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Topological Model and Analysis of the P2P BitTorrent Protocol

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***Abstract* - In this paper, we propose a directed and weighted graph model for describing P2P networks using BitTorrent protocol. Then, we analysis the topological properties of the model based on complex networks theory. We find that the node strength follows a power-law distribution and that there exists a positive correlation between flow betweenness and out-strength.**

Index Terms - complex networks, BitTorrent, P2P, simulation, clustering, betweenness.

I. INTRODUCTION

A peer-to-peer (P2P) overlay network is composed of participants that make a portion of their resources directly available to others in the same network, without the need for central coordination instances[1]. Due to the increasing number of Internet users globally and fast development of digital multimedia technology, P2P has become dominating in dealing with file-sharing and content delivery such as on-line TV broadcasting, video-on-demand (VOD) and etc. , for which traditional client-server model showed its limit and weakness with respect to bandwidth, storage space and computing power.

Among many of the popular P2P applications, BitTorrent is one of the most widely used protocols. Originally designed for file-sharing, BitTorrent is a robust P2P protocol that takes advantage of peers' bandwidth to efficiently replicate and transfer content without adding too much load to the servers. It has been shown to be very efficient with featuring a game theoretical incentive mechanism which is used to ensure the fair distribution of content and prevent selfish peer behavior. In BitTorrent networks, each peer has to maximize their uploading capacity while owning a wish to maximize the downloading speed, which essentially contributes to the connectivity and continuity of the networks. Therefore, studying the BitTorrent networks is important in order to investigate how this protocol works and can be improved. The network topology has significant impact on how peers interact and cooperate with each other, which can be a very essential issue on the performance, functionality, efficiency, and resource cost of P2P networks. It makes sense to study the overlay topology of P2P networks in purpose of uncovering the inner characteristics of certain networks that concern about their sophisticated behaviors.

In previous works based on the topology of P2P systems, it has been found that the communication efficiency is improved due to the topological model used in P2P networks[2]. Network topology has also been proved to be relevant to the accessibility of resources on P2P networks[3]. A simple scheme for participants to build P2P networks with topological properties such as low diameter has been proposed[4]. Some specific protocols such as Small World Overlay Protocol[5] have also been proposed, which were shown to be helpful for improving the object lookup performance and dealing with flash crowd efficiently in P2P networks. Also, it has been investigated how the performance and network resilience of the P2P communication network depends on the topology in a two coupled network model[6]. In another work, the performance of P2P video streaming system has been studied under several basic topology model such as random, small-world and scale-free graphs, which indicates that a more connected graph does not necessarily imply a higher streaming rate[7]. And for another P2P protocol named Gnutella, a complex network model of P2P network structure has been constituted with some basic analysis [8]. There is another paper[9] which has proposed a way for studying the overlay topologies of BitTorrent networks by doing experiments on actual networks.

However, most of these works assume that the P2P network is an un-weighted or un-directed graph. One defect of these models is that links without weight or direction are not clearly describing the relation between the corresponding peers. It can be assumed that there is a data transfer if there exists a link between a pair of peers. However, the amount of data transferred and from which peer to another the data was transferred are considered to be trivial, which includes significant information in communication networks such as P2P.

In this paper, our objective is to model and analysis an overlay P2P network using BitTorrent protocol. In order to characterize the BitTorrent networks, we use complex networks theory[10] which exhibits to be useful in analyzing the topology of large networks, since structure affects function[11]. We propose a directed and weighted graph model to describe the peers' behavior in P2P networks derived from BitTorrent protocol. Not only the amount of data transferred but also the direction of the transfer have been taken into consideration in the analysis of evaluating parameters such as clustering coefficient and betweenness in our model.

Therefore, our results reflect the behavior of BitTorrent networks more precisely. As far as we know, this should be the first work that uses both directed and weighted graph to model the topology of P2P networks and give statistical analysis. These analysis techniques such as betweenness may provide a way for P2P protocol designers to solve some open issues such as selection of best peers[12] in the networks. We believe that our methodology of modeling and analysis could also be a good choice for other communication networks.

