Deterministic Disturbance Modeling Framework for Residual-Envelope Fusion Systems

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Abstract

This paper establishes a Deterministic Disturbance Modeling Framework (DDMF) for residual-envelope fusion systems, including Drift–Slew Fusion Bootstrap (DSFB) and Hierarchical Residual-Envelope Trust (HRET). Disturbances are defined as discrete-time signal classes without probabilistic assumptions and are rigorously categorized into pointwise-bounded, drift-type, slew-rate-bounded, impulsive, and group-correlated structures. Envelope-admissibility is formalized through boundedness of the residual-envelope recursion, and explicit conditions are provided under which disturbances induce bounded response, sustained suppression, or exponential recovery of trust weights. The framework separates admissible and inadmissible disturbance regimes and provides the disturbance-side theoretical foundation required for deterministic residual-envelope fusion architectures. The framework applies only to disturbances that satisfy the explicitly defined deterministic admissibility conditions and does not provide guarantees under unbounded or statistically modeled noise regimes.

Contents

1 Introduction

Residual-envelope fusion systems modulate channel trust weights through deterministic functions of residual magnitude and its filtered history. In frameworks such as Drift–Slew Fusion Bootstrap (DSFB) [3] and Hierarchical Residual-Envelope Trust (HRET) [1], envelope dynamics govern suppression and recovery behavior under disturbance. While prior formulations assume bounded disturbances to guarantee envelope stability, the underlying disturbance classes and their admissibility conditions have not been formally specified.

This paper introduces the Deterministic Disturbance Modeling Framework (DDMF), which rigorously defines admissible disturbance signal classes for residual-envelope fusion architectures. Disturbances are categorized according to structural properties—such as pointwise boundedness, drift, slew-rate limitation, impulsive support, and group correlation—and are analyzed with respect to their effect on envelope boundedness and trust-weight dynamics.

In contrast to stochastic filtering frameworks such as the Kalman filter [6] and H_∞ robust control formulations [7], the present framework operates entirely under deterministic bounded disturbance assumptions. Although residual-based weighting shares structural similarities with robust M-estimation [8], the envelope recursion considered here is not derived from a statistical cost functional and does not rely on distributional modeling.

The objective is to establish precise deterministic conditions under which residual envelopes remain bounded, suppression regimes arise, and recovery occurs. No probabilistic noise models, distributional assumptions, or statistical optimality claims are introduced.

2 Signal Model

Let $n \in \mathbb{N}$ denote discrete time. For each measurement channel $k \in \{1, \dots, M\}$, define a disturbance sequence

$$d_k : \mathbb{N} \to \mathbb{R}.$$

Residuals are decomposed as

$$r_k[n] = \varepsilon_k[n] + d_k[n], \tag{1}$$

where $\varepsilon_k[n]$ represents modeling and estimation error contributions, and $d_k[n]$ denotes the disturbance component to be classified.

Throughout this paper, disturbances are treated as deterministic sequences without distributional assumptions. The effect of disturbances is analyzed through their influence on the residual-envelope recursion

$$s_k[n+1] = \rho s_k[n] + (1-\rho)|r_k[n]|, \quad \rho \in (0,1), \tag{2}$$

which defines the channel-level envelope state used in residual-envelope fusion systems.

The boundedness and qualitative behavior of $s_k[n]$ under different disturbance classes form the basis of the admissibility analysis developed in subsequent sections.

3 Pointwise-Bounded Disturbances

Definition 1 (Pointwise-Bounded Disturbance Class)**.** *For a bound $D_k \geq 0$, define the disturbance class*

$$\mathcal{D}_k^\infty(D_k) = \{d_k : \mathbb{N} \to \mathbb{R} \mid |d_k[n]| \leq D_k \ \ \forall n \in \mathbb{N}\}.$$

Assumption 1 (Bounded Residual Contribution). *There exists $E_k \geq 0$ such that*

$$|\varepsilon_k[n]| \leq E_k \quad \forall n \in \mathbb{N}.$$

Under this assumption, residual magnitudes satisfy

$$|r_k[n]| = |\varepsilon_k[n] + d_k[n]| \leq |\varepsilon_k[n]| + |d_k[n]| \leq E_k + D_k = R_k. \tag{3}$$

Thus, pointwise-bounded disturbances together with bounded residual contributions imply uniformly bounded residual magnitude.

Proposition 1 (Envelope Boundedness Under Pointwise Bounds). *If $|r_k[n]| \leq R_k$ for all n, then the envelope recursion*

$$s_k[n+1] = \rho s_k[n] + (1-\rho)|r_k[n]|$$

satisfies

$$0 \leq s_k[n] \leq \max\{s_k[0], R_k\} \quad \forall n \in \mathbb{N}.$$

Proof. Non-negativity follows directly from $s_k[0] \geq 0$ and $|r_k[n]| \geq 0$.

Let $S := \max\{s_k[0], R_k\}$. We prove by induction that $s_k[n] \leq S$ for all n.

Base case: $s_k[0] \leq S$ by definition.

Inductive step: Assume $s_k[n] \leq S$. Then

$$s_k[n+1] = \rho s_k[n] + (1-\rho)|r_k[n]| \leq \rho S + (1-\rho)R_k \leq \rho S + (1-\rho)S = S.$$

Thus $s_k[n] \leq S$ for all n, establishing boundedness. □

Lemma 1 (Fixed-Point Convergence Under Constant Residual). *Assume $|r_k[n]| = R_k$ for all $n \geq 0$, with $R_k < \infty$. Then the envelope recursion*

$$s_k[n+1] = \rho s_k[n] + (1-\rho)R_k$$

admits a unique fixed point $s_k^* = R_k$, and $s_k[n]$ converges exponentially to s_k^*.

Proof. Define $e[n] := s_k[n] - R_k$. Then

$$e[n+1] = \rho e[n].$$

Thus $e[n] = \rho^n e[0]$, which converges to zero since $\rho \in (0,1)$. □

4 Drift Disturbances

Drift-type disturbances model slowly varying bias terms whose magnitude and rate of change are both bounded.

Definition 2 (Drift-Type Disturbance Class). *For constants $B_k \geq 0$ and $S_k \geq 0$, define*

$$\mathcal{D}_k^{\text{drift}}(B_k, S_k) = \left\{ d_k : \mathbb{N} \to \mathbb{R} \;\middle|\; \begin{array}{c} |d_k[n]| \leq B_k, \\ |d_k[n+1] - d_k[n]| \leq S_k \end{array} \; \forall n \in \mathbb{N} \right\}.$$

The bound B_k constrains absolute disturbance magnitude, while S_k limits inter-sample variation. This class captures deterministic bias terms that evolve gradually over time.

