What is the force of tension on the first five strings of a set of Ernie Ball 9-42 guitar strings on a 25.5” scale?

Introduction

The electric guitar has been a cultural force for decades. Many aspiring musicians, myself included, have attempted to replicate their hero’s music, such as the iconic riff of Bryan Adams’ “Run to you” and the dueling guitars of Joe Walsh and Don Felder on the outro of “Hotel California”. However, there is a more interesting force involving the guitar, which is the force of tension. The higher the tension on a guitar, the higher the pitch of the note will be, and vice versa. Guitar players adjust the amount of tension on the guitar when they are tuning so that they can play in tune (Martin, 1998). While playing, guitar players can change the tension of strings by pressing on the tremolo arm or by “bending” the strings, which will then change the pitch of the guitar (Kemp, 2017).

Knowing the value for the force of tension on a guitar is very important. As the guitar is primarily made of somewhat malleable wood, the force of tension of the strings would easily bend this wood to the point where the guitar becomes unplayable. A truss rod, which is simply a reinforcing metal rod, is put into the guitar and adjusted based on the amount of tension in the guitar in order to provide the most suitable playing surface possible (Martin, 1998). As well, the “feel” of the strings when playing is primarily based on its tension. Thus, knowing the force of tension exerted on guitar strings will allow an easier switch to different tunings, string gauges (diameters), and scale lengths (Kemp, 2017).

Research question

What happens to the frequency of a plucked guitar string when its wavelength is shortened?

Background information

It is difficult to get a direct accurate measurement of the force of tension on guitar strings, as any measurement would significantly change the tension due to the way the guitar is set up. It is also virtually impossible to get a direct reading of the wave speed on a guitar string, although we can measure the frequency and the length of the part of the string that resonates. We can also find the mass of the string. Thus, it seems logical that the formula that will be used will have to involve Newton’s second law in some form, as well as the wave equation.

Assumptions:

1. The string is a standing wave undergoing simple harmonic motion.
2. The string is resonating at the first harmonic.

3. The mass of the strings is negligible. Thus, we can ignore any force of gravity in the free body diagram (the data will show that the force of gravity is three orders of magnitude smaller than the force of tension)
4. The strings have a uniform linear density.
   Through these assumptions, the force of tension on the strings is assumed to be constant.

Figure 1 is showing a free body diagram part of the top half of a standing wave of a guitar string (the wave is reflected and the force of tension remains the same so it does not matter).

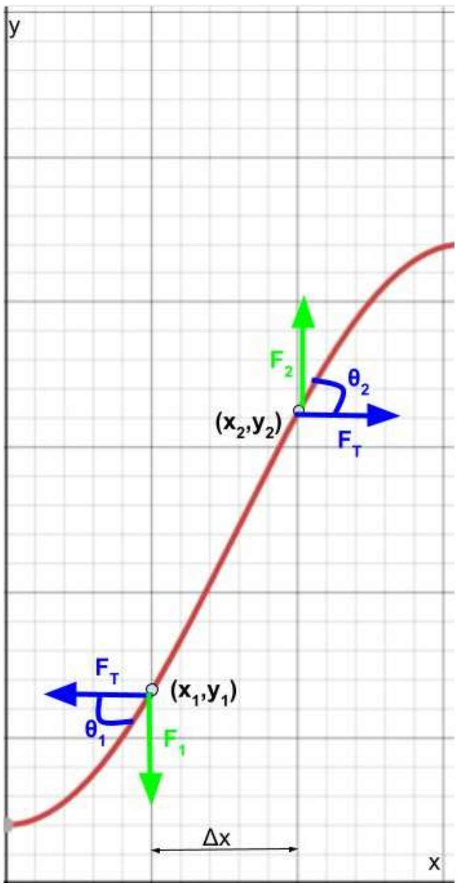


Figure 1 Free body diagram (graph created on desmos.com)

We can see that $tan\theta_1 = \frac{-F_1}{F_T}$ and $tan\theta_2 = \frac{F_2}{F_T}$. Since the tangent of a point is also the instantaneous slope at a point, we can take the partial derivative of y in terms of x.