P2P is such a hot subject that it arouses lots of research activities, which vary from search algorithms[13] to simulation methods[14]. Generally to say, testing on real and active P2P networks would not be easy task due to its large in peers amount, highly distributing and ad-hoc characteristics, as well as the risk of affecting the network when doing experiments on it is too high[15]. According to the BitTorrent Specification[16], although it is possible for one peer to know which peers it has connection with, it is still very difficult to obtain the data amount transferred between all pair of peers, which is needed for constructing the both weighted and directed graph model. Till now, we haven't found a good way to collect data from actual BT networks for analysis. Therefore, we use the popular P2P simulator Peersim[17-18] for simulations of the BitTorrent protocol on varieties of networks which are arranged from small ones with dozens of nodes to large ones with thousands of nodes. Several topological parameters such as node strength distribution, clustering coefficient[19], shortest path length and betweenness[20] will be calculated to get the network anatomy. Finally, we include some findings like the scale-free topology and positive correlation between flow betweenness and out-strength of BitTorrent networks.

II. BitTorrent

For better understanding of our work, a brief introduction of how BitTorrent protocol works is given as follows. More details about how this protocol works can be found in BitTorrent Specification[16].

The network swarm constructed by BitTorrent protocol is constituted with three different types of components: peers, seeders, and trackers. Peers are the majority of terminals which joined in the P2P networks to download files or other resources while contributing to other participants by uploading the parts they already obtained. Peers become seeders as soon as they have finished downloading and continued uploading to others. The resource file which is being shared in the network is divided into many small pieces which have a fixed size such as 512KB or 256KB. Each piece can then be divided into smaller blocks of certain size such as 16KB, which can be transferred separately and assembled after downloading all of them.

In traditional file-sharing systems, all peers get the list of seeders and other participants with their file status from a (or several) tracker server(s) whose URL or IP address could be found in a file called the torrent file. This torrent file also contains information of the file or resource being shared in the network. Therefore it is necessary for BitTorrent users to obtain the torrent file before they participate in the network. Because the torrent file is small in size (usually dozens of KB) and does not contain sensitive information, it can be easily put on websites by resource publishers for downloading. Similar techniques could be found in other BitTorrent-like systems. Clearly, there must be at least one available seeder in the network for making resource sharing possible. This first seeder is often provided by the publisher when the BitTorrent network is created. Peers which joined in afterwards will find the seeder from the tracker server and send it a request for transferring. They will become seeders after successfully downloading all the resources. As new peers appear and seeders quit, the network stays robust and stable by its game mechanism which encourages uploading and refuses selfish participants.

III. Simulation Model

The objective of this paper is to construct a directed and weighted graph model in order to analyze the BitTorrent protocol using complex network theory. Although network protocols are usually designed to be independent of the network topology, topology in many cases has a major impact on the performance of network protocols[21]. Therefore, we will evaluate the topological characteristics of overlay BitTorrent networks and make comparisons by adjusting parameters such as network size, original seeder proportion, resource file size, swarm size and etc.

Any P2P system relies on a network of peers within which requests and messages must be routed with efficiency and fault tolerance, and through which peers and content can be efficiently located. The total time for downloading each block of the sharing resource file is decided by the bandwidth and latency of the corresponding pair of peers according to the router topology. In order to describe peers' capacity more accurately, we constructed a router network by using router-level Internet topology in which every node represents a router locating in a real geographic place on the earth with longitude and latitude parameters. Each link's weight represents the distance between the two nodes, which can be calculated using geographic parameters. Using the internet topology generator Igen[22], some heuristics in a real router network such as the geographic distribution, identification of points of presence which are the access points to the Internet, topology of backbone routers, and link capacity, were considered in constructing the router-graph. The total number of routers in the network was denoted Num_r.

Each peer should be assigned randomly to one router in the router-network. Suppose that the size of the BitTorrent network is Num_p. Then Num_p peers will be assigned to Num_r routers. Two or more peers are allowed to associate with the same router. The latency between two peers is usually caused by the number of hops on the router trace, which is decided by certain interior/Border Gateway Protocols. For simplicity as well as being accurate, the sum of latency is calculated in the shortest path of the router graph as for any pair of routers. A local latency is set for the case in which source and destination router is the same. We set the allowed transferring bandwidth value to be 1Mb/s, 2Mb/s, 4Mb/s and 10Mb/s. Each peer is randomly assigned to one of those values above.

Some other parameters have to be adjusted before performing simulations and making comparison of BitTorrent protocol under different circumstances. These parameters include original seeder proportion, resource file size, swarm size and etc. With communication capacity allocated to each peer, we can then simulate the BitTorrent protocol using Peersim simulator[17-18]. The simulation should record the amount of data transferred between all pairs of peers from the first downloading action till the accomplishment of the whole network.