If $B_k < \infty$ and Assumption 1 holds for the residual contribution $\varepsilon_k[n]$, then $|r_k[n]|$ remains uniformly bounded. Consequently, by Proposition 1, drift-type disturbances are envelope-admissible.

Drift disturbances therefore induce gradual envelope elevation under persistent bias but do not lead to envelope divergence.

5 Slew-Rate-Bounded Disturbances

Slew-rate-bounded disturbances constrain the incremental variation of the disturbance sequence without necessarily imposing a magnitude bound.

Definition 3 (Slew-Rate-Bounded Disturbance Class). *For $S_k \geq 0$, define*

$$\mathcal{D}_k^{\text{slew}}(S_k) = \{d_k : \mathbb{N} \to \mathbb{R} \mid |d_k[n+1] - d_k[n]| \leq S_k \ \ \forall n \in \mathbb{N}\}\,.$$

The parameter S_k limits inter-sample variation and therefore controls the temporal smoothness of the disturbance. In isolation, this condition does not prevent unbounded growth of $d_k[n]$; magnitude may accumulate over time even when the increment constraint is satisfied.

For residual-envelope fusion systems, admissible disturbances are typically drawn from the intersection

$$\mathcal{D}_k^{\infty}(D_k) \ \cap \ \mathcal{D}_k^{\text{slew}}(S_k),$$

which enforces both pointwise boundedness and bounded rate of change. Under this intersection, residual magnitudes remain uniformly bounded and envelope admissibility follows from Proposition 1.

Slew-aware deterministic modeling has been previously employed in oscillatory estimation settings [5].

6 Impulsive Disturbances

Impulsive disturbances model finite-duration corruption events, such as transient sensor blackout or burst interference.

Definition 4 (Finite-Support Impulsive Disturbance). *Let $D_k \geq 0$. A disturbance sequence $d_k[n]$ is said to be impulsive if there exists a finite index set $\mathcal{I}_k \subset \mathbb{N}$ such that*

$$|d_k[n]| \leq D_k \quad \forall n \in \mathbb{N},$$

and

$$d_k[n] = 0 \quad \forall n \notin \mathcal{I}_k.$$

Thus, impulsive disturbances are uniformly bounded in magnitude and nonzero only over a finite time interval.

Proposition 2 (Exponential Envelope Recovery After Impulse). *Assume $|r_k[n]| \leq R_k$ for all n, and that there exists $N \in \mathbb{N}$ such that*

$$|r_k[n]| \leq R_k^{\text{nom}} \quad \forall n \geq N,$$

for some nominal bound R_k^{nom}. Then for $n \geq N$, the envelope satisfies

$$s_k[n] \leq \rho^{n-N} s_k[N] + \left(1 - \rho^{n-N}\right) R_k^{\text{nom}},$$

and converges exponentially to a bounded level not exceeding $\max\{s_k[N], R_k^{\text{nom}}\}$.

Proof. For $n \geq N$, the recursion becomes

$$s_k[n+1] = \rho s_k[n] + (1-\rho)|r_k[n]| \leq \rho s_k[n] + (1-\rho)R_k^{\text{nom}}.$$

Define $e_k[n] := s_k[n] - R_k^{\text{nom}}$. Then

$$e_k[n+1] \leq \rho e_k[n].$$

Iterating yields

$$e_k[n] \leq \rho^{n-N} e_k[N].$$

Rewriting in terms of $s_k[n]$ gives the stated bound, establishing exponential convergence with rate ρ. □

7 Group-Correlated Disturbances

Group-correlated disturbances model common-mode corruption affecting multiple channels simultaneously.

Let $\mathcal{G}_g \subset \{1, \ldots, M\}$ denote a predefined channel group. A disturbance is group-correlated on $\mathcal{G}_g$ if there exists a scalar sequence $\phi_g[n]$ such that

$$d_k[n] = \phi_g[n], \qquad k \in \mathcal{G}_g. \tag{4}$$

The sequence $\phi_g[n]$ may belong to any of the disturbance classes introduced in Sections 3–6 (e.g., pointwise-bounded, drift-type, slew-rate-bounded, or impulsive).

If $\phi_g[n] \in \mathcal{D}_g^{\infty}(D_g)$ for some $D_g \geq 0$, then residual magnitudes for all channels in $\mathcal{G}_g$ remain uniformly bounded. Consequently, by Proposition 1, the channel envelopes $s_k[n]$ remain bounded for all $k \in \mathcal{G}_g$.

In hierarchical residual-envelope systems such as HRET, the corresponding group envelope recursion

$$s_g[n+1] = \rho_g s_g[n] + (1-\rho_g)\frac{1}{|\mathcal{G}_g|}\sum_{k \in \mathcal{G}_g} |r_k[n]| \tag{5}$$

inherits boundedness under the same pointwise bound on $\phi_g[n]$. Thus, group-correlated disturbances remain envelope-admissible whenever the underlying common-mode signal is bounded.

Table 1: Deterministic Disturbance Classes in DDMF

Class	**Magnitude Bound**	**Rate Constraint**	**Support**
Pointwise-Bounded	$\lvert d_k[n]\rvert \leq D_k$	None	Infinite
Drift-Type	$\lvert d_k[n]\rvert \leq B_k$	$\lvert d_k[n+1] - d_k[n]\rvert \leq S_k$	Infinite
Slew-Rate-Bounded	None required	$\lvert d_k[n+1] - d_k[n]\rvert \leq S_k$	Infinite
Impulsive	$\lvert d_k[n]\rvert \leq D_k$	None required	Finite
Group-Correlated	$\lvert \phi_g[n]\rvert \leq D_g$	Optional	Structured subset

8 Envelope Admissibility

Envelope admissibility formalizes when a disturbance class is compatible with residual-envelope fusion dynamics.

Operator-Theoretic Formulation

Define the causal operator

$$\mathcal{E}_\rho : \ell_\infty \to \ell_\infty$$

by

$$(\mathcal{E}_\rho u)[n] = \rho^n s[0] + \sum_{i=0}^{n-1} (1-\rho)\rho^i u[n-1-i],$$

where $u[n] := |r[n]|$.

The operator $\mathcal{E}_\rho$ is linear, causal, time-invariant, and bounded with operator norm equal to 1.

