## Содержание

## final/template/vimrc.txt

```
map <F9> :wall! <CR> :!g++ -Wall -Wextra -Wshadow -Wno-unused-result -o %:r % -std=c++14 -DHOME -D_GLIBCXX_DEBUG -fsanitize=address <CR>
map <F7> :wall! <CR> :!g++ -Wall -Wextra -Wshadow -Wno-unused-result -o %:r % -std=c++14 -DHOME -O2 <CR>
map <F8> :wall! <CR> :!ulimit -s 500000 && ./%:r <CR>

inoremap {<CR> {<CR>}<ESC>O
map <c-a> ggVG

set nu
set rnu
syntax on

map <c-t> :tabnew <CR>
map <c-l> :tabn <CR>
map <c-h> :tabp <CR>

set cin
set sw=4
set so=99
set bs=2
set et
set sts=4
```

## final/template/template.cpp

```cpp
// team : SPb ITMO University 1
#include <bits/stdc++.h>

#define F first
#define S second
#define pb push_back
#define forn(i, n) for(int i = 0 ; (i) < (n) ; ++i)
#define eprintf(...) fprintf(stderr, __VA_ARGS__),fflush(stderr)
#define sz(a) ((int)(a).size())
#define all(a) (a).begin(),a.end()
#define pw(x) (1LL<<(x))

using namespace std;

typedef long long ll;
typedef double dbl;
typedef vector<int> vi;
typedef pair<int, int> pi;

const int INF = 1.01e9;
const dbl eps = 1e-9;

/* --- main part --- */







int main()
{
#define TASK ""
#ifdef home
  assert(freopen(TASK".in", "r", stdin));
  //assert(freopen(TASK".out", "w", stdout));
#endif





#ifdef home
  eprintf("time = %d ms\n", (int)(clock() * 1000. / CLOCKS_PER_SEC));
#endif
  return 0;
}
```

## final/template/fastIO.cpp

```cpp
#include <cstdio>
#include <algorithm>

/** Interface */

inline int readInt();
inline int readUInt();
inline bool isEof();

/** Read */

static const int buf_size = 100000;
static char buf[buf_size];
static int buf_len = 0, pos = 0;

inline bool isEof() {
  if (pos == buf_len) {
    pos = 0, buf_len = fread(buf, 1, buf_size, stdin);
    if (pos == buf_len) return 1;
  }
  return 0;
}

inline int getChar() { return isEof() ? -1 : buf[pos++]; }

inline int readChar() {
  int c = getChar();
  while (c != -1 && c <= 32) c = getChar();
  return c;
}

inline int readUInt() {
  int c = readChar(), x = 0;
  while ('0' <= c && c <= '9') x = x * 10 + c - '0', c = getChar();
  return x;
}

inline int readInt() {
  int s = 1, c = readChar();
  int x = 0;
  if (c == '-') s = -1, c = getChar();
  while ('0' <= c && c <= '9') x = x * 10 + c - '0', c = getChar();
  return s == 1 ? x : -x;
}


// 10M int [0..1e9)
// cin 3.02
// scanf 1.2
// cin sync_with_stdio(false) 0.71
// fastRead getchar 0.53
// fastRead fread 0.15
```

## final/template/hashTable.cpp

```cpp
template <const int max_size, class HashType, class Data, const Data default_value>
struct hashTable {
  HashType hash[max_size];
  Data f[max_size];
  int size;

  int position(HashType H ) const {
    int i = H % max_size;
    while (hash[i] && hash[i] != H)
      if (++i == max_size)
        i = 0;
    return i;
  }

  Data & operator [] (HashType H ) {
    assert(H != 0);
    int i = position(H);
    if (!hash[i]) {
      hash[i] = H;
      f[i] = default_value;
      size++;
    }
    return f[i];
  }
};

hashTable<13, int, int, 0> h;

#include "ext/pb_ds/assoc_container.hpp"
using namespace __gnu_pbds;

template <typename T> using ordered_set = tree<T, null_type, less<T>, rb_tree_tag, tree_order_statistics_node_update>;
template <typename K, typename V> using ordered_map = tree<K, V, less<K>, rb_tree_tag, tree_order_statistics_node_update>;

// HOW TO USE ::
// -- order_of_key(10) returns the number of elements in set/map strictly less than 10
// -- *find_by_order(10) returns 10-th smallest element in set/map (0-based)
```

## final/template/optimizations.cpp

```cpp
// from anta code http://codeforces.com/contest/755/submission/23864531

#pragma GCC optimize ("O3")
#pragma GCC target ("sse4")
inline void fasterLLDivMod(unsigned long long x, unsigned y, unsigned &out_d, unsigned &out_m) {
  unsigned xh = (unsigned)(x >> 32), xl = (unsigned)x, d, m;
#ifdef __GNUC__
  asm(
    "divl %4; \n\t"
    : "=a" (d), "=d" (m)
    : "d" (xh), "a" (xl), "r" (y)
  );
#else
  __asm {
    mov edx, dword ptr[xh];
    mov eax, dword ptr[xl];
    div dword ptr[y];
    mov dword ptr[d], eax;
    mov dword ptr[m], edx;
  };
#endif
  out_d = d; out_m = m;
}


// have no idea what sse flags are really cool; list of some of them
// -- very good with bitsets
#pragma GCC optimize("O3")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4,popcnt,abm,mmx")
```

## final/template/Template.java

```java
import java.util.*;
import java.io.*;

public class Template {
  FastScanner in;
  PrintWriter out;

  public void solve() throws IOException {
    int n = in.nextInt();
    out.println(n);
  }

  public void run() {
    try {
      in = new FastScanner();
      out = new PrintWriter(System.out);

      solve();

      out.close();
    } catch (IOException e) {
      e.printStackTrace();
    }
  }

  class FastScanner {
    BufferedReader br;
    StringTokenizer st;

    FastScanner() {
      br = new BufferedReader(new InputStreamReader(System.in));
    }

    String next() {
      while (st == null || !st.hasMoreTokens()) {
        try {
          st = new StringTokenizer(br.readLine());
        } catch (IOException e) {
          e.printStackTrace();
        }
      }
      return st.nextToken();
    }

    int nextInt() {
      return Integer.parseInt(next());
    }
  }

  public static void main(String[] arg) {
    new Template().run();
  }
}
```

## final/numeric/fft.cpp

```cpp
namespace fft
{
  const int maxBase = 21;
  const int maxN = 1 << maxBase;

  struct num
  {
    dbl x, y;
    num(){}
    num(dbl xx, dbl yy): x(xx), y(yy) {}
    num(dbl alp): x(cos(alp)), y(sin(alp)) {}
  };

  inline num operator + (num a, num b) { return num(a.x + b.x, a.y + b.y); }
  inline num operator - (num a, num b) { return num(a.x - b.x, a.y - b.y); }
  inline num operator * (num a, num b) { return num(a.x * b.x - a.y * b.y, a.x * b.y + a.y * b.x); }
  inline num conj(num a) { return num(a.x, -a.y); }

  const dbl PI = acos(-1);

  num root[maxN];
  int rev[maxN];
  bool rootsPrepared = false;

  void prepRoots()
  {
    if (rootsPrepared) return;
    rootsPrepared = true;
    root[1] = num(1, 0);
    for (int k = 1; k < maxBase; ++k)
    {
      num x(2 * PI / pw(k + 1));
      for (int i = pw(k - 1); i < pw(k); ++i)
      {
        root[2 * i] = root[i];
        root[2 * i + 1] = root[i] * x;
      }
    }
  }

  int base, N;

  int lastRevN = -1;
  void prepRev()
  {
    if (lastRevN == N) return;
    lastRevN = N;
    forn(i, N) rev[i] = (rev[i >> 1] >> 1) + ((i & 1) << (base - 1));
  }

  void fft(num *a, num *f)
  {
    forn(i, N) f[i] = a[rev[i]];
    for (int k = 1; k < N; k <<= 1) for (int i = 0; i < N; i += 2 * k) forn(j, k)
    {
      num z = f[i + j + k] * root[j + k];
      f[i + j + k] = f[i + j] - z;
      f[i + j] = f[i + j] + z;
    }
  }

  num a[maxN], b[maxN], f[maxN], g[maxN];
  ll A[maxN], B[maxN], C[maxN];

  void _multMod(int mod)
  {
    forn(i, N)
    {
      int x = A[i] % mod;
      a[i] = num(x & (pw(15) - 1), x >> 15);
    }
    forn(i, N)
    {
      int x = B[i] % mod;
      b[i] = num(x & (pw(15) - 1), x >> 15);
    }
    fft(a, f);
    fft(b, g);

    forn(i, N)
    {
      int j = (N - i) & (N - 1);
      num a1 = (f[i] + conj(f[j])) * num(0.5, 0);
      num a2 = (f[i] - conj(f[j])) * num(0, -0.5);
      num b1 = (g[i] + conj(g[j])) * num(0.5 / N, 0);
      num b2 = (g[i] - conj(g[j])) * num(0, -0.5 / N);
      a[j] = a1 * b1 + a2 * b2 * num(0, 1);
      b[j] = a1 * b2 + a2 * b1;
    }

    fft(a, f);
    fft(b, g);

    forn(i, N)
    {
      ll aa = f[i].x + 0.5;
      ll bb = g[i].x + 0.5;
      ll cc = f[i].y + 0.5;
      C[i] = (aa + bb % mod * pw(15) + cc % mod * pw(30)) % mod;
    }
  }

  void prepAB(int n1, int n2)
  {
    base = 1;
    N = 2;
    while (N < n1 + n2) base++, N <<= 1;

    for (int i = n1; i < N; ++i) A[i] = 0;
    for (int i = n2; i < N; ++i) B[i] = 0;

    prepRoots();
    prepRev();
  }

  void mult(int n1, int n2)
  {
    prepAB(n1, n2);
    forn(i, N) a[i] = num(A[i], B[i]);
    fft(a, f);
    forn(i, N)
    {
      int j = (N - i) & (N - 1);
      a[i] = (f[j] * f[j] - conj(f[i] * f[i])) * num(0, -0.25 / N);
    }
    fft(a, f);
    forn(i, N) C[i] = (ll)round(f[i].x);
  }


  void multMod(int n1, int n2, int mod)
  {
    prepAB(n1, n2);
    _multMod(mod);
  }

  int D[maxN];

  void multLL(int n1, int n2)
  {
    prepAB(n1, n2);

    int mod1 = 1.5e9;
    int mod2 = mod1 + 1;

    _multMod(mod1);

    forn(i, N) D[i] = C[i];

    _multMod(mod2);

    forn(i, N)
    {
      C[i] = D[i] + (C[i] - D[i] + (ll)mod2) * (ll)mod1 % mod2 * mod1;
    }
  }
  // HOW TO USE ::
  // -- set correct maxBase
  // -- use mult(n1, n2), multMod(n1, n2, mod) and multLL(n1, n2)
  // -- input : A[], B[]
  // -- output : C[]
}
```

