

# Treatment of less-known CMB instrumental systematics

CosmoForward 2026/02/09  
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Lead, SO LAT PS pipeline AWG

# The End

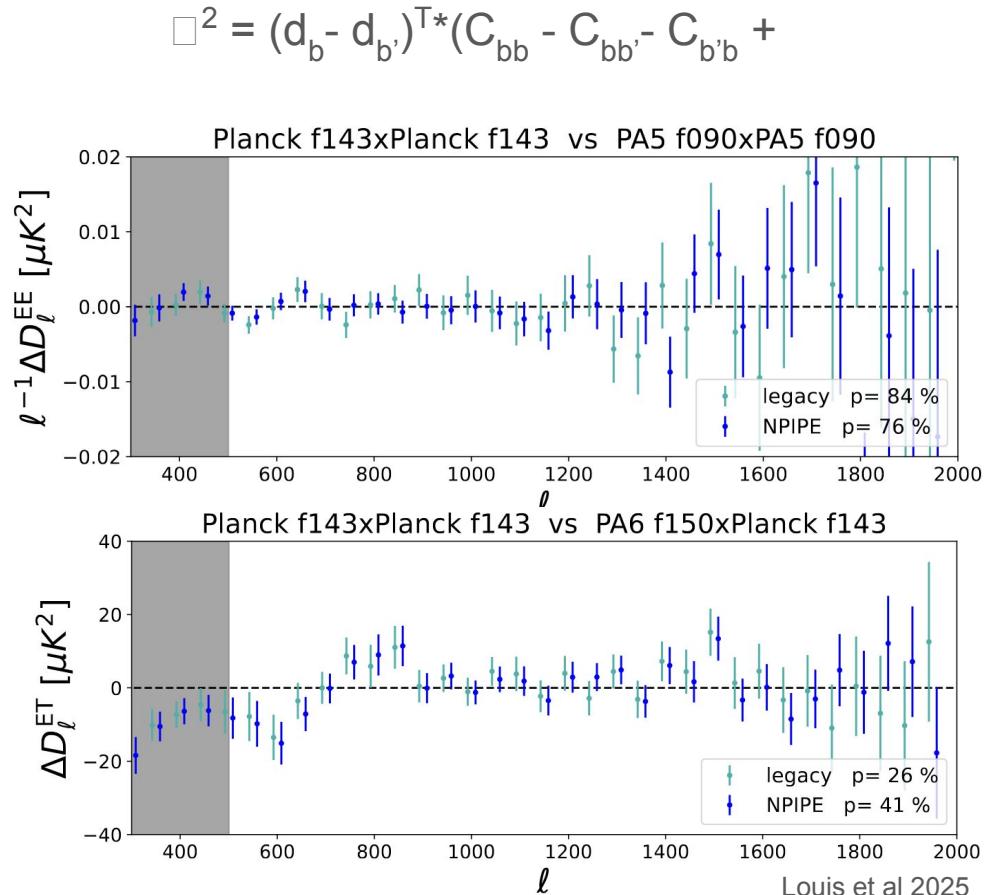
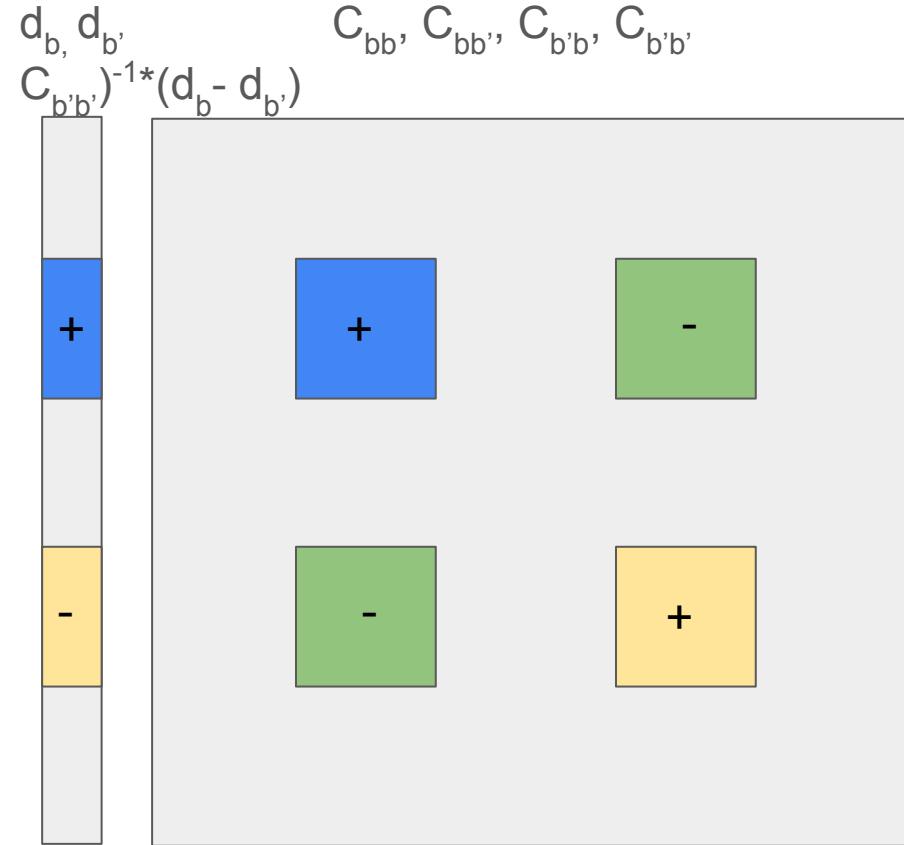
# Treatment of ~~less~~- unknown CMB instrumental systematics

# ~~Treatment of less-known CMB instrumental systematics~~

## What/how/why of null tests?

- Comparison of 2+ measurements of the same thing
- How example:  $(ps1\_ell - ps2\_ell) / cov \rightarrow \chi^2$
- Measure systematics (or don't)
- Internal consistency of dataset
- Validate software pipeline
- ...
- (~)Independent of cosmology/sky model

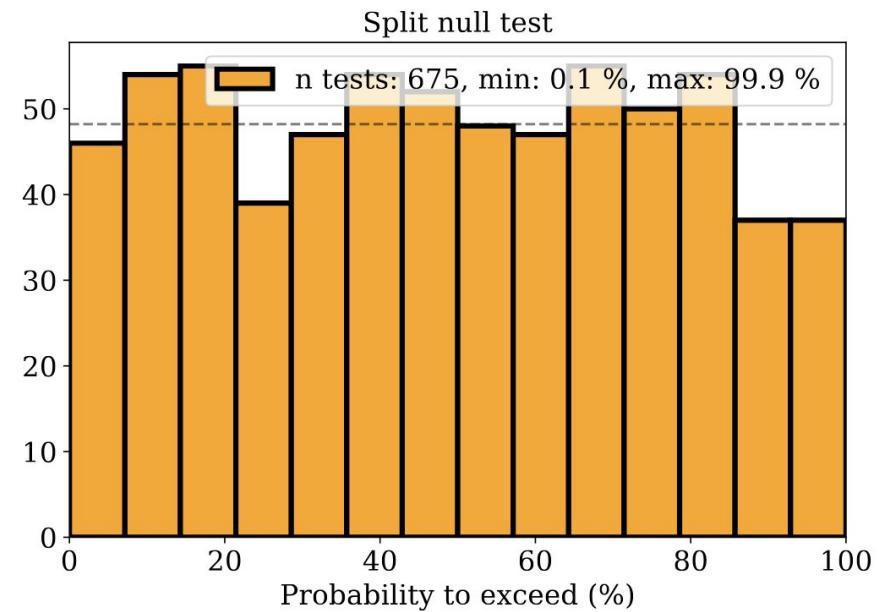
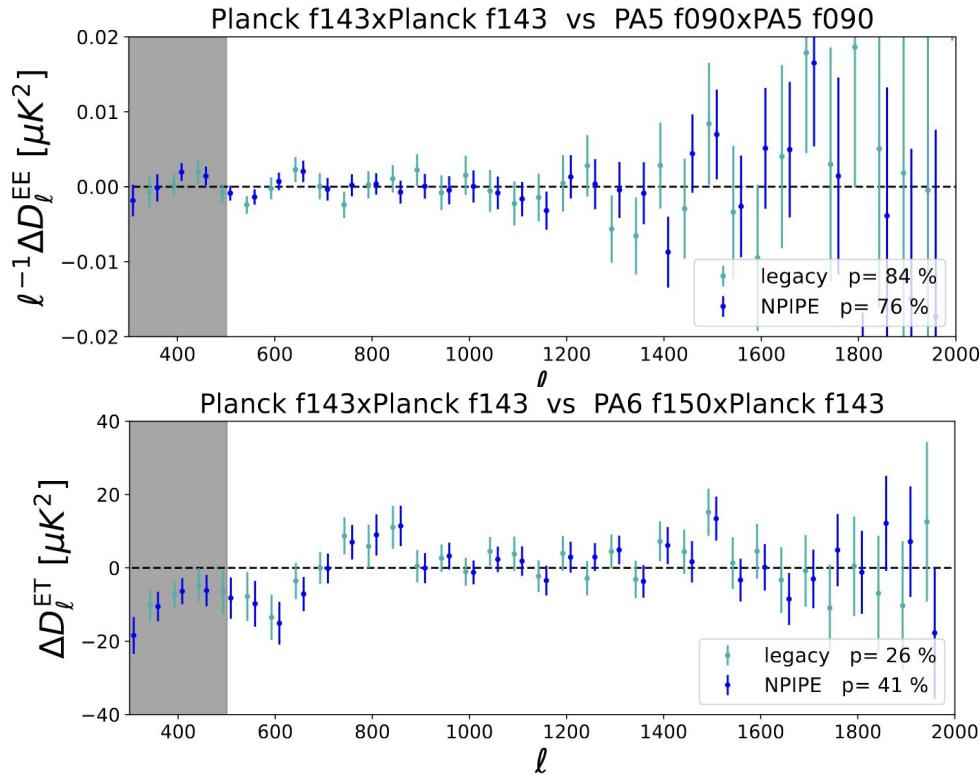
# Quick example of classic null tests (1)