To verify the operator norm bound, observe that for $u \in \ell_\infty$,

$$|(E_\rho u)[n]| \le \rho^n |s[0]| + \sum_{i=0}^{n-1} (1-\rho)\rho^i \|u\|_\infty.$$

Since

$$\sum_{i=0}^{n-1} (1-\rho)\rho^i = 1 - \rho^n \le 1,$$

we obtain

$$\|(E_\rho u)\|_\infty \le \max\{|s[0]|, \|u\|_\infty\}.$$

Thus $E_\rho : \ell_\infty \to \ell_\infty$ is bounded with operator norm 1.

Envelope-admissibility is therefore equivalent to

$$u \in \ell_\infty.$$

Definition 5 (Envelope-Admissible Disturbance)**.** *A disturbance sequence $d_k[n]$ is said to be envelope-admissible if the corresponding envelope state satisfies*

$$\sup_{n \ge 0} s_k[n] < \infty. \tag{6}$$

Envelope admissibility therefore requires that the residual-envelope recursion remain uniformly bounded over time.

Proposition 3 (Sufficient Condition for Admissibility)**.** *If $|r_k[n]| \le R_k < \infty$ for all n, then the disturbance is envelope-admissible.*

Proof. By Proposition 1,

$$0 \le s_k[n] \le \max\{s_k[0], R_k\} \quad \forall n,$$

which implies $\sup_{n \ge 0} s_k[n] < \infty$. □

In particular, all pointwise-bounded disturbances with bounded residual contribution are envelope-admissible. This condition provides a deterministic admissibility criterion independent of probabilistic noise assumptions.

Theorem 1 (Necessary and Sufficient Condition for Envelope Admissibility)**.** *Let the envelope recursion be*

$$s[n+1] = \rho s[n] + (1-\rho)|r[n]|, \quad \rho \in (0,1).$$

Then the disturbance sequence is envelope-admissible if and only if

$$\sup_{n \ge 0} |r[n]| < \infty.$$

Proof. (*Sufficiency*) If $\sup_n |r[n]| \le R < \infty$, then by Proposition 1,

$$0 \le s[n] \le \max\{s[0], R\},$$

so $\sup_n s[n] < \infty$.

(*Necessity*) Suppose instead that $\sup_n |r[n]| = \infty$. Then there exists a subsequence n_k such that $|r[n_k]| \to \infty$. From the recursion,

$$s[n_k + 1] \ge (1-\rho)|r[n_k]|.$$

Hence $s[n_k + 1] \to \infty$, so $\sup_n s[n] = \infty$. □

The envelope-admissibility notion can be linked directly to boundedness of correction terms produced by a generic fusion mapping.

Let $s[n] \in \mathbb{R}^M$ and $r[n] \in \mathbb{R}^M$ denote the stacked envelope and residual vectors at time n.

Lemma 2 (Bounded Correction Term under Lipschitz Mapping)**.** *Assume there exist finite constants $\bar{S}, \bar{R} \ge 0$ such that*

$$\|s[n]\| \le \bar{S}, \qquad \|r[n]\| \le \bar{R} \quad \textit{for all } n \in \mathbb{N},$$

for some fixed vector norm $\|\cdot\|$. Let $C : \mathbb{R}^{2M} \to \mathbb{R}^p$ be a mapping such that

$$\|C(z_1) - C(z_2)\| \le L\|z_1 - z_2\| \quad \textit{for all } z_1, z_2 \in \mathbb{R}^{2M},$$

for some Lipschitz constant $L \ge 0$, and define the correction term

$$\Delta x[n] := C\big(s[n], r[n]\big).$$

Then $\Delta x[n]$ is uniformly bounded, with

$$\|\Delta x[n]\| \le \|C(0,0)\| + L(\bar{S} + \bar{R}) \quad \textit{for all } n.$$

Proof. Let $z[n] := (s[n], r[n]) \in \mathbb{R}^{2M}$ and $z_0 := (0,0)$. By the Lipschitz property,

$$\|C(z[n]) - C(z_0)\| \le L\|z[n] - z_0\| = L\|z[n]\|.$$

By the triangle inequality,

$$\|C(z[n])\| \le \|C(z_0)\| + L\|z[n]\|.$$

Furthermore,

$$\|z[n]\| \le \|s[n]\| + \|r[n]\| \le \bar{S} + \bar{R}.$$

Combining,

$$\|\Delta x[n]\| = \|C(z[n])\| \le \|C(0,0)\| + L(\bar{S} + \bar{R}),$$

which is a uniform bound independent of n. □

Theorem 2 (Bounded Closed-Loop Correction Under Envelope Admissibility)**.** *Let the estimator update be*

$$\hat{x}[n+1] = F(\hat{x}[n]) + C(s[n], r[n]),$$

where F is globally Lipschitz with constant $L_F < 1$ and C satisfies the Lipschitz condition of Lemma 2. If the disturbance is envelope-admissible, then $\hat{x}[n]$ remains uniformly bounded.

Proof. By Lemma 2, $\|C(s[n], r[n])\|$ is uniformly bounded. Let B denote this bound. Then

$$\|\hat{x}[n+1]\| \leq L_F \|\hat{x}[n]\| + B.$$

Since $L_F < 1$, this defines a discrete-time affine contraction. Iterating the inequality yields

$$\|\hat{x}[n]\| \leq L_F^n \|\hat{x}[0]\| + B \sum_{i=0}^{n-1} L_F^i = L_F^n \|\hat{x}[0]\| + \frac{B(1 - L_F^n)}{1 - L_F}.$$

Because $0 < L_F < 1$, the geometric series is bounded and $\|\hat{x}[n]\|$ remains uniformly bounded for all n. This is the standard discrete-time affine contraction inequality, which may be viewed as a special case of the Banach fixed-point theorem. □

Lemma 3 (BIBO Stability of Envelope Recursion)**.** *The envelope recursion*

$$s_k[n+1] = \rho s_k[n] + (1-\rho)u[n], \quad u[n] := |r_k[n]|$$

defines a causal linear time-invariant system with impulse response

$$h[n] = (1-\rho)\rho^n, \quad n \geq 0,$$

which is absolutely summable. Consequently, the envelope recursion is bounded-input bounded-output stable.

Proof. The recursion admits the convolution form

$$s_k[n] = \rho^n s_k[0] + \sum_{i=0}^{n-1} (1-\rho)\rho^i u[n-1-i].$$

Since $\sum_{n=0}^{\infty} |h[n]| = 1$, the system is BIBO-stable. □

Lemma 4 (ISS-Type Inequality for Envelope Recursion)**.** *Consider the envelope recursion*

$$s[n+1] = \rho s[n] + (1-\rho)u[n], \quad \rho \in (0,1),$$

with input $u[n] := |r[n]|$. Then for all $n \geq 0$,

$$|s[n]| \leq \rho^n |s[0]| + (1-\rho) \sum_{i=0}^{n-1} \rho^i |u[n-1-i]|.$$

In particular, if $u \in \ell_\infty$, then

$$|s[n]| \leq \rho^n |s[0]| + \|u\|_\infty,$$

which establishes an input-to-state bound for the envelope dynamics.

