Spatial IIR – Thesis proposal

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Abstract

The analogy between spatially distributed sensors processing (beam-forming) and a finite-impulse-response (FIR) filter [7] laid the grounds for the "spatial-processing" concept which enables the filtering (or nulling) signals according to their direction-of-arrival (DOA). [9] had further developed the concept to approximation of the "spatial-IIR" concept using shifted-sub-arrays to generate the delays elements in the regressive part of the filter. Other papers ([4, 6, 5, 1, 8, 2]) are taking a different approach of 2D filtering, in both spatial and temporal domains simultaneously, using the known plane-wave characteristic of equally-phased planes $x(x, ct) = x(-sin(\theta)x + ct)$ where θ is the DOA of the signal. All mentioned approaches actually enable frame-by-frame processing and not an actual IIR which is influenced from all it's input's history, which limits the processing power of the beam-former. Our proposed approach is a source-sensor-cooperated-IIR-beam-former which assumes that the source can also transmit a synthetic signal which will be sent from the sensor in addition to its own signal, thus, as will be explained, enabling real IIR transfer function purely in the spatial domain.

Signal model

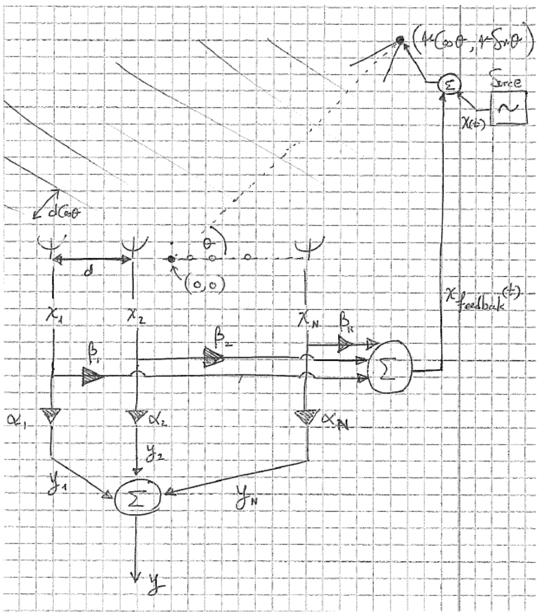

Figure 1: Signal model

In Figure 1, the assumed scenario is described. A source of far-field signal, which is received by a uniform-linear-array (ULA), also transmits a synthetic signal which is generated by the sensor array and transmitted back to the source. After defining $x(t)$ as the source signal, $x_n(t)$ as the n'th sensor received signal, N as the number of sensors in the ULA, r as the range of the source, c as the speed of signal propagation, $\tau_{pd} = \frac{r}{c}$ as the propagation delay, τ_{tx} as the feedback transmission delay, θ as the

DOA of the signal, and $\tau_\theta = \dfrac{d\cos(\theta)}{c}$ as the relative delay between the signal arrival to the sensors, One can extract the temporal relation which defines the overall system

$$x_n(t,\theta) = x(t-\tau_{pd}-(N-n)\tau_\theta) + \sum_{m=1}^{N}\beta_m x_m(t-\tau_{pd}-\tau_{tx}-(N-m)\tau_\theta)$$

which transforms to

$$x_n(\omega,\theta) = X(\omega)e^{j\omega(-\tau_{pd}-(N-n)\tau_\theta)} + \sum_{m=1}^{N}\beta_m x_m(\omega)e^{j\omega(-\tau_{pd}-\tau_{tx}-(N-m)\tau_\theta)}$$

under the Fourier transform. Using the notation of regular letters (x) for scalars, over-lined letters for vectors ($\overline{x}$) and double-underlined letters ($\underline{\underline{x}}$) for matrices and considering all sensors, one can write

$$\overline{X}(\omega,\theta) = X(\omega)\overline{d}(\theta)e^{-j\omega\tau_{pd}} + \underline{\underline{\beta}}(\theta)\overline{X}(\omega) \tag{1}$$

where $d(\theta) = \begin{bmatrix} 1 \\ e^{j\omega\tau_\theta} \\ e^{j2\omega\tau_\theta} \\ \vdots \\ e^{j(N-1)\omega\tau_\theta} \end{bmatrix}$ is the well known "steering-vector" of the array and given the coefficients vector $\overline{\beta} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_N \end{bmatrix}$, the matrix $\underline{\underline{\beta}}(\theta)$ can be represented as

$$\underline{\underline{\beta}}(\theta) = \mathbf{1}_{N\times 1}\overline{\beta}^T diag(\overline{d}(\theta))e^{-j\omega(\tau_{pd}+\tau_{tx})} \tag{2}$$

where the ^{T} represents the transpose operator and $diag()$ generates a matrix that its main diagonal is filled with the values of the argument vector and all the rest elements are zeros. This relation leads to finding the transfer function between the source signal and the sensors output

$$\overline{X}(\omega,\theta) = (I-\underline{\underline{\beta}}(\theta))^{-1}\overline{d}(\theta)X(\omega)e^{-j\omega(\tau_{pd})}$$

which, combined with the coefficients vector $\overline{\alpha} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_N \end{bmatrix}$, results in the general transfer function of the system

$$Y(\omega,\theta) = \overline{\alpha}^T(I-\underline{\underline{\beta}}(\theta))^{-1}\overline{d}(\theta)X(\omega)e^{-j\omega(\tau_{pd})} \tag{3}$$

.