## final/numeric/fftint.cpp

```cpp
namespace fft
{
  const int mod = 998244353;
  const int base = 20;
  const int N = 1 << base;
  const int ROOT = 646;

  int root[N];
  int rev[N];

  void init()
  {
    forn(i, N) rev[i] = (rev[i >> 1] >> 1) + ((i & 1) << (base - 1));
    int NN = N >> 1;
    int z = 1;
    forn(i, NN)
    {
      root[i + NN] = z;
      z = z * (ll)ROOT % mod;
    }
    for (int i = NN - 1; i > 0; --i) root[i] = root[2 * i];
  }

  void fft(int *a, int *f)
  {
    forn(i, N) f[i] = a[rev[i]];
    for (int k = 1; k < N; k <<= 1) for (int i = 0; i < N; i += 2 * k) forn(j, k)
    {
      int z = f[i + j + k] * (ll)root[j + k] % mod;
      f[i + j + k] = (f[i + j] - z + mod) % mod;
      f[i + j] = (f[i + j] + z) % mod;
    }
  }

  int A[N], B[N], C[N];
  int F[N], G[N];

  void _mult(int eq)
  {
    fft(A, F);
    if (eq) forn(i, N) G[i] = F[i];
    else fft(B, G);
    int invN = inv(N);
    forn(i, N) A[i] = F[i] * (ll)G[i] % mod * invN % mod;
    reverse(A + 1, A + N);
    fft(A, C);
  }

  void mult(int n1, int n2, int eq = 0)
  {
    for (int i = n1; i < N; ++i) A[i] = 0;
    for (int i = n2; i < N; ++i) B[i] = 0;

    _mult(eq);

    //forn(i, n1 + n2) C[i] = 0;
    //forn(i, n1) forn(j, n2) C[i + j] = (C[i + j] + A[i] * (ll)B[j]) % mod;
  }
}
```

## final/numeric/blackbox.cpp

```cpp
namespace blackbox
{
  int A[N];
  int B[N];
  int C[N];

  int magic(int k, int x)
  {
    B[k] = x;
    C[k] = (C[k] + A[0] * (ll)B[k]) % mod;
    int z = 1;
    if (k == N - 1) return C[k];
    while ((k & (z - 1)) == (z - 1))
    {
      //mult B[k - z + 1 ... k] x A[z .. 2 * z - 1]
      forn(i, z) fft::A[i] = A[z + i];
      forn(i, z) fft::B[i] = B[k - z + 1 + i];
      fft::multMod(z, z, mod);
      forn(i, 2 * z - 1) C[k + 1 + i] = (C[k + 1 + i] + fft::C[i]) % mod;
      z <<= 1;
    }
    return C[k];
  }
  // A -- constant array
  // magic(k, x):: B[k] = x, returns C[k]
  // !! WARNING !! better to set N twice the size needed
}
```

## final/numeric/crt.cpp

```cpp
int rev(int x, int m)
{
  if (x == 1) return 1;
  return (1 - rev(m % x, x) * (ll)m) / x + m;
}

int CRT(int a1, int m1, int a2, int m2)
{
  return (a1 - a2 % m1 + m1) * (ll)rev(m2, m1) % m1 * m2 + a2;
}
```

## final/numeric/pollard.cpp

```cpp
namespace pollard
{
  using math::p;

  vector<pair<ll, int>> getFactors(ll N)
  {
    vector<ll> primes;

    const int MX = 1e5;
    const ll MX2 = MX * (ll)MX;

    assert(MX <= math::maxP && math::pc > 0);

    function<void(ll)> go = [&go, &primes](ll n)
    {
      for (ll x : primes) while (n % x == 0) n /= x;
      if (n == 1) return;
      if (n > MX2)
      {
        auto F = [&](ll x) {
          ll k = ((long double)x * x) / n;
          ll r = (x * x - k * n + 3) % n;
          return r < 0 ? r + n : r;
        };
        ll x = mt19937_64()() % n, y = x;
        const int C = 3 * pow(n, 0.25);

        ll val = 1;
        forn(it, C)
        {
          x = F(x), y = F(F(y));
          if (x == y) continue;
          ll delta = abs(x - y);
          ll k = ((long double)val * delta) / n;
          val = (val * delta - k * n) % n;
          if (val < 0) val += n;
          if (val == 0)
          {
            ll g = __gcd(delta, n);
            go(g), go(n / g);
            return;
          }
          if ((it & 255) == 0)
          {
            ll g = __gcd(val, n);
            if (g != 1)
            {
              go(g), go(n / g);
              return;
            }
          }
        }
      }
      primes.pb(n);
    };

    ll n = N;

    for (int i = 0; i < math::pc && p[i] < MX; ++i) if (n % p[i] == 0)
    {
      primes.pb(p[i]);
      while (n % p[i] == 0) n /= p[i];
    }

    go(n);

    sort(primes.begin(), primes.end());

    vector<pair<ll, int>> res;
    for (ll x : primes)
    {
      int cnt = 0;
      while (N % x == 0)
      {
        cnt++;
        N /= x;
      }
      res.push_back({x, cnt});
    }
    return res;
  }
}
```

## final/numeric/poly.cpp

```cpp
struct poly
{
  vi v;
  poly() {}
  poly(vi vv)
  {
    v = vv;
  }
  int size()
  {
    return (int)v.size();
  }
  poly cut(int maxLen)
  {
    if (maxLen < sz(v)) v.resize(maxLen);
    return *this;
  }
  poly norm()
  {
    while (sz(v) > 1 && v.back() == 0) v.pop_back();
    return *this;
  }
  inline int& operator [] (int i)
  {
    return v[i];
  }
  void out(string name="")
  {
    stringstream ss;
    if (sz(name)) ss << name << "=";
    int fst = 1;
    forn(i, sz(v)) if (v[i])
    {
      int x = v[i];
      int sgn = 1;
      if (x > mod / 2) x = mod-x, sgn = -1;
      if (sgn == -1) ss << "-";
      else if (!fst) ss << "+";
      fst = 0;
      if (!i || x != 1)
      {
        ss << x;
        if (i > 0) ss << "*x";
        if (i > 1) ss << "^" << i;
      }
      else
      {
        ss << "x";
        if (i > 1) ss << "^" << i;
      }
    }
    if (fst) ss <<"0";
    string s;
    ss >> s;
    eprintf("%s\n", s.data());
  }
};

poly operator + (poly A, poly B)
{
  poly C;
  C.v = vi(max(sz(A), sz(B)));
  forn(i, sz(C))
  {
    if (i < sz(A)) C[i] = (C[i] + A[i]) % mod;
    if (i < sz(B)) C[i] = (C[i] + B[i]) % mod;
  }
  return C.norm();
}

poly operator - (poly A, poly B)
{
  poly C;
  C.v = vi(max(sz(A), sz(B)));
  forn(i, sz(C))
  {
    if (i < sz(A)) C[i] = (C[i] + A[i]) % mod;
    if (i < sz(B)) C[i] = (C[i] + mod - B[i]) % mod;
  }
  return C.norm();
}

poly operator * (poly A, poly B)
{
  poly C;
  C.v = vi(sz(A) + sz(B) - 1);

  forn(i, sz(A)) fft::A[i] = A[i];
  forn(i, sz(B)) fft::B[i] = B[i];
  fft::multMod(sz(A), sz(B), mod);
  forn(i, sz(C)) C[i] = fft::C[i];
  return C.norm();
}

poly inv(poly A, int n) // returns A^-1 mod x^n
{
  assert(sz(A) && A[0] != 0);
  A.cut(n);

  auto cutPoly = [](poly &from, int l, int r)
  {
    poly R;
    R.v.resize(r - l);
    for (int i = l; i < r; ++i)
    {
      if (i < sz(from)) R[i - l] = from[i];
    }
    return R;
  };

  function<int(int, int)> rev = [&rev](int x, int m)->int
  {
    if (x == 1) return 1;
    return (1 - rev(m % x, x) * (ll)m) / x + m;
  };

  poly R({rev(A[0], mod)});
  for (int k = 1; k < n; k <<= 1)
  {
    poly A0 = cutPoly(A, 0, k);
    poly A1 = cutPoly(A, k, 2 * k);
    poly H = A0 * R;
    H = cutPoly(H, k, 2 * k);
    poly R1 = (((A1 * R).cut(k) + H) * (poly({0}) - R)).cut(k);
    R.v.resize(2 * k);
    forn(i, k) R[i + k] = R1[i];
  }
  return R.cut(n).norm();
}

pair<poly, poly> divide(poly A, poly B)
{
  if (sz(A) < sz(B)) return {poly({0}), A};

  auto rev = [](poly f)
  {
    reverse(all(f.v));
    return f;
  };

  poly q = rev((inv(rev(B), sz(A) - sz(B) + 1) * rev(A)).cut(sz(A) - sz(B) + 1));
  poly r = A - B * q;

  return {q, r};
}
```

