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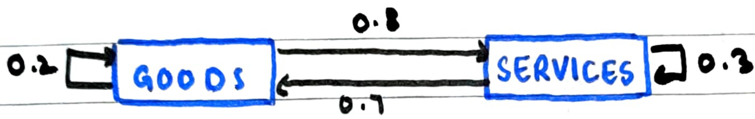
Date:

Assignment #01 (Exercise 1.7)

QUESTION # 1

Suppose an economy has two sectors, Goods and Services. Each year, goods sells 80% of its output to Services and keeps the rest. Services sells 70% of its output to ~~ser~~ goods and keeps the rest. Find the equilibrium prices



Denoting the ~~prices~~ annual output of Goods by P_G and of Services by P_S.

→ Total income of Goods = P_G

→ Total income of Services = P_S

Using the values given,

$$0.2P_G + 0.7P_S = P_G \quad \text{—— A}$$

$$0.8P_G + 0.3P_S = P_S. \quad \text{—— B}$$

Equations A and B can also be written as.

$$0.2P_G + 0.7P_S - P_G = 0 \Rightarrow -0.8P_G + 0.7P_S = 0$$

$$0.8P_G + 0.3P_S - P_S = 0 \Rightarrow 0.8P_G - 0.7P_S = 0.$$

→ Writing the equations in augmented Matrix form.

$$\left[\begin{array}{cc|c} -0.8 & 0.7 & 0 \\ 0.8 & -0.7 & 0 \end{array}\right]$$

Applying Row Reduction Algorithm

$R_2 = R_1 + R_2.$

$$\left[\begin{array}{cc|c} -0.8 & 0.7 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$R_1 = R_1 \times \frac{1}{-0.8}$

$$\left[\begin{array}{cc|c} 1 & -0.875 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

P_S → Free variable

$P_G - 0.875P_S = 0 \rightarrow \boxed{P_G = 0.875P_S}$

General Solution: $P_G = 0.875P_S$

Date:

QUESTION # 2

Find another set of equilibrium prices for the economy in Ex 1. Suppose the same economy used Japanese yen instead of dollars to measure the value of various sectors output. Would this change the problem in any ways? Discuss.

Coal	Electric	Steel	Purchased by
0.0	0.4	0.6	Coal
0.6	0.1	0.2	Electric
0.4	0.5	0.2	Steel

According to Ex 1. The equilibrium price vector for the economy has the form

$$P = \begin{bmatrix} P_C \\ P_E \\ P_S \end{bmatrix} = \begin{bmatrix} 0.94P_S \\ 0.85P_E \\ 1P_S \end{bmatrix} = P_S \begin{bmatrix} 0.94 \\ 0.85 \\ 1 \end{bmatrix}$$

When P_S is \$100\$ million, P_C is \$94 Million and P_E is \$85 million.

→ For a new set of Prices, lets suppose P_S = 500 Million Dollars.

Then, according to the equilibrium price ratio,

$P_C = 0.94 \times 500 =$ \$470 Million

$P_E = 0.85 \times 500 =$ \$425 Million

→ Changing the currency will not change the ratio of the prices. It will have the same affect as multiplying/dividing the equilibrium prices by a constant number. It doesn't matter what the number is as long as it is non-negative and all of the prices are being multiplied/divided by it.

Date: ______

QUESTION #7

Alka-Seltzer contains sodium bicarbonate ($NaHCO_3$) and citric acid ($H_3C_6H_5O_7$). When a tablet is dissolved in water the following reaction occurs.

$$NaHCO_3 + H_3C_6H_5O_7 \rightarrow Na_3C_6H_5O_7 + H_2O + CO_2$$

The number of each atom in a molecule can be written as.

$$\begin{array}{c} Na \\ H \\ C \\ O \end{array} \quad NaHCO_3 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix}, \; H_3C_6H_5O_7 \begin{bmatrix} 0 \\ 8 \\ 6 \\ 7 \end{bmatrix}, \; Na_3C_6H_5O_7 \begin{bmatrix} 3 \\ 5 \\ 6 \\ 7 \end{bmatrix}$$

$$H_2O \begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix} \quad CO_2 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}$$

Denoting the molecules by $x_1 \cdots x_5$ notations.

$$x_1 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 8 \\ 6 \\ 7 \end{bmatrix} \rightarrow x_3 \begin{bmatrix} 3 \\ 5 \\ 6 \\ 7 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}$$

① $x_1 = 3x_3 \rightarrow x_1 - 3x_3 = 0.$

② $x_1 + 8x_2 = 5x_3 + 2x_4 \rightarrow x_1 + 8x_2 - 5x_3 - 2x_4 = 0$

③ $x_1 + 6x_2 = 6x_3 + x_5 \rightarrow x_1 + 6x_2 - 6x_3 - x_5 = 0.$

④ $3x_1 + 7x_2 = 7x_3 + x_4 + 2x_5 \rightarrow 3x_1 + 7x_2 - 7x_3 - x_4 - 2x_5 = 0$

Transforming the equations to augumented matrix

$$\left[\begin{array}{ccccc:c} 1 & 0 & -3 & 0 & 0 & 0 \\ 1 & 8 & -5 & -2 & 0 & 0 \\ 1 & 6 & -6 & 0 & -1 & 0 \\ 3 & 7 & -7 & -1 & -2 & 0 \end{array}\right]$$

Date:

