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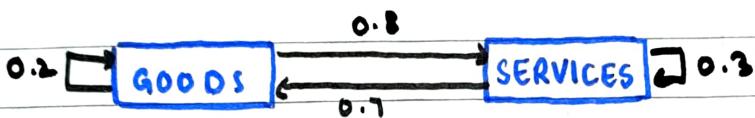
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Assignment #01 (Exercise 1.6)

QUESTION #1

Suppose an economy has two sectors, Goods and Services

Each year, goods sells 80% of its output to services and keeps the rest. Services sells 70% of its output to goods and keeps the rest. Find the equilibrium prices



Denoting the ~~per~~ annual output of Goods by P_G and of Services by P_S .

$$\rightarrow \text{Total income of Goods} = P_G$$

$$\rightarrow \text{Total income of Services} = P_S$$

Using the values given,

$$0.2P_G + 0.7P_S = P_G \quad \text{--- (A)}$$

$$0.8P_G + 0.3P_S = P_S \quad \text{--- (B)}$$

Equations (A) and (B) can also be written as.

$$0.2P_G + 0.7P_S - P_G = 0 \Rightarrow -0.8P_G + 0.7P_S = 0$$

$$0.8P_G + 0.3P_S - P_S = 0 \Rightarrow 0.8P_G - 0.7P_S = 0$$

\rightarrow Writing the equations in augmented Matrix form.

$$\left[\begin{array}{cc|c} -0.8 & 0.7 & 0 \\ 0.8 & -0.7 & 0 \end{array} \right] \text{ Applying Row Reduction Algorithm}$$
$$R_2 = R_1 + R_2$$

$$\left[\begin{array}{cc|c} -0.8 & 0.7 & 0 \\ 0 & 0 & 0 \end{array} \right] \text{ } R_1 = R_1 \times \frac{1}{-0.8}$$

$$\left[\begin{array}{cc|c} 1 & -0.875 & 0 \\ 0 & 0 & 0 \end{array} \right] \text{ } P_S \rightarrow \text{Free variable}$$

$$P_G - 0.875P_S = 0 \rightarrow P_G = 0.875P_S$$

$$\text{General Solution: } P_G = 0.875P_S$$

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QUESTION # 2

Find another set of equilibrium prices for the economy in

Ex 1. Suppose the same economy used Japanese yen instead of dollars to measure the value of various sector's output. Would this change the problem in any way?

Discuss.

Coal	Electric	Steel	Purchased by
0.0	0.4	0.6	Coal
0.6	0.1	0.2	Electric
0.4	0.5	0.2	Steel

According to Ex 1. The equilibrium price vector for the economy has the form

$$P = \begin{bmatrix} P_C \\ P_E \\ P_S \end{bmatrix} = \begin{bmatrix} 0.94 P_S \\ 0.85 P_E \\ 1 P_S \end{bmatrix} = P_S \begin{bmatrix} 0.94 \\ 0.85 \\ 1 \end{bmatrix}$$

When P_S is \$100 million, P_C is \$94 million and P_E is \$85 million.

→ For a new set of Prices, let's suppose $P_S = 500$ Million Dollars.

Then, according to the equilibrium price ratio,

$$P_C = 0.94 \times 500 = \$470 \text{ Million}$$

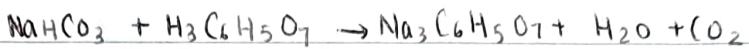
$$P_E = 0.85 \times 500 = \$425 \text{ Million}$$

→ Changing the currency will not change the ratio of the prices. It will have the same effect as multiplying/dividing the equilibrium prices by a constant number. It doesn't matter what the number is as long as it is non-negative and all of the prices are being multiplied/divided by it.

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QUESTION #7

Alka-Seltzer contains sodium bicarbonate (NaHCO_3) and citric acid ($\text{H}_3\text{C}_6\text{H}_5\text{O}_7$). When a tablet is dissolved in water, the following reaction occurs.



The number of each atom in a molecule can be written as.

Na	1	0	3
H	NaHCO_3	1, $\text{H}_3\text{C}_6\text{H}_5\text{O}_7$	8, $\text{Na}_3\text{C}_6\text{H}_5\text{O}_7$
C	1	6	6
O	3	7	7

H_2O	0	CO_2	0
	2		0
	0		1
	2		2

Denoting the molecules by $x_1 \dots x_5$ notations.

x_1	1	0	3	0	0
	1	8	5	2	0
	1	6	6	0	1
	3	7	7	1	2

$$① \quad x_1 = 3x_3 \rightarrow x_1 - 3x_3 = 0$$

$$② \quad x_1 + 8x_2 = 5x_3 + 2x_4 \rightarrow x_1 + 8x_2 - 5x_3 - 2x_4 = 0$$

$$③ \quad x_1 + 6x_2 = 6x_3 + x_5 \rightarrow x_1 + 6x_2 - 6x_3 - x_5 = 0$$

$$④ \quad 3x_1 + 7x_2 = 7x_3 + x_4 + 2x_5 \rightarrow 3x_1 + 7x_2 - 7x_3 - x_4 - 2x_5 = 0$$

Transforming the equations to augmented matrix

$$\left| \begin{array}{cccc|c} 1 & 0 & -3 & 0 & 0 & 0 \\ 1 & 8 & -5 & -2 & 0 & 0 \\ 1 & 6 & -6 & 0 & -1 & 0 \\ 3 & 7 & -7 & -1 & -2 & 0 \end{array} \right|$$