## final/numeric/simplex.cpp

```cpp

namespace simplex {
  const int MAX_N = -1; // number of variables
  const int MAX_M = -1; // number of inequalities
  dbl a[MAX_M][MAX_N];
  dbl b[MAX_M];
  dbl c[MAX_N];
  dbl v;
  ll n, m;
  int left[MAX_M];
  int up[MAX_N];
  int pos[MAX_N];
  dbl res[MAX_N];

  void init(int nn, int mm) {
    n = nn;
    m = mm;
    v = 0;
    for (int i = 0; i < m; i++)
      for (int j = 0; j < n; j++)
        a[i][j] = 0;
    for (int i = 0; i < m; i++)
      b[i] = 0;
    for (int i = 0; i < n; i++)
      c[i] = 0;
  }

  void pivot(int x, int y) {
    swap(left[x], up[y]);
    dbl k = a[x][y];
    a[x][y] = 1;
    b[x] /= k;
    int cur = 0;
    for (int i = 0; i < n; i++) {
      a[x][i] = a[x][i] / k;
      if (!eq(a[x][i], 0))
        pos[cur++] = i;
    }

    for (int i = 0; i < m; i++) {
      if (i == x || eq(a[i][y], 0)) continue;
      dbl cof = a[i][y];
      b[i] -= cof * b[x];
      a[i][y] = 0;
      for (int j = 0; j < cur; j++)
        a[i][pos[j]] -= cof * a[x][pos[j]];
    }
    dbl cof = c[y];
    v += cof * b[x];
    c[y] = 0;
    for (int i = 0; i < cur; i++) {
      c[pos[i]] -= cof * a[x][pos[i]];
    }
  }

  void solve() {
    for (int i = 0; i < n; i++)
      up[i] = i;
    for (int i = 0; i < m; i++)
      left[i] = i + n;

    while (1) {
      int x = -1;
      for (int i = 0; i < m; i++)
        if (ls(b[i], 0) && (x == -1 || b[i] < b[x]))
     {
          x = i;
        }
      if (x == -1) break;
      int y = -1;
      for (int j = 0; j < n; j++)
        if (ls(a[x][j], 0)) {
          y = j;
          break;
        }
      if (y == -1) {
        assert(false); // no solution
      }
      pivot(x, y);
    }
    while (1) {
      int y = -1;
      for (int i = 0; i < n; i++)
        if (ls(0, c[i]) && (y == -1 || (c[i] > c[y]))) {
          y = i;
        }
      if (y == -1) break;
      int x = -1;
      for (int i = 0; i < m; i++) {
        if (ls(0, a[i][y])) {
          if (x == -1 || (b[i] / a[i][y] < b[x] / a[x][y])) {
            x = i;
          }
        }
      }
      if (y == -1) {
        assert(false); // infinite solution
      }
      pivot(x, y);
    }

    memset(res, 0, sizeof(res));

    for (int i = 0; i < m; i++) {
      if (left[i] < n) {
        res[left[i]] = b[i];
      }
    }
  }
  // HOW TO USE ::
  // -- call init(n, m)
  // -- call solve()
  // -- variables in "up" equals to zero
  // -- variables in "left" equals to b
  // -- max: c * x
  // -- b[i] >= a[i] * x
  // -- answer in "v"
  // -- sertificate in "res"
};
```

## final/geom/commonTangents.cpp

```


vector<Line> commonTangents(pt A, dbl rA, pt B, dbl rB) {
  vector<Line> res;
  pt C = B - A;
  dbl z = C.len2();
  for (int i = -1; i <= 1; i += 2) {
    for (int j = -1; j <= 1; j += 2) {
      dbl r = rB * j - rA * i;
      dbl d = z - r * r;
      if (ls(d, 0)) continue;
      d = sqrt(max(0.01, d));
      pt magic = pt(r, d) / z;
      pt v(magic % C, magic * C);
      dbl CC = (rA * i - v % A) / v.len2();
      pt O = v * -CC;
      res.pb(Line(O, O + v.rotate()));
    }
  }
  return res;
}

// HOW TO USE ::
// --   *D*----------------*F*
// --   *...*-          -*...*
// --  *.....* -      - *.....*
// --  *.......* -  - *.......*
// --  *...A...*  --  *...B...*
// --  *.......* -  - *.......*
// --  *.....* -      - *.....*
// --   *...*-          -*...*
// --   *C*----------------*E*
// -- res = {CE, CF, DE, DF}
```

## final/geom/halfplaneIntersection.cpp

```
int getPart(pt v) {
  return less(0, v.y) || (equal(0, v.y) && less(v.x, 0));
}

int cmpV(pt a, pt b) {
  int partA = getPart(a);
  int partB = getPart(b);
  if (partA < partB) return -1;
  if (partA > partB) return 1;
  if (equal(0, a * b)) return 0;
  if (0 < a * b) return -1;
  return 1;
}

double planeInt(vector<Line> l) {
  int n = l.size();
  sort(all(l), [](Line a, Line b) {
      int r = cmpV(a.v, b.v);
      if (r != 0) return r < 0;
      return a.O % a.v.rotate() < b.O % a.v.rotate();
    });

  int cur = 0;
  for (int i = 0; i < n; ) {
    int j = i;
    for (; i < n && cmpV(l[j].v, l[i].v) == 0 && cmpV(l[i].v, l[j].v) == 0; i++);
    l[cur++] = l[i - 1];
  }
  n = cur;

  for (int i = 0; i < n; i++)
    l[i].id = i;

  int flagUp = 0;
  int flagDown = 0;
  for (int i = 0; i < n; i++) {
    int part = getPart(l[i].v);
    if (part == 1) flagUp = 1;
    if (part == 0) flagDown = 1;
  }
  if (!flagUp || !flagDown) return -1;

  for (int i = 0; i < n; i++) {
    pt v = l[i].v;
    pt u = l[(i + 1) % n].v;
    if (equal(0, v * u) && less(v % u, 0)) {
      pt dir = l[i].v.rotate();
      if (lessE(l[(i + 1) % n].O % dir, l[i].O % dir↩
  )) return 0;
      return -1;
    }
    if (less(v * u, 0))
      return -1;
  }

  cur = 0;
  vector<Line> st(n * 2);
  for (int tt = 0; tt < 2; tt++) {
    for (int i = 0; i < n; i++) {
      for (; cur >= 2; cur--) {
        pt G = st[cur - 1] * l[i];
        if (!lessE(st[cur - 2].v * (G - st[cur - 2].↩
  O), 0))
          break;
      }
      st[cur++] = l[i];
      if (cur >= 2 && lessE(st[cur - 2].v * st[cur -↩
   1].v, 0)) return 0;
    }
  }
  vector < int > use(n, -1);
  int left = -1, right = -1;
  for (int i = 0; i < cur; i++) {
    if (use[st[i].id] == -1) {
      use[st[i].id] = i;
    }
    else {
      left = use[st[i].id];
      right = i;
      break;
    }
  }
  vector < Line > tmp;
  for (int i = left; i < right; i++)
    tmp.pb(st[i]);
  vector < pt > res;
  for (int i = 0; i < (int)tmp.size(); i++)
    res.pb(tmp[i] * tmp[(i + 1) % tmp.size()]);
  double area = 0;
  for (int i = 0; i < (int)res.size(); i++)
    area += res[i] * res[(i + 1) % res.size()];
  return area / 2;
}
```

## final/geom/minDisc.cpp

```

pair<pt, dbl> minDisc(vector<pt> p) {
  int n = p.size();
  pt O = pt(0, 0);
  dbl R = 0;
  random_shuffle(all(p));
  for (int i = 0; i < n; i++) {
    if (ls(R, (O - p[i]).len())) {
      O = p[i];
      R = 0;
      for (int j = 0; j < i; j++) {
        if (ls(R, (O - p[j]).len())) {
          O = (p[i] + p[j]) / 2;
          R = (p[i] - p[j]).len() / 2;
          for (int k = 0; k < j; k++) {
            if (ls(R, (O - p[k]).len())) {
              Line l1((p[i] + p[j]) / 2, (p[i] + p[j↩
  ]) / 2 + (p[i] - p[j]).rotate());
              Line l2((p[k] + p[j]) / 2, (p[k] + p[j↩
  ]) / 2 + (p[k] - p[j]).rotate());
              O = l1 * l2;
              R = (p[i] - O).len();
            }
          }
        }
      }
    }
  }
  return {O, R};
}
```

## final/geom/plane3DInt.cpp

```
//(A, v) * (B, u) -> (O, n)

pt n = v * u;
pt m = v * n;
double t = (B - A) % u / (u % m);
pt O = A - m * t;
```

## final/geom/polygonArcCut.cpp

```

struct Meta {
  int type; // 0 - seg, 1 - circle
  pt O;
  dbl R;
};

const Meta SEG = {0, pt(0, 0), 0};

vector<pair<pt, Meta>> cut(vector<pair<pt, Meta>> p,↩
     Line l) {
  vector<pair<pt,Meta>> res;
  int n = p.size();
  for (int i = 0; i < n; i++) {
    pt A = p[i].F;
    pt B = p[(i + 1) % n].F;
    if (le(0, l.v * (A - l.O))) {
      if (eq(0, l.v * (A - l.O)) && p[i].S.type == 1↩
   && ls(0, l.v % (p[i].S.O - A)))
        res.pb({A, SEG});
      else
        res.pb(p[i]);
    }
    if (p[i].S.type == 0) {
      if (sign(l.v * (A - l.O)) * sign(l.v * (B - l.↩
 O)) == -1) {
        pt FF = Line(A, B) * l;
        res.pb(make_pair(FF, SEG));
      }
    }
    else {
      pt E, F;
      if (intCL(p[i].S.O, p[i].S.R, l, E, F)) {
        if (onArc(p[i].S.O, A, E, B))
          res.pb({E, SEG});
        if (onArc(p[i].S.O, A, F, B))
          res.pb({F, p[i].S});
      }
    }
  }
  return res;
}
```

## final/strings/eertree.cpp

```cpp
namespace eertree {
  const int INF = 1e9;
  const int N = 5e6 + 10;
  char _s[N];
  char *s = _s + 1;
  int to[N][2];
  int suf[N], len[N];
  int sz, last;

  const int odd = 1, even = 2, blank = 3;

  void go(int &u, int pos) {
    while (u != blank && s[pos - len[u] - 1] != s[pos]) {
      u = suf[u];
    }
  }

  int add(int pos) {
    go(last, pos);
    int u = suf[last];
    go(u, pos);
    int c = s[pos] - 'a';
    int res = 0;
    if (!to[last][c]) {
      res = 1;
      to[last][c] = sz;
      len[sz] = len[last] + 2;
      suf[sz] = to[u][c];
      sz++;
    }
    last = to[last][c];
    return res;
  }

  void init() {
    to[blank][0] = to[blank][1] = even;
    len[blank] = suf[blank] = INF;
    len[even] = 0, suf[even] = odd;
    len[odd] = -1, suf[odd] = blank;
    last = even;
    sz = 4;
  }
}
```