Applying row reduction techniques on the matrix

→ $R_2 = R_2 - R_1$
$R_3 = R_3 - R_1$
$R_4 = R_4 - 3R_1$

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 8 & -2 & -2 & 0 & 0 \\ 0 & 6 & -3 & 0 & -1 & 0 \\ 0 & 7 & 2 & -1 & -2 & 0 \end{bmatrix}$$

→ $R_2 = R_2/8 = \begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 1 & -1/4 & -1/4 & 0 & 0 \\ 0 & 6 & -3 & 0 & -1 & 0 \\ 0 & 7 & 2 & -1 & -2 & 0 \end{bmatrix}$

→ $R_3 = R_3 - 6R_2$, $R_4 = R_4 - 7R_2$

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 1 & -1/4 & -1/4 & 0 & 0 \\ 0 & 0 & -3/2 & 3/2 & -1 & 0 \\ 0 & 0 & 15/4 & 3/4 & -2 & 0 \end{bmatrix}$$

$R_3 = -\frac{2}{3}R_3 = \begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 1 & -1/4 & -1/4 & 0 & 0 \\ 0 & 0 & 1 & -1 & 2/3 & 0 \\ 0 & 0 & 15/4 & 3/4 & -2 & 0 \end{bmatrix}$

$R_1 = R_1 + 3R_3$ $\begin{bmatrix} 1 & 0 & 0 & -3 & 2 & 0 \\ 0 & 1 & -1/4 & -1/4 & 0 & 0 \\ 0 & 0 & 1 & -1 & 2/3 & 0 \\ 0 & 0 & 15/4 & 3/4 & -2 & 0 \end{bmatrix}$

$R_2 = R_2 + \frac{R_3}{4}$, $R_4 = R_4 - 15\frac{R_3}{4}$

$$\begin{bmatrix} 1 & 6 & 0 & -3 & 2 & 0 \\ 0 & 1 & 0 & -1/2 & 1/6 & 0 \\ 0 & 0 & 1 & -1 & 2/3 & 0 \\ 0 & 0 & 0 & 9/2 & -9/2 & 0 \end{bmatrix}$$

Date:

→ $R_4 = \frac{2R_4}{9}$ $\begin{bmatrix} 1 & 0 & 0 & -3 & 2 & 0 \\ 0 & 1 & 0 & -1/2 & 1/6 & 0 \\ 0 & 0 & 1 & -1 & 2/3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$

→ $R_1 = R_1 + 3R_4 = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1/2 & 1/6 & 0 \\ 0 & 0 & 1 & -1 & 2/3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$

→ $R_2 = R_2 + R_4/2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1/3 & 0 \\ 0 & 0 & 1 & -1 & 2/3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

→ $R_3 = R_3 + R_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1/3 & 0 \\ 0 & 0 & 1 & 0 & -4/3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

Date:

$x_1 = x_5$

$x_2 = \left(\frac{1}{3}\right) x_5$

$x_3 = \left(\frac{1}{3}\right) x_5$

$x_4 = x_5$

$x_5 \longrightarrow$ free variable.

$\rightarrow$ lets suppose

$x_5 = 3$

$x_1 = x_4 = 3.$

$x_2 = x_3 = 1.$

$3NaHCO_3 + H_3C_6H_5O_7 \rightarrow Na_3C_6H_5O_7 + 3H_2O + 3CO_2$

Solution: $3NaHCO_3 + H_3C_6H_5O_7 \rightarrow Na_3C_6H_5O_7 + 3H_2O + 3CO_2$

QUESTION # 8

$$KMnO_4 + MnSO_4 + H_2O \longrightarrow MnO_2 + K_2SO_4 + H_2SO_4$$

$$\begin{array}{c} K \\ Mn \\ O \\ S \\ H \end{array} \quad KMnO_4\,(x_1)\begin{bmatrix}1\\1\\4\\0\\0\end{bmatrix} + MnSO_4\,(x_2)\begin{bmatrix}0\\1\\4\\1\\0\end{bmatrix} + H_2O\,(x_3)\begin{bmatrix}0\\0\\1\\0\\2\end{bmatrix} \rightarrow MnO_2\,(x_4)\begin{bmatrix}0\\1\\2\\0\\0\end{bmatrix} + K_2SO_4\,(x_5)\begin{bmatrix}2\\0\\4\\1\\0\end{bmatrix} + H_2SO_4\,(x_6)\begin{bmatrix}0\\0\\4\\1\\2\end{bmatrix}$$

~~The~~ We can write the following equations from the vectors

→ $x_1 = 2x_5 \rightarrow x_1 - 2x_5 = 0.$

→ $x_1 + 4x_2 = x_4 \rightarrow x_1 + 4x_2 - x_4 = 0$

→ $4x_1 + 4x_2 + x_3 = 2x_4 + 4x_5 + 4x_6 \rightarrow 4x_1 + 4x_2 + x_3 - 2x_4 - 4x_5 - 4x_6 = 0$

→ $x_2 = x_5 + x_6 \rightarrow x_2 - x_5 - x_6 = 0$

→ $2x_4 = 2x_6 \rightarrow 2x_4 - 2x_6 = 0.$

Writing the following equations in an augmented Matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 1 & 1 & 0 & -1 & 0 & 0 & 0 \\ 4 & 4 & 1 & -2 & -4 & -4 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 2 & 0 & 0 & -2 & 0 \end{bmatrix}$$

