{T_{D,\max}} = \underline{0.0798}$$

(c) The loss in torque due to friction, windage, and stray load is a small percentage of developed torque. Hence, little error is introduced if Eq. (5–27) (for developed torque) is used to determine the load torque. Thus, using proportionality (5–29) to set up a ratio,

$$\frac{T_{D1}}{T_{D2}} = \frac{[s/(R_2 + R'_{\text{rheo}})]_1}{[s/(R_2 + R'_{\text{rheo}})]_2} \quad\Rightarrow\quad s_2 = s_1 \times \frac{T_{D2}}{T_{D1}} \times \frac{[(R_2 + R'_{\text{rheo}})]_2}{[(R_2 + R'_{\text{rheo}})]_1}$$

$$s = 0.0159 \times \frac{0.5T_{\text{rated}}}{T_{\text{rated}}} \times \frac{0.00613}{0.00613} = 0.00795$$

$$n_s = \frac{120f}{P} = \frac{120(50)}{4} = 1500 \text{ r/min}$$

$$n_r = n_s(1 - s) = 1500(1 - 0.00795) = \underline{1488 \text{ r/min}}$$

(d) $$s = \frac{n_s - n_r}{n_s} = \frac{1500 - 1000}{1500} = 0.3333$$

$$\frac{T_{D1}}{T_{D2}} = \frac{[s/(R_2 + R'_{\text{rheo}})]_1}{[s/(R_2 + R'_{\text{rheo}})]_2} \quad \Rightarrow \quad R'_{\text{rheo},2} = \frac{s_2}{s_1} \times \frac{T_{D1}}{T_{D2}} \times [R_2 + R'_{\text{rheo}}]_1 - R_2$$

$$R'_{\text{rheo},2} = \frac{0.3333}{0.0159} \times \frac{T_{\text{rated}}}{0.5T_{\text{rated}}} \times (0.00613 + 0.0) - 0.00613 = \underline{0.2509\ \Omega}$$

$$R_{\text{rheo}} = \frac{R'_{\text{rheo}}}{a^2} = \frac{0.2509}{2^2} = \underline{0.0627\ \Omega/\text{phase}}$$

(e) $$n_r = n_s(1 - s) = 1500(1 - 0.0159) = 1476.2 \text{ r/min}$$

$$\text{hp} = \frac{Tn}{5252} \quad \Rightarrow \quad 400 = \frac{T(1476.2)}{5252}$$

$$T = \underline{1423 \text{ lb-ft}}$$

EXAMPLE 5.12 A 50-hp, 10-pole, 60-Hz, 575-V, wye-connected, three-phase, wound-rotor induction motor, operating with the rotor rheostat in the circuit develops its maximum torque at 45 percent slip. Determine the percentage increase or decrease in rotor circuit resistance required to cause $T_{D,\max}$ to occur at 80 percent slip.

Solution

From proportionality (5–25),

$$\frac{s_{T_{D,\max 1}}}{s_{T_{D,\max 2}}} = \frac{[R_2 + R'_{\text{rheo}}]_1}{[R_2 + R'_{\text{rheo}}]_2} \quad \Rightarrow \quad \left[\frac{0.45}{0.80}\right] = \frac{[R_2 + R'_{\text{rheo}}]_1}{[R_2 + R'_{\text{rheo}}]_2}$$

$$[R_2 + R'_{\text{rheo}}]_2 = 1.78[R_2 + R_{\text{rheo}}]_1 = \underline{78\% \text{ increase}}$$

Thus, a 78 percent increase in equivalent rotor circuit resistance is required to cause $T_{D,\max}$ to occur at 80 percent slip.

5.11 MOTOR NAMEPLATE DATA

Nameplate data offer very pertinent information on the limits, operating range, and general characteristics of electrical apparatus. Interpretation of these data and adherence to their specifications are vital to the successful operating and servicing of such equipment. Correspondence with the manufacturer should always be accompanied by the complete nameplate data of the apparatus. Figure 5.11 illustrates a typical nameplate for an induction motor.

The nameplate lists the *rated operating conditions* of the motor as guaranteed by the manufacturer. For example, for the three-phase motor represented by the nameplate in Figure 5.11, if exactly 460 V, 60 Hz, and three phases are supplied to the stator, and the motor is located in a 40°C ambient region, with the shaft loaded to exactly

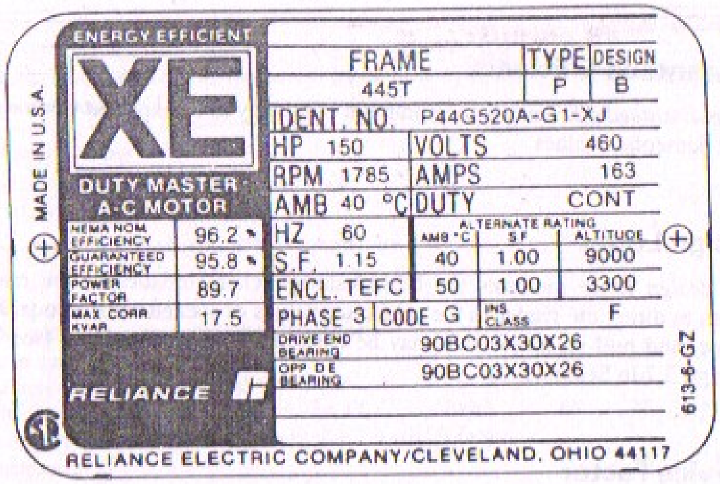


FIGURE 5.11
Induction-motor nameplate. (Courtesy Reliance Electric Company)

150 hp, the motor will run at approximately 1785 r/min, draw a line current of approximately 163 A, and have a guaranteed efficiency of 95.8 percent.

Motors rarely, if ever, operate at the exact rated conditions specified by the manufacturer, however. The utilization voltage (voltage at the apparatus) rarely corresponds exactly to the motor nameplate voltage, the motor rarely operates at exactly rated horsepower, and the ambient temperature is rarely 40°C. Although the system frequency most often matches the rated frequency of the motor, there are instances, especially in isolated generator systems (offshore drilling rigs or ships), where the frequency is subject to change.

The nameplate acts as a guide to motor applications, and satisfactory performance is assured if the applied voltage is approximately rated voltage, the frequency is approximately rated frequency, the shaft load does not exceed the service factor rating, and the temperature of the ambient is within the limits indicated on the nameplate. The horsepower rating for each speed of a multispeed motor is based on the type of industrial application: constant horsepower, constant torque, or variable torque.[7]

Nominal Efficiency[8]

The *nominal efficiency* indicated on the nameplate is the average efficiency of a large number of motors of the same design.

[7] See Appendix C.

[8] See Reference [9] for tables of nominal and minimum efficiencies of NEMA-design induction motors.

Guaranteed Efficiency

The *guaranteed efficiency* is the minimum efficiency to be expected when operating at rated nameplate values.

Design Letter

The *design letter* indicates the NEMA-design characteristics of the machine and serves to direct the reader to the minimum values of locked-rotor torque, breakdown torque, and pull-up torque that may be expected from the machine (see Tables 5.1 through 5.7 in Section 5.3).

Service Factor

The *service factor* (S.F.) of a motor is a multiplier that, when multiplied by the rated power, indicates the permissible loading, provided that the voltage and frequency are maintained at the value specified on the nameplate. Note, however, that if induction motors are operated at a service factor greater than 1.0, the efficiency, power factor, and speed will be different from those at rated load.

Insulation Class

The letter designating *insulation class* specifies the maximum allowable temperature rise above the temperature of the cooling medium for motor windings, based on a maximum ambient temperature of 40°C. *All winding temperatures are to be determined by winding resistance measurement.* A list of maximum allowable winding-temperature rise above the ambient temperature for different classes of insulation systems used in medium single-phase and polyphase induction motors is given in Table 5.8. The maximum winding temperatures are for continuous-duty motors, or motors with short-time ratings of 5, 15, 30, and 60 min. *Note:* Table 5.8 is not valid for ambient temperatures above 40°C. See References [9] and [5] for more detailed information regarding ambient temperatures above 40°C, high altitude, etc. The motor whose nameplate is shown in Figure 5.11, has Class F insulation, a service factor of 1.15, and is rated for continuous duty. Thus, based on a 40°C ambient, the maximum allowable temperature rise is 115°C, as obtained from Table 5.8.

Frame Number

The *frame number*, for example, 445T in Figure 5.11, determines the critical mounting dimensions of the motor, including shaft length, shaft diameter, shaft height, location of mounting holes, etc. Mounting dimensions for specific frame sizes are given in Reference [9].

TABLE 5.8
Maximum allowable temperature rise for medium single-phase and polyphase induction motors in °C, based on a maximum ambient temperature of 40°C[a]

Class of insulation system (see MG 1-1.65)	A	B	F[b]	H[b,c]
Time rating (shall be continuous or any short-time rating given in MG 1-10.36)				
Temperature rise (based on a maximum ambient temperature of 40°C), °C				
1. Windings, by resistance method				
(a) Motors with 1.0 service factor other than those given in items 1(c) and 1(d)	60	80	105	125
(b) All motors with 1.15 or higher service factor	70	90	115	—
(c) Totally enclosed nonventilated motors with 1.0 service factor	65	85	110	135
(d) Motors with encapsulated windings and with 1.0 service factor, all enclosures	65	85	110	—
2. The temperatures attained by cores, squirrel-cage windings, commutators, collector rings, and miscellaneous parts (such as brushholders, brushes, pole tips, uninsulated shading coils) shall not injure the insulation or the machine in any respect.				

[a] Reproduced by permission of the National Electrical Manufacturers Association from *NEMA Standards Publication MG 1-1998, Motors & Generators*. Copyright 1999 by NEMA, Washington, DC.
[b] Where a Class F or H insulation system is used, special consideration should be given to bearing temperatures, lubrication, etc. (Footnote approved as Authorized Engineering Information.)
[c] This column applies to polyphase induction motors only.

Enclosure

The *enclosure* (TEFC in Figure 5.11) indicates that the motor is totally enclosed and is fan cooled by an external fan mounted on the motor shaft.

Code Letter

The *code letter* provides a means for determining the expected locked-rotor *in-rush current* to the stator when starting the motor with rated voltage and rated frequency applied directly to the stator terminals. The code letter directs the reader to a table of locked-rotor kVA per horsepower, from which the in-rush current may be calculated (see Table 5.9).

5.12 LOCKED-ROTOR IN-RUSH CURRENT

At locked rotor each phase of an induction motor acts as a simple R–L series circuit. Closing the switch to such a circuit causes a combined transient and steady-state current [3].

TABLE 5.9

NEMA code letters for locked-rotor kVA per horsepower[a]

Code Letter	kVA/hp[b]	Code Letter	kVA/hp[b]
A	0.0–3.15	K	8.0–9.0
B	3.15–3.55	L	9.0–10.0
C	3.55–4.0	M	10.0–11.2
D	4.0–4.5	N	11.2–12.5
E	4.5–5.0	P	12.5–14.0
F	5.0–5.6	R	14.0–16.0
G	5.6–6.3	S	16.0–18.0
H	6.3–7.1	T	18.0–20.0
J	7.1–8.0	U	20.0–22.4
		V	22.4 and up

[a]Reprinted by permission of the National Electrical Manufacturers Association from *NEMA Standards Publication MG 1-1998, Motors & Generators*. Copyright 1999 by NEMA, Washington, DC.

[b]Locked kVA per horsepower range includes the lower figure up to, but not including, the higher figure. For example, 3.14 is designated by letter A, and 3.15 by letter B.

Thus, the locked-rotor in-rush current to an induction motor consists of a steady-state component ($i_{lr,ss}$), called the *normal in-rush current*, and a transient component ($i_{lr,tr}$) that decays to insignificance in a very short time. Unless otherwise specified, the term *locked-rotor current* always refers to the steady-state component.

The steady-state component for a specific motor may be obtained from the manufacturer or approximated from data usually stamped on the motor nameplate. The steady-state component may also be calculated from the machine parameters and the rated voltage as outlined in Section 5.4. Using machine parameters and solving on a per-phase basis,

$$\mathbf{I}_{lr,ss} = \left[\frac{\mathbf{V}_{phase}}{\mathbf{Z}_{in}}\right]_{s=1.0} \qquad (5\text{–}30)$$

Expressed in the time domain as a sinusoidal current,

$$i_{lr,ss} = \sqrt{2}\left|\frac{\mathbf{V}_{phase}}{\mathbf{Z}_{in}}\right|_{s=1.0} \sin(2\pi ft - \theta_z) \qquad (5\text{–}31)$$

The transient component of locked-rotor in-rush current per phase may be approximated by the following exponential function:

$$i_{lr,tr} = A\varepsilon^{-(R/L)t} \qquad (5\text{–}32)$$

where: $R = Z_{in}\cos\theta_z$

$L = (Z_{in}\sin\theta_z)/(2\pi f)$

A = coefficient whose value depends on the magnitude and phase angle of the applied voltage at the instant the switch is closed:

$$Z_{in}\angle\theta_z = Z_{in}\cos\theta_z + jZ_{in}\sin\theta_z = R + jX_L$$

Combining the steady-state component and the transient component,

$$i_{lr} = i_{lr,ss} + i_{lr,tr} \qquad (5\text{–}33)$$

$$i_{lr} = \sqrt{2}\left|\frac{\mathbf{V}_{phase}}{\mathbf{Z}_{in}}\right|_{s=1.0} \sin(2\pi ft - \theta_z) + A\varepsilon^{-(R/L)t} \qquad (5\text{–}34)$$

Since the three-phase voltages are 120° apart, their voltage zeros occur at different instants of time. Hence, the transient behavior indicated by Eq. (5–34) is per phase.

If the motor is at rest and the switch is closed at the instant that the applied voltage has its maximum value, the transient component of locked-rotor current will be zero. If the switch is closed at the instant that the voltage wave is passing through zero, the transient component will be a maximum, as shown in Figure 5.12 for one phase of a representative motor.[9]

In the preceding discussion of locked-rotor in-rush current, and in Figure 5.12, it was assumed that the rotor was blocked and could not rotate. For normal motor operation, with relatively low inertia loads, however, the decrease in slip during acceleration would cause a rapid reduction in locked-rotor current, decaying to rated current at rated load in 2 s to 5 s or less.

[9]See Reference [3] for transients in driven systems.

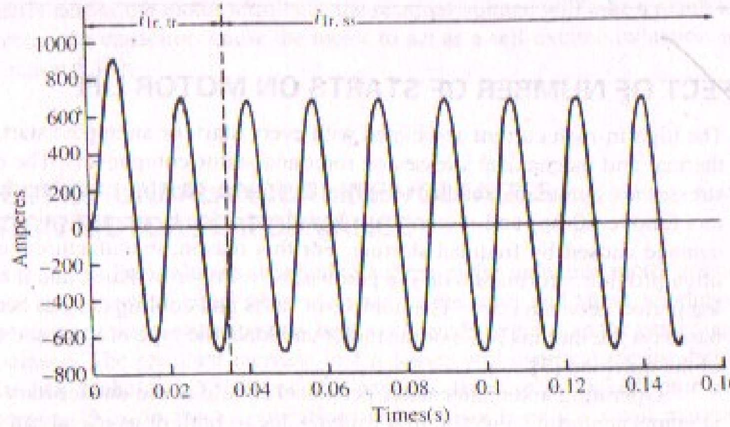


FIGURE 5.12

Locked-rotor in-rush current

E For the motor whose nameplate is shown in Figure 5.11, determine the expected in-rush current.