# Quick example of classic null tests (2)

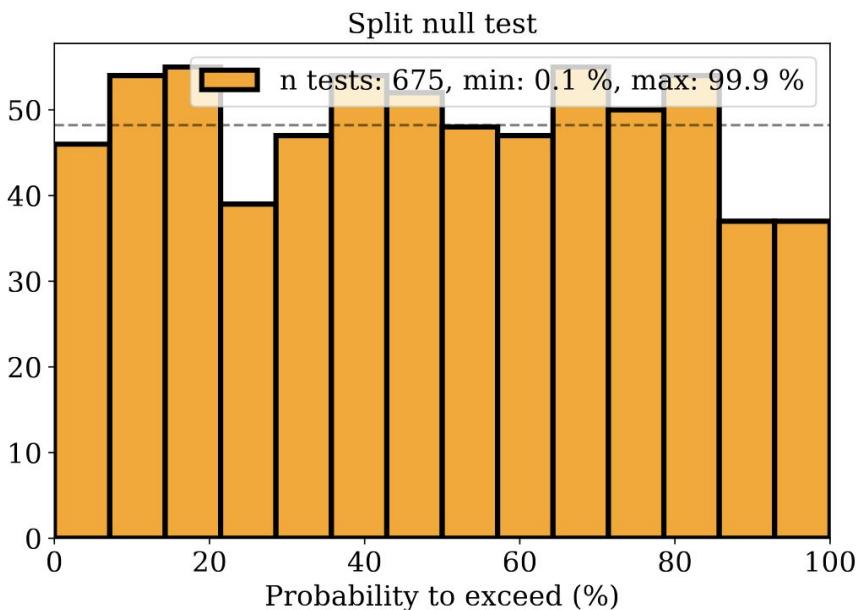
$$\chi^2 = (\mathbf{d}_b - \mathbf{d}_{b'})^\top (\mathbf{C}_{bb} - \mathbf{C}_{bb'} - \mathbf{C}_{b'b} + \mathbf{C}_{b'b'})^{-1} (\mathbf{d}_b - \mathbf{d}_{b'})$$

Uniform?

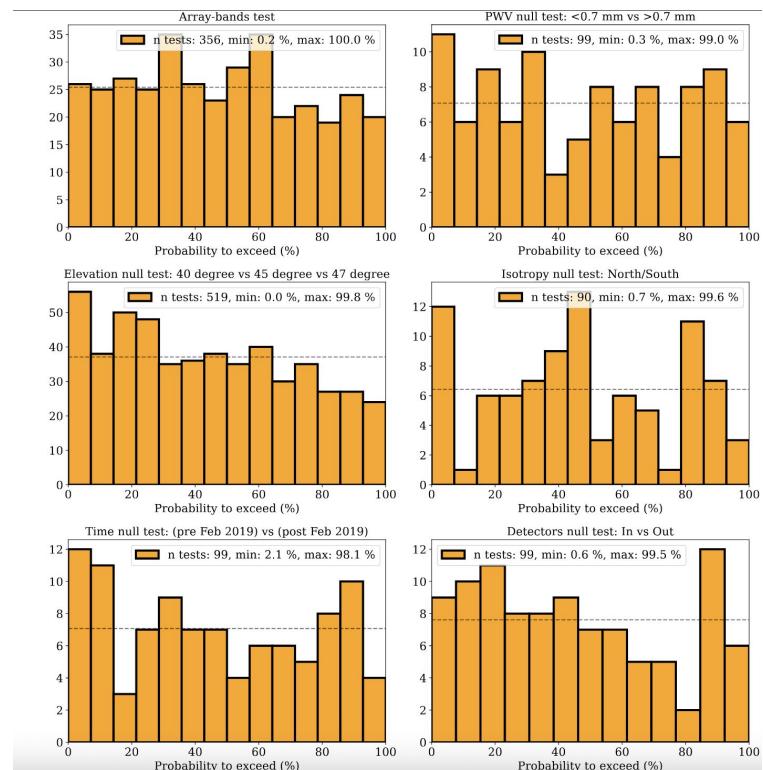


# Quick example of classic null tests (3)

Uniform?



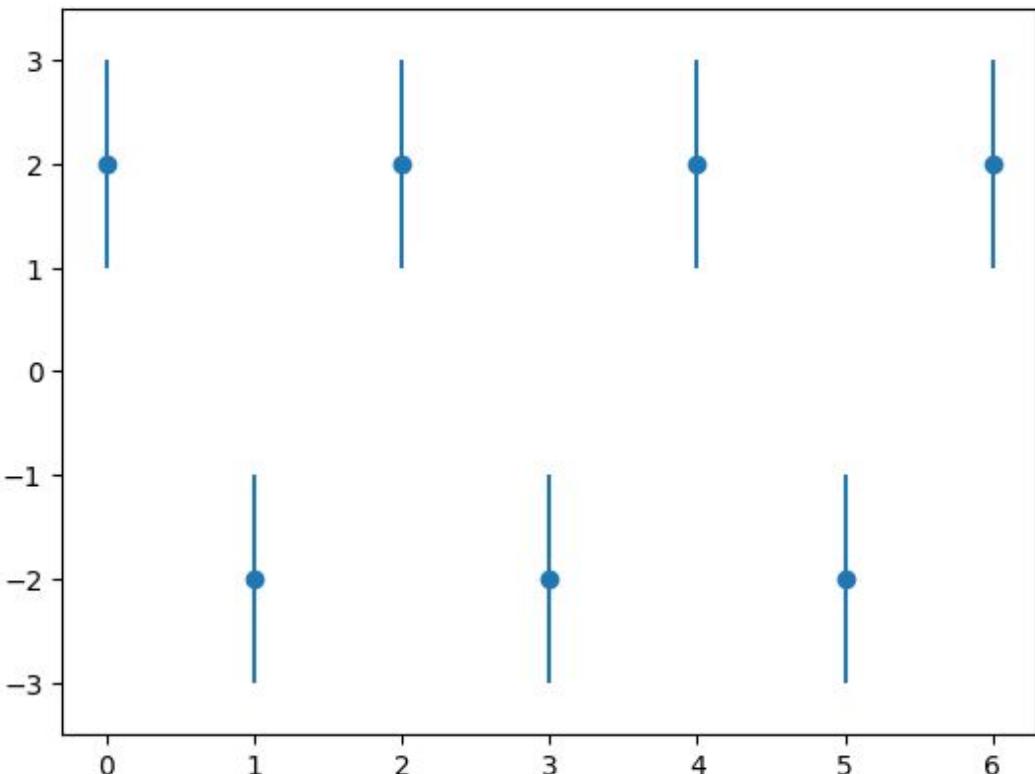
All uniform?



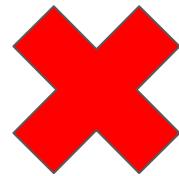
# Null tests in the literature

Experiment	Type of null	Summary of null	Summary of summaries	Joint summaries of null	Joint summaries of summaries
ACT DR6	Spectrum	PTE( $\square^2$ )	Rank statistics		
SPT DR1	Spectrum, Conditional	PTE( $\square^2$ )	Rank statistics		
Planck 2018	Spectrum, Conditional	PTE( $\square^2$ )			
BICEP/Keck 2021	Map	PTE( $\square^2$ ), PTE( $\square$ )	Rank statistics, KS, $\Sigma(\square^2)$	~100s map sims	
POLARBEAR 2022	Map	PTE( $\square^2$ ), PTE( $\square$ )	5 (incl. summary of summary of summaries)	~100s maps sims	~100s maps sims
SPIDER 2021	Map	PTE( $\square^2$ )	Rank statistics, KS	~100s maps sims, ~1000s spectra sims	

# Choice of summary of null



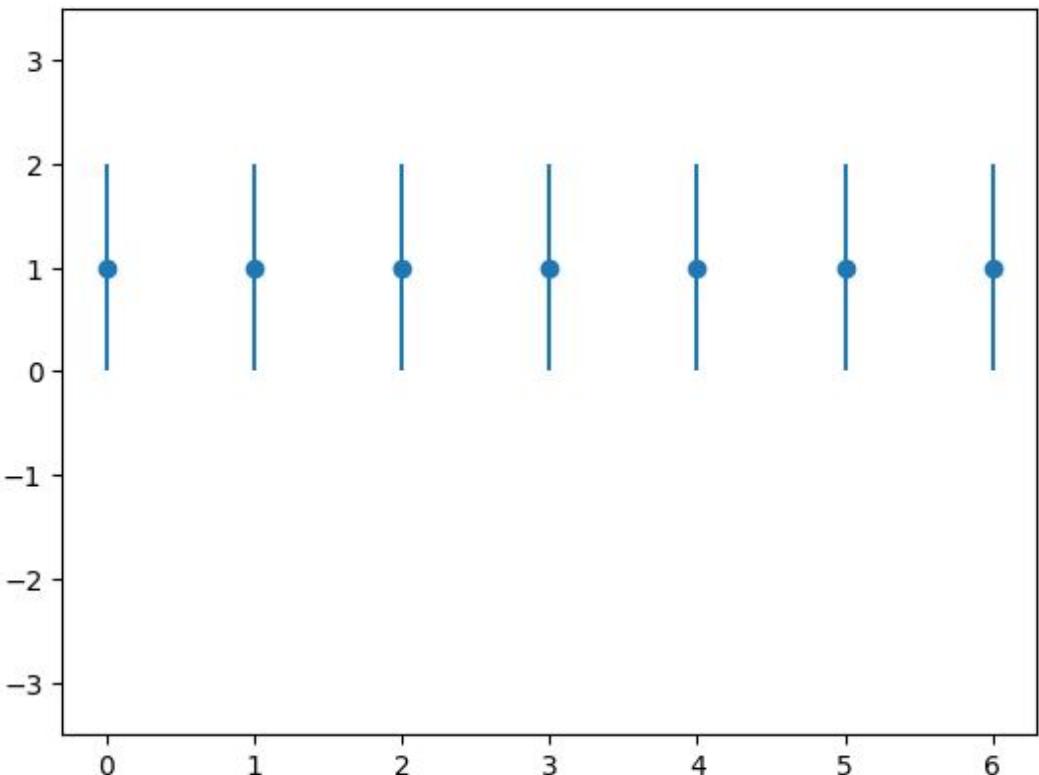
$$\chi^2 = \vec{d}^T \mathbf{C}^{-1} \vec{d}$$



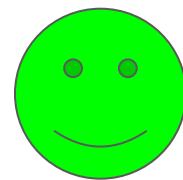
$$\chi = \vec{1}^T \mathbf{C}^{-\frac{1}{2}} \vec{d}$$