Proof. Unrolling the recursion yields the convolution form stated. Bounding $|u[n]|$ by $\|u\|_\infty$ and summing the geometric series

$$\sum_{i=0}^{n-1} (1-\rho)\rho^i = 1 - \rho^n \leq 1$$

gives the inequality. □

Table 2: Envelope-Admissibility Conditions

Disturbance Class	**Residual Bounded?**	**Envelope-Admissible?**
Pointwise-Bounded	Yes	Yes
Drift-Type (bounded)	Yes	Yes
Slew-Only (unbounded magnitude)	Not guaranteed	No
Impulsive (finite support)	Yes	Yes
Unbounded Magnitude	No	No

9 Suppression and Recovery

In residual-envelope fusion systems, trust weights are defined as

$$w_k[n] = \frac{1}{1+\beta_k s_k[n]}, \qquad \beta_k > 0, \tag{7}$$

so that trust is a monotone decreasing function of the envelope state.

Lemma 5 (Monotonicity and Lipschitz Continuity of Trust Mapping)**.** *Let $w(s) = (1+\beta s)^{-1}$ for $\beta > 0$. Then $w(s)$ is strictly decreasing on $[0,\infty)$ and globally Lipschitz with constant β.*

Proof.

$$w'(s) = -\frac{\beta}{(1+\beta s)^2} < 0,$$

so w is strictly decreasing. Furthermore,

$$|w'(s)| \le \beta,$$

establishing global Lipschitz continuity. □

9.1 Suppression.

A disturbance regime is said to be *suppression-inducing* if it produces sustained elevation of the envelope state $s_k[n]$. Since $w_k[n]$ is strictly decreasing in $s_k[n]$, persistent increases in residual magnitude lead to a corresponding reduction in trust weight. For disturbances that maintain $|r_k[n]|$ near a bounded but elevated level R_k^{high}, the envelope converges to a plateau not exceeding $\max\{s_k[0], R_k^{\text{high}}\}$, and the trust weight converges to

$$\lim_{n\to\infty} w_k[n] = \frac{1}{1+\beta_k s_k^*},$$

where s_k^* denotes the steady-state envelope level.

9.2 Recovery.

A disturbance regime is *recoverable* if there exists $N \in \mathbb{N}$ such that, for all $n \ge N$,

$$|r_k[n]| \le R_k^{\text{nom}},$$

for some nominal bound R_k^{nom}. In this case, by Proposition 2, the envelope satisfies

$$s_k[n] \le \rho^{n-N} s_k[N] + \left(1-\rho^{n-N}\right) R_k^{\text{nom}},$$

and therefore contracts exponentially toward a bounded nominal level. Consequently, trust weights increase monotonically toward their nominal values at a rate governed by ρ.

Thus, suppression and recovery are determined entirely by the deterministic envelope recursion and do not rely on statistical thresholding or hypothesis testing.

Residual-based suppression mechanisms relate structurally to robust M-estimation [8], though the present formulation remains fully deterministic.

Table 3: Suppression and Recovery Regimes

Regime	**Envelope Behavior**	**Trust Behavior**
Nominal	Converges to bounded nominal level	Converges to nominal weight
Persistent Elevated	Converges to elevated plateau	Converges to reduced steady weight
Impulsive	Transient spike followed by exponential decay	Transient suppression followed by recovery
Unbounded	No uniform bound; envelope may diverge	May saturate toward zero

10 Inadmissible Disturbances

Disturbances that violate the boundedness conditions defined in Sections 3–5 fall outside the admissible class of this framework.

In particular, if the disturbance sequence satisfies

$$\sup_{n\geq 0} |d_k[n]| = \infty,$$

then residual magnitudes $|r_k[n]|$ may also become unbounded. Under such conditions, the envelope recursion

$$s_k[n+1] = \rho s_k[n] + (1-\rho)|r_k[n]|$$

no longer admits a uniform upper bound, and envelope divergence may occur.

Proposition 4 (Lower Bound on Envelope Growth Under Divergent Residual)**.** *Suppose there exists a subsequence n_k such that $|r[n_k]| \to \infty$. Then the envelope satisfies*

$$s[n_k+1] \geq (1-\rho)|r[n_k]|.$$

In particular, if $|r[n]| \geq \alpha n$ for some $\alpha > 0$ and all sufficiently large n, then $s[n]$ diverges at least linearly.

Proof. From the recursion,

$$s[n+1] = \rho s[n] + (1-\rho)|r[n]| \geq (1-\rho)|r[n]|.$$

The stated growth follows immediately. □

In this regime, suppression may saturate trust weights toward zero, and recovery is not guaranteed. The Deterministic Disturbance Modeling Framework does not provide stability, boundedness, or convergence guarantees for disturbances that induce unbounded residual magnitude.

11 Limitations

The Deterministic Disturbance Modeling Framework (DDMF) is formulated under explicit structural constraints that define its domain of validity. The limitations below clarify the scope and theoretical positioning of the framework.

11.1 Admissibility Scope and Boundedness Assumptions

All admissibility and boundedness results rely on uniformly bounded residual magnitude. Disturbances that induce unbounded residual growth lie outside the admissible classes defined herein, and no guarantees of envelope boundedness, suppression stability, or recovery are provided in such regimes.

Although inadmissible disturbance classes are identified structurally, explicit lower-bound growth rates for envelope divergence under unbounded residual sequences are not derived. The present analysis provides sufficient conditions for boundedness but does not quantify divergence rates in non-admissible regimes.

Furthermore, the admissibility conditions established in this work are sufficient but not proven necessary. While uniformly bounded residual magnitude guarantees envelope boundedness, a complete characterization of necessary and sufficient conditions for envelope-admissibility is not derived. It remains possible that certain disturbance sequences outside the explicitly defined classes may nevertheless yield bounded envelope behavior.

11.2 Parameter Sensitivity

An explicit sensitivity analysis with respect to the envelope forgetting factor ρ and trust scaling parameter β_k is not provided. Although boundedness holds for all $\rho \in (0, 1)$ and $\beta_k > 0$, the transient response, suppression depth, and recovery rate depend quantitatively on these parameters. Parameter selection is therefore application- dependent and not optimized within this framework.