For the purpose of inverting the matrix $(I - \underline{\underline{\beta}}(\theta))$ one can use the celebrated woodbury matrix identity [10], which degenerates to the special case discussed in [3] that deals with inverting the sum of two matrices $(\underline{\underline{G}} + \underline{\underline{H}})$ where $\underline{\underline{G}}$ is invertible and $\underline{\underline{H}}$ is of rank 1 and concludes that

$$(\underline{\underline{G}} + \underline{\underline{H}})^{-1} = \underline{\underline{G}}^{-1} - \frac{1}{1+g}\underline{\underline{G}}^{-1}\underline{\underline{H}}\underline{\underline{G}}^{-1}$$

where $g = tr(\underline{\underline{H}}\underline{\underline{G}}^{-1})$. Inserting $\underline{\underline{I}}$ as $\underline{\underline{G}}$ and $-\underline{\underline{\beta}}(\theta)$ as $\underline{\underline{H}}$, which clearly has rank 1, results in

$$(\underline{\underline{I}} - \underline{\underline{\beta}}(\theta))^{-1} = \underline{\underline{I}} + \frac{1}{1+g}\underline{\underline{\beta}}(\theta)$$

where $g(\theta) = tr(\underline{\underline{\beta}}(\theta)) = tr\left(\mathbf{1}_{N\times 1}\overline{\beta}^T diag(\overline{d}(\theta))e^{-j\omega(\tau_{pd}+\tau_{tx})}\right) = e^{-j\omega(\tau_{pd}+\tau_{tx})}\sum_{n=1}^{N}\beta_n e^{j\omega(n-1)\tau_\theta}$ and when combining the last result with (2) and (3) we remain with

$$Y(\omega,\theta) = \overline{\alpha}^T\left(\underline{\underline{I}} + \frac{1}{1+g(\theta)}\left(\mathbf{1}_{N\times 1}\overline{\beta}^T diag(\overline{d}(\theta))e^{-j\omega(\tau_{pd}+\tau_{tx})}\right)\right)\overline{d}(\theta)X(\omega)e^{-j\omega(\tau_{pd})}$$

. After some math one gets

$$Y(\omega,\theta) = \frac{e^{-j\omega\tau_{pd}}\sum_{n=1}^{N}\alpha_n d_n(\theta) + e^{-j\omega\left(2\tau_{pd}+\tau_{tx}\right)}\left[\sum_{m=1}^{N}\sum_{l=1}^{N}\beta_m d_m(\theta)\alpha_l d_l(\theta) + \sum_{k=1}^{N}\sum_{l=1}^{r}\beta_k\alpha_r d_k^2(\theta)\right]}{1 + e^{-j\omega(\tau_{pd}+\tau_{tx})}\sum_{n=1}^{N}\beta_n e^{j\omega(n-1)\tau_\theta}}X(\omega) \quad (4)$$

. Let $d_n(\theta) = e^{j\omega(n-1)\tau_\theta} \triangleq z^{n-1}$ and we get an auto-regressive (IIR-like) transfer function with respect to θ, which is the goal of our proposed system. The transfer function in the Z-domain is

$$H(\omega,z) = \frac{e^{-j\omega\tau_{pd}}\sum_{n=1}^{N}\alpha_n z^{n-1} + e^{-j\omega\left(2\tau_{pd}+\tau_{tx}\right)}\left[\sum_{m=1}^{N}\sum_{l=1}^{N}\beta_m\alpha_l z^{m+l-2} + \sum_{k=1}^{N}\sum_{l=1}^{r}\beta_k\alpha_r z^{2k-2}\right]}{1 + e^{-j\omega(\tau_{pd}+\tau_{tx})}\sum_{n=1}^{N}\beta_n z^{n-1}}X(\omega) \quad (5)$$

and the purpose of out thesis proposal is to investigate and develop methods of choosing the coefficients $\overline{\alpha}_{opt}$ and $\overline{\beta}_{opt}$ when a goal beam-pattern, $H_{wanted}(\omega, z)$, is presented to achieve

$$H(\omega, z, \overline{\alpha}_{opt}, \overline{\beta}_{opt}) \approx H_{wanted}(\omega, z)$$

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