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Applying row reduction techniques on the matrix

$$\rightarrow R_2 = R_2 - R_1 \quad \left[\begin{array}{cccccc} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 1 & -2 & -2 & 0 & 0 \\ 0 & 6 & -3 & 0 & -1 & 0 \\ 0 & 7 & 2 & -1 & -2 & 0 \end{array} \right]$$

$$R_3 = R_3 - R_1$$

$$R_4 = R_4 - 3R_1$$

$$\rightarrow R_2 = R_2 / 8 = \left[\begin{array}{cccccc} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 1 & -1/4 & -1/4 & 0 & 0 \\ 0 & 6 & -3 & 0 & -1 & 0 \\ 0 & 7 & 2 & -1 & -2 & 0 \end{array} \right]$$

$$\rightarrow R_3 = R_3 - 6R_2, \quad R_4 = R_4 - 7R_2$$

$$\left[\begin{array}{cccccc} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 1 & -1/4 & -1/4 & 0 & 0 \\ 0 & 0 & -3/4 & 3/2 & -1 & 0 \\ 0 & 0 & 15/4 & 3/4 & -2 & 0 \end{array} \right]$$

$$R_3 = -\frac{2}{3}R_3 = \left[\begin{array}{cccccc} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 1 & -1/4 & -1/4 & 0 & 0 \\ 0 & 0 & 1 & -1 & 2/3 & 0 \\ 0 & 0 & 15/4 & 3/4 & -2 & 0 \end{array} \right]$$

$$R_1 = R_1 + 3R_3 \quad \left[\begin{array}{cccccc} 1 & 0 & 0 & -3 & 2 & 0 \\ 0 & 1 & -1/4 & -1/4 & 0 & 0 \\ 0 & 0 & 1 & -1 & 2/3 & 0 \\ 0 & 0 & 15/4 & 3/4 & -2 & 0 \end{array} \right]$$

$$R_2 = R_2 + \frac{R_3}{4}, \quad R_4 = R_4 - 15 \frac{R_3}{4}$$

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & -3 & 2 & 0 \\ 0 & 1 & 0 & -1/2 & 1/6 & 0 \\ 0 & 0 & 1 & -1 & 2/3 & 0 \\ 0 & 0 & 0 & 9/2 & -9/2 & 0 \end{array} \right]$$

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$$\rightarrow R_4 = \frac{R_4}{9}$$
$$\left[\begin{array}{cccccc} 1 & 0 & 0 & -3 & 2 & 0 \\ 0 & 1 & 0 & -1/2 & 1/6 & 0 \\ 0 & 0 & 1 & -1 & 2/3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{array} \right]$$

$$\rightarrow R_1 = R_1 + 3R_4 =$$
$$\left[\begin{array}{cccccc} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1/2 & 1/6 & 0 \\ 0 & 0 & 1 & -1 & 2/3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{array} \right]$$

$$\rightarrow R_2 = R_2 + R_4 / 2$$
$$\left[\begin{array}{cccccc} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1/3 & 0 \\ 0 & 0 & 1 & -1 & 2/3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{array} \right]$$

$$\rightarrow R_3 = R_3 + R_4$$

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1/3 & 0 \\ 0 & 0 & 1 & 0 & -4/3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{array} \right]$$

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$$x_1 = x_5$$

$$x_2 = \left(\frac{1}{3}\right) x_5$$

$$x_3 = \left(\frac{1}{3}\right) x_5$$

$$x_4 = x_5$$

$x_5 \rightarrow$ free variable -

→ let's suppose

$$x_5 = 3$$

$$x_1 = x_4 = 3.$$

$$x_2 = x_3 = 1.$$





QUESTION # 8

	K	Mn	O	S	H	K	Mn	O	S	H
K	1	1	4	0	0	0	1	1	2	0
Mn	KMnO_4	1	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
O	(x_1)	4	(x_2)	$4(x_3)$	1	(x_4)	2	(x_5)	4	(x_6)
S	0	0	1	0	0	0	0	1	0	1
H	0	0	0	2	0	0	0	0	0	2

We can write the following equations from the vectors

$$\rightarrow x_1 = 2x_5 \rightarrow x_1 - 2x_5 = 0.$$

$$\rightarrow x_1 + 4x_2 = x_4 \rightarrow x_1 + 4x_2 - x_4 = 0$$

$$\rightarrow 4x_1 + 4x_2 + x_3 = 2x_4 + 4x_5 + 4x_6 \rightarrow 4x_1 + 4x_2 + x_3 - 2x_4 - 4x_5 - 4x_6 = 0$$

$$\rightarrow x_2 = x_5 + x_6 \rightarrow x_2 - x_5 - x_6 = 0$$

$$\rightarrow 2x_4 = 2x_1 \rightarrow 2x_4 - 2x_1 = 0.$$

Writing the following equations in an augmented Matrix

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 1 & 1 & 0 & -1 & 0 & 0 & 0 \\ 4 & 4 & 1 & -2 & -4 & -4 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 2 & 0 & 0 & -2 & 0 \end{array} \right]$$

Reducing the Matrix

$$R_2 = R_2 - R_1, \quad R_3 = R_3 - 4R_1$$

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 2 & 0 & 0 \\ 0 & 4 & 1 & -2 & 4 & -4 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 2 & 0 & 0 & -2 & 0 \end{array} \right]$$

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$$R_3 = R_3 - 4R_2$$

$$\left[\begin{array}{ccccccc} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & -4 & -4 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 2 & 0 & 0 & -2 & 0 \end{array} \right]$$

$$R_4 = R_4 - R_2 = \left[\begin{array}{ccccccc} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & -4 & -4 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 \\ 0 & 0 & 2 & 0 & 0 & -2 & 0 \end{array} \right]$$

$$R_5 = R_5 - 2R_3, \quad R_2 = R_2 + R_4$$

$$\left[\begin{array}{ccccccc} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 2 & -4 & -4 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 \\ 0 & 0 & 0 & -4 & 8 & 6 & 0 \end{array} \right]$$

$$R_3 = R_3 - 2R_4, \quad R_5 = R_5 + 4R_4.$$

$$\left[\begin{array}{ccccccc} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 \\ 0 & 0 & 0 & 0 & -4 & 2 & 0 \end{array} \right]$$

$$R_3 = -\frac{R_3}{4} = \left[\begin{array}{ccccccc} 1 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 2 & -2 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1/2 & 0 \end{array} \right]$$

