

# Hierarchical Residual-Envelope Trust: A Deterministic Framework for Grouped Multi-Sensor Fusion

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## Abstract

Hierarchical Residual-Envelope Trust (HRET) extends the Drift-Slew Fusion Bootstrap (DSFB) framework to structured multi-sensor systems with explicit sensor grouping and correlated disturbance modeling. Unlike stochastic or covariance-based approaches, HRET operates under deterministic bounded disturbance assumptions and modulates trust through residual-envelope dynamics at both channel and group levels. Each sensor channel maintains a residual envelope governing channel-level trust, while group envelopes aggregate residual magnitudes across predefined sensor clusters to capture correlated degradation. Trust weights are composed multiplicatively and normalized to preserve strict convexity, ensuring positive weights and unity sum at all times. The framework guarantees bounded envelope dynamics under bounded disturbances and enables coordinated suppression and recovery of sensor groups under correlated corruption. HRET reduces to DSFB as a special case when each channel forms its own singleton group. The formulation is fully deterministic, convexity-preserving, and implementation-aligned with the existing DSFB architecture. The framework assumes bounded disturbances and predefined grouping, and does not claim statistical optimality.

## 1 Introduction

Hierarchical Residual-Envelope Trust (HRET) formalizes a structured extension of the Drift-Slew Fusion Bootstrap (DSFB) framework for systems in which sensing channels are naturally organized into groups. While DSFB defines deterministic residual-envelope modulation at the individual channel level, HRET introduces an additional aggregation layer that captures correlated behavior across predefined sensor clusters.

The central objective of HRET is to introduce a hierarchical trust architecture that:

- augments channel-level residual envelopes with group-level envelope dynamics,
- composes channel and group trust multiplicatively in a deterministic manner,
- preserves strict convex normalization of fusion weights at all times,
- maintains bounded envelope and weight evolution under bounded disturbances.

The formulation assumes the residual definition, channel envelope recursion, and deterministic trust mapping established in the DSFB Core Framework. HRET does not alter the underlying disturbance model or channel-level structure; it adds a structured hierarchical layer that enables coordinated suppression and recovery in the presence of correlated sensor degradation.

## 2 System and Grouping Model

Consider a discrete-time nonlinear dynamical system

$$x[n+1] = f(x[n], u[n]), \quad (1)$$

where  $x[n] \in \mathbb{R}^p$  is the system state and  $u[n]$  is a known input.

The system is observed through  $M$  measurement channels,

$$y_k[n] = h_k(x[n]) + d_k[n], \quad k = 1, \dots, M, \quad (2)$$

where  $h_k(\cdot)$  denotes the channel measurement map and  $d_k[n]$  represents a deterministic disturbance term.

**Assumption 1** (Bounded Deterministic Disturbances). *For each channel  $k$ , there exists a constant  $D_k \geq 0$  such that*

$$|d_k[n]| \leq D_k \quad \text{for all } n \geq 0. \quad (3)$$

*No probabilistic structure is assumed for  $d_k[n]$ .*

Let  $\hat{x}[n]$  denote the estimator state. The channel residuals are defined as

$$r_k[n] = y_k[n] - h_k(\hat{x}[n]), \quad k = 1, \dots, M. \quad (4)$$

To introduce hierarchical structure, the index set of channels is partitioned into  $G$  disjoint groups,

$$\mathcal{G}_g \subset \{1, \dots, M\}, \quad g = 1, \dots, G, \quad (5)$$

satisfying

$$\mathcal{G}_g \cap \mathcal{G}_{g'} = \emptyset \quad (g \neq g'), \quad \bigcup_{g=1}^G \mathcal{G}_g = \{1, \dots, M\}. \quad (6)$$

Each channel belongs to exactly one group; denote by  $g(k)$  the unique group index such that

$$k \in \mathcal{G}_{g(k)}. \quad (7)$$

Table 1: Primary Symbols and Parameters

Symbol	Definition	Constraint
$x[n]$	System state	$\mathbb{R}^p$
$r_k[n]$	Channel residual	—
$s_k[n]$	Channel residual envelope	$s_k[n] \geq 0$
$s_g[n]$	Group residual envelope	$s_g[n] \geq 0$
$\rho, \rho_g$	Forgetting factors	$(0, 1)$
$\beta_k, \beta_g$	Trust sensitivity gains	$> 0$
$w_k[n]$	Channel trust weight	$(0, 1]$
$w_g[n]$	Group trust weight	$(0, 1]$
$\tilde{w}_k[n]$	Normalized hierarchical weight	$\sum \tilde{w}_k = 1$
$\mathcal{G}_g$	Channel group	Partition of $\{1, \dots, M\}$
$g(k)$	Group index of channel $k$	Unique mapping

