Notes

Pontus Vikstål[a)]
Wallenberg Centre for Quantum Technology, Department of Microtechnology and Nanoscience, Chalmers University of Technology, 412 96 Gothenburg, Sweden

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I. INTRODUCTION

We consider a driven weakly anharmonic qubit whose Hamiltonian in lab frame can be written as

$$\frac{H}{\hbar} = \omega_q a^\dagger a + \alpha a^\dagger a^\dagger a a + \mathcal{E}(t) a^\dagger + \mathcal{E}(t)^* a, \tag{1}$$

where $\omega_q \equiv \omega_q^{0\to1}$ is the qubit frequency and $\alpha = \omega_q^{1\to2} - \omega_q^{0\to1}$ is the anharmonicity. The driving and control is given by

$$\mathcal{E}(t) = \begin{cases} \Omega^x(t)\cos(\omega_d t) + \Omega^y(t)\sin(\omega_d t), & 0 < t < t_g, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Here $\Omega^x(t)$ and $\Omega^y(t)$ are two independent quadrature controls. t_g is the total gate-time, and ω_d is the drive frequency. Next we move into the rotating frame of the drive by performing the following unitary transformation $U(t) = e^{i\omega_r t a^\dagger a}$, where ω_r is close to the qubit frequency. The Hamiltonian in the rotating frame after having performed the rotating wave approximation reads

$$\begin{aligned} \frac{H^R}{\hbar} = \Delta a^\dagger a + \alpha a^{\dagger 2} a^2 &+ \left(\frac{\Omega^x(t)}{2}\cos((\omega_r - \omega_d)t) - \frac{\Omega^y(t)}{2}\sin((\omega_r - \omega_d)t)\right)(a^\dagger + a) \\ &+ \left(\frac{\Omega^x(t)}{2}\sin((\omega_r - \omega_d)t) + \frac{\Omega^y(t)}{2}\cos((\omega_r - \omega_d)t)\right)(ia^\dagger - ia), \end{aligned} \tag{3}$$

where $\Delta = \omega_q - \omega_r$ is the qubit detuning. For the pulse we use the following pulse shapes

$$\Omega^x(t) = B e^{-\frac{(t-t_g)^2}{2\sigma^2}}, \quad \Omega^y(t) = q\sigma \frac{d\Omega^x(t)}{dt} \tag{4}$$

where q is a scale parameter that needs to be optimized. Interchanging $\Omega^x(t)$ and $\Omega^y(t)$ in equation above corresponds to DRAG pulsing the $\Omega^y(t)$ component. The amplitude B is fixed such that

$$\left| \int_0^{t_g} [\Omega^x(t) + i\Omega^y(y)] \mathrm{d}t \right| = \pi. \tag{5}$$

for a π-pulse.

[a)] e-mail: vikstal@chalmers.se

Appendix A: Rotating Frame and RWA

The Hamiltonian in the Lab frame is given by,

$$\frac{H}{\hbar} = \omega_q a^\dagger a + \alpha a^{\dagger 2} a^2 + \mathcal{E}(t) a^\dagger + \mathcal{E}(t)^* a. \tag{A1}$$

Next we move into the rotating frame by performing the following unitary transformation $U(t) = e^{i\omega_r t a^\dagger a}$, where ω_r is a frequency close to the qubit frequency. Under a unitary transformation the state vector $|\psi(t)\rangle$ transforms according to

$$|\tilde{\psi}(t)\rangle = U(t)\,|\psi(t)\rangle \to |\psi(t)\rangle = U^\dagger(t)\,|\tilde{\psi}(t)\rangle\,. \tag{A2}$$