Reducing the Matrix

$R_2 = R_2 - R_1$, $R_3 = R_3 - 4R_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 2 & 0 & 0 \\ 0 & 4 & 1 & -2 & 4 & -4 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 2 & 0 & 0 & -2 & 0 \end{bmatrix}$$

Date:

$R_3 = R_3 - 4R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & -4 & -4 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 2 & 0 & 0 & -2 & 0 \end{bmatrix}$$

$$R_4 = R_4 - R_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & -4 & -4 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 \\ 0 & 0 & 2 & 0 & 0 & -2 & 0 \end{bmatrix}$$

$R_5 = R_5 - 2R_3$, $R_2 = R_2 + R_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 2 & -4 & -4 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 \\ 0 & 0 & 0 & -4 & 8 & 6 & 0 \end{bmatrix}$$

$R_3 = R_3 - 2R_4$, $R_5 = R_5 + 4R_4$.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 \\ 0 & 0 & 0 & 0 & -4 & 2 & 0 \end{bmatrix}$$

$$R_5 = -\frac{R_5}{4} = \begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1/2 & 0 \end{bmatrix}$$

$R_1 = R_1 + 2R_5$, $R_2 = R_2 + R_5$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -3/2 & 0 \\ 0 & 0 & 1 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1/2 & 0 \end{bmatrix}$$

$$R_3 = R_3 - 2R_5 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -3/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1/2 & 0 \end{bmatrix}$$

$$R_4 = R_4 + 3R_5 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -3/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -5/2 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1/2 & 0 \end{bmatrix}$$

$x_1 = x_6$

$x_2 = (1.5)x_6$

$x_3 = x_6$

$x_4 = (2.5)x_6$

$x_5 = 0.5(x_6)$

$x_6 =$ free.

Taking $x_6 = 2$,

$x_1 = 2$

$x_3 = 2$

$x_2 = 3$

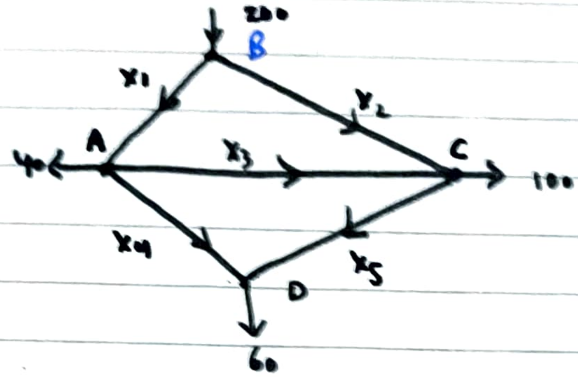
$x_4 = 5$

$x_5 = 1$.

$$\boxed{2KMnO_4 + 3MnSO_4 + 2H_2O \rightarrow 5MnO_4 + K_2SO_4 + 2H_2SO_4}$$

Date:

QUESTION # 12



a) Find the general traffic pattern in freeway network shown in the figure

Node	Flowing in	Flowing out
A	x_1	$x_4 + x_3 + 40$
B	200	$x_1 + x_2$
C	$x_2 + x_3$	$100 + x_5$
D	$x_5 + x_4$	60
Total	200	200

We can rearrange the equations,

→ $x_1 - x_4 - x_3 = 40$

$x_1 + x_2 = 200$

$x_2 + x_3 - x_5 = 100$

$x_5 + x_4 = 60$

→ Writing the following equations in Augmented Matrix

$$\left[\begin{array}{ccccc|c} 1 & 0 & -1 & -1 & 0 & 40 \\ 1 & 1 & 0 & 0 & 0 & 200 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{array}\right]$$

Date:

$\rightarrow$ $R_2 = R_2 - R_1$
$$\begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{bmatrix}$$

$R_3 = R_3 - R_2$
$$\begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 0 & 0 & -1 & -1 & -60 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{bmatrix}$$

~~Swapping~~ $R_3 = R_3 \times -1$; $R_1 = R_1 + R_3$
$$\left[\begin{array}{ccccc:c} 1 & 0 & -1 & 0 & 1 & 100 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 0 & 0 & 1 & 1 & 60 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{array}\right]$$

$R_2 = R_2 - R_3$
$$\left[\begin{array}{ccccc:c} 1 & 0 & -1 & 0 & 1 & 100 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{array}\right]$$

$R_4 = R_4 - R_3$ =
$$\begin{bmatrix} 1 & 0 & -1 & 0 & 1 & 100 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 60 \end{bmatrix}$$

x_5 = free variable

x_3 = free variable

$x_1 = 100 - x_5 + x_3$

$x_2 = 100 - x_3 + x_5$

$x_4 = 60 - x_5$.

b) x_4 is closed $\longrightarrow x_4 = 0$.

$x_5 = 60$.