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$$R_1 = R_1 + 2R_5, \quad R_2 = R_2 + R_5$$

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 & -3/2 & 0 \\ 0 & 0 & 1 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1/2 & 0 \end{array} \right]$$

$$R_3 = R_3 - 2R_5 = \left[\begin{array}{cccccc} 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -3/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & -3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1/2 & 0 \end{array} \right]$$

$$R_4 = R_4 + 3R_5 = \left[\begin{array}{cccccc} 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -3/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -5/2 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1/2 & 0 \end{array} \right]$$

$$Y_1 = Y_6$$

$$Y_2 = (1.5)Y_6$$

$$Y_3 = Y_6$$

$$Y_4 = (2.5)Y_6$$

$$Y_5 = 0.5(Y_6)$$

$$Y_6 = \text{free.}$$

$$\text{Taking } Y_6 = 2.$$

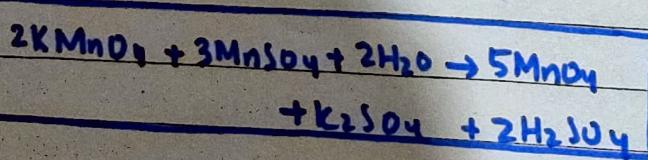
$$Y_1 = 2$$

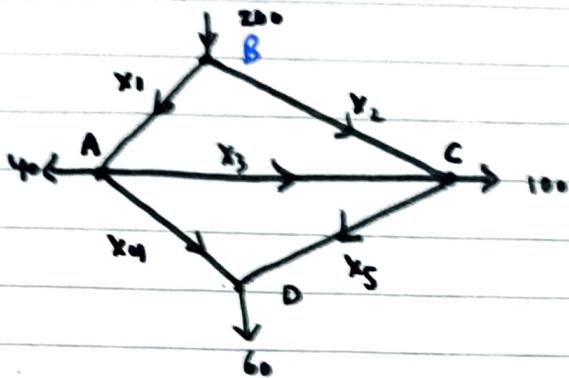
$$Y_3 = 2$$

$$Y_2 = 3$$

$$Y_4 = 5$$

$$Y_5 = 1.$$



QUESTION # 12

- a) Find the general traffic pattern in freeway network shown in the figure

Node	Flowing in	Flowing out
A	x_1	$x_4 + x_3 + 40$
B	200	$x_1 + x_2$
C	$x_2 + x_3$	$100 + x_5$
D	$x_5 + x_4$	60
Total	200	200

We can rearrange the equations,

$$\rightarrow x_1 - x_4 - x_3 = 40$$

$$x_1 + x_2 = 200$$

$$x_2 + x_3 - x_5 = 100$$

$$x_5 + x_4 = 60$$

- \rightarrow Writing the following equations in Augmented Matrix

$$\left[\begin{array}{cccccc|c} 1 & 0 & -1 & -1 & 0 & : & 40 \\ 1 & 1 & 0 & 0 & 0 & : & 200 \\ 0 & 1 & 1 & 0 & -1 & : & 100 \\ 0 & 0 & 0 & 1 & 1 & : & 60 \end{array} \right]$$

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$$\rightarrow R_2 = R_2 - R_1 \quad \left| \begin{array}{cccccc} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{array} \right|$$

$$R_3 = R_3 - R_2 \quad \left| \begin{array}{cccccc} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 0 & 0 & -1 & -1 & -60 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{array} \right|$$

~~Swapping~~ $R_3 = R_3 \times -1 ; R_1 = R_1 + R_3$

$$\left| \begin{array}{ccccc} 1 & 0 & -1 & 0 & 1 : 100 \\ 0 & 1 & 1 & 1 & 0 : 160 \\ 0 & 0 & 0 & 1 & 1 : 60 \\ 0 & 0 & 0 & 1 & 1 : 60 \end{array} \right|$$

$$R_2 = R_2 - R_3 \quad \left| \begin{array}{ccccc} 1 & 0 & -1 & 0 & 1 : 100 \\ 0 & 1 & 1 & 0 & -1 : 100 \\ 0 & 0 & 0 & 1 & 1 : 60 \\ 0 & 0 & 0 & 1 & 1 : 60 \end{array} \right|$$

$$R_4 = R_4 - R_3 = \left| \begin{array}{ccccc} 1 & 0 & -1 & 0 & 1 : 100 \\ 0 & 1 & 1 & 0 & -1 : 100 \\ 0 & 0 & 0 & 1 & 1 : 60 \\ 0 & 0 & 0 & 0 & 0 : 60 \end{array} \right|$$

x_5 = free variable

x_3 = free variable.

$$x_1 = 100 - x_5 + x_3$$

$$x_2 = 160 - x_3 + x_5$$

$$x_4 = 60 - x_5.$$

b) x_4 is closed $\rightarrow x_4 = 0$.

$$x_5 = 60.$$

$$x_1 = 40 + x_3$$

$$x_2 = 160 - x_3$$

$x_3 \rightarrow$ Free variable.

c) $x_4 = 0$, minval of x_1

graphically

$$\therefore x_1 = 100 + x_3 - x_5$$

$$x_1 = 40 + x_3$$

\hookrightarrow The minimum value is 40

calls per minute since

we cannot x_3 to be

negative (i.e. $x_3 = 0$)

Q.3 Part a:

Distribution of output from:

Chemical & Metal	Fuel & Power	Machinery	Purchased by
• 2	• 8	• 4	chemical
• 3	• 1	• 4	Fuel
• 5	• 1	• 2	Machinery