## final/strings/sufAutomaton.cpp

```cpp
namespace SA {
  const int MAXN = 1 << 18;
  const int SIGMA = 26;

  int sz, last;
  int nxt[MAXN][SIGMA];
  int link[MAXN], len[MAXN], pos[MAXN];

  void init() {
    memset(nxt, -1, sizeof(nxt));
    memset(link, -1, sizeof(link));
    memset(len, 0, sizeof(len));
    last = 0;
    sz = 1;
  }

  void add(int c) {
    int cur = sz++;
    len[cur] = len[last] + 1;
    pos[cur] = len[cur];
    int p = last;
    last = cur;
    for (; p != -1 && nxt[p][c] == -1; p = link[p]) nxt[p][c] = cur;
    if (p == -1) {
      link[cur] = 0;
      return;
    }
    int q = nxt[p][c];
    if (len[p] + 1 == len[q]) {
      link[cur] = q;
      return;
    }
    int clone = sz++;
    memcpy(nxt[clone], nxt[q], sizeof(nxt[q]));
    len[clone] = len[p] + 1;
    pos[clone] = pos[q];
    link[clone] = link[q];
    link[q] = link[cur] = clone;
    for (; p != -1 && nxt[p][c] == q; p = link[p]) nxt[p][c] = clone;
  }

  int n;
  string s;
  int l[MAXN], r[MAXN];
  int e[MAXN][SIGMA];

  void getSufTree(string _s) {
    memset(e, -1, sizeof(e));
    s = _s;
    n = s.length();
    reverse(s.begin(), s.end());
    init();
    for (int i = 0; i < n; i++) add(s[i] - 'a');
    reverse(s.begin(), s.end());
    for (int i = 1; i < sz; i++) {
      int j = link[i];
      l[i] = n - pos[i] + len[j];
      r[i] = n - pos[i] + len[i];
      e[j][s[l[i]] - 'a'] = i;
    }
  }
}

namespace duval {
  string s;
  int n = (int) s.length();
  int i=0;
  while (i < n) {
    int j=i+1, k=i;
    while (j < n && s[k] <= s[j]) {
      if (s[k] < s[j])
        k = i;
      else
        ++k;
      ++j;
    }
    while (i <= k) {
      cout << s.substr (i, j-k) << ' ';
      i += j - k;
    }
  }
}
```

## final/graphs/centroid.cpp

```cpp
// original author: burunduk1, rewritten by me (enot110)
// !!! warning !!! this code is not tested well
const int N = 1e5, K = 17;

int pivot, level[N], parent[N];
vector<int> v[N];

int get_pivot( int x, int xx, int n ) {
  int size = 1;
  for (int y : v[x])
  {
    if (y != xx && level[y] == -1) size += get_pivot(y, x, n);
  }
  if (pivot == -1 && (size * 2 >= n || xx == -1)) pivot = x;
  return size;
}

void build( int x, int xx, int dep, int size ) {
  assert(dep < K);
  pivot = -1;
  get_pivot(x, -1, size);
  x = pivot;
  level[x] = dep, parent[x] = xx;
  for (int y : v[x]) if (level[y] == -1)
  {
    build(y, x, dep + 1, size / 2);
  }
}
```

## final/graphs/dinica.cpp

```cpp
namespace flow
{
  const int maxn = 1e5 + 10;
  const int maxe = 2 * maxn;

  int head[maxn], next[maxe], to[maxe], f[maxe], ec = 1;
  int ST, EN, N = maxn;

  inline void setN(int n)
  {
    ST = n;
    EN = n + 1;
    N = n + 2;
  }

  inline void _add(int x, int y, int ff)
  {
    ++ec;
    to[ec] = y;
    next[ec] = head[x];
    head[x] = ec;
    f[ec] = ff;
  }

  inline int add(int x, int y, int ff)
  {
    _add(x, y, ff);
    _add(y, x, 0);
    return ec - 1;
  }

  void clear()
  {
    forn(i, N) head[i] = 0;
    ec = 1;
  }

  int d[maxn];
  int q[maxn], st = 0, en = 0;

  int bfs()
  {
    forn(i, N) d[i] = 1e9;
    st = 0, en = 0;
    d[ST] = 0;
    q[en++] = ST;
    while (st < en)
    {
      int x = q[st++];
      if (x == EN) return 1;
      for (int e = head[x]; e; e = next[e])
      {
        int y = to[e];
        if (d[y] == 1e9 && f[e])
        {
          d[y] = d[x] + 1;
          q[en++] = y;
        }
      }
    }
    return 0;
  }

  int pushed;
  int fst[maxn];

  int dfs(int x, int flow = 1e9)
  {
    if (x == EN)
    {
      pushed = flow;
      return 1;
    }
    for (; fst[x]; fst[x] = next[fst[x]])
    {
      int e = fst[x];
      int y = to[e];
      if (d[y] == d[x] + 1 && f[e] && dfs(y, min(f[e], flow)))
      {
        f[e] -= pushed;
        f[e ^ 1] += pushed;
        return 1;
      }
    }
    return 0;
  }


  ll calcFlow()
  {
    ll res = 0;
    while (bfs())
    {
      forn(i, N) fst[i] = head[i];
      while (dfs(ST))
      {
        res += pushed;
      }
    }
    return res;
  }

  // HOW TO USE ::
  // -- set maxn and maxe (special for izban)
  // -- add adges using add(x, y, f), call setN(n)
  // -- run calcFlow
}
```

## final/graphs/dominatorTree.cpp

```cpp
namespace domtree {
  const int MAXN = 300100;
  int n;
  vector<int> e[MAXN];
  vector<int> g[MAXN];
  int par[MAXN];
  int in[MAXN], rin[MAXN], tmr;
  int dom[MAXN], sdom[MAXN], cmn[MAXN];
  int p[MAXN];
  int adom[MAXN];
  vector<int> vct[MAXN];

  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; i++) {
      e[i].clear();
    }
  }

  void addEdge(int from, int to) {
    e[from].push_back(to);
  }

  void dfs(int v) {
    in[v] = tmr++;
    rin[in[v]] = v;
    for (int to : e[v]) {
      if (in[to] == -1) {
        dfs(to);
        par[in[to]] = in[v];
      }
      g[in[to]].push_back(in[v]);
    }
  }

  int get(int u, int x = 0) {
    if (u == p[u]) return x ? -1 : u;
    int v = get(p[u], x + 1);
    if (v < 0) return u;
    if (sdom[cmn[p[u]]] < sdom[cmn[u]]) cmn[u] = cmn[p[u]];
    p[u] = v;
    return x ? v : cmn[u];
  }

  void uni(int u, int v) {
    p[v] = u;
  }

  void calc() {
    for (int i = 0; i < n; i++) {
      in[i] = -1;
      adom[i] = -1;
      dom[i] = sdom[i] = p[i] = cmn[i] = i;
      vct[i].clear();
      g[i].clear();
    }
    tmr = 0;
    dfs(0);
    for (int i = tmr - 1; i >= 0; i--) {
      for (int to : g[i]) sdom[i] = min(sdom[i], sdom[get(to)]);
      if (i > 0) vct[sdom[i]].push_back(i);
      for (int w : vct[i]) {
        int v = get(w);
        if (sdom[v] == sdom[w]) dom[w] = sdom[w];
        else dom[w] = v;
      }
      if (i > 0) uni(par[i], i);
    }
    for (int i = 1; i < tmr; i++) {
      if (dom[i] != sdom[i]) dom[i] = dom[dom[i]];
      adom[rin[i]] = rin[dom[i]];
    }
  }
}
```

## final/graphs/fenwick-min.cpp

```cpp
const int inf = 1.01e9;
const int maxn = 1e5;

namespace fenwik
{
  const int N = maxn + 1;

  int a[N], l[N], r[N];

  void modify(int q, int v) {
   q++;
   a[q] = min(a[q], v);
   int x = q;
   while (x < N) {
    l[x] = min(l[x], v);
    x = (x | (x - 1)) + 1;
   }
   x = q;
   while (x > 0) {
    r[x] = min(r[x], v);
    x &= x - 1;
   }
  }

  int find_min(int ll, int rr) {
   ll++;
   rr++;
   int res = inf;
   int x = ll;
   while ((x | (x - 1)) + 1 <= rr) {
    res = min(res, r[x]);
    x = (x | (x - 1)) + 1;
   }
   res = min(res, a[x]);
   x = rr;
   while ((x & (x - 1)) >= ll) {
    res = min(res, l[x]);
    x &= x - 1;
   }
   return res;
  }

  // indexes 0 .. maxn-1
  // (!) to init fill (a, l, r) with INF
  // (!) modify supports only decreasing of the value
  // find_min [l, r] (both inclusive)
}
```

## final/graphs/generalMatching.cpp

```cpp
//COPYPASTED FROM E-MAXX
namespace GeneralMatching {
  const int MAXN = 256;
  int n;
  vector<int> g[MAXN];
  int match[MAXN], p[MAXN], base[MAXN], q[MAXN];
  bool used[MAXN], blossom[MAXN];

  int lca (int a, int b) {
    bool used[MAXN] = { 0 };
    for (;;) {
      a = base[a];
      used[a] = true;
      if (match[a] == -1) break;
      a = p[match[a]];
    }
    for (;;) {
      b = base[b];
      if (used[b]) return b;
      b = p[match[b]];
    }
  }

  void mark_path (int v, int b, int children) {
    while (base[v] != b) {
      blossom[base[v]] = blossom[base[match[v]]] = true;
      p[v] = children;
      children = match[v];
      v = p[match[v]];
    }
  }

  int find_path (int root) {
    memset (used, 0, sizeof used);
    memset (p, -1, sizeof p);
    for (int i=0; i<n; ++i)
      base[i] = i;

    used[root] = true;
    int qh=0, qt=0;
    q[qt++] = root;
    while (qh < qt) {
      int v = q[qh++];
      for (size_t i=0; i<g[v].size(); ++i) {
        int to = g[v][i];
        if (base[v] == base[to] || match[v] == to) continue;
        if (to == root || (match[to] != -1 && p[match[to]] != -1)) {
          int curbase = lca (v, to);
          memset (blossom, 0, sizeof blossom);
          mark_path (v, curbase, to);
          mark_path (to, curbase, v);
          for (int i=0; i<n; ++i)
            if (blossom[base[i]]) {
              base[i] = curbase;
              if (!used[i]) {
                used[i] = true;
                q[qt++] = i;
              }
            }
        }
        else if (p[to] == -1) {
          p[to] = v;
          if (match[to] == -1)
            return to;
          to = match[to];
          used[to] = true;
          q[qt++] = to;
        }
      }
    }
    return -1;
  }

  vector<pair<int, int> > solve(int _n, vector<pair<int, int> > edges) {
    n = _n;
    for (int i = 0; i < n; i++) g[i].clear();
    for (auto o : edges) {
      g[o.first].push_back(o.second);
      g[o.second].push_back(o.first);
    }
    memset (match, -1, sizeof match);
    for (int i=0; i<n; ++i) {
      if (match[i] == -1) {
        int v = find_path (i);
        while (v != -1) {
          int pv = p[v], ppv = match[pv];
          match[v] = pv, match[pv] = v;
          v = ppv;
        }
      }
    }
    vector<pair<int, int> > ans;
    for (int i = 0; i < n; i++) {
      if (match[i] > i) {
        ans.push_back(make_pair(i, match[i]));
      }
    }
    return ans;
  }
}
```