Solution

The motor is a NEMA design *B* machine rated at 150 hp at 460 V, 60 Hz. It has a rated current of 163 A, a nominal efficiency of 96.2 percent, and a Code G designation for locked-rotor kVA/hp. Referring to Table 5.9, the expected range of starting kVA/hp for a Code G machine is

$$5.6 \le \text{kVA/hp} < 6.3$$

The expected range of locked-rotor current can be determined from the following apparent-power equation:

$$\text{kVA/hp} \times \text{hp} \times 1000 = \sqrt{3} \times V_{\text{line}} \times I_{\text{line}}$$

Thus, for the lower limit:

$$5.6 \times 150 \times 1000 = \sqrt{3} \times 460 I_{\text{line}}$$

$$I_{\text{lr,ss}} = 1054 \text{ A}$$

For the upper limit:

$$6.3 \times 150 \times 1000 = \sqrt{3} \times 460 I_{\text{line}}$$

$$I_{\text{lr,ss}} = 1186 \text{ A}$$

Thus, the expected range of in-rush current at locked rotor, with rated voltage and rated frequency applied to the stator, is

$$1054 \text{ A} \le I_{\text{lr,ss}} < 1186 \text{ A}$$

This is a lot of current for a motor whose full-load rating is 163 A.

EFFECT OF NUMBER OF STARTS ON MOTOR LIFE

The high in-rush current associated with every start, or attempted start, causes severe thermal and mechanical stresses on rotor and stator components. The effects of these stresses are cumulative and adversely affect the service life of the machine. Large motors (above 50 hp) and motors with high inertia loads are particularly susceptible to damage caused by frequent starting. For this reason, manufacturers of large motors often provide information on the permissible number of starts, and the required waiting period between starts. The number of starts and cooling periods between starts are based on the inertia (WK^2) of the motor and load, the type of load, and the temperature of the machine [9].

Operating and maintenance personnel should avoid unnecessary starts: repeated attempts at starting a motor after tripping due to fault or overload can cause extensive damage to the machine. The reason for tripping or failure to start must be determined and corrected before attempting a restart [4].

5.14 RECLOSING OUT-OF-PHASE SCENARIO

When the stator of an induction motor is disconnected from the line, the closed circuit formed by the rotor bars and end rings prevents the quick collapse of flux. The flux in the revolving rotor induces a three-phase voltage in the stator windings that appears at the stator terminals. This *residual voltage* decays at a rate determined by the open-circuit time constant of the motor. In an elapsed time equal to one time constant, the residual voltage will decay to 36.8 percent of line voltage. Open-circuit time constants of 0.3 s are not uncommon.

If after a power interruption the motor is reconnected to the power line, with the residual voltage out of phase with the line voltage, the in-rush current will be higher than if the motor were started from a stopped position. The worst possible condition would be reclosure immediately after a power interruption, with the residual voltage almost equal to the line voltage and nearly 180° out of phase; the in-rush current would be almost double the locked-rotor value, and severe damage to the motor could occur. If the machine is large, a power blackout may occur.

Reclosing with high residual voltage and out of phase is likely to occur when switching rapidly from a failed power supply to a standby or emergency supply. Rapid and safe reclosing to alternate power supplies may be accomplished with an in-phase monitor, which measures the phase angle between the source voltage and the motor residual voltage, and initiates reclosure when the phase angle approaches zero.

Permanently connecting a voltmeter across the motor terminals and manually reclosing when the residual voltage drops below 20 percent or less of rated voltage is another method that will enable a relatively quick restart while avoiding an abnormally high in-rush current. If reclosed at 20 percent voltage and 180° out of phase, the net voltage applied to the stator windings will be only 120 percent rated voltage. *Note:* In those applications where capacitors for power-factor improvement are connected directly across the motor terminals, the residual voltage will take a much longer time to decay; the capacitors cause the motor to act as a self-excited induction generator (see Section 5.18).

5.15 EFFECT OF UNBALANCED LINE VOLTAGES ON INDUCTION MOTOR PERFORMANCE

If the three line voltages supplied to a three-phase induction motor are not equal, they not only cause unequal phase currents in the rotor and stator windings but the percentage current unbalance may be 6 to 10 times larger than the percentage voltage unbalance. The resultant increase in I^2R losses will overheat the insulation, shortening its life. Unbalanced voltages also cause a decrease in locked-rotor and breakdown torque. Thus, in those applications where there is only a small margin between the locked-rotor torque and the load torque, severe voltage unbalance may prevent the motor from starting. The full-load speed of running motors is reduced slightly by voltage unbalance.

Percent voltage unbalance is defined by NEMA as the maximum line voltage deviation from the average value of the three line voltages, times 100. Expressed as an equation,

$$\%\text{UBV} = \frac{V_{\text{max dev}}}{V_{\text{avg}}} \cdot 100 \tag{5–35}$$

where: $\%\text{UBV}$ = percent unbalanced voltage
$V_{\text{avg}} = (V_1 + V_2 + V_3)/3$
$V_{\text{max dev.}}$ = maximum volt deviation between a line voltage and V_{avg}

Note: Voltage measurements, taken for the purpose of determining voltage unbalance, should be made as close as possible to the motor terminals, and the readings should be taken with a digital voltmeter for greater accuracy.

Empirical results obtained from laboratory tests indicate that the percentage increase in motor temperature, due to voltage unbalance, is approximately equal to two times the square of the percentage voltage unbalance [2]. Expressed as an equation,

$$\%\Delta T \cong 2(\%\text{UBV})^2 \tag{5–36}$$

where $\%\Delta T$ = percent increase in motor temperature. Thus, assuming operation at rated load, the expected temperature rise caused by voltage unbalance is

$$T_{\text{UBV}} \cong T_{\text{rated}} \cdot \left(1 + \frac{\%\Delta T}{100}\right) \tag{5–37}$$

where: T_{rated} = expected temperature rise from Table 5.8 (C°)
T_{UBV} = expected temperature rise due to voltage unbalance (C°)

The effect of higher operating temperatures on the life of electrical insulation may be approximated by the *ten-degree rule*. This rule, developed by A. M. Montsinger in a classic study, demonstrated that insulation life is approximately halved with each 10°C increase in temperature, and conversely is doubled with each 10°C decrease in temperature [1]. Although not precise, this ten-degree rule has been substantiated over time, and does provide a rough approximation of the effect of motor temperature on insulation life. Expressing the ten-degree rule in equation form,

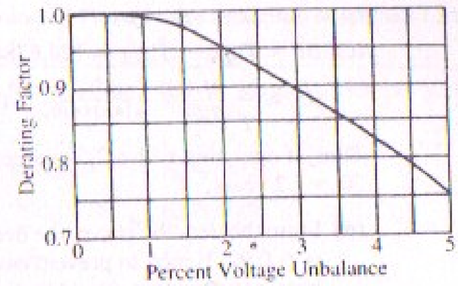
$$\text{RL} \cong \frac{1}{2^{(\delta T/10)}} \tag{5–38}$$

where: RL = relative life of insulation
$\delta T = T_{\text{UBV}} - T_{\text{rated}}$

Thus, a motor with an expected life of 20 years operating with the same rated load, but with a phase unbalance that causes $\delta T = 15°\text{C}$, would, from Eq. (5–38), have a relative life of RL = 0.35. The expected life of the insulation, assuming continued operation at the higher temperature, would be 0.35 × 20 = 7 years.

In those applications where voltage unbalance cannot be corrected, the motor should be *derated*, that is, operated at a lower horsepower. The derating curve shown

FIGURE 5.13
Derating curve for induction motors. (Courtesy National Electrical Manufacturers Association. From *NEMA Standards Publication MG1-1998*. Copyright 1999 by NEMA)



in Figure 5.13 should be used to determine the required derating [9]. A 1 percent unbalance will not cause significant problems. Operating a motor with a voltage unbalance greater than 5 percent is not recommended, however.

EXAMPLE 5.14 A 30-hp, design *B*, 460-V, 60-Hz, four-pole, totally enclosed nonventilated induction motor with Class F insulation, and a service factor of 1.0, is to be operated at rated power from an unbalanced three-phase system. The expected life of the motor under normal operating conditions is 20 years. If the three line-to-line voltages are 460 V, 455 V, and 440 V, determine (a) the percent voltage unbalance; (b) the expected approximate temperature rise if operating at rated load in a 40°C ambient; (c) the expected insulation life; (d) the required derating of the motor to prevent shortening insulation life.

Solution

(a) $$V_{\text{avg}} = \frac{460 + 455 + 440}{3} = 451.667\text{ V}$$

The voltage deviations from the average are:

$$|460 - 451.667| = 8.333\text{ V}$$
$$|455 - 451.667| = 3.333\text{ V}$$
$$|440 - 451.667| = 11.333\text{ V}$$

$$\%\text{UBV} = \frac{V_{\text{max dev}}}{V_{\text{avg}}} \cdot 100 = \frac{11.667}{451.667} \cdot 100 = 2.5831 \quad \Rightarrow \quad \underline{2.58}$$

(b) $$\%\Delta T \cong 2(\%\text{UBV})^2 = 2(2.5831)^2 = 13.344 \quad \Rightarrow \quad \underline{13.34}$$

The rated temperature rise (from Table 5.8) for a totally enclosed nonventilated motor with Class F insulation and 1.0 service factor is 110°C. Thus,

$$T_{\text{UBV}} \cong T_{\text{rated}} \cdot \left(1 + \frac{\%\Delta T}{100}\right) = 110\left(1 + \frac{13.344}{100}\right) = 124.6784 \quad \Rightarrow \quad \underline{125°\text{C}}$$

(c) $\delta T = T_{UBV} - T_{rated} = 124.6784 - 110 = 14.6784°C$

$$RL \cong \frac{1}{2^{(\delta T/10)}} = \frac{1}{2^{(14.6784/10)}} = 0.361$$

Thus, if operating with a 2.58 percent voltage unbalance, the expected life is $0.361 \times 20 \approx$ 7.2 years.

(d) From the derating curve, the derating factor for a 2.58 percent voltage unbalance is ≈ 0.92. Hence, to prevent excessive heating and a shortened life, the shaft horsepower output should be limited to $30 \times 0.92 =$ 27.6 hp.

R-UNIT VALUES OF INDUCTION-MOTOR ERS

Manufacturers of electrical machinery often publish induction-motor parameters as per-unit impedance values instead of actual impedance values. Per-unit values are useful in machine design in that a wide range of machine sizes have approximately the same values. This makes it easy to detect gross errors in design calculations and also provides a convenient means for comparing the relative performance of machines without regard to machine size. For example, machines with higher per-unit rotor resistance will have $T_{D,max}$ occur at a greater slip.

Per-unit values of induction-motor parameters are defined as a ratio of the actual value of the respective parameters divided by a common base impedance. The base impedance for induction motors is calculated from *output power* rather than input power to avoid having to make assumptions of power factor and efficiency [6].

Defining base values for a three-phase induction motor:

P_{base} = base power/phase = rated shaft power output ÷ 3 (W)
V_{base} = base voltage = rated voltage/phase (V)
I_{base} = base current = rated current/phase (A)

$$I_{base} = \frac{P_{base}}{V_{base}} \tag{5–39}$$

$$Z_{base} = \frac{V_{base}}{I_{base}} \tag{5–40}$$

Expressing the motor parameters in terms of per-unit values:

$$\left.\begin{array}{lll} r_1 = \dfrac{R_1}{Z_{base}} & r_2 = \dfrac{R_2}{Z_{base}} & r_{fe} = \dfrac{R_{fe}}{Z_{base}} \\ x_1 = \dfrac{X_1}{Z_{base}} & x_2 = \dfrac{X_2}{Z_{base}} & x_M = \dfrac{X_M}{Z_{base}} \end{array}\right\} \tag{5–41}$$

Note: Z_{base} may also be expressed in terms of P_{base} as derived below:

$$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{V_{base}}{P_{base}/V_{base}} = \frac{V_{base}^2}{P_{base}} \tag{5–41a}$$

EXAMPLE 5.15 Given the following per-unit values for a 50-hp, 460-V, six-pole, 60-Hz, wye-connected induction motor:

$$r_1 = 0.021 \quad r_2 = 0.020 \quad r_{fe} = 20.0$$
$$x_1 = 0.100 \quad x_2 = 0.0178 \quad x_M = 3.68$$

Determine the machine parameters in ohms.

Solution

$$V_{base} = \frac{460}{\sqrt{3}} = 265.58 \text{ V}$$

$$P_{base} = 50 \times 746 \div 3 = 12{,}433.33 \text{ W}$$

$$Z_{base} = \frac{V_{base}^2}{P_{base}} = \frac{(265.58)^2}{12{,}433.33} = 5.67\ \Omega$$

From equation set (5–41),

$$R_1 = r_1 Z_{base} = 0.021(5.67) = \underline{0.119\ \Omega}$$
$$X_1 = x_1 Z_{base} = 0.100(5.67) = \underline{0.567\ \Omega}$$
$$R_2 = r_2 Z_{base} = 0.020(5.67) = \underline{0.113\ \Omega}$$
$$X_2 = x_2 Z_{base} = 0.0178(5.67) = \underline{0.101\ \Omega}$$
$$R_{fe} = r_{fe} Z_{base} = 20.0(5.67) = \underline{113.40\ \Omega}$$
$$X_M = x_M Z_{base} = 3.68(5.67) = \underline{20.87\ \Omega}$$

5.17 DETERMINATION OF INDUCTION-MOTOR PARAMETERS

In those cases where induction motor parameters are not readily available from the manufacturer, they can be approximated from a DC test, a no-load test, and a blocked-rotor test [7].

DC Test

The purpose of the DC test is to determine R_1. This is accomplished by connecting any two stator leads to a variable-voltage DC source as shown in Figure 5.14(a). The DC source is adjusted to provide approximately rated stator current, and the resistance between the two stator leads is determined from voltmeter and ammeter readings. Thus, from Figure 5.14(a),

$$R_{DC} = \frac{V_{DC}}{I_{DC}}$$

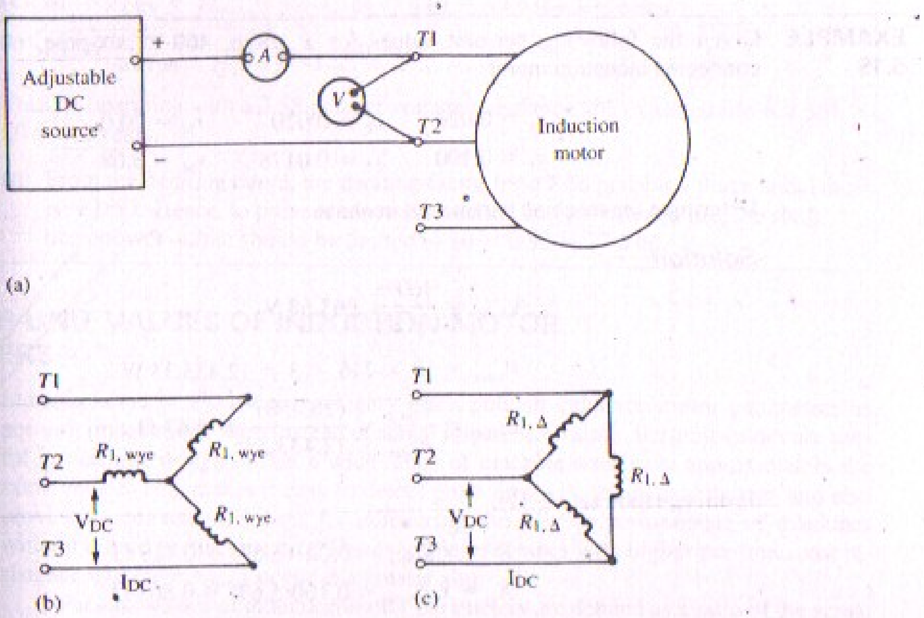


FIGURE 5.14
Circuits for DC test to determine parameter R_1.

If the stator is wye connected,[10] as shown in Figure 5.14(b),

$$R_{DC} = 2R_{1,\text{wye}}$$

$$R_{1,\text{wye}} = \frac{R_{DC}}{2} \tag{5-42}$$

If the stator is delta connected, as shown in Figure 5.14(c),

$$R_{DC} = \frac{R_{1\Delta} \cdot 2R_{1\Delta}}{R_{1\Delta} + 2R_{1\Delta}} = \frac{2}{3}R_{1\Delta}$$

$$R_{1\Delta} = 1.5R_{DC} \tag{5-43}$$

Blocked-Rotor Test

The blocked-rotor test is used to determine X_1 and X_2. When combined with data from the DC test, it also determines R_2.