After the simulation has been completed, a directed and weighted graph can finally be constructed to analyze the BitTorrent protocol. One peer P_i in the network is represented by a node N_i in the graph. There exists an arc A_{ij} from N_i to N_j if there is data transferred between the corresponding peers of P_i and P_j. The data amount transferred between P_i and P_j is denoted L_{ij}. We assign arc A_{ij} a weight W_{ij} which is proportional to L_{ij}. L_{ij} is divided with the sharing file size *Res_Size* for comparing networks with different resource size. Therefore arc A_{ij} is evaluated with a weight of W_{ij} that is equal to L_{ij}/ *Res_Size*.

IV. RESULTS

In this section we present the results from our simulations and complex network analysis of BitTorrent networks.

A. Node strength distribution

One node's strength is defined as the sum of weights of all the arcs that are either from or to this node. The in-strength of a node is the sum of weights of all the arcs that point to it, and the out-strength of a node is the sum of weight of all the arcs that point to other nodes from this node. Since every peer's total downloaded data amount (in-strength) should be the same for one BitTorrent network, we can calculate the out-strength D_i of node N_i as the sum of each peer's uploaded data:

$$D_i = \sum_{j=1}^{n} L_{ij} \big/ Res\_Size \qquad (1)$$

We performed simulations on a BitTorrent network of 1000 peers with varying resource file size from 10 MBytes to 1000 Mbytes. 5 original seeders were provided in the beginning of each experiment. Other peers were added afterwards simultaneously. Results were analyzed when every peer had completed downloading. The resulting cumulative out-strength distribution is shown Figure 1.

Figure 2 shows the same result when the resource file size is 10 Mbytes. As can be seen, when $x > 3$ approximately the cumulative out-strength distribution obeys a power-law function of $P(X > x) = x^{-\alpha}$ for $\alpha = -2.1825$. Then the out-strength's distribution should be $P(x) \sim x^{-(\alpha+1)} = x^{-3.1825}$ when x>3 approximately.

By comparing results from BitTorrent networks with different *Res_Size*, we find that a higher value of α is obtained for a larger *Res_Size*, which can be explained with that peers have a higher possibility to get a small out-strength and a lower possibility to get a large out-strength when downloading larger resource files.

B. Clustering

Clustering coefficient is a popular measure to describe the degree to which nodes tend to cluster together. In non-weighted and for undirected networks the global clustering coefficient is defined as the ratio between the number of closed triplets and the number of all triplets. A triplet is composed of three nodes that have at least two links connecting them. A closed triplet is one with three links which forms a triangle centered in a specific node.

For BitTorrent networks which are both weighted and directed, we use a method called generalized clustering coefficient proposed in [19]. It is based on the classical global clustering coefficient with several modifications.

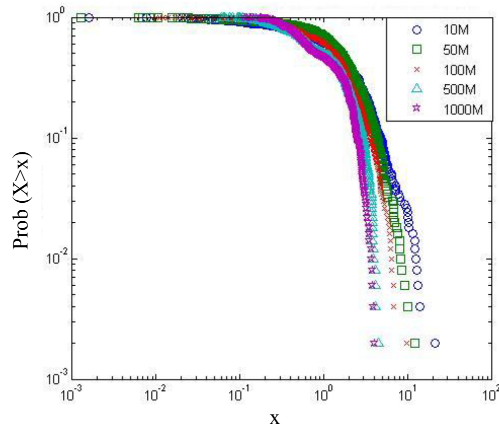

Figure 1 Cumulative out-strength distribution of 1000-peer BitTorrent networks with different resource file sizes.

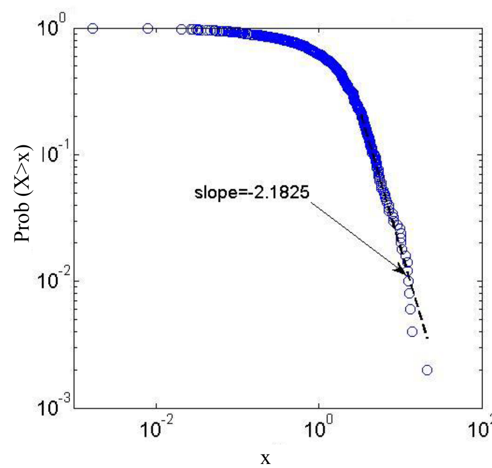

Figure 2 Cumulative out-strength distribution when resource file size is 10M. As can be seen from the figure, it obeys a power-law distribution when approximately x>3.

