MBW damage model history and outlook

- Monotonic loading case

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This version was presented on 20.03.17.
Updates and corrections will be added in future versions.

Subgroup
MBW damage model
Discussion held
on Nov 2 2016

MBW Damage model history & outlook / monotonic

Versions	Features	WMAT	IS	loc.
MBW1.0(M)	1.2, 2.1, 3.1, 4.1 5.1, 6.1, 7.1.	✓	Expl.	O
MBW2.0(M)				o
MBW3.0(M)				o
MBW3.0(P.Y)	1.3*, 2.1, 3.3, 4.2, 5.1	✓	Expl.	O
MBW4.0(Bo)	1.2, 2.2, 3.4, 4.3, 5.2 6.1, 7.1	✓	"	O
MBW5.0(J)	1.2, 2.1, 3.1, 4.1, 5.1 6.4, 7.2	✓	"	O
MBW6.0(G)	1.3, 2.1, 3.4, 4.2, 5.1 6.3, 7.1	✓	"	G

- $\dot{\underline{\varepsilon}} = \dot{\underline{\varepsilon}}^e + \dot{\underline{\varepsilon}}^p$

- $\underline{\Phi} = \bar{\sigma} - (1-D)\sigma_y \leq 0$

$\sigma_y = \bar{\sigma}_y(\bar{\varepsilon}^p) \cdot \cancel{f(y)} \cdot f(\bar{\theta})$ (1,1)

$\sigma_y = \bar{\sigma}(\bar{\varepsilon}^p, \dot{\bar{\varepsilon}}^p) \cdot f(y) \cdot f(\bar{\theta}) \cdot f(T)$ (1,2)

$* \bar{\sigma}(\bar{\varepsilon}^p, \dot{\bar{\varepsilon}}^p) = \bar{\sigma}_y(\bar{\varepsilon}^p) \cdot (C_1^{\dot{\varepsilon}^p} \ln \dot{\bar{\varepsilon}}^p + C_2^{\dot{\varepsilon}^p}) + C_3^{\dot{\varepsilon}^p} \cdot \dot{\bar{\varepsilon}}^p$

$f(T) = C_1^T \cdot \exp(C_2^T \cdot T) + C_3^T$

$* \bar{\sigma}(\bar{\varepsilon}^p, \dot{\bar{\varepsilon}}^p) \Rightarrow \sigma(\bar{\varepsilon}^p, f(\dot{\bar{\varepsilon}}^p, \bar{\varepsilon}^p))$ (1,3)

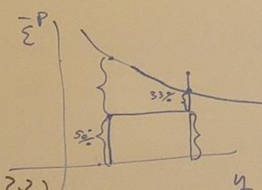
$f(T) \Rightarrow f(T, \bar{\varepsilon}^p)$

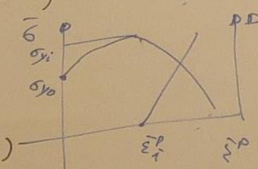
- DDI

. $\bar{\varepsilon}^p = \bar{\varepsilon}^p_i$ (2,1)

$\bar{\varepsilon}^p_{ddi} = (C_1 - C_6)(y, \bar{\theta})$

. $I_{dd} = \int \frac{d\bar{\varepsilon}^p}{\bar{\varepsilon}^p_{ddi}(y, \bar{\theta})} = 1$ (2,2)





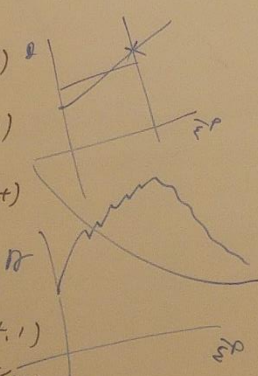
- DDE

. $\dot{D} = \frac{\sigma_{yi} \cdot L}{2G_f} \cdot \dot{\bar{\varepsilon}}^p$ (3,1)

. $\dot{D} = \frac{\sigma_{yo} \cdot L}{2G_f} \cdot \dot{\bar{\varepsilon}}^p$ (3,2)

. $\dot{D} = \frac{\sigma_{yi}(\sigma_{yo})}{2G_f} \cdot \dot{\bar{\varepsilon}}^p$ (3,3)

. $\dot{D} = \frac{\sigma_{yi}(\sigma_{yo})}{G_f} \cdot \dot{\bar{\varepsilon}}^p$ (3,4)



- DF

. $D = D_{cr}$ (constant) (4,1)

. $D_{cr} = C_1^{cr} \cdot \bar{\theta}^2 + C_2^{cr}$ (4,2)

. $D_{cr} = (D_1 - D_6)(y, \bar{\theta})$ (4,3)

. $I_{df} = \int \frac{dD}{D_{cr}(y, \bar{\theta})} = 1$

- Cut-off values.