The test is performed by blocking the rotor so that it cannot turn, and measuring the line voltage, line current, and three-phase power input to the stator. Connections for the test are shown in Figure 5.15. An adjustable voltage AC supply (not shown) is

[10] If there is no indication as to whether the connections are wye or delta, assume an equivalent wye connection and proceed as outlined.

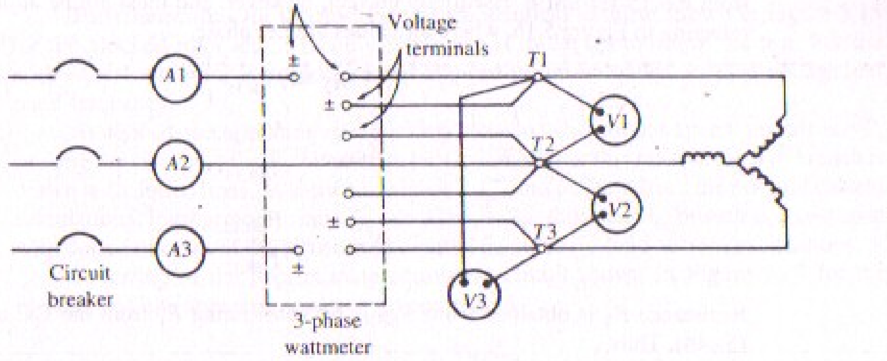


FIGURE 5.15
Basic circuit for blocked-rotor test and no-load test.

used to adjust the blocked-rotor current to approximately rated current. If instrument transformers and single-phase wattmeters are used, the effect of transformer ratios and the direction of the wattmeter readings (whether positive or negative) must be considered [3].

Since the exciting current (I_0) at blocked rotor is considerably less than the rotor current (I_2), the exciting current may be neglected, enabling a simplification of the equivalent circuit. This is shown in Figure 5.16, where X_M and R_{fe} are drawn with dotted lines and omitted when making blocked-rotor calculations.

To minimize errors that would otherwise be introduced by magnetic saturation and resistance skin effect, if tested at rated voltage and rated frequency, *the IEEE test code recommends that the blocked-rotor test be made using 25 percent rated frequency with the test voltage adjusted to obtain approximately rated current* [7]. Thus, a 60-Hz motor would use a 15-Hz test voltage. The total reactance calculated from the 15-Hz test is then corrected to 60 Hz by multiplying by 60/15. The total resistance calculated

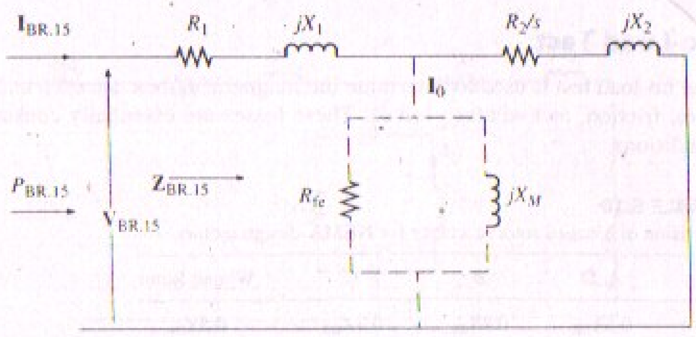


FIGURE 5.16
Equivalent circuit per phase for blocked-rotor test.

from the 15-Hz test is essentially correct, however, and must not be adjusted. Thus, referring to Figure 5.16, where all values are per phase,

$$R_1 + R_2 = R_{BR,15} \tag{5–44}$$

$$Z_{BR,15} = \frac{V_{BR,15}}{I_{BR,15}} \tag{5–45}$$

$$R_{BR,15} = \frac{P_{BR,15}}{I^2_{BR,15}} \tag{5–46}$$

Resistance R_2 is obtained from $R_{BR,15}$ by substituting R_1 from the DC test into Eq. (5–44). Thus,

$$R_2 = R_{BR,15} - R_1 \tag{5–47}$$

From Figure 5.16,

$$Z_{BR,15} = \sqrt{R^2_{BR,15} + X^2_{BR,15}}$$

$$X_{BR,15} = \sqrt{Z^2_{BR,15} - R^2_{BR,15}} \tag{5–48}$$

Converting $X_{BR,15}$ to 60 Hz,

$$X_{BR,60} = \frac{60}{15} X_{BR,15} \tag{5–49}$$

where:

$$X_{BR,60} = X_1 + X_2 \tag{5–50}$$

If the NEMA-design letter of the induction motor is known, the division of blocked-rotor reactance between X_1 and X_2, as shown in Eq. (5–50), may be determined from Table 5.10 [7]. If the NEMA design letter is not known, an equal division between X_1 and X_2 is generally assumed.

No-Load Test

The no-load test is used to determine the magnetizing reactance X_M and the combined core, friction, and windage losses. These losses are essentially constant for all load conditions.

TABLE 5.10
Division of blocked-rotor reactance for NEMA-design motors

	A, D	B	C	Wound Rotor
X_1	$0.5X_{BR}$	$0.4X_{BR}$	$0.3X_{BR}$	$0.5X_{BR}$
X_2	$0.5X_{BR}$	$0.6X_{BR}$	$0.7X_{BR}$	$0.5X_{BR}$

The connections for the no-load test are identical to those shown in Figure 5.15 for the blocked-rotor test. The only difference is in the operation of the test: For the no-load test, the rotor is unblocked and allowed to run unloaded at rated voltage and rated frequency.

At no load, the operating speed is very close to synchronous speed; the slip is ≈ 0, causing the current in the R_2/s branch to be very small. For this reason, the R_2/s branch is drawn with dotted lines, as shown in Figure 5.17, and omitted from the no-load current calculations. Furthermore, since $I_M >> I_{fe}$, $I_0 \approx I_M$; thus, the R_{fe} branch is also drawn with dotted lines, as in the figure, and omitted from the no-load current calculations.

Referring to the approximate equivalent circuit shown in Figure 5.17 for the no-load test, the apparent power input per phase is

$$S_{NL} = V_{NL} I_{NL}$$

The reactive power per phase is determined from

$$S_{NL} = \sqrt{P^2_{NL} + Q^2_{NL}} \tag{5–51}$$

Solving for Q_{NL},

$$Q_{NL} = \sqrt{S^2_{NL} - P^2_{NL}} \tag{5–52}$$

Expressing the reactive power in terms of current and reactance, and solving for the equivalent reactance at no load,

$$Q_{NL} = I^2_{NL} X_{NL} \tag{5–53}$$

Thus,

$$X_{NL} = \frac{Q_{NL}}{I^2_{NL}} \tag{5–54}$$

where, as indicated in Figure 5.17,

$$X_{NL} = X_1 + X_M \tag{5–55}$$

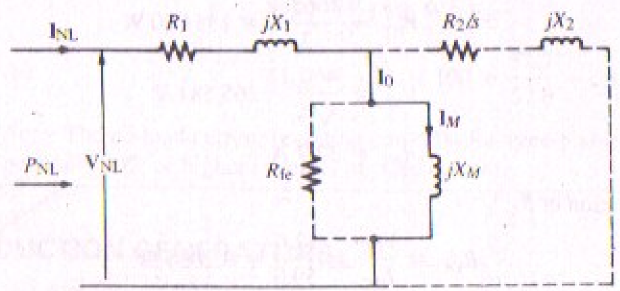


FIGURE 5.17
Equivalent circuit per phase for no-load test.

Substituting X_1 as determined from the blocked-rotor test. into Eq. (5–55), permits the determination of X_M.

The input power per phase at no load includes the core loss, stator copper loss, windage loss, and friction loss (all per phase). That is,

$$P_{NL} = I_{NL}^2 R_1 + P_{core} + P_{w,f} \tag{5–56}$$

Separation of friction and windage losses from the no-load loss may be accomplished by plotting the no-load power vs. voltage squared for low values of voltage and then extrapolating to zero voltage [7].

EXAMPLE 5.16 The following data were obtained from no-load, blocked-rotor, and DC tests of a three-phase, wye-connected, 40-hp, 60-Hz, 460-V, design *B* induction motor whose rated current is 57.8 A. The blocked-rotor test was made at 15 Hz.

Blocked Rotor	No-Load	DC
$V_{line} = 36.2$ V	$V_{line} = 460.0$ V	$V_{DC} = 12.0$ V
$I_{line} = 58.0$ A	$I_{line} = 32.7$ A	$I_{DC} = 59.0$ A
$P_{3\,phase} = 2573.4$ W	$P_{3\,phase} = 4664.4$ W	

(a) Determine R_1, X_1, R_2, X_2, X_M, and the combined core, friction, and windage loss.

(b) Express the no-load current as a percent of rated current.

Solution

(a) Converting the AC test data to corresponding phase values for a wye-connected motor,

$$P_{BR,15} = \frac{2573.4}{3} = 857.80 \text{ W}$$

$$V_{BR,15} = \frac{36.2}{\sqrt{3}} = 20.90 \text{ V}$$

$$I_{BR,15} = 58.0 \text{ A}$$

$$P_{NL} = \frac{4664.4}{3} = 1554.80 \text{ W}$$

$$V_{NL} = \frac{460}{\sqrt{3}} = 265.581 \text{ V}$$

$$I_{NL} = 32.7 \text{ A}$$

Determination of R_1:

$$R_{DC} = \frac{V_{DC}}{I_{DC}} = \frac{12.0}{59.0} = 0.2034\ \Omega$$

$$R_{1,wye} = \frac{R_{DC}}{2} = \frac{0.2034}{2} = \underline{0.102\ \Omega/\text{phase}}$$

Determination of R_2:

$$Z_{BR,15} = \frac{V_{BR,15}}{I_{BR,15}} = \frac{20.90}{58.0} = 0.3603\ \Omega$$

$$R_{BR,15} = \frac{P_{BR,15}}{I_{BR,15}^2} = \frac{857.8}{(58)^2} = 0.2550\ \Omega/\text{phase}$$

$$R_2 = R_{BR,15} - R_{1,wye} = 0.2550 - 0.102 = \underline{0.153\ \Omega/\text{phase}}$$

Determination of X_1 and X_2:

$$X_{BR,15} = \sqrt{Z_{BR,15}^2 - R_{BR,15}^2} = \sqrt{(0.3603)^2 - (0.255)^2} = 0.2545\ \Omega$$

$$X_{BR,60} = \frac{60}{15} X_{BR,15} = \frac{60}{15}(0.2545) = 1.0182\ \Omega$$

From Table 5.10, for a design *B* machine,

$$X_1 = 0.4X_{BR,60} = 0.4(1.0182) = \underline{0.4073\ \Omega/\text{phase}}$$

$$X_2 = 0.6X_{BR,60} = 0.6(1.0182) = \underline{0.6109\ \Omega/\text{phase}}$$

Determination of X_M:

$$S_{NL} = V_{NL} I_{NL} = 265.581(32.7) = 8684.50 \text{ VA}$$

$$Q_{NL} = \sqrt{S_{NL}^2 - P_{NL}^2} = \sqrt{(8684.50)^2 - (1554.8)^2} = 8544.19 \text{ var}$$

$$X_{NL} = \frac{Q_{NL}}{I_{NL}^2} = \frac{8544.19}{(32.7)^2} = 7.99\ \Omega$$

$$X_{NL} = X_1 + X_M \quad \Rightarrow \quad 7.99 = 0.4073 + X_M$$

$$X_M = \underline{7.58\ \Omega/\text{phase}}$$

Determination of combined friction, windage, and core loss:

$$P_{NL} = I_{NL}^2 R_{1,wye} + P_{core} + P_{f,w}$$

$$1554.8 = (32.7)^2(0.102) + P_{core} + P_{f,w}$$

$$P_{core} + P_{f,w} = \underline{1446 \text{ W/phase}}$$

(b) $$\%I_{NL} = \frac{I_{NL}}{I_{rated}} \times 100 = \frac{32.7}{57.8} = \underline{56.6\%}$$

Note: The no-load current (exciting current) of a three-phase induction motor is large, generally 40% or higher in terms of rated current.

5.18 INDUCTION GENERATORS

Induction generators have the same basic construction as squirrel-cage induction motors. In fact, all induction motors can be operated very effectively as induction generators by driving them at a speed greater than synchronous speed. In induction generator applications, however, where the machine is not started as a motor, and hence does not require a high starting torque, induction generators are generally designed with lower resistance values to provide a lower slip and a higher efficiency at rated load.

Induction generators are suitable for operation by wind turbines, hydraulic turbines, steam turbines, and gas engines powered by natural gas or biogas. They can range in size from a few kilowatts to 10 MW or higher, and are used extensively in cogeneration operations. *Cogeneration* is the sequential production of two forms of energy, usually steam for process operations and electricity for plant use and for sale to utilities.

Motor-to-Generator Transition

Figure 5.18 shows an induction motor connected to a three-phase system, with its shaft mechanically coupled to a steam turbine. Assume the turbine valve is closed so that no steam enters the turbine, and the motor is started at full voltage by closing the circuit breaker. The induction motor accelerates and drives the turbine at a speed somewhat less than the synchronous speed of the rotating stator flux. Gradually opening the turbine valve causes a gradual buildup of turbine torque, adding to that developed by the induction motor, resulting in an increase in rotor speed. When the speed of the turbine-motor set reaches the synchronous speed of the stator, the slip becomes zero, R_2/s becomes infinite, rotor current I_2 becomes zero, and no motor torque is developed. At zero slip, the induction machine is neither a motor nor a generator; it is "floating" on the bus. The only current in the stator is the exciting current that supplies the rotating magnetic field and the iron losses.

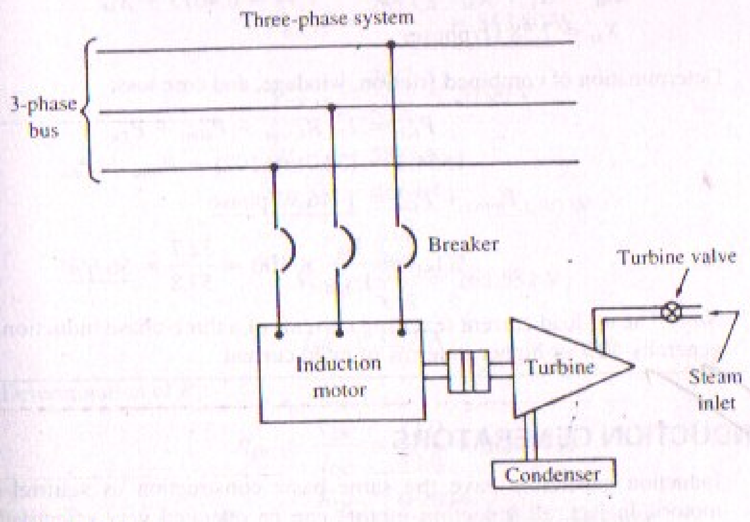


FIGURE 5.18
Induction motor coupled to a turbine for induction-generator operation.

Prime movers, such as turbines, diesels, and waterwheels, can only supply active power. Hence, when connected to an energized system, the magnetizing current I_M is supplied by the power system whether the machine operates as a motor or a generator.

The speed of the rotating flux is independent of the rotor speed; it is a function of the number of stator poles and the frequency of the power system to which the stator is connected. *Increasing the rotor speed above synchronous by increasing the energy input to the turbine will cause the slip to become negative*, reversing the direction of the air-gap power ($P_{gap} = P_{rcl}/s$). Thus, instead of active power being transferred across the air gap from the stator to the rotor, as in motor action, a negative slip causes the power to be transferred in the reverse direction, from the rotor to the stator, and on into the distribution system.