$$\frac{F_1}{F_T} = -\left(\frac{\partial y}{\partial x}\right)_{x_1} \quad \text{Eq. 1}$$

$$\frac{F_2}{F_T} = \left(\frac{\partial y}{\partial x}\right)_{x_2} \qquad \text{Eq. 2}$$

The only forces that do not get cancelled out are the forces in the y direction, so the net force will only consist of those forces.

$$F_{net} = -F_1 + F_2 \qquad \text{Eq. 3}$$

Substituting in (1) and (2) into (3) we get (4).

$$F_{net} = F_T \left[\left(\frac{\partial y}{\partial x}\right)_{x_2} - \left(\frac{\partial y}{\partial x}\right)_{x_1}\right] \qquad \text{Eq. 4}$$

We now use Newton's second law of motion. The mass in this case will just be the mass per unit length, μ, multiplied by the length of the relevant part of the string, in this case Δx. The acceleration is just the second partial derivative of y with respect to time (Ling et al. 2018).

$$F_T \left[\left(\frac{\partial y}{\partial x}\right)_{x_2} - \left(\frac{\partial y}{\partial x}\right)_{x_1}\right] = \mu \Delta \mathrm{x} \left(\frac{\partial^2 \mathrm{y}}{\partial t^2}\right) \qquad \text{Eq. 5}$$

(5) can be rearranged into (6).

$$\frac{\left[\left(\frac{\partial y}{\partial x}\right)_{x_2} - \left(\frac{\partial y}{\partial x}\right)_{x_1}\right]}{\Delta \mathrm{x}} = \frac{\mu}{F_T} \left(\frac{\partial^2 \mathrm{y}}{\partial t^2}\right) \qquad \text{Eq. 6}$$

Since we are assuming the force of tension is uniform, the term on the left side can be expressed as the limit as Δx approaches zero. The term on the left then becomes the textbook definition of a second derivative (Kreyszig, 1999) giving us (7).

$$\frac{\partial^2 \mathrm{y}}{\partial x^2} = \frac{\mu}{F_T} \left(\frac{\partial^2 \mathrm{y}}{\partial t^2}\right) \qquad \text{Eq. 7}$$

Given the one-dimensional wave equation (8) (Ling et al. 2018), we can simply rearrange (7) to get (9).

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y(x,t)}{\partial t^2} \qquad \text{Eq. 8}$$

$$\frac{\mu}{F_T} = \frac{1}{v^2} \qquad \text{Eq. 9}$$

Using the substitution of $v = f\lambda$, we can rewrite (9) into the variables we can actually measure.

$$F_T = \mu f^2 \lambda^2 \qquad \text{Eq. 10}$$

We can then linearize this equation so that the force of tension will be the slope of the linearized equation.

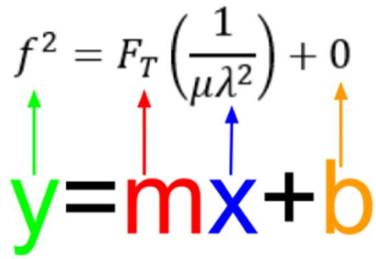


Figure 2 Linearized equation (made with google drawings)

Variables

Type of variable	Variable	Symbol	Description
Independent	Wavelength (m)	λ	The string is assumed to be vibrating at the first harmonic. The wavelength then becomes 2 times the string length, measured by a measuring tape. The minimum wavelength is 0.91 m and the maximum is 1.29 m. All the raw measurements are in cm +/- 0.1cm).
Dependent	Frequency (Hz)	f	The pitch of the note generated by playing the string. The frequency is measured by a Snark supertight ST-8HZ tuner, which directly measures the vibrations from the strings onto the headstock of the guitar, rather than from a microphone. I emailed Snark to ask for the precision on the tuner and they said it was +/-0.1 Hz.
Controlled (1)	String mass per unit length (kg/m)	μ	The string mass is measured using a scale (g +/- 0.001g), and then divided by length of the string cut. The string were kept constant, with the same gauge and the same brand.

Controlled (3)	Scale length	-	The scale length was kept constant, which was measured with a measuring tape.
Controlled (4)	Capo force	-	The force exerted by the capo could not be measured, however it is the same spring and there was no reason for it to change.