In our model, clustering coefficient (or transitivity) helps describe the capacity of the transferred data from some peers (like seeders that do not download when uploading) to others (like leechers that do not upload at all when downloading), and then to the other of the others until all peers have obtained the data. Therefore a network with higher clustering coefficient has a better capacity to propagate data from the original seeders to all other peers.

We start the simulation of BitTorrent networks with two different parameters, network size and proportion of original seeders in all peers. The first parameter is for analyzing how the number of participants in the BitTorrent network will affect the clustering. The objective of the second parameter is to investigate if the popularity of certain BitTorrent networks will have an effect to its structure since more seeders exist in more popular networks. No dynamics of adding or deleting peers is considered in this step. The generalized clustering coefficient C_ω for varieties of BitTorrent networks is showed in Figure 3.

First, from Figure 3(a), we can see that the clustering coefficient of the BitTorrent network which have 5% of its node members being original seeders gets smaller from approximately 0.6 to 0.06 when the size of the network increases from 100 to 1000. All other networks with different original seeder proportion have similar results in our experiments. This means that BitTorrent networks with fewer peers are more clustered together than those with more peers. This is similar to many other networks such as random graphs for it tends to be "easier" for smaller networks to cluster together than larger ones. Second, we find that the clustering coefficient decreases gently when the proportion of original seeders increases except for very few cases. We believe that this is due to the decreasing number of arcs when more peers are seeders originally who do not need to create a connection to download data from each other.

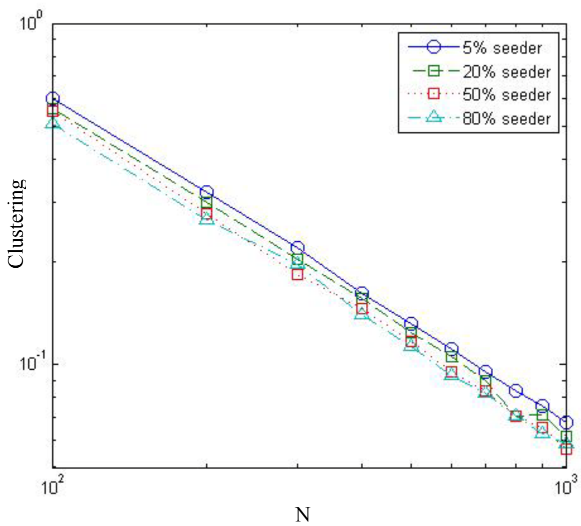

Figure 3 (a) Global clustering coefficient for BitTorrent networks with different size and proportion of original seeders.

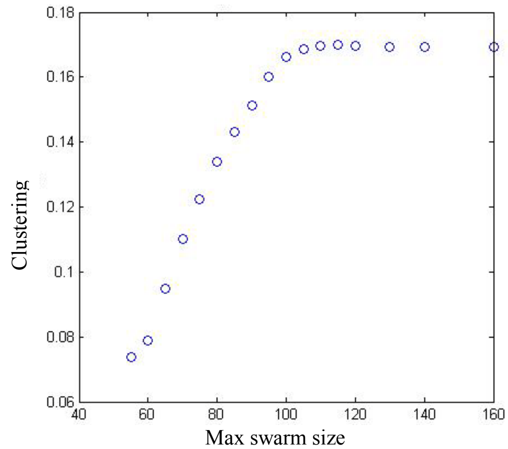

Figure 3 (b) Global clustering coefficient for max swarm size varying from 50 to 160 in a 500-peer network. All other parameters are set to be unchanged.

Then Figure 3(b) shows the clustering of BitTorrent networks with different maximum swarm size. Here, maximum swarm size refers to the upper limit of the number of the neighbor peers that one peer downloads data from in the BitTorrent network. This is a parameter which could be set by BitTorrent users, since larger maximum swarm size may bring faster downloading speed with more peering sending data simultaneously while satisfying more system resource for dealing with the communication which gets denser. From the results of our experiments, it can be seen that clustering increases from 0.07 to 0.17 as maximum swarm size grows from 50 to 100. But the clustering stays stable as the maximum swarm size continues growing. It can be interpreted that, when a higher value of maximum swarm size is set, the networks tend to cluster more if maximum swarm size is comparably small. When maximum swarm size grows to some extent, like 100 in our 500-peer network, the clustering stops to increase and stays stable. We can explain this as larger swarm size may require more resources such as calculation and bandwidth which may have its own limit and therefore peers could not have too large number of neighbors simultaneously, even if the maximum swarm size is large. We can know from the result that one cannot always get a better connected network (with higher clustering coefficient) by setting a larger swarm size.