Figure 5.19 shows plots of air-gap power, developed torque, and line current vs. rotor speed for induction-machine operation, as a motor below synchronous speed and as a generator above synchronous speed. The curves were plotted from calculated data using the four-pole, 40-hp, 460-V, 60-Hz motor in Example 5.3. Note the smooth transition of power from motor action to generator action as the shaft speed is raised above synchronous speed.

When operating as an induction generator, the interaction of the magnetic flux of the stator and the magnetic flux of the rotor produce a countertorque in opposition to the driving torque of the prime mover. As the speed of the prime mover is increased, the increase in electrical power supplied to the distribution system by the induction generator causes an increase in countertorque. The countertorque increases with increasing speed until it attains a maximum value called the *pushover torque*. The pushover torque when operating as an induction generator corresponds to the breakdown torque when operating as an induction motor.

As shown in Figure 5.19, increasing the prime-mover speed beyond the pushover point causes the power output to decrease. This decreases the load (countertorque) on the prime mover, causing a rapid increase in speed. The resultant overspeed can damage both the induction machine and the prime mover. Automatic closure of the turbine valve is necessary to prevent damaging overspeed when overloading past the pushover point. A similar effect occurs when an induction generator is loaded and the breaker connecting the generator to the power system is tripped. The sudden loss of load causes the turbine to overspeed.

To assure some degree of protection against damage due to accidental overspeed, NEMA specifications require that squirrel-cage and wound-rotor motors (except crane motors) be so constructed that, in an emergency, they will be able to withstand without mechanical injury the respective overspeeds listed in Table 5.11 [9].

Induction-Generator Starting and Operation with Other Three-Phase Sources

Normal procedure for starting induction generators to be paralleled with the bus is to use the prime mover to accelerate the machine to synchronous speed or slightly above, and then close the circuit breaker. No special care is required, since it generates no measurable voltage until the breaker is closed; closing the breaker causes the three-phase bus to supply the magnetizing current necessary to establish the rotating field.

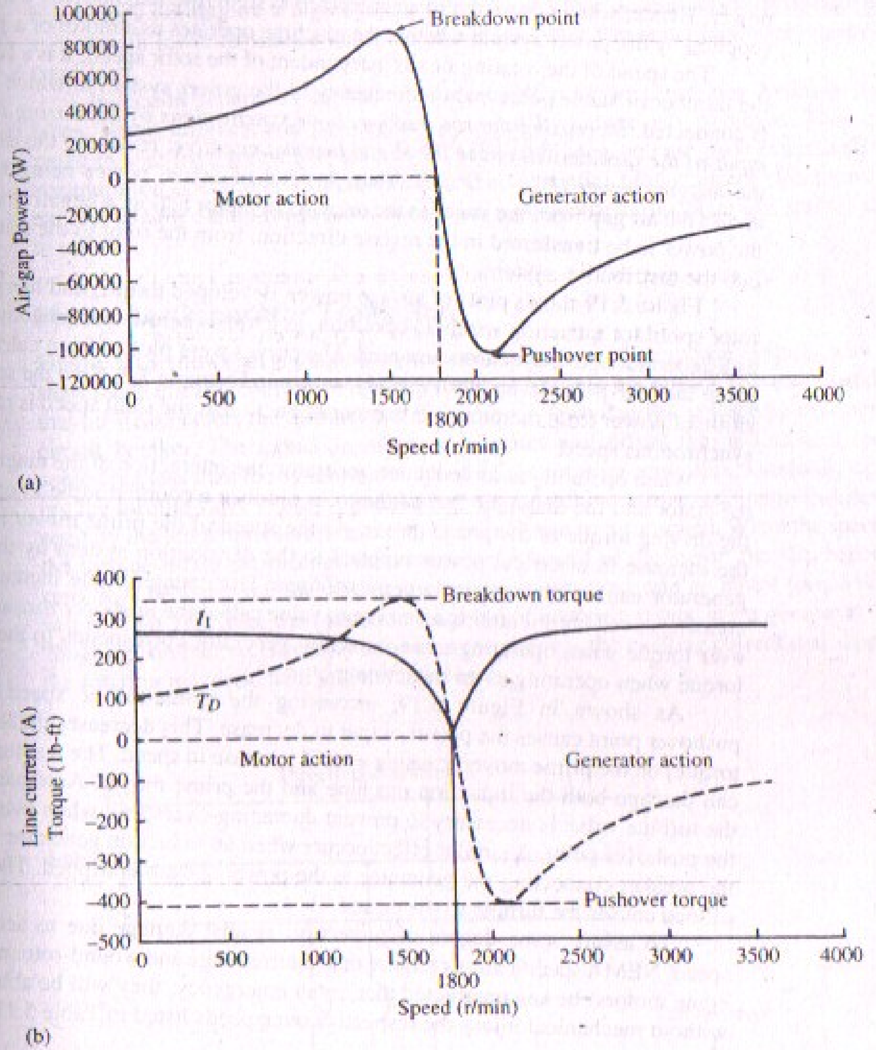


FIGURE 5.19
Plots of air-gap power, developed torque, and line current vs. rotor speed for induction-machine operation.

The voltage and frequency of an induction generator that is connected to an energized power system cannot be adjusted; its frequency and voltage are the respective frequency and voltage of the electrical system to which it is connected. The only control over the induction generator is load control through speed adjustment of the prime mover.

TABLE 5.11
Allowable emergency overspeed of squirrel-cage and wound-rotor motors

hp	Synchronous Speed (r/min)	Percent above Synchronous Speed
≤200	1201 and over	25
	1200 and below	50
250–500, inclusive	1801 and over	20
	1800 and below	25

EXAMPLE 5.17 A three-phase, six-pole, 460-V, 60-Hz induction motor rated at 15-hp, 1182 r/min i driven by a turbine at 1215 r/min, as shown in Figure 5.18. The equivalent circuit dia gram is shown in Figure 5.20 and the motor parameters (in ohms) are:

$$R_1 = 0.200 \quad R_2 = 0.250 \quad R_{fe} = 317$$
$$X_1 = 1.20 \quad X_2 = 1.29 \quad X_M = 42.0$$

Determine the active power that the motor, driven as an induction generator, delivers to the system. *Note:* This is the same machine driven as a motor in Example 5.2.

Solution

$$s = \frac{n_s - n_r}{n_s} = \frac{1200 - 1215}{1200} = -0.0125$$

$$Z_2 = \frac{R_2}{s} + jX_2 = -0.20 + j1.29 = 20.0416\underline{/176.30^\circ}\ \Omega$$

Note: The apparent equivalent resistance is negative ($R_2/s = -20$) when operating an induction generator.

$$Z_0 = \frac{R_{fe} \cdot jX_M}{R_{fe} + jX_M} = \frac{317(42.0\underline{/90^\circ})}{317 + j42.0} = 41.6361\underline{/82.4527^\circ} = 5.4687 + j41.2754$$

$$Z_P = \frac{Z_2 \cdot Z_0}{Z_2 + Z_0} = \frac{(20.0416\underline{/176.30^\circ})(41.6361\underline{/82.4527^\circ})}{(-0.20 + j1.29) + 5.4687 + j41.2754)}$$

$$Z_P = 18.5527\underline{/149.91^\circ} = -16.0530 + j9.301 \quad \Omega$$

$$Z_{in} = Z_1 + Z_P = (0.2 + j1.2) + (-16.0530 + j9.301) = 19.0153\underline{/146.4802^\circ}$$

$$I_1 = \frac{V}{Z_{in}} = \frac{460/\sqrt{3}\underline{/0^\circ}}{19.0153\underline{/146.4802^\circ}} = 13.9667\underline{/-146.4802^\circ} \quad \text{A}$$

$$S = 3VI_1^* = 3(460/\sqrt{3}\underline{/0^\circ})(13.9667\underline{/-146.4802^\circ})^* = 11{,}127.87\underline{/146.48^\circ}$$

$$S = -9277 + j6145 \quad \text{VA}$$

$$P = \underline{-9277\ \text{W}}$$

The negative sign indicates power flow *from* the induction machine *to* the electr distribution system.

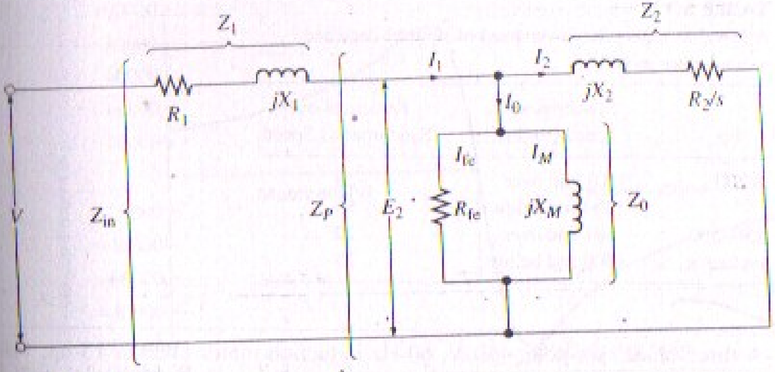


FIGURE 5.20
Equivalent circuit for Example 5.17.

Isolated-Generator Operation

The buildup of voltage in an induction generator that is isolated from other power sources requires the use of capacitors in parallel with the stator windings, as shown in Figure 5.21. The capacitors provide the magnetizing current that is necessary for the

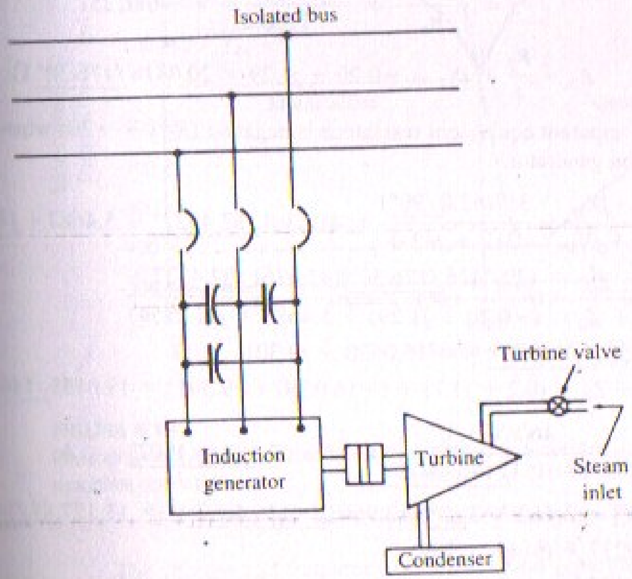


FIGURE 5.21
Capacitors in parallel with stator windings for single induction-generator operation.

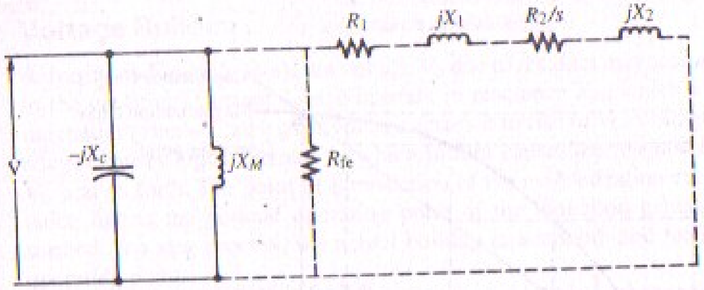


FIGURE 5.22
Approximate equivalent circuit for a self-excited induction generator.

buildup of a rotating magnetic field. Induction generators that do not depend on the power system to establish a rotating magnetic field are called *self-excited* generators.

The theory of voltage buildup is explained with the aid of the approximate equivalent-circuit diagram in Figure 5.22, which shows a capacitor connected across the output terminals. Resistance R_{fe}, shown with dotted lines, is relatively high compared to X_M and can be ignored, since it has very little effect on voltage buildup. Furthermore, with no load on the machine, the slip is zero, causing R_2/s to be infinite, and no current appears in the branch formed by R_1, jX_1, R_2/s, and jX_2, so this branch is also drawn with dotted lines and ignored.

When the prime mover is started, the residual magnetism[11] in the revolving rotor generates a low voltage in magnetizing reactance X_M. This low voltage appears across the capacitor, causing a small current in the circuit formed by X_M and the capacitor. The small current in X_M causes voltage $I_M X_M$ that appears at the terminals as output voltage V; this voltage is higher than the residual voltage, causing a higher capacitor current, which also appears in X_M, causing a further voltage buildup, and so forth.

The process of voltage buildup is illustrated in Figure 5.23(a), which shows the magnetization curve and capacitance line for a representative induction generator. The magnetization curve is a plot of voltage vs. current for magnetizing reactance X_M, and is obtained by running the induction machine as a motor with no load, as shown in Figure 5.23(b), and plotting applied voltage vs. current as the voltage is raised in steps from zero voltage to rated voltage. The nonlinearity of the magnetization curve is caused by magnetic saturation effects in the stator and rotor iron. The capacitance line is an Ohm's law plot of voltage vs. current for the capacitor when measured alone, as shown in Figure 5.23(c). For the capacitor line shown in Figure 5.23(a), $V = IX_C$ where X_C is the slope of the line.

[11] An induction generator that has lost its residual magnetism will not build up voltage if operating as a single generator. Switching on a capacitor bank with some residual voltage, however, or running the machine briefly as motor, will be sufficient to build up flux.

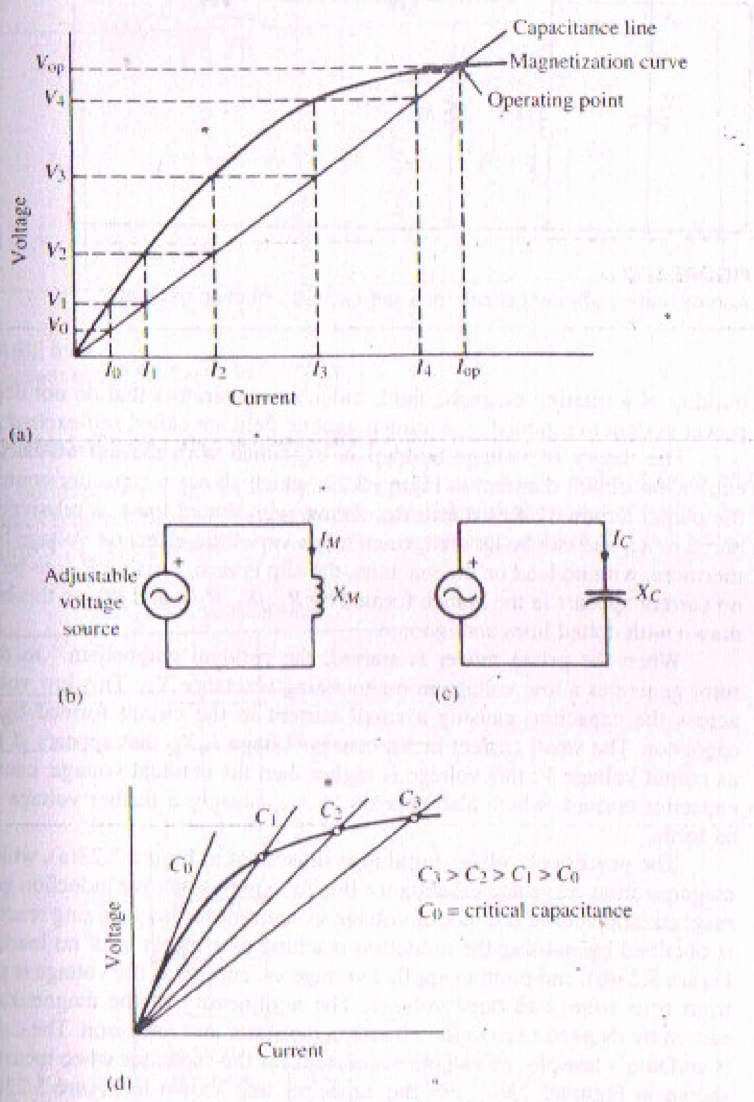


FIGURE 5.23
(a) Process of voltage buildup in a self-excited induction generator; (b) circuit for obtaining magnetization curve; (c) circuit for obtaining capacitance line; (d) operating points for several values of capacitance.