Part b:

Let P_c , P_f , P_m be total price output of chemicals, fuel and machinery respectively. Thus,

$$\text{Chemicals: } P_c = -2P_c + 8P_f + 4P_m \quad (i)$$

$$\text{Fuels : } P_f = 3P_c + 1P_f + 4P_m \quad (ii)$$

$$\text{Machinery : } P_m = 5P_c + 1P_f + 2P_m \quad (iii)$$

\Rightarrow Eq (i) becomes:

$$-2P_c - P_c + 8P_f + 4P_m = 0$$

$$-3P_c + 8P_f + 4P_m = 0$$

\Rightarrow Eq (ii) becomes:

$$3P_c + 1P_f - P_f + 4P_m = 0$$

$$3P_c - 9P_f + 4P_m = 0$$

\Rightarrow Eq (iii) becomes:

$$5P_c + 1P_f + 2P_m - P_m = 0$$

$$5P_c + 1P_f - 8P_m = 0$$

System of Equations:

$$-3P_c + 8P_f + 4P_m = 0$$

$$3P_c - 9P_f + 4P_m = 0$$

$$5P_c + 1P_f - 8P_m = 0$$

Augmented Matrix:

$$\begin{bmatrix} -0.8 & 0.8 & 0.4 & 0 \\ 0.3 & -0.9 & 0.4 & 0 \\ 0.5 & 0.1 & -0.8 & 0 \end{bmatrix}$$

Reduced echelon form Calculation:

Multiplying matrix by 10 to get whole numbers.

$$\sim \begin{bmatrix} -8 & 8 & 4 & 0 \\ 3 & -9 & 4 & 0 \\ 5 & 1 & -8 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & -0.5 & 0 \\ 0 & 6 & -5.5 & 0 \\ 5 & 1 & -8 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & -0.5 & 0 \\ 0 & 6 & -5.5 & 0 \\ 0 & -6 & 5.5 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & -0.5 & 0 \\ 0 & 1 & -0.916 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1.416 & 0 \\ 0 & 1 & -0.916 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The general solution is

$$P_C = 1.416 P_M$$

$$P_F = 0.916 P_M$$

P_M is free

Part c:

equilibrium price vector has form:

$$P = \begin{bmatrix} P_C \\ P_F \\ P_M \end{bmatrix} = \begin{bmatrix} 1.416 P_M \\ 0.916 P_M \\ P_M \end{bmatrix} = \begin{bmatrix} 0.916 \\ 1 \\ 1 \end{bmatrix}$$

If $P_M = 100$, then $P_C = 141.6$ and $P_F = 91.6$. Hence, when Machinery output is 100 units, chemicals output is 141.6 units and fuels output is 91.6 units.

[Q.5]



Following vector lists number of atoms of boron (B), sulfur(s), hydrogen(H) and oxygen(O):

$$B_2S_3 : \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix}, H_2O : \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix}, H_3BO_3 : \begin{bmatrix} 1 \\ 0 \\ 3 \\ 3 \end{bmatrix}, H_2S : \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix} \leftarrow \begin{array}{l} \text{boron} \\ \text{sulfur} \\ \text{hydrogen} \\ \text{oxygen} \end{array}$$

Let x_1, x_2, x_3, x_4 be co-efficients of each term in eq(i), then co-efficients must satisfy:

$$x_1 \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 0 \\ 3 \\ 3 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}$$

Its augmented form becomes after moving x_3, x_4 to left side

$$\begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$

$$n \begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 3 & 0 & 0 & -1 & 0 \end{bmatrix} n \begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & -3 & 2 & 0 \\ 0 & 0 & -3 & 2 & 0 \end{bmatrix}$$

$$n \begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & -3 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$n \begin{bmatrix} 1 & 0 & -1/2 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} n \begin{bmatrix} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The general solution becomes:

$$x_1 = \frac{1}{3} x_4, x_2 = 2x_4, x_3 = \frac{2}{3} x_4, x_4 \text{ is free.}$$

Taking $x_4 = 3$ to convert co-efficients from fraction to whole numbers; we have:

$$x_1 = 1, x_2 = 6, x_3 = 2,$$

Chemical equation with co-efficients:



Balanced equation:



Q. 14

Equations for each node:

Nodes Flow in Flow out

$$A : x_1 = x_2 + 150$$

$$B : x_2 + 50 = x_3$$

$$C : x_3 = x_4$$

$$D : x_4 + 150 = x_5 + 120$$

$$E : x_5 = x_6 + 80$$

$$F : x_6 + 100 = x_1$$

Thus,

$$x_1 - x_2 = 150$$

$$x_2 - x_3 = -50$$

$$x_3 - x_4 = 120$$

$$x_4 - x_5 = -150$$

$$x_5 - x_6 = 80$$

$$x_6 = -100$$

$$-x_1$$

In augmented form we have:

$$\left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 0 & 100 & 1 \\ 0 & 1 & -1 & 0 & 0 & -50 & 0 \\ 0 & 0 & 1 & -1 & 0 & 120 & 0 \\ 0 & 0 & 0 & 1 & -1 & -180 & 0 \\ -1 & 0 & 0 & 0 & 1 & -100 & 0 \end{array} \right] \sim \left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 0 & 100 & 1 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & -50 \\ 0 & 0 & 0 & 1 & -1 & 0 & 120 \\ 0 & 0 & 0 & 0 & 1 & -100 & 0 \end{array} \right] \sim \left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 0 & 100 & 1 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & -50 \\ 0 & 0 & 0 & 1 & -1 & 0 & 120 \\ 0 & 0 & 0 & 0 & 1 & -100 & 0 \end{array} \right] \sim \left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 0 & 100 & 1 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & -50 \\ 0 & 0 & 0 & 1 & -1 & 0 & 120 \\ 0 & 0 & 0 & 0 & 1 & -100 & 0 \end{array} \right]$$

