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final/template/vimrc.txt

```

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map <F9> :wall! <CR> :!g++ -Wall -Wextra -Wshadow -Wno-unused-result -o %:r % -std=c++14 -DHOME -D_GLIBCXX_DEBUG -fsanitize=address <CR>
map <F7> :wall! <CR> :!g++ -Wall -Wextra -Wshadow -Wno-unused-result -o %:r % -std=c++14 -DHOME -D_GLIBCXX_DEBUG -fsanitize=address -g && gdb <CR>
map <F8> :wall! <CR> :!ulimit -s 500000 && ./%:r <CR>
map <F10> :wall! <CR> :!g++ -Wall -Wextra -Wshadow -Wno-unused-result -o %:r % -std=c++14 -DHOME -D_GLIBCXX_DEBUG -fsanitize=address -g && gdb <CR>

inoremap {<CR> {<CR>}<ESC>O
map <c-a> ggVG

set nu
set rnu
syntax on

map <c-t> :tabnew <CR>
map <c-l> :tabn <CR>
map <c-h> :tabp <CR>

set cin
set sw=4
set so=99
set bs=2
set et
set sts=4

```

final/template/template.cpp

```

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31
32
33
34
35
36
37
38
39
// team : SPb ITMO University 1
#include <bits/stdc++.h>

#define F first
#define S second
#define X first
#define Y second
#define pb push_back
#define sz(a) (int)(a).size()
#define all(a) (a).begin(), a.end()
#define pw(x) (1LL<<(x))

#define db(x) cerr << #x << " = " << x << endl
#define db2(x, y) cerr << "(" << #x << ", " << #y << "\n" << "(" << x << ", " << y << ")" << "\n";
#define db3(x, y, z) cerr << "(" << #x << ", " << #y << ", " << #z << ")" << "\n";
#define dbv(a) cerr << #a << " = "; for (auto xxxx: a) cerr << xxxx << " "; cerr << endl

using namespace std;

typedef long long ll;
typedef double dbl;
const int INF = 1.01e9;

int main() {
#define TASK ""
#ifndef HOME
assert(freopen(TASK".in", "r", stdin));
#endif

#ifdef HOME
cerr << "time: " << clock() * 1.0 / CLOCKS_PER_SEC << endl;
#endif
return 0;
}

```

Practice round

1. Посабмитить задачи каждому человеку
2. IDE для джавы
3. Сравнить скорость локального компьютера и сервера
4. Проверить __int128
5. Проверить прагмы (например на битсетах)
6. Узнать максимально возможный размер отправляемого кода

final/template/fastIO.cpp

```

1 #include <cstdio>
2 #include <algorithm>
3
4 /** Interface */
5
6 inline int readInt();
7 inline int readUInt();
8 inline bool isEof();
9
10 /** Read */
11
12 static const int buf_size = 100000;
13 static char buf[buf_size];
14 static int buf_len = 0, pos = 0;
15
16 inline bool isEof() {
17     if (pos == buf_len) {
18         pos = 0, buf_len = fread(buf, 1, buf_size, stdin);
19     }
20     if (pos == buf_len) return 1;
21     return 0;
22 }
23
24 inline int getChar() { return isEof() ? -1 : buf[pos++]; }
25
26 inline int readChar() {
27     int c = getChar();
28     while (c != -1 && c <= 32) c = getChar();
29     return c;
30 }
31
32 inline int readUInt() {
33     int c = readChar(), x = 0;
34     while ('0' <= c && c <= '9') x = x * 10 + c - '0', c = getChar();
35     return x;
36 }
37
38 inline int readInt() {
39     int s = 1, c = readChar();
40     int x = 0;
41     if (c == '-') s = -1, c = getChar();
42     while ('0' <= c && c <= '9') x = x * 10 + c - '0', c = getChar();
43     return s == 1 ? x : -x;
44 }
45
46 // 10M int [0..1e9)
47 // cin 3.02
48 // scanf 1.2
49 // cin_sync_with_stdio(false) 0.71
50 // fastRead_getchar 0.53
51 // fastRead_fread 0.15

```

final/template/hashTable.cpp

```

1 template <const int max_size, class HashType, class Data, const Data default_value>
2 struct hashTable {
3     HashType hash[max_size];
4     Data f[max_size];
5     int size;
6
7     int position(HashType H) const {
8         int i = H % max_size;
9         while (hash[i] && hash[i] != H)
10             if (++i == max_size)
11                 i = 0;
12         return i;
13     }
14
15     Data & operator [] (HashType H) {
16         assert(H != 0);
17         int i = position(H);
18         if (!hash[i]) {
19             hash[i] = H;
20             f[i] = default_value;
21             size++;
22         }
23         return f[i];
24     }
25 };
26
27 hashTable<13, int, int, 0> h;

```

final/template/optimizations.cpp

```

1 inline void fasterLLDivMod(unsigned long long x, unsigned y, unsigned &out_d, unsigned &out_m) {
2     unsigned xh = (unsigned)(x >> 32), xl = (unsigned)x,
3         d, m;
4     #ifdef __GNUC__
5         asm(
6             "divl %4; \n\t"
7             : "=a" (d), "=d" (m)
8             : "d" (xh), "a" (xl), "r" (y)
9         );
10    #else
11        __asm {
12            mov edx, dword ptr[xh];
13            mov eax, dword ptr[xl];
14            div dword ptr[y];
15            mov dword ptr[d], eax;
16            mov dword ptr[m], edx;
17        };
18    #endif
19    out_d = d; out_m = m;
20 }
21
22 // have no idea what sse flags are really cool; list of some of them
23 // — very good with bitsets
24 #pragma GCC optimize("O3")
25 #pragma GCC target("sse,sse2,sse3,ssse3,sse4,popcnt,abm,mmx")

```

final/template/useful.cpp

```

1 #include "ext/pb_ds/assoc_container.hpp"
2 using namespace __gnu_pbds;
3
4 template <typename T> using ordered_set = tree<T, null_type, less<T>, rb_tree_tag, tree_order_statistics_node_update>;
5 template <typename K, typename V> using ordered_map = tree<K, V, less<K>, rb_tree_tag, tree_order_statistics_node_update>;
6
7 // HOW TO USE ::
8 // — order_of_key(10) returns the number of elements in set/map strictly less than 10
9 // — *find_by_order(10) returns 10-th smallest element in set/map (0-based)
10
11 bitset<N> a;
12 for (int i = a._Find_first(); i != a.size(); i = a._Find_next(i)) {

```

```

13 cout << i << endl;
14 }

```

final/numeric/fft.cpp

final/template/Template.java

```

1 import java.util.*;
2 import java.io.*;
3
4 public class Template {
5     FastScanner in;
6     PrintWriter out;
7
8     public void solve() throws IOException {
9         int n = in.nextInt();
10        out.println(n);
11    }
12
13    public void run() {
14        try {
15            in = new FastScanner();
16            out = new PrintWriter(System.out);
17
18            solve();
19
20            out.close();
21        } catch (IOException e) {
22            e.printStackTrace();
23        }
24    }
25
26    class FastScanner {
27        BufferedReader br;
28        StringTokenizer st;
29
30        FastScanner() {
31            br = new BufferedReader(new InputStreamReader(←
32            System.in));
33        }
34
35        String next() {
36            while (st == null || !st.hasMoreTokens()) {
37                try {
38                    st = new StringTokenizer(br.readLine());
39                } catch (IOException e) {
40                    e.printStackTrace();
41                }
42            }
43            return st.nextToken();
44        }
45
46        int nextInt() {
47            return Integer.parseInt(next());
48        }
49
50        public static void main(String[] arg) {
51            new Template().run();
52        }
53    }

```

```

1 namespace fft
2 {
3     const int maxBase = 21;
4     const int maxN = 1 << maxBase;
5
6     struct num
7     {
8         dbl x, y;
9         num() {}
10        num(dbl xx, dbl yy): x(xx), y(yy) {}
11        num(dbl alp): x(cos(alp)), y(sin(alp)) {}
12    };
13
14    inline num operator + (num a, num b) { return num(←
15        a.x + b.x, a.y + b.y); }
16    inline num operator - (num a, num b) { return num(←
17        a.x - b.x, a.y - b.y); }
18    inline num operator * (num a, num b) { return num(←
19        a.x * b.x - a.y * b.y, a.x * b.y + a.y * b.x); ←
20    }
21    inline num conj(num a) { return num(a.x, -a.y); }
22
23    const dbl PI = acos(-1);
24
25    num root[maxN];
26    int rev[maxN];
27    bool rootsPrepared = false;
28
29    void prepRoots()
30    {
31        if (rootsPrepared) return;
32        rootsPrepared = true;
33        root[1] = num(1, 0);
34        for (int k = 1; k < maxBase; ++k)
35        {
36            num x(2 * PI / pw(k + 1));
37            for (int i = pw(k - 1); i < pw(k); ++i)
38            {
39                root[2 * i] = root[i];
40                root[2 * i + 1] = root[i] * x;
41            }
42        }
43    }
44
45    int base, N;
46
47    int lastRevN = -1;
48    void prepRev()
49    {
50        if (lastRevN == N) return;
51        lastRevN = N;
52        forn(i, N) rev[i] = (rev[i >> 1] >> 1) + ((i & ←
53        1) << (base - 1));
54    }
55
56    void fft(num *a, num *f)
57    {
58        forn(i, N) f[i] = a[rev[i]];
59        for (int k = 1; k < N; k <= 1) for (int i = 0; ←
60        i < N; i += 2 * k) forn(j, k)
61        {
62            num z = f[i + j + k] * root[j + k];
63            f[i + j + k] = f[i + j] - z;
64            f[i + j] = f[i + j] + z;
65        }
66    }
67
68    num a[maxN], b[maxN], f[maxN], g[maxN];
69    A[maxN], B[maxN], C[maxN];
70
71    void _multMod(int mod)
72    {
73        forn(i, N)
74        {
75            int x = A[i] % mod;
76            a[i] = num(x & (pw(15) - 1), x >> 15);
77        }
78        forn(i, N)
79        {
80            int x = B[i] % mod;
81            b[i] = num(x & (pw(15) - 1), x >> 15);
82        }
83        fft(a, f);
84        fft(b, g);
85
86        forn(i, N)
87        {
88            int j = (N - i) & (N - 1);
89
90

