

# Simplifying Mathematical Notations in Recurrence Equations

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April 2022

## 1 Introduction

In the book "Introduction to Algorithms" written by CLRS, section 4.4, the following recurrence equation is given:

$$T(n) = \begin{cases} \Theta(1), & \text{if } n = 1 \\ 3T(\lfloor n/4 \rfloor) + \Theta(n^2), & \text{otherwise} \end{cases} \quad (1)$$

which later on was simplified as

$$T(n) = \begin{cases} \Theta(1), & \text{if } n = 1 \\ 3T(\lfloor n/4 \rfloor) + cn^2, & \text{otherwise} \end{cases} \quad (2)$$

we will argue that even if  $\Theta(n^2)$  is a set, and  $c * n^2$  is a scalar and they generally cannot be swapped one with the other, in this case it can be done and will help the equation to be more tractable.

## 2 Proof

Let's consider the general case only, so we have

$$3T(\lfloor n/4 \rfloor) + \Theta(n^2)$$

which expanded is

$$3T(\lfloor n/4 \rfloor) + f(n) \text{ where } f(n) \in \Theta(n^2)$$

$f(n) \in \Theta(n^2)$  means that

$\exists c_1 > 0, c_2 > 0, n_0 > 0$  such that  $c_1 * n^2 \leq f(n) \leq c_2 * n^2 \forall n \geq n_0$

So we have

$$\begin{aligned}
 3T(\lfloor n/4 \rfloor) + c_1 * n^2 &\leq 3T(\lfloor n/4 \rfloor) + f(n) \leq 3T(\lfloor n/4 \rfloor) + c_2 * n^2 \quad \forall n \geq n_0 \\
 &= \\
 \underbrace{3T(\lfloor n/4 \rfloor) + c_1 * n^2}_{\alpha} &\leq T(n) \leq \underbrace{3T(\lfloor n/4 \rfloor) + c_2 * n^2}_{\beta} \quad \forall n \geq n_0
 \end{aligned}$$

So if we resolve  $\alpha$  only, we'll have a lower bound for  $T(n)$ . If we resolve  $\beta$  only, we'll have an upper bound for  $T(n)$ .

But  $\alpha$  and  $\beta$  are asymptotically equivalent, that means that  $T(n)$  is asymptotically equivalent to  $\alpha$  and  $\beta$  too.

So we just need to resolve either  $\alpha$  or  $\beta$  to obtain the asymptotic value of  $T(n)$ , so we can just resolve for a generic  $c$ :

$$T(n) = \begin{cases} \Theta(1), & \text{if } n = 1 \\ 3T(\lfloor n/4 \rfloor) + cn^2, & \text{otherwise} \end{cases} \quad (3)$$

since using a generic constant won't change the asymptotic value.

### 3 Conclusions

The concept described in this document and the relative proof can be easily extended to any similar recurrence, and generalized.