Voltage Buildup

Referring to Figure 5.23(a), the voltage V_0 due to residual magnetism causes current I_0 in the capacitor. Current I_0 also appears in reactance X_M, which as indicated on the magnetizing curve, causes the voltage across it to rise to V_1. Voltage V_1 across the capacitor causes higher current I_1, which in turn causes the reactance voltage to rise to V_2, and so forth. The point of intersection of the magnetization curve and the capacitance line is the no-load operating point of the induction generator. Although described as a step process, the actual buildup is a smooth and fairly rapid rise to the operating point.

Adjustment of the no-load voltage is accomplished by changing the slope of the capacitance line. Since X_C is the slope of the capacitance line, and $X_C = 1/(2\pi f C)$, increasing the capacitance decreases the slope and therefore increases the voltage. The operating points for several values of capacitance are shown in Figure 5.23(d). The value of capacitance that causes the capacitance line to be tangent to the magnetization curve is called the critical capacitance. Values of capacitance less than the critical capacitance C_0 will prevent the buildup of voltage.

The frequency of an induction generator when operating self-excited may be determined from the capacitive reactance at the operating point. Thus, from Figure 5.23(a) and Ohm's law,

$$X_C = \frac{V_{op}}{I_{op}} \quad \Rightarrow \quad \frac{1}{2\pi f C} = \frac{V_{op}}{I_{op}}$$

$$f = \frac{I_{op}}{V_{op}} \cdot \frac{1}{2\pi C} \tag{5-57}$$

The principal disadvantage of self-excited induction generators is the rapid falloff in voltage when load is applied, requiring significantly higher capacitance with higher and more lagging power-factor loads. Despite this drawback, however, self-excited induction generators are very effective for battery-charging systems and other applications where radical changes in terminal voltage are not disturbing to equipment operation.

5.19 DYNAMIC BRAKING OF INDUCTION MOTORS

Dynamic braking is the slowing down of a machine by converting the kinetic energy stored in the rotating mass to heat energy in the rotor and/or stator windings. To do this, the motor is switched from the line to a braking circuit that causes the motor to behave as a generator with a connected load; the load is the resistance of the rotor and/or stator windings. Since the only source of energy is the rotating parts of the induction motor and the driven equipment, the machine will slow down. Dynamic braking of induction motors may be accomplished by DC injection and/or capacitive braking. Note, however, that this type of braking does not provide holding torque at the end of the braking period. Hence, where required, as in hoists or other applications where rolling is not permitted after a stop, a mechanical brake must be used to hold the shaft.

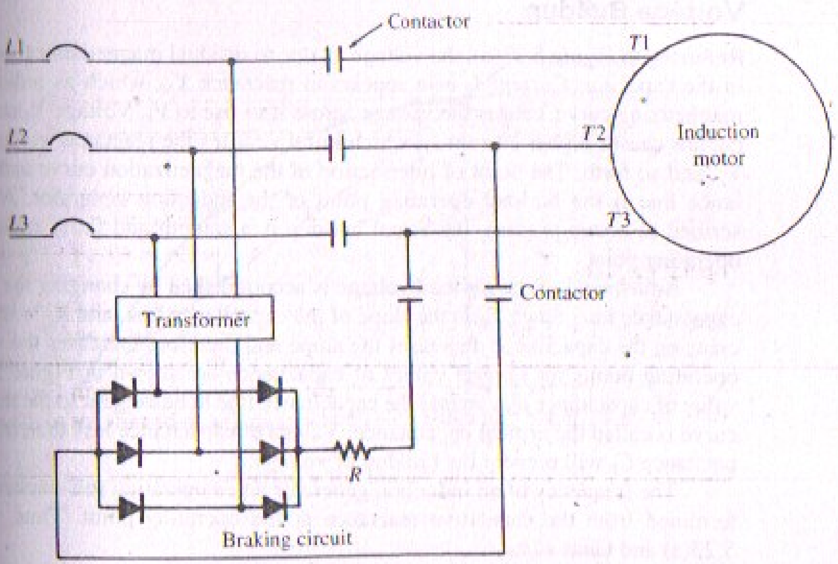


FIGURE 5.24
Dynamic braking using DC injection.

DC Injection

In DC injection the motor is disconnected from the line, and a DC source supplied by a rectifier is then connected to any two terminals of the stator through a current-limiting resistor, as shown in Figure 5.24. The direct current in the stator winding sets up a stationary magnetic field that generates a voltage in the windings of the spinning rotor. The resultant current in the closed loops formed by the squirrel cage (or wound rotor) dissipates the rotational energy as I^2R losses, rapidly slowing the rotor.

The rate of deceleration by DC injection may be adjusted by adjusting resistor R in Figure 5.24, by using a variable ratio transformer, or by using a thyristor (SCR) control circuit in place of the resistor **[10]**.

Capacitor Braking

In capacitor braking, the motor is disconnected from the line, and a capacitor bank is then connected to the stator terminals, as shown in Figure 5.25. When braking, the motor behaves as a self-excited induction generator as described in Section 5.18. During capacitive braking, the rotational energy is dissipated as I^2R losses in both stator and rotor windings. The braking effect can be increased by adding a resistor load, as shown with broken lines in Figure 5.25.

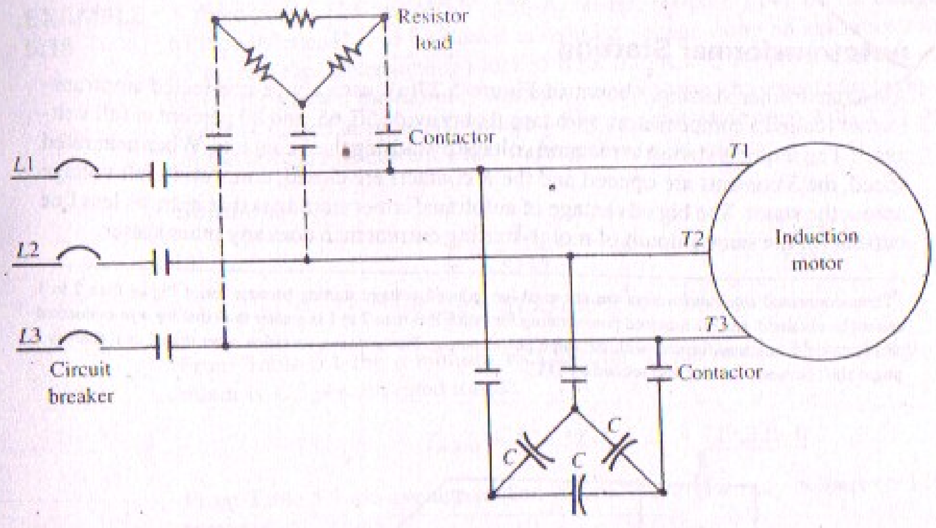


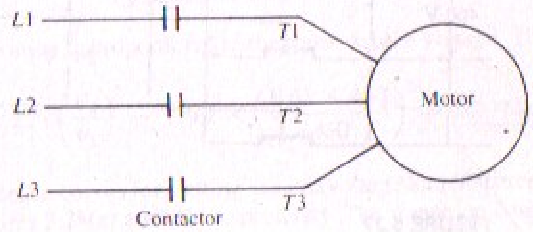
FIGURE 5.25
Dynamic braking using capacitors.

5.20 INDUCTION-MOTOR STARTING

Induction motors of almost any horsepower may be started by connecting them across full voltage, as shown in Figure 5.26, and most are started that way. In many cases, however, the high *in-rush current* associated with full-voltage starting can cause large voltage dips in the distribution system; lights may dim or flicker, unprotected control systems may drop out due to low voltage, and unprotected computers may go off line or lose data. Furthermore, the impact torque that occurs when starting at full voltage can, if high enough, damage gears and other components of the driven equipment.

The methods commonly used for reducing in-rush current are reduced voltage starting using autotransformers, current limiting through wye-delta connections of stator windings, part-winding connections, series impedance, and solid-state control.

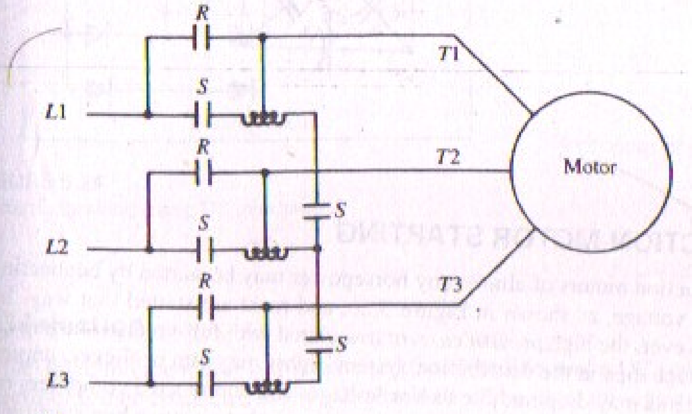
FIGURE 5.26
Full-voltage starting.



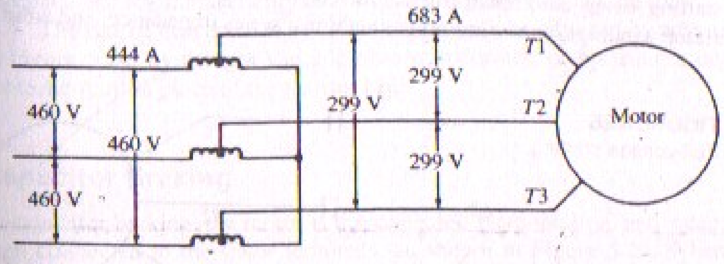
Autotransformer Starting

Autotransformer starting, shown in Figure 5.27(a), uses a wye-connected autotransformer (called a compensator) with taps that provide 50, 65, and 80 percent of full voltage.[12] The motor is started at reduced voltage by closing the *S* contacts. When near rated speed, the *S* contacts are opened and the *R* contacts are closed, connecting full voltage across the stator. The big advantage of autotransformer starting is that it draws less line current for the same amount of motor-starting current than does any other starter.

[12]Delta-connected autotransformers are not used for reduced-voltage starting because ratios higher than 2 to 1 cannot be obtained, and the required power rating for ratios less than 2 to 1 is greater than that for wye-connected and open-delta autotransformers with the same power output. Furthermore, for ratios other than 2 to 1, there is a phase shift between primary and secondary [11].



(a)



(b)

FIGURE 5.27
(a) Circuit for autotransformer starting; (b) current and voltage distributions for Example 5.18.

EXAMPLE 5.18 A three-phase, 125-hp, 460-V, 156-A, 60-Hz, six-pole 1141 r/min, design *B* motor with code letter H is to be started at reduced voltage using an autotransformer with a 65 percent tap. Determine (a) locked-rotor torque and expected average in-rush current to the stator if the motor is started at rated voltage; (b) repeat part (a) assuming the motor is started at reduced voltage using an autotransformer with a 65 percent tap; (c) the in-rush line current when starting at reduced voltage.

Solution

(a)

$$P = \frac{Tn}{5252} \quad \Rightarrow \quad 125 = \frac{T(1141)}{5252}$$

$$T_{\text{rated}} = 575.37 \text{ lb-ft}$$

From Table 5.1 the minimum locked-rotor torque for a 125-hp design *B*, six-pole motor is 125 percent rated torque.

$$T_{lr,460} = 575.37 \times 1.25 = \underline{719.2 \text{ lb-ft}}$$

From Table 5.9 the average locked-rotor kVA/hp that may be expected for a code H motor is

$$\frac{6.3 + 7.1}{2} = 6.70 \text{ kVA/hp}$$

The expected average in-rush current to the stator is

$$\text{kVA/hp} \times 1000 \times \text{hp} = \sqrt{3}\, V_{\text{line}} I_{\text{line}}$$

$$6.70 \times 1000 \times 125 = \sqrt{3} \times 460 \times I_{lr,460}$$

$$I_{lr,460} = \underline{1051 \text{ A}}$$

(b) With the 65 percent tap, the voltage impressed across the stator at locked rotor is

$$V_2 = 0.65 \times 460 = 299 \text{ V}$$

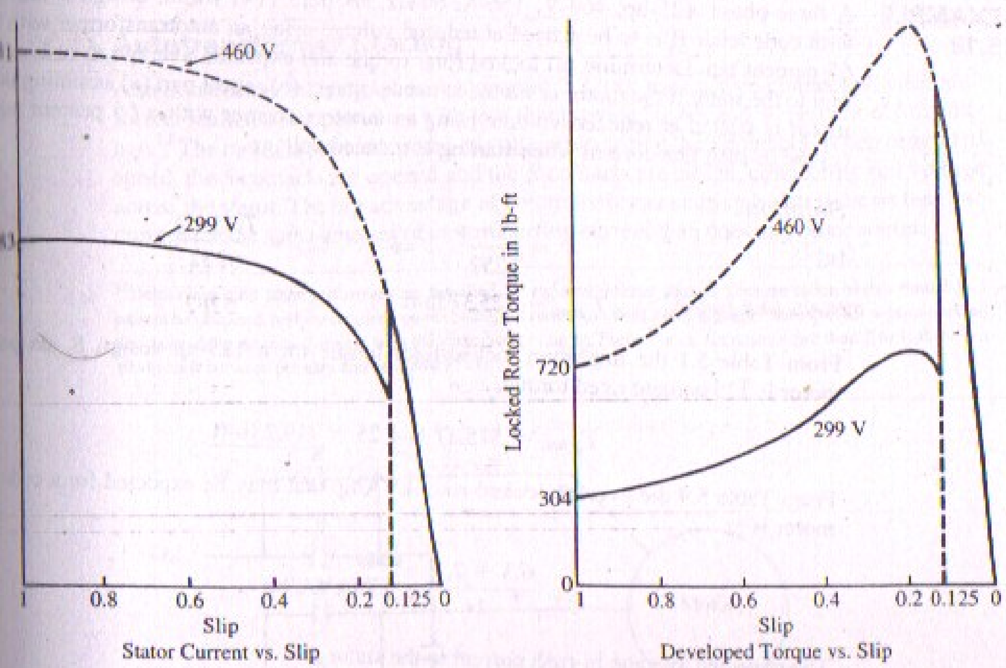
Since the motor input impedance is constant at locked rotor, the average in-rush current to the stator will be proportional to the voltage applied to the stator. Thus,

$$I = 1051 \times 0.65 = 683 \text{ A}$$

The locked-rotor torque is proportional to the square of the voltage. Thus,

$$T_2 = T_1 \cdot \left(\frac{V_2}{V_1}\right)^2 = 719.2 \times \left(\frac{0.65 \times 460}{460}\right)^2 = \underline{303.9 \text{ lb-ft}}$$

The current and torque curves for starting at rated voltage and 65 percent rated voltage are shown in Figures 5.28(a) and (b), respectively. The transition from low voltage to



URE 5.28
rrent and torque curves for Example 5.18.

rated voltage for this starter is assumed to occur at $s = 0.125$. This corresponds to a shaft speed of

$$n_r = n_s(1 - s) = 1200(1 - 0.125) = 1050 \text{ r/min}$$

The solid lines indicate the overall behavior of the machine when using autotransformer starting.