```

```

83     num a1 = (f[i] + conj(f[j])) * num(0.5, 0);
84     num a2 = (f[i] - conj(f[j])) * num(0, -0.5);
85     num b1 = (g[i] + conj(g[j])) * num(0.5 / N, 0) ←
;
86     num b2 = (g[i] - conj(g[j])) * num(0, -0.5 / N ←
);
87     a[j] = a1 * b1 + a2 * b2 * num(0, 1);
88     b[j] = a1 * b2 + a2 * b1;
89 }
90
91 fft(a, f);
92 fft(b, g);
93
94 forn(i, N)
95 {
96     ll aa = f[i].x + 0.5;
97     ll bb = g[i].x + 0.5;
98     ll cc = f[i].y + 0.5;
99     C[i] = (aa + bb % mod * pw(15) + cc % mod * pw ←
(30)) % mod;
100 }
101
102 void prepAB(int n1, int n2)
103 {
104     base = 1;
105     N = 2;
106     while (N < n1 + n2) base++, N <= 1;
107
108     for (int i = n1; i < N; ++i) A[i] = 0;
109     for (int i = n2; i < N; ++i) B[i] = 0;
110
111     prepRoots();
112     prepRev();
113 }
114
115 void mult(int n1, int n2)
116 {
117     prepAB(n1, n2);
118     forn(i, N) a[i] = num(A[i], B[i]);
119     fft(a, f);
120     forn(i, N)
121     {
122         int j = (N - i) & (N - 1);
123         a[i] = (f[j] * f[j] - conj(f[i] * f[i])) * num ←
(0, -0.25 / N);
124     }
125     fft(a, f);
126     forn(i, N) C[i] = (ll)round(f[i].x);
127 }
128
129 void multMod(int n1, int n2, int mod)
130 {
131     prepAB(n1, n2);
132     _multMod(mod);
133 }
134
135 int D[maxN];
136
137 void multLL(int n1, int n2)
138 {
139     prepAB(n1, n2);
140
141     int mod1 = 1.5e9;
142     int mod2 = mod1 + 1;
143
144     _multMod(mod1);
145
146     forn(i, N) D[i] = C[i];
147
148     _multMod(mod2);
149
150     forn(i, N)
151     {
152         C[i] = D[i] + (C[i] - D[i] + (ll)mod2) * (ll) ←
mod1 % mod2 * mod1;
153     }
154 }
155
156 // HOW TO USE ::
157 // — set correct maxBase
158 // — use mult(n1, n2), multMod(n1, n2, mod) and ←
multLL(n1, n2)
159 // — input : A[], B[]
160 // — output : C[]
161 }
162

```

final/numeric/fftint.cpp

```

1 namespace fft
2 {
3     const int mod = 998244353;
4     const int base = 20;
5     const int N = 1 << base;
6     const int ROOT = 646;
7
8     int root[N];
9     int rev[N];
10
11 void init()
12 {
13     forn(i, N) rev[i] = (rev[i >> 1] >> 1) + ((i & ←
1) << (base - 1));
14     int NN = N >> 1;
15     int z = 1;
16     forn(i, NN)
17     {
18         root[i + NN] = z;
19         z = z * (ll)ROOT % mod;
20     }
21     for (int i = NN - 1; i > 0; --i) root[i] = root ←
[2 * i];
22 }
23
24 void fft(int *a, int *f)
25 {
26     forn(i, N) f[i] = a[rev[i]];
27     for (int k = 1; k < N; k <= 1) for (int i = 0; ←
i < N; i += 2 * k) forn(j, k)
28     {
29         int z = f[i + j + k] * (ll)root[j + k] % mod;
30         f[i + j + k] = (f[i + j] - z + mod) % mod;
31         f[i + j] = (f[i + j] + z) % mod;
32     }
33 }
34
35 int A[N], B[N], C[N];
36 int F[N], G[N];
37
38 void _mult(int eq)
39 {
40     fft(A, F);
41     if (eq) forn(i, N) G[i] = F[i];
42     else fft(B, G);
43     int invN = inv(N);
44     forn(i, N) A[i] = F[i] * (ll)G[i] % mod * invN % ←
mod;
45     reverse(A + 1, A + N);
46     fft(A, C);
47 }
48
49 void mult(int n1, int n2, int eq = 0)
50 {
51     for (int i = n1; i < N; ++i) A[i] = 0;
52     for (int i = n2; i < N; ++i) B[i] = 0;
53
54     _mult(eq);
55
56     //forn(i, n1 + n2) C[i] = 0;
57     //forn(i, n1) forn(j, n2) C[i + j] = (C[i + j] + ←
A[i] * (ll)B[j]) % mod;
58 }
59 }

```

final/numeric/blackbox.cpp

```

1 namespace blackbox
2 {
3     int A[N];
4     int B[N];
5     int C[N];
6
7     int magic(int k, int x)
8     {
9         B[k] = x;
10        C[k] = (C[k] + A[0] * (ll)B[k]) % mod;
11        int z = 1;
12        if (k == N - 1) return C[k];
13        while ((k & (z - 1)) == (z - 1))
14        {
15            //mult B[k - z + 1 ... k] x A[z .. 2 * z - 1]
16            forn(i, z) fft::A[i] = A[z + i];
17            forn(i, z) fft::B[i] = B[k - z + 1 + i];
18            fft::multMod(z, z, mod);
19            forn(i, 2 * z - 1) C[k + 1 + i] = (C[k + 1 + i] ←
+ fft::C[i]) % mod;

```

```

20     z <=<= 1;
21     }
22     return C[k];
23 }
24 // A — constant array
25 // magic(k, x):: B[k] = x, returns C[k]
26 // !! WARNING !! better to set N twice the size ←
27 // needed
28 }

```

final/numeric/crt.cpp

```

1 int CRT(int a1, int m1, int a2, int m2) {
2     return (a1 - a2 % m1 + m1) * (ll)rev(m2, m1) % m1 ←
3     * m2 + a2;
4 }

```

final/numeric/mulMod.cpp

```

1 ll mul(ll a, ll b, ll m) { // works for MOD 8e18
2     ll k = (ll)((long double)a * b / m);
3     ll r = a * b - m * k;
4     if (r < 0) r += m;
5     if (r >= m) r -= m;
6     return r;
7 }

```

final/numeric/modReverse.cpp

```

1 int rev(int x, int m) {
2     if (x == 1) return 1;
3     return (1 - rev(m % x, x) * (ll)m) / x + m;
4 }

```

final/numeric/pollard.cpp

```

1 namespace pollard
2 {
3     using math::p;
4
5     vector<pair<ll, int>> getFactors(ll N)
6     {
7         vector<ll> primes;
8
9         const int MX = 1e5;
10        const ll MX2 = MX * (ll)MX;
11
12        assert(MX <= math::maxP && math::pc > 0);
13
14        function<void(ll)> go = [&go, &primes](ll n)
15        {
16            for (ll x : primes) while (n % x == 0) n /= x;
17            if (n == 1) return;
18            if (n > MX2)
19            {
20                auto F = [&](ll x) {
21                    ll k = ((long double)x * x) / n;
22                    ll r = (x * x - k * n + 3) % n;
23                    return r < 0 ? r + n : r;
24                };
25                ll x = mt19937_64()() % n, y = x;
26                const int C = 3 * pow(n, 0.25);
27
28                ll val = 1;
29                for(it, C)
30                {
31                    x = F(x), y = F(y);
32                    if (x == y) continue;
33                    ll delta = abs(x - y);
34                }
35            }
36        }
37    }
38 }

```

```

34 ll k = ((long double)val * delta) / n;
35 val = (val * delta - k * n) % n;
36 if (val < 0) val += n;
37 if (val == 0)
38 {
39     ll g = __gcd(delta, n);
40     go(g), go(n / g);
41     return;
42 }
43 if ((it & 255) == 0)
44 {
45     ll g = __gcd(val, n);
46     if (g != 1)
47     {
48         go(g), go(n / g);
49         return;
50     }
51 }
52 }
53 }
54 primes.pb(n);
55 };
56
57 ll n = N;
58
59 for (int i = 0; i < math::pc && p[i] < MX; ++i) ←
60 if (n % p[i] == 0)
61 {
62     primes.pb(p[i]);
63     while (n % p[i] == 0) n /= p[i];
64 }
65
66 go(n);
67
68 sort(primes.begin(), primes.end());
69
70 vector<pair<ll, int>> res;
71 for (ll x : primes)
72 {
73     int cnt = 0;
74     while (N % x == 0)
75     {
76         cnt++;
77         N /= x;
78     }
79     res.push_back({x, cnt});
80 }
81 return res;
82 }

```

final/numeric/poly.cpp

```

1 struct poly
2 {
3     vi v;
4     poly() {}
5     poly(vi vv)
6     {
7         v = vv;
8     }
9     int size()
10    {
11        return (int)v.size();
12    }
13    poly cut(int maxLen)
14    {
15        if (maxLen < sz(v)) v.resize(maxLen);
16        return *this;
17    }
18    poly norm()
19    {
20        while (sz(v) > 1 && v.back() == 0) v.pop_back();
21        return *this;
22    }
23    inline int& operator [] (int i)
24    {
25        return v[i];
26    }
27    void out(string name="")
28    {
29        stringstream ss;
30        if (sz(name)) ss << name << "=";
31        int fst = 1;
32        for(it, sz(v)) if (v[i])
33        {
34            int x = v[i];
35        }
36    }
37 }

```

```

35     int sgn = 1;
36     if (x > mod / 2) x = mod - x, sgn = -1;
37     if (sgn == -1) ss << "-";
38     else if (!fst) ss << "+";
39     fst = 0;
40     if (!i || x != 1)
41     {
42         ss << x;
43         if (i > 0) ss << "*x";
44         if (i > 1) ss << "^" << i;
45     }
46     else
47     {
48         ss << "x";
49         if (i > 1) ss << "^" << i;
50     }
51 }
52 if (fst) ss << "0";
53 string s;
54 ss >> s;
55 eprintf("%s\n", s.data());
56 }
57 };
58
59 poly operator + (poly A, poly B)
60 {
61     poly C;
62     C.v = vi(max(sz(A), sz(B)));
63     forn(i, sz(C))
64     {
65         if (i < sz(A)) C[i] = (C[i] + A[i]) % mod;
66         if (i < sz(B)) C[i] = (C[i] + B[i]) % mod;
67     }
68     return C.norm();
69 }
70
71 poly operator - (poly A, poly B)
72 {
73     poly C;
74     C.v = vi(max(sz(A), sz(B)));
75     forn(i, sz(C))
76     {
77         if (i < sz(A)) C[i] = (C[i] + A[i]) % mod;
78         if (i < sz(B)) C[i] = (C[i] + mod - B[i]) % mod;
79     }
80     return C.norm();
81 }
82
83 poly operator * (poly A, poly B)
84 {
85     poly C;
86     C.v = vi(sz(A) + sz(B) - 1);
87
88     forn(i, sz(A)) fft::A[i] = A[i];
89     forn(i, sz(B)) fft::B[i] = B[i];
90     fft::multMod(sz(A), sz(B), mod);
91     forn(i, sz(C)) C[i] = fft::C[i];
92     return C.norm();
93 }
94
95 poly inv(poly A, int n) // returns A^{-1} mod x^n
96 {
97     assert(sz(A) && A[0] != 0);
98     A.cut(n);
99
100     auto cutPoly = [](poly &from, int l, int r)
101     {
102         poly R;
103         R.v.resize(r - l);
104         for (int i = l; i < r; ++i)
105         {
106             if (i < sz(from)) R[i - l] = from[i];
107         }
108         return R;
109     };
110
111     function<int(int, int)> rev = [&rev](int x, int m) -> int
112     {
113         if (x == 1) return 1;
114         return (1 - rev(m % x, x) * (ll)m) / x + m;
115     };
116
117     poly R({rev(A[0], mod)});
118     for (int k = 1; k < n; k <= 1)
119     {
120         poly A0 = cutPoly(A, 0, k);
121         poly A1 = cutPoly(A, k, 2 * k);
122         poly H = A0 * R;
123         H = cutPoly(H, k, 2 * k);
124         poly R1 = (((A1 * R).cut(k) + H) * (poly({0}) - -> R)).cut(k);
125         R.v.resize(2 * k);