Here $|\tilde{\psi}\rangle$ is the transformed state vector. Substituting the transformed state vector into the Schrödinger equation we obtain

$$i\partial_t\,|\tilde{\psi}(t)\rangle = i\partial_t U(t)\,|\psi(t)\rangle + U(t) i\partial_t\,|\psi(t)\rangle = i\partial_t U(t)\,|\psi(t)\rangle + U(t)H(t)\,|\psi(t)\rangle\,,$$

by using Eq. (A2) we replace $|\psi(t)\rangle$ with $U^\dagger(t)\,|\tilde{\psi}(t)\rangle$ and get

$$i\partial_t\,|\tilde{\psi}(t)\rangle = \left(i\partial_t U(t)U^\dagger(t) + U(t)H(t)U^\dagger(t)\right)|\tilde{\psi}(t)\rangle = \tilde{H}(t)\,|\tilde{\psi}(t)\rangle\,,$$

where

$$\tilde{H}(t) \equiv i\partial_t U(t)U^\dagger(t) + U(t)H(t)U^\dagger(t) \tag{A3}$$

is the transformed Hamiltonian. We proceed by evaluating the first term in this expression

$$i\partial_t U(t)U^\dagger(t) = -\omega_r a^\dagger a.$$

We then calculate the second term of Eq. (A3) by invoking the *Baker-Hausdorff* lemma,

$$e^A a e^{-A} = a + [A, a] + \frac{1}{2!}[A, [A, a]] + \ldots$$

where A is a Hermitian operator. Applying this formula we get that each operator a and $a^\dagger$ transform according to

$$\begin{aligned} e^{i\omega_r t a^\dagger a} a e^{-i\omega_r t a^\dagger a} &= a\left(1 - i\omega_r t + \frac{(-i\omega_r t)^2}{2!} + \ldots\right) = a e^{-i\omega_r t}, \\ e^{i\omega_r t a^\dagger a} a^\dagger e^{-i\omega_r t a^\dagger a} &= a^\dagger\left(1 + i\omega_r t + \frac{(i\omega_r t)^2}{2!} + \ldots\right) = a^\dagger e^{i\omega_r t}, \end{aligned}$$

and thus the Hamiltonian in the rotating frame is given by

$$\frac{H^R}{\hbar} = \Delta a^\dagger a + \alpha a^{\dagger 2} a^2 + \mathcal{E}(t) e^{i\omega_r t} a^\dagger + \mathcal{E}(t)^* e^{-i\omega_r t} a, \tag{A4}$$

where $\Delta \equiv \omega_q - \omega_r$ is the frequency detuning. If we write the drive as

$$\mathcal{E}(t) = \Omega^x(t)\cos(\omega_d t) + \Omega^y(t)\sin(\omega_d t) = \frac{\Omega(t)}{2} e^{-i\omega_d t} + \frac{\Omega(t)^*}{2} e^{i\omega_d t} \tag{A5}$$

where $\Omega(t) = \Omega^x(t) + i\Omega^y(t)$. Multiplying and doing the rotating wave approximation yields

$$\frac{H^R}{\hbar} = \Delta a^\dagger a + \alpha a^{\dagger 2} a^2 + \frac{\Omega(t)}{2} e^{i(\omega_r - \omega_d)t} a^\dagger + \frac{\Omega(t)^*}{2} e^{-i(\omega_r - \omega_d)t} a. \tag{A6}$$

The part corresponding to the drive can be written as

$$\frac{\Omega^x(t)}{2}(e^{i(\omega_r-\omega_d)t}a^\dagger + e^{-i(\omega_r-\omega_d)t}a) + \frac{\Omega^y(t)}{2}(ie^{i(\omega_r-\omega_d)t}a^\dagger - ie^{-i(\omega_r-\omega_d)t}a). \tag{A7}$$

In trigonometric form this is equivalent to

$$\left(\frac{\Omega^x(t)}{2}\cos((\omega_r-\omega_d)t) - \frac{\Omega^y(t)}{2}\sin((\omega_r-\omega_d)t)\right)(a^\dagger + a) \\ + \left(\frac{\Omega^x(t)}{2}\sin((\omega_r-\omega_d)t) + \frac{\Omega^y(t)}{2}\cos((\omega_r-\omega_d)t)\right)(ia^\dagger - ia), \tag{A8}$$

which for the special case when $\omega_r - \omega_d = 0$ reduces to

$$\frac{\Omega^x(t)}{2}(a^\dagger + a) + \frac{\Omega^y(t)}{2}(ia^\dagger - ia). \tag{A9}$$