Exercise 2.3-3

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Reformulating the problem slightly, we wish to prove that for all $n \geq 1$, if

$$T(2^n) = \begin{cases} 2 & \text{if } n = 1 \\ 2T(2^{n-1}) + 2^n & \text{if } n > 1 \end{cases}$$

then $T(2^n) = 2^n lg(2^n) = n2^n$.

First, if $n = 1$, then $T(2) = 2 = 1 * 2^1$.

Suppose that $T(2^n) = n2^n$ for some $n \geq 1$, and consider $T(2^{n+1})$. Since $n + 1 > 1$, we have

$$T(2^{n+1}) = 2T(2^n) + 2^{n+1} = 2n2^n + 2^{n+1} = n2^{n+1} + 2^{n+1} = (n + 1)2^{n+1}.$$

Therefore, by induction, $T(2^n) = n2^n$ for all $n \geq 1$.