Notes*

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(Dated: January 28, 2020)

I. SINGLE QUBIT GATES

These notes are mostly based on the Ref. [1].

We consider a driven weakly anharmonic qubit whose Hamiltonian in lab frame can be written as

$$\frac{H}{\hbar} = \omega_q a^\dagger a + \frac{\alpha}{2} a^\dagger a^\dagger a a + \mathcal{E}(t) a^\dagger + \mathcal{E}(t)^* a, \tag{1}$$

where $\omega_q \equiv \omega_q^{0\to 1}$ is the qubit frequency and $\alpha = \omega_q^{1\to 2} - \omega_q^{0\to 1}$ is the anharmonicity. The driving and control is given by

$$\mathcal{E}(t) = \begin{cases} \Omega^x(t)\cos(\omega_d t) + \Omega^y(t)\sin(\omega_d t), & 0 < t < t_g, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Here $\Omega^x(t)$ and $\Omega^y(t)$ are two independent quadrature controls. t_g is the total gate-time, and ω_d is the drive frequency. Next we move into the rotating frame of the drive by performing the following unitary transformation $U(t) = e^{i\omega_r t a^\dagger a}$, where ω_r is close to the qubit frequency, see Appendix A. The Hamiltonian in the rotating frame after having performed the rotating wave approximation reads

$$\begin{aligned}\frac{H^R}{\hbar} = \Delta a^\dagger a + \frac{\alpha}{2} a^{\dagger 2} a^2 &+ \left(\frac{\Omega^x(t)}{2}\cos([\omega_r - \omega_d]t) - \frac{\Omega^y(t)}{2}\sin([\omega_r - \omega_d]t)\right)(a^\dagger + a) \\ &+ \left(\frac{\Omega^x(t)}{2}\sin([\omega_r - \omega_d]t) + \frac{\Omega^y(t)}{2}\cos([\omega_r - \omega_d]t)\right)(ia^\dagger - ia),\end{aligned} \tag{3}$$

where $\Delta \equiv \omega_q - \omega_r$ is the qubit detuning.

As a concrete example, assume that we apply a pulse at the qubit frequency $\omega_d = \omega_q$, and choose the rotating frame of the drive $\omega_r = \omega_d$. Then,

$$\frac{H^R}{\hbar} = \frac{\alpha}{2} a^{\dagger 2} a^2 + \frac{\Omega^x(t)}{2}(a^\dagger + a) + \frac{\Omega^y(t)}{2}(ia^\dagger - ia). \tag{4}$$

If we treat the Hamiltonian as an effective two level system (ignoring the anharmonic term) and make the replacement $(a^\dagger + a) \to \sigma_x$ and $(ia^\dagger - ia) \to \sigma_y$, we obtain

$$\frac{H^R}{\hbar} = \frac{\Omega^x(t)}{2}\sigma_x + \frac{\Omega^y(t)}{2}\sigma_y, \tag{5}$$

showing that an in-phase pulse (i.e. the $\Omega^x(t)$ quadrature component) corresponds to a rotation around the x-axis while the out-of-phase pulse (i.e. the $\Omega^y(t)$ quadrature component),

* Thanks to Christian Kraglund Andersen, Shahnawaz Ahmed and Christopher Warren.

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corresponds to rotations about the y-axis. As a concrete example of an in-phase pulse, writing out the unitary evolution operator yields,

$$U^R(t) = \exp\left(\left[-\frac{i}{2}\int_0^t \Omega^x(t')\mathrm{d}t'\right]\sigma_x\right). \tag{6}$$

By defining the angle

$$\Theta(t) = \int_0^t \Omega^x(t')\mathrm{d}t', \tag{7}$$

which is the angle a state is rotated given a waveform envelope $\Omega^x(t)$. This means that to implement a π-pulse on the x-axis one would solve $\Theta(t) = \pi$ and output the signal in-phase with the qubit drive.