$x_1 = 40 + x_3$

$x_2 = 160 - x_3$

$x_3 \rightarrow$ free variable.

c) $x_4 = 0$, min val of x_1

& $x_1 = 100 + x_3 - x_5$

$x_1 = 40 + x_3$

↳ The minimum value is 40 cars per minute since we cannot x_3 to be negative (i.e $x_3 = 0$)

HUNIA NADEEM (FA20-BCS-024)

$\boxed{Q.3}$ Part a:

Distribution of output from:

Chemical & Metal	Fuel & Power	Machinery	Purchased by
.2	.8	.4	chemical
.3	.1	.4	Fuel
.5	.1	.2	Machinery

Part b:

Let P_c, P_f, P_m be total price output of chemicals, Fuels and Machinery respectively. Thus,

Chemicals: $P_c = .2P_c + .8P_f + .4P_m$ — (i)

Fuels : $P_f = .3P_c + .1P_f + .4P_m$ — (ii)

Machinery: $P_m = .5P_c + .1P_f + .2P_m$ — (iii)

$\Rightarrow$ Eq (i) becomes:

$.2P_c - P_c + .8P_f + .4P_m = 0$

$-0.8P_c + .8P_f + .4P_m = 0$

$\Rightarrow$ Eq (ii) becomes:

$.3P_c + .1P_f - P_f + .4P_m = 0$

$.3P_c - .9P_f + .4P_m = 0$

$\Rightarrow$ Eq (iii) becomes:

$.5P_c + .1P_f + .2P_m - P_m = 0$

$.5P_c + .1P_f - .8P_m = 0$

System of Equations:

$-.8P_c + .8P_f + .4P_m = 0$

$.3P_c - .9P_f + .4P_m = 0$

$.5P_c + .1P_f - .8P_m = 0$

Augmented Matrix:

$$\begin{bmatrix} -.8 & .8 & .4 & 0 \\ .3 & -.9 & .4 & 0 \\ .5 & .1 & -.8 & 0 \end{bmatrix}$$

Reduced Echelon form Calculation:

Multiplying matrix by 10 to get whole numbers.

$$\begin{bmatrix} -8 & 8 & 4 & 0 \\ 3 & -9 & 4 & 0 \\ 5 & 1 & -8 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & -.5 & 0 \\ 3 & -9 & 4 & 0 \\ 5 & 1 & -8 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & -.5 & 0 \\ 0 & 6 & -5.5 & 0 \\ 5 & 1 & -8 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & -.5 & 0 \\ 0 & 6 & -5.5 & 0 \\ 0 & -6 & 5.5 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & -.5 & 0 \\ 0 & 1 & -.916 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1.416 & 0 \\ 0 & 1 & -0.916 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The general solution is

$P_c = 1.416 P_m$

$P_f = 0.916\, P_m$

P_m is free

Part c:

Equilibrium price vector has form:

$$P = \begin{bmatrix} P_c \\ P_f \\ P_m \end{bmatrix} = \begin{bmatrix} 1.416 P_m \\ 0.916 P_m \\ P_m \end{bmatrix} = P_m \begin{bmatrix} 1.416 \\ 0.916 \\ 1 \end{bmatrix}$$

if $P_m = 100$, then $P_c = 141.6$ and $P_f = 91.6$

Hence, when Machinery output is 100 units, Chemicals output is 141.6 units and Fuels output is 91.6 units.

$\boxed{Q.5}$

$$B_2S_3 + H_2O \rightarrow H_3BO_3 + H_2S \quad \text{— (i)}$$

Following vector lists number of atoms of boron (B), sulfur(S), hydrogen(H) and oxygen(O):

$$B_2S_3 : \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix}, \quad H_2O : \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \quad H_3BO_3 : \begin{bmatrix} 1 \\ 0 \\ 3 \\ 3 \end{bmatrix}, \quad H_2S : \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix} \begin{array}{l} \leftarrow \text{boron} \\ \leftarrow \text{sulfur} \\ \leftarrow \text{hydrogen} \\ \leftarrow \text{oxygen.} \end{array}$$

let x_1, x_2, x_3, x_4 be co-efficients of each term in eq (i), then co-efficients must satisfy:

$$x_1 \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 0 \\ 3 \\ 3 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}$$

Its augmented form becomes after moving x_3, x_4 to left side

$$\begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 3 & 0 & 0 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 0 & -3 & 2 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & -3 & 2 & 0 \\ 0 & 0 & -3 & 2 & 0 \end{bmatrix} \sim \begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & -3 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -1/2 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The general solution becomes:

$x_1 = \frac{1}{3} x_4$, $x_2 = 2x_4$, $x_3 = \frac{2}{3} x_4$, x_4 is free.

Taking $x_4 = 3$ to convert co-efficients from fraction to whole numbers; we have:

$x_1 = 1$, $x_2 = 6$, $x_3 = 2$,

Chemical equation with co-efficients:

$x_1(B_2S_3) + x_2(H_2O) \longrightarrow x_3(H_3BO_3) + x_4(H_2S)$

Balanced equation:

$B_2S_3 + 6H_2O \longrightarrow 2H_3BO_3 + 3H_2S$

Q. 14

Equations for each node:

Nodes		Flow in		Flow out
A	:	x_1	=	x_2+100
B	:	x_2+50	=	x_3
C	:	x_3	=	x_4+120
D	:	x_4+150	=	x_5
E	:	x_5	=	x_6+80
F	:	x_6+100	=	x_1

Thus,

$$x_1 - x_2 = 100$$
$$x_2 - x_3 = -50$$
$$x_3 - x_4 = 120$$
$$x_4 - x_5 = -150$$
$$x_5 - x_6 = 80$$
$$-x_1 + x_6 = -100$$