. $y \leq y_c = 0 \Rightarrow \dot{D} = 0$ (5,1)

. $y \leq y_c = -\frac{1}{3} \Rightarrow \dot{I} = 0$ & $\dot{D}_F = 0$ (5,2)

. $y_c = f(\bar{\theta})$ (5,3)

- Cleavage Fracture iniciation

. $\sigma_1 = \sigma_c$ (cons.) (6,1)

. $\sigma_1 = \sigma_c$ (cons.) $\wedge$ $\bar{\varepsilon}^p = \bar{\varepsilon}^p_{mdi}$ (6,2)

$* \bar{\varepsilon}^p_{mdi} = (C_1 - C_6)(y, \bar{\theta})$

$* \bar{\varepsilon}^p_{mdi} = \bar{\varepsilon}_u \left(\ln \left(\frac{1}{1 - P_{mdi}} \right) \right)^{1/m}$ (6,3)

$\bar{\varepsilon}_u = (C_1 - C_6)(y, \bar{\theta})$

. $\sigma_1 = \sigma_c$ (cons.) $\wedge$ $I_{CF} = \int \frac{d\bar{\varepsilon}^p}{\bar{\varepsilon}^p_{mdi}} = 1$ (6,4)

- Ductile & Cleavage interaction

$$D = \begin{cases} 0 & (\bar{\varepsilon}^p < \bar{\varepsilon}^p_{mdi}) \quad I_{CF} < 1 \\ 0 & (\bar{\varepsilon}^p_{mdi} \leq \bar{\varepsilon}^p < \bar{\varepsilon}^p_{ddi} \wedge \sigma_1 < \sigma_c) \\ 1 & (I_{CF} = 1 \wedge I_{dd} < 1, \wedge \sigma_1 \geq \sigma_c) \\ 0 & (I_{dd} < 1) \\ D & (I_{dd} \geq 1, I_{df} < 1) \\ 1 & (I_{df} = 1) \end{cases}$$ (7,1)

Basic equations in MBW

$$\varepsilon = \varepsilon^{\mathrm{e}} + \varepsilon^{\mathrm{p}}$$

Yield function.

$$\Phi = \bar{\sigma}(\boldsymbol{\sigma}) - (1 - D)\sigma_{\mathrm{y}} \leq 0$$

1. Flow curves

The original one without strain rate and T effects.

$$\sigma_{\mathrm{y}} = \bar{\sigma}_{\mathrm{y}}(\bar{\varepsilon}^{\mathrm{p}}) \cdot f(\eta) \cdot f(\bar{\theta}) \qquad 1.1$$

The one with strain rate and T effects.

$$\sigma_{\mathrm{y}} = \bar{\sigma}_{\mathrm{y}}(\bar{\varepsilon}^{\mathrm{p}}, \dot{\varepsilon}^{\mathrm{p}}) \cdot f(\eta) \cdot f(\bar{\theta}) \cdot f(T) \qquad 1.2$$

- $\bar{\sigma}_{\mathrm{y}}\left(\bar{\varepsilon}^{\mathrm{p}}, \dot{\varepsilon}^{\mathrm{p}}\right) = \bar{\sigma}_{\mathrm{y}}\left(\bar{\varepsilon}^{\mathrm{p}}\right) \cdot \left(C_1^{\dot{\varepsilon}^{\mathrm{p}}} \ln\left(\dot{\varepsilon}^{\mathrm{p}}\right) + C_2^{\dot{\varepsilon}^{\mathrm{p}}}\right) + C_3^{\dot{\varepsilon}^{\mathrm{p}}} \cdot \left(\dot{\varepsilon}^{\mathrm{p}}\right)$
- $f(T) = C_1^T \cdot exp(C_2^T \cdot T) + C_3^T$