(c) The bank ratio for a wye-connected autotransformer equals the turns ratio. Thus,

$$a = \frac{V_{\text{HS,line}}}{V_{\text{LS,line}}} = \frac{V_{\text{HS,line}}}{0.65 \cdot V_{\text{HS,line}}} = \frac{1}{0.65}$$

$$I_{\text{HS}} = \frac{I_{\text{LS}}}{a} = 683 \times 0.65 = \underline{444 \text{ A}}$$

Wye-Delta Starting

The circuit for wye-delta starting, also called star-delta starting, is shown in Figure 5.29. The three phases of the stator are connected in wye during start-up, and then

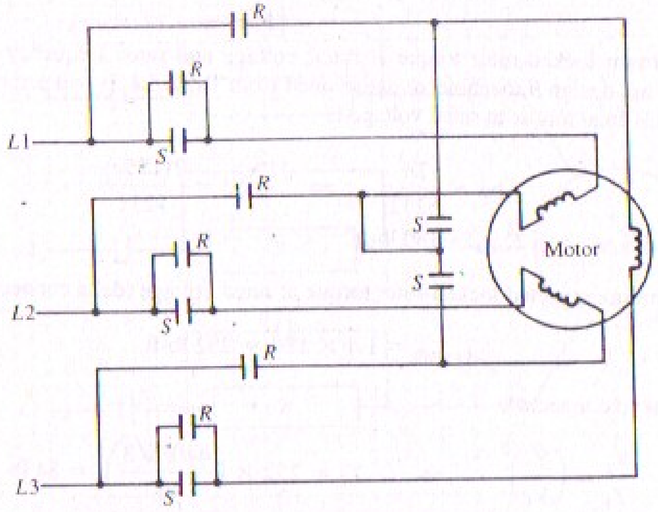


FIGURE 5.29
Wye-delta starter.

reconnected in delta when the starting current is sufficiently decreased. All motors connected for wye-delta starting are actually delta-connected machines that are only wye connected for starting purposes. The S contacts provide the wye connection, and the R contacts provide the delta connection. When connected in wye, the voltage across each phase of the stator winding is $V_{\text{line}}/\sqrt{3}$.

EXAMPLE 5.19 A 60-hp, 460-V, 60-Hz, 77-A, three-phase, 1750 r/min, design B motor has a locked-rotor impedance of $0.547\angle 69.1^\circ$ Ω/phase. Assuming the machine is connected for wye-delta starting, determine (a) the locked-rotor current per phase and the expected minimum locked-rotor torque when starting; (b) the locked-rotor current per phase assuming the motor is started delta connected; (c) the code letter.

Solution

(a) The voltage per phase when wye connected is

$$\frac{460}{\sqrt{3}} = 265.6 \text{ V}$$

The corresponding locked-rotor current per phase is

$$I_{lr} = \frac{V}{Z} = \frac{460/\sqrt{3}}{0.547} = \underline{485.5 \text{ A}}$$

The minimum locked-rotor torque at rated voltage and rated frequency for a 1750 r/min, 60-hp, design *B* machine, as determined from Table 5.1, is 140 percent full-load torque. Full-load torque at rated voltage is

$$\text{hp} = \frac{Tn}{5252} \quad \Rightarrow \quad 60 = \frac{T(1750)}{5252}$$

$$T_{\text{rated}} = 180 \text{ lb-ft}$$

The minimum expected locked-rotor torque at rated voltage (delta connected) is

$$T_{\text{lr}(460)} = 1.4 \times 180 = 252 \text{ lb-ft}$$

Thus, if wye connected,

$$\frac{T_2}{T_1} = \left(\frac{V_2}{V_1}\right)^2 \quad \Rightarrow \quad T_2 = 252 \times \left(\frac{460/\sqrt{3}}{460}\right)^2 = \underline{84 \text{ lb-ft}}$$

(b) If started delta connected (full-voltage starting), the locked-rotor current per phase would be

$$I_{\text{lr},\Delta} = \frac{V}{Z} = \frac{460}{0.547} = 840.95 \text{ A/phase}$$

The corresponding line current is

$$840.95 \times \sqrt{3} = \underline{1457 \text{ A}}$$

(c) The code letter is determined at rated voltage. Thus,

$$S_{\text{lr}} = \sqrt{3} \times 460 \times \frac{1457}{1000} = 1161 \text{ kVA}$$

$$\text{kVA/hp} = 1161/60 = 19.4$$

This corresponds to Code T in Table 5.9.

Note: The current and torque curves for wye-delta starting are similar to those in Figure 5.28 for autotransformer starting.

Series-Impedance Starting

The series-impedance starter, shown in Figure 5.30(a), uses a resistor or an inductor in series with each phase of the stator windings to limit the current during start-up. The running contacts (*R*) are open when starting, to limit the in-rush current, and are closed to short out the impedance when the motor is near rated speed. The ohmic value of the resistor or reactor is generally selected to provide approximately 70 percent

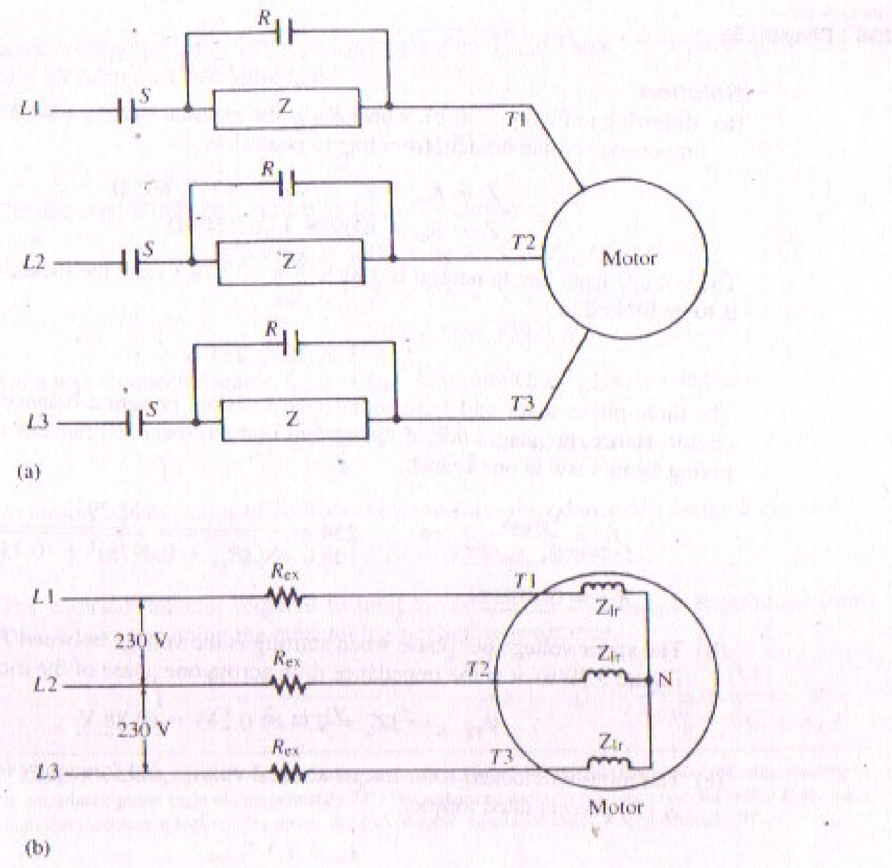


FIGURE 5.30
(a) Series-impedance starter; (b) circuit for Example 5.20.

rated voltage at the motor terminals when starting. The series-impedance starter provides smooth acceleration and is the simplest method for starting induction motors.

EXAMPLE 5.20 A 30-hp, wye-connected, three-phase, 230-V, 60-Hz, 78-A 1748 r/min, design *B* motor has a locked-rotor impedance per phase of $0.273\underline{/69^\circ}\ \Omega$ at rated temperature and frequency. The motor is to be started using series resistors in each line. Determine (a) the resistance of the resistors required to limit the locked-rotor current to three times rated current; (b) the *stator voltage* per phase at locked rotor; (c) the expected minimum locked-rotor torque when starting as a percent of rated torque.

Solution

(a) Referring to Figure 5.30(b), where R_{ex} is the external starting resistance, the impedance of one branch, from line to neutral, is

$$\mathbf{Z} = R_{ex} + \mathbf{Z}_{lr} = R_{ex} + 0.273\underline{/69^\circ}\ \Omega$$
$$\mathbf{Z} = R_{ex} + 0.0978 + j0.2549\ \Omega$$

The voltage from line to neutral is $230/\sqrt{3} = 132.79$ V, and the locked-rotor current is to be limited to

$$I_{lr} = 3 \times 78 = 234 \text{ A}$$

The three-phase stator and series-connected resistors present a balanced three-phase circuit. Hence, the magnitude of the current in the respective branches are equal. Applying Ohm's law to one branch,

$$I_{lr} = \frac{V_{branch}}{Z_{branch}} \Rightarrow 234 = \frac{132.79}{\sqrt{(R_{ex} + 0.0978)^2 + (0.2549)^2}}$$
$$R_{ex} = \underline{0.4093\ \Omega}$$

(b) The stator voltage per phase when starting is the voltage between $T1$ and N in Figure 5.30(b); it is the impedance drop across one phase of the motor winding:

$$V_{T1-N} = IZ_{lr} = 234 \times 0.273 = \underline{63.88 \text{ V}}$$

(c) The minimum locked-rotor torque at rated voltage and frequency from Table 5.1 is 150 percent rated torque.

$$T_{lr} = \underline{1.5\ T_{rated}}$$

Note: Calculations must always be made on the basis of the *actual voltage applied to the stator.* Thus,

$$\frac{1.5T_{rated}}{T_{63.88}} = \left[\frac{132.79}{63.88}\right]^2$$
$$T_{lr,63.88} = 0.347T_{rated} \quad \text{or} \quad 34.7\%\ T_{rated}$$

MPLE A certain 208-V, 7.5-hp, 60-Hz, four-pole, wye-connected, design B induction motor with Code letter H, has a rated current of 24 A at 1722 r/min. The motor is to be started using the series-impedance method with inductors in each line. Calculate the inductance and voltage rating of each series-connected inductor required to limit the starting current to approximately $2 \times I_{rated}$.

Solution

The circuit is similar to that shown in Figure 5.30(b), with R_{ex} replaced by jX_{ex}. A rough approximation of the motor locked-rotor impedance may be calculated from

motor horsepower, code letter, and voltage rating. From Table 5.9, the average locked-rotor kVA/hp for Code letter H is

$$\frac{6.3 + 7.1}{2} = 6.7 \text{ kVA/hp}$$

The corresponding approximate locked-rotor current is

$$\text{kVA/hp} \times 1000 \times \text{hp} = \sqrt{3}\, V_{line} I_{line}$$
$$6.7 \times 1000 \times 7.5 = \sqrt{3} \times 208\, I_{lr}$$
$$I_{lr} = 139.5 \text{ A}$$

For a wye-connected stator, $I_{phase} = I_{line}$. Applying Ohm's law to one phase,

$$Z_{lr} \cong \frac{V_{phase}}{I_{lr}} = \frac{208/\sqrt{3}}{139.5} = \underline{0.861\ \Omega}$$

Assuming a phase angle of 70° for the locked-rotor impedance of a design B machine,[13]

$$\mathbf{Z}_{lr} \approx 0.861\underline{/70^\circ} \approx 0.294 + j0.809\ \Omega$$

The external inductor required to limit the current to $2 \times I_{rated}$ is determined from Ohm's law. Assuming the inductor has negligible resistance,

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{\mathbf{V}}{R_{lr} + jX_{lr} + jX_{ex}} \Rightarrow |\mathbf{I}| = \frac{|\mathbf{V}|}{\sqrt{R_{lr}^2 + (X_{lr} + X_{ex})^2}}$$

[13] Design A and design B machines have relatively low resistance and high reactance at locked rotor, resulting in an impedance phase angle of approximately 75°. The higher resistance to reactance ratio for design D and other high-slip machines at locked rotor results in a locked-rotor impedance angle of approximately 50°.

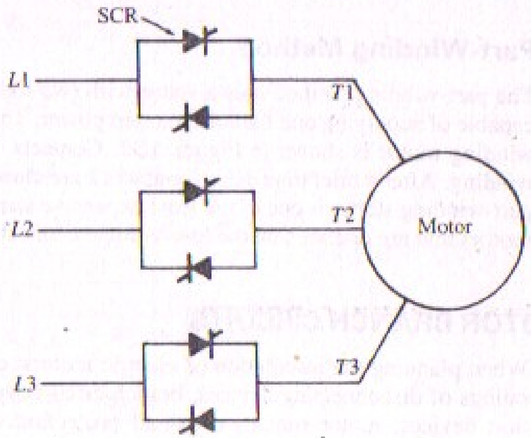


FIGURE 5.31
Solid-state starter.

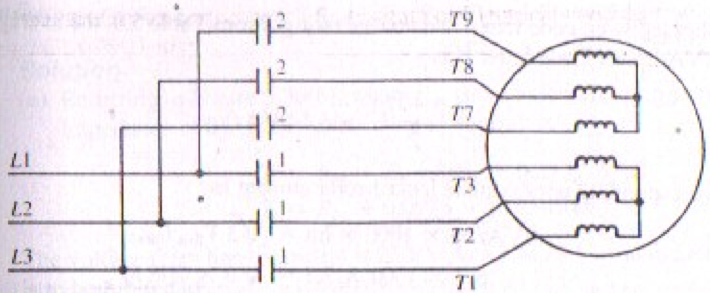


FIGURE 5.32
Part-winding starter.

$$2 \times 24 = \frac{208/\sqrt{3}}{\sqrt{(0.294)^2 + (0.809 + X_{ex})^2}}$$

$$X_{ex} = 1.675\ \Omega$$

$$X_{ex} = 2\pi fL \quad \Rightarrow \quad 1.675 = 2\pi 60L \quad \Rightarrow \quad L = \underline{4.44\text{ mH}}$$

$$\text{Voltage rating} = IX_L = 2 \times 24 \times 1.675 = \underline{80.4\text{ V}}$$

Solid-State Starting

A solid-state starter, shown in Figure 5.31, uses back-to-back thyristors (SCRs) to limit the current. The control circuitry (not shown) allows a gradual buildup of current. The smooth buildup permits a soft start with no impact loading and no significant voltage dips. Solid-state starters can be designed to incorporate many special features, such as speed control, power factor control, protection against overload, and single phasing.

Part-Winding Method

The part-winding method uses a stator with two identical three-phase windings, each capable of supplying one-half of the rated power. The power circuit for starting a part-winding motor is shown in Figure 5.32. Contacts 1 are closed first, energizing one winding. After a brief time delay, contacts 2 are closed, energizing both windings. The part-winding starter is one of the least expensive starters, but is limited to dual-voltage motors that are operated on the low-voltage connections.

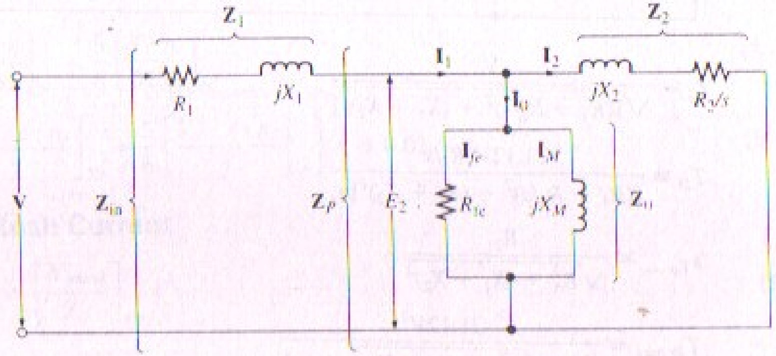
MOTOR BRANCH CIRCUITS

When planning the installation of electric motors, consideration must be given to the ratings of disconnecting devices, branch-circuit overcurrent and ground-fault protection devices, motor running-overload protection devices, motor, motor-controller, rheostat if wound rotor, and all connecting conductors. The correct selection of protective devices, cable sizes, and disconnecting devices for a particular motor application requires familiarization with Article 430 of the National Electric Code (NEC) **[8]**. Adherence to the NEC requirements is absolutely essential; failure to do so may result in damage to equipment and injury to personnel.

SUMMARY OF EQUATIONS FOR PROBLEM SOLVING

Note: If wound-rotor, replace R_2 with $(R_2 + R'_{rheo})$.