### 3 Channel Residual Envelope

For each measurement channel  $k$ , a residual-envelope state  $s_k[n]$  is maintained to capture the recent magnitude of residual activity in a deterministic manner. The envelope evolves according to the first-order recursion

$$s_k[n+1] = \rho s_k[n] + (1 - \rho)|r_k[n]|, \quad k = 1, \dots, M, \quad (8)$$

where  $\rho \in (0, 1)$  is a forgetting factor and  $s_k[n] \geq 0$  for all  $n$  provided  $s_k[0] \geq 0$ .

The envelope  $s_k[n]$  serves as a state variable governing channel-level trust. A deterministic trust weight is assigned through the monotone mapping

$$w_k[n] = \frac{1}{1 + \beta_k s_k[n]}, \quad \beta_k > 0. \quad (9)$$

Since  $s_k[n] \geq 0$  and  $\beta_k > 0$ , it follows directly that

$$0 < w_k[n] \leq 1 \quad \text{for all } n. \quad (10)$$

Thus, each channel contributes to fusion through a strictly positive weight that decreases smoothly as residual magnitude accumulates and recovers as the envelope decays.

### 4 Group Residual Envelope Dynamics

To capture correlated residual behavior within predefined sensor clusters, each group  $g$  is assigned a group-level residual envelope state  $s_g[n]$ . This envelope aggregates residual magnitudes across all channels in  $\mathcal{G}_g$  and evolves according to

$$s_g[n+1] = \rho_g s_g[n] + (1 - \rho_g) \frac{1}{|\mathcal{G}_g|} \sum_{k \in \mathcal{G}_g} |r_k[n]|, \quad g = 1, \dots, G, \quad (11)$$

where  $\rho_g \in (0, 1)$  is a group-level forgetting factor and  $s_g[n] \geq 0$  provided  $s_g[0] \geq 0$ .

The group envelope defines a deterministic trust modulation at the cluster level via

$$w_g[n] = \frac{1}{1 + \beta_g s_g[n]}, \quad \beta_g > 0. \quad (12)$$

Since  $s_g[n] \geq 0$  and  $\beta_g > 0$ , the group weights satisfy

$$0 < w_g[n] \leq 1 \quad \text{for all } n. \quad (13)$$

The group envelope thus provides a coordinated mechanism for attenuating trust across all channels within a cluster when their residual magnitudes increase coherently.

## 5 Hierarchical Trust Composition

Channel-level and group-level trust variables are composed multiplicatively to form hierarchical weights. For each channel  $k$ , define the unnormalized hierarchical weight

$$\hat{w}_k[n] = w_k[n] w_{g(k)}[n], \quad k = 1, \dots, M, \quad (14)$$

where  $g(k)$  denotes the group index of channel  $k$ .

Normalization across all channels yields the hierarchical fusion weights

$$\tilde{w}_k[n] = \frac{\hat{w}_k[n]}{\sum_{j=1}^M \hat{w}_j[n]}. \quad (15)$$

The multiplicative structure ensures that channel trust is modulated both by its individual residual envelope and by the collective residual behavior of its group.

**Proposition 1** (Convexity Preservation). *Suppose  $w_k[n] > 0$  for all channels  $k$  and  $w_g[n] > 0$  for all groups  $g$ . Then for all  $n$ ,*

$$\tilde{w}_k[n] > 0, \quad \sum_{k=1}^M \tilde{w}_k[n] = 1. \quad (16)$$

*Proof.* Since  $w_k[n] > 0$  and  $w_{g(k)}[n] > 0$ , it follows that  $\hat{w}_k[n] > 0$  for all  $k$ . Hence

$$\sum_{j=1}^M \hat{w}_j[n] > 0. \quad (17)$$

Normalization by this strictly positive quantity yields  $\tilde{w}_k[n] > 0$ . Furthermore,

$$\sum_{k=1}^M \tilde{w}_k[n] = \frac{\sum_{k=1}^M \hat{w}_k[n]}{\sum_{j=1}^M \hat{w}_j[n]} = 1. \quad (18)$$

Thus, the hierarchical weights form a strictly positive convex combination.  $\square$

## 6 Hierarchical Fusion Update

Let  $K_k$  denote the deterministic correction gain associated with channel  $k$ , consistent with the DSFB Core Framework. The hierarchical fusion mechanism aggregates channel corrections through the normalized weights defined in Section 5.