The new one with strain rate and T effects, which are dependent on $\bar{\varepsilon}^{\mathrm{p}}$.

$$\sigma_{\mathrm{y}} = \bar{\sigma}_{\mathrm{y}}(\bar{\varepsilon}^{\mathrm{p}}, \dot{\varepsilon}^{\mathrm{p}}) \cdot f(\eta) \cdot f(\bar{\theta}) \cdot f(T) \qquad 1.3$$

- $\bar{\sigma}_{\mathrm{y}}\left(\bar{\varepsilon}^{\mathrm{p}}, \dot{\varepsilon}^{\mathrm{p}}\right) = \bar{\sigma}_{\mathrm{y}}\left(\bar{\varepsilon}^{\mathrm{p}}, f\left(\dot{\varepsilon}^{\mathrm{p}}, \bar{\varepsilon}^{\mathrm{p}}\right)\right)$
- $f(T) = f\left(T, \bar{\varepsilon}^{\mathrm{p}}\right)$

2. Ductile damage initiation (DDI)

The original one without loading history dependence.

$$\bar{\varepsilon}^{\mathrm{p}} = \bar{\varepsilon}^{\mathrm{p}}_{\mathrm{ddi}} \tag{2.1}$$

- $\bar{\varepsilon}^{\mathrm{p}}_{\mathrm{ddi}} = f(\eta, \bar{\theta} | C_1 \sim C_6)$

The one with loading history dependence.

$$I_{\mathrm{dd}} = \int \frac{d\bar{\varepsilon}^{\mathrm{p}}}{\bar{\varepsilon}^{\mathrm{p}}_{\mathrm{ddi}}(\eta, \bar{\theta})} = 1 \tag{2.2}$$

3. Ductile damage evolution (DDE)

$$\dot{D} = \frac{\sigma_{\mathrm{yi}} \cdot L}{2 \cdot G_f} \cdot \dot{\varepsilon}^p \tag{3.1}$$

$$\dot{D} = \frac{\sigma_{\mathrm{y0}} \cdot L}{2 \cdot G_{\mathrm{f}}} \cdot \dot{\varepsilon}^{\mathrm{p}} \tag{3.2}$$

$$\dot{D} = \frac{\sigma_{\mathrm{yi}}\left(\sigma_{\mathrm{y0}}\right)}{2 \cdot G_{\mathrm{f}}} \cdot \dot{\varepsilon}^{\mathrm{p}} \tag{3.3}$$

$$\dot{D} = \frac{\sigma_{\mathrm{yi}}\left(\sigma_{\mathrm{y0}}\right)}{G_{\mathrm{f}}} \cdot \dot{\varepsilon}^{\mathrm{p}} \tag{3.4}$$

4. Ductile fracture (DF)

The original one without stress state dependence.

$$D = D_{\mathrm{cr}} \tag{4.1}$$

- $D_{\mathrm{cr}} = \mathrm{constant}$

The new one with stress state dependence.

$$D = D_{\mathrm{cr}} \tag{4.2}$$

- $D_{\mathrm{cr}} = C_1^{\mathrm{cr}} \cdot \bar{\theta}^2 + C_2^{\mathrm{cr}}$

The new one with stress state dependence and considering loading history effects.

$$I_{\mathrm{df}} = \int \frac{dD}{D_{\mathrm{cr}}(\eta, \bar{\theta})} = 1 \tag{4.3}$$

- $D_{\mathrm{cr}} = f(\eta, \bar{\theta} | C_1 \sim C_6)$

5. Cut-off values

$$none \tag{5.0}$$

$$\eta \leq \eta_{\mathrm{c}} = 0 \qquad \Rightarrow \dot{D} = 0 \tag{5.1}$$

$$\eta \leq \eta_{\mathrm{c}} = -\frac{1}{3} \qquad \Rightarrow \dot{I} = 0 \quad \& \quad \dot{D}_{\mathrm{f}} = 0 \tag{5.2}$$

$$\eta \leq \eta_{\mathrm{c}} = f(\bar{\theta}) \tag{5.3}$$

5. Cut-off values

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$$\eta \leq \eta_{\mathrm{c}} = f(\bar{\theta}) \tag{5.3}$$