In this simple example we assumed that we could ignore the higher levels of the qubit. In general leakage errors which take the qubit out of the computational subspace as well as phase errors can occur. To combat theses errors the so-called DRAG[2–5] procedure (Derivative Reduction by Adiabatic Gate) is used. In doing so we apply an extra signal in the out-of-phase component, such that

$$\Omega^x(t) = Be^{-\frac{(t-t_g)^2}{2\sigma^2}}, \quad \Omega^y(t) = q\sigma\frac{d\Omega^x(t)}{dt} \tag{8}$$

where q is a scale parameter that needs to be optimized. Interchanging $\Omega^x(t)$ and $\Omega^y(t)$ in the equation above corresponds to DRAG pulsing the $\Omega^y(t)$ component. The amplitude B is fixed such that

$$\left|\int_0^t [\Omega^x(t') + i\Omega^y(t')]\mathrm{d}t'\right| = \pi. \tag{9}$$

for a π-pulse with DRAG.

II. TWO QUBIT GATES

We consider two transmons that are capacitively coupled together. In the Lab frame this can be written as

$$\frac{H}{\hbar} = \sum_{n=1,2}\left(\omega_n a_n^\dagger a_n + \frac{\alpha_n}{2}a_n^{\dagger 2}a_n^2\right) + J(a_1^\dagger a_2 + a_2^\dagger a_1), \tag{10}$$

where ω_n, α_n is the qubit frequency and anharmonicity for the n:th qubit respectively and J is the coupling.

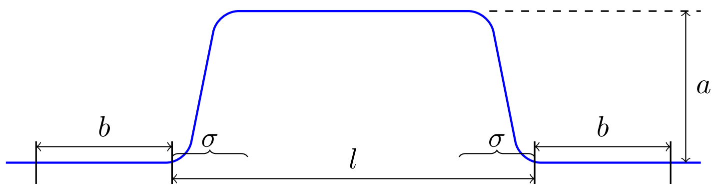

FIG. 1. Shape of the two qubit pulse (blue), where b is a zero amplitude buffer, l is the length of the square pulse, a is the amplitude, and σ is the Gaussian filter std.

The two qubit gate is performed by tuning the frequency of one of the qubits such that the $|11\rangle$ state become resonant with the $|20\rangle$ state between the qubits. We model the tuning of the qubit frequency as

$$H_{J,n} = -\Delta(t) a_n^\dagger a_n, \tag{11}$$

with

$$\Delta(t) = \left(\omega_n - \omega_n \sqrt[4]{\cos(\Phi(t) + \phi_0)^2 + d^2 \sin(\Phi(t) + \phi_0)^2} \right), \tag{12}$$

where the pulse shape is given by:

$$\Phi(t) = C \cdot (A(t) * f(t)). \tag{13}$$

Where $A(t)$ is a square pulse of length l and amplitude a and $f(t)$ is a Gaussian with mean 0 and sigma σ, see Fig. 1.

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Appendix A: Rotating Frame and RWA

The Hamiltonian in the Lab frame is given by,

$$\frac{H}{\hbar} = \omega_q a^\dagger a + \frac{\alpha}{2} a^{\dagger 2} a^2 + \mathcal{E}(t) a^\dagger + \mathcal{E}(t)^* a. \tag{A1}$$

Next we move into the rotating frame by performing the following unitary transformation $U(t) = e^{i\omega_r t a^\dagger a}$, where ω_r is a frequency close to the qubit frequency. Under a unitary transformation the state vector $|\psi(t)\rangle$ transforms according to

$$|\tilde{\psi}(t)\rangle = U(t)\,|\psi(t)\rangle \rightarrow |\psi(t)\rangle = U^\dagger(t)\,|\tilde{\psi}(t)\rangle\,. \tag{A2}$$

Here $|\tilde{\psi}\rangle$ is the transformed state vector. Substituting the transformed state vector into the Schrödinger equation we obtain

$$i\partial_t\,|\tilde{\psi}(t)\rangle = i\partial_t U(t)\,|\psi(t)\rangle + U(t) i\partial_t\,|\psi(t)\rangle = i\partial_t U(t)\,|\psi(t)\rangle + U(t)H(t)\,|\psi(t)\rangle\,,$$

by using Eq. (A2) we replace $|\psi(t)\rangle$ with $U^\dagger(t)\,|\tilde{\psi}(t)\rangle$ and get