Exact Solutions



$$s = (n_s - n_r)/n_s \qquad n_r = n_s(1 - s)$$

$$\mathbf{Z}_2 = \frac{R_2}{s} + jX_2 \qquad \mathbf{Z}_0 = \frac{R_{fe} \cdot jX_M}{R_{fe} + jX_M}$$

$$\mathbf{Z}_P = \frac{\mathbf{Z}_2 \cdot \mathbf{Z}_0}{\mathbf{Z}_2 + \mathbf{Z}_0} \qquad \mathbf{Z}_{in} = \mathbf{Z}_1 + \mathbf{Z}_P$$

$$\mathbf{I}_1 = \frac{\mathbf{V}}{\mathbf{Z}_{in}} \qquad \mathbf{E}_2 = \mathbf{I}_1 \cdot \mathbf{Z}_P \qquad \mathbf{I}_2 = \frac{\mathbf{E}_2}{\mathbf{Z}_2}$$

$$P_{scl} = 3I_1^2R_1 \qquad P_{rcl} = 3I_2^2R_2 \qquad P_{core} = 3E_2^2/R_{fe}$$

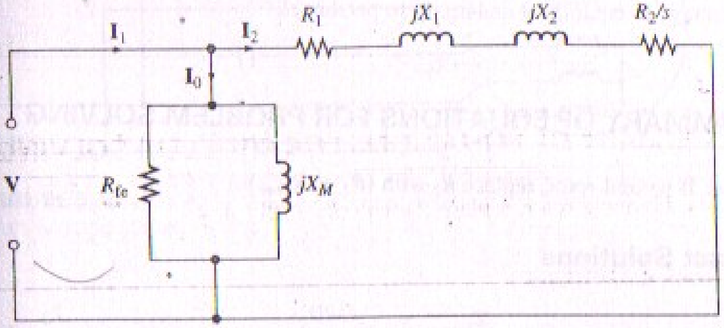
$$P_{gap} = \frac{P_{rcl}}{s} \qquad P_{mech} = \frac{P_{rcl}(1 - s)}{s}$$

$$\text{Loss} = P_{scl} + P_{rcl} + P_{core} + P_{f,w} + P_{stray}$$

$$T_D = \frac{21.12 \cdot I_2^2R_2}{s \cdot n_s} \qquad \text{hp} = \frac{Tn}{5252} \tag{5–1}$$

Close Approximations Using Approximate Equivalent Circuit

Note: If wound rotor, replace R_2 with $(R_2 + R'_{\text{rheo}})$.



$$I_2 \cong \frac{V}{\sqrt{[(R_1 + R_2/s)^2 + (X_1 + X_2)^2]}} \tag{5–4}$$

$$T_D \cong \frac{21.12V^2R_2/s}{[(R_1 + R_2/s)^2 + (X_1 + X_2)^2]n_s} \tag{5–5}$$

$$s_{T_{D,\max}} \cong \frac{R_2}{\sqrt{R_1^2 + (X_1 + X_2)^2}} \tag{5–7}$$

$$T_{D,\max} \cong \frac{21.12V^2}{2n_s[\sqrt{R_1^2 + (X_1 + X_2)^2} + R_1]} \tag{5–10}$$

Approximations for Normal Running and Overload Conditions

Squirrel Cage with s ≤ 0.03

$$\underset{s\le 0.03}{I_2} \cong \frac{V \cdot s}{R_2} \qquad \underset{s\le 0.03}{T_D} \cong \frac{21.12V^2 \cdot s}{R_2 n_s} \qquad \text{(5–11, 5–12)}$$

$$\underset{s\le 0.03}{I_2} \propto s \qquad \underset{s\le 0.03}{T_D} \propto s \qquad \text{(5–13, 5–14)}$$

Wound-Rotor Motor

$$s_{T_{D,\max}} = \frac{R_2 + R'_{\text{rheo}}}{\sqrt{R_1^2 + (X_1 + X_2)^2}} \tag{5–24}$$

$$R'_{\text{rheo}} = a^2 \cdot R_{\text{rheo}} \tag{5–24a}$$

$$\underset{\text{linear}}{I_2} \propto \frac{s}{R_2 + R'_{\text{rheo}}} \qquad \underset{\text{linear}}{T_D} \propto \frac{s}{R_2 + R'_{\text{rheo}}} \qquad \text{(5–28, 5–29)}$$

Approximations for Off-Rated Voltage and Frequency

$$\underset{s\le 0.03}{T_D} \propto \frac{V^2 \cdot s}{f} \tag{5–16}$$

$$\underset{s=1.0,\theta_z\ge 75^\circ}{I_{\text{lr}}} \cong I_2 \propto \frac{V}{f} \qquad \theta_z = \arctan\left(\frac{X_1 + X_2}{R_1 + R_2}\right) \qquad \text{(5–23, 5–18)}$$

60-Hz Motors on 50-Hz Systems

$$V_{50} = \frac{5}{6} V_{60} \tag{5–23c}$$

$$\text{hp}_{50} = \frac{5}{6} \text{hp}_{60} \tag{5–23d}$$

$$\left[\frac{T \cdot n_r}{5252}\right]_{50} = \frac{5}{6}\left[\frac{T \cdot n_r}{5252}\right]_{60} \tag{5–23e}$$

$$\left.\left[\frac{V^2 \cdot s \cdot n_r}{f}\right]_{50} = \frac{5}{6}\left[\frac{V^2 \cdot s \cdot n_r}{f}\right]_{60}\right\} s \le 0.03 \tag{5–23f}$$

In-Rush Current

$$\mathbf{I}_{\text{lr,ss}} = \left[\frac{\mathbf{V}_{\text{phase}}}{\mathbf{Z}_{\text{in}}}\right]_{s=1.0} \tag{5–30}$$

$$i_{\text{lr,ss}} = \sqrt{2}\left|\frac{\mathbf{V}_{\text{phase}}}{\mathbf{Z}_{\text{in}}}\right|_{s=1.0} \sin(2\pi f t - \theta_z) \tag{5–31}$$

Effect of Unbalanced Line Voltages

$$\%\text{UBV} = \frac{V_{\max\text{ dev}}}{V_{\text{avg}}} \cdot 100 \qquad \%\Delta T \cong 2(\%\text{UBV})^2 \qquad \text{(5–35, 5–36)}$$

$$T_{\text{UBV}} \cong T_{\text{rated}} \cdot \left(1 + \frac{\%\Delta T}{100}\right) \qquad \delta T = T_{\text{UBV}} - T_{\text{rated}} \tag{5–37}$$

$$\text{RL} \cong \frac{1}{2^{(\delta T/10)}} \tag{5–38}$$

Per-Unit Conversions

P_{base} = rated three-phase shaft-power output in watts ÷ 3

V_{base} = rated voltage/phase

I_{base} = base current = rated current/phase

$$I_{\text{base}} = \frac{P_{\text{base}}}{V_{\text{base}}} \qquad Z_{\text{base}} = \frac{V_{\text{base}}}{I_{\text{base}}} = \frac{V_{\text{base}}}{P_{\text{base}}/V_{\text{base}}} = \frac{V^2_{\text{base}}}{P_{\text{base}}} \tag{5–39, 5–41a}$$

$$\left.\begin{array}{lll} r_1 = \dfrac{R_1}{Z_{\text{base}}} & r_2 = \dfrac{R_2}{Z_{\text{base}}} & r_{fe} = \dfrac{R_{fe}}{Z_{\text{base}}} \\[2ex] x_1 = \dfrac{X_1}{Z_{\text{base}}} & x_2 = \dfrac{X_2}{Z_{\text{base}}} & x_M = \dfrac{X_M}{Z_{\text{base}}} \end{array}\right\} \tag{5–41}$$

Determination of Induction Motor Parameters

DC Test

$$R_{1,\text{wye}} = \frac{R_{\text{DC}}}{2} \qquad R_{1,\Delta} = 1.5R_{\text{DC}} \tag{5–42, 5–43}$$

Blocked-Rotor Test

$$Z_{\text{BR},15} = \frac{V_{\text{BR},15}}{I_{\text{BR},15}} \qquad R_{\text{BR},15} = \frac{P_{\text{BR},15}}{I^2_{\text{BR},15}} \tag{5–45, 5–46}$$

$$R_2 = R_{\text{BR},15} - R_1 \qquad X_{\text{BR},15} = \sqrt{Z^2_{\text{BR},15} - R^2_{\text{BR},15}} \tag{5–47, 5–48}$$

$$X_{\text{BR},60} = \tfrac{60}{15}X_{\text{BR},15} \qquad X_{\text{BR},60} = X_1 + X_2 \tag{5–49, 5–50}$$

No-Load Test

$$S_{\text{NL}} = V_{\text{NL}}I_{\text{NL}} \qquad Q_{\text{NL}} = \sqrt{S^2_{\text{NL}} - P^2_{\text{NL}}} \qquad X_{\text{NL}} = \frac{Q_{\text{NL}}}{I^2_{\text{NL}}} \tag{5–52, 5–54}$$

$$X_{\text{NL}} = X_1 + X_M \qquad P_{\text{NL}} = I^2_{\text{NL}}R_1 + P_{\text{core}} + P_{w,f} \tag{5–55, 5–56}$$

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REVIEW QUESTIONS

1. Sketch (on one set of coordinate axes) the torque-speed characteristics of design *A, B, C,* and *D* motors, and state an application for each design.
2. Sketch representative cross sections of design *A, B, C,* and *D* rotors, and explain why the respective construction gives the desired characteristics.
3. Sketch the complete equivalent circuit of a squirrel-cage motor and label all components.
4. How does the developed torque vary with the applied stator voltage?
5. What effect does increasing rotor-circuit resistance have on the breakdown torque and on the slip at which breakdown occurs?

6. What is the approximate slip range for NEMA-design squirrel-cage motors operating between no-load and rated service factor load? Assume rated voltage and rated frequency.
7. What is the effect of off-rated voltage and off-rated frequency on running torque for $s \leq 0.03$?
8. Under what constraints may a 60-Hz motor be operated from a 50-Hz system?
9. What are the maximum permissible frequency and voltage variations that a NEMA-design motor can tolerate and still operate successfully at rated load?
10. (a) Sketch a family of torque-speed curves for a wound-rotor induction motor and indicate the relative values of rotor resistance for each curve. (b) If operating at no load, what effect does adjusting the rheostat have on the motor speed? (c) State an application for a wound-rotor motor.
11. Define utilization voltage, nominal efficiency, code letter, design letter, service factor, and insulation class, and state how this information can be used.
12. (a) What is in-rush current and what causes it? (b) Under what conditions is the transient in-rush a minimum and (c) a maximum?
13. State how to approximate in-rush current from nameplate data.
14. What specific adverse effect does in-rush current have on the service life of an induction motor?
15. (a) What is meant by "reclosing out of phase" and how does it affect an induction motor? (b) What can be done to avoid reclosing out of phase?
16. (a) How does NEMA define voltage unbalance? (b) What is the maximum permissible voltage unbalance?
17. (a) What adverse effects do unbalanced line voltages have on induction-motor performance? (b) If the voltage unbalance cannot be corrected, what should be done to protect the machine?
18. What is the ten-degree rule as it pertains to electrical insulation?
19. What are some of the advantages of expressing induction-motor parameters in terms of per-unit values?
20. What is an induction generator? How does it operate? What are its principal fields of application?
21. Sketch air-gap power vs. speed for an induction machine, showing its behavior as it goes from zero speed to twice synchronous speed. Indicate the breakdown point and the pushover point.
22. What determines the output voltage and frequency of an induction generator in parallel with other three-phase sources?
23. What is the normal procedure for starting and then paralleling an induction generator with a live three-phase bus?
24. Using suitable diagrams, explain how an isolated induction generator builds up voltage as a self-excited machine.
25. Explain how dynamic braking is accomplished, using (a) DC injection; (b) capacitors.
26. If induction motors of almost any power rating can be started by connecting them across full voltage, why use reduced voltage starters?

PROBLEMS

5–1/3 A three-phase, 230-V, 25-hp, 60-Hz, two-pole, NEMA design A induction motor runs at 3564 r/min when operating at rated conditions. Determine, in lb-ft, the minimum expected values of (a) locked-rotor torque; (b) breakdown torque; (c) pull-up torque.

5–2/3 A 50-Hz, design B, three-phase, 380-V, four-pole induction motor, with rated shaft output of 45 kW, has a speed of 1490 r/min. Determine, in lb-ft, the minimum expected values of (a) locked-rotor torque; (b) breakdown torque; (c) pull-up torque.

5–3/3 Determine, in lb-ft, the minimum values of locked-rotor torque, breakdown torque, and pull-up torque that can be expected from a 5-hp, four-pole, 440-V, 60-Hz, three-phase, design C induction motor whose rated speed is 1776 r/min.

5–4/3 (a) Prepare a table listing the NEMA minimum expected values of locked-rotor torque, breakdown torque, and pull-up torque for six-pole, 100-hp, 240-V, 60-Hz, three-phase induction motors, designs A, B, C, D, and E. (b) Referring to the table in part (a), under what conditions would it be safe to substitute a design E motor for design B motor.

5–5/4 A 25-hp, 60-Hz, 575-V, six-pole motor is operating at a slip of 0.030. The stray power loss and the combined windage and friction loss at this load are 230.5 W and 115.3 W, respectively. The motor is wye connected, and the motor parameters in ohms/phase are

$$R_1 = 0.3723 \qquad R_2 = 0.390 \qquad X_M = 26.59$$
$$X_1 = 1.434 \qquad X_2 = 2.151 \qquad R_{fe} = 354.6$$

Determine (a) motor input impedance per phase; (b) line current; (c) active, reactive, and apparent power and power factor; (d) equivalent rotor current; (e) stator copper loss; (f) rotor copper loss; (g) core loss; (h) air-gap power; (i) mechanical power developed; (j) developed torque; (k) shaft horsepower; (l) shaft torque; (m) efficiency. (n) Sketch the power-flow diagram and enter all data.

5–6/4 The shaft load on a 40-hp, 60-Hz, 460-V, four-pole induction motor is such as to cause the machine to operate at 1447 r/min. The stray load loss and combined windage and friction loss, when operating at this load, are 450 W and 220 W, respectively. The motor parameters in ohms/phase are

$$R_1 = 0.1418 \qquad R_2 = 1.10 \qquad X_M = 21.27$$
$$X_1 = 0.7273 \qquad X_2 = 0.7284 \qquad R_{fe} = 212.73$$

The motor is NEMA design D and wye connected. Determine (a) motor input impedance per phase; (b) line current; (c) active, reactive, and apparent power and power factor; (d) equivalent rotor current; (e) stator copper loss; (f) rotor copper loss; (g) core loss; (h) air-gap power; (i) mechanical power developed; (j) developed torque; (k) shaft horsepower; (l) shaft torque; (m) efficiency. (n) Sketch the power-flow diagram and enter all data. (o) If the speed at rated load is 1190 r/min, determine the expected minimum locked-rotor torque.

5–7/4 A three-phase, eight-pole induction motor, rated at 847 r/min, 30 hp, 60 Hz, 460 V, operating at reduced load, has a shaft speed of 880 r/min. The combined stray power loss, windage loss, and friction loss is 350 W. The motor parameters in ohms/phase are

$$R_1 = 0.1891 \qquad R_2 = 0.191 \qquad X_M = 14.18$$
$$X_1 = 1.338 \qquad X_2 = 0.5735 \qquad R_{fe} = 189.1$$

The motor is NEMA design *C* and wye connected. Determine (a) motor input impedance per phase; (b) line current; (c) active, reactive, and apparent power and power factor; (d) equivalent rotor current; (e) stator copper loss; (f) rotor copper loss; (g) core loss; (h) air-gap power; (i) mechanical power developed; (j) developed torque; (k) shaft horsepower; (l) shaft torque; (m) efficiency. (n) Sketch the power-flow diagram and enter all data. (o) Determine the expected minimum locked-rotor torque, breakdown torque, and pull-up torque.

5–8/5 For the motor in Problem 5–5/4, determine (a) the shaft speed at maximum torque; (b) the value of the maximum torque.

5–9/5 For the motor in Problem 5–7/4, determine (a) the shaft speed at maximum torque; and (b) the value of the maximum torque.