6. Cleavage fracture initiation (CFI)

The original one.

$$\sigma_1 = \sigma_\mathrm{c} \tag{6.1}$$

- $\sigma_\mathrm{c} = \mathrm{constant}$

The new one with stress state dependence.

$$\sigma_1 = \sigma_\mathrm{c} \quad \wedge \quad \bar{\varepsilon}^\mathrm{p} = \bar{\varepsilon}^\mathrm{p}_\mathrm{mdi} \tag{6.2}$$

- $\sigma_\mathrm{c} = \mathrm{constant}$
- $\bar{\varepsilon}^\mathrm{p}_\mathrm{mdi} = f(\eta, \bar{\theta} | C_1 \sim C_6)$

The new one with stress state dependence and P value.

$$\sigma_1 = \sigma_\mathrm{c} \quad \wedge \quad \bar{\varepsilon}^\mathrm{p} = \bar{\varepsilon}^\mathrm{p}_\mathrm{mdi} \tag{6.3}$$

- $\sigma_\mathrm{c} = \mathrm{constant}$
- $\bar{\varepsilon}^\mathrm{p}_\mathrm{mdi} = \bar{\varepsilon}^\mathrm{p}_\mathrm{u} \cdot \left(\ln \left(\frac{1}{1 - P_\mathrm{mdi}} \right) \right)^{1/m}$
- $\bar{\varepsilon}^\mathrm{p}_\mathrm{u} = f(\eta, \bar{\theta} | C_1 \sim C_6)$

The new one with loading history dependence.

$$\sigma_1 = \sigma_\mathrm{c} \quad \wedge \quad I_\mathrm{cf} = \int \frac{d\bar{\varepsilon}^\mathrm{p}}{\bar{\varepsilon}^\mathrm{p}_\mathrm{mdi}(\eta, \bar{\theta})} = 1 \tag{6.4}$$

- $\sigma_\mathrm{c} = \mathrm{constant}$
- $\bar{\varepsilon}^\mathrm{p}_\mathrm{mdi} = f(\eta, \bar{\theta} | C_1 \sim C_6)$

7. Ductile & cleavage interaction

The original one.

$$D = \begin{cases} \begin{cases} 0 & \bar{\varepsilon}^{\mathrm{p}} < \bar{\varepsilon}^{\mathrm{p}}_{\mathrm{mdi}} \\ 0 & \bar{\varepsilon}^{\mathrm{p}}_{\mathrm{mdi}} < \bar{\varepsilon}^{\mathrm{p}} < \bar{\varepsilon}^{\mathrm{p}}_{\mathrm{ddi}} \quad \wedge \quad \sigma_1 < \sigma_{\mathrm{c}} \\ 1 & \bar{\varepsilon}^{\mathrm{p}}_{\mathrm{mdi}} < \bar{\varepsilon}^{\mathrm{p}} < \bar{\varepsilon}^{\mathrm{p}}_{\mathrm{ddi}} \quad \wedge \quad \sigma_1 \geq \sigma_{\mathrm{c}} \end{cases} \\ 0 \quad I_{\mathrm{dd}} < 1 \\ 0 \quad I_{\mathrm{dd}} \geq 1 \quad \wedge \quad I_{\mathrm{df}} < 1 \\ 1 \quad I_{\mathrm{df}} = 1 \end{cases} \tag{7.1}$$

The new one.

$$D = \begin{cases} \begin{cases} 0 & I_{\mathrm{cf}} < 1 \\ 0 & I_{\mathrm{cf}} = 1, I_{\mathrm{dd}} < 1 \quad \wedge \quad \sigma_1 < \sigma_{\mathrm{c}} \\ 1 & I_{\mathrm{cf}} = 1, I_{\mathrm{dd}} < 1 \quad \wedge \quad \sigma_1 \geq \sigma_{\mathrm{c}} \end{cases} \\ 0 \quad I_{\mathrm{dd}} < 1 \\ 0 \quad I_{\mathrm{dd}} \geq 1 \quad \wedge \quad I_{\mathrm{df}} < 1 \\ 1 \quad I_{\mathrm{df}} = 1 \end{cases} \tag{7.2}$$