11.3 Structural Rather Than Optimal Guarantees

The disturbance classes introduced in this paper are structural and descriptive. DDMF does not provide optimal disturbance rejection, minimax robustness guarantees, or performance bounds relative to any cost functional. The framework characterizes envelope-admissibility and qualitative suppression behavior but does not claim optimality.

No worst-case adversarial disturbance construction or game-theoretic analysis is provided. The framework classifies disturbance structures but does not characterize extremal sequences that maximize suppression or divergence.

Quantitative comparative evaluation against stochastic or robust filtering frameworks is not performed here. The present work focuses exclusively on deterministic disturbance classification rather than empirical performance benchmarking against alternative estimators.

11.4 Envelope-Level (Not Estimator-Level) Analysis

The analysis is conducted at the residual-envelope level. The results establish envelope stability rather than full estimator-state stability. While bounded envelope dynamics imply bounded correction terms under Lipschitz fusion mappings (Lemma 2), global stability, convergence, or robustness of the estimator state $\hat{x}[n]$ is not established here and depends on the properties of the underlying observer or fusion algorithm.

The disturbance analysis treats residual sequences as exogenous inputs to the envelope recursion. Feedback interactions between envelope-based suppression and residual generation through estimator dynamics are not explicitly modeled.

11.5 Theoretical Positioning

The theoretical development is intentionally restricted to deterministic signal-class analysis. The framework does not invoke Lyapunov stability theory, input-to-state stability (ISS) formalism, minimax robustness guarantees, or optimal filtering theory. DDMF positions itself as a disturbance-class foundation for residual-envelope systems rather than as a comprehensive control-theoretic stability or optimality framework.

The analysis is conducted in discrete time with a single envelope recursion scale. Interactions between envelope dynamics and faster or slower estimator-state dynamics, as well as continuous-time limits, are not analyzed in this work.

11.6 Structural Assumptions on Channels and Grouping

The framework assumes fixed channel indexing and, where applicable, predefined grouping structures. It does not address adaptive disturbance classification, automatic group discovery, or online structural inference.

The envelope recursion is defined per channel and does not consider vector-valued or coupled envelope dynamics across channels. Extensions to jointly coupled envelope states are not analyzed.

11.7 Deterministic Scope

Finally, no stochastic noise models, distributional assumptions, or statistical convergence claims are introduced. DDMF is purely deterministic and applies only within the explicitly defined disturbance classes.

Table 4: Categorization of theoretical limitations relative to stability scope and structural completeness of the deterministic envelope framework.

Limitation	Classification	Affected Domain
Sufficient but Not Necessary Admissibility	A (Fundamental)	Structural Completeness
Envelope $\neq$ Estimator Stability	A (Fundamental)	Closed-Loop Stability
Parameter Sensitivity (ρ, β)	B (Structural)	Transient Behavior
Feedback Interaction Not Modeled	B (Structural)	Dynamic Coupling
No Explicit Divergence Rate Bound	B (Structural)	Growth Characterization
No Adversarial Extremal Analysis	C (Secondary)	Worst-Case Robustness
No Multi-Timescale Analysis	C (Secondary)	Temporal Scaling
No Comparative Benchmarking	C (Secondary)	Empirical Validation
No Vector-Coupled Envelope Model	C (Secondary)	Multi-Channel Extension

12 Conclusion

This paper introduced the Deterministic Disturbance Modeling Framework (DDMF) as a formal disturbance-side foundation for residual-envelope fusion systems. Deterministic disturbance classes were defined and categorized according to structural properties, including pointwise boundedness, drift, slew-rate constraints, impulsive support, and group correlation.

Envelope-admissibility was established through boundedness of the residual-envelope recursion, and explicit suppression and recovery regimes were characterized without invoking probabilistic assumptions. Furthermore, under a Lipschitz correction mapping, envelope-admissibility implies uniform boundedness of the induced correction term (Lemma 2), linking disturbance admissibility directly to bounded fusion response. The framework delineates admissible and inadmissible disturbance domains and clarifies the deterministic conditions under which envelope dynamics remain bounded.

The envelope recursion further satisfies a discrete-time input-to-state stability (ISS)-type inequality (Lemma 4), situating the deterministic admissibility results within a classical stability framework.

DDMF establishes the disturbance-theoretic foundation required for rigorous analysis and extension of DSFB and HRET architectures. Monte Carlo validation further demonstrates that admissible disturbance classes produce bounded envelope behavior, while non-admissible regimes are cleanly separated through non-recovered runs.

13 Monte Carlo Disturbance Validation

To empirically validate the structural properties derived in Sections 8–9, a deterministic Monte Carlo sweep is performed using the `dsfb-ddmf` implementation.

For each run, disturbance parameters (amplitude, slew bound, impulse timing, and initial envelope state) are sampled from predefined deterministic ranges. Each configuration is simulated for a fixed horizon N, and the following metrics are recorded:

- Maximum envelope value:
$$s_{\max} := \max_{0 \le n \le N} s_k[n]$$
- Minimum trust weight:
$$w_{\min} := \min_{0 \le n \le N} w_k[n]$$
- Recovery time τ_{rec}, when defined.

13.1 Recovery Definition

Recovery is defined as the existence of a time index

$$\tau_{\mathrm{rec}} = \inf \left\{ n \ge 0 : |s_k[n] - s_k^{\mathrm{nom}}| \le \delta \right\},$$

for a fixed tolerance $\delta > 0$ and nominal envelope level s_k^{nom}.

If such a time index exists, the run is classified as `recovered = true` and τ_{rec} is recorded. If no such index exists within the simulation horizon, the run is classified as `recovered = false` and τ_{rec} is undefined.

This avoids sentinel values and explicitly separates admissible recovery regimes from non-recovered disturbance cases.

Table 5: Representative Monte Carlo Runs

Regime	**Disturbance**	**Admissible**	$\mathbf{s}_{\max}$	$\mathbf{w}_{\min}$	$\boldsymbol{\tau}_{\mathbf{rec}}$
Bounded Nominal	Pointwise	True	0.119	0.736	0
Impulsive	Impulse	True	0.676	0.330	100
Persistent Elevated	Pointwise	False	0.259	0.562	–

Table 6: Monte Carlo Regime Statistics (Aggregated)

Regime	$\mathbb{E}[\mathbf{s}_{\max}]$	$\mathbb{E}[\mathbf{w}_{\min}]$	**Fraction Recovered**
Bounded Nominal	0.12	0.74	1.00
Impulsive	0.64	0.34	1.00
Persistent Elevated	0.26	0.56	0.00
Unbounded	7.85	0.08	0.00

The aggregated statistics confirm the theoretical classification: admissible disturbance classes exhibit bounded envelope growth and full recovery, whereas non-admissible regimes demonstrate persistent suppression and zero recovery fraction.