$$\begin{bmatrix} 1 & -100 & 0 & 0 & 100 \\ 0 & 1 & -100 & 0 & -50 \\ 0 & 0 & 1 & -10 & 0 \\ 0 & 0 & 0 & 1 & -100 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -10 & 0 & -50 \\ 0 & 0 & 1 & -10 & 0 \\ 0 & 0 & 0 & 1 & -100 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 100 \\ 0 & 1 & -1 & 0 & 0 & -50 \\ 0 & 0 & 1 & -1 & 0 & 120 \\ 0 & 0 & 0 & 1 & -1 & 80 \\ 0 & 0 & -1 & 0 & 0 & -50 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 100 \\ 0 & 1 & -1 & 0 & 0 & -50 \\ 0 & 0 & 1 & -1 & 0 & 120 \\ 0 & 0 & 0 & 1 & -1 & 80 \\ 0 & 0 & 0 & 0 & 1 & 70 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 100 \\ 0 & 1 & -1 & 0 & 0 & -50 \\ 0 & 0 & 1 & -1 & 0 & 120 \\ 0 & 0 & 0 & 1 & -1 & 80 \\ 0 & 0 & 0 & 0 & 1 & 70 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 100 \\ 0 & 1 & -1 & 0 & 0 & -50 \\ 0 & 0 & 1 & -1 & 0 & 120 \\ 0 & 0 & 0 & 1 & -1 & 80 \\ 0 & 0 & 0 & 0 & 1 & 70 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 50 \\ 0 & 1 & -1 & 0 & 0 & -50 \\ 0 & 0 & 1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & -80 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 170 \\ 0 & 1 & 0 & 0 & 0 & 0 & 70 \\ 0 & 0 & 1 & -1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 0 & 1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 0 & 1 & -80 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

So the general solution is:

$$x_1 = 100 + x_6$$

$$x_2 = x_6$$

$$x_3 = 50 + x_6$$

$$x_4 = x_6 - 70$$

$$x_5 = 80 + x_6$$

x_6 is free

As x_4 has to be non-negative, x_6 value should be $x_6 \geq 70$. Hence, smallest possible value for x_6 is 70.

Q.6



Let x_1, x_2, x_3, x_4 be coefficients of each term:
 $x_1(\text{Na}_3\text{PO}_4) + x_2(\text{Ba}(\text{NO}_3)_2) \rightarrow x_3(\text{Ba}_2(\text{PO}_4)_2) + x_4(\text{NaNO}_3)$

Following vector lists number of atoms of sodium (Na), phosphorus (P), Oxygen(O), barium(Ba) and nitrogen(N):

$$\text{Na}_3\text{PO}_4: \begin{bmatrix} 3 \\ 1 \\ 4 \\ 0 \\ 0 \end{bmatrix}, \text{Ba}(\text{NO}_3)_2: \begin{bmatrix} 0 \\ 0 \\ 6 \\ 1 \\ 2 \end{bmatrix}, \text{Ba}_2(\text{PO}_4)_2: \begin{bmatrix} 0 \\ 2 \\ 8 \\ 3 \\ 0 \end{bmatrix} \leftarrow \begin{array}{l} \text{sodium} \\ \text{phosphorus} \\ \text{oxygen} \\ \text{barium} \\ \text{nitrogen} \end{array}$$

$$\text{NaNO}_3: \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \\ 1 \end{bmatrix} \leftarrow \begin{array}{l} \text{sodium} \\ \text{phosphorus} \\ \text{oxygen} \\ \text{barium} \\ \text{nitrogen} \end{array}$$

The co-efficients must satisfy:

$$x_1 \begin{bmatrix} 3 \\ 1 \\ 4 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 6 \\ 1 \\ 2 \end{bmatrix} = x_3 \begin{bmatrix} 0 \\ 2 \\ 8 \\ 3 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \\ 1 \end{bmatrix}$$

Its augmented form becomes:

$$\begin{pmatrix} 3 & 0 & 0 & -1 & 0 \\ 1 & 0 & -2 & 0 & 0 \\ 4 & 6 & -8 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{pmatrix}$$

$$\sim \left[\begin{array}{ccccc} 1 & 0 & -2 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 4 & 6 & -8 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{array} \right] \sim \left[\begin{array}{ccccc} 1 & 0 & -2 & 0 & 0 \\ 0 & 0 & +6 & -1 & 0 \\ 4 & 6 & -8 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{array} \right]$$

$$\sim \left[\begin{array}{ccccc} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 4 & 6 & -8 & -3 & 0 \\ 0 & 0 & 6 & -1 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{array} \right] \sim \left[\begin{array}{ccccc} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & -6 & 0 & 3 & 0 \\ 0 & 0 & 6 & -1 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{array} \right]$$

$$\sim \left[\begin{array}{ccccc} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6 & -10 & 0 \\ 0 & 2 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \sim \left[\begin{array}{ccccc} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 6 & -10 & 0 \\ 0 & 2 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \sim \left[\begin{array}{ccccc} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 6 & -10 & 0 \\ 0 & 0 & -6 & +10 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$N \left[\begin{array}{ccccc} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 6 & -1 & 0 \end{array} \right] \sim \left[\begin{array}{ccccc} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{6} & 0 \end{array} \right] \sim \left[\begin{array}{ccccc} 1 & 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{6} & 0 \end{array} \right]$$

$$\sim \left[\begin{array}{cccc} 1 & 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{1}{6} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

The general solution is

$$x_1 = \frac{1}{3} x_4$$

$$x_2 = \frac{1}{2} x_4$$

$$x_3 = \frac{1}{6} x_4$$

x_4 is free.