In Augmented form, we have:

$$\left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 0 & 0 & 100 \\ 0 & 1 & -1 & 0 & 0 & 0 & -50 \\ 0 & 0 & 1 & -1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ -1 & 0 & 0 & 0 & 0 & 1 & -100 \end{array}\right] \sim \left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 0 & 0 & 100 \\ 0 & 1 & -1 & 0 & 0 & 0 & -50 \\ 0 & 0 & 1 & -1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$
\sim \left[\begin{array}{cccccc|c}
1 & -1 & 0 & 0 & 0 & 0 & 100 \\
0 & 1 & -1 & 0 & 0 & 0 & -50 \\
0 & 0 & 1 & -1 & 0 & 0 & 120 \\
0 & 0 & 0 & 1 & -1 & 0 & -150 \\
0 & 0 & 0 & 0 & 1 & -1 & 80 \\
0 & 0 & -1 & 0 & 0 & 1 & -50
\end{array}\right]
\sim \left[\begin{array}{cccccc|c}
1 & -1 & 0 & 0 & 0 & 0 & 100 \\
0 & 1 & -1 & 0 & 0 & 0 & -50 \\
0 & 0 & 1 & -1 & 0 & 0 & 120 \\
0 & 0 & 0 & 1 & -1 & 0 & -150 \\
0 & 0 & 0 & 0 & 1 & -1 & 80 \\
0 & 0 & 0 & -1 & 0 & 1 & 70
\end{array}\right]
$$

$$
\sim \left[\begin{array}{cccccc|c}
1 & -1 & 0 & 0 & 0 & 0 & 100 \\
0 & 1 & -1 & 0 & 0 & 0 & -50 \\
0 & 0 & 1 & -1 & 0 & 0 & 120 \\
0 & 0 & 0 & 1 & -1 & 0 & -150 \\
0 & 0 & 0 & 0 & 1 & -1 & 80 \\
0 & 0 & 0 & 0 & -1 & 1 & -80
\end{array}\right]
\sim \left[\begin{array}{cccccc|c}
1 & -1 & 0 & 0 & 0 & 0 & 100 \\
0 & 1 & -1 & 0 & 0 & 0 & -50 \\
0 & 0 & 1 & -1 & 0 & 0 & 120 \\
0 & 0 & 0 & 1 & -1 & 0 & -150 \\
0 & 0 & 0 & 0 & 1 & -1 & 80 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
$$

$$
\sim \left[\begin{array}{cccccc|c}
1 & 0 & -1 & 0 & 0 & 0 & 50 \\
0 & 1 & -1 & 0 & 0 & 0 & -50 \\
0 & 0 & 1 & 0 & 0 & 0 & 120 \\
0 & 0 & 0 & 1 & 0 & 0 & -150 \\
0 & 0 & 0 & 0 & 1 & -1 & 80 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
\sim \left[\begin{array}{cccccc|c}
1 & 0 & 0 & -1 & 0 & 0 & 170 \\
0 & 1 & 0 & -1 & 0 & 0 & 70 \\
0 & 0 & 1 & -1 & 0 & 0 & 120 \\
0 & 0 & 0 & 1 & 0 & 0 & -150 \\
0 & 0 & 0 & 0 & 1 & -1 & 80 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
$$

$$
\sim \left[\begin{array}{cccccc|c}
1 & 0 & 0 & 0 & -1 & 0 & 20 \\
0 & 1 & 0 & 0 & -1 & 0 & -80 \\
0 & 0 & 1 & 0 & -1 & 0 & -30 \\
0 & 0 & 0 & 1 & -1 & 0 & -150 \\
0 & 0 & 0 & 0 & 1 & -1 & 80 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
\sim \left[\begin{array}{cccccc|c}
1 & 0 & 0 & 0 & 0 & -1 & 100 \\
0 & 1 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 1 & 0 & 0 & -1 & 50 \\
0 & 0 & 0 & 1 & 0 & -1 & -70 \\
0 & 0 & 0 & 0 & 1 & -1 & 80 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
$$

So the general solution is:

$x_1 = 100 + x_6$

$x_2 = x_6$

$x_3 = 50 + x_6$

$x_4 = x_6 - 70$

$x_5 = 80 + x_6$

x_6 is free

As x_4 has to be non-negative, x_6 value should be $x_6 \geq 70$. Hence, smallest possible value for x_6 is 70.

$\boxed{Q.6}$

$$Na_3PO_4 + Ba(NO_3)_2 \longrightarrow Ba_2(PO_4)_2 + NaNO_3$$

let x_1, x_2, x_3, x_4 be co-efficients of each term:

$$x_1(Na_3PO_4) + x_2(Ba(NO_3)_2) \rightarrow x_3(Ba_2(PO_4)_2) + x_4(NaNO_3)$$

Following vector lists number of atoms of sodium (Na), phosphorus (P), Oxygen (O), barium (Ba) and nitrogen (N):

$$Na_3PO_4 : \begin{bmatrix} 3 \\ 1 \\ 4 \\ 0 \\ 0 \end{bmatrix}, \quad Ba(NO_3)_2 : \begin{bmatrix} 0 \\ 0 \\ 6 \\ 1 \\ 2 \end{bmatrix}, \quad Ba_2(PO_4)_2 : \begin{array}{l} \begin{bmatrix} 0 \\ 2 \\ 8 \\ 3 \\ 0 \end{bmatrix} \end{array} \begin{array}{l} \leftarrow \text{sodium} \\ \leftarrow \text{phosphorous} \\ \leftarrow \text{oxygen} \\ \leftarrow \text{barium} \\ \leftarrow \text{nitrogen} \end{array}$$

$$NaNO_3 : \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \\ 1 \end{bmatrix} \begin{array}{l} \leftarrow \text{sodium} \\ \leftarrow \text{phosphorous} \\ \leftarrow \text{oxygen} \\ \leftarrow \text{barium} \\ \leftarrow \text{nitrogen.} \end{array}$$