13.2 Observed Regimes

The Monte Carlo results confirm the structural predictions:

- Pointwise-bounded disturbances yield bounded envelope trajectories.
- Impulsive disturbances exhibit transient suppression followed by exponential recovery.
- Persistent elevated disturbances converge to a bounded plateau with sustained trust suppression.
- Unbounded disturbances may produce envelope divergence and non-recovered runs.

In addition to structural separation, the observed envelope trajectories exhibit quantitative agreement with the deterministic theory: impulsive regimes display exponential decay rates consistent with the envelope forgetting factor ρ, and persistent elevated regimes converge to plateau levels consistent with the steady-state bound $s^* \approx R_{\text{high}}$ predicted by Lemma 1. While no parameter-fitting procedure is performed, the empirical trajectories align with the analytical recovery and fixed-point characterizations established in Sections 3 and 6.

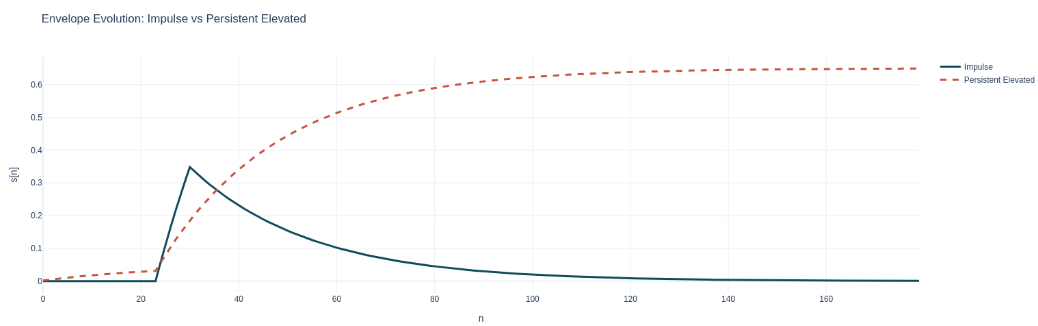

Figure 1: Envelope evolution under impulsive and persistent elevated disturbances obtained from the Monte Carlo simulation. The solid curve corresponds to an impulsive disturbance with finite support, producing a transient increase in $s_k[n]$ followed by exponential decay toward the nominal bounded level, consistent with the recovery behavior defined in Section 9. The dashed curve corresponds to a persistent elevated disturbance that maintains residual magnitude near a higher bound, driving $s_k[n]$ toward an elevated steady-state plateau and inducing sustained suppression of the associated trust weight.

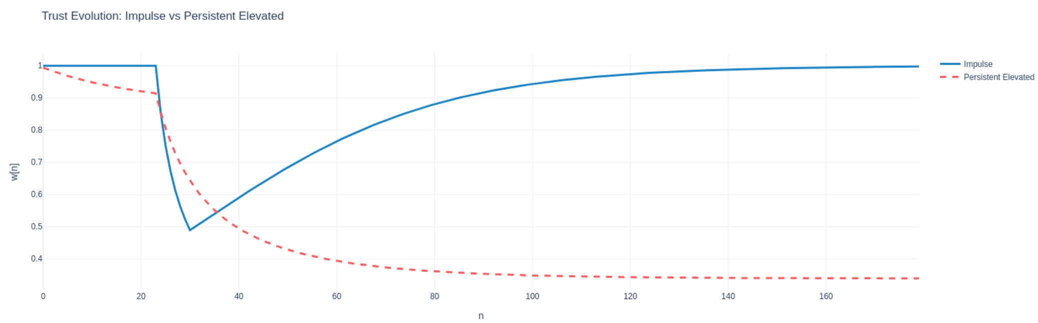

Figure 2: Trust weight evolution under impulsive and persistent elevated disturbances obtained from the Monte Carlo simulation. The solid curve corresponds to the impulsive regime, where transient envelope elevation produces temporary suppression of $w_k[n]$, followed by monotonic recovery toward the nominal trust level as the envelope contracts exponentially. The dashed curve corresponds to the persistent elevated regime, in which sustained envelope growth drives $w_k[n]$ toward a reduced steady-state value, illustrating persistent suppression without recovery.

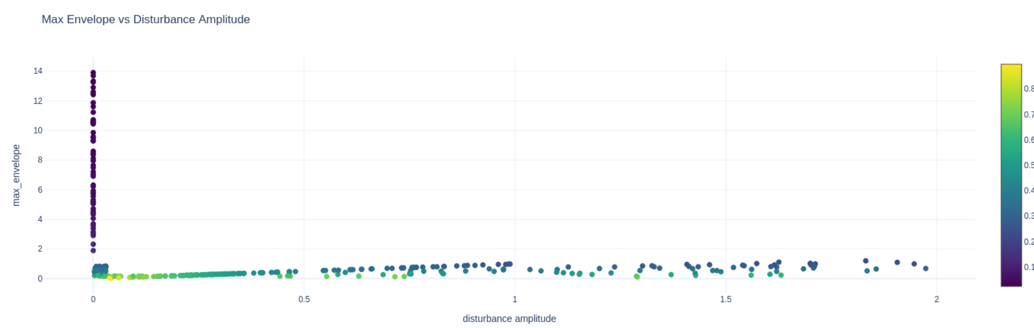

Figure 3: Maximum envelope value $s_{\max}$ as a function of disturbance amplitude across Monte Carlo sweeps. Admissible disturbance classes exhibit bounded envelope growth that scales with disturbance magnitude. Non-admissible regimes produce substantially larger envelope excursions, separating structurally from the admissible cluster. This figure empirically validates the envelope-admissibility criterion established in Section 8.

