Welcome to Calculus BC!

Presentation slides made by itsdawei

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Review: Integration Rules

1. $\int f(x)dx = F(x) + C$ if and only if $F'(x) = f(x)$
2. $\int af(x)dx = a\int f(x)dx$
3. $\int -f(x)dx = -\int f(x)dx$
4. $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$

REMARK

$$\int f(x) \cdot g(x)dx \neq \int f(x)dx \cdot \int g(x)dx$$

BAD EXAMPLE

(32) Evaluate $\int x^2 \cos x dx$.

$$\int x^2 \cdot \int \cos x dx = \frac{x^3}{3} \cdot \sin x + C$$

The U-Substitution Method

The Chain Rule for Differentiation

$$\frac{d}{dx}F(g(x)) = f(g(x))g'(x), \text{ where } F' = f$$

The Integral of a Composite Function

$$\int f(g(x))g'(x)dx = F(g(x)) + C$$

Making a U-Substitution

Let $u = g(x)$, then $du = g'(x)dx$

$$\int f(g(x))g'(x)dx = \int f(u)du = F(u) + C = F(g(x)) + C$$

Procedure for U-Substitution

1. Given $f(g(x))$; let $u = g(x)$.
2. Differentiate: $du = g'(x)dx$.
3. Rewrite the integral in terms of u.
4. Evaluate the integral.
5. Replace u by $g(x)$.
6. Check your result by taking the derivative of the answer.

Try it yourself

(29) If $f(x)$ is an antiderivative of $\frac{e^x}{e^x+1}$ and $f(0) = \ln(2)$, find $f(\ln 2)$.

Evaluate $\int \frac{x^2}{(x^3-8)^5}dx$.

Evaluate $\int 3(\sec^2 x)\sqrt{\tan x}dx$

Evaluate $\int 2x^2 \cos(x^3)dx$

Integration by Parts

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$
$$uv = \int udv + \int vdu$$
$$\int udv = uv - \int vdu$$

How to Choose u and dv? (LIPET)

- **L**ogarithmic
- **I**nverse Trig.
- **P**olynomial
- **E**xponential
- **T**rig.

Try it yourself

$\int xe^{-x}dx$

Let $u = x$ and $dv = e^{-x}dx$ since x is a **P**olynomial, which comes before **E**xponential in **LIPET**.

$$\int xe^{-x}dx = -xe^{-x} - \int -e^{-x}dx = -xe^{-x} - e^{-x} + C$$

$\int x\sin(4x)dx$

Let $u = x$ and $dv = \sin(4x)dx$, since x is a **P**olynomial, which comes before **T**rig in **LIPET**.

$$\int x\sin(4x)dx = \frac{-x}{4}\cos(4x) + \frac{1}{4}\int \cos(4x)dx = \frac{-x}{4}\cos 4x + \frac{1}{16}\sin(4x) + C$$

Integration by Partial Fractions

$$\int \frac{dx}{x^2+3x-4}$$

$$\int \frac{x^5+2x^2+1}{x^3-x}$$

Integration Practice

1. Evaluate $\int \frac{1}{x^2} dx$.
2. Evaluate $\int \frac{x^3-1}{x} dx$.
3. Evaluate $\int x\sqrt{x^2 - 1}dx$.
4. Evaluate $\int \sin x dx$
5. Evaluate $\int \cos(2x) dx$
6. Evaluate $\int \frac{\ln x}{x} dx$
7. Evaluate $\int xe^{x^2} dx$
8. $\int x \cos x dx$
9. $\int \frac{5}{(x+3)(x-7)} dx$

Evaluate $\int \frac{1}{x^2} dx$.

Rewrite as $\int x^{-2} dx = \frac{x^{-1}}{-1} + C = -\frac{1}{x} + C$.

Evaluate $\int \frac{x^3 - 1}{x} dx$.

Rewrite as $\int (x^2 - \frac{1}{x}) dx = \frac{x^3}{3} - \ln|x| + C$.

Evaluate $\int x\sqrt{x^2 - 1} dx$.

Rewrite as $\int x(x^2 - 1)^{1/2} dx$. Let $u = x^2 - 1$.

Thus, $\frac{du}{2} = xdx \to \frac{1}{2} \int u^{1/2} du = \frac{u^{3/2}}{2^{3/2}} + C = \frac{1}{3}(x^2 - 1)^{3/2} + C$.

Evaluate $\int \sin x dx$

$-\cos x + C$

Evaluate $\int \cos(2x) dx$

Let $u = 2x$ and obtain $\frac{1}{2} \sin 2x + C$.

Evaluate $\int \frac{\ln x}{x} dx$

Let $u = \ln x; du = \frac{1}{x} dx$ and obtain $\frac{(\ln x)^2}{2} + C$.

Evaluate $\int xe^{x^2}dx$

Let $u = x^2$; $\frac{du}{2} = xdx$ and obtain $\frac{e^{x^2}}{2} + C$

$\int x \cos xdx$

Let $u = x$, $du = dx$, $dv = \cos xdx$, and $v = \sin x$,

then $\int x \cos xdx = x \sin x - \int \sin xdx = x \sin x + \cos x + C$

$\int \frac{5}{(x+3)(x-7)}dx$

Rewrite as $\int (\frac{-1/2}{x+3} + \frac{1/2}{x-7})$, then solve:

$$\begin{aligned}\int (\frac{-1/2}{x+3} + \frac{1/2}{x-7}) &= -\frac{1}{2}\ln|x+3| + \frac{1}{2}\ln|x-7| + C \\ &= \frac{1}{2}\ln|\frac{x-7}{x+3}| + C\end{aligned}$$

Homework

Practice Problems 1 to 25