5–10/5 A three-phase, 60 Hz, 75-hp, 460-V, six-pole, design *C* induction motor, operating at rated conditions, has a slip of 0.041. Determine (a) the expected minimum locked-rotor torque; (b) the expected minimum locked-rotor torque if the stator is connected to a 400-V, 60-Hz supply; (c) the percent change in applied voltage; (d) the resultant percent change in locked-rotor torque.

5–11/5 A three-phase, wye-connected, design *A*, eight-pole motor rated at 50 Hz, 240 V, 20 hp has a slip of 1.80 percent when operating at rated conditions. The motor is to drive a constant-torque load of 145 lb-ft from a 208-V, 50-Hz supply. The breakaway torque required to get the motor started is 155 lb-ft. Will the motor start? Show all work!

5–12/6 A 75-hp, two-pole, wye-connected motor, operating from a 60-Hz, 2300-V line, is delivering 75.4 hp at 3500 r/min and 18.9 A. Determine (a) the new shaft speed if the torque load is reduced by 25 percent; (b) the rotor current; (c) the air-gap power. The motor parameters expressed in ohms per phase are

$$R_1 = 1.08 \qquad R_2 = 2.14 \qquad R_{fe} = 1892$$
$$X_1 = 8.14 \qquad X_2 = 3.24 \qquad X_M = 147.5$$

5–13/6 A 250-hp, four-pole, wye-connected motor, operating from a 60-Hz, 460-V line, is delivering 255 hp at 1777 r/min. If the shaft torque load is reduced by

25 percent, determine the approximate values of (a) slip; (b) shaft speed; (c) rotor current; (d) shaft horsepower. The motor parameters expressed in ohms/phase are:

$$R_1 = 0.0626 \qquad R_2 = 0.0118 \qquad R_{fe} = 32.25$$
$$X_1 = 0.027 \qquad X_2 = 0.040 \qquad X_M = 2.465$$

5–14/6 A 10-hp, 440-V, 60-Hz, two-pole, induction motor operating at rated load develops 15.5 lb-ft torque at 3492 r/min. The motor parameters expressed in ohms/phase are:

$$R_1 = 0.740 \qquad R_2 = 0.647 \qquad R_{fe} = \text{not known}$$
$$X_1 = 1.33 \qquad X_2 = 2.01 \qquad X_M = 77.6$$

If the load torque is reduced by 30 percent, determine the approximate values of (a) slip; (b) shaft speed; (c) rotor current; (d) shaft horsepower.

5–15/6 A 15-hp, 440-V, 60-Hz, six-pole motor operates at 1173 r/min when carrying rated load. The motor parameters expressed in ohms/phase are:

$$R_1 = 0.301 \qquad R_2 = 0.327 \qquad R_{fe} = 496$$
$$X_1 = 0.833 \qquad X_2 = 1.25 \qquad X_M = 30.3$$

If a 15 percent torque overload occurs, determine the approximate values of (a) speed; (b) rotor current; (c) shaft power.

5–16/8 A 150-hp, three-phase, wye-connected, 60-Hz, 4000-V, six-pole induction motor, operating at rated conditions from an isolated power system, has a shaft speed of 1175 r/min. A very heavy power demand on the electrical system caused the voltage and frequency to decrease by 15 percent and 5 percent, respectively. To compensate for this off-normal condition, the torque load on the motor was reduced to 82 percent of rated torque. Determine the operating speed for the new conditions.

5–17/8 A three-phase, wye-connected, 50-hp, 60-Hz, 460-V, four-pole induction motor, operating at rated conditions, has an efficiency, power factor, and slip of 89.6 percent, 79.5 percent, and 3.0 percent, respectively. Operating the motor from a 430-V, 55-Hz supply results in a shaft speed of 1750 r/min. Determine the resultant shaft horsepower for the new operating conditions. Assume the windage, friction, and stray load losses are the same.

5–18/8 A four-pole, 60-hp, 440-V, 60-Hz, 1760 r/min, three-phase induction motor driving a loaded conveyer develops a locked-rotor torque equal to 161 percent rated torque when rated voltage and rated frequency are applied. The *load torque* at start-up is 114 percent rated motor torque. To compensate for inertia and static friction, the developed torque at locked rotor must be at least 15 percent greater than the load torque. If a very heavy power demand on the electrical system causes the voltage to decrease by 15 percent and the frequency to decrease by 3 percent, will the motor start? Show all work.

5–19/8 A three-phase, 125-hp, 60-Hz, eight-pole, 575-V, design *B* induction motor is to be operated from a 50-Hz system. Determine (a) allowable voltage at 50 Hz; (b) allowable shaft load in horsepower; (c) new synchronous speed; (d) shaft speed assuming a slip of 2.1 percent; (e) shaft torque at 2.1 percent slip.

5–20/9 A three-phase, wye-connected, 25-hp, 60-Hz, 575-V, six-pole, wound-rotor motor, operating at nameplate conditions, runs at 1164 r/min with the rheostat shorted. The stator/rotor turns ratio is 2.15, and the motor parameters in ohms/phase are

$$R_1 = 0.3723 \qquad R_2 = 0.390 \qquad R_{fe} = 26.59$$
$$X_1 = 1.434 \qquad X_2 = 2.151 \qquad X_m = 354.6$$

Determine (a) slip at which $T_{D,max}$ occurs; (b) $T_{D,max}$; (c) the rheostat resistance/phase required to operate the machine at rated torque load and 1074 r/min.

5–21/9 A 40-hp, 60-Hz, 460-V, four-pole, wye-connected wound-rotor motor, with slip rings shorted, has its breakdown torque occur at 25 percent slip. The rotor impedance in ohms/phase referred to the stator is $0.158 + j0.623$, and the turns ratio is 1.28. Determine the rheostat resistance required to cause the breakdown torque to occur at 60 percent slip.

5–22/12 A 75-hp, 460-V, six-pole NEMA design *A* machine with Code letter H is operating at rated load, is 89 percent efficient, has a power factor of 84 percent, and has a slip of 2.36 percent. Determine (a) rated current; (b) the expected range of locked-rotor current with rated voltage and rated frequency applied.

5–23/12 A 30-hp, 60-Hz, 230-V, two-pole, design *E* motor, with Code letter C, operating at rated conditions has an efficiency of 91.8 percent and a power factor of 86.2 percent. Determine (a) rated current; (b) the expected range of locked-rotor current.

5–24/12 A 150-hp, 60-Hz, 460-V, four-pole, design *B* motor, with Code letter R, operating at rated conditions has an efficiency of 94.5 percent and a power factor of 86.2 percent. Determine (a) rated current; (b) the expected range of locked-rotor current.

5–25/15 A 60-hp, design *C*, 230-V, 60-Hz, six-pole motor with a 1.15 service factor and Class B insulation is operating at rated horsepower from an unbalanced three-phase system. The three line-to-line voltages are 232 V, 238 V, and 224 V. The machine is new and has an expected life of 20 years. Determine (a) the percent voltage unbalance; (b) the expected approximate temperature rise if operating at rated load in a 40°C ambient situation, with the percent unbalance in (a); (c) the expected insulation life; (d) the required rerating, if any, to prevent shortening the life of the insulation.

5–26/15 A three-phase, 30-hp, 460-V, 60-Hz, 1770 r/min, design *B* induction motor with Class F insulation and service factor 1.0 is operating at rated shaft load in a 40°C ambient situation, and has an expected 20-year life. A preventive maintenance check shows line-to-line voltages to be 449.2 V, 431.3 V, and 462.4 V. Determine (a) percent voltage unbalance; (b) expected temperature rise; (c) expected insulation life; (d) rerating of motor, if any to prevent shortening insulation life.

5–27/16 A three-phase, wye-connected 10-hp, 60-Hz, 230-V, four-pole induction motor has the following per-unit parameters:

$$R_1 = 0.0358 \qquad R_2 = 0.0264 \qquad R_{fe} = \text{not known}$$
$$X_1 = 0.0964 \qquad X_2 = 0.1450 \qquad X_M = 3.02$$

Determine the machine parameters in ohms/phase.

5–28/16 A three-phase, 460-V, wye-connected, 200-hp, 60-Hz, eight-pole, design *B*, squirrel-cage induction motor, has the following per-unit parameters:

$$R_1 = 0.011 \qquad R_2 = 0.011 \qquad R_{fe} = \text{not known}$$
$$X_1 = 0.123 \qquad X_2 = 0.210 \qquad X_M = 2.994$$

Determine the corresponding ohmic values.

5–29/16 A three-phase, wye-connected, 100-hp, 60-Hz, 440-V, 10-pole induction motor has the following parameters in ohms:

$$R_1 = 0.0864 \qquad R_2 = 0.078 \qquad R_{fe} = 110$$
$$X_1 = 0.146 \qquad X_2 = 0.218 \qquad X_M = 3.185$$

Determine the corresponding per-unit values.

5–30/17 A three-phase, design *B*, wye-connected, 25-hp, 575-V, 60-Hz induction motor, operating at rated conditions, draws a line current of 27 A. Data from a 15-Hz blocked-rotor test, a 60-Hz no-load test, and a DC test are:

Blocked Rotor	No-Load	DC
$V_{line} = 54.7$ V	$V_{line} = 575$ V	$V_{DC} = 20$ V
$I_{line} = 27.0$ A	$I_{line} = 11.8$ A	$I_{DC} = 27$ A
$P_{3\text{-phase}} = 1653$ W	$P_{3\text{-phase}} = 1264.5$ W	

Determine R_1, R_2, X_1, X_2, X_M, and the combined core, friction, and windage loss.

5–31/17 The following data were obtained from a no-load test, a 15-Hz blocked-rotor test, and a DC test on a three-phase, wye-connected, four-pole, 30-hp, 460-V, 60-Hz, 40-A, design *C* induction motor:

Blocked Rotor	No-Load	DC
$V_{line} = 42.39$ V	$V_{line} = 458.6$ V	$V_{DC} = 15.4$ V
$I_{line} = 40$ A	$I_{line} = 17.0$ A	$I_{DC} = 40.2$ A
$P_{3\text{-phase}} = 1828.8$ W	$P_{3\text{-phase}} = 1381.4$ W	

Determine R_1, R_2, X_1, X_2, X_M, and the combined core, friction, and windage loss.

5–32/17 A three-phase, design *A*, wye-connected, 15-hp, 460-V, 60-Hz induction motor draws a line current of 14 A when operating at rated conditions. A

60-Hz no-load test, a 15-Hz blocked-rotor test, and a DC test provide the following data:

Blocked Rotor	No-Load	DC
$V_{line} = 18.5$ V	$V_{line} = 459.8$ V	$V_{DC} = 5.6$ V
$I_{line} = 13.9$ A	$I_{line} = 6.2$ A	$I_{DC} = 14.0$ A
$P_{3\text{-phase}} = 264.6$ W	$P_{3\text{-phase}} = 799.5$ W	

Determine R_1, R_2, X_1, X_2, X_M, and the combined core, friction, and windage loss.

5–33/18 A 75-hp, 2300-V, 60-Hz, two-pole, wye-connected motor, driven at 3650 r/min by a steam turbine, is connected to a 2300-V, 60-Hz distribution system. The motor parameters in ohms are:

$$R_1 = 1.08 \quad R_2 = 2.14 \quad R_{fe} = 1892$$
$$X_1 = 8.14 \quad X_2 = 3.24 \quad X_M = 187.5$$

Determine the active power that the machine delivers to the system.

5–34/18 A 60-Hz, 15-hp, 460-V, six-pole, wye-connected, three-phase induction motor is connected to a 460-V distribution system and driven at 1210 r/min by a diesel engine. The motor parameters in ohms are:

$$R_1 = 0.200 \quad R_2 = 0.250 \quad X_M = 42.0$$
$$X_1 = 1.20 \quad X_2 = 1.29 \quad R_{fe} = 317$$

Determine the active power that the machine delivers to the system.

5–35/18 A three-phase, wye-connected, 400-hp, four-pole, 380-V, 50-Hz induction motor is driven by a wind turbine at 1515 r/min. The motor parameters in ohms are:

$$R_1 = 0.00536 \quad R_2 = 0.00613 \quad R_{fe} = 7.66$$
$$X_1 = 0.0383 \quad X_2 = 0.0383 \quad X_M = 0.5743$$

Determine the active power that the machine delivers to the system.

5–36/20 A 200-hp, 1150 r/min, 440-V, 60-Hz pump motor uses a wye-connected autotransformer for reduced voltage starting. The transformer has a 65 percent tap. The starting torque at rated voltage is 150 percent rated torque. Sketch a one-line diagram and determine the blocked-rotor torque when (a) starting at rated voltage; (b) starting at reduced voltage.

5–37/20 A three-phase 12-pole, 220-V, 60-Hz, 50-hp squirrel-cage motor, operating at rated load, has a speed of 595 r/min, an efficiency of 89 percent, and a power factor of 81 percent lagging. The locked-rotor current, and locked-rotor torque at rated voltage and rated frequency are 725 A and 120 percent rated torque, respectively. Determine (a) rated line current; (b) rated torque; (c) minimum voltage required to obtain a blocked-rotor torque equal to 70 percent rated torque; (d) autotransformer bank ratio required to provide the minimum voltage in (c); (e) stator current at blocked rotor with autotransformer in circuit; (f) corresponding input current to the transformer when starting the motor at the voltage in (c).

5–38/20 A 50-hp, six-pole, three-phase, 450-V, 60-Hz, 1120 r/min induction motor, operating at rated conditions, has an efficiency of 91 percent and a power factor of 89 percent. When starting at rated voltage, the motor develops 170 percent rated torque and draws five times rated current. Determine (a) slip at rated conditions; (b) rated torque; (c) blocked-rotor torque; (d) rated line current. (e) Design a V–V transformer bank of the correct turns ratio that will limit the stator line current to 200 percent rated current. (f) Determine the primary line current for the conditions in (e); (g) motor line current when starting at the reduced voltage.

5–39/20 A three-phase, four-pole, 460-V, 60-Hz, 200-hp, design *B* induction motor has a locked-rotor torque equal to 125 percent rated torque when rated voltage is applied. The efficiency power factor and slip at rated conditions are 92, 82, and 2.0 percent, respectively. The locked-rotor current at rated voltage is 1450 A. The motor is to be used to drive a centrifugal pump whose specifications require the motor to have a minimum starting torque of 357 ft-lb. Determine (a) rated line current; (b) rated torque; (c) minimum stator voltage required to start the load; (d) required transformer bank ratio; (e) and then sketch the circuit showing the motor and wye-connected autotransformer.

5–40/20 A three-phase, design *C*, 25-hp, 60-Hz, 27-A, 575-V, six-pole motor has a rated speed of 1164 r/min. The locked-rotor impedance is $3.49\angle 78.18°$ Ω/phase, and the motor was designed for wye-delta starting. Determine (a) locked-rotor line current and phase current when wye starting; (b) locked-rotor line current and phase current when delta starting; (c) the expected minimum locked-rotor torque if delta connected; (d) the expected minimum locked-rotor torque if wye connected.

5–41/20 A four-pole, 30-hp, 60-Hz, 460-V, design *B*, wye-connected induction motor, operating at rated conditions, draws 40 A and has a slip of 2.89 percent. The locked rotor impedance is $1.93\angle 79.02°$ Ω/phase. (a) Design a series-resistor starter that will limit the starting current to 200 percent rated current; (b) determine the stator voltage per phase at locked rotor; (c) determine the expected minimum locked-rotor torque when using the starting resistance.

5–42/20 A 75-hp, 60-Hz, 2300-V, 20-A, two-pole, wye-connected, design *B* motor with Code K delivers rated horsepower at 3500 r/min. Assume the phase angle of the locked-rotor impedance is 75°. Determine (a) the required reactance of a series-reactor starter that will limit the starting current to 350 percent of rated current; (b) the expected minimum locked-rotor torque as a percent of rated torque when starting with the series reactor.