C. *Shortest path length*

The shortest path has long played an essential role in the research of complex networks. In telecommunication networks such as BitTorrent, the length of shortest paths between these pairs of peers reflects the dependence between them. The shorter the length is, the more dependent the two nodes are, and vice versa. In our model, we find shortest paths in the following steps.

1. Invert the weight of all the arcs (dividing them with 1). We do so because in weighted networks, the transaction between two nodes might be quicker along arcs with higher weight which could be interpreted as a stronger or closer connection between them than with lower weight.
2. Find all pairs' shortest paths and calculate their lengths by adding the inverted weight of each arc along the paths.

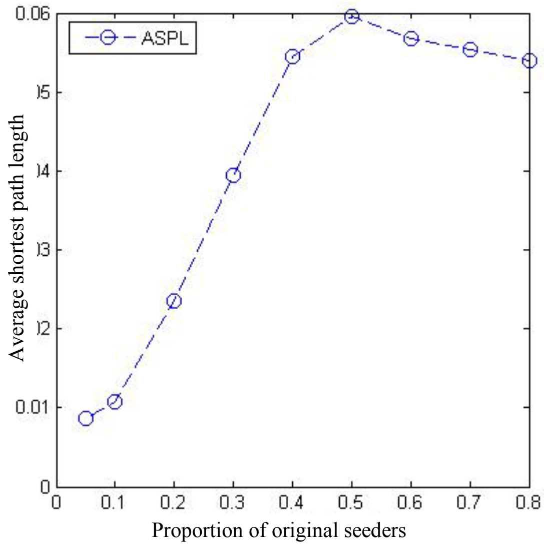

Figure 4 Average shortest path length with different proportion of original seeders. All other conditions are set to be unchanged.

Figure 4 gives the average shortest path length (ASPL) of nine 500-peer BitTorrent networks with different original seeder proportion, which varies from 5% to 80%. We can see that the ASPL increases when seeder proportion varies from 5% to 50%. And then ASPL decreases gently after seeder proportion gets larger than 50%. It could be explained as, when seeder proportion increases, one peer tends to receive data from a larger number of seeders rather than from very few of them. Therefore, the amount of data which comes from one seeder tends to decrease for common peers in the result of the decreasing of weight along one path. Since the weight is inverted in our model, the ASPL increases correspondingly as seeder proportion increases. However, when there is enough number of seeders, the number of arcs in the path decreases as well as the number of seeders continues growing. Consider the extreme case when there is only 1 peer with all partners being seeders in which all paths consist of only 1 seeder-to-peer arc. This contributes to the decrease of ASPL when seeder proportion is larger than 50%. This implies that one cannot always get a shorter ASPL of BitTorrent networks by setting a larger maximum swarm size.

D. Betweenness

Betweenness[20] is one of the structural indicators which relates to the centrality of different nodes. In a network, certain nodes occupy advantageous positions, whereas some other nodes may rely on them to connect to further ones. The extent to which a node contributes itself to the communications of others can be studied using the betweenness measure.

In our model, we propose to use flow betweenness[23] to see how peers' performance affects the data flow of the whole BitTorrent networks. First we define the capacity *Cap(i,j)* of the network connection between nodes *i* and *j* as

$$Cap(i,j) = \max_{Path(i,j)\in Paths(i,j)} \min_{A_{uv}\in Path(i,j)} W_{uv} \tag{2}$$

which gives the maximum flow between nodes *i* and *j*. The capacity between nodes *i* and *j* in a network with an assuming that node *k* has been wiped off from the network is defined as

$$Cap(i\text{-}k\text{-}j) = \max_{\substack{Path(i,j)\in Paths(i,j)\\ k\notin Path(i,j)}} \min_{A_{uv}\in Path(i,j)} W_{uv} \tag{3}$$

where A_{uv} is the arc from node *u* to *v* and W_{uv} is the weight of A_{uv}. *Paths(i,j)* is the set of all the possible simple paths between nodes *i* and *j*. *Path(i,j)* denotes as one of them. For directed networks, *Cap(i,j)* distincts from *Cap(j,i)*. If there is no connection between nodes *i* and *j* then *Cap(i,j)* is set equal to 0. The same is valid when *i* equals to *j*. When there is only one possible path between two nodes this definition could be comprehended as how strong a chain is always relying on its weakest part.

