Notes

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I. SINGLE QUBIT GATES

These notes are mostly based on the Ref.[1].

We consider a driven weakly anharmonic qubit whose Hamiltonian in lab frame can be written as

$$\frac{H}{\hbar} = \omega_q a^\dagger a + \alpha a^\dagger a^\dagger a a + \mathcal{E}(t) a^\dagger + \mathcal{E}(t)^* a, \tag{1}$$

where $\omega_q \equiv \omega_q^{0\to 1}$ is the qubit frequency and $\alpha = \omega_q^{1\to 2} - \omega_q^{0\to 1}$ is the anharmonicity. The driving and control is given by

$$\mathcal{E}(t) = \begin{cases} \Omega^x(t)\cos(\omega_d t) + \Omega^y(t)\sin(\omega_d t), & 0 < t < t_g, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Here $\Omega^x(t)$ and $\Omega^y(t)$ are two independent quadrature controls. t_g is the total gate-time, and ω_d is the drive frequency. Next we move into the rotating frame of the drive by performing the following unitary transformation $U(t) = e^{i\omega_r t a^\dagger a}$, where ω_r is close to the qubit frequency, see Appendix A. The Hamiltonian in the rotating frame after having performed the rotating wave approximation reads

$$\begin{aligned} \frac{H^R}{\hbar} = \Delta a^\dagger a + \alpha a^{\dagger 2} a^2 &+ \left(\frac{\Omega^x(t)}{2}\cos((\omega_r - \omega_d)t) - \frac{\Omega^y(t)}{2}\sin((\omega_r - \omega_d)t)\right)(a^\dagger + a) \\ &+ \left(\frac{\Omega^x(t)}{2}\sin((\omega_r - \omega_d)t) + \frac{\Omega^y(t)}{2}\cos((\omega_r - \omega_d)t)\right)(ia^\dagger - ia), \end{aligned} \tag{3}$$

where $\Delta = \omega_q - \omega_r$ is the qubit detuning.

As a concrete example, assume that we apply a pulse at the qubit frequency $\omega_d = \omega_q$, and choose the rotating frame of the drive $\omega_r = \omega_d$. Then,

$$\frac{H^R}{\hbar} = \alpha a^{\dagger 2} a^2 + \frac{\Omega^x(t)}{2}(a^\dagger + a) + \frac{\Omega^y(t)}{2}(ia^\dagger - ia). \tag{4}$$

If we treat the Hamiltonian as an effective two level system (ignoring the anharmonic term) and make the replacement $(a^\dagger + a) \to \sigma_x$ and $(ia^\dagger - ia) \to \sigma_y$, we obtain

$$\frac{H^R}{\hbar} = \frac{\Omega^x(t)}{2}\sigma_x + \frac{\Omega^y(t)}{2}\sigma_y, \tag{5}$$

showing that an in-phase pulse (i.e. the $\Omega^x(t)$ quadrature component) corresponds to a rotation around the x-axis while the out-of-phase pulse (i.e. the $\Omega^y(t)$ quadrature component), corresponds to rotations about the y-axis. As a concrete example of an in-phase pulse, writing out the unitary evolution operator yields,

$$U^R(t) = \exp\left(\left[-\frac{i}{2}\int_0^t \Omega^x(t')\mathrm{d}t'\right]\sigma_x\right). \tag{6}$$

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By defining the angle

$$\Theta(t) = \int_0^t \Omega^x(t')\mathrm{d}t', \tag{7}$$

which is the angle a state is rotated given a waveform envelope $\Omega^x(t)$. This means that to implement a π-pulse on the x-axis one would solve $\Theta(t) = \pi$ and output the signal in-phase with the qubit drive.

In this simple example we assumed that we could ignore the higher levels of the qubit. In general leakage errors which take the qubit out of the computational subspace as well as phase errors can occur. Leakage errors can occur because a qubit in the state $|1\rangle$ may be excited to $|2\rangle$ as a π pulse is applied, or be excited directly from the $|0\rangle$, since the qubit spends some amount of time in the $|1\rangle$ state during the π pulse. Phase errors occurs because the presence of the drive results in a repulsion between the $|1\rangle$ and $|2\rangle$. The so-called DRAG[2–5] procedure (Derivative Reduction by Adiabatic Gate) seeks to combat leakage errors which take the qubit out of the computational subspace, and phase errors. The trick to combat these errors is to applying an extra signal in the out-of-phase component, such that

$$\Omega^x(t) = Be^{-\frac{(t-t_g)^2}{2\sigma^2}}, \quad \Omega^y(t) = q\sigma\frac{d\Omega^x(t)}{dt} \tag{8}$$

where q is a scale parameter that needs to be optimized. Interchanging $\Omega^x(t)$ and $\Omega^y(t)$ in equation above corresponds to DRAG pulsing the $\Omega^y(t)$ component. The amplitude B is fixed such that

$$\left|\int_0^t [\Omega^x(t') + i\Omega^y(t')]\mathrm{d}t'\right| = \pi. \tag{9}$$

for a π-pulse with DRAG.