Basic equations in MBW

$$\varepsilon = \varepsilon^{\mathrm{e}} + \varepsilon^{\mathrm{p}}$$

Yield function.

$$\Phi = \bar{\sigma}(\boldsymbol{\sigma}) - (1 - D)\sigma_{\mathrm{y}} \leq 0$$

1. Flow curves

The original one without strain rate and T effects.

$$\sigma_{\mathrm{y}} = \bar{\sigma}_{\mathrm{y}}(\bar{\varepsilon}^{\mathrm{p}}) \cdot f(\eta) \cdot f(\bar{\theta}) \quad 1.1$$

The one with strain rate and T effects.

$$\sigma_{\mathrm{y}} = \bar{\sigma}_{\mathrm{y}}(\bar{\varepsilon}^{\mathrm{p}}, \dot{\bar{\varepsilon}}^{\mathrm{p}}) \cdot f(\eta) \cdot f(\bar{\theta}) \cdot f(T) \quad 1.2$$

- $\bar{\sigma}_{\mathrm{y}}(\bar{\varepsilon}^{\mathrm{p}}, \dot{\bar{\varepsilon}}^{\mathrm{p}}) = \bar{\sigma}_{\mathrm{y}}(\bar{\varepsilon}^{\mathrm{p}}) \cdot \left(C_1^{\dot{\bar{\varepsilon}}\mathrm{p}} \ln(\dot{\bar{\varepsilon}}^{\mathrm{p}}) + C_2^{\dot{\bar{\varepsilon}}\mathrm{p}}\right) + C_3^{\dot{\bar{\varepsilon}}\mathrm{p}} \cdot (\dot{\bar{\varepsilon}}^{\mathrm{p}})$
- $f(T) = C_1^T \cdot exp(C_2^T \cdot T) + C_3^T$

The new one with strain rate and T effects, which are dependent on $\bar{\varepsilon}^{\mathrm{p}}$.

$$\sigma_{\mathrm{y}} = \bar{\sigma}_{\mathrm{y}}(\bar{\varepsilon}^{\mathrm{p}}, \dot{\bar{\varepsilon}}^{\mathrm{p}}) \cdot f(\eta) \cdot f(\bar{\theta}) \cdot f(T) \quad 1.3$$

- $\bar{\sigma}_{\mathrm{y}}(\bar{\varepsilon}^{\mathrm{p}}, \dot{\bar{\varepsilon}}^{\mathrm{p}}) = \bar{\sigma}_{\mathrm{y}}\left(\bar{\varepsilon}^{\mathrm{p}}, f(\dot{\bar{\varepsilon}}^{\mathrm{p}}, \bar{\varepsilon}^{\mathrm{p}})\right)$
- $f(T) = f(T, \bar{\varepsilon}^{\mathrm{p}})$

2. Ductile damage initiation (DDI)

The original one without loading history dependence.

$$\bar{\varepsilon}^{\mathrm{p}} = \bar{\varepsilon}^{\mathrm{p}}_{\mathrm{ddi}} \quad 2.1$$

- $\bar{\varepsilon}^{\mathrm{p}}_{\mathrm{ddi}} = f(\eta, \bar{\theta} | C_1 \sim C_6)$

The one with loading history dependence.

$$I_{\mathrm{dd}} = \int \frac{d\bar{\varepsilon}^{\mathrm{p}}}{\bar{\varepsilon}^{\mathrm{p}}_{\mathrm{ddi}}(\eta, \bar{\theta})} = 1 \quad 2.2$$

3. Ductile damage evolution (DDE)

$$\dot{D} = \frac{\sigma_{\mathrm{yi}} \cdot L}{2 \cdot G_f} \cdot \dot{\bar{\varepsilon}}^{\mathrm{p}} \quad 3.1$$

$$\dot{D} = \frac{\sigma_{\mathrm{y0}} \cdot L}{2 \cdot G_{\mathrm{f}}} \cdot \dot{\bar{\varepsilon}}^{\mathrm{p}} \quad 3.2$$

$$\dot{D} = \frac{\sigma_{\mathrm{yi}}(\sigma_{\mathrm{y0}})}{2 \cdot G_{\mathrm{f}}} \cdot \dot{\bar{\varepsilon}}^{\mathrm{p}} \quad 3.3$$

$$\dot{D} = \frac{\sigma_{\mathrm{yi}}(\sigma_{\mathrm{y0}})}{G_{\mathrm{f}}} \cdot \dot{\bar{\varepsilon}}^{\mathrm{p}} \quad 3.4$$