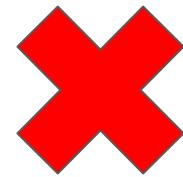
# Choice of summary of null



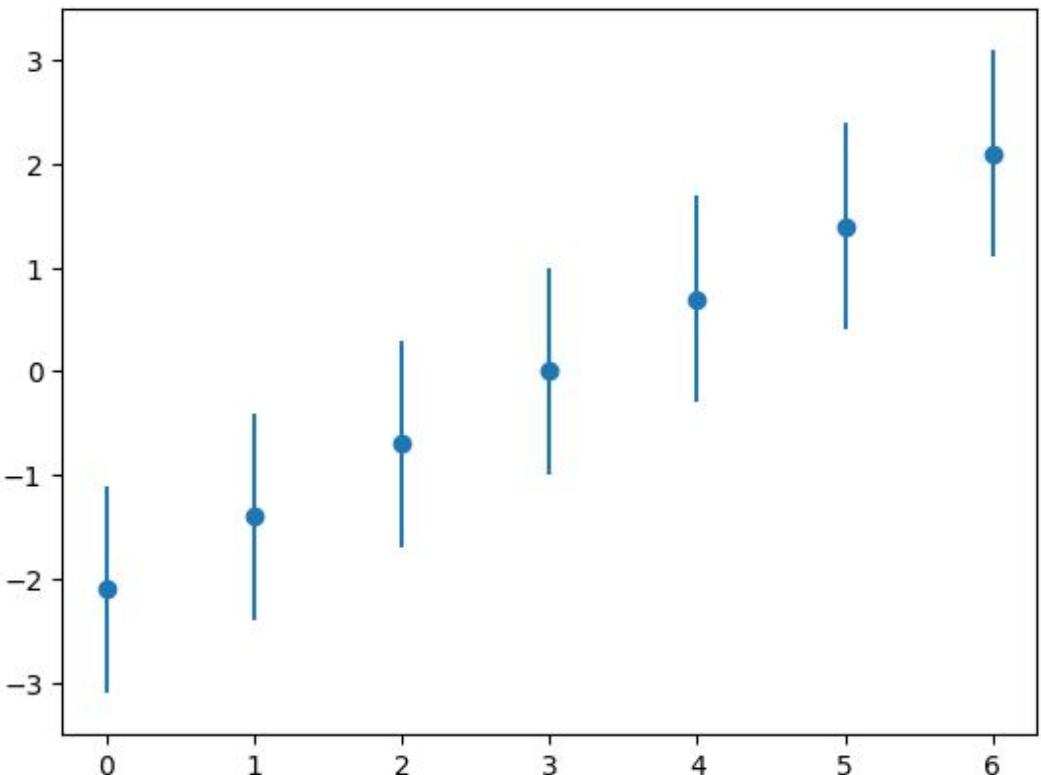
$$\chi^2 = \vec{d}^T \mathbf{C}^{-1} \vec{d}$$



$$\chi = \vec{1}^T \mathbf{C}^{-\frac{1}{2}} \vec{d}$$



# Choice of summary of null



$$\chi^2 = \vec{d}^T \mathbf{C}^{-1} \vec{d}$$



$$\chi = \vec{1}^T \mathbf{C}^{-\frac{1}{2}} \vec{d}$$



# Choice of summary of summaries + joint distributions

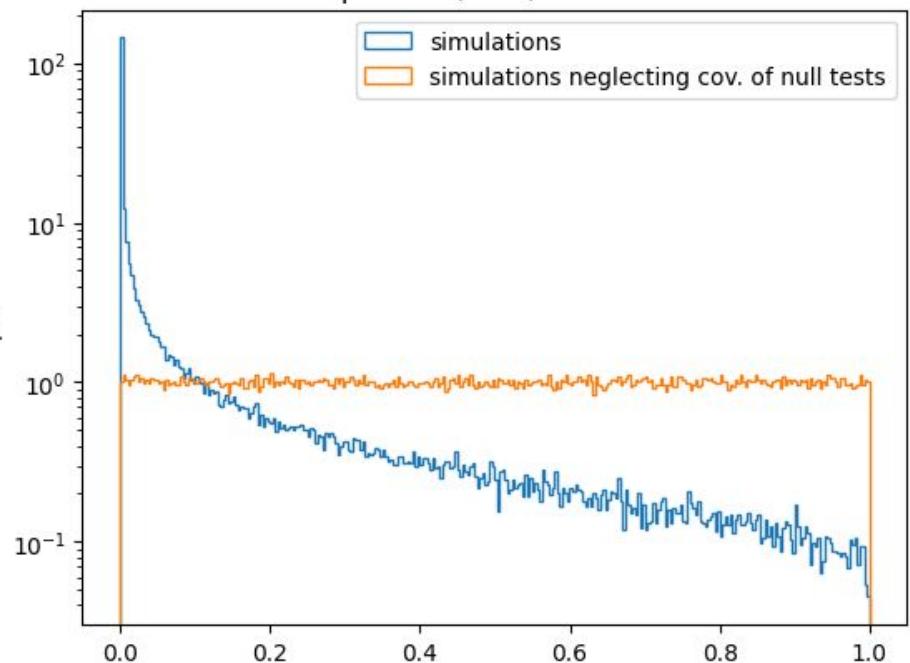
Setup mock PS covariance matrix (slices of ACT DR6 cov):

- Only TT TE EE + assume each spectrum shares sky model
- Tile this for 10 blocks

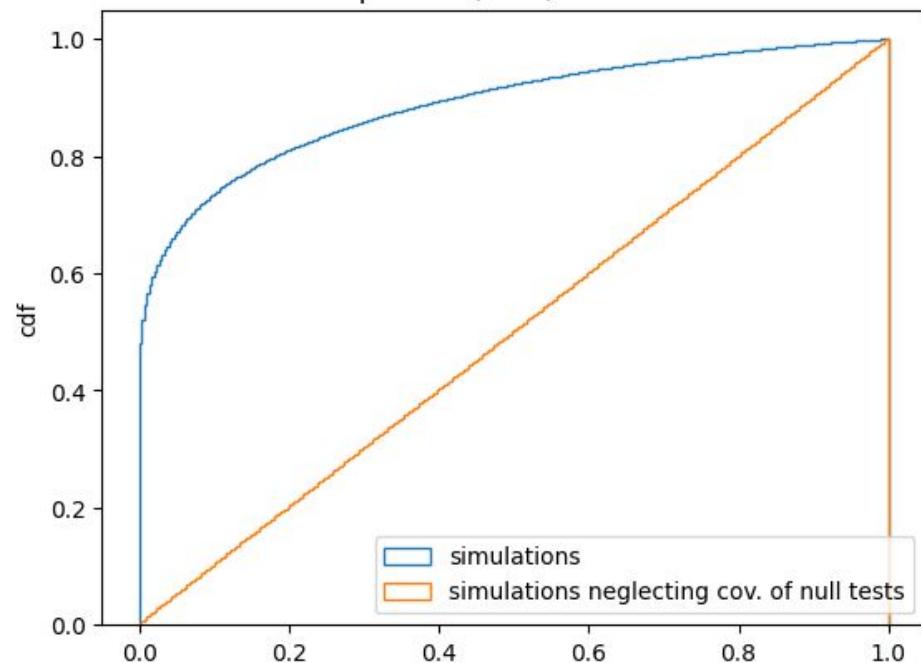
Draw 100,000 realizations from covariance:

- Form all spectrum null tests (for 10 blocks, 135 tests), calculate PTE( $\chi^2$ ) for each (135 x 100,000  $\chi^2$  values)
- For each simulation, calculate : KS test p-value, rank statistic 1/135, rank statistic 7/135 (~5th percentile)