```

```

126         forn(i, k) R[i + k] = R1[i];
127     }
128     return R.cut(n).norm();
129 }
130
131 pair<poly, poly> divide(poly A, poly B)
132 {
133     if (sz(A) < sz(B)) return {poly({0}), A};
134
135     auto rev = [](poly f)
136     {
137         reverse(all(f.v));
138         return f;
139     };
140
141     poly q = rev((inv(rev(B), sz(A) - sz(B) + 1) * rev(A)).cut(sz(A) - sz(B) + 1));
142     poly r = A - B * q;
143
144     return {q, r};
145 }

```

final/numeric/simplex.cpp

```

1 vector<double> simplex(vector<vector<double>> > a) {
2     int n = a.size() - 1;
3     int m = a[0].size() - 1;
4     vector<int> left(n + 1), up(m + 1);
5     iota(up.begin(), up.end(), 0);
6     iota(left.begin(), left.end(), m);
7     auto pivot = [&](int x, int y) {
8         swap(left[x], up[y]);
9         double k = a[x][y];
10        a[x][y] = 1;
11        vector<int> vct;
12        for (int j = 0; j <= m; j++) {
13            a[x][j] /= k;
14            if (!eq(a[x][j], 0)) vct.push_back(j);
15        }
16        for (int i = 0; i <= n; i++) {
17            if (eq(a[i][y], 0) || i == x) continue;
18            k = a[i][y];
19            a[i][y] = 0;
20            for (int j : vct) a[i][j] -= k * a[x][j];
21        }
22    };
23    while (1) {
24        int x = -1;
25        for (int i = 1; i <= n; i++) if (ls(a[i][0], 0) <=
26            && (x == -1 || a[i][0] < a[x][0])) x = i;
27        if (x == -1) break;
28        int y = -1;
29        for (int j = 1; j <= m; j++) if (ls(a[x][j], 0) <=
30            && (y == -1 || a[x][j] < a[x][y])) y = j;
31        if (y == -1) assert(0); // infeasible
32        pivot(x, y);
33    }
34    while (1) {
35        int y = -1;
36        for (int j = 1; j <= m; j++) if (ls(0, a[0][j]) <=
37            && (y == -1 || a[0][j] > a[0][y])) y = j;
38        if (y == -1) break;
39        int x = -1;
40        for (int i = 1; i <= n; i++) if (ls(0, a[i][y]) <=
41            && (x == -1 || a[i][0] / a[i][y] < a[x][0] / a[x][y])) x = i;
42        if (x == -1) assert(0); // unbounded
43        pivot(x, y);
44    }
45    vector<double> ans(m + 1);
46    for (int i = 1; i <= n; i++) if (left[i] <= m) ans[
47        left[i]] = a[i][0];
48    ans[0] = -a[0][0];
49    return ans;
50 }
51 // j=1..m: x[j]>=0
52 // i=1..n: sum(j=1..m) A[i][j]*x[j] <= A[i][0]
53 // max sum(j=1..m) A[0][j]*x[j]
54 // res[0] is answer
55 // res[1..m] is certificate

```

final/numeric/sumLine.cpp

```

1 // sum(i=0..n-1) (a+b*i) div m
2 ll solve(ll n, ll a, ll b, ll m) {
3     if (b == 0) return n * (a / m);
4     if (a >= m) return n * (a / m) + solve(n, a % m, b ←
5         , m);
6     if (b >= m) return n * (n - 1) / 2 * (b / m) + ←
7         solve(n, a, b % m, m);
8     return solve((a + b * n) / m, (a + b * n) % m, m, ←
9         b);
10 }

```

```

68     n >= 1;
69 }
70 return res;
71 }
72
73 int f(vector<int> t, int m) {
74     vector<int> v = berlekamp(t);
75     vector<int> o = bin(m - 1, v);
76     int res = 0;
77     for (int i = 0; i < (int)o.size(); i++) res = (res ←
78         + 1LL * o[i] * t[i]) % MOD;
79     return res;
80 }

```

final/numeric/berlekamp.cpp

```

1 vector<int> berlekamp(vector<int> s) {
2     int l = 0;
3     vector<int> la(1, 1);
4     vector<int> b(1, 1);
5     for (int r = 1; r <= (int)s.size(); r++) {
6         int delta = 0;
7         for (int j = 0; j <= l; j++) {
8             delta = (delta + 1LL * s[r - 1 - j] * la[j]) % ←
9                 MOD;
10        }
11        b.insert(b.begin(), 0);
12        if (delta != 0) {
13            vector<int> t(max(la.size(), b.size()));
14            for (int i = 0; i < (int)t.size(); i++) {
15                if (i < (int)la.size()) t[i] = (t[i] + la[i ←
16                    ]) % MOD;
17                if (i < (int)b.size()) t[i] = (t[i] - 1LL * ←
18                    delta * b[i] % MOD + MOD) % MOD;
19            }
20            if (2 * l <= r - 1) {
21                b = la;
22                int od = inv(delta);
23                for (int &x : b) x = 1LL * x * od % MOD;
24                l = r - 1;
25            }
26            la = t;
27        }
28        assert((int)la.size() == l + 1);
29        assert(l * 2 + 30 < (int)s.size());
30        reverse(la.begin(), la.end());
31        return la;
32    }
33
34    vector<int> mul(vector<int> a, vector<int> b) {
35        vector<int> c(a.size() + b.size() - 1);
36        for (int i = 0; i < (int)a.size(); i++) {
37            for (int j = 0; j < (int)b.size(); j++) {
38                c[i + j] = (c[i + j] + 1LL * a[i] * b[j]) % ←
39                    MOD;
40            }
41        }
42        vector<int> res(c.size());
43        for (int i = 0; i < (int)res.size(); i++) res[i] = ←
44            c[i] % MOD;
45        return res;
46    }
47
48    vector<int> mod(vector<int> a, vector<int> b) {
49        if (a.size() < b.size()) a.resize(b.size() - 1);
50
51        int o = inv(b.back());
52        for (int i = (int)a.size() - 1; i >= (int)b.size() ←
53            - 1; i--) {
54            if (a[i] == 0) continue;
55            int coef = 1LL * o * (MOD - a[i]) % MOD;
56            for (int j = 0; j < (int)b.size(); j++) {
57                a[i - (int)b.size() + 1 + j] = (a[i - (int)b ←
58                    .size() + 1 + j] + 1LL * coef * b[j]) % MOD;
59            }
60        }
61        while (a.size() >= b.size()) {
62            assert(a.back() == 0);
63            a.pop_back();
64        }
65        return a;
66    }
67
68    vector<int> bin(int n, vector<int> p) {
69        vector<int> res(1, 1);
70        vector<int> a(2); a[1] = 1;
71        while (n) {
72            if (n & 1) res = mod(mul(res, a), p);
73            a = mod(mul(a, a), p);
74        }
75    }

```

final/geom/commonTangents.cpp

```

1  vector<Line> commonTangents(pt A, dbl rA, pt B, dbl rB) {
2
3      vector<Line> res;
4      pt C = B - A;
5      dbl z = C.len2();
6      for (int i = -1; i <= 1; i += 2) {
7          for (int j = -1; j <= 1; j += 2) {
8              dbl r = rB * j - rA * i;
9              dbl d = z - r * r;
10             if (ls(d, 0)) continue;
11             d = sqrt(max(0.01, d));
12             pt magic = pt(r, d) / z;
13             pt v(magic % C, magic * C);
14             dbl CC = (rA * i - v % A) / v.len2();
15             pt O = v * -CC;
16             res.pb(Line(O, 0 + v.rotate()));
17         }
18     }
19     return res;
20 }
21
22 // HOW TO USE ::
23 // --- *D*-----*F*
24 // --- *...* - - - *...*
25 // --- *.....* - - *.....*
26 // --- *.....* - - *.....*
27 // --- *.....* - - *.....*
28 // --- *...A...* - - *...B...*
29 // --- *.....* - - *.....*
30 // --- *.....* - - *.....*
31 // --- *...* - - *...*
32 // --- *C*-----*E*
33 // --- res = {CE, CF, DE, DF}

```

final/geom/halfplaneIntersection.cpp

```

1  int getPart(pt v) {
2      return less(0, v.y) || (equal(0, v.y) && less(v.x, 0));
3  }
4
5  int cmpV(pt a, pt b) {
6      int partA = getPart(a);
7      int partB = getPart(b);
8      if (partA < partB) return -1;
9      if (partA > partB) return 1;
10     if (equal(0, a * b)) return 0;
11     if (0 < a * b) return -1;
12     return 1;
13 }
14
15 double planeInt(vector<Line> l) {
16     int n = l.size();
17     sort(all(l), [](Line a, Line b) {
18         int r = cmpV(a.v, b.v);
19         if (r != 0) return r < 0;
20         return a.O % a.v.rotate() < b.O % a.v.rotate();
21     });
22
23     int cur = 0;
24     for (int i = 0; i < n; i++) {
25         int j = i;
26         for (; i < n && cmpV(l[j].v, l[i].v) == 0 && cmpV(l[i].v, l[j].v) == 0; i++) {
27             l[cur++] = l[i - 1];
28         }
29         n = cur;
30
31         for (int i = 0; i < n; i++)
32             l[i].id = i;
33
34         int flagUp = 0;
35         int flagDown = 0;
36         for (int i = 0; i < n; i++) {
37             int part = getPart(l[i].v);
38             if (part == 1) flagUp = 1;
39             if (part == 0) flagDown = 1;
40         }
41         if (!flagUp || !flagDown) return -1;
42     }

```

```

43     for (int i = 0; i < n; i++) {
44         pt v = l[i].v;
45         pt u = l[(i + 1) % n].v;
46         if (equal(0, v * u) && less(v % u, 0)) {
47             pt dir = l[i].v.rotate();
48             if (lessE(l[(i + 1) % n].O % dir, l[i].O % dir))
49                 return 0;
50             return -1;
51         }
52         if (less(v * u, 0))
53             return -1;
54     }
55
56     cur = 0;
57     vector<Line> st(n * 2);
58     for (int tt = 0; tt < 2; tt++) {
59         for (int i = 0; i < n; i++) {
60             for (; cur >= 2; cur--) {
61                 pt G = st[cur - 1] * l[i];
62                 if (!lessE(st[cur - 2].v * (G - st[cur - 2].v), 0))
63                     break;
64                 st[cur++] = l[i];
65                 if (cur >= 2 && lessE(st[cur - 2].v * st[cur - 1].v, 0)) return 0;
66             }
67         }
68         vector<int> use(n, -1);
69         int left = -1, right = -1;
70         for (int i = 0; i < cur; i++) {
71             if (use[st[i].id] == -1) {
72                 use[st[i].id] = i;
73             }
74             else {
75                 left = use[st[i].id];
76                 right = i;
77                 break;
78             }
79         }
80         vector<Line> tmp;
81         for (int i = left; i < right; i++)
82             tmp.pb(st[i]);
83         vector<pt> res;
84         for (int i = 0; i < (int)tmp.size(); i++)
85             res.pb(tmp[i] * tmp[(i + 1) % tmp.size()]);
86         double area = 0;
87         for (int i = 0; i < (int)res.size(); i++)
88             area += res[i] * res[(i + 1) % res.size()];
89         return area / 2;
90     }