Define the composite correction term

$$\Delta x[n] = \sum_{k=1}^M \tilde{w}_k[n] K_k r_k[n]. \quad (19)$$

The estimator state is updated according to

$$\hat{x}[n+1] = f(\hat{x}[n], u[n]) + \Delta x[n]. \quad (20)$$

Since  $\tilde{w}_k[n]$  are strictly positive and sum to unity (Proposition 1), the correction term is a convex combination of channel-specific correction contributions. The hierarchical structure therefore modulates the influence of each channel without altering the deterministic gain structure or introducing non-convex aggregation.

## 7 Boundedness and Suppression Properties

Assume Assumption 1 holds and that the estimation error remains bounded so that each residual sequence  $r_k[n]$  is bounded. Then there exists  $R_k \geq 0$  such that

$$|r_k[n]| \leq R_k \quad \text{for all } n. \quad (21)$$

Under this condition, the channel envelope recursion

$$s_k[n+1] = \rho s_k[n] + (1-\rho)|r_k[n]| \quad (22)$$

implies that  $s_k[n]$  remains bounded for all  $n$ , with an upper bound determined by  $\rho$ ,  $s_k[0]$ , and  $\sup_n |r_k[n]|$ . An analogous argument applies to each group envelope  $s_g[n]$ , since it evolves as a convex combination of its previous value and bounded residual magnitudes.

Consequently:

- Channel weights  $w_k[n]$  and group weights  $w_g[n]$  remain strictly positive for all  $n$ .
- If residual magnitudes within a group remain persistently elevated, the corresponding group envelope  $s_g[n]$  increases, reducing  $w_g[n]$  and attenuating all channels in that group.
- When residual magnitudes decrease to nominal bounded levels,  $s_g[n]$  decays exponentially with rate  $\rho_g$ , and the associated group weight recovers accordingly.

Thus, hierarchical suppression and recovery are governed by bounded deterministic envelope dynamics. A detailed classification of admissible disturbance types is provided in the deterministic disturbance modeling framework paper.

## 8 Correlated Disturbance Structures

To formalize correlated degradation within a sensor cluster, consider a disturbance structure of the form

$$d_k[n] = \phi_g[n], \quad k \in \mathcal{G}_g, \quad (23)$$

where  $\phi_g[n]$  is a bounded deterministic signal associated with group  $g$ . This model represents a common-mode disturbance affecting all channels within the same group.

Under such a disturbance pattern, residuals within  $\mathcal{G}_g$  exhibit coherent growth in magnitude. Since the group envelope update depends on the average absolute residual over the group,

$$\frac{1}{|\mathcal{G}_g|} \sum_{k \in \mathcal{G}_g} |r_k[n]|, \quad (24)$$

persistent correlated residuals drive  $s_g[n]$  upward.

As  $s_g[n]$  increases, the group-level weight  $w_g[n]$  decreases, uniformly attenuating the hierarchical weights of all channels in  $\mathcal{G}_g$ . This mechanism produces coordinated suppression across the affected cluster without requiring probabilistic correlation modeling or covariance estimation.

## 9 Relationship to DSFB Core Framework

The Drift-Slew Fusion Bootstrap (DSFB) framework is obtained as a limiting case of HRET when no higher-level aggregation beyond individual channels is introduced. Specifically, let

$$G = M, \quad \mathcal{G}_g = \{g\}, \quad g = 1, \dots, M. \quad (25)$$

Under this construction, each channel forms a singleton group. The group envelope recursion becomes identical to the channel envelope recursion, and the group-level weight coincides with the channel-level weight up to parameter choice.

Consequently, the hierarchical weight reduces to the normalized channel weight structure defined in the DSFB Core Framework. In this sense, HRET strictly generalizes DSFB by introducing an additional aggregation layer without altering the underlying deterministic residual-envelope mechanism.

Table 2: Structural Comparison Between DSFB and HRET

Component	DSFB	HRET
Channel residual envelope $s_k[n]$	Yes	Yes
Group residual envelope $s_g[n]$	No	Yes
Channel trust weight $w_k[n]$	Yes	Yes
Group trust weight $w_g[n]$	No	Yes
Trust composition	Channel-only	Channel $\times$ Group
Convex normalization	Yes	Yes
Correlated disturbance handling	Implicit	Explicit group suppression
Reduction case	—	$G = M$ , singleton groups

## 10 Implementation Notes

The hierarchical extension is designed to integrate directly with the existing `dsfb` Rust crate without altering the channel-level residual-envelope machinery.