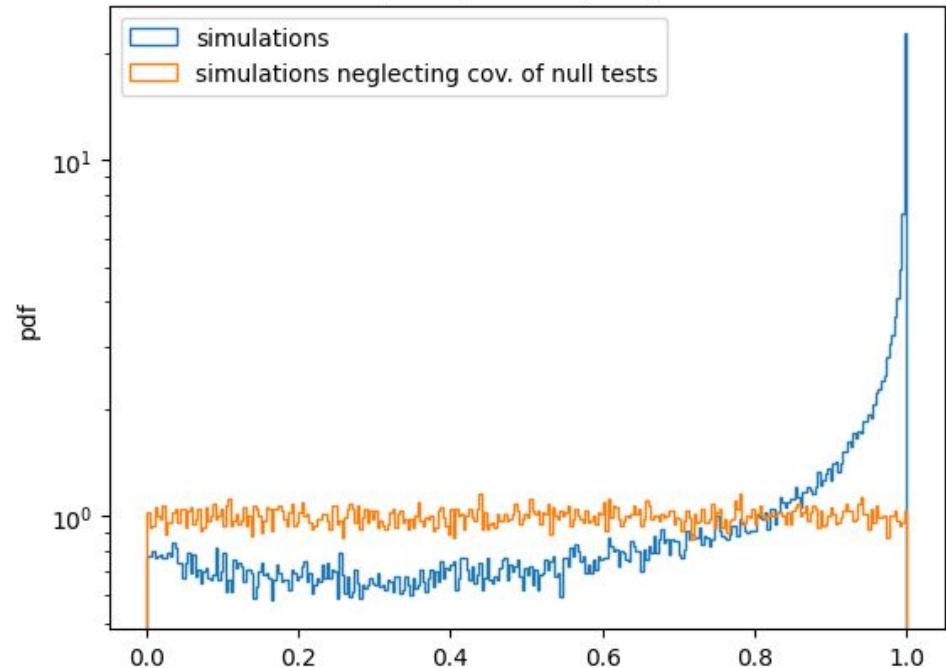
KS test p-values, 100,000 simulations



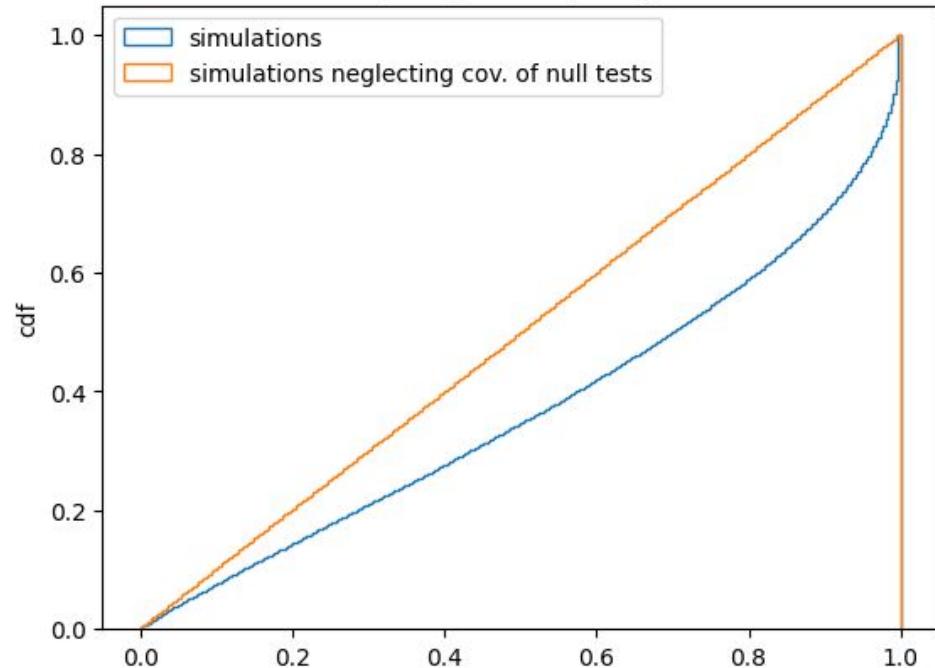
KS test p-values, 100,000 simulations



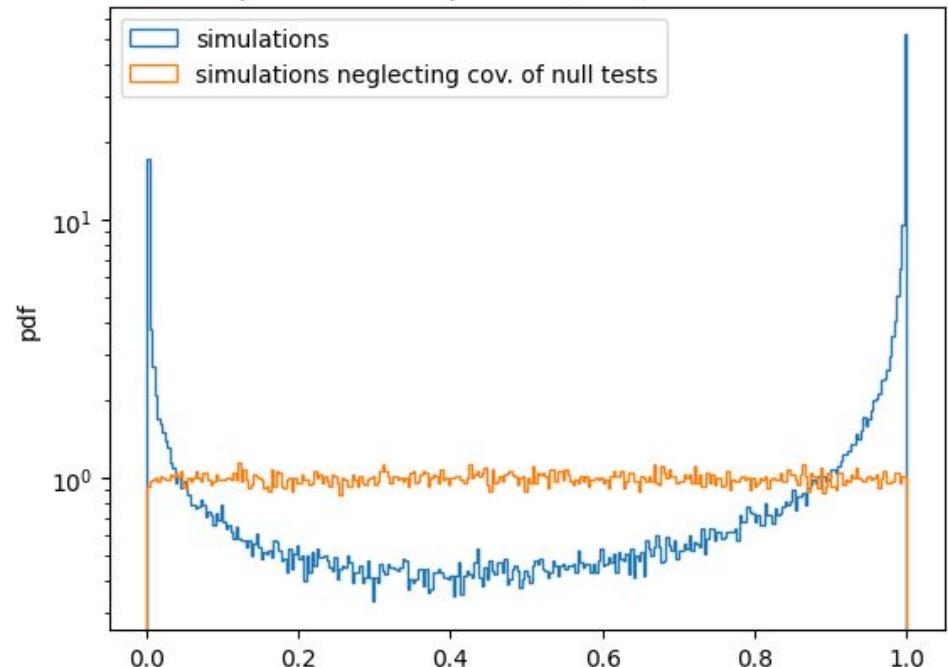
Rank statistic 1/135 p-values, 100,000 simulations



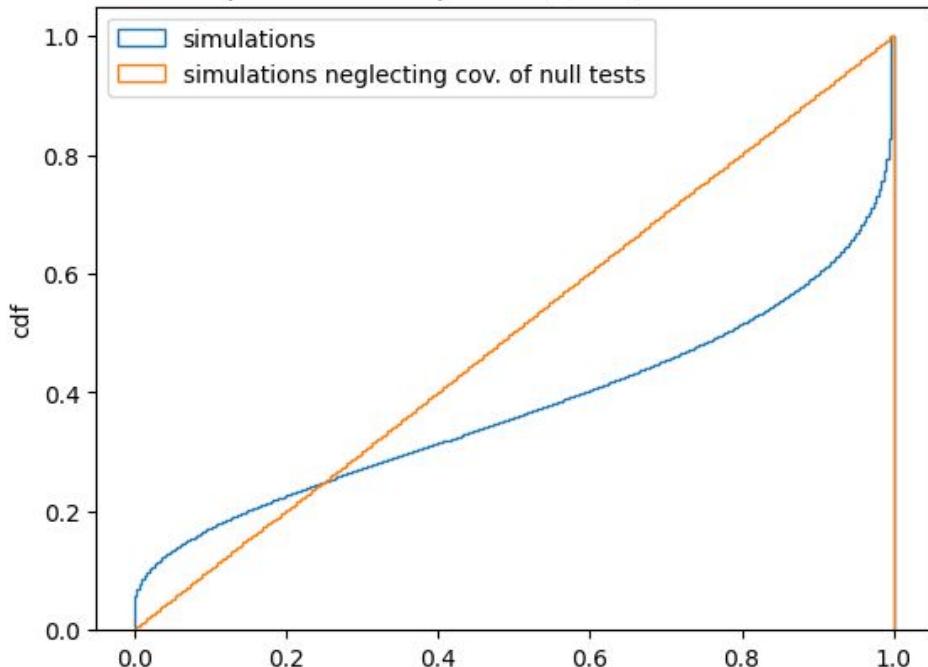
Rank statistic 1/135 p-values, 100,000 simulations