Apparatus

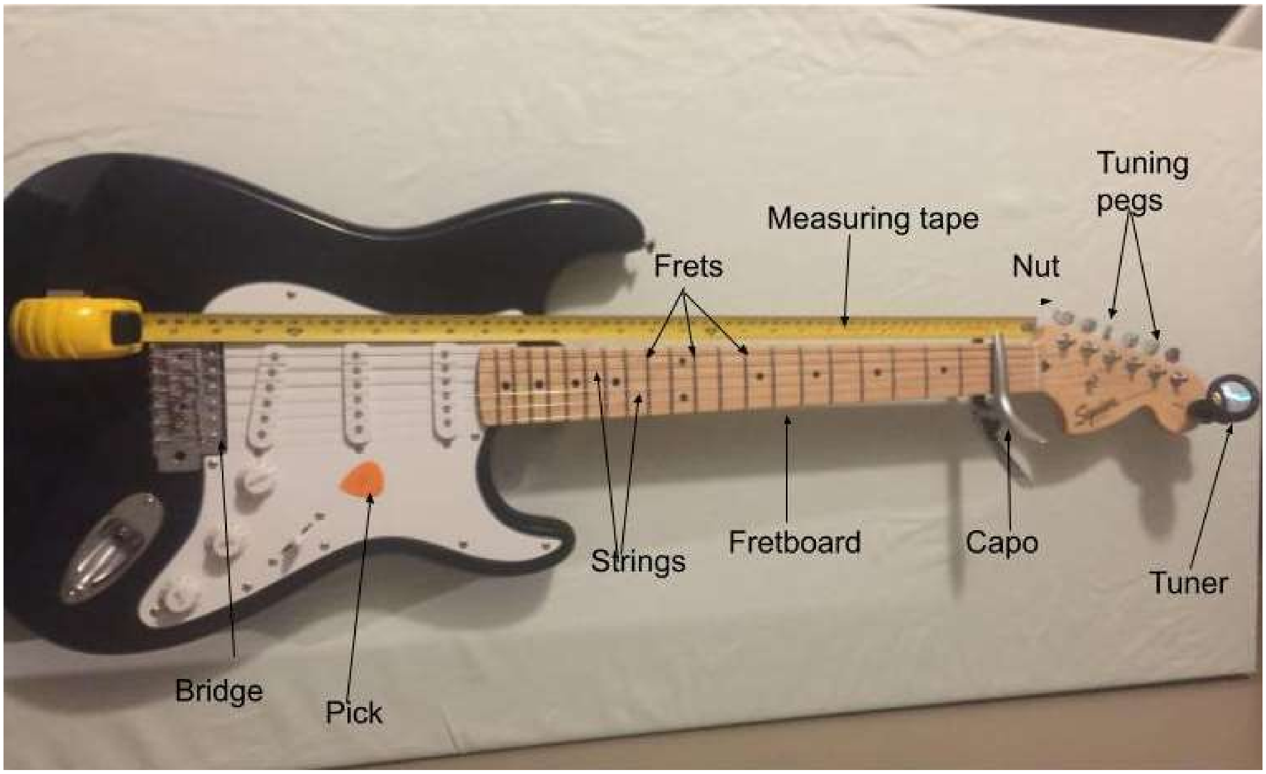


Figure 3 Apparatus

Apparatus	Description
Bridge	The bridge is an endpoint of the standing wave. It is also the other end measurement for the length of the wave. The bridge length is not constant for each string.
Pick	The pick was used to increase the repeatability of results when plucking the strings. There is more of the ironically named "pick attack", which is when the beginning of the note has a higher pitch than expected, when not using a pick.
Strings	The string order goes from left to right: low E (henceforth referred to as the E string), A, D, G, B, and high E. As they go from left to right the mass decreases and the pitch increases on a standard guitar.