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[4] J. M. Gambetta, F. Motzoi, S. T. Merkel, and F. K. Wilhelm, Phys. Rev. A **83**, 012308 (2011).
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Appendix A: Rotating Frame and RWA

The Hamiltonian in the Lab frame is given by,

$$\frac{H}{\hbar} = \omega_q a^\dagger a + \alpha a^{\dagger 2} a^2 + \mathcal{E}(t)a^\dagger + \mathcal{E}(t)^* a. \tag{A1}$$

Next we move into the rotating frame by performing the following unitary transformation $U(t) = e^{i\omega_r t a^\dagger a}$, where ω_r is a frequency close to the qubit frequency. Under a unitary transformation the state vector $|\psi(t)\rangle$ transforms according to

$$|\tilde{\psi}(t)\rangle = U(t)\,|\psi(t)\rangle \rightarrow |\psi(t)\rangle = U^\dagger(t)\,|\tilde{\psi}(t)\rangle\,. \tag{A2}$$

Here $|\tilde{\psi}\rangle$ is the transformed state vector. Substituting the transformed state vector into the Schrödinger equation we obtain

$$i\partial_t\,|\tilde{\psi}(t)\rangle = i\partial_t U(t)\,|\psi(t)\rangle + U(t)i\partial_t\,|\psi(t)\rangle = i\partial_t U(t)\,|\psi(t)\rangle + U(t)H(t)\,|\psi(t)\rangle\,,$$

by using Eq. (A2) we replace $|\psi(t)\rangle$ with $U^\dagger(t)\,|\tilde{\psi}(t)\rangle$ and get

$$i\partial_t\,|\tilde{\psi}(t)\rangle = \left(i\partial_t U(t)U^\dagger(t) + U(t)H(t)U^\dagger(t)\right)|\tilde{\psi}(t)\rangle = \tilde{H}(t)\,|\tilde{\psi}(t)\rangle\,,$$

where

$$\tilde{H}(t) \equiv i\partial_t U(t)U^\dagger(t) + U(t)H(t)U^\dagger(t) \tag{A3}$$

is the transformed Hamiltonian. We proceed by evaluating the first term in this expression

$$i\partial_t U(t)U^\dagger(t) = -\omega_r a^\dagger a.$$

We then calculate the second term of Eq. (A3) by invoking the *Baker-Hausdorff* lemma,

$$e^A a e^{-A} = a + [A,a] + \frac{1}{2!}[A,[A,a]] + \ldots$$

where A is a Hermitian operator. Applying this formula we get that each operator a and $a^\dagger$ transform according to

$$e^{i\omega_r t a^\dagger a} a e^{-i\omega_r t a^\dagger a} = a\left(1 - i\omega_r t + \frac{(-i\omega_r t)^2}{2!} + \ldots\right) = a e^{-i\omega_r t},$$
$$e^{i\omega_r t a^\dagger a} a^\dagger e^{-i\omega_r t a^\dagger a} = a^\dagger\left(1 + i\omega_r t + \frac{(i\omega_r t)^2}{2!} + \ldots\right) = a^\dagger e^{i\omega_r t},$$

and thus the Hamiltonian in the rotating frame is given by

$$\frac{H^R}{\hbar} = \Delta a^\dagger a + \alpha a^{\dagger 2} a^2 + \mathcal{E}(t) e^{i\omega_r t} a^\dagger + \mathcal{E}(t)^* e^{-i\omega_r t} a, \tag{A4}$$

where $\Delta \equiv \omega_q - \omega_r$ is the frequency detuning. If we write the drive as

$$\mathcal{E}(t) = \Omega^x(t)\cos(\omega_d t) + \Omega^y(t)\sin(\omega_d t) = \frac{\Omega(t)}{2}e^{-i\omega_d t} + \frac{\Omega(t)^*}{2}e^{i\omega_d t} \tag{A5}$$

where $\Omega(t) = \Omega^x(t) + i\Omega^y(t)$. Multiplying and doing the rotating wave approximation yields

$$\frac{H^R}{\hbar} = \Delta a^\dagger a + \alpha a^{\dagger 2} a^2 + \frac{\Omega(t)}{2}e^{i(\omega_r - \omega_d)t} a^\dagger + \frac{\Omega(t)^*}{2}e^{-i(\omega_r - \omega_d)t} a. \tag{A6}$$

The part corresponding to the drive can be written as

$$\frac{\Omega^x(t)}{2}(e^{i(\omega_r-\omega_d)t}a^\dagger + e^{-i(\omega_r-\omega_d)t}a) + \frac{\Omega^y(t)}{2}(ie^{i(\omega_r-\omega_d)t}a^\dagger - ie^{-i(\omega_r-\omega_d)t}a). \tag{A7}$$

In trigonometric form this is equivalent to

$$\begin{aligned}\left(\frac{\Omega^x(t)}{2}\cos((\omega_r-\omega_d)t) - \frac{\Omega^y(t)}{2}\sin((\omega_r-\omega_d)t)\right)&(a^\dagger + a)\\ + \left(\frac{\Omega^x(t)}{2}\sin((\omega_r-\omega_d)t) + \frac{\Omega^y(t)}{2}\cos((\omega_r-\omega_d)t)\right)&(ia^\dagger - ia),\end{aligned} \tag{A8}$$

which for the special case when $\omega_r - \omega_d = 0$ reduces to

$$\frac{\Omega^x(t)}{2}(a^\dagger + a) + \frac{\Omega^y(t)}{2}(ia^\dagger - ia). \tag{A9}$$