```

final/geom/minDisc.cpp

```

1  pair<pt, dbl> minDisc(vector<pt> p) {
2      int n = p.size();
3      pt O = pt(0, 0);
4      dbl R = 0;
5      random_shuffle(all(p));
6      for (int i = 0; i < n; i++) {
7          if (ls(R, (O - p[i]).len())) {
8              O = p[i];
9              R = 0;
10             for (int j = 0; j < i; j++) {
11                 if (ls(R, (O - p[j]).len())) {
12                     O = (p[i] + p[j]) / 2;
13                     R = (p[i] - p[j]).len() / 2;
14                     for (int k = 0; k < j; k++) {
15                         if (ls(R, (O - p[k]).len())) {
16                             Line l1((p[i] + p[j]) / 2, (p[i] + p[j] - p[k]).rotate());
17                             Line l2((p[k] + p[j]) / 2, (p[k] + p[j] - p[i]).rotate());
18                             O = l1 * l2;
19                             R = (p[i] - O).len();
20                         }
21                     }
22                 }
23             }
24         }
25     }
26     return {O, R};
27 }

```


final/geom/convexHull3D-N2.cpp

```

1
2 struct Plane {
3     pt O, v;
4     vector<int> id;
5 };
6
7 vector<Plane> convexHull3(vector<pt> p) {
8     vector<Plane> res;
9     int n = p.size();
10    for (int i = 0; i < n; i++)
11        p[i].id = i;
12    for (int i = 0; i < 4; i++) {
13        vector<pt> tmp;
14        for (int j = 0; j < 4; j++)
15            if (i != j)
16                tmp.pb(p[j]);
17        res.pb({tmp[0], (tmp[1] - tmp[0]) * (tmp[2] - tmp[0]), {tmp[0].id, tmp[1].id, tmp[2].id}});
18        if ((p[i] - res.back().O) % res.back().v > 0) {
19            res.back().v = res.back().v * -1;
20            swap(res.back().id[0], res.back().id[1]);
21        }
22    }
23    vector<vector<int>> use(n, vector<int>(n, 0));
24    int tmr = 0;
25    for (int i = 4; i < n; i++) {
26        int cur = 0;
27        tmr++;
28        vector<pair<int, int>> curEdge;
29        for (int j = 0; j < sz(res); j++) {
30            if ((p[i] - res[j].O) % res[j].v > 0) {
31                for (int t = 0; t < 3; t++) {
32                    int v = res[j].id[t];
33                    int u = res[j].id[(t + 1) % 3];
34                    use[v][u] = tmr;
35                    curEdge.pb({v, u});
36                }
37            }
38            else {
39                res[cur++] = res[j];
40            }
41        }
42        res.resize(cur);
43        for (auto x: curEdge) {
44            if (use[x.S][x.F] == tmr) continue;
45            res.pb({p[i], (p[x.F] - p[i]) * (p[x.S] - p[i] ←
46        )}, {x.F, x.S, i});
47        }
48    }
49    return res;
50 }
51 // plane in 3d
52 // (A, v) * (B, u) -> (O, n)
53
54 pt n = v * u;
55 pt m = v * n;
56 double t = (B - A) % u / (u % m);
57 pt O = A - m * t;

```

final/geom/polygonArcCut.cpp

```

1
2 struct Meta {
3     int type; // 0 - seg, 1 - circle
4     pt O;
5     dbl R;
6 };
7
8 const Meta SEG = {0, pt(0, 0), 0};
9
10 vector<pair<pt, Meta>> cut(vector<pair<pt, Meta>> p, ←
    Line l) {
11     vector<pair<pt, Meta>> res;
12     int n = p.size();
13     for (int i = 0; i < n; i++) {
14         pt A = p[i].F;
15         pt B = p[(i + 1) % n].F;
16         if (le(0, l.v * (A - l.O))) {
17             if (eq(0, l.v * (A - l.O)) && p[i].S.type == 1 ←
18                 && ls(0, l.v % (p[i].S.O - A)))
19                 res.pb({A, SEG});

```

```

19         else
20             res.pb(p[i]);
21     }
22     if (p[i].S.type == 0) {
23         if (sign(l.v * (A - l.O)) * sign(l.v * (B - l. ←
24             0)) == -1) {
25             pt FF = Line(A, B) * l;
26             res.pb(make_pair(FF, SEG));
27         }
28     }
29     else {
30         pt E, F;
31         if (intCL(p[i].S.O, p[i].S.R, l, E, F)) {
32             if (onArc(p[i].S.O, A, E, B))
33                 res.pb({E, SEG});
34             if (onArc(p[i].S.O, A, F, B))
35                 res.pb({F, p[i].S});
36         }
37     }
38 }
39 return res;

```

final/strings/eertree.cpp

```

1 namespace eertree {
2     const int INF = 1e9;
3     const int N = 5e6 + 10;
4     char _s[N];
5     char *s = _s + 1;
6     int to[N][2];
7     int suf[N], len[N];
8     int sz, last;
9
10    const int odd = 1, even = 2, blank = 3;
11
12    void go(int &u, int pos) {
13        while (u != blank && s[pos - len[u] - 1] != s[ ←
14            pos]) {
15            u = suf[u];
16        }
17    }
18
19    int add(int pos) {
20        go(last, pos);
21        int u = suf[last];
22        go(u, pos);
23        int c = s[pos] - 'a';
24        int res = 0;
25        if (!to[last][c]) {
26            res = 1;
27            to[last][c] = sz;
28            len[sz] = len[last] + 2;
29            suf[sz] = to[u][c];
30            sz++;
31        }
32        last = to[last][c];
33        return res;
34    }
35
36    void init() {
37        to[blank][0] = to[blank][1] = even;
38        len[blank] = suf[blank] = INF;
39        len[even] = 0, suf[even] = odd;
40        len[odd] = -1, suf[odd] = blank;
41        last = even;
42        sz = 4;
43    }

```

final/strings/sufAutomaton.cpp

```

1 namespace SA {
2     const int MAXN = 1 << 18;
3     const int SIGMA = 26;
4
5     int sz, last;
6     int nxt[MAXN][SIGMA];
7     int link[MAXN], len[MAXN], pos[MAXN];
8
9     void init() {

```

```

10  memset(nxt, -1, sizeof(nxt));
11  memset(link, -1, sizeof(link));
12  memset(len, 0, sizeof(len));
13  last = 0;
14  sz = 1;
15  }
16
17  void add(int c) {
18      int cur = sz++;
19      len[cur] = len[last] + 1;
20      pos[cur] = len[cur];
21      int p = last;
22      last = cur;
23      for (; p != -1 && nxt[p][c] == -1; p = link[p]) ←
24          nxt[p][c] = cur;
25      if (p == -1) {
26          link[cur] = 0;
27          return;
28      }
29      int q = nxt[p][c];
30      if (len[p] + 1 == len[q]) {
31          link[cur] = q;
32          return;
33      }
34      int clone = sz++;
35      memcpy(nxt[clone], nxt[q], sizeof(nxt[q]));
36      len[clone] = len[p] + 1;
37      pos[clone] = pos[q];
38      link[clone] = link[q];
39      link[q] = link[cur] = clone;
40      for (; p != -1 && nxt[p][c] == q; p = link[p]) ←
41          nxt[p][c] = clone;
42  }
43
44  int n;
45  string s;
46  int l[MAXN], r[MAXN];
47  int e[MAXN][SIGMA];
48
49  void getSufTree(string _s) {
50      memset(e, -1, sizeof(e));
51      s = _s;
52      n = s.length();
53      reverse(s.begin(), s.end());
54      init();
55      for (int i = 0; i < n; i++) add(s[i] - 'a');
56      reverse(s.begin(), s.end());
57      for (int i = 1; i < sz; i++) {
58          int j = link[i];
59          l[i] = n - pos[i] + len[j];
60          r[i] = n - pos[i] + len[i];
61          e[j][s[l[i]] - 'a'] = i;
62      }
63  }

```

final/strings/duval.cpp

```

1  void duval(string s) {
2      int n = (int) s.length();
3      int i=0;
4      while (i < n) {
5          int j=i+1, k=i;
6          while (j < n && s[k] <= s[j]) {
7              if (s[k] < s[j])
8                  k = j;
9              else
10                 ++k;
11             ++j;
12         }
13         while (i <= k) {
14             cout << s.substr(i, j-k) << ' ';
15             i += j - k;
16         }
17     }
18 }

```

final/graphs/centroid.cpp

```

1  // original author: burunduk1, rewritten by me (←
2  // !!! warning !!! this code is not tested well
3  const int N = 1e5, K = 17;
4
5  int pivot, level[N], parent[N];
6  vector<int> v[N];
7
8  int get_pivot(int x, int xx, int n) {
9      int size = 1;
10     for (int y : v[x])
11     {
12         if (y != xx && level[y] == -1) size += get_pivot←
13             (y, x, n);
14     }
15     if (pivot == -1 && (size * 2 >= n || xx == -1)) ←
16         pivot = x;
17     return size;
18 }
19
20 void build(int x, int xx, int dep, int size) {
21     assert(dep < K);
22     pivot = -1;
23     get_pivot(x, -1, size);
24     x = pivot;
25     level[x] = dep, parent[x] = xx;
26     for (int y : v[x]) if (level[y] == -1)
27     {
28         build(y, x, dep + 1, size / 2);
29     }
30 }

```

final/graphs/dominatorTree.cpp

```

1  namespace domtree {
2      const int K = 18;
3      const int N = 1 << K;
4
5      int n, root;
6      vector<int> e[N], g[N];
7      int sdom[N], dom[N];
8      int p[N][K], h[N], pr[N];
9      int in[N], out[N], tmr, rev[N];
10
11     void init(int _n, int _root) {
12         n = _n;
13         root = _root;
14         tmr = 0;
15         for (int i = 0; i < n; i++) {
16             e[i].clear();
17             g[i].clear();
18             in[i] = -1;
19         }
20     }
21
22     void addEdge(int u, int v) {
23         e[u].push_back(v);
24         g[v].push_back(u);
25     }
26
27     void dfs(int v) {
28         in[v] = tmr++;
29         for (int to : e[v]) {
30             if (in[to] != -1) continue;
31             pr[to] = v;
32             dfs(to);
33         }
34         out[v] = tmr - 1;
35     }
36
37     int lca(int u, int v) {
38         if (h[u] < h[v]) swap(u, v);
39         for (int i = 0; i < K; i++) if ((h[u] - h[v]) & ←
40             (1 << i)) u = p[u][i];
41         if (u == v) return u;
42         for (int i = K - 1; i >= 0; i--) {
43             if (p[u][i] != p[v][i]) {
44                 u = p[u][i];
45                 v = p[v][i];
46             }
47         }
48         return p[u][0];
49     }
50 }

```

```

48 }
49
50 void solve(int _n, int _root, vector<pair<int, int> > _edges) {
51     init(_n, _root);
52     for (auto ed : _edges) addEdge(ed.first, ed.second);
53
54     dfs(root);
55     for (int i = 0; i < n; i++) if (in[i] != -1) rev[in[i]] = i;
56     segtree tr(tmr); // a[i] := min(a[i], x) and return a[i]
57     for (int i = tmr - 1; i >= 0; i--) {
58         int v = rev[i];
59         int cur = i;
60         for (int to : g[v]) {
61             if (in[to] == -1) continue;
62             if (in[to] < in[v]) cur = min(cur, in[to]);
63             else cur = min(cur, tr.get(in[to]));
64         }
65         sdom[v] = rev[cur];
66         tr.upd(in[v], out[v], in[sdom[v]]);
67     }
68     for (int i = 0; i < tmr; i++) {
69         int v = rev[i];
70         if (i == 0) {
71             dom[v] = v;
72             h[v] = 0;
73         } else {
74             dom[v] = lca(sdom[v], pr[v]);
75             h[v] = h[dom[v]] + 1;
76         }
77         p[v][0] = dom[v];
78         for (int j = 1; j < K; j++) p[v][j] = p[p[v][j-1]][j-1];
79     }
80     for (int i = 0; i < n; i++) if (in[i] == -1) dom[i] = -1;
81 }
82