## final/graphs/heavyLight.cpp

```
namespace hld {
  const int N = 1 << 17;
  int par[N], heavy[N], h[N];
  int root[N], pos[N];
  int n;
  vector<vector<int> > e;
  segtree tree;

  int dfs(int v) {
    int sz = 1, mx = 0;
    for (int to : e[v]) {
      if (to == par[v]) continue;
      par[to] = v;
      h[to] = h[v] + 1;
      int cur = dfs(to);
      if (cur > mx) heavy[v] = to, mx = cur;
      sz += cur;
    }
    return sz;
  }

  template <typename T>
  void path(int u, int v, T op) {
    for (; root[u] != root[v]; v = par[root[v]]) {
      if (h[root[u]] > h[root[v]]) swap(u, v);
      op(pos[root[v]], pos[v] + 1);
    }
    if (h[u] > h[v]) swap(u, v);
    op(pos[u], pos[v] + 1);
  }

  void init(vector<vector<int> > _e) {
    e = _e;
    n = e.size();
    tree = segtree(n);
    memset(heavy, -1, sizeof(heavy[0]) * n);
    par[0] = -1;
    h[0] = 0;
    dfs(0);
    for (int i = 0, cpos = 0; i < n; i++) {
      if (par[i] == -1 || heavy[par[i]] != i) {
        for (int j = i; j != -1; j = heavy[j]) {
          root[j] = i;
          pos[j] = cpos++;
        }
      }
    }
  }

  void add(int v, int x) {
    tree.add(pos[v], x);
  }

  int get(int u, int v) {
    int res = 0;
    path(u, v, [&](int l, int r) {
      res = max(res, tree.get(l, r));
    });
    return res;
  }
}
```

## final/graphs/hungary.cpp

```
namespace hungary
{
  const int N = 210;

  int a[N][N];
  int ans[N];

  int calc(int n, int m)
  {
    ++n, ++m;
    vi u(n), v(m), p(m), prev(m);
    for (int i = 1; i < n; ++i)
    {
      p[0] = i;
      int x = 0;
      vi mn(m, inf);
      vi was(m, 0);
      while (p[x])
      {
        was[x] = 1;
        int ii = p[x], dd = inf, y = 0;
        for (int j = 1; j < m; ++j) if (!was[j])
        {
          int cur = a[ii][j] - u[ii] - v[j];
          if (cur < mn[j]) mn[j] = cur, prev[j] = x;
          if (mn[j] < dd) dd = mn[j], y = j;
        }
        forn(j, m)
        {
          if (was[j]) u[p[j]] += dd, v[j] -= dd;
          else mn[j] -= dd;
        }
        x = y;
      }
      while (x)
      {
        int y = prev[x];
        p[x] = p[y];
        x = y;
      }
    }
    for (int j = 1; j < m; ++j)
    {
      ans[p[j]] = j;
    }
    return -v[0];
  }
  // HOW TO USE ::
  // -- set values to a[1..n][1..m] (n <= m)
  // -- run calc(n, m) to find MINIMUM
  // -- to restore permutation use ans[]
  // -- everything works on negative numbers
  //
  // !! i don't understand this code, it's copypasted from e-maxx (and rewritten by enot110)
}
```

## final/graphs/max-flow-min-cost.cpp

```
namespace flow
{
  const int maxn = 2e5 + 10;
  const int maxe = 2 * maxn;
```

```cpp

  int head[maxn], next[maxe], to[maxe], flow[maxe], cost[maxe], ec = 1;
  int ST, EN, N = maxn;

  inline void setN(int n)
  {
    ST = n;
    EN = n + 1;
    N = n + 2;
  }

  inline void _add(int x, int y, int f, int c)
  {
    ++ec;
    to[ec] = y;
    next[ec] = head[x];
    head[x] = ec;
    flow[ec] = f;
    cost[ec] = c;
  }

  inline int add(int x, int y, int f, int c)
  {
    _add(x, y, f, c);
    _add(y, x, 0, -c);
    return ec - 1;
  }

  void clear()
  {
    forn(i, N) head[i] = 0;
    ec = 1;
  }

  ll d[maxn], p[maxn];
  int last[maxn];
  int used[maxn];

  pair<ll, ll> _calc(int flag)
  {
    const ll INF = 1e12;
    forn(i, N) p[i] = INF;
    p[ST] = 0;
    forn(_, N) forn(x, N) for (int e = head[x]; e; e = next[e]) if (flow[e] > 0)
    {
      int y = to[e];
      if (p[y] > p[x] + cost[e])
      {
        p[y] = p[x] + cost[e];
      }
    }

    ll resFlow = 0, resCost = 0;
    while (1)
    {
      forn(i, N) d[i] = INF, used[i] = 0;
      d[ST] = 0;
      forn(_, N)
      {
        int x = -1;
        forn(i, N) if (!used[i] && (x == -1 || d[x] > d[i])) x = i;
        used[x] = 1;
        if (d[x] == INF) break;
        for (int e = head[x]; e; e = next[e]) if (flow[e] > 0)
        {
          int y = to[e];
          ll len = cost[e] + p[x] - p[y];
          if (d[y] > d[x] + len)
          {
            d[y] = d[x] + len;
            last[y] = e;
          }
        }
      }

      if (d[EN] == INF) break;

      ll realCost = d[EN] + p[EN] - p[ST];
      if (flag && realCost > 0) break;

      int pushed = inf;
      int x = EN;
      while (x != ST)
      {
        int e = last[x];
        pushed = min(pushed, flow[e]);
        x = to[e ^ 1];
      }

      resCost += realCost * pushed;
      resFlow += pushed;

      x = EN;
      while (x != ST)
      {
        int e = last[x];
        flow[e] -= pushed;
        flow[e ^ 1] += pushed;
        x = to[e ^ 1];
      }

      forn(i, N) p[i] += d[i];
    }
    return mp(resFlow, resCost);
  }

  pair<ll, ll> maxFlow()
  {
    return _calc(0);
  }

  pair<ll, ll> minCost()
  {
    return _calc(1);
  }

  // HOW TO USE::
  // -- add adges using add(x, y, f, c), call setN(n)
  // -- run maxFlow/minCost, returns pair(flow, cost)
}
```

## final/graphs/retro.cpp

```cpp
namespace retro
{
  const int N = 4e5 + 10;

  vi v[N];
  vi vrev[N];

  void add(int x, int y)
  {
    v[x].pb(y);
    vrev[y].pb(x);
  }

  const int UD = 0;
  const int WIN = 1;
  const int LOSE = 2;

  int res[N];
  int moves[N];
  int deg[N];
  int q[N], st, en;

  void calc(int n)
  {
    forn(i, n) deg[i] = sz(v[i]);
    st = en = 0;
    forn(i, n) if (!deg[i])
    {
      q[en++] = i;
      res[i] = LOSE;
    }
    while (st < en)
    {
      int x = q[st++];
      for (int y : vrev[x])
      {
        if (res[y] == UD && (res[x] == LOSE || (--deg[y] == 0 && res[x] == WIN)))
        {
          res[y] = 3 - res[x];
          moves[y] = moves[x] + 1;
          q[en++] = y;
        }
      }
    }
  }
}
```

## final/graphs/smith.cpp

```cpp
const int N = 1e5 + 10;

struct graph
{
  int n;

  vi v[N];
  vi vrev[N];

  void read()
  {
    int m;
    scanf("%d%d", &n, &m);
    forn(i, m)
    {
      int x, y;
      scanf("%d%d", &x, &y);
      --x, --y;
      v[x].pb(y);
      vrev[y].pb(x);
    }
  }

  int deg[N], cnt[N], used[N], f[N];
  int q[N], st, en;

  set<int> s[N];

  void calc()
  {
    forn(x, n) f[x] = -1, cnt[x] = 0;
    int val = 0;
    while (1)
    {
      st = en = 0;
      forn(x, n)
      {
        deg[x] = 0;
        used[x] = 0;
        for (int y : v[x]) if (f[y] == -1) deg[x]++;
      }
      forn(x, n) if (!deg[x] && f[x] == -1 && cnt[x]↩
 == val)
      {
        q[en++] = x;
        f[x] = val;
      }
      if (!en) break;
      while (st < en)
      {
        int x = q[st];
        st++;
        for (int y : vrev[x])
        {
          if (used[y] == 0 && f[y] == -1)
          {
            used[y] = 1;
            cnt[y]++;
            for (int z : vrev[y])
            {
              deg[z]--;
              if (f[z] == -1 && deg[z] == 0 && cnt[z↩
] == val)
              {
                f[z] = val;
                q[en++] = z;
              }
            }
          }
        }
      }
      val++;
    }
    forn(x, n) eprintf("%d%c", f[x], " \n"[x + 1 == ↩
n]);
    forn(x, n) if (f[x] == -1)
    {
      for (int y : v[x]) if (f[y] != -1) s[x].insert↩
(f[y]);
    }
  }
} g1, g2;

string get(int x, int y)
{
  int f1 = g1.f[x], f2 = g2.f[y];
  if (f1 == -1 && f2 == -1) return "draw";
  if (f1 == -1) {
    if (g1.s[x].count(f2)) return "first";
    return "draw";
  }
  if (f2 == -1) {
    if (g2.s[y].count(f1)) return "first";
    return "draw";
  }
  if (f1 ^ f2) return "first";
  return "second";
}
```