Taking $x_4 = 6$ to make co-efficients whole numbers;
we have:

$$x_1 = 2, x_2 = 3, x_3 = 1,$$

Balanced equation becomes:



Question - 4

(a)

Distribution output form

A E M T

Output	0.65	0.30	0.30	0.20	Input A
	• 10	0.10	0.15	0.10	E
	0.25	- 0.35	- 0.15	0.30	M
	0	0.25	0.40	0.40	T

(b)

Find set of equilibrium prices in economy

$$P_A = 0.65P_A + 0.30P_E + 0.30P_M + 0.20P_T$$

$$P_E = 0.10P_A + 0.10P_E + 0.15P_M + 0.10P_T$$

$$P_M = 0.25P_A + 0.35P_E + 0.15P_M + 0.30P_T$$

$$P_T = 0.25P_E + 0.40P_M + 0.40P_T$$

Moving all the equations to the left

$$0.35P_A - 0.30P_E - 0.30P_M - 0.20P_T = 0$$

$$- 0.10P_A + 0.90P_E - 0.15P_M - 0.30P_T = 0$$

$$- 0.25P_A - 0.35P_E + 0.85P_M - 0.30P_T = 0$$

$$- 0.25P_E - 0.40P_M + 0.60P_T = 0$$

Converting to Augmented matrix form

$$\left[\begin{array}{cccc|c} .30 & -0.30 & -0.30 & .20 & 0 \\ -0.10 & .90 & -0.15 & -0.30 & 0 \\ -0.25 & -0.35 & .85 & -0.30 & 0 \\ 0 & -0.25 & -0.40 & 0.60 & 0 \end{array} \right]$$

Multiply all rows by 100 we get;

$$\left[\begin{array}{ccccc} 35 & -30 & -30 & -20 & 0 \\ -10 & 90 & -15 & -10 & 0 \\ -25 & -35 & 85 & -30 & 0 \\ 0 & -25 & -40 & 60 & 0 \end{array} \right]$$

$$\sim \left[\begin{array}{ccccc} 35 & -30 & -30 & -20 & 0 \\ -10 & 90 & -15 & -10 & 0 \\ -25 & -35 & 85 & -30 & 0 \\ 0 & -25 & -40 & 60 & 0 \end{array} \right] \sim \left[\begin{array}{ccccc} 35 & -30 & -30 & -20 & 0 \\ 0 & 81 & 24 & -16 & 0 \\ 0 & 0 & 1 & -1.17 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\sim \left[\begin{array}{ccccc} 35 & -30 & 0 & -55 & 0 \\ 0 & 81 & 0 & -43 & 0 \\ 0 & 0 & 1 & -1.17 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

Divide all rows by 100

$$\sim \left[\begin{array}{ccccc} 0.35 & 0 & 0 & -0.71 & 0 \\ 0 & 1 & 0 & -0.53 & 0 \\ 0 & 0 & 1 & -1.17 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] R_2 \div 0.81$$

$$0.35 p_A - 0.71 p_T = 0$$

$$1 p_E - 0.53 p_T = 0$$

$$p_m + 1.17 p_T = 0$$

p_T is free variable

$$p_A = 2.02 p_T$$

$$p_E = 0.53 p_T$$

$$p_m = -1.17 p_T$$

If $P_T = 100$

then, $P_A = 200.03$

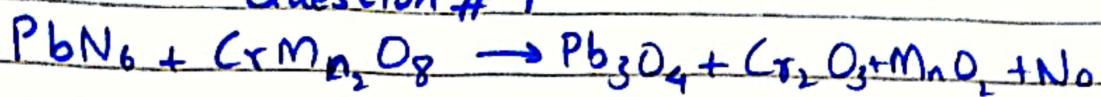
$P_B = 53$

$P_m = 120$

Answers



Question # 9



$$\begin{array}{|c|c|c|c|c|c|c|c|} \hline & x_1 & 1 & & & & & \\ \hline & & + x_2 & 0 & x_3 & 3 & + x_4 & 0 \\ \hline & & 6 & 0 & 0 & 0 & 0 & 0 \\ \hline & & & 1 & 0 & 2 & 0 & 1 \\ \hline & & & 0 & 0 & 0 & 1 & 0 \\ \hline & & & 2 & 0 & 0 & 0 & 0 \\ \hline & & & 8 & 4 & 3 & 2 & 1 \\ \hline \end{array}$$

The augmented matrix form we get;

$$\sim \begin{array}{|c|c|c|c|c|c|c|c|} \hline & 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ \hline & 6 & 0 & 0 & 0 & 0 & -1 & 0 \\ \hline & 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ \hline & 0 & 2 & 0 & 0 & -1 & 0 & 0 \\ \hline & 0 & 8 & -4 & -3 & -2 & -1 & 0 \\ \hline \end{array} \sim \begin{array}{|c|c|c|c|c|c|c|c|} \hline & 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ \hline & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ \hline & 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ \hline & 0 & 2 & 0 & 0 & -1 & 0 & 0 \\ \hline & 0 & 8 & -4 & -3 & -2 & -1 & 0 \\ \hline \end{array}$$

$$\sim \begin{array}{|c|c|c|c|c|c|c|c|} \hline & 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ \hline & 0 & 1 & 0 & -2 & 0 & -1 & 0 \\ \hline & 0 & 0 & 3 & 0 & 0 & -1 & 0 \\ \hline & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ \hline & 0 & 8 & -4 & -3 & -2 & -1 & 0 \\ \hline \end{array} \sim \begin{array}{|c|c|c|c|c|c|c|c|} \hline & 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ \hline & 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ \hline & 0 & 0 & 3 & 0 & 0 & -1 & 0 \\ \hline & 0 & 2 & 0 & 0 & -1 & 0 & 0 \\ \hline & 0 & 8 & -4 & -3 & -2 & -1 & 0 \\ \hline \end{array}$$