```

final/graphs/generalMatching.cpp

```

1 //COPYPASTED FROM E-MAXX
2 namespace GeneralMatching {
3     const int MAXN = 256;
4     int n;
5     vector<int> g[MAXN];
6     int match[MAXN], p[MAXN], base[MAXN], q[MAXN];
7     bool used[MAXN], blossom[MAXN];
8
9     int lca(int a, int b) {
10         bool used[MAXN] = {0};
11         for (;;) {
12             a = base[a];
13             used[a] = true;
14             if (match[a] == -1) break;
15             a = p[match[a]];
16         }
17         for (;;) {
18             b = base[b];
19             if (used[b]) return b;
20             b = p[match[b]];
21         }
22     }
23
24     void mark_path(int v, int b, int children) {
25         while (base[v] != b) {
26             blossom[base[v]] = blossom[base[match[v]]] = true;
27             p[v] = children;
28             children = match[v];
29             v = p[match[v]];
30         }
31     }
32
33     int find_path(int root) {
34         memset(used, 0, sizeof used);
35         memset(p, -1, sizeof p);
36         for (int i=0; i<n; ++i)
37             base[i] = i;
38
39         used[root] = true;
40         int qh=0, qt=0;
41         q[qt++] = root;
42         while (qh < qt) {

```

```

43         int v = q[qh++];
44         for (size_t i=0; i<g[v].size(); ++i) {
45             int to = g[v][i];
46             if (base[v] == base[to] || match[v] == to) continue;
47             if (to == root || (match[to] != -1 && p[match[to]] != -1)) {
48                 int curbase = lca(v, to);
49                 memset(blossom, 0, sizeof blossom);
50                 mark_path(v, curbase, to);
51                 mark_path(to, curbase, v);
52                 for (int i=0; i<n; ++i)
53                     if (blossom[base[i]]) {
54                         base[i] = curbase;
55                         if (!used[i]) {
56                             used[i] = true;
57                             q[qt++] = i;
58                         }
59                     }
60             }
61             else if (p[to] == -1) {
62                 p[to] = v;
63                 if (match[to] == -1)
64                     return to;
65                 to = match[to];
66                 used[to] = true;
67                 q[qt++] = to;
68             }
69         }
70     }
71     return -1;
72 }
73
74 vector<pair<int, int> > solve(int _n, vector<pair<int, int> > edges) {
75     n = _n;
76     for (int i = 0; i < n; i++) g[i].clear();
77     for (auto o : edges) {
78         g[o.first].push_back(o.second);
79         g[o.second].push_back(o.first);
80     }
81     memset(match, -1, sizeof match);
82     for (int i=0; i<n; ++i) {
83         if (match[i] == -1) {
84             int v = find_path(i);
85             while (v != -1) {
86                 int pv = p[v], ppv = match[pv];
87                 match[v] = pv, match[pv] = v;
88                 v = ppv;
89             }
90         }
91     }
92     vector<pair<int, int> > ans;
93     for (int i = 0; i < n; i++) {
94         if (match[i] > i) {
95             ans.push_back(make_pair(i, match[i]));
96         }
97     }
98     return ans;
99 }
100

```

final/graphs/heavyLight.cpp

```

1 namespace hld {
2     const int N = 1 << 17;
3     int par[N], heavy[N], h[N];
4     int root[N], pos[N];
5     int n;
6     vector<vector<int> > e;
7     segtree tree;
8
9     int dfs(int v) {
10         int sz = 1, mx = 0;
11         for (int to : e[v]) {
12             if (to == par[v]) continue;
13             par[to] = v;
14             h[to] = h[v] + 1;
15             int cur = dfs(to);
16             if (cur > mx) heavy[v] = to, mx = cur;
17             sz += cur;
18         }
19         return sz;
20     }
21
22     template <typename T>
23     void path(int u, int v, T op) {

```

```

24 for (; root[u] != root[v]; v = par[root[v]]) {
25     if (h[root[u]] > h[root[v]]) swap(u, v);
26     op(pos[root[v]], pos[v] + 1);
27 }
28 if (h[u] > h[v]) swap(u, v);
29 op(pos[u], pos[v] + 1);
30 }
31
32 void init(vector<vector<int>> &e) {
33     e = _e;
34     n = e.size();
35     tree = segtree(n);
36     memset(heavy, -1, sizeof(heavy[0]) * n);
37     par[0] = -1;
38     h[0] = 0;
39     dfs(0);
40     for (int i = 0, cpos = 0; i < n; i++) {
41         if (par[i] == -1 || heavy[par[i]] != i) {
42             for (int j = i; j != -1; j = heavy[j]) {
43                 root[j] = i;
44                 pos[j] = cpos++;
45             }
46         }
47     }
48 }
49
50 void add(int v, int x) {
51     tree.add(pos[v], x);
52 }
53
54 int get(int u, int v) {
55     int res = 0;
56     path(u, v, [&](int l, int r) {
57         res = max(res, tree.get(l, r));
58     });
59     return res;
60 }
61 }

```

final/graphs/hungary.cpp

```

1 namespace hungary
2 {
3     const int N = 210;
4
5     int a[N][N];
6     int ans[N];
7
8     int calc(int n, int m)
9     {
10         ++n, ++m;
11         vi u(n), v(m), p(m), prev(m);
12         for (int i = 1; i < n; ++i)
13         {
14             p[0] = i;
15             int x = 0;
16             vi mn(m, inf);
17             vi was(m, 0);
18             while (p[x])
19             {
20                 was[x] = 1;
21                 int ii = p[x], dd = inf, y = 0;
22                 for (int j = 1; j < m; ++j) if (!was[j])
23                 {
24                     int cur = a[ii][j] - u[ii] - v[j];
25                     if (cur < mn[j]) mn[j] = cur, prev[j] = x;
26                     if (mn[j] < dd) dd = mn[j], y = j;
27                 }
28                 forn(j, m)
29                 {
30                     if (was[j]) u[p[j]] += dd, v[j] -= dd;
31                     else mn[j] -= dd;
32                 }
33                 x = y;
34             }
35             while (x)
36             {
37                 int y = prev[x];
38                 p[x] = p[y];
39                 x = y;
40             }
41         }
42         for (int j = 1; j < m; ++j)
43         {
44             ans[p[j]] = j;
45         }
46         return -v[0];
47     }
48 }

```

```

47 }
48 // HOW TO USE ::
49 // --- set values to a[1..n][1..m] (n <= m)
50 // --- run calc(n, m) to find MINIMUM
51 // --- to restore permutation use ans[]
52 // --- everything works on negative numbers
53
54 // !! i don't understand this code, it's ←
55 // copped from e-maxx (and rewritten by enot110←)

```

final/graphs/retro.cpp

```

1 namespace retro
2 {
3     const int N = 4e5 + 10;
4
5     vi v[N];
6     vi vrev[N];
7
8     void add(int x, int y)
9     {
10         v[x].pb(y);
11         vrev[y].pb(x);
12     }
13
14     const int UD = 0;
15     const int WIN = 1;
16     const int LOSE = 2;
17
18     int res[N];
19     int moves[N];
20     int deg[N];
21     int q[N], st, en;
22
23     void calc(int n)
24     {
25         forn(i, n) deg[i] = sz(v[i]);
26         st = en = 0;
27         forn(i, n) if (!deg[i])
28         {
29             q[en++] = i;
30             res[i] = LOSE;
31         }
32         while (st < en)
33         {
34             int x = q[st++];
35             for (int y : vrev[x])
36             {
37                 if (res[y] == UD && (res[x] == LOSE || (--deg[y] == 0 && res[x] == WIN)))
38                 {
39                     res[y] = 3 - res[x];
40                     moves[y] = moves[x] + 1;
41                     q[en++] = y;
42                 }
43             }
44         }
45     }
46 }

```

final/graphs/smith.cpp

```

1 const int N = 1e5 + 10;
2
3 struct graph
4 {
5     int n;
6
7     vi v[N];
8     vi vrev[N];
9
10    void read()
11    {
12        int m;
13        scanf("%d%d", &n, &m);
14        forn(i, m)
15        {
16            int x, y;
17            scanf("%d%d", &x, &y);
18        }
19    }
20 }

```

```

18     --x, --y;
19     v[x].pb(y);
20     vrev[y].pb(x);
21 }
22 }
23
24 int deg[N], cnt[N], used[N], f[N];
25 int q[N], st, en;
26
27 set<int> s[N];
28
29 void calc()
30 {
31     forn(x, n) f[x] = -1, cnt[x] = 0;
32     int val = 0;
33     while (1)
34     {
35         st = en = 0;
36         forn(x, n)
37         {
38             deg[x] = 0;
39             used[x] = 0;
40             for (int y : v[x]) if (f[y] == -1) deg[x]++;
41         }
42         forn(x, n) if (!deg[x] && f[x] == -1 && cnt[x] <= val)
43         {
44             q[en++] = x;
45             f[x] = val;
46         }
47         if (!en) break;
48         while (st < en)
49         {
50             int x = q[st];
51             st++;
52             for (int y : vrev[x])
53             {
54                 if (used[y] == 0 && f[y] == -1)
55                 {
56                     used[y] = 1;
57                     cnt[y]++;
58                     for (int z : vrev[y])
59                     {
60                         deg[z]--;
61                         if (f[z] == -1 && deg[z] == 0 && cnt[z] <= val)
62                         {
63                             f[z] = val;
64                             q[en++] = z;
65                         }
66                     }
67                 }
68             }
69             val++;
70         }
71     }
72     forn(x, n) eprintf("%d%c", f[x], " \n"[x + 1 == n]);
73     forn(x, n) if (f[x] == -1)
74     {
75         for (int y : v[x]) if (f[y] != -1) s[x].insert(f[y]);
76     }
77 }
78 } g1, g2;
79
80 string get(int x, int y)
81 {
82     int f1 = g1.f[x], f2 = g2.f[y];
83     if (f1 == -1 && f2 == -1) return "draw";
84     if (f1 == -1) {
85         if (g1.s[x].count(f2)) return "first";
86         return "draw";
87     }
88     if (f2 == -1) {
89         if (g2.s[y].count(f1)) return "first";
90         return "draw";
91     }
92     if (f1 ^ f2) return "first";
93     return "second";
94 }

```

final/graphs/twoChineseFast.cpp

```

1 namespace twoc {
2     struct Heap {
3         static Heap* null;