Implementation proceeds by introducing a group-indexed layer alongside the existing channel structures:

- Channel envelope states  $s_k[n]$  and channel weights  $w_k[n]$  remain unchanged from the DSFB Core implementation.
- Group envelope states  $s_g[n]$  and corresponding weights  $w_g[n]$  are stored in a separate group-indexed container.
- A static mapping structure, e.g. `group_of_channel[k] = g(k)`, encodes the partition of channels into disjoint groups.

At each update step:

1. Channel residuals  $r_k[n]$  are computed.
2. Channel envelopes  $s_k[n]$  and group envelopes  $s_g[n]$  are updated.
3. Unnormalized hierarchical weights  $\hat{w}_k[n] = w_k[n]w_{g(k)}[n]$  are formed.

4. A single normalization pass produces  $\tilde{w}_k[n]$ .

This structure preserves determinism, convex normalization, and compatibility with existing DSFB data flows. Detailed API-level interfaces and reproducibility workflows are specified in the implementation companion paper.

## Algorithmic Summary

1. Compute residuals  $r_k[n] = y_k[n] - h_k(\hat{x}[n])$ .
2. Update channel envelopes:

$$s_k[n+1] = \rho s_k[n] + (1 - \rho)|r_k[n]|.$$

3. Update group envelopes:

$$s_g[n+1] = \rho_g s_g[n] + (1 - \rho_g) \frac{1}{|\mathcal{G}_g|} \sum_{k \in \mathcal{G}_g} |r_k[n]|.$$

4. Compute channel and group weights:

$$w_k[n], \quad w_g[n].$$

5. Form  $\hat{w}_k[n] = w_k[n]w_{g(k)}[n]$ .
6. Normalize  $\tilde{w}_k[n]$ .
7. Update state:

$$\hat{x}[n+1] = f(\hat{x}[n], u[n]) + \sum_{k=1}^M \tilde{w}_k[n] K_k r_k[n].$$

## 11 Limitations

HRET assumes:

- Bounded deterministic disturbances.
- Assumed bounded residual magnitude.
- Predefined and fixed sensor grouping.

The framework does not provide:

- Statistical optimality guarantees.
- Automatic group discovery.
- Stability proofs under unbounded disturbances.

Performance depends on appropriate selection of  $\rho$ ,  $\rho_g$ ,  $\beta_k$ , and  $\beta_g$ . Parameter tuning remains application-dependent.

## 12 Conclusion

Hierarchical Residual-Envelope Trust (HRET) introduces an explicit grouping layer into the deterministic residual-envelope framework established by DSFB. By combining channel-level and group-level envelope dynamics through multiplicative trust composition and convex normalization, HRET provides coordinated suppression and recovery under structured disturbance patterns.

The formulation preserves strict positivity of weights, unity-sum normalization, and bounded envelope evolution under bounded disturbances. DSFB is recovered as a special case when grouping is degenerate, establishing HRET as a structural generalization rather than a replacement of the core framework.

## A Structural Summary

### Residual Definition

$$r_k[n] = y_k[n] - h_k(\hat{x}[n])$$

### Channel Envelope

$$s_k[n+1] = \rho s_k[n] + (1 - \rho)|r_k[n]|$$

### Group Envelope

$$s_g[n+1] = \rho_g s_g[n] + (1 - \rho_g) \frac{1}{|\mathcal{G}_g|} \sum_{k \in \mathcal{G}_g} |r_k[n]|$$

### Trust Mapping

$$w_k[n] = \frac{1}{1 + \beta_k s_k[n]}, \quad w_g[n] = \frac{1}{1 + \beta_g s_g[n]}$$

### Hierarchical Composition

$$\hat{w}_k[n] = w_k[n] w_{g(k)}[n], \quad \tilde{w}_k[n] = \frac{\hat{w}_k[n]}{\sum_{j=1}^M \hat{w}_j[n]}$$

### Fusion Update

$$\Delta x[n] = \sum_{k=1}^M \tilde{w}_k[n] K_k r_k[n]$$

### Properties

- Deterministic disturbance model (bounded case analyzed)
- Strictly positive weights
- Unity-sum convex normalization
- Bounded envelope dynamics
- Coordinated suppression under correlated disturbance
- Exponential recovery under nominal residual levels

### Reduction

$$G = M, \quad \mathcal{G}_g = \{g\} \Rightarrow \text{HRET} \equiv \text{DSFB}$$

## References

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