Exercise 2.2-3

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Assuming that we're searching a list l for an element v, and assume that the probability of any element in l equaling v is $\frac{1}{n}$ (where l has n elements). Let X be the random variable denoting the number of elements of l that needs to be searched to find v via linear search. Then, we have

$$E(X) = \sum_{i=1}^{n} iP(X = i) = 1 \cdot \frac{1}{n} + \sum_{i=2}^{n} iP(\text{the first } i-1 \text{ elements of } l \text{ are not } v)$$

$$= \frac{1}{n} + \sum_{i=2}^{n} i \left(1 - \frac{1}{n}\right)^{i-1}.$$

Now, consider the polynomial $f(x) = \sum_{i=2}^{n} i(1-x)^{i-1}$. We have,

$$\int f(x)dx = C - \sum_{i=2}^{n} (1-x)^i = C + 1 + x - \sum_{i=0}^{n} (1-x)^i$$

$$= C + 1 + x - \frac{1 - (1-x)^{n+1}}{1 - (1-x)} = C + 1 + x - \frac{1 - (1-x)^{n+1}}{x}$$

(for $x \neq 0$, obviously).

Thus,

$$f(x) = \frac{d}{dx}\left(1 + x - \frac{1 - (1-x)^{n+1}}{x}\right) = \frac{-nx(1-x)^n - (1-x)^n + x^2 + 1}{x^2}$$