```

```

4         ll x, xadd;
5         int ver, h;
6         /* ANS */ int ei;
7         Heap *l, *r;
8         Heap(ll xx, int vv) : x(xx), xadd(0), ver(vv), h(1), l(null), r(null) {}
9         Heap(const char*) : x(0), xadd(0), ver(0), h(0), l(this), r(this) {}
10        void add(ll a) { x += a; xadd += a; }
11        void push() {
12            if (l != null) l->add(xadd);
13            if (r != null) r->add(xadd);
14            xadd = 0;
15        }
16    };
17    Heap *Heap::null = new Heap("wqeqw");
18    Heap* merge(Heap *l, Heap *r) {
19        if (l == Heap::null) return r;
20        if (r == Heap::null) return l;
21        l->push(); r->push();
22        if (l->x > r->x)
23            swap(l, r);
24        l->r = merge(l->r, r);
25        if (l->l->h < l->r->h)
26            swap(l->l, l->r);
27        l->h = l->r->h + 1;
28        return l;
29    }
30    Heap *pop(Heap *h) {
31        h->push();
32        return merge(h->l, h->r);
33    }
34    const int N = 666666;
35    struct DSU {
36        int p[N];
37        void init(int nn) { iota(p, p + nn, 0); }
38        int get(int x) { return p[x] == x ? x : p[x] = get(p[x]); }
39        void merge(int x, int y) { p[get(y)] = get(x); }
40    } dsu;
41    Heap *eb[N];
42    int n;
43    /* ANS */ struct Edge {
44        /* ANS */ int x, y;
45        /* ANS */ ll c;
46        /* ANS */ };
47    /* ANS */ vector<Edge> edges;
48    /* ANS */ int answer[N];
49    void init(int nn) {
50        n = nn;
51        dsu.init(n);
52        fill(eb, eb + n, Heap::null);
53        edges.clear();
54    }
55    void addEdge(int x, int y, ll c) {
56        Heap *h = new Heap(c, x);
57        /* ANS */ h->ei = sz(edges);
58        /* ANS */ edges.push_back({x, y, c});
59        eb[y] = merge(eb[y], h);
60    }
61    ll solve(int root = 0) {
62        ll ans = 0;
63        static int done[N], pv[N];
64        memset(done, 0, sizeof(int) * n);
65        done[root] = 1;
66        int tt = 1;
67        /* ANS */ int cnum = 0;
68        /* ANS */ static vector<ipair> eout[N];
69        /* ANS */ for (int i = 0; i < n; ++i) eout[i].clear();
70        for (int i = 0; i < n; ++i) {
71            int v = dsu.get(i);
72            if (done[v]) continue;
73            ++tt;
74            while (true) {
75                done[v] = tt;
76                int nv = -1;
77                while (eb[v] != Heap::null) {
78                    nv = dsu.get(eb[v]->ver);
79                    if (nv == v) {
80                        eb[v] = pop(eb[v]);
81                        continue;
82                    }
83                }
84                break;
85            }
86            if (nv == -1)
87                return LINF;
88            ans += eb[v]->x;
89            eb[v]->add(-eb[v]->x);
90            /* ANS */ int ei = eb[v]->ei;
91            /* ANS */ eout[edges[ei].x].push_back({++cnum, ei});

```

```

92     if (!done[nv]) {
93         pv[v] = nv;
94         v = nv;
95         continue;
96     }
97     if (done[nv] != tt)
98         break;
99     int v1 = nv;
100    while (v1 != v) {
101        eb[v] = merge(eb[v], eb[v1]);
102        dsu.merge(v, v1);
103        v1 = dsu.get(pv[v1]);
104    }
105 }
106
107 /* ANS */ memset(answer, -1, sizeof(int) * n);
108 /* ANS */ answer[root] = 0;
109 /* ANS */ set<ipair> es(all(eout[root]));
110 /* ANS */ while (!es.empty()) {
111     /* ANS */ auto it = es.begin();
112     /* ANS */ int ei = it->second;
113     /* ANS */ es.erase(it);
114     /* ANS */ int nv = edges[ei].y;
115     /* ANS */ if (answer[nv] != -1)
116         continue;
117     /* ANS */ answer[nv] = ei;
118     /* ANS */ es.insert(all(eout[nv]));
119 }
120 /* ANS */ answer[root] = -1;
121 return ans;
122 }
123
124 /* Usage: twoc::init(vertex_count);
125 * twoc::addEdge(v1, v2, cost);
126 * twoc::solve(root); - returns cost or LINF
127 * twoc::answer contains index of ingoing edge for<=
128 * each vertex
129 */

```

final/graphs/linkcut.cpp

```

1  #include <iostream>
2  #include <cstdio>
3  #include <cassert>
4
5  using namespace std;
6
7  // BEGIN ALGO
8
9  const int MAXN = 110000;
10
11  typedef struct _node{
12      _node *l, *r, *p, *pp;
13      int size; bool rev;
14      _node();
15      explicit _node(nullptr_t){
16          l = r = p = pp = this;
17          size = rev = 0;
18      }
19      void push(){
20          if (rev){
21              l->rev ^= 1; r->rev ^= 1;
22              rev = 0; swap(l, r);
23          }
24      }
25      void update();
26  }* node;
27  node None = new _node(nullptr);
28  node v2n[MAXN];
29  _node::_node(){
30      l = r = p = pp = None;
31      size = 1; rev = false;
32  }
33  void _node::update(){
34      size = (this != None) + l->size + r->size;
35      l->p = r->p = this;
36  }
37  void rotate(node v){
38      assert(v != None && v->p != None);
39      assert(!v->rev); assert(!v->p->rev);
40      node u = v->p;
41      if (v == u->l)
42          u->l = v->r, v->r = u;
43      else
44          u->r = v->l, v->l = u;
45      swap(u->p, v->p); swap(v->pp, u->pp);
46      if (v->p != None){

```

```

47          assert(v->p->l == u || v->p->r == u);
48          if (v->p->r == u) v->p->r = v;
49          else v->p->l = v;
50      }
51      u->update(); v->update();
52  }
53  void bigRotate(node v){
54      assert(v->p != None);
55      v->p->p->push();
56      v->p->push();
57      v->push();
58      if (v->p->p != None){
59          if ((v->p->l == v) ^ (v->p->p->r == v->p))
60              rotate(v->p);
61          else
62              rotate(v);
63      }
64      rotate(v);
65  }
66  inline void Splay(node v){
67      while (v->p != None) bigRotate(v);
68  }
69  inline void splitAfter(node v){
70      v->push();
71      Splay(v);
72      v->r->p = None;
73      v->r->pp = v;
74      v->r = None;
75      v->update();
76  }
77  void expose(int x){
78      node v = v2n[x];
79      splitAfter(v);
80      while (v->pp != None){
81          assert(v->p == None);
82          splitAfter(v->pp);
83          assert(v->pp->r == None);
84          assert(v->pp->p == None);
85          assert(!v->pp->rev);
86          v->pp->r = v;
87          v->pp->update();
88          v = v->pp;
89          v->r->pp = None;
90      }
91      assert(v->p == None);
92      Splay(v2n[x]);
93  }
94  inline void makeRoot(int x){
95      expose(x);
96      assert(v2n[x]->p == None);
97      assert(v2n[x]->pp == None);
98      assert(v2n[x]->r == None);
99      v2n[x]->rev ^= 1;
100  }
101  inline void link(int x, int y){
102      makeRoot(x); v2n[x]->pp = v2n[y];
103  }
104  inline void cut(int x, int y){
105      expose(x);
106      Splay(v2n[y]);
107      if (v2n[y]->pp != v2n[x]){
108          swap(x, y);
109          expose(x);
110          Splay(v2n[y]);
111          assert(v2n[y]->pp == v2n[x]);
112      }
113      v2n[y]->pp = None;
114  }
115  inline int get(int x, int y){
116      if (x == y) return 0;
117      makeRoot(x);
118      expose(y); expose(x);
119      Splay(v2n[y]);
120      if (v2n[y]->pp != v2n[x]) return -1;
121      return v2n[y]->size;
122  }
123  // END ALGO
124
125  _node mem[MAXN];
126
127
128  int main(){
129      freopen("linkcut.in", "r", stdin);
130      freopen("linkcut.out", "w", stdout);
131
132      int n, m;
133      scanf("%d %d", &n, &m);
134
135      for (int i = 0; i < n; i++)
136          v2n[i] = &mem[i];
137
138      for (int i = 0; i < m; i++){
139          int a, b;

```

```

140 if (scanf(" link %d %d",&a,&b) == 2)
141 link(a-1,b-1);
142 else if (scanf(" cut %d %d",&a,&b) == 2)
143 cut(a-1,b-1);
144 else if (scanf(" get %d %d",&a,&b) == 2)
145 printf("%d\n",get(a-1,b-1));
146 else
147 assert(false);
148 }
149 return 0;
150 }

```

final/graphs/chordaltree.cpp

```

1 void chordaltree(vector<vector<int>> e) {
2     int n = e.size();
3
4     vector<int> mark(n);
5     set<pair<int, int>> st;
6     for (int i = 0; i < n; i++) st.insert({-mark[i], i});
7
8     vector<int> vct(n);
9     vector<pair<int, int>> ted;
10    vector<vector<int>> who(n);
11    vector<vector<int>> verts(1);
12    vector<int> cliq(n, -1);
13    cliq.push_back(0);
14    vector<int> last(n + 1, n);
15    int prev = n + 1;
16    for (int i = n - 1; i >= 0; i--) {
17        int x = st.begin()->second;
18        st.erase(st.begin());
19        if (mark[x] <= prev) {
20            vector<int> cur = who[x];
21            cur.push_back(x);
22            verts.push_back(cur);
23            ted.push_back({cliq[last[x]], (int)verts.size() - 1});
24        } else {
25            verts.back().push_back(x);
26        }
27        for (int y : e[x]) {
28            if (cliq[y] != -1) continue;
29            who[y].push_back(x);
30            st.erase({-mark[y], y});
31            mark[y]++;
32            st.insert({-mark[y], y});
33            last[y] = x;
34        }
35        prev = mark[x];
36        vct[i] = x;
37        cliq[x] = (int)verts.size() - 1;
38    }
39
40    int k = verts.size();
41    vector<int> pr(k);
42    vector<vector<int>> g(k);
43    for (auto o : ted) {
44        pr[o.second] = o.first;
45        g[o.first].push_back(o.second);
46    }
47 }

```

```

dbl Simpson() { return (F(-1) + 4 * F(0) + F(1)) / 6;
} dbl Runge2() { return (F(-sqrtl(1.0 / 3)) + F(sqrtl(1.0 / 3))) / 2; }
dbl Runge3() { return (F(-sqrtl(3.0 / 5)) * 5 + F(0) * 8 + F(sqrtl(3.0 / 5)) * 5) / 18; }

```

Simpson и Runge2 – точны для полиномов степени ≤ 3
Runge3 – точен для полиномов степени ≤ 5

Явный Рунге-Кутты четвертого порядка, ошибка $O(h^4)$

$y' = f(x, y)$ $y_{n+1} = y_n + (k_1 + 2 * k_2 + 2 * k_3 + k_4) * h / 6$

$k_1 = f(x_n, y_n)$ $k_2 = f(x_n + h/2, y_n + h/2 * k_1)$ $k_3 = f(x_n + h/2, y_n + h/2 * k_2)$ $k_4 = f(x_n + h, y_n + h * k_3)$

Методы Адамса-Башфорта

$y_{n+3} = y_{n+2} + h * (23/12 * f(x_{n+2}, y_{n+2}) - 4/3 * f(x_{n+1}, y_{n+1}) + 5/12 * f(x_n, y_n))$
 $y_{n+4} = y_{n+3} + h * (55/24 * f(x_{n+3}, y_{n+3}) - 59/24 * f(x_{n+2}, y_{n+2}) + 37/24 * f(x_{n+1}, y_{n+1}) - 3/8 * f(x_n, y_n))$
 $y_{n+5} = y_{n+4} + h * (1901/720 * f(x_{n+4}, y_{n+4}) - 1387/360 * f(x_{n+3}, y_{n+3}) + 109/30 * f(x_{n+2}, y_{n+2}) - 637/360 * f(x_{n+1}, y_{n+1}) + 251/720 * f(x_n, y_n))$

Извлечение корня по простому модулю (от Серёжи) $3 \leq p$, $1 \leq a < p$, найти $x^2 = a$

1) Если $a^{((p-1)/2)} \neq 1$, return -1
2) Выбрать случайный $1 \leq i < p$
3) $T(x) = (x+i)^{((p-1)/2)} \bmod (x^2 - a) = bx + c$
4) Если $b \neq 0$ то вернуть c/b , иначе к шагу 2)

Иногда вместо того чтобы считать первообразный у простого числа, можно написать чекер ответа и перебирать случайный первообразный.