$$\sim \begin{array}{|c|c|c|c|c|c|c|c|} \hline & 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ \hline & 0 & 1 & 0 & -2 & 0 & -1 & 0 \\ \hline & 0 & 0 & 3 & 0 & 0 & -1 & 0 \\ \hline & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ \hline & 0 & 1 & -\frac{1}{2} & -\frac{3}{8} & -\frac{1}{4} & -\frac{1}{8} & 0 \\ \hline \end{array} \sim \begin{array}{|c|c|c|c|c|c|c|c|} \hline & 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ \hline & 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ \hline & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ \hline & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ \hline & 0 & 0 & -\frac{1}{2} & \frac{13}{8} & -\frac{1}{4} & -\frac{1}{8} & 0 \\ \hline \end{array}$$

$$\sim \begin{array}{|c|c|c|c|c|c|c|c|} \hline & 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ \hline & 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ \hline & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ \hline & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ \hline & 0 & 0 & -\frac{1}{2} & \frac{13}{8} & -\frac{1}{4} & -\frac{1}{8} & 0 \\ \hline \end{array} \sim \begin{array}{|c|c|c|c|c|c|c|c|} \hline & 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ \hline & 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ \hline & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ \hline & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ \hline & 0 & 0 & 0 & -\frac{13}{4} & \frac{1}{2} & \frac{11}{36} & 0 \\ \hline \end{array}$$

$$\left[\begin{array}{cccccc} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -5/18 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 0 & 0 \end{array} \right] \sim \left[\begin{array}{cccccc} 1 & 0 & 0 & 0 & 0 & -\frac{1}{6} & 0 \\ 0 & 1 & 0 & 0 & 0 & -\frac{22}{6} & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{45}{18} & 0 \\ 0 & 0 & 0 & 1 & 0 & -\frac{11}{45} & 0 \\ 0 & 0 & 0 & 0 & 1 & -\frac{44}{45} & 0 \end{array} \right]$$

In general equation are:

$$x_1 = (1/6)x_6$$

$$x_2 = (22/45)x_6$$

$$x_3 = (1/18)x_6$$

$$x_4 = (11/45)x_6$$

$$x_5 = (44/45)x_6$$

x_6 is free variable

if $x_6 = 90 \rightarrow$ to remove fraction
then

$$x_1 = 15$$

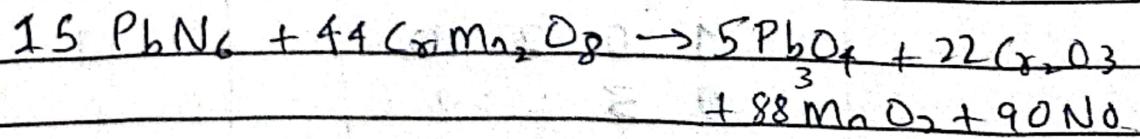
$$x_2 = 44$$

$$x_3 = 5$$

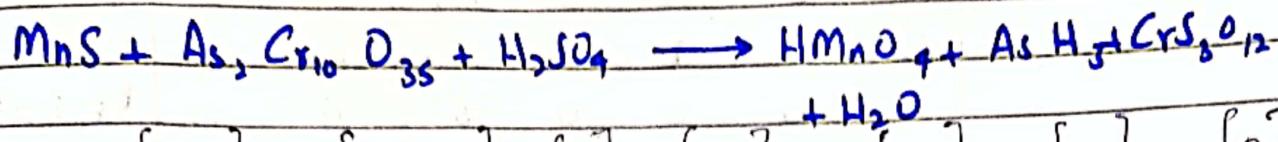
$$x_4 = 22$$

$$x_5 = 88$$

The balanced eq. is:



Question - 10



$$\begin{array}{c}
 \text{+ H}_2\text{O} \\
 \left[\begin{array}{ccccccccc}
 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
 x_1 & 1 & x_2 & 0 & +x_3 & 1 & -x_4 & 0 & +x_5 \\
 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
 0 & 10 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
 0 & 35 & 4 & 4 & 0 & 0 & 12 & 1 & 1 \\
 0 & 0 & 2 & 1 & 3 & 0 & 0 & 0 & 2
 \end{array} \right]
 \end{array}$$

The augmented matrix form is

$$\left[\begin{array}{ccccccc} 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & -3 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & -1 & 0 \\ 0 & 35 & 4 & -4 & 0 & -12 & -1 \\ 0 & 0 & 20 & -1 & -3 & 0 & -2 \end{array} : \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array} \right] \quad R_2 \neq R_2 = 0$$

1	0	0	-2	0	0	0	0
0	0	1	-1	0	-3	0	0
0	2	0	0	-1	0	0	0
0	10	0	0	0	-1	0	0
0	35	4	-4	0	-12	-1	0
0	0	2	-1	-3	0	-2	0

$$R_2 \leftrightarrow R_3 \quad \& \quad R_2 \cdot 1 \cdot 2 \quad \& \quad 10R_2 - R_4 \xrightarrow{R_4} R_4 \quad \& \quad 35R_2 - R_5 \xrightarrow{R_5} R_5$$

$$n \left| \begin{array}{ccccccc} 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5 & 0 & 0 & 0 \\ 0 & 0 & 4 & -4 & -5 & -12 & -1 & 0 \\ 0 & 0 & 2 & -1 & -3 & 0 & -2 & 0 \end{array} \right| \quad \begin{array}{l} 4R_3 - R_5 \rightarrow R_5 \\ 2R_5 - R_6 \rightarrow R_6 \end{array}$$

~	1	0	0	-1	0	0	0	0
	0	1	0	0	-1/2	0	0	0
	0	0	1	-1	0	-3	0	0
	0	0	0	0	-5	0	0	0
	0	0	0	0	-5	0	-1	0
	0	0	0	0	+3	-6	2	0