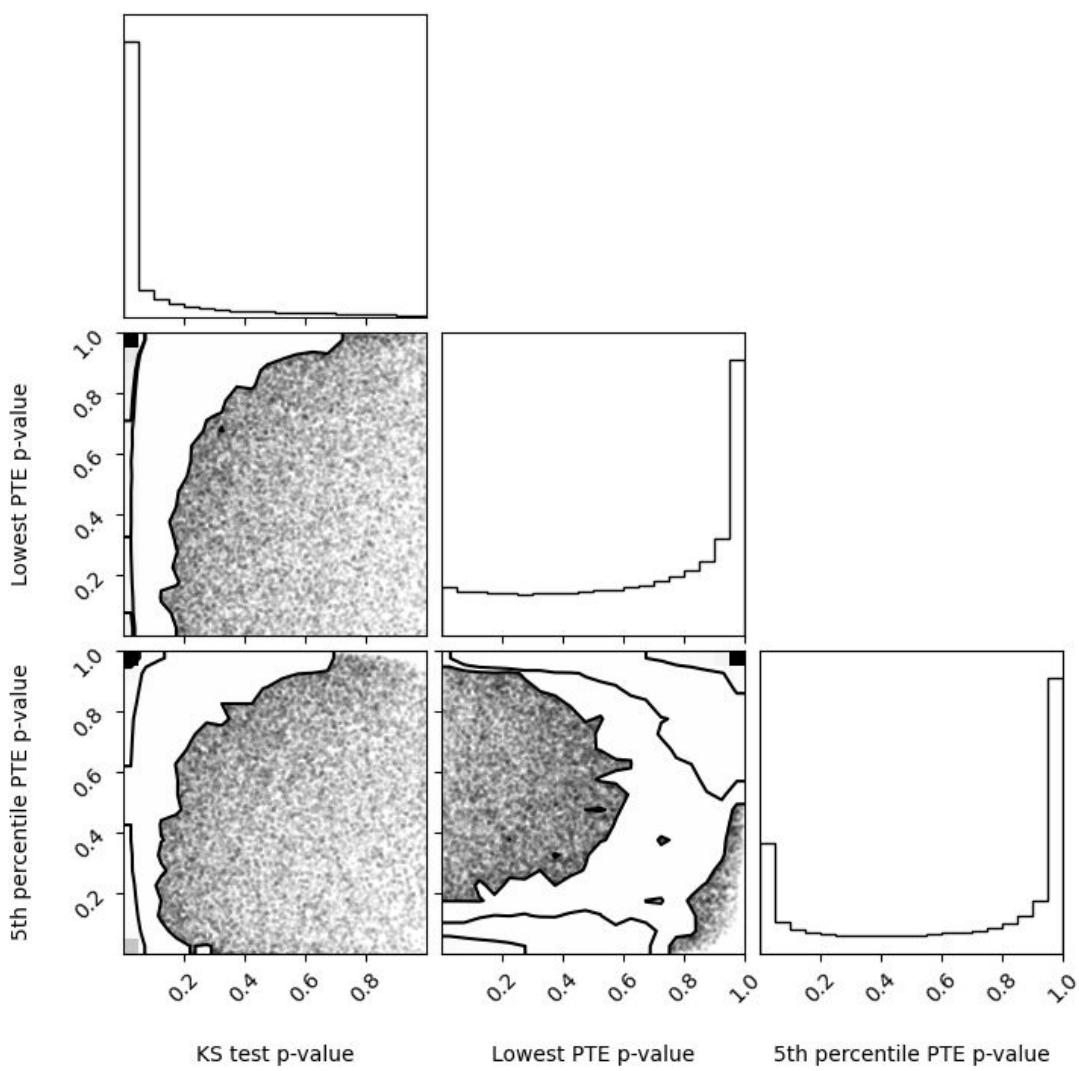


5th percentile PTE p-values, 100,000 simulations



5th percentile PTE p-values, 100,000 simulations



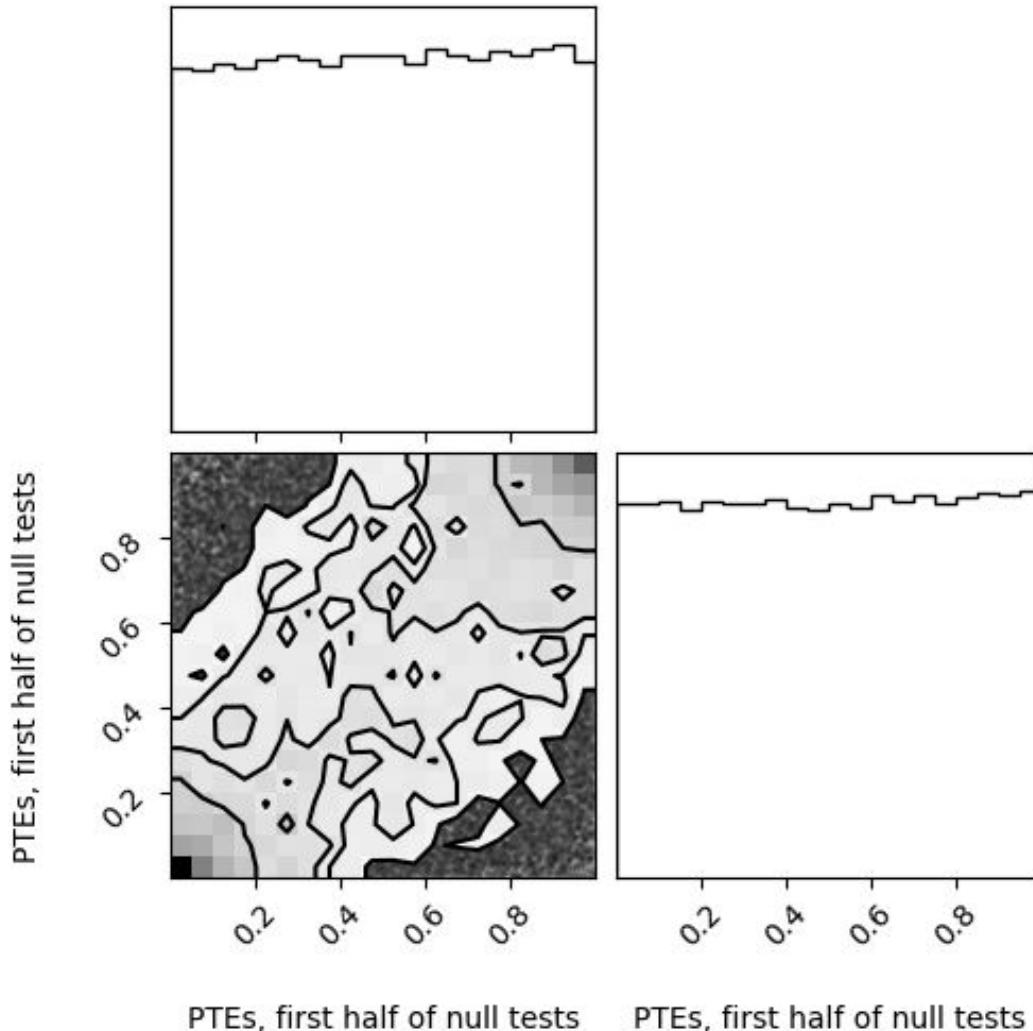


# Choice of summary of summaries + joint distributions

Same setup as before except:

- Tile cov template for 20 blocks
- Form all spectrum null tests for first 10 blocks and second 10 blocks separately for 1,000 simulations
- For each null test in each half of null tests, calculate PTE( $\square^2$ ) ( $2 \times 135 \times 1,000$ )
- (Don't push through to summary of summaries, just plot raw PTE( $\square^2$ ))

Simulates a pair of “systematics-targeting” null tests (e.g., PWV and Elevation)



# Null test design choices...summary

Choice of summary of null: throw kitchen sink at problem?

Choice of summary of summaries: throw kitchen sink at problem?

Modeling joint distributions of summaries (of summaries) important

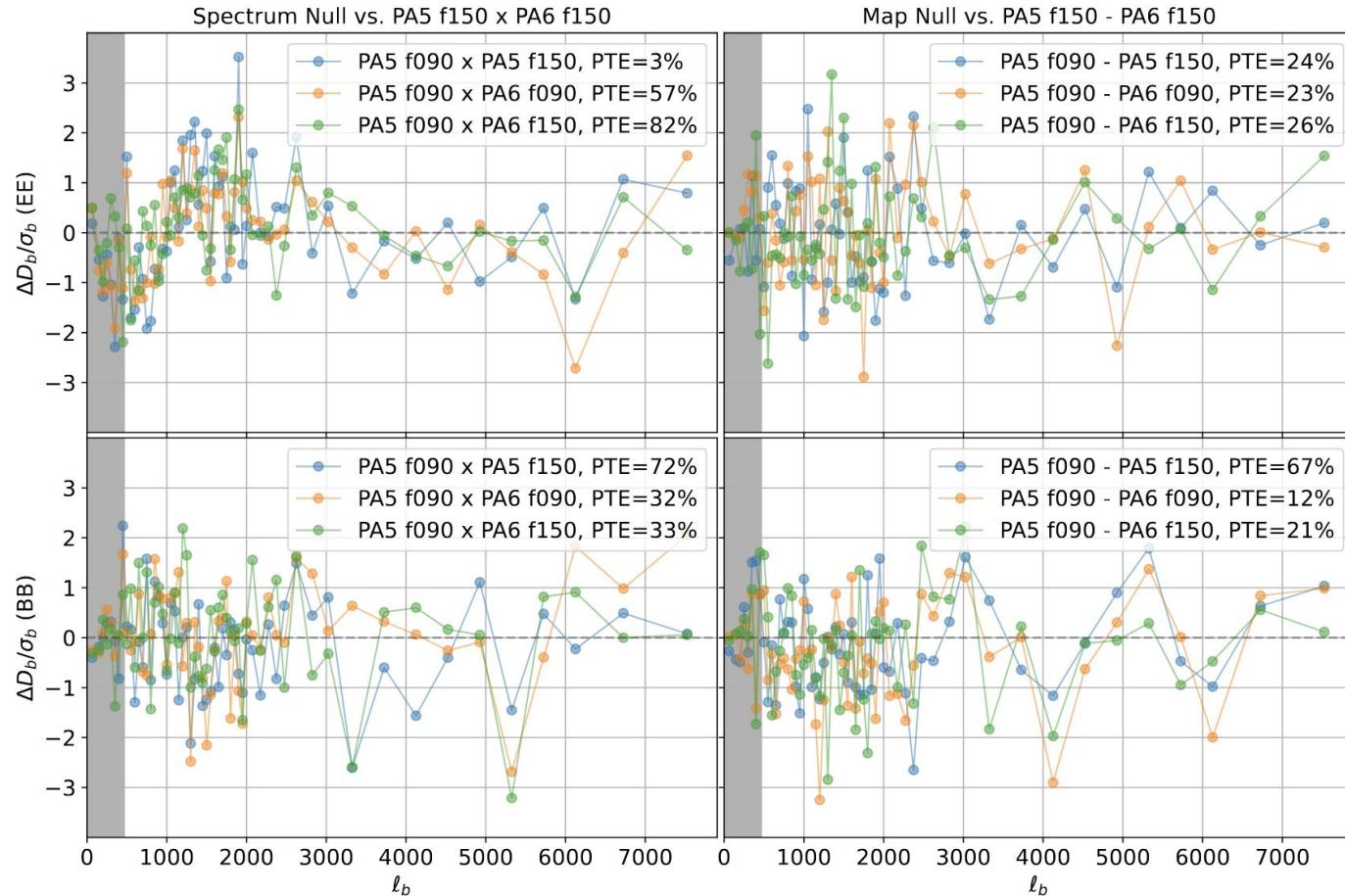
Why do test → summaries → summaries of summaries → etc. in the first place?

Ultimately, we want:

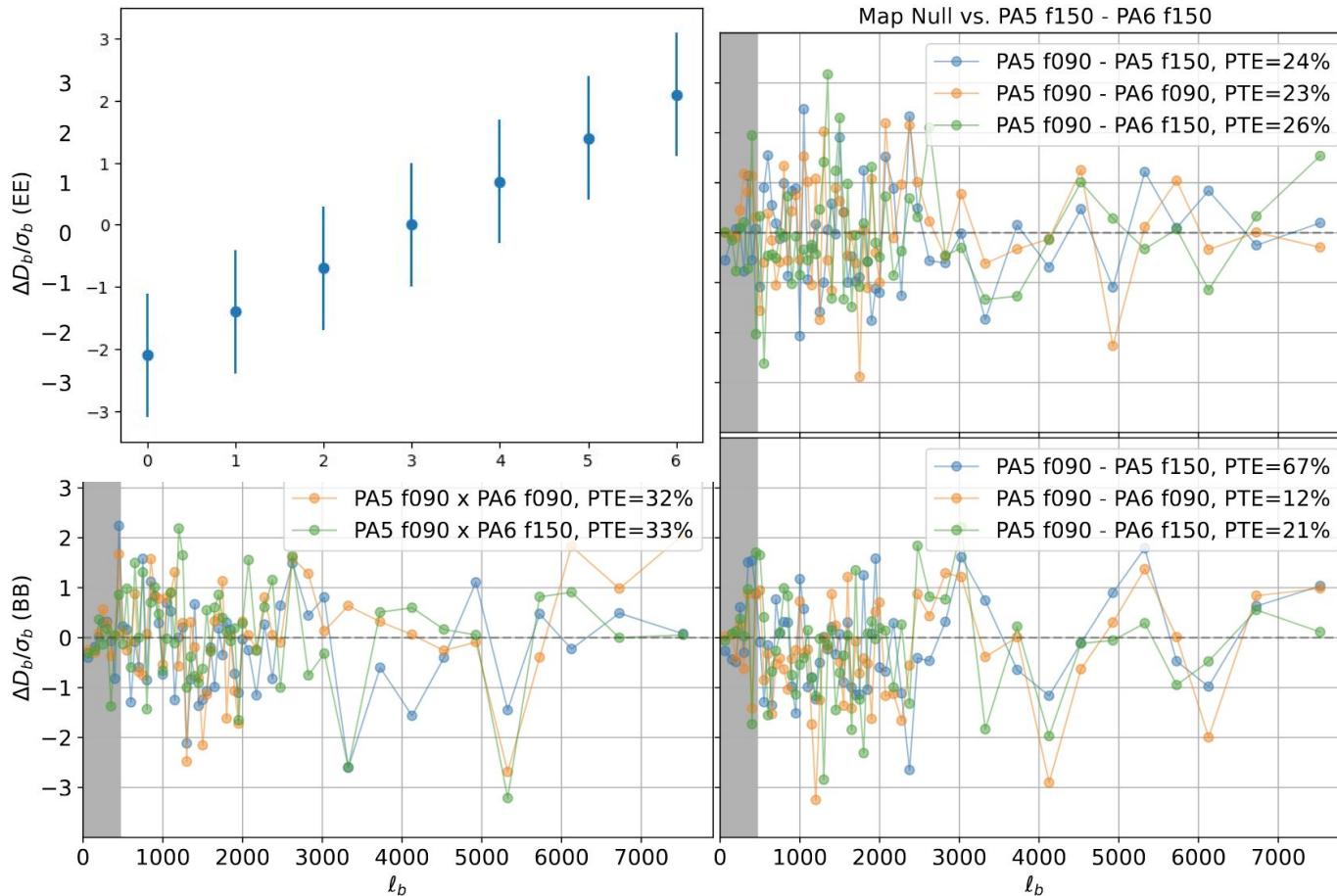
→ Evaluate likelihood  $P(d|\text{Model excl. cosmology})$