## final/graphs/twoChinese.cpp

```cpp
const int INF = 1e9;
struct Edge {
  int from, to, w, id;
};
namespace dmst {
  int n;
  vector<int> p;
  vector<Edge> edges;

  int get(int x) {
    if (x == p[x]) return x;
    return p[x] = get(p[x]);
  }

  void uni(int u, int v) {
    p[get(v)] = get(u);
  }

  vector<Edge> solve() {
    vector<int> id(n, -1);
    vector<int> vert;
    int cn = 0;
    for (int i = 0; i < n; i++) if (get(i) == i) {
      vert.push_back(i);
      id[i] = cn++;
    }
    if (cn == 1) return vector<Edge>();

    vector<vector<int> > e(cn);
    for (int i = 0; i < (int)edges.size(); i++) {
      if (get(edges[i].to) != get(edges[i].from)) {
        e[id[get(edges[i].to)]].push_back(i);
      }
    }

    vector<int> nxtId(cn, -1);
    for (int i = 0; i < cn; i++) {
      int mn = INF;
      for (int id : e[i]) mn = min(mn, edges[id].w);
      for (int id : e[i]) {
        edges[id].w -= mn;
        if (edges[id].w == 0) nxtId[i] = id;
      }
    }

    vector<char> vis(cn);
    vis[0] = 1;
    int cur = 1;
    while (!vis[cur]) {
      vis[cur] = 1;
      cur = id[get(edges[nxtId[cur]].from)];
    }
    vector<Edge> ans;
    if (cur == 0) {
      for (int i = 0; i < cn; i++) {
        if (vis[i] && i != 0) {
          ans.push_back(edges[nxtId[i]]);
          uni(0, vert[i]);
        }
      }
      auto nans = solve();
      for (auto ee : nans) ans.push_back(ee);
      return ans;
    }
    vector<int> cp = p;
    int o = cur;
    while (1) {
      uni(vert[o], vert[cur]);
      ans.push_back(edges[nxtId[cur]]);
      int to = id[get(edges[nxtId[cur]].from)];
      if (to == o) break;
      cur = to;
    }
    vector<Edge> nedges = solve();
    p = cp;
    vector<char> covered(cn);
    for (auto ee : nedges) covered[id[get(ee.to)]] = 1;
    for (auto ee : ans) if (!covered[id[get(ee.to)]]) nedges.push_back(ee);
    return nedges;
  }

  // root is 0
  vector<Edge> getMst(int _n, vector<Edge> _edges) {
    n = _n;
    edges = _edges;
    p.resize(n);
    for (int i = 0; i < n; i++) p[i] = i;

    return solve();
  }
}
```

## final/graphs/linkcut.cpp

```cpp
#include <iostream>
#include <cstdio>
#include <cassert>

using namespace std;

// BEGIN ALGO

const int MAXN = 110000;

typedef struct _node{
 _node *l, *r, *p, *pp;
 int size; bool rev;
 _node();
 explicit _node(nullptr_t){
  l = r = p = pp = this;
  size = rev = 0;
 }
 void push(){
  if (rev){
   l->rev ^= 1; r->rev ^= 1;
   rev = 0; swap(l,r);
  }
 }
 void update();
}* node;
node None = new _node(nullptr);
node v2n[MAXN];
_node::_node(){
 l = r = p = pp = None;
 size = 1; rev = false;
}
void _node::update(){
 size = (this != None) + l->size + r->size;
 l->p = r->p = this;
}
void rotate(node v){
 assert(v != None && v->p != None);
 assert(!v->rev); assert(!v->p->rev);
 node u = v->p;
 if (v == u->l)
  u->l = v->r, v->r = u;
 else
  u->r = v->l, v->l = u;
 swap(u->p,v->p); swap(v->pp,u->pp);
 if (v->p != None){
  assert(v->p->l == u || v->p->r == u);
  if (v->p->r == u) v->p->r = v;
  else v->p->l = v;
 }
 u->update(); v->update();
}
void bigRotate(node v){
 assert(v->p != None);
 v->p->p->push();
 v->p->push();
 v->push();
 if (v->p->p != None){
  if ((v->p->l == v) ^ (v->p->p->r == v->p))
   rotate(v->p);
  else
   rotate(v);
 }
 rotate(v);
}
inline void Splay(node v){
 while (v->p != None) bigRotate(v);
}
inline void splitAfter(node v){
 v->push();
 Splay(v);
 v->r->p = None;
 v->r->pp = v;
 v->r = None;
 v->update();
}
void expose(int x){
 node v = v2n[x];
 splitAfter(v);
 while (v->pp != None){
  assert(v->p == None);
  splitAfter(v->pp);
  assert(v->pp->r == None);
  assert(v->pp->p == None);
  assert(!v->pp->rev);
  v->pp->r = v;
  v->pp->update();
  v = v->pp;
  v->r->pp = None;
 }
 assert(v->p == None);
 Splay(v2n[x]);
}
inline void makeRoot(int x){
 expose(x);
 assert(v2n[x]->p == None);
 assert(v2n[x]->pp == None);
 assert(v2n[x]->r == None);
 v2n[x]->rev ^= 1;
}
inline void link(int x,int y){
 makeRoot(x); v2n[x]->pp = v2n[y];
}
inline void cut(int x,int y){
 expose(x);
 Splay(v2n[y]);
 if (v2n[y]->pp != v2n[x]){
  swap(x,y);
  expose(x);
  Splay(v2n[y]);
  assert(v2n[y]->pp == v2n[x]);
 }
 v2n[y]->pp = None;
}
inline int get(int x,int y){
 if (x == y) return 0;
 makeRoot(x);
 expose(y); expose(x);
 Splay(v2n[y]);
 if (v2n[y]->pp != v2n[x]) return -1;
 return v2n[y]->size;
}
// END ALGO

_node mem[MAXN];


int main(){
 freopen("linkcut.in","r",stdin);
 freopen("linkcut.out","w",stdout);

 int n,m;
 scanf("%d %d",&n,&m);

 for (int i = 0; i < n; i++)
  v2n[i] = &mem[i];

 for (int i = 0; i < m; i++){
  int a,b;
  if (scanf(" link %d %d",&a,&b) == 2)
   link(a-1,b-1);
  else if (scanf(" cut %d %d",&a,&b) == 2)
   cut(a-1,b-1);
  else if (scanf(" get %d %d",&a,&b) == 2)
   printf("%d\n",get(a-1,b-1));
  else
   assert(false);
 }
 return 0;
}
```

dbl Simpson() { return (F(-1) + 4 * F(0) + F(1)) / 6; } dbl Runge2() { return (F(-sqrtl(1.0 / 3)) + F(sqrtl(1.0 / 3))) / 2; } dbl Runge3() { return (F(-sqrtl(3.0 / 5)) * 5 + F(0) * 8 + F(sqrtl(3.0 / 5)) * 5) / 18; }

Simpson и Runge2 – точны для полиномов степени <= 3 Runge3 – точен для полиномов степени <= 5

—

Явный Рунге-Кутт четвертого порядка, ошибка O(h^4)

y' = f(x, y) y_(n+1) = y_n + (k1 + 2 * k2 + 2 * k3 + k4) * h / 6

k1 = f(xn, yn) k2 = f(xn + h/2, yn + h/2 * k1) k3 = f(xn + h/2, yn + h/2 * k2) k4 = f(xn + h, yn + h * k3)

Методы Адамса-Башфорта

y_n+3 = y_n+2 + h * (23/12 * f(x_n+2,y_n+2) - 4/3 * f(x_n+1,y_n+1) + 5/12 * f(x_n,y_n)) y_n+4 = y_n+3 + h * (55/24 * f(x_n+3,y_n+3) - 59/24 * f(x_n+2,y_n+2) + 37/24 * f(x_n+1,y_n+1) - 3/8 * f(x_n,y_n)) y_n+5 = y_n+4 + h * (1901/720 * f(x_n+4,y_n+4) - 1387/360 * f(x_n+3,y_n+3) + 109/30 * f(x_n+2,y_n+2) - 637/360 * f(x_n+1,y_n+1) + 251/720 * f(x_n,y_n))

—

Извлечение корня по простому модулю (от Сережи) 3 <= p, 1 <= a < p, найти x^2 = a

1) Если a^((p - 1)/2) != 1, return -1 2) Выбрать случайный 1 <= i < p 3) T(x) = (x + i)^((p - 1)/2) mod (x^2 - a) = bx + c 4) Если b != 0 то вернуть c/b, иначе к шагу 2)

—

Иногда вместо того чтобы считать первообразный у простого числа, можно написать чекер ответа и перебирать случайный первообразный.

Не заходит FFT по TL-ю – чекнуть что стоит double, а не long double

mt19937 генерит случайный unsigned int, если хочется больше есть mt19937_64

Иногда можно представить ответ в виде многочлена и вместо подсчета самих к-тов посчитать значения и проинтерполировать

Перед сабмитом чекнуть что все выводится в printf, а не eprintf!!!