$R_4 \rightarrow R_4 - 5R_5 \rightarrow R_4$
 $R_4 \leftrightarrow R_5 \rightarrow R_5$
 $R_4 - R_5 \rightarrow R_5$
 $3R_4 + 5R_6 \rightarrow R_6$

~	1	0	0	-1	0	0	0	0
	0	1	0	0	-1/2	0	0	0
	0	0	1	-1	0	-3	0	0
	0	0	0	0	-5	0	0	0
	0	0	0	0	0	0	1	0
	0	0	0	0	0	30	10	0

$R_5 \leftrightarrow R_6$

- ~	1	0	0	-1	0	0	0	0
	0	1	0	0	-1/2	0	0	0
	0	0	1	-1	0	-3	0	0
	0	0	0	0	-5	0	0	0
	0	0	0	0	0	30	10	0
	0	0	0	0	0	0	1	0

So in rational format

$$x_1 = \frac{1}{2}x_2 + \frac{1}{10}(3x_7)$$

$$x_2 = (3/2)x_7$$

$$x_3 = x_4 + 3x_7$$

$$x_5 = 0$$

$$30x_6 = -10x_7$$

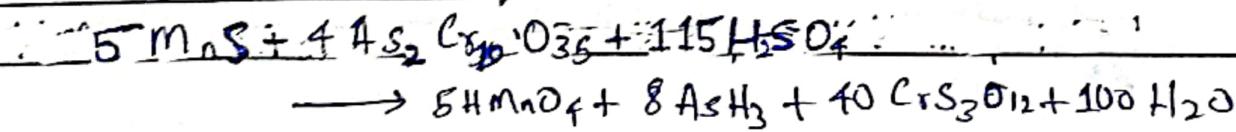
$$x_7 = 0 \quad \dots$$

$x_2 \rightarrow$ free variable

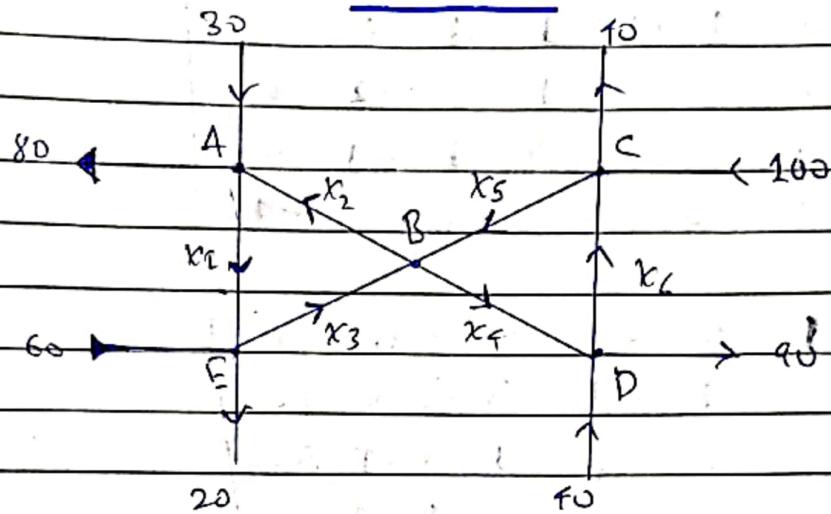
if $x_7 = 327$

~~x_2~~

The balanced eq. is:



-Question 13



Flow in Flow out

$$A \quad x_2 + 30 = x_1 + 80$$

$$B \quad x_3 + x_5 = x_2 + x_4$$

$$C \quad x_6 + 100 = x_5 + 40$$

$$D \quad x_4 + 40 = x_6 + 90$$

$$E \quad x_1 + 60 = x_3 + 20$$

$$\text{Total flow} \quad 230 = 230$$

By re arranging

$$x_1 - x_2 = -50$$

$$x_2 - x_3 + x_4 - x_5 = 0$$

$$x_5 - x_6 = 60$$

$$x_4 - x_6 = 50$$

$$x_1 - x_3 = -40$$

The augmented matrix form is

$$\left[\begin{array}{ccccccc|c}
 1 & -1 & 0 & 0 & 0 & 0 & -50 \\
 0 & 1 & -1 & 1 & -1 & 0 & 0 \\
 0 & 0 & 0 & 0 & 1 & -1 & 10 \\
 0 & 0 & 0 & 1 & 0 & -1 & 50 \\
 1 & 0 & -1 & 0 & 0 & 0 & -40
 \end{array} \right] \sim$$

$$R_2 + R_5 \rightarrow R_6 \text{ & } R_3 \leftrightarrow R_4 \text{ & } R_3 - R_5 \rightarrow R_5$$

$$\sim \left[\begin{array}{ccccccc} 1 & -1 & 0 & 0 & 0 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 60 \\ 0 & 0 & 0 & 0 & 1 & -1 & 50 \\ 0 & 0 & 0 & 0 & 0 & 0 & -10 \end{array} \right]$$

$$\sim \left[\begin{array}{ccccccc} 1 & 0 & -1 & 0 & 0 & 0 & -10 \\ 0 & 1 & -1 & 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

General Solution is:

$$x_1 = x_3 - 40$$

$$x_2 = x_3 + 10$$

x_3 is free

$$x_4 = x_6 + 50$$

$$x_5 = x_6 + 60$$

x_6 is free.

(b)

To find the minimum flows, note that since x_1 cannot be negative, $x_3 \geq 40$, this implies that $x_2 \geq 50$. Also since x_4 cannot be negative, $x_4 \geq 50$ and $x_5 \geq 60$. The minimum flows are:

$$x_2 = 50,$$

$$x_3 = 40$$

$$x_4 = 50$$

$$\text{and } x_5 = 60$$

(when $x_1 \geq 0$ and $x_6 \geq 0$)