Не заходит FFT по TL-ю – чекнуть что стоит double, а не long double

mt19937 генерит случайный unsigned int, если хочется больше есть mt19937_64

Иногда можно представить ответ в виде многочлена и вместо подсчета самих k-тов посчитать значения и проинтерполировать

Перед сабмитом чекнуть что все выводится в printf, а не eprintf!!!

Лемма Бернсайда:

Группа G действует на множество X Тогда число классов эквивалентности $= (\sum |f(g)| \text{ for } g \in G) / |G|$ где $f(g) = \text{число } x \text{ (из } X) : g(x) = x$

Число простых быстрее $O(n)$:

$dp(n, k)$ – число чисел от 1 до n в которых все простые $\geq p[k]$
 $dp(n, 1) = n$
 $dp(n, j) = dp(n, j+1) + dp(n / p[j], j)$, т. е. $dp(n, j+1) = dp(n, j) - dp(n / p[j], j)$

Если $p[j], p[k] > \sqrt{n}$ то $dp(n, j) + j == dp(n, k) + k$

Хуяришь все оптимайзы сверху, но не считаешь глубже $dp(n, k)$, $n < K$ Потом фенвиком+сортировкой подсчитываешь за $(K+Q)\log$ все эти запросы Хуяришь во второй раз, но на этот раз берешь прекальканные значения

Если $\sqrt{n} < p[k] < n$ то (число простых до n) $= dp(n, k) + k - 1$

Чиселки:

Table of Integrals*

Basic Forms

$$\int x^n dx = \frac{1}{n+1} x^{n+1} \quad (1)$$

$$\int \frac{1}{x} dx = \ln |x| \quad (2)$$

$$\int u dv = uv - \int v du \quad (3)$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln |ax+b| \quad (4)$$

Integrals of Rational Functions

$$\int \frac{1}{(x+a)^2} dx = -\frac{1}{x+a} \quad (5)$$

$$\int (x+a)^n dx = \frac{(x+a)^{n+1}}{n+1}, n \neq -1 \quad (6)$$

$$\int x(x+a)^n dx = \frac{(x+a)^{n+1}((n+1)x-a)}{(n+1)(n+2)} \quad (7)$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x \quad (8)$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} \quad (9)$$

$$\int \frac{x}{a^2+x^2} dx = \frac{1}{2} \ln |a^2+x^2| \quad (10)$$

$$\int \frac{x^2}{a^2+x^2} dx = x - a \tan^{-1} \frac{x}{a} \quad (11)$$

$$\int \frac{x^3}{a^2+x^2} dx = \frac{1}{2} x^2 - \frac{1}{2} a^2 \ln |a^2+x^2| \quad (12)$$

$$\int \frac{1}{ax^2+bx+c} dx = \frac{2}{\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} \quad (13)$$

$$\int \frac{1}{(x+a)(x+b)} dx = \frac{1}{b-a} \ln \frac{a+x}{b+x}, a \neq b \quad (14)$$

$$\int \frac{x}{(x+a)^2} dx = \frac{a}{a+x} + \ln |a+x| \quad (15)$$

$$\int \frac{x}{ax^2+bx+c} dx = \frac{1}{2a} \ln |ax^2+bx+c| - \frac{b}{a\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} \quad (16)$$

Integrals with Roots

$$\int \sqrt{x-a} dx = \frac{2}{3} (x-a)^{3/2} \quad (17)$$

$$\int \frac{1}{\sqrt{x \pm a}} dx = 2\sqrt{x \pm a} \quad (18)$$

$$\int \frac{1}{\sqrt{a-x}} dx = -2\sqrt{a-x} \quad (19)$$

$$\int x\sqrt{x-a} dx = \frac{2}{3} a(x-a)^{3/2} + \frac{2}{5} (x-a)^{5/2} \quad (20)$$

$$\int \sqrt{ax+bd} dx = \left(\frac{2b}{3a} + \frac{2x}{3} \right) \sqrt{ax+b} \quad (21)$$

$$\int (ax+b)^{3/2} dx = \frac{2}{5a} (ax+b)^{5/2} \quad (22)$$

$$\int \frac{x}{\sqrt{x \pm a}} dx = \frac{2}{3} (x \mp 2a) \sqrt{x \pm a} \quad (23)$$

$$\int \sqrt{\frac{x}{a-x}} dx = -\sqrt{x(a-x)} - a \tan^{-1} \frac{\sqrt{x(a-x)}}{x-a} \quad (24)$$

$$\int \sqrt{\frac{x}{a+x}} dx = \sqrt{x(a+x)} - a \ln [\sqrt{x} + \sqrt{x+a}] \quad (25)$$

$$\int x\sqrt{ax+bd} dx = \frac{2}{15a^2} (-2b^2 + abx + 3a^2x^2) \sqrt{ax+b} \quad (26)$$

$$\int \sqrt{x(ax+b)} dx = \frac{1}{4a^{3/2}} \left[(2ax+b) \sqrt{ax(ax+b)} - b^2 \ln |a\sqrt{x} + \sqrt{a(ax+b)}| \right] \quad (27)$$

$$\int \sqrt{x^3(ax+b)} dx = \left[\frac{b}{12a} - \frac{b^2}{8a^2x} + \frac{x}{3} \right] \sqrt{x^3(ax+b)} + \frac{b^3}{8a^{5/2}} \ln |a\sqrt{x} + \sqrt{a(ax+b)}| \quad (28)$$

$$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} x \sqrt{x^2 \pm a^2} \pm \frac{1}{2} a^2 \ln |x + \sqrt{x^2 \pm a^2}| \quad (29)$$

$$\int \sqrt{a^2 - x^2} dx = \frac{1}{2} x \sqrt{a^2 - x^2} + \frac{1}{2} a^2 \tan^{-1} \frac{x}{\sqrt{a^2 - x^2}} \quad (30)$$

$$\int x\sqrt{x^2 \pm a^2} dx = \frac{1}{3} (x^2 \pm a^2)^{3/2} \quad (31)$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln |x + \sqrt{x^2 \pm a^2}| \quad (32)$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} \quad (33)$$

$$\int \frac{x}{\sqrt{x^2 \pm a^2}} dx = \sqrt{x^2 \pm a^2} \quad (34)$$

$$\int \frac{x}{\sqrt{a^2 - x^2}} dx = -\sqrt{a^2 - x^2} \quad (35)$$

$$\int \frac{x^2}{\sqrt{x^2 \pm a^2}} dx = \frac{1}{2} x \sqrt{x^2 \pm a^2} \mp \frac{1}{2} a^2 \ln |x + \sqrt{x^2 \pm a^2}| \quad (36)$$

$$\int \sqrt{ax^2+bx+cd} dx = \frac{b+2ax}{4a} \sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8a^{3/2}} \ln |2ax+b+2\sqrt{a(ax^2+bx+c)}| \quad (37)$$

$$\int x\sqrt{ax^2+bx+c} dx = \frac{1}{48a^{5/2}} \left(2\sqrt{a}\sqrt{ax^2+bx+c} \times (-3b^2+2abx+8a(c+ax^2)) + 3(b^3-4abc) \ln |b+2ax+2\sqrt{a}\sqrt{ax^2+bx+c}| \right) \quad (38)$$

$$\int \frac{1}{\sqrt{ax^2+bx+c}} dx = \frac{1}{\sqrt{a}} \ln |2ax+b+2\sqrt{a(ax^2+bx+c)}| \quad (39)$$

$$\int \frac{x}{\sqrt{ax^2+bx+c}} dx = \frac{1}{a} \sqrt{ax^2+bx+c} - \frac{b}{2a^{3/2}} \ln |2ax+b+2\sqrt{a(ax^2+bx+c)}| \quad (40)$$

$$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}} \quad (41)$$

Integrals with Logarithms

$$\int \ln ax dx = x \ln ax - x \quad (42)$$

$$\int \frac{\ln ax}{x} dx = \frac{1}{2} (\ln ax)^2 \quad (43)$$

$$\int \ln(ax+b) dx = \left(x + \frac{b}{a} \right) \ln(ax+b) - x, a \neq 0 \quad (44)$$

$$\int \ln(x^2+a^2) dx = x \ln(x^2+a^2) + 2a \tan^{-1} \frac{x}{a} - 2x \quad (45)$$

$$\int \ln(x^2-a^2) dx = x \ln(x^2-a^2) + a \ln \frac{x+a}{x-a} - 2x \quad (46)$$

$$\int \ln(ax^2+bx+c) dx = \frac{1}{a} \sqrt{4ac-b^2} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} - 2x + \left(\frac{b}{2a} + x \right) \ln(ax^2+bx+c) \quad (47)$$

$$\int x \ln(ax+b) dx = \frac{bx}{2a} - \frac{1}{4} x^2 + \frac{1}{2} \left(x^2 - \frac{b^2}{a^2} \right) \ln(ax+b) \quad (48)$$

$$\int x \ln(a^2-b^2x^2) dx = -\frac{1}{2} x^2 + \frac{1}{2} \left(x^2 - \frac{a^2}{b^2} \right) \ln(a^2-b^2x^2) \quad (49)$$

Integrals with Exponentials

$$\int e^{ax} dx = \frac{1}{a} e^{ax} \quad (50)$$

$$\int \sqrt{x} e^{ax} dx = \frac{1}{a} \sqrt{x} e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}} \operatorname{erf}(i\sqrt{ax}),$$

where $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$ (51)

$$\int x e^x dx = (x-1)e^x \quad (52)$$

$$\int x e^{ax} dx = \left(\frac{x}{a} - \frac{1}{a^2} \right) e^{ax} \quad (53)$$

$$\int x^2 e^x dx = (x^2 - 2x + 2) e^x \quad (54)$$

$$\int x^2 e^{ax} dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3} \right) e^{ax} \quad (55)$$

$$\int x^3 e^x dx = (x^3 - 3x^2 + 6x - 6) e^x \quad (56)$$

$$\int x^n e^{ax} dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} dx \quad (57)$$

$$\int x^n e^{ax} dx = \frac{(-1)^n}{a^{n+1}} \Gamma[1+n, -ax],$$

where $\Gamma(a, x) = \int_x^\infty t^{a-1} e^{-t} dt$ (58)

$$\int e^{ax^2} dx = -\frac{i\sqrt{\pi}}{2\sqrt{a}} \operatorname{erf}(i\sqrt{a}x) \quad (59)$$

$$\int e^{-ax^2} dx = \frac{\sqrt{\pi}}{2\sqrt{a}} \operatorname{erf}(x\sqrt{a}) \quad (60)$$

$$\int x e^{-ax^2} dx = -\frac{1}{2a} e^{-ax^2} \quad (61)$$

$$\int x^2 e^{-ax^2} dx = \frac{1}{4} \sqrt{\frac{\pi}{a^3}} \operatorname{erf}(x\sqrt{a}) - \frac{x}{2a} e^{-ax^2} \quad (62)$$