→ Provide feedback to instrumentalists etc. before unblinding

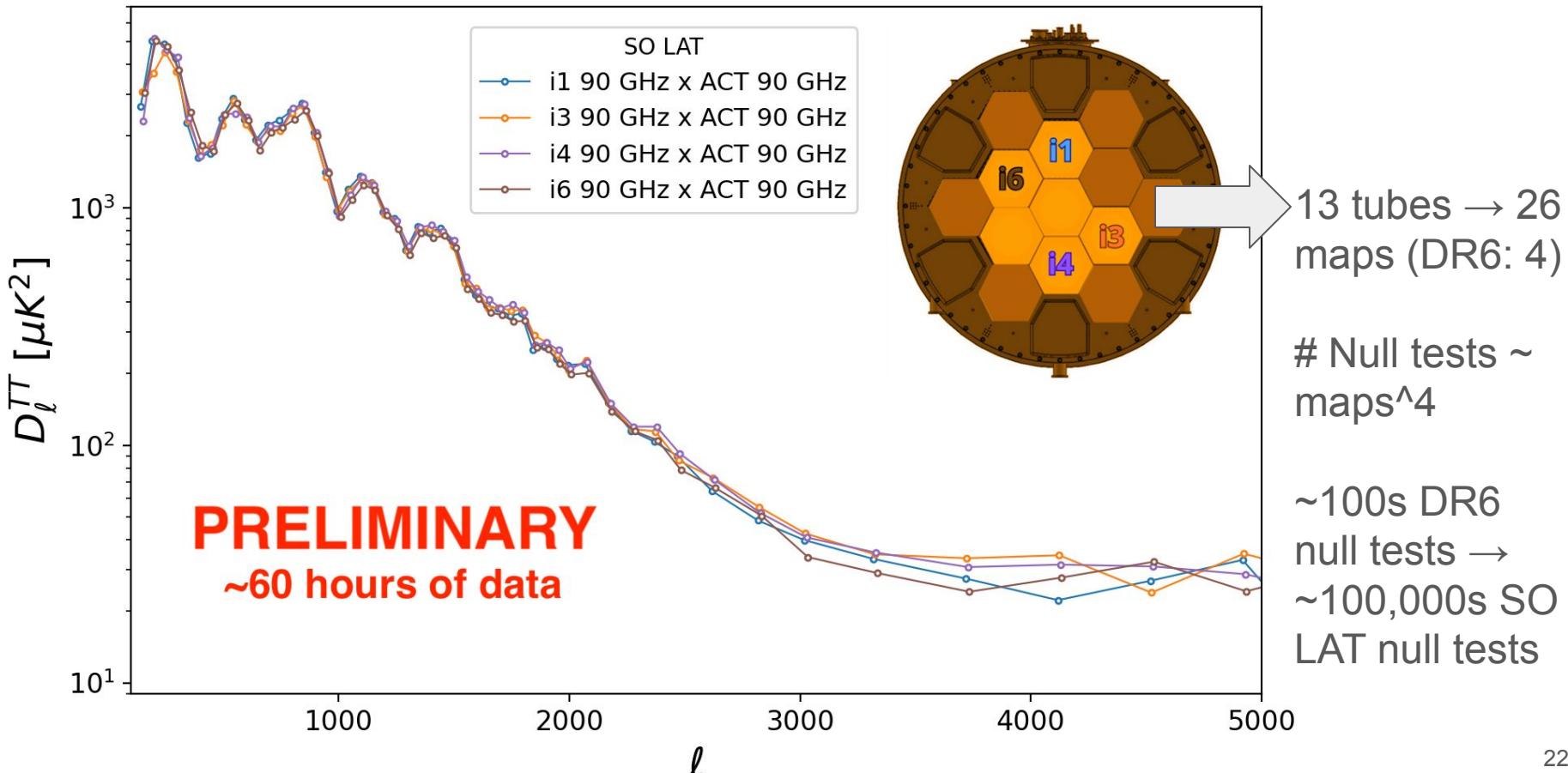
# A complementary null test: motivation from ACT DR6



# A complementary null test: motivation from ACT DR6



# A complementary null test: motivation from SO



# A map-level, linear summary statistic

Assume some baseline sky model, model map-level perturbations:

$$\begin{pmatrix} \tilde{T}_{\ell m}^{\alpha} \\ \tilde{E}_{\ell m}^{\alpha} \end{pmatrix} = \begin{pmatrix} 1 + \delta_{\ell}^{T_{\alpha}} & \gamma_{\ell}^{\alpha} \\ \gamma_{\ell}^{\alpha} & 1 + \delta_{\ell}^{E_{\alpha}} \end{pmatrix} \begin{pmatrix} T_{\ell m}^{\alpha} \\ E_{\ell m}^{\alpha} \end{pmatrix}$$

Effect on spectra:

$$\begin{pmatrix} \tilde{C}_{\ell}^{T_{\alpha}, T_{\beta}} \\ \tilde{C}_{\ell}^{T_{\alpha}, E_{\beta}} \\ \tilde{C}_{\ell}^{E_{\alpha}, T_{\beta}} \\ \tilde{C}_{\ell}^{E_{\alpha}, E_{\beta}} \end{pmatrix} = \begin{pmatrix} (1 + \delta_{\ell}^{T_{\alpha}})(1 + \delta_{\ell}^{T_{\beta}}) & (1 + \delta_{\ell}^{T_{\alpha}})\gamma_{\ell}^{\beta} & \gamma_{\ell}^{\alpha}(1 + \delta_{\ell}^{T_{\beta}}) & \gamma_{\ell}^{\alpha}\gamma_{\ell}^{\beta} \\ (1 + \delta_{\ell}^{T_{\alpha}})\gamma_{\ell}^{\beta} & (1 + \delta_{\ell}^{T_{\alpha}})(1 + \delta_{\ell}^{E_{\beta}}) & \gamma_{\ell}^{\alpha}\gamma_{\ell}^{\beta} & \gamma_{\ell}^{\alpha}(1 + \delta_{\ell}^{E_{\beta}}) \\ \gamma_{\ell}^{\alpha}(1 + \delta_{\ell}^{T_{\beta}}) & \gamma_{\ell}^{\alpha}\gamma_{\ell}^{\beta} & (1 + \delta_{\ell}^{E_{\alpha}})(1 + \delta_{\ell}^{T_{\beta}}) & (1 + \delta_{\ell}^{E_{\alpha}})\gamma_{\ell}^{\beta} \\ \gamma_{\ell}^{\alpha}\gamma_{\ell}^{\beta} & \gamma_{\ell}^{\alpha}(1 + \delta_{\ell}^{E_{\beta}}) & (1 + \delta_{\ell}^{E_{\alpha}})\gamma_{\ell}^{\beta} & (1 + \delta_{\ell}^{E_{\alpha}})(1 + \delta_{\ell}^{E_{\beta}}) \end{pmatrix} \begin{pmatrix} C_{\ell}^{T_{\alpha}, T_{\beta}} \\ C_{\ell}^{T_{\alpha}, E_{\beta}} \\ C_{\ell}^{E_{\alpha}, T_{\beta}} \\ C_{\ell}^{E_{\alpha}, E_{\beta}} \end{pmatrix}$$

Linearize:

$$\begin{pmatrix} \tilde{C}_{\ell}^{T_{\alpha}, T_{\beta}} \\ \tilde{C}_{\ell}^{T_{\alpha}, E_{\beta}} \\ \tilde{C}_{\ell}^{E_{\alpha}, T_{\beta}} \\ \tilde{C}_{\ell}^{E_{\alpha}, E_{\beta}} \end{pmatrix} = \begin{pmatrix} 1 + \delta_{\ell}^{T_{\alpha}} + \delta_{\ell}^{T_{\beta}} & \gamma_{\ell}^{\beta} & \gamma_{\ell}^{\alpha} & 0 \\ \gamma_{\ell}^{\beta} & 1 + \delta_{\ell}^{T_{\alpha}} + \delta_{\ell}^{E_{\beta}} & 0 & \gamma_{\ell}^{\alpha} \\ \gamma_{\ell}^{\alpha} & 0 & 1 + \delta_{\ell}^{E_{\alpha}} + \delta_{\ell}^{T_{\beta}} & \gamma_{\ell}^{\beta} \\ 0 & \gamma_{\ell}^{\alpha} & \gamma_{\ell}^{\beta} & 1 + \delta_{\ell}^{E_{\alpha}} + \delta_{\ell}^{E_{\beta}} \end{pmatrix} \begin{pmatrix} C_{\ell}^{T_{\alpha}, T_{\beta}} \\ C_{\ell}^{T_{\alpha}, E_{\beta}} \\ C_{\ell}^{E_{\alpha}, T_{\beta}} \\ C_{\ell}^{E_{\alpha}, E_{\beta}} \end{pmatrix}$$

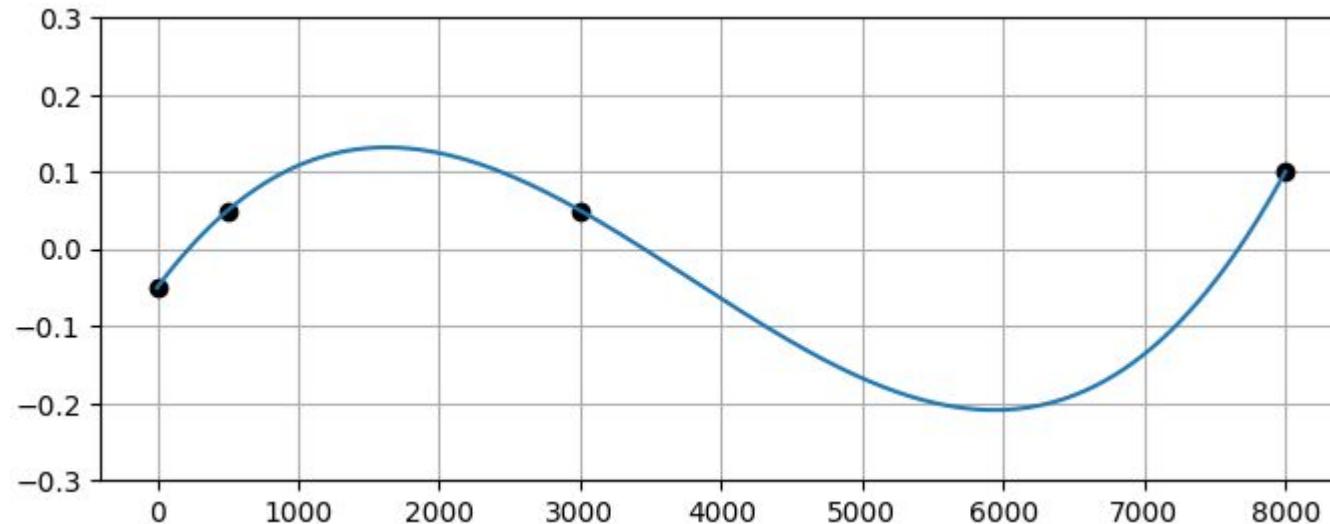
# Choose parameterization for $\delta_\ell^T, \gamma_\ell, \delta_\ell^E$