$$i\partial_t\,|\tilde{\psi}(t)\rangle = \left(i\partial_t U(t)U^\dagger(t) + U(t)H(t)U^\dagger(t)\right)|\tilde{\psi}(t)\rangle = \tilde{H}(t)\,|\tilde{\psi}(t)\rangle\,,$$

where

$$\tilde{H}(t) \equiv i\partial_t U(t)U^\dagger(t) + U(t)H(t)U^\dagger(t) \tag{A3}$$

is the transformed Hamiltonian. We proceed by evaluating the first term in this expression

$$i\partial_t U(t)U^\dagger(t) = -\omega_r a^\dagger a.$$

We then calculate the second term of Eq. (A3) by invoking the *Baker-Hausdorff* lemma,

$$e^A a e^{-A} = a + [A, a] + \frac{1}{2!}[A, [A, a]] + \ldots$$

where A is a Hermitian operator. Applying this formula we get that each operator a and $a^\dagger$ transform according to

$$e^{i\omega_r t a^\dagger a} a e^{-i\omega_r t a^\dagger a} = a\left(1 - i\omega_r t + \frac{(-i\omega_r t)^2}{2!} + \ldots\right) = a e^{-i\omega_r t},$$
$$e^{i\omega_r t a^\dagger a} a^\dagger e^{-i\omega_r t a^\dagger a} = a^\dagger\left(1 + i\omega_r t + \frac{(i\omega_r t)^2}{2!} + \ldots\right) = a^\dagger e^{i\omega_r t},$$

and thus the Hamiltonian in the rotating frame is given by

$$\frac{H^R}{\hbar} = \Delta a^\dagger a + \frac{\alpha}{2} a^{\dagger 2} a^2 + \mathcal{E}(t) e^{i\omega_r t} a^\dagger + \mathcal{E}(t)^* e^{-i\omega_r t} a, \tag{A4}$$

where $\Delta \equiv \omega_q - \omega_r$ is the frequency detuning. If we write the drive as

$$\mathcal{E}(t) = \Omega^x(t)\cos(\omega_d t) + \Omega^y(t)\sin(\omega_d t) = \frac{\Omega(t)}{2} e^{-i\omega_d t} + \frac{\Omega(t)^*}{2} e^{i\omega_d t} \tag{A5}$$

where $\Omega(t) = \Omega^x(t) + i\Omega^y(t)$. Multiplying and doing the rotating wave approximation yields

$$\frac{H^R}{\hbar} = \Delta a^\dagger a + \frac{\alpha}{2} a^{\dagger 2} a^2 + \frac{\Omega(t)}{2} e^{i(\omega_r - \omega_d)t} a^\dagger + \frac{\Omega(t)^*}{2} e^{-i(\omega_r - \omega_d)t} a. \tag{A6}$$

The part corresponding to the drive can be written as

$$\frac{\Omega^x(t)}{2}\left(e^{i(\omega_r - \omega_d)t} a^\dagger + e^{-i(\omega_r - \omega_d)t} a\right) + \frac{\Omega^y(t)}{2}\left(i e^{i(\omega_r - \omega_d)t} a^\dagger - i e^{-i(\omega_r - \omega_d)t} a\right). \tag{A7}$$

In trigonometric form this is equivalent to

$$\begin{aligned}&\left(\frac{\Omega^x(t)}{2}\cos((\omega_r - \omega_d)t) - \frac{\Omega^y(t)}{2}\sin((\omega_r - \omega_d)t)\right)(a^\dagger + a) \\ &\qquad + \left(\frac{\Omega^x(t)}{2}\sin((\omega_r - \omega_d)t) + \frac{\Omega^y(t)}{2}\cos((\omega_r - \omega_d)t)\right)(ia^\dagger - ia),\end{aligned} \tag{A8}$$

which for the special case when $\omega_r - \omega_d = 0$ reduces to

$$\frac{\Omega^x(t)}{2}(a^\dagger + a) + \frac{\Omega^y(t)}{2}(ia^\dagger - ia). \tag{A9}$$