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Integrals with Trigonometric Functions

$$\begin{aligned}
 & \text{Фибоначчи } 45: 1134903170 \quad 46: 1836311903 \\
 & 47: 2971215073 \quad 91: 4660046610375330309 \\
 & 754011380474634642993: 12200160415121876738 \\
 & \text{Числа с кучей делителей } 20: d(12)=6 \quad 50: d(48)=10 \\
 & 100: d(60)=12 \quad 1000: d(840)=32 \quad 10^4: d(9240)=64 \quad 10^5: \\
 & d(83160)=128 \quad 10^6: d(720720)=240 \quad 10^7: d(8648640)=448 \\
 & 10^8: d(91891800)=768 \quad 10^9: d(931170240)=1344 \quad 10^{11}: \\
 & d(97772875200)=4032 \quad 10^{12}: d(963761198400)=6720 \\
 & 10^{15}: d(866421317361600)=26880 \quad 10^{18}: \\
 & d(897612484786617600)=103680 \\
 & \text{Bell numbers: } 4a-0: 1, 1, 3, 5, 15, 52, 203, 877, 4140, 21147, 115975, \\
 & 11: 678570, 12: 4213597, 13: 27644437, 14: 190899322, \\
 & 15: 1382958545, 16: 10480142147, 17: 82864869804, \\
 & 18: 682076806159, 19: 5832742205057, 20: 51724158235372, \\
 & 21: 474869816156751, 22: 4506715738447323, \\
 & 23: 4452005855084346 \\
 & \text{Catalan numbers: } 3: 0, 1, 2, 3, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \\
 & 12: 208012, 13: 742900, 14: 2674440, 15: 9694845, \\
 & 16: 35357670, 17: 129644790, 18: 477638700, 19: 1767263190, \\
 & 20: 6564120420, 21: 24466267020, 22: 91482563640, \\
 & 23: 343059613650, 24: 1289904147324, 25: 4861946401452
 \end{aligned}$$

$$\int \frac{\cos((a-b)x)}{2(a-b)} - \frac{\cos((a+b)x)}{2(a+b)} dx = \frac{\sin((a-b)x)}{2(a-b)} - \frac{\sin((a+b)x)}{2(a+b)} \quad (71)$$

$$\begin{aligned}
 \int \sin^2 ax \cos bxdx &= -\frac{\sin[(2a-b)x]}{4(2a-b)} \\
 &+ \frac{\sin bx}{2b} - \frac{\sin[(2a+b)x]}{4(2a+b)} \quad (72)
 \end{aligned}$$

$$\int \sin^2 x \cos x dx = \frac{1}{3} \sin^3 x \quad (73)$$

$$\begin{aligned}
 \int \cos^2 ax \sin bxdx &= \frac{\cos[(2a-b)x]}{4(2a-b)} - \frac{\cos bx}{2b} \\
 &- \frac{\cos[(2a+b)x]}{4(2a+b)} \quad (74)
 \end{aligned}$$

$$\int \cos^2 ax \sin ax dx = -\frac{1}{3a} \cos^3 ax \quad (75)$$

$$\begin{aligned}
 \int \sin^2 ax \cos^2 bxdx &= \frac{x}{4} - \frac{\sin 2ax}{8a} - \frac{\sin[2(a-b)x]}{16(a-b)} \\
 &+ \frac{\sin 2bx}{8b} - \frac{\sin[2(a+b)x]}{16(a+b)} \quad (76)
 \end{aligned}$$

$$\int \sin^2 ax \cos^2 ax dx = \frac{x}{8} - \frac{\sin 4ax}{32a} \quad (77)$$

$$\int \tan ax dx = -\frac{1}{a} \ln \cos ax \quad (78)$$

$$\int \tan^2 ax dx = -x + \frac{1}{a} \tan ax \quad (79)$$

$$\begin{aligned}
 \int \tan^n ax dx &= \frac{\tan^{n+1} ax}{a(n+1)} \times \\
 &{}_2F_1\left(\frac{n+1}{2}, 1, \frac{n+3}{2}, -\tan^2 ax\right) \quad (80)
 \end{aligned}$$

$$\int \tan^3 ax dx = \frac{1}{a} \ln \cos ax + \frac{1}{2a} \sec^2 ax \quad (81)$$

$$\int \sec x dx = \ln |\sec x + \tan x| = 2 \tanh^{-1} \left(\tan \frac{x}{2} \right) \quad (82)$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax \quad (83)$$

$$\int \sec^3 x dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x + \tan x| \quad (84)$$

$$\int \sec x \tan x dx = \sec x \quad (85)$$

$$\int \sec^2 x \tan x dx = \frac{1}{2} \sec^2 x \quad (86)$$

$$\int \sec^n x \tan x dx = \frac{1}{n} \sec^n x, n \neq 0 \quad (87)$$

$$\int \csc x dx = \ln \left| \tan \frac{x}{2} \right| = \ln |\csc x - \cot x| + C \quad (88)$$

$$\int \csc^2 x dx = -\frac{1}{a} \cot ax \quad (89)$$

$$\int \csc x dx = -\frac{1}{2} \cot x \csc x + \frac{1}{2} \ln |\csc x - \cot x| \quad (90)$$

$$\int \csc^n x \cot x dx = -\frac{1}{n} \csc^n x, n \neq 0 \quad (91)$$

$$\int \csc x dx = \ln |\tan x| \quad (92)$$

Products of Trigonometric Functions and Monomials

$$\int x \cos x dx = \cos x + x \sin x \quad (93)$$

$$\int x \cos ax dx = \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax \quad (94)$$

$$\int x^2 \cos x dx = 2x \cos x + (x^2 - 2) \sin x \quad (95)$$

$$\int x^2 \cos ax dx = \frac{2x \cos ax}{a^2} + \frac{a^2 x^2 - 2}{a^3} \sin ax \quad (96)$$

$$\begin{aligned}
 \int x^n \cos x dx &= -\frac{1}{2} (i)^{n+1} [\Gamma(n+1, -ix) \\
 &+ (-1)^n \Gamma(n+1, ix)] \quad (97)
 \end{aligned}$$

$$\begin{aligned}
 \int x^n \cos ax dx &= \frac{1}{2} (ia)^{1-n} [(-1)^n \Gamma(n+1, -iax) \\
 &- \Gamma(n+1, iax)] \quad (98)
 \end{aligned}$$

$$\int x \sin x dx = -x \cos x + \sin x \quad (99)$$

$$\int x \sin ax dx = -\frac{x \cos ax}{a} + \frac{\sin ax}{a^2} \quad (100)$$

$$\int x^2 \sin x dx = (2 - x^2) \cos x + 2x \sin x \quad (101)$$

$$\int x^2 \sin ax dx = \frac{2 - a^2 x^2}{a^3} \cos ax + \frac{2x \sin ax}{a^2} \quad (102)$$

$$\begin{aligned}
 \int x^n \sin x dx &= -\frac{1}{2} (i)^n [\Gamma(n+1, -ix) - (-1)^n \Gamma(n+1, -ix)] \\
 &\quad (103)
 \end{aligned}$$

Products of Trigonometric Functions and Exponentials

$$\int e^x \sin x dx = \frac{1}{2} e^x (\sin x - \cos x) \quad (104)$$

$$\int e^{bx} \sin ax dx = \frac{1}{a^2 + b^2} e^{bx} (b \sin ax - a \cos ax) \quad (105)$$

$$\int e^x \cos x dx = \frac{1}{2} e^x (\sin x + \cos x) \quad (106)$$

$$\int e^{bx} \cos ax dx = \frac{1}{a^2 + b^2} e^{bx} (a \sin ax + b \cos ax) \quad (107)$$

$$\int x e^x \sin x dx = \frac{1}{2} e^x (\cos x - x \cos x + x \sin x) \quad (108)$$

$$\int x e^x \cos x dx = \frac{1}{2} e^x (x \cos x - \sin x + x \sin x) \quad (109)$$

Integrals of Hyperbolic Functions

$$\int \cosh ax dx = \frac{1}{a} \sinh ax \quad (110)$$

$$\begin{aligned}
 \int e^{ax} \cosh bxdx &= \\
 \begin{cases} \frac{e^{ax}}{a^2 - b^2} [a \cosh bx - b \sinh bx] & a \neq b \\ \frac{e^{2ax}}{4a} + \frac{x}{2} & a = b \end{cases} \quad (111)
 \end{aligned}$$

$$\int \sinh ax dx = \frac{1}{a} \cosh ax \quad (112)$$

$$\begin{aligned}
 \int e^{ax} \sinh bxdx &= \\
 \begin{cases} \frac{e^{ax}}{a^2 - b^2} [-b \cosh bx + a \sinh bx] & a \neq b \\ \frac{e^{2ax}}{4a} - \frac{x}{2} & a = b \end{cases} \quad (113)
 \end{aligned}$$

$$\begin{aligned}
 \int e^{ax} \tanh bxdx &= \\
 \begin{cases} \frac{e^{(a+2b)x}}{(a+2b)} {}_2F_1\left[1 + \frac{a}{2b}, 1, 2 + \frac{a}{2b}, -e^{2bx}\right] \\ \quad - \frac{1}{2} e^{ax} {}_2F_1\left[\frac{a}{2b}, 1, 1E, -e^{2bx}\right] & a \neq b \\ \frac{e^{ax} - 2 \tan^{-1}[e^{ax}]}{a} & a = b \end{cases} \quad (114)
 \end{aligned}$$

$$\int \tanh ax dx = \frac{1}{a} \ln \cosh ax \quad (115)$$

$$\begin{aligned}
 \int \cos ax \cosh bxdx &= \frac{1}{a^2 + b^2} [a \sin ax \cosh bx \\
 &+ b \cos ax \sinh bx] \quad (116)
 \end{aligned}$$

$$\begin{aligned}
 \int \cos ax \sinh bxdx &= \frac{1}{a^2 + b^2} [b \cos ax \cosh bx + \\
 &a \sin ax \sinh bx] \quad (117)
 \end{aligned}$$

$$\begin{aligned}
 \int \sin ax \cosh bxdx &= \frac{1}{a^2 + b^2} [-a \cos ax \cosh bx + \\
 &b \sin ax \sinh bx] \quad (118)
 \end{aligned}$$

$$\begin{aligned}
 \int \sin ax \sinh bxdx &= \frac{1}{a^2 + b^2} [b \cosh bx \sin ax - \\
 &a \cos ax \sinh bx] \quad (119)
 \end{aligned}$$

$$\int \sinh ax \cosh ax dx = \frac{1}{4a} [-2ax + \sinh 2ax] \quad (120)$$

$$\begin{aligned}
 \int \sinh ax \cosh bxdx &= \frac{1}{b^2 - a^2} [b \cosh bx \sinh ax \\
 &- a \cosh ax \sinh bx] \quad (121)
 \end{aligned}$$