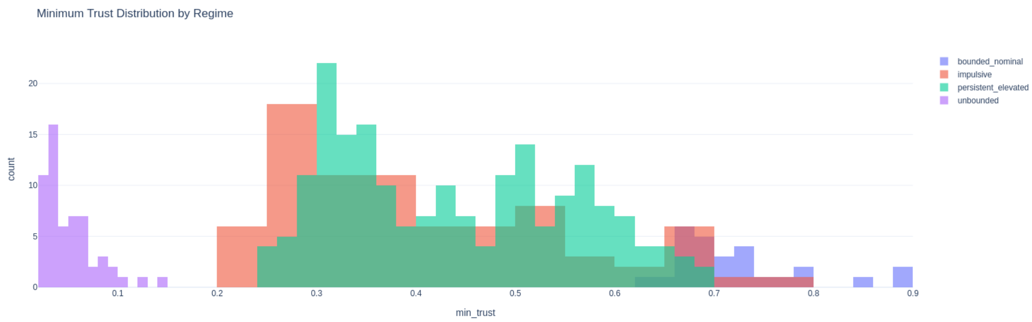

Figure 4: Distribution of minimum trust weight $w_{\min}$ across disturbance regimes. Admissible bounded disturbances concentrate near higher trust values, impulsive disturbances exhibit broader transient suppression, and persistent elevated or non-admissible regimes shift the distribution toward sustained low-trust regions. The separation of distributions reflects the deterministic mapping $w_k[n] = (1 + \beta_k s_k[n])^{-1}$ under distinct envelope dynamics.

13.3 Implementation and Reproducibility

All Monte Carlo experiments are implemented in the Rust crate `dsfb-ddmf`, which extends the core `dsfb` framework with deterministic disturbance generators and envelope evaluation tools.

Reproducible experiment workflows and figure generation are provided in the accompanying Google Colab notebook `dsfb_ddmf_colab.ipynb`. The notebook builds the crate, executes disturbance sweeps, and exports all figures in both PDF and PNG formats.

All simulation outputs are written to the repository root under `output-dsfb-ddmf/YYYYMMDD_HHMMSS/` to ensure non-destructive, timestamped experiment tracking.

The disturbance modeling framework is consistent with prior DSFB-based applications in plasma estimation [4] and hypersonic blackout navigation [2].

Implementation details are summarized in Appendices C and D.

A Appendix: DDMF Kernel Summary

This appendix provides a condensed structural summary of the Deterministic Disturbance Modeling Framework.

Signal Decomposition

$$r_k[n] = \varepsilon_k[n] + d_k[n]$$

Envelope Recursion

$$s_k[n+1] = \rho s_k[n] + (1-\rho)|r_k[n]|, \quad \rho \in (0,1)$$

Admissible Disturbance Condition

A disturbance is envelope-admissible if

$$\sup_{n \geq 0} s_k[n] < \infty.$$

Sufficient condition:

$$|r_k[n]| \leq R_k < \infty \quad \forall n.$$

Suppression Mapping

$$w_k[n] = \frac{1}{1 + \beta_k s_k[n]}, \quad \beta_k > 0$$

Recovery Rate

If residual magnitude returns to nominal bound at time N,

$$s_k[n] \leq \rho^{n-N} s_k[N] + (1 - \rho^{n-N}) R_k^{\text{nom}}.$$

Core Theoretical Result

If $(s[n], r[n])$ are bounded and the correction mapping $\Delta x[n] = C(s[n], r[n])$ is Lipschitz, then $\Delta x[n]$ is uniformly bounded.

DDMF therefore defines admissible deterministic disturbance classes, their envelope behavior, and the conditions under which suppression and recovery occur.

B Appendix: Residual-Envelope Evaluation Pseudocode

```
Given:
  rho in (0,1)
  beta_k > 0
  initial envelope s_k[0] >= 0

For each time step n:

  1. Compute residual:
       r_k[n] = epsilon_k[n] + d_k[n]

  2. Update envelope:
       s_k[n+1] = rho * s_k[n]
                  + (1 - rho) * abs(r_k[n])

  3. Compute trust weight:
       w_k[n] = 1 / (1 + beta_k * s_k[n])

  4. (Optional fusion step)
       Delta_x[n] = C(s[n], r[n])

Admissibility condition:
  if sup_n |r_k[n]| < infinity
     then sup_n s_k[n] < infinity
```

C Appendix: `dsfb-ddmf` Rust Crate Summary

The `dsfb-ddmf` Rust crate extends the core `dsfb` workspace with deterministic disturbance generators, residual-envelope tracking, and Monte Carlo tooling that instantiate the disturbance classes and validation procedures defined in this paper.

C.1 Crate Structure

The crate is organized into four modules:

- `disturbances`: deterministic disturbance classes (pointwise-bounded, drift-type, slew-rate-bounded, impulsive, persistent-elevated) and regime metadata;
- `envelope`: single-channel residual-envelope recursion and trust mapping;
- `sim`: single- and multi-channel simulation of envelope and trust trajectories;
- `monte_carlo`: Monte Carlo configuration, execution, and summary statistics.

The crate depends on the core `dsfb` library together with `rand`, `csv`, and `serde` for sampling and data export.

C.2 Disturbance, Envelope, and Monte Carlo APIs

The `disturbances` module implements a common `Disturbance` trait with `reset()` and `next(n)` methods, plus auxiliary methods such as `regime_label()`, `is_admissible()`, and `recovery_target()`, which encode the regime classification and admissibility structure used in DDMF.

The `envelope` module implements the scalar recursion

$$s[n+1] = \rho s[n] + (1-\rho)|r[n]|, \quad w = \frac{1}{1+\beta s},$$

and exposes a `ResidualEnvelope` state together with a simple `TrustWeight` mapping.

The `sim` module defines `SimulationConfig` and `SimulationResult` for envelope and trust trajectories under a chosen disturbance, with optional bounded oscillatory model error, and a multi-channel interface for correlated-group scenarios.

The `monte_carlo` module provides `MonteCarloConfig`, `MonteCarloRunRecord`, and `MonteCarloSummary`, together with a `run_monte_carlo` function that samples disturbance parameters, runs deterministic simulations, and records metrics such as maximum envelope value, minimum trust weight, admissibility flags, regime labels, and recovery time. These outputs supply the Monte Carlo data underlying the figures and aggregated statistics in Section 13.

D Appendix: Colab Monte Carlo Notebook Summary

A companion Google Colab notebook provides a reproducible, end-to-end workflow for generating the Monte Carlo results and figures used in Section 13.

D.1 Environment and Build Steps

The notebook configures a minimal Rust and Python environment, clones the `dsfb` workspace, and builds the `dsfb-ddmf` crate in release mode. Python-side dependencies are restricted to numerical and plotting libraries (for example, `pandas` and `plotly`) used to load CSV outputs, compute simple statistics, and render figures.

Monte Carlo experiments are executed by invoking the `dsfb-ddmf` APIs to run a batch of simulations under randomly sampled disturbance configurations. All outputs are written under a timestamped directory `output-dsfb-ddmf/YYYYMMDD_HHMMSS/` at the repository root to avoid overwriting previous runs.