4. Ductile fracture (DF)

The original one without stress state dependence.

$$D = D_{\mathrm{cr}} \quad 4.1$$

- $D_{\mathrm{cr}} = \mathrm{constant}$

The new one with stress state dependence.

$$D = D_{\mathrm{cr}} \quad 4.2$$

- $D_{\mathrm{cr}} = C_1^{\mathrm{cr}} \cdot \bar{\theta}^2 + C_2^{\mathrm{cr}}$

The new one with stress state dependence and considering loading history effects.

$$I_{\mathrm{df}} = \int \frac{dD}{D_{\mathrm{cr}}(\eta, \bar{\theta})} = 1 \quad 4.3$$

- $D_{\mathrm{cr}} = f(\eta, \bar{\theta} | C_1 \sim C_6)$

5. Cut-off values

$none$ 5.0

$\eta \leq \eta_c = 0 \qquad \Rightarrow \dot{D} = 0$ 5.1

$\eta \leq \eta_c = -\frac{1}{3} \qquad \Rightarrow \dot{I} = 0 \quad \& \quad \dot{D}_{\mathrm{f}} = 0$ 5.2

$\eta \leq \eta_c = f(\bar{\theta})$ 5.3

6. Cleavage fracture initiation (CFI)

The original one.

$$\sigma_1 = \sigma_{\mathrm{c}} \quad 6.1$$

- $\sigma_{\mathrm{c}} = \mathrm{constant}$

The new one with stress state dependence.

$$\sigma_1 = \sigma_{\mathrm{c}} \quad \wedge \quad \bar{\varepsilon}^{\mathrm{p}} = \bar{\varepsilon}^{\mathrm{p}}_{\mathrm{mdi}} \quad 6.2$$

- $\sigma_{\mathrm{c}} = \mathrm{constant}$
- $\bar{\varepsilon}^{\mathrm{p}}_{\mathrm{mdi}} = f(\eta, \bar{\theta} | C_1 \sim C_6)$

The new one with stress state dependence and P value

$$\sigma_1 = \sigma_{\mathrm{c}} \quad \wedge \quad \bar{\varepsilon}^{\mathrm{p}} = \bar{\varepsilon}^{\mathrm{p}}_{\mathrm{mdi}} \quad 6.3$$

- $\sigma_{\mathrm{c}} = \mathrm{constant}$
- $\bar{\varepsilon}^{\mathrm{p}}_{\mathrm{mdi}} = \bar{\varepsilon}^{\mathrm{p}}_{\mathrm{u}} \cdot \left(\ln\left(\frac{1}{1 - P_{\mathrm{mdi}}}\right)\right)^{1/m}$
- $\bar{\varepsilon}^{\mathrm{p}}_{\mathrm{u}} = f(\eta, \bar{\theta} | C_1 \sim C_6)$

The new one with loading history dependence.

$$\sigma_1 = \sigma_{\mathrm{c}} \quad \wedge \quad I_{\mathrm{cf}} = \int \frac{d\bar{\varepsilon}^{\mathrm{p}}}{\bar{\varepsilon}^{\mathrm{p}}_{\mathrm{mdi}}(\eta, \bar{\theta})} = 1 \quad 6.4$$

- $\sigma_{\mathrm{c}} = \mathrm{constant}$
- $\bar{\varepsilon}^{\mathrm{p}}_{\mathrm{mdi}} = f(\eta, \bar{\theta} | C_1 \sim C_6)$

7. Ductile & cleavage interaction

The original one.

