N1

1) $\|x\|_2 = \sqrt{\sum_{i=1}^{m} x_i^2}$

$\|x\|_\infty \cdot \sqrt{m(\max|x_i|)^2} \geq \sqrt{\sum_i^m x_i} \Rightarrow \|x_i\| \leq \sqrt{m}\,\|x\|_\infty$

Насколько: $x = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$ $\quad \|x\|_2 = \sqrt{m}$

$\|x\|_\infty = 1$ $\quad \Rightarrow \|x\|_2 = \sqrt{m}\,\|x_\infty\|$

2) $\|A\|_2 = \max\limits_{\|x\|_2 = 1} \|Ax\|_2 = \sigma_1$ $\qquad \sigma_i \geq \sigma_{i+1}$

$\|A\|_\infty = \max \|Ax\|_\infty \leq \max \sqrt{\sum \sigma_1^2} \leq \sqrt{n}\,\sigma \Rightarrow$

$\Rightarrow \|A\|_\infty \leq \sqrt{n}\,\|A\|_2$

N2

1) $A = \begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}$ $\qquad \sigma_{1,2} = 3, 2$ $\qquad \begin{pmatrix} 0 & 0 \\ 0 & -5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$AA^T = \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix}$

$x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$y = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

2) $A = \begin{pmatrix} 0 & 2 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$ $\quad AA^T = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\cancel{A}$

$\sigma_1 = 4 \quad \sigma_2 = 0 \quad \sigma_3 = 0$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow x = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \to y = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

3) $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ $\quad AA^T = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$

$\sigma_1 = 2 \quad \sigma_2 = 0$

$$\begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow x = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$y = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$A = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

NH

$$f(A) = A^{-1} X (X^T A^{-1} X)^{-1}$$

$$A f(A) = X (X^T A^{-1} X)^{-1}$$

$$X^T A f(A) = (X^T A^{-1} X)^{-1}$$

$$X^T A^{-1} X X^T A f(A) = E$$

$$X^T f(A) = E$$

$f(A) = X^{T+} \Rightarrow f(A)$ не зав. от A

$$\Rightarrow \begin{cases} f(\Delta) = X^{T+} \\ f(X \Omega X^T + \Delta) = X^{T+} \end{cases} \Rightarrow f(\Delta) = f(X \Omega X^T + \Delta)$$