—

Лемма Бернсайда:

Группа G действует на множество X Тогда число классов эквивалентности = (sum |f(g)| for g in G) / |G| где f(g) = число x (из X) : g(x) == x

—

Число простых быстрее O(n):

dp(n, k) – число чисел от 1 до n в которых все простые >= p[k] dp(n, 1) = n dp(n, j) = dp(n, j + 1) + dp(n / p[j], j), т. е. dp(n, j + 1) = dp(n, j) - dp(n / p[j], j)

Если p[j], p[k] > sqrt(n) то dp(n, j) + j == dp(n, k) + k

Хуяришь все оптимайзы сверху, но не считаешь глубже dp(n, k), n < K Потом фенвиком+сортировкой подсчитываешь за (K+Q)log все эти запросы Хуяришь во второй раз, но на этот раз берешь прекальканные значения

Если sqrt(n) < p[k] < n то (число простых до n)=dp(n, k) + k - 1

—

Чиселки:

Фибоначчи 45: 1134903170 46: 1836311903 47: 2971215073 91: 4660046610375530309 92: 7540113804746346429 93: 12200160415121876738

Числа с кучей делителей 20: d(12)=6 50: d(48)=10 100: d(60)=12 1000: d(840)=32 10^4: d(9240)=64 10^5: d(83160)=128 10^6: d(720720)=240 10^7: d(8648640)=448 10^8: d(91891800)=768 10^9: d(931170240)=1344 10^{11}: d(97772875200)=4032 10^{12}: d(963761198400)=6720 10^{15}: d(866421317361600)=26880 10^{18}: d(897612484786617600)=103680

Bell numbers: 0:1, 1:1, 2:2, 3:5, 4:15, 5:52, 6:203, 7:877, 8:4140, 9:21147, 10:115975, 11:678570, 12:4213597, 13:27644437, 14:190899322, 15:1382958545, 16:10480142147, 17:82864869804, 18:682076806159, 19:5832742205057, 20:51724158235372, 21:474869816156751, 22:4506715738447323, 23:44152005855084346

Catalan numbers: 0:1, 1:1, 2:2, 3:5, 4:14, 5:42, 6:132, 7:429, 8:1430, 9:4862, 10:16796, 11:58786, 12:208012, 13:742900, 14:2674440, 15:9694845, 16:35357670, 17:129644790, 18:477638700, 19:1767263190, 20:6564120420, 21:24466267020, 22:91482563640, 23:343059613650, 24:1289904147324, 25:4861946401452

# Table of Integrals*

**Basic Forms**

$$\int x^n dx = \frac{1}{n+1}x^{n+1} \tag{1}$$

$$\int \frac{1}{x}dx = \ln|x| \tag{2}$$

$$\int udv = uv - \int vdu \tag{3}$$

$$\int \frac{1}{ax+b}dx = \frac{1}{a}\ln|ax+b| \tag{4}$$

**Integrals of Rational Functions**

$$\int \frac{1}{(x+a)^2}dx = -\frac{1}{x+a} \tag{5}$$

$$\int (x+a)^n dx = \frac{(x+a)^{n+1}}{n+1}, n \neq -1 \tag{6}$$

$$\int x(x+a)^n dx = \frac{(x+a)^{n+1}((n+1)x-a)}{(n+1)(n+2)} \tag{7}$$

$$\int \frac{1}{1+x^2}dx = \tan^{-1} x \tag{8}$$

$$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a} \tag{9}$$

$$\int \frac{x}{a^2+x^2}dx = \frac{1}{2}\ln|a^2+x^2| \tag{10}$$

$$\int \frac{x^2}{a^2+x^2}dx = x - a\tan^{-1}\frac{x}{a} \tag{11}$$

$$\int \frac{x^3}{a^2+x^2}dx = \frac{1}{2}x^2 - \frac{1}{2}a^2\ln|a^2+x^2| \tag{12}$$

$$\int \frac{1}{ax^2+bx+c}dx = \frac{2}{\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} \tag{13}$$

$$\int \frac{1}{(x+a)(x+b)}dx = \frac{1}{b-a}\ln\frac{a+x}{b+x},\ a \neq b \tag{14}$$

$$\int \frac{x}{(x+a)^2}dx = \frac{a}{a+x} + \ln|a+x| \tag{15}$$

$$\int \frac{x}{ax^2+bx+c}dx = \frac{1}{2a}\ln|ax^2+bx+c| - \frac{b}{a\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} \tag{16}$$

**Integrals with Roots**

$$\int \sqrt{x-a}dx = \frac{2}{3}(x-a)^{3/2} \tag{17}$$

$$\int \frac{1}{\sqrt{x\pm a}}dx = 2\sqrt{x\pm a} \tag{18}$$

$$\int \frac{1}{\sqrt{a-x}}dx = -2\sqrt{a-x} \tag{19}$$

$$\int x\sqrt{x-a}dx = \frac{2}{3}a(x-a)^{3/2} + \frac{2}{5}(x-a)^{5/2} \tag{20}$$

$$\int \sqrt{ax+b}dx = \left(\frac{2b}{3a} + \frac{2x}{3}\right)\sqrt{ax+b} \tag{21}$$

$$\int (ax+b)^{3/2}dx = \frac{2}{5a}(ax+b)^{5/2} \tag{22}$$

$$\int \frac{x}{\sqrt{x\pm a}}dx = \frac{2}{3}(x\mp 2a)\sqrt{x\pm a} \tag{23}$$

$$\int \sqrt{\frac{x}{a-x}}dx = -\sqrt{x(a-x)} - a\tan^{-1}\frac{\sqrt{x(a-x)}}{x-a} \tag{24}$$

$$\int \sqrt{\frac{x}{a+x}}dx = \sqrt{x(a+x)} - a\ln\left[\sqrt{x} + \sqrt{x+a}\right] \tag{25}$$

$$\int x\sqrt{ax+b}dx = \frac{2}{15a^2}(-2b^2 + abx + 3a^2x^2)\sqrt{ax+b} \tag{26}$$

$$\int \sqrt{x(ax+b)}dx = \frac{1}{4a^{3/2}}\left[(2ax+b)\sqrt{ax(ax+b)} - b^2\ln\left|a\sqrt{x} + \sqrt{a(ax+b)}\right|\right] \tag{27}$$

$$\int \sqrt{x^3(ax+b)}dx = \left[\frac{b}{12a} - \frac{b^2}{8a^2x} + \frac{x}{3}\right]\sqrt{x^3(ax+b)} + \frac{b^3}{8a^{5/2}}\ln\left|a\sqrt{x} + \sqrt{a(ax+b)}\right| \tag{28}$$

$$\int \sqrt{x^2\pm a^2}dx = \frac{1}{2}x\sqrt{x^2\pm a^2} \pm \frac{1}{2}a^2\ln\left|x + \sqrt{x^2\pm a^2}\right| \tag{29}$$

$$\int \sqrt{a^2-x^2}dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{1}{2}a^2\tan^{-1}\frac{x}{\sqrt{a^2-x^2}} \tag{30}$$

$$\int x\sqrt{x^2\pm a^2}dx = \frac{1}{3}\left(x^2\pm a^2\right)^{3/2} \tag{31}$$

$$\int \frac{1}{\sqrt{x^2\pm a^2}}dx = \ln\left|x + \sqrt{x^2\pm a^2}\right| \tag{32}$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a} \tag{33}$$

$$\int \frac{x}{\sqrt{x^2\pm a^2}}dx = \sqrt{x^2\pm a^2} \tag{34}$$

$$\int \frac{x}{\sqrt{a^2-x^2}}dx = -\sqrt{a^2-x^2} \tag{35}$$

$$\int \frac{x^2}{\sqrt{x^2\pm a^2}}dx = \frac{1}{2}x\sqrt{x^2\pm a^2} \mp \frac{1}{2}a^2\ln\left|x + \sqrt{x^2\pm a^2}\right| \tag{36}$$

$$\int \sqrt{ax^2+bx+c}dx = \frac{b+2ax}{4a}\sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8a^{3/2}}\ln\left|2ax + b + 2\sqrt{a(ax^2+bx^+c)}\right| \tag{37}$$

$$\int x\sqrt{ax^2+bx+c} = \frac{1}{48a^{5/2}}\left(2\sqrt{a}\sqrt{ax^2+bx+c} \times \left(-3b^2 + 2abx + 8a(c+ax^2)\right) + 3(b^3-4abc)\ln\left|b + 2ax + 2\sqrt{a}\sqrt{ax^2+bx+c}\right|\right) \tag{38}$$

$$\int \frac{1}{\sqrt{ax^2+bx+c}}dx = \frac{1}{\sqrt{a}}\ln\left|2ax + b + 2\sqrt{a(ax^2+bx+c)}\right| \tag{39}$$

$$\int \frac{x}{\sqrt{ax^2+bx+c}}dx = \frac{1}{a}\sqrt{ax^2+bx+c} - \frac{b}{2a^{3/2}}\ln\left|2ax + b + 2\sqrt{a(ax^2+bx+c)}\right| \tag{40}$$

$$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}} \tag{41}$$

**Integrals with Logarithms**

$$\int \ln ax dx = x\ln ax - x \tag{42}$$

$$\int \frac{\ln ax}{x}dx = \frac{1}{2}(\ln ax)^2 \tag{43}$$

$$\int \ln(ax+b)dx = \left(x + \frac{b}{a}\right)\ln(ax+b) - x, a \neq 0 \tag{44}$$

$$\int \ln(x^2+a^2)\,\mathrm{d}x = x\ln(x^2+a^2) + 2a\tan^{-1}\frac{x}{a} - 2x \tag{45}$$

$$\int \ln(x^2-a^2)\,\mathrm{d}x = x\ln(x^2-a^2) + a\ln\frac{x+a}{x-a} - 2x \tag{46}$$

$$\int \ln\left(ax^2+bx+c\right)dx = \frac{1}{a}\sqrt{4ac-b^2}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} - 2x + \left(\frac{b}{2a} + x\right)\ln\left(ax^2+bx+c\right) \tag{47}$$

$$\int x\ln(ax+b)dx = \frac{bx}{2a} - \frac{1}{4}x^2 + \frac{1}{2}\left(x^2 - \frac{b^2}{a^2}\right)\ln(ax+b) \tag{48}$$

$$\int x\ln\left(a^2-b^2x^2\right)dx = -\frac{1}{2}x^2 + \frac{1}{2}\left(x^2 - \frac{a^2}{b^2}\right)\ln\left(a^2-b^2x^2\right) \tag{49}$$