Want model that is (1) linear, (2) expressive, (3) robust

Cubic spline: (1) and (2)

Linear spline: (1) and (3)

Pick cubic spline:



# Make model parameters indep. of cosmology template

Setup:

- 3 kinds of systematics:  $\delta_\ell^T, \gamma_\ell, \delta_\ell^E$
- Each map gets one of each (Nmaps):  $\delta_\ell^{T_\alpha}, \gamma_\ell^\alpha, \delta_\ell^{E_\alpha}$
- For fixed spline x-values, fit the y-values (Nknots):  $\delta_{\{i\}}^{T_\alpha}, \gamma_{\{i\}}^\alpha, \delta_{\{i\}}^{E_\alpha}$

Independent from cosmology (to 1st order) if (e.g.):  $\Sigma_\alpha \delta_{\{i\}}^{T_\alpha} = 0$

- Enforce with Nmaps x (Nmaps - 1) matrix
- Nmaps - 1 latent parameters ~ “linearly indep. differences vs. the mean”

Total params = 3 \* (Nmaps - 1) \* Knots

# Distribution of Model

Total model = Linear → Just a matrix! → **knts2spec** (d.size x # total params)

Model distribution:

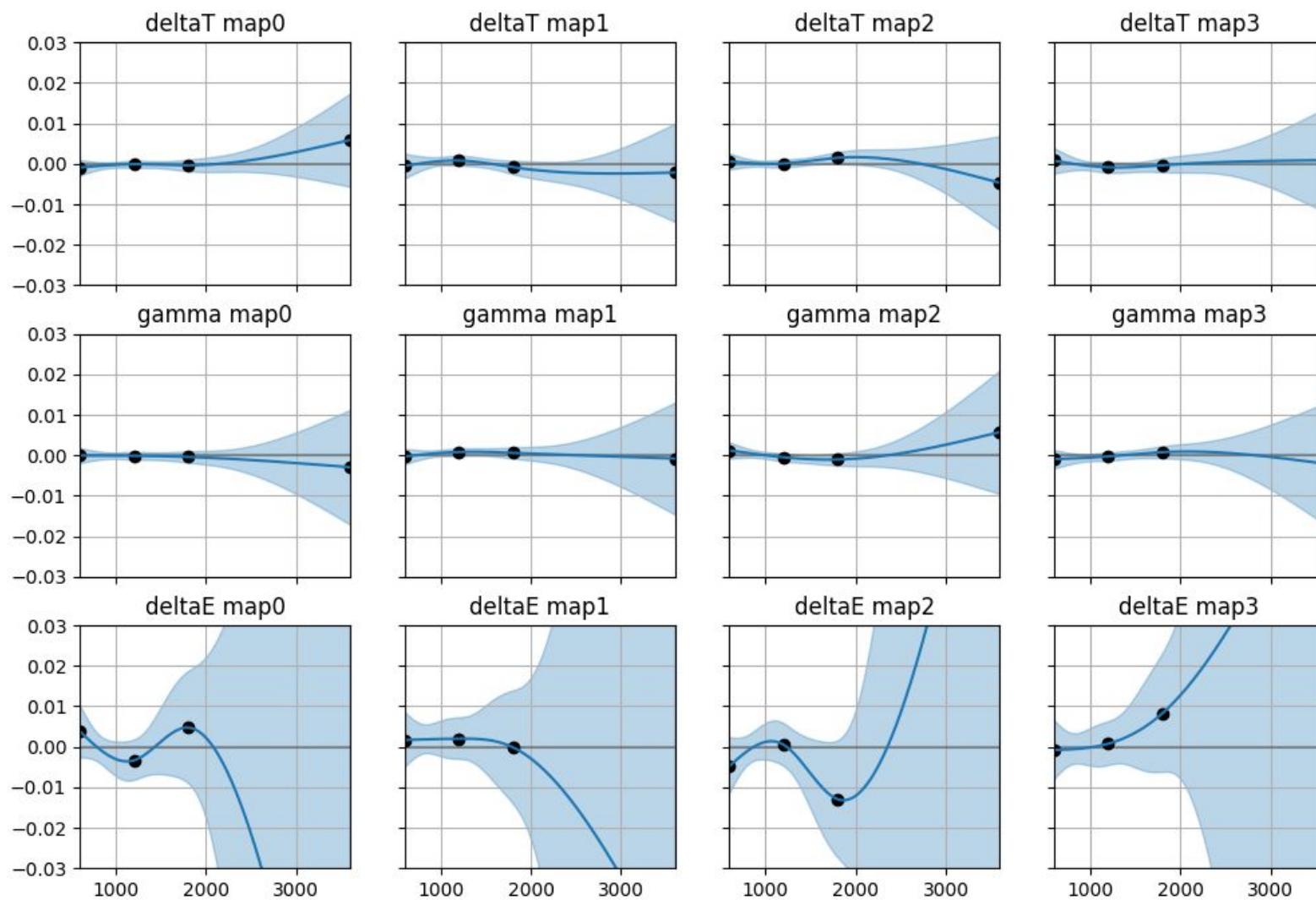
- mean:  $(\text{knts2spec} * \text{cov}^{-1} * \text{knts2spec.T})^{-1} * \text{knts2spec.T} * \text{cov}^{-1} * \mathbf{d}$
- covariance:  $(\text{knts2spec} * \text{cov}^{-1} * \text{knts2spec.T})^{-1}$

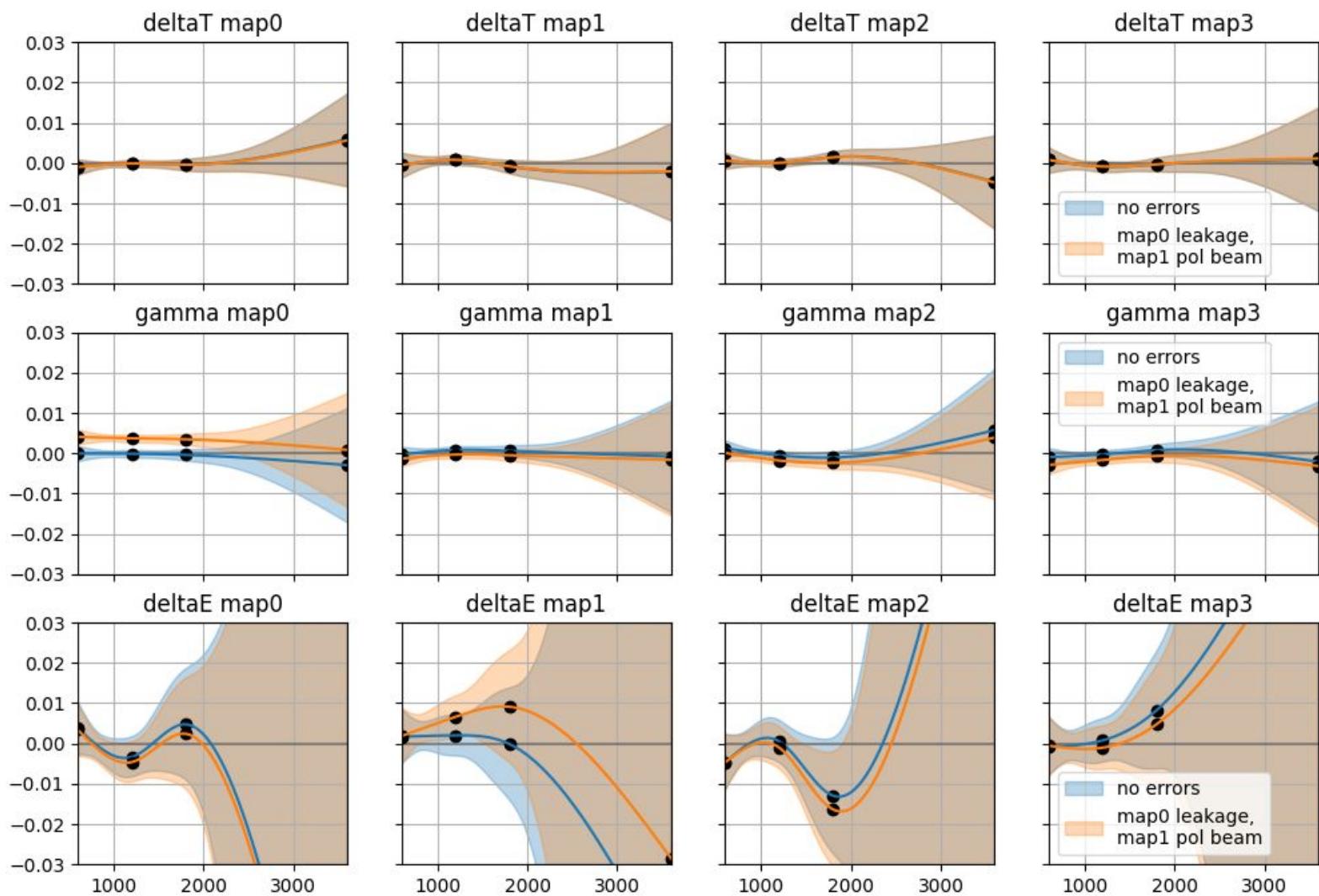
Fast, interpretable, low-dimensional!