The flow betweenness of node *k*, $b_w(k)$, is given as:

$$b_w(k) = \frac{\sum_{j\neq k,k\neq i,j\neq i}\left(Cap(i,j) - Cap(i-k-j)\right)}{\sum_{j\neq k,k\neq i,j\neq i} Cap(i,j)} \tag{4}$$

which can be interpreted as the ratio of the sum of capacity's increment with the contribution of node *k* to the sum of the capacity of all. The more the capacity increases with the participant of node *k*, the higher $b_w(k)$ is, the more important role node *k* plays in the communication of the whole network. It is not difficult to see that flow betweenness is always a value between 0 and 1. For the nodes which only act as the first or last node of the path, we consider them to be trivial. For example, the flow betweenness of all the seeder and leecher nodes in BitTorrent networks are set equal to 0.

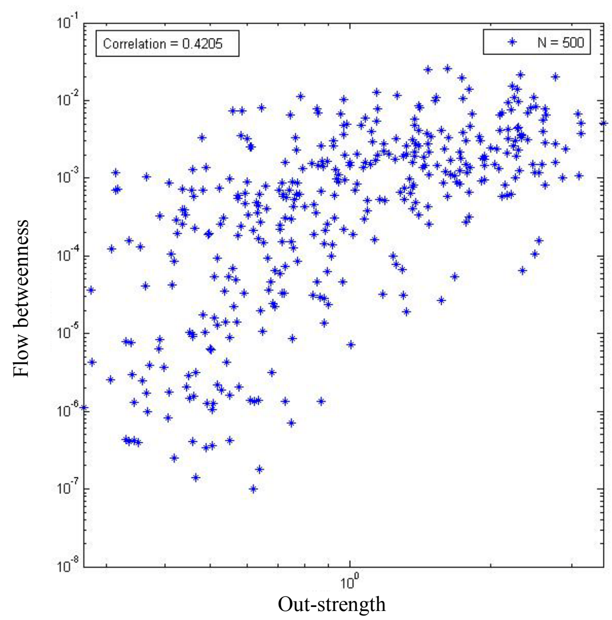

Figure 5 This figure shows the correlation between node strength and flow betweenness of each node in a 500-peer BitTorrent network. With Pearson correlation value being 0.4205, a positive (increasing) relationship between the two variables could be seen in the figure above.

Figure 5 shows the correlation between node strength and flow betweenness in a 500-peer BitTorrent network. The Pearson correlation[24] value is 0.4205, which indicates a positive (increasing) relationship between the two variables. We can see from the figure that, the FB value of nodes with small out-strength varies widely, which means that a small out-strength doesn't decide the FB value to be low or high at all. However, when out-strength grows larger, the lower limit of the FB value grows as well. Since the upper limit of the FB value doesn't change much, it can be concluded that for nodes with large out-strength, their FB value is comparably high as well.

We can see that in our model flow Betweenness is such a measure that it helps describing the contribution of certain peer to the communication of whole network. We believe that peers with high value of flow betweenness have played an more important role in the spreading of data from seeders to peers in the network than those with low value of flow betweenness. These peers with high flow betweenness value should then be encouraged and become very good candidates for other peers to choose to download data from with high priority. Therefore, betweenness may be a good measure for selection of the best peers, which is one of the opening issues in the improvements of P2P protocols[12]. However, there are still some difficulties such as computing complexity that needs to be resolved before betweenness can be applied into the improvement of the BitTorrent protocol. We hope more progress can be made in the future.

V. Conclusion

In this work, we have proposed a plane graph model to exhibit the topology of P2P communication networks using BitTorrent protocol. We found that the node strength of BitTorrent networks follows a power-law distribution. When network size and seeder proportion grows, BitTorrent networks tend to be less clustered. However, a higher clustering value is obtained when the maximum swarm size is set to be larger, which allows peers to have more adjacent neighbors. The average shortest path length grows as network size expands, which is quite a common phenomenon in real networks. But the growing of seeders' proportion affects the ASPL (firstly increase, then decrease) as well since the seeder can only act as the first peer in paths both of whose length and number gets smaller. Finally a positive correlation between flow betweenness and node strength is found in our work.

Acknowledgment

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