Appendix B: Two qubit gate

We expand the Identity and the annihilation and creation operators in the Fock-basis spanned by the Qutrit

$$I = |0\rangle\langle 0| + |1\rangle\langle 1| + |2\rangle\langle 2|\,, \quad a = |0\rangle\langle 1| + \sqrt{2}\,|1\rangle\langle 2|\,, \quad a^\dagger = |1\rangle\langle 0| + \sqrt{2}\,|2\rangle\langle 1|\,.$$

Let $a_1 \equiv a \otimes I$, and $a_2 \equiv I \otimes a$. Then we have that $a_1^\dagger a_1 = (|1\rangle\langle 1| + 2\,|2\rangle\langle 2|) \otimes I$ and $a_1^{\dagger 2} a_1^2 = 2\,|2\rangle\langle 2| \otimes I$. Thus get that the qubit Hamiltonian in the Fock-basis is given by

$$\begin{aligned}H_{\text{qubits}} &= \omega_1 a_1^\dagger a_1 + \frac{\alpha_1}{2} a_1^{\dagger 2} a_1^2 + \omega_2 a_2^\dagger a_2 + \frac{\alpha_2}{2} a_2^{\dagger 2} a_2^2 \\ &= \omega_1\,|1\rangle\langle 1| \otimes I + (2\omega_1 + \alpha_1)\,|2\rangle\langle 2| \otimes I + \omega_2 I \otimes |1\rangle\langle 1| + (2\omega_2 + \alpha_2) I \otimes |2\rangle\langle 2|\end{aligned}$$

Likewise for the coupling term we get

$$\begin{aligned} H_{\text{coupling}} &= g\left(a_1^\dagger a_2 + a_2^\dagger a_1\right) \\ &= g\left(|10\rangle\langle 01| + \sqrt{2}\,|20\rangle\langle 11| + \sqrt{2}\,|11\rangle\langle 02| + 2\,|21\rangle\langle 12| + \text{h.c.}\right) \end{aligned}$$

Thus the spectrum for the total Hamiltonian $H = H_{\text{qubits}} + H_{\text{coupling}}$ written in the $\{|01\rangle, |02\rangle, |10\rangle, |11\rangle, |20\rangle\}$-basis is given by

$$H_{\text{excitations}} = \begin{array}{c} \\ \langle 01| \\ \langle 02| \\ \langle 10| \\ \langle 11| \\ \langle 20| \end{array} \!\!\begin{array}{c} \begin{array}{ccccc} |01\rangle & |02\rangle & |10\rangle & |11\rangle & |20\rangle \end{array} \\ \begin{pmatrix} \omega_2 & 0 & g & 0 & 0 \\ 0 & \alpha_2 + 2\omega_2 & 0 & \sqrt{2}g & 0 \\ g & 0 & \omega_1 & 0 & 0 \\ 0 & \sqrt{2}g & 0 & \omega_1 + \omega_2 & \sqrt{2}g \\ 0 & 0 & 0 & \sqrt{2}g & \alpha_1 + 2\omega_1 \end{pmatrix} \end{array} \tag{B1}$$

Appendix C: Qubit Parameters

	qubit 1	qubit 2	qubit 3	qubit 4
Qubit frequency, $\omega_q/2\pi$ (GHz)	5.708390	5.202204	4.877051	4.383388
Anharmonicity $\alpha/2\pi$ (MHz)	-261.081457	-275.172227	-277.331082	-286.505059
Lifetime, T_1 (μs)	25.111199	17.987260	25.461490	46.371173
Ramsey decay time, T_2^* (μs)	23.872090	10.778744	17.271457	10.838021

TABLE I. Parameters for the qubits.

Description	QB1 – QB2	QB2 – QB3	QB3 – QB4
Sweet spot frequency, $\omega_n/2\pi$ (GHz)	5.708	4.877	-
Josephson Junction assymetry, d	0.78	0.78	0.78
Conversion constant, C (Φ_0/V)	0.47703297	0.53	-
Offset, ϕ_0 (Φ_0)	0	$\pi/2$	-
Amplitude for CZ-gate, a (V),	1.44	0.475	-
Length for CZ-gate, l (ns)	96.00	108	-
Gaussian filter std. σ (ns)	1.00	1.00	1.00
Buffer length, b (ns)	15	15	15

TABLE II. Two qubit gate parameters.