The co-efficients must satisfy:

$$x_1 \begin{bmatrix} 3 \\ 1 \\ 4 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 6 \\ 1 \\ 2 \end{bmatrix} = x_3 \begin{bmatrix} 0 \\ 2 \\ 8 \\ 3 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \\ 1 \end{bmatrix}$$

Its augmented form becomes:

$$\begin{bmatrix} 3 & 0 & 0 & -1 & 0 \\ 1 & 0 & -2 & 0 & 0 \\ 4 & 6 & -8 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 4 & 6 & -8 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 0 & +6 & -1 & 0 \\ 4 & 6 & -8 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 4 & 6 & -8 & -3 & 0 \\ 0 & 0 & 6 & -1 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & -6 & 0 & 3 & 0 \\ 0 & 0 & 6 & -1 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6 & -1 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 6 & -1 & 0 \\ 0 & 2 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 6 & -1 & 0 \\ 0 & 0 & -6 & +1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 6 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & -1/6 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & -1/6 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & -1/2 & 0 \\ 0 & 0 & 1 & -1/6 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The general solution is

$$x_1 = \frac{1}{3} x_4$$

$$x_2 = \frac{1}{2} x_4$$

$$x_3 = \frac{1}{6} x_4$$

x_4 is free.

Taking $x_4 = 6$ to make co-efficients whole numbers; we have:

$$x_1 = 2, \quad x_2 = 3, \quad x_3 = 1,$$

Balanced equation becomes:

$$2Na_3PO_4 + 3Ba(NO_3)_2 \longrightarrow Ba_3(PO_4)_2 + 6NaNO_3$$

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Question - 4

(a)

Distribution output form

	A	E	M	T	
Output	0.65	.30	.30	.20	$\xrightarrow{\text{input}}$ A
	.10	.10	.15	.10	E
	.25	.35	.15	.30	M
	0	.25	.40	.40	T

(b)

Find set of equilibrium prices in economy

$$p_A = 0.65p_A + 0.30p_E + 0.30p_m + 0.20p_T$$
$$p_E = 0.10p_A + 0.10p_E + 0.15p_m + 0.10p_T$$
$$p_m = 0.25p_A + 0.35p_E + 0.15p_m + 0.30p_T$$
$$p_T = 0.25p_E + 0.40p_m + 0.40p_T$$

Moving all the equations to the left

$$0.35p_A - 0.30p_E - 0.30p_m - 0.20p_T = 0$$
$$-0.10p_A + 0.90p_E - 0.15p_m - 0.10p_T = 0$$
$$-0.25p_A - 0.35p_E + 0.85p_m - 0.30p_T = 0$$
$$-.25p_E - .40p_m + 0.60p_T = 0$$

Converting to Augmented matrix form

$$\left[\begin{array}{cccc|c} .30 & -0.30 & -.30 & .20 & 0 \\ -.10 & .90 & -.15 & -.10 & 0 \\ -.25 & -.35 & .85 & -.30 & 0 \\ 0 & -.25 & -.40 & .60 & 0 \end{array}\right]$$

Multiply all rows by 100 we get;

$$\begin{bmatrix} 35 & -30 & -30 & -20 & 0 \\ -10 & 90 & -15 & -10 & 0 \\ -25 & -35 & 85 & -30 & 0 \\ 0 & -25 & -40 & 60 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 35 & -30 & -30 & -20 & 0 \\ -10 & 90 & -15 & -10 & 0 \\ -25 & -35 & 85 & -30 & 0 \\ 0 & -25 & -40 & 60 & 0 \end{bmatrix} \sim \begin{bmatrix} 35 & -30 & -30 & -20 & 0 \\ 0 & 81 & 24 & -16 & 0 \\ 0 & 0 & 1 & 117 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 35 & -30 & 0 & -55 & 0 \\ 0 & 81 & 0 & -43 & 0 \\ 0 & 0 & 1 & 117 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Divide all rows by 100

$$\sim \begin{bmatrix} .35 & 0 & 0 & -.71 & 0 \\ 0 & 1 & 0 & .53 & 0 \\ 0 & 0 & 1 & 1.17 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad R_2 \div 0.81$$

$$0.35\, p_A - 0.71\, p_T = 0$$

$$p_E - 0.53\, p_T = 0$$

$$p_m + 1.17\, p_T = 0$$

p_T is free variable

$p_A = 2.02\, p_T$

$p_E = 0.53\, p_T$

$p_m = -1.17\, p_T$

If $p_T = 100$

then, $p_A = 200.03$

$p_B = 53$

$p_m = 120$

<u>Answer</u>

Question # 9

$PbN_6 + CrMn_2O_8 \longrightarrow Pb_3O_4 + Cr_2O_3 + MnO_2 + NO$

$$x_1\begin{bmatrix}1\\6\\0\\0\\0\end{bmatrix} + x_2\begin{bmatrix}0\\0\\1\\2\\8\end{bmatrix} = x_3\begin{bmatrix}3\\0\\0\\0\\4\end{bmatrix} + x_4\begin{bmatrix}0\\0\\2\\0\\3\end{bmatrix} + x_5\begin{bmatrix}0\\0\\0\\1\\2\end{bmatrix} + x_6\begin{bmatrix}0\\1\\0\\0\\1\end{bmatrix}$$