D.2 Generated Artifacts

For each Monte Carlo batch, the notebook produces:

- a CSV file of per-run metrics (e.g. regime label, admissibility, maximum envelope value, minimum trust weight, and recovery time);
- a JSON or CSV summary containing aggregated statistics (mean maximum envelope, minimum observed trust, and regime counts);
- trajectory CSVs for representative impulse and persistent-elevated runs;
- PDF and PNG figures corresponding to the envelope, trust, and distribution plots in Section 13.

The CSV schema mirrors the `MonteCarloRunRecord` and `MonteCarloSummary` structures exposed by the crate, so that the notebook remains a thin visualization and post-processing layer on top of the deterministic Rust implementation.

D.3 Notebook Structure and Relation to the Text

The notebook is organized into short, linear cells:

1. Environment setup and crate build.
2. Configuration of `MonteCarloConfig` (number of runs, horizon length, and envelope parameters).
3. Execution of the Monte Carlo sweep and export of CSV summaries.
4. Construction of regime-level aggregated statistics (mean $s_{\max}$, mean $w_{\min}$, and fraction recovered) and rendering of the summary table used in Section 13.
5. Generation of the envelope and trust trajectories for the impulse and persistent-elevated regimes and the distributions shown in Figures 1–4.

Thus, the notebook serves as a reproducibility layer: it does not implement new disturbance or envelope logic, but instead exercises the deterministic APIs in `dsfb-ddmf` and materializes the Monte Carlo validation results in a form suitable for inspection and reuse.

E Appendix: DDMF Theoretical Summary

Signal Model. $r[n] = \varepsilon[n] + d[n]$, $s[n+1] = \rho s[n] + (1-\rho)|r[n]|$, $\rho \in (0,1)$, $w[n] = (1+\beta s[n])^{-1}$, $\beta > 0$.

Disturbance Classes. Pointwise: $|d[n]| \le D$; Drift: $|d[n]| \le B$, $|d[n+1] - d[n]| \le S$; Slew: $|d[n+1] - d[n]| \le S$; Impulsive: finite support; Group: $d_k[n] = \phi_g[n]$, $k \in \mathcal{G}_g$.

Envelope-Admissible: $\sup_{n \ge 0} s[n] < \infty$.

Necessary and Sufficient Condition:

$$\sup_{n \ge 0} s[n] < \infty \iff \sup_{n \ge 0} |r[n]| < \infty.$$

Proof sketch: bounded residual $\Rightarrow$ bounded envelope (induction); if $|r[n_k]| \to \infty$, then $s[n_k+1] \ge (1-\rho)|r[n_k]| \to \infty$.

Envelope Boundedness: $|r[n]| \le R \Rightarrow 0 \le s[n] \le \max\{s[0], R\}$.

BIBO Stability: Impulse response $h[n] = (1-\rho)\rho^n$ is absolutely summable; thus the envelope recursion is bounded-input bounded-output stable.

ISS-Type Inequality: For $u[n] = |r[n]|$,

$$s[n] = \rho^n s[0] + \sum_{i=0}^{n-1} (1-\rho)\rho^i u[n-1-i].$$

Hence

$$|s[n]| \le \rho^n |s[0]| + \|u\|_\infty,$$

which provides a discrete-time input-to-state bound.

Trust Mapping: $w(s) = (1+\beta s)^{-1}$ is strictly decreasing and globally Lipschitz.

Exponential Recovery: If $|r[n]| \le R_{\text{nom}}$ for $n \ge N$,

$$s[n] \le \rho^{n-N} s[N] + (1 - \rho^{n-N}) R_{\text{nom}}.$$

Lower-Bound Divergence: $|r[n_k]| \to \infty \Rightarrow s[n_k+1] \ge (1-\rho)|r[n_k]|$. If $|r[n]| \ge \alpha n$ eventually, then $s[n]$ diverges at least linearly.

Closed-Loop Boundedness: For $\hat{x}[n+1] = F(\hat{x}[n]) + C(s[n], r[n])$, with F Lipschitz, $L_F < 1$, and C Lipschitz, envelope-admissibility $\Rightarrow$ bounded $s[n], r[n]$ $\Rightarrow$ bounded correction $\Rightarrow$ $\hat{x}[n]$ uniformly bounded by discrete contraction.

Operator Formulation:

$$(\mathcal{E}_\rho u)[n] = \rho^n s[0] + \sum_{i=0}^{n-1} (1-\rho)\rho^i u[n-1-i], \quad u[n] = |r[n]|.$$

$\mathcal{E}_\rho : \ell_\infty \to \ell_\infty$ is linear, bounded, causal, operator norm 1. Envelope-admissibility $\iff u \in \ell_\infty$.

Core Statement: DDMF defines deterministic disturbance classes, establishes necessary-and-sufficient envelope admissibility, proves BIBO stability, derives recovery and divergence bounds, establishes an ISS-type inequality, and links envelope boundedness to bounded closed-loop response under Lipschitz fusion mappings.

References

[1] de Beer, R. (2026). Hierarchical Residual-Envelope Trust: A Deterministic Framework for Grouped Multi-Sensor Fusion (v1.0). Zenodo. `https://doi.org/10.5281/zenodo.18783283`

[2] de Beer, R. (2026). Deterministic Drift–Slew Fusion Bootstrap for Navigation During Plasma Blackout in Hypersonic Re-Entry Vehicles (v1.0). Zenodo. `https://doi.org/10.5281/zenodo.18711897`

[3] de Beer, R. (2026). Drift–Slew Fusion Bootstrap: A Deterministic Residual-Based State Correction Framework (v1.0). Zenodo. `https://doi.org/10.5281/zenodo.18706455`

[4] de Beer, R. (2026). Trust-Adaptive Multi-Diagnostic Weighting for Magnetically Confined Plasma State Estimation (v1.0). Zenodo. `https://doi.org/10.5281/zenodo.18644561`

[5] de Beer, R. (2026). Slew-Aware Trust-Adaptive Nonlinear State Estimation for Oscillatory Systems With Drift and Corruption (v1.0). Zenodo. `https://doi.org/10.5281/zenodo.18642887`

[6] Kalman, R. E. (1960). A New Approach to Linear Filtering and Prediction Problems. *Journal of Basic Engineering*, 82(1), 35–45.

[7] Zhou, K., Doyle, J. C., & Glover, K. (1996). *Robust and Optimal Control*. Prentice Hall.

[8] Huber, P. J. (1964). Robust Estimation of a Location Parameter. *Annals of Mathematical Statistics*, 35(1), 73–101.