$$D = \begin{cases} 0 & \bar{\varepsilon}^{\mathrm{p}} < \bar{\varepsilon}^{\mathrm{p}}_{\mathrm{mdi}} \\ 0 & \bar{\varepsilon}^{\mathrm{p}}_{\mathrm{mdi}} < \bar{\varepsilon}^{\mathrm{p}} < \bar{\varepsilon}^{\mathrm{p}}_{\mathrm{ddi}} \quad \wedge \quad \sigma_1 < \sigma_{\mathrm{c}} \\ 1 & \bar{\varepsilon}^{\mathrm{p}}_{\mathrm{mdi}} < \bar{\varepsilon}^{\mathrm{p}} < \bar{\varepsilon}^{\mathrm{p}}_{\mathrm{ddi}} \quad \wedge \quad \sigma_1 \geq \sigma_{\mathrm{c}} \\ 0 & I_{\mathrm{dd}} < 1 \\ 0 & I_{\mathrm{dd}} \geq 1 \quad \wedge \quad I_{\mathrm{df}} < 1 \\ 1 & I_{\mathrm{df}} = 1 \end{cases} \quad 7.1$$

$$D = \begin{cases} 0 & I_{\mathrm{cf}} < 1 \\ 0 & I_{\mathrm{cf}} = 1, I_{\mathrm{dd}} < 1 \quad \wedge \quad \sigma_1 < \sigma_{\mathrm{c}} \\ 1 & I_{\mathrm{cf}} = 1, I_{\mathrm{dd}} < 1 \quad \wedge \quad \sigma_1 \geq \sigma_{\mathrm{c}} \\ 0 & I_{\mathrm{dd}} < 1 \\ 0 & I_{\mathrm{dd}} \geq 1 \quad \wedge \quad I_{\mathrm{df}} < 1 \\ 1 & I_{\mathrm{df}} = 1 \end{cases} \quad 7.2$$

Internal name	Features	Subroutine	IS	Location	Developer	Comment
MBW 1.0	1.1; 2.1; 3.1; 4.1; 5.0	VUMAT	Expl.	O	Sharaf/Lian	
MBW 2.0	1.2; 2.1; 3.1; 4.1; 5.1; 6.1; 7.1	VUMAT	Expl.	O	Sharaf/Lian	Equivalent to bw-001.f
MBW 2.1	1.1; 2.1; 3.1; 4.1; 5.1; 6.1; 7.1	VUMAT	Impl.	O	Sharaf/Lian	Unverified
MBW 3.0	1.2; 2.1; 3.4; 4.2; 5.1	VUMAT	Expl.	O	Novokshanov	
MBW 4.0	1.2; 2.2; 3.4; 4.3; 5.2; 6.1; 7.1	VUMAT	Expl.	O	Wu	
MBW 5.0	1.2; 2.1; 3.1; 4.1; 5.1; 6.4; 7.2	VUMAT	Expl.	O	He	
MBW 6.0	1.3; 2.1; 3.4; 4.2; 5.1; 6.3; 7.1	VUMAT	Expl.	GG	Golisch	
MBW 7.0	1.2; 2.2; 3.4; 4.3; 5.2; 6.4; 7.2	VUMAT	Expl.	O	Wu/He/Shen	

Internal name	Features	Subroutine	IS	Location	Developer	Comment
MBW 1.0	1.1; 2.1; 3.1; 4.1; 5.0	VUMAT	Expl.	O	Sharaf/Lian	
MBW 2.0	1.2; 2.1; 3.1; 4.1; 5.1; 6.1; 7.1	VUMAT	Expl.	O	Sharaf/Lian	Equivalent to bw-001.f
MBW 2.1	1.1; 2.1; 3.1; 4.1; 5.1; 6.1; 7.1	VUMAT	Impl.	O	Sharaf/Lian	Unverified
MBW 3.0	1.2; 2.1; 3.4; 4.2; 5.1	VUMAT	Expl.	O	Novokshanov	
MBW 4.0	1.2; 2.2; 3.4; 4.3; 5.2; 6.1; 7.1	VUMAT	Expl.	O	Wu	
MBW 5.0	1.2; 2.1; 3.1; 4.1; 5.1; 6.4; 7.2	VUMAT	Expl.	O	He	
MBW 6.0	1.3; 2.1; 3.4; 4.2; 5.1; 6.3; 7.1	VUMAT	Expl.	GG	Golisch	
MBW 7.0	1.2; 2.2; 3.4; 4.3; 5.2; 6.4; 7.2	VUMAT	Expl.	O	Wu/He/Shen	