The augmented matrix form we get;

$$\begin{bmatrix}1&0&-3&0&0&0&0\\6&0&0&0&0&-1&0\\0&1&0&-2&0&0&0\\0&2&0&0&-1&0&0\\0&8&-4&-3&-2&-1&0\end{bmatrix} \sim \begin{bmatrix}1&0&-3&0&0&0&0\\1&0&0&0&0&-1/6&0\\0&1&0&-2&0&0&0\\0&2&0&0&-1&0&0\\0&8&-4&-3&-2&-1&0\end{bmatrix}$$

$$\sim \begin{bmatrix}1&0&-3&0&0&0&0\\0&0&3&0&0&-1/6&0\\0&1&0&-2&0&0&0\\0&2&0&0&-1&0&0\\0&8&-4&-3&-2&-1&0\end{bmatrix} \sim \begin{bmatrix}1&0&-3&0&0&0&0\\0&1&0&-2&0&0&0\\0&0&3&0&0&-1/6&0\\0&2&0&0&-1&0&0\\0&8&-4&-3&-2&-1&0\end{bmatrix}$$

$$\sim \begin{bmatrix}1&0&-3&0&0&0&0\\0&1&0&-2&0&0&0\\0&0&3&0&0&-1/6&0\\0&1&0&0&-1/2&0&0\\0&1&-\frac{1}{2}&-\frac{3}{8}&-\frac{1}{4}&-\frac{1}{8}&0\end{bmatrix} \sim \begin{bmatrix}1&0&-3&0&0&0&0\\0&1&0&-2&0&0&0\\0&0&3&0&0&-1/6&0\\0&0&0&2&-1/2&0&0\\0&0&-\frac{1}{2}&\frac{13}{8}&-\frac{1}{4}&-\frac{1}{8}&0\end{bmatrix}$$

$$\sim \begin{bmatrix}1&0&-3&0&0&0&0\\0&1&0&-2&0&0&0\\0&0&1&0&0&-1/18&0\\0&0&0&2&-\frac{1}{2}&0&0\\0&0&-\frac{1}{2}&-\frac{3}{8}&-\frac{1}{4}&-\frac{1}{8}&0\end{bmatrix} \sim \begin{bmatrix}1&0&-3&0&0&0&0\\0&1&0&-2&0&0&0\\0&0&1&0&0&-1/18&0\\0&0&0&2&-\frac{1}{2}&0&0\\0&0&0&-\frac{13}{4}&\frac{1}{2}&\frac{11}{36}&0\end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{2}{13} & -\frac{11}{117} & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -\frac{1}{6} & 0 \\ 0 & 1 & 0 & 0 & 0 & -\frac{22}{45} & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 1 & 0 & -11/45 & 0 \\ 0 & 0 & 0 & 0 & 1 & -44/45 & 0 \end{bmatrix}$$

In general equation are:

$x_1 = (1/6)x_6$

$x_2 = (22/45)x_6$

$x_3 = (1/18)x_6$

$x_4 = (11/45)x_6$

$x_5 = (44/45)x_6$

x_6 is free variable

if $x_6 = 90$ → to remove fraction

then

$x_1 = 15$

$x_2 = 44$

$x_3 = 5$

$x_4 = 22$

$x_5 = 88$

The balanced eq. is:

$$15\,PbN_6 + 44\,CrMn_2O_8 \rightarrow 5\,Pb_3O_4 + 22\,Cr_2O_3 + 88\,MnO_2 + 90\,NO$$

Question - 10

$$MnS + As_2Cr_{10}O_{35} + H_2SO_4 \longrightarrow HMnO_4 + AsH_3 + CrS_3O_{12} + H_2O$$

$$x_1\begin{bmatrix}1\\1\\0\\0\\0\\0\end{bmatrix} + x_2\begin{bmatrix}0\\0\\2\\10\\35\\0\end{bmatrix} + x_3\begin{bmatrix}0\\1\\0\\0\\4\\2\end{bmatrix} = x_4\begin{bmatrix}1\\0\\0\\0\\4\\1\end{bmatrix} + x_5\begin{bmatrix}0\\0\\1\\0\\0\\3\end{bmatrix} + x_6\begin{bmatrix}0\\3\\0\\1\\12\\0\end{bmatrix} + x_7\begin{bmatrix}0\\0\\0\\0\\1\\2\end{bmatrix}$$

The augmented matrix form is

$$\left[\begin{array}{ccccccc|c}
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0\\
1 & 0 & 1 & 0 & 0 & -3 & 0 & 0\\
0 & 2 & 0 & 0 & -1 & 0 & 0 & 0\\
0 & 10 & 0 & 0 & 0 & -1 & 0 & 0\\
0 & 35 & 4 & -4 & 0 & -12 & -1 & 0\\
0 & 0 & 2 & -1 & -3 & 0 & -2 & 0
\end{array}\right] \quad R_1 \mp R_2 = 0$$

$$\begin{bmatrix}
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0\\
0 & 0 & 1 & -1 & 0 & -3 & 0 & 0\\
0 & 2 & 0 & 0 & -1 & 0 & 0 & 0\\
0 & 10 & 0 & 0 & 0 & -1 & 0 & 0\\
0 & 35 & 4 & -4 & 0 & -12 & -1 & 0\\
0 & 0 & 2 & -1 & -3 & 0 & -2 & 0
\end{bmatrix}$$