**Integrals with Exponentials**

$$\int e^{ax}dx = \frac{1}{a}e^{ax} \tag{50}$$

$$\int \sqrt{x}e^{ax}dx = \frac{1}{a}\sqrt{x}e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}}\mathrm{erf}\left(i\sqrt{ax}\right), \text{ where } \mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt \tag{51}$$

$$\int xe^x dx = (x-1)e^x \tag{52}$$

$$\int xe^{ax}dx = \left(\frac{x}{a} - \frac{1}{a^2}\right)e^{ax} \tag{53}$$

$$\int x^2e^x dx = \left(x^2 - 2x + 2\right)e^x \tag{54}$$

$$\int x^2e^{ax}dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right)e^{ax} \tag{55}$$

$$\int x^3e^x dx = \left(x^3 - 3x^2 + 6x - 6\right)e^x \tag{56}$$

$$\int x^n e^{ax}\,\mathrm{d}x = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\,\mathrm{d}x \tag{57}$$

$$\int x^n e^{ax}\,\mathrm{d}x = \frac{(-1)^n}{a^{n+1}}\Gamma[1+n, -ax], \text{ where } \Gamma(a,x) = \int_x^\infty t^{a-1}e^{-t}\,\mathrm{d}t \tag{58}$$

$$\int e^{ax^2}\,\mathrm{d}x = -\frac{i\sqrt{\pi}}{2\sqrt{a}}\mathrm{erf}\left(ix\sqrt{a}\right) \tag{59}$$

$$\int e^{-ax^2}\,\mathrm{d}x = \frac{\sqrt{\pi}}{2\sqrt{a}}\mathrm{erf}\left(x\sqrt{a}\right) \tag{60}$$

$$\int xe^{-ax^2}\,\mathrm{d}x = -\frac{1}{2a}e^{-ax^2} \tag{61}$$

$$\int x^2e^{-ax^2}\,\mathrm{d}x = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}\mathrm{erf}(x\sqrt{a}) - \frac{x}{2a}e^{-ax^2} \tag{62}$$

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**Integrals with Trigonometric Functions**

$$\int \sin ax dx = -\frac{1}{a}\cos ax \tag{63}$$

$$\int \sin^2 ax dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \tag{64}$$

$$\int \sin^n ax dx = -\frac{1}{a}\cos ax \; {}_2F_1\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3}{2}, \cos^2 ax\right] \tag{65}$$

$$\int \sin^3 ax dx = -\frac{3\cos ax}{4a} + \frac{\cos 3ax}{12a} \tag{66}$$

$$\int \cos ax dx = \frac{1}{a}\sin ax \tag{67}$$

$$\int \cos^2 ax dx = \frac{x}{2} + \frac{\sin 2ax}{4a} \tag{68}$$

$$\int \cos^p ax dx = -\frac{1}{a(1+p)}\cos^{1+p} ax \times {}_2F_1\left[\frac{1+p}{2}, \frac{1}{2}, \frac{3+p}{2}, \cos^2 ax\right] \tag{69}$$

$$\int \cos^3 ax dx = \frac{3\sin ax}{4a} + \frac{\sin 3ax}{12a} \tag{70}$$

$$\int \cos ax \sin bx dx = \frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)}, a \neq b \tag{71}$$

$$\int \sin^2 ax \cos bx dx = -\frac{\sin[(2a-b)x]}{4(2a-b)} + \frac{\sin bx}{2b} - \frac{\sin[(2a+b)x]}{4(2a+b)} \tag{72}$$

$$\int \sin^2 x \cos x dx = \frac{1}{3}\sin^3 x \tag{73}$$

$$\int \cos^2 ax \sin bx dx = \frac{\cos[(2a-b)x]}{4(2a-b)} - \frac{\cos bx}{2b} - \frac{\cos[(2a+b)x]}{4(2a+b)} \tag{74}$$

$$\int \cos^2 ax \sin ax dx = -\frac{1}{3a}\cos^3 ax \tag{75}$$

$$\int \sin^2 ax \cos^2 bx dx = \frac{x}{4} - \frac{\sin 2ax}{8a} - \frac{\sin[2(a-b)x]}{16(a-b)} + \frac{\sin 2bx}{8b} - \frac{\sin[2(a+b)x]}{16(a+b)} \tag{76}$$

$$\int \sin^2 ax \cos^2 ax dx = \frac{x}{8} - \frac{\sin 4ax}{32a} \tag{77}$$

$$\int \tan ax dx = -\frac{1}{a}\ln\cos ax \tag{78}$$

$$\int \tan^2 ax dx = -x + \frac{1}{a}\tan ax \tag{79}$$

$$\int \tan^n ax dx = \frac{\tan^{n+1} ax}{a(1+n)} \times {}_2F_1\left(\frac{n+1}{2}, 1, \frac{n+3}{2}, -\tan^2 ax\right) \tag{80}$$

$$\int \tan^3 ax dx = \frac{1}{a}\ln\cos ax + \frac{1}{2a}\sec^2 ax \tag{81}$$

$$\int \sec x dx = \ln|\sec x + \tan x| = 2\tanh^{-1}\left(\tan\frac{x}{2}\right) \tag{82}$$

$$\int \sec^2 ax dx = \frac{1}{a}\tan ax \tag{83}$$

$$\int \sec^3 x \,\mathrm{dx} = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| \tag{84}$$

$$\int \sec x \tan x dx = \sec x \tag{85}$$

$$\int \sec^2 x \tan x dx = \frac{1}{2}\sec^2 x \tag{86}$$

$$\int \sec^n x \tan x dx = \frac{1}{n}\sec^n x, n \neq 0 \tag{87}$$

$$\int \csc x dx = \ln\left|\tan\frac{x}{2}\right| = \ln|\csc x - \cot x| + C \tag{88}$$

$$\int \csc^2 ax dx = -\frac{1}{a}\cot ax \tag{89}$$

$$\int \csc^3 x dx = -\frac{1}{2}\cot x \csc x + \frac{1}{2}\ln|\csc x - \cot x| \tag{90}$$

$$\int \csc^n x \cot x dx = -\frac{1}{n}\csc^n x, n \neq 0 \tag{91}$$

$$\int \sec x \csc x dx = \ln|\tan x| \tag{92}$$

**Products of Trigonometric Functions and Monomials**

$$\int x\cos x dx = \cos x + x\sin x \tag{93}$$

$$\int x\cos ax dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax \tag{94}$$

$$\int x^2 \cos x dx = 2x\cos x + \left(x^2 - 2\right)\sin x \tag{95}$$

$$\int x^2 \cos ax dx = \frac{2x\cos ax}{a^2} + \frac{a^2x^2 - 2}{a^3}\sin ax \tag{96}$$

$$\int x^n cosxdx = -\frac{1}{2}(i)^{n+1}\left[\Gamma(n+1, -ix) + (-1)^n\Gamma(n+1, ix)\right] \tag{97}$$

$$\int x^n cosaxdx = \frac{1}{2}(ia)^{1-n}\left[(-1)^n\Gamma(n+1, -iax) - \Gamma(n+1, ixa)\right] \tag{98}$$

$$\int x\sin x dx = -x\cos x + \sin x \tag{99}$$

$$\int x\sin ax dx = -\frac{x\cos ax}{a} + \frac{\sin ax}{a^2} \tag{100}$$

$$\int x^2 \sin x dx = \left(2 - x^2\right)\cos x + 2x\sin x \tag{101}$$

$$\int x^2 \sin ax dx = \frac{2 - a^2x^2}{a^3}\cos ax + \frac{2x\sin ax}{a^2} \tag{102}$$

$$\int x^n \sin x dx = -\frac{1}{2}(i)^n\left[\Gamma(n+1, -ix) - (-1)^n\Gamma(n+1, -ix)\right] \tag{103}$$

**Products of Trigonometric Functions and Exponentials**

$$\int e^x \sin x dx = \frac{1}{2}e^x(\sin x - \cos x) \tag{104}$$

$$\int e^{bx}\sin ax dx = \frac{1}{a^2 + b^2}e^{bx}(b\sin ax - a\cos ax) \tag{105}$$

$$\int e^x \cos x dx = \frac{1}{2}e^x(\sin x + \cos x) \tag{106}$$

$$\int e^{bx}\cos ax dx = \frac{1}{a^2 + b^2}e^{bx}(a\sin ax + b\cos ax) \tag{107}$$

$$\int xe^x \sin x dx = \frac{1}{2}e^x(\cos x - x\cos x + x\sin x) \tag{108}$$

$$\int xe^x \cos x dx = \frac{1}{2}e^x(x\cos x - \sin x + x\sin x) \tag{109}$$

**Integrals of Hyperbolic Functions**

$$\int \cosh ax dx = \frac{1}{a}\sinh ax \tag{110}$$

$$\int e^{ax}\cosh bx dx = \begin{cases} \dfrac{e^{ax}}{a^2 - b^2}[a\cosh bx - b\sinh bx] & a \neq b \\ \dfrac{e^{2ax}}{4a} + \dfrac{x}{2} & a = b \end{cases} \tag{111}$$

$$\int \sinh ax dx = \frac{1}{a}\cosh ax \tag{112}$$

$$\int e^{ax}\sinh bx dx = \begin{cases} \dfrac{e^{ax}}{a^2 - b^2}[-b\cosh bx + a\sinh bx] & a \neq b \\ \dfrac{e^{2ax}}{4a} - \dfrac{x}{2} & a = b \end{cases} \tag{113}$$

$$\int e^{ax}\tanh bx dx = \begin{cases} \dfrac{e^{(a+2b)x}}{(a+2b)}{}_2F_1\left[1 + \dfrac{a}{2b}, 1, 2 + \dfrac{a}{2b}, -e^{2bx}\right] \\ \qquad -\dfrac{1}{a}e^{ax}{}_2F_1\left[\dfrac{a}{2b}, 1, 1E, -e^{2bx}\right] & a \neq b \\ \dfrac{e^{ax} - 2\tan^{-1}[e^{ax}]}{a} & a = b \end{cases} \tag{114}$$

$$\int \tanh ax \, dx = \frac{1}{a}\ln\cosh ax \tag{115}$$

$$\int \cos ax \cosh bx dx = \frac{1}{a^2 + b^2}\left[a\sin ax \cosh bx + b\cos ax \sinh bx\right] \tag{116}$$

$$\int \cos ax \sinh bx dx = \frac{1}{a^2 + b^2}\left[b\cos ax \cosh bx + a\sin ax \sinh bx\right] \tag{117}$$

$$\int \sin ax \cosh bx dx = \frac{1}{a^2 + b^2}\left[-a\cos ax \cosh bx + b\sin ax \sinh bx\right] \tag{118}$$

$$\int \sin ax \sinh bx dx = \frac{1}{a^2 + b^2}\left[b\cosh bx \sin ax - a\cos ax \sinh bx\right] \tag{119}$$

$$\int \sinh ax \cosh ax dx = \frac{1}{4a}\left[-2ax + \sinh 2ax\right] \tag{120}$$

$$\int \sinh ax \cosh bx dx = \frac{1}{b^2 - a^2}\left[b\cosh bx \sinh ax - a\cosh ax \sinh bx\right] \tag{121}$$