N4

$$(1-\varepsilon)^2 - 10\delta \Rightarrow \varepsilon^2 - 2\varepsilon + 1 - 10\delta = 0 \Rightarrow$$

$$\Rightarrow D = 4 - 4 + 40\delta \Rightarrow \varepsilon = \underline{2 \pm 2\sqrt{10\delta}}$$

$$A = \begin{pmatrix} 1 & 10 \\ \delta & 1 \end{pmatrix} \Rightarrow \varepsilon(\delta) = 1 + \sqrt{10\delta} \Rightarrow k = \frac{d\varepsilon(\delta)^2}{d\delta} = \frac{1}{2}\sqrt{\frac{10}{\delta}} \Rightarrow$$

$$\Rightarrow k(10) = \frac{1}{2}\ ;\ k(0.1) = 5$$

N1

$$I_n(\alpha) = \int_0^1 \frac{x^n}{x+\alpha}\,dx$$

$$\begin{array}{l|l} -\,x^n & x+\alpha \\ \hline \;\; x^n + \alpha x^{n-1} & x^{n-1} - \alpha x^{n-2} + \alpha^2 x^{n-3} - \alpha^3 x^{n-4} + \dots \\ \;\; -\alpha x^{n-1} & \\ -\;\; -\alpha x^{n-1} - \alpha^2 x^{n-2} & \\ \;\;\;\;\; \alpha^2 x^{n-2} & \\ -\;\;\;\; \alpha^2 x^{n-2} + \alpha^3 x^{n-3} & \\ \;\;\;\;\;\;\; \dots & \end{array}$$

$$\text{Т.е.}: I_n(\alpha) = \int_0^1 x^{n-1} - \alpha \cdot I_{n-1} = \underline{\frac{1}{n} - \alpha I_{n-1}}$$

$$\underline{I_{n-1} = \frac{1}{n\alpha} - \frac{1}{\alpha} I_n}$$

N3

$$I_n = -I_{n-1} + 6 I_{n-2}$$

$$1 = -\lambda^{-1} + 6\lambda^{-2}$$

$$\lambda^2 + \lambda - 6 = 0$$

$$\lambda = \frac{-1 \pm \sqrt{25}}{2} = 2;\ -3$$

$$\begin{cases} I_n = a \cdot 2^n + b \cdot (-3)^n \\ I_0 = 1 \\ I_1 = -3 \end{cases} \Rightarrow \begin{cases} a = 0 \\ b = 1 \end{cases} \Rightarrow I_n = (-3)^n$$