$R_2 \leftrightarrow R_3$ & $R_2 \% 2$ & $10R_2 - R_4 \to R_4$ & $35R_2 - R_5 \to R_5$

$$\sim \begin{bmatrix}
1 & 0 & 0 & -1 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & -1/2 & 0 & 0 & 0\\
0 & 0 & 1 & -1 & 0 & -3 & 0 & 0\\
0 & 0 & 0 & 0 & -5 & 0 & 0 & 0\\
0 & 0 & 4 & -4 & -5 & -12 & -1 & 0\\
0 & 0 & 2 & -1 & -3 & 0 & -2 & 0
\end{bmatrix} \quad \begin{array}{l} \\ \\ 4R_3 - R_5 \to R_5\\ 2R_3 - R_6 \to R_6 \\ \\ \\ \end{array}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & +3 & -6 & 2 & 0 \end{bmatrix} \quad \begin{array}{l} \cancel{R_4 \div -5 \to R_4} \\ \cancel{5R_4 + R_5 \to R_5} \\ R_4 - R_5 \to R_5 \\ 3R_4 + 5R_6 \to R_6 \end{array}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 30 & 10 & 0 \end{bmatrix} \quad R_5 \leftrightarrow R_6$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 30 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

So in ~~r~~ational format

$\cancel{x_1 = 16(1/327)x_7}$ $\quad x_1 = x_4$

$\cancel{x_2 = ((3/327))x_7}$ $\quad x_2 = (1/2)x_5$

$x_3 = x_4 + 3x_6$

$x_5 = 0$

$30x_6 = -10x_7$

$x_7 = 0$

$x_7 \Rightarrow$ free variable

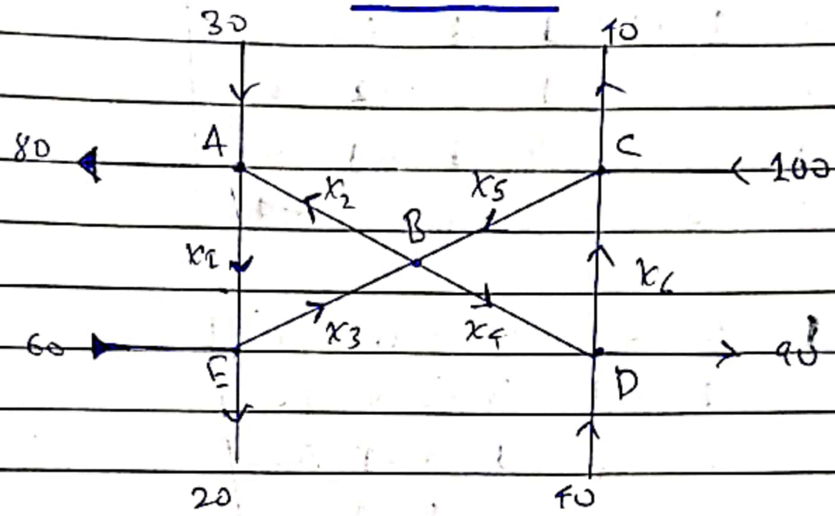
if $x_7 = 327$

~~x_7~~

The balanced eq. is:

$$5MnS + 4As_2Cr_{10}O_{35} + 115H_2SO_4 \longrightarrow 5HMnO_4 + 8AsH_3 + 40CrS_3O_{12} + 100H_2O$$

Question 13



	Flow in		Flow out
A	$x_2 + 30$	$=$	$x_1 + 80$
B	$x_3 + x_5$	$=$	$x_2 + x_4$
C	$x_6 + 100$	$=$	$x_5 + 40$
D	$x_4 + 40$	$=$	$x_6 + 90$
E	$x_1 + 60$	$=$	$x_3 + 20$
Total flow	230	$=$	230

By re arranging

$$x_1 - x_2 = -50$$
$$x_2 - x_3 + x_4 - x_5 = 0$$
$$x_5 - x_6 = 60$$
$$x_4 - x_6 = 50$$
$$x_1 - x_3 = -40$$

The augmented matrix form is

$$\left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 0 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 1 & 0 & -1 & 0 & 0 & 0 & -40 \end{array}\right] \sim$$

$R_1 + R_5 \to R_5$ & $R_3 \leftrightarrow R_4$ & $R_3 - R_5 \to R_5$

$$\sim \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 60 \\ 0 & 0 & 0 & 0 & 1 & -1 & 50 \\ 0 & 0 & 0 & 0 & 0 & 0 & -10 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & -40 \\ 0 & 1 & -1 & 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

General Solution is:

$x_1 = x_3 - 40$
$x_2 = x_3 + 10$
x_3 is free
$x_4 = x_6 + 50$
$x_5 = x_6 + 60$
x_6 is free.

(b)

To find the minimum flows, note that since x_1 cannot be negative, $x_3 \geq 40$, This implies that $x_2 \geq 50$. Also since x_6 cannot be negative, $x_4 \geq 50$ and $x_5 \geq 60$. The **minimum** flows are

$$\boxed{\begin{aligned} x_2 &= 50, \\ x_3 &= 40 \\ x_4 &= 50 \\ \text{and } x_5 &= 60 \end{aligned}}$$

(when $x_1 = 0$ and $x_6 = 0$)