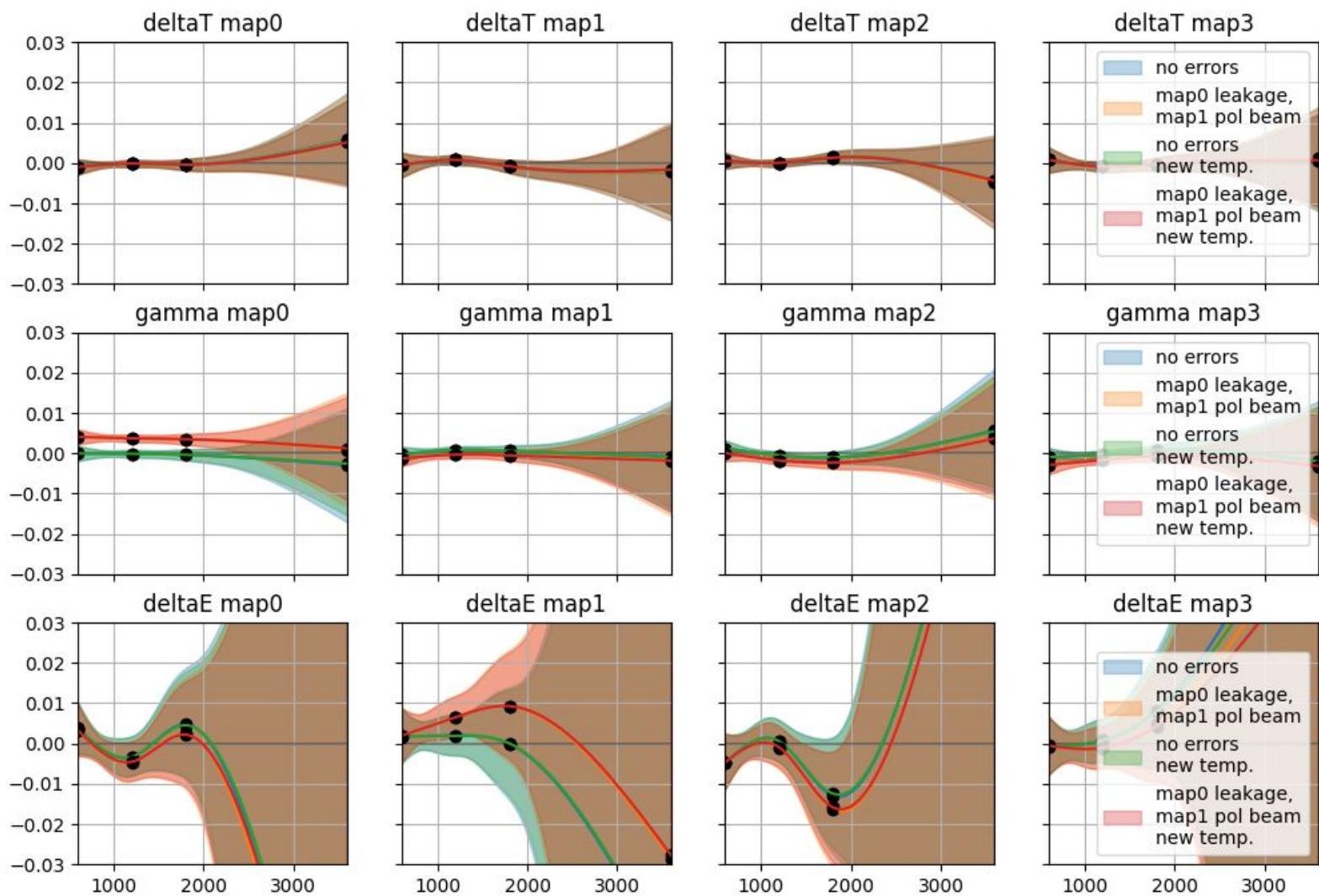
# Let's try it out

Setup:

- Borrow the ACT DR6 covariance matrix\*
- Draw 10,000 simulations with no errors and a “correct” cosmology template
- Repeat, add 0.5% leakage to one map and 1%/1000 ell pol beam error to another map in each simulation
- Repeat both with a “very wrong” cosmology template ( $\text{Cl} += 5\% / 1000 \text{ ell}$ )



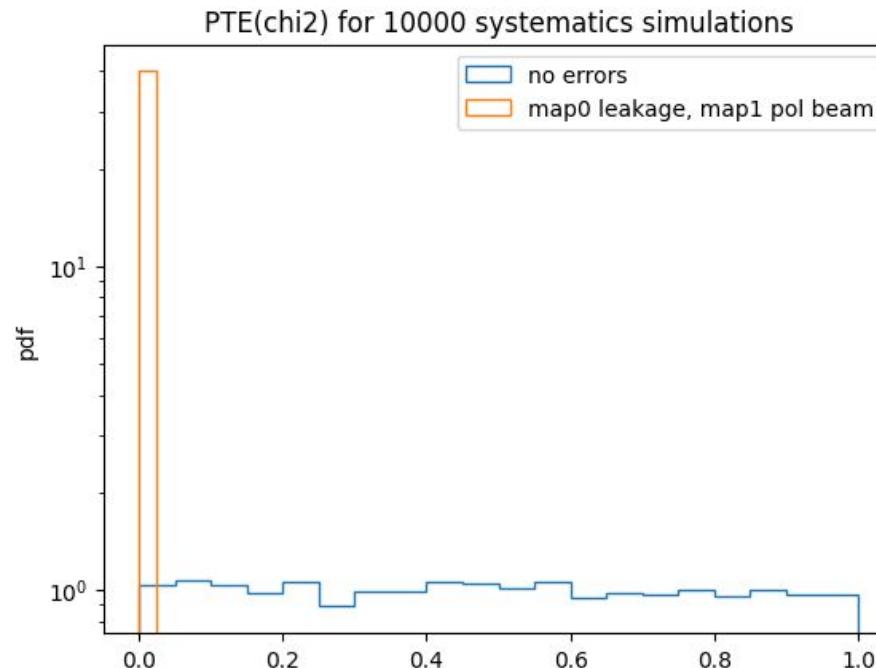




# Last note on this

Need to be careful with look-elsewhere effect when inspecting specific parameters!

“Global” PTE of the parameter vector still sensible though:



# Summary to end all summaries...

1. Current null test “paradigm” a bit contrived in my opinion, “best practices” not well studied
2. More work needed on modeling joint distribution of summary statistics
3. Interesting new pathways for null tests (map-level)

Null tests independent of cosmology/shared sky model

= Null tests **cannot** measure or rule-out systematics common to observation channels

Three important ways around this:

1. Calibration
2. Calibration
3. Calibration

# Backup

# A lot to discuss

Topic is open-ended (and under-studied)

Can't put all my thoughts in one talk, not sure what all my thoughts are

Excited to discuss more with anyone!

# Cubic splines as a matrix

This link <https://www.math.ntnu.no/emner/TMA4215/2008h/cubicsplines.pdf>

Combine slides 16 and 19

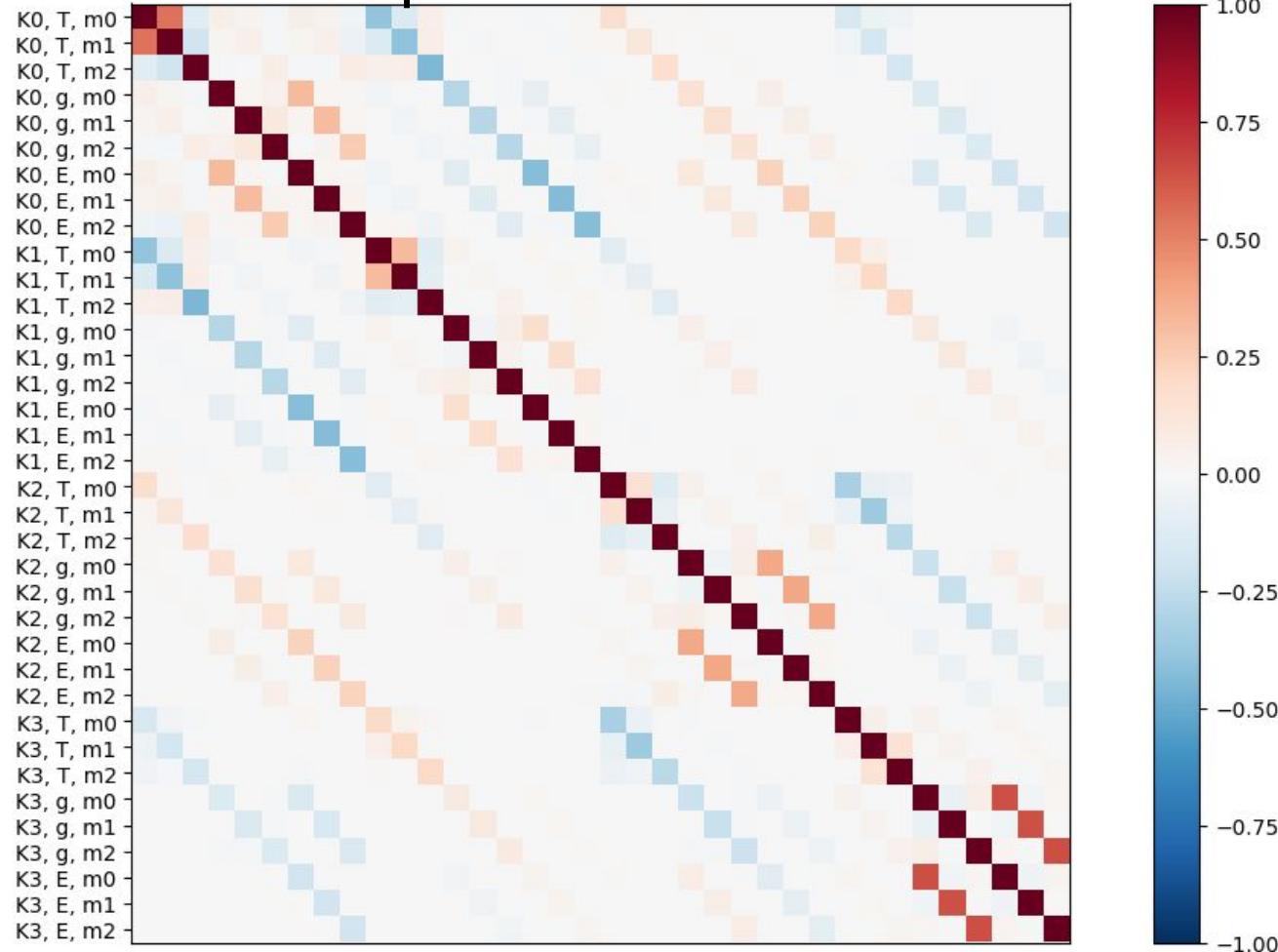
# The magic $N \times N - 1$ matrix

$\mathbf{C}_n \equiv \mathbf{I}_n - (1/n)\mathbf{J}_n$ , where  $\mathbf{J}_n$  is the  $n \times n$  matrix full of ones.

$\mathbf{C}_n \equiv \mathbf{P}_n \mathbf{E}_n \mathbf{P}_n^T$ : singular value corresponds to the null space of  $\mathbf{C}_n$ ,  $\vec{\mathbf{l}}_n$

define  $\mathbf{Z}_n$  instead: the matrix composed of the  $n - 1$  non-singular column vectors of  $\mathbf{P}_n$

## Correlation matrix of params



# SPT D1 equivalent to ACT DR6 null tests