Frets	The frets stick out of the guitar, they mark semitone intervals. Pushing down behind the fret will still make the string vibration stop at the fret, but with an increased force and thus an increased pitch. The frets become endpoints, replacing the nut, when pressed down.
Fretboard	The fretboard is the entire length of the playable surface that the strings are played on. Note that it is not the length of the standing wave, the wave continues past the end of the fretboard.
Measuring tape	The measuring tape has an uncertainty of +/- 0.05 cm for each reading however since the 0 cm mark was sketchy, I used the 1 cm mark which would make the uncertainty +/- 0.1 cm.
Capo	The capo presses down on strings with a constant force. The capo was placed on the center of the fret as that is where it is supposed to be placed to be the most in tune.
Nut	The nut is the other endpoint of the standing wave as it stops the vibration of the string. It is the end measurement for the total length of the wave.
Tuning Pegs	The tuning pegs control the amount of tension and the tuning of a string. They are rotated to adjust the elasticity of the string.
Tuner	The tuner senses vibrations from the guitar and it measures the frequency. Note that this tuner was only designed to read notes near the open notes, and thus the fret order will be (Nut, 1, 4, 5, 6) for the E, A, D, and B string and the order will be (Nut, 1, 3, 4, 5) for the G string. The tuner cannot read 5 separate notes for the high E string.

Method

1. Tune the guitar to the standard E tuning[1] on a new set of strings.
2. Use the pick to pluck[2] the low e string and record the frequency on the tuner once the value settles.
3. Repeat step 2 for 3 total trials then do the same for the next 4 strings.
4. Place the capo on the first fret.
5. Repeat steps 2-3 then move the capo to the next fret.
6. Once done, measure the distance from each fret to the bridge for each string.
7. Put a wrench on the nut to clamp down the strings.
8. Wearing goggles, cut the strings with heavy duty clippers at the bridge and the nut.
9. Record the mass of the cut strings.

Risk assessment

When cutting the guitar strings to get an accurate length, I wore goggles and a long-sleeve sweater as they were cut under tension and there was the possibility of the string snapping up or small string

[1] Specific instructions on tuning: https://www.snarktuners.com/wp-content/uploads/2013/08/SN8-revised.pdf
[2] Specific video on how to play guitar strings https://www.youtube.com/watch?v=w4a2ge9N31E&ab_channel=The-Art-of-Guitar

fragments flying around. There is also the environmental issue in that an essentially new pack of strings was thrown out. They typically go through weeks of playing rather than minutes. The strings are made mostly out of nickel, which is difficult (but not impossible) to recycle due to its high melting point (deBarbadillo, 1983). There are no ethical issues in this experiment.

Data tables

The frequencies read are expressed as $f_{n\,(high/low)}$, where n is the trial number and high or low was either the highest reading or the lowest reading. All the frequencies are expressed in hertz (Hz +/- 0.1 Hz). This is only the table for the E string; the others are available in the appendices.

Fret	Fret to bridge distance (cm +/- 0.1 cm)	$f_{1\,(low)}$	$f_{1\,(high)}$	$f_{2\,(low)}$	$f_{2\,(high)}$	$f_{3\,(low)}$	$f_{3\,(high)}$	f_{mean}	Δf_{mean}
Nut	64.9	82.2	82.4	82.2	82.5	82.3	82.5	82.4	0.3
1	61.2	85.4	85.5	85.6	85.6	85.4	85.6	85.5	0.2
4	51.5	103.1	103.3	103.1	103.3	103.3	103.4	103.3	0.3
5	48.6	109.9	110.1	109.9	110.0	109.9	109.9	110.0	0.2
6	45.9	116.1	116.3	116.1	116.4	116.0	116.3	116.2	0.3

Sample error calculation

As I had a range of values with no consistent timings for their duration, I decided to take the value the furthest from the mean and add 0.1 (as the value could have been that according to the uncertainty of the tuner). For the E string at the nut the most extreme value was 82.2 Hz and the mean was 82.4 Hz.

$$|82.2 - 82.4| + 0.1 = 0.3 \text{ Hz}$$

String masses

String	E	A	D	G	B	Weigh boat
Mass (g +/- 0.001 g)	5.893	4.224	3.298	2.668	2.314	2.006
Nut to bridge distance (m +/- 0.001 m)	0.649	0.647	0.645	0.647	0.645	-

Processed data

String	E	A	D	G	B
Mass of string only (kg +/- 0.000002 kg)	0.003887	0.002218	0.001292	0.000662	0.000308

Mass per unit length ($kg\ m^{-1}$)	0.00599	0.00343	0.00201	0.00102	0.000478
Relative uncertainty	0.21%	0.24%	0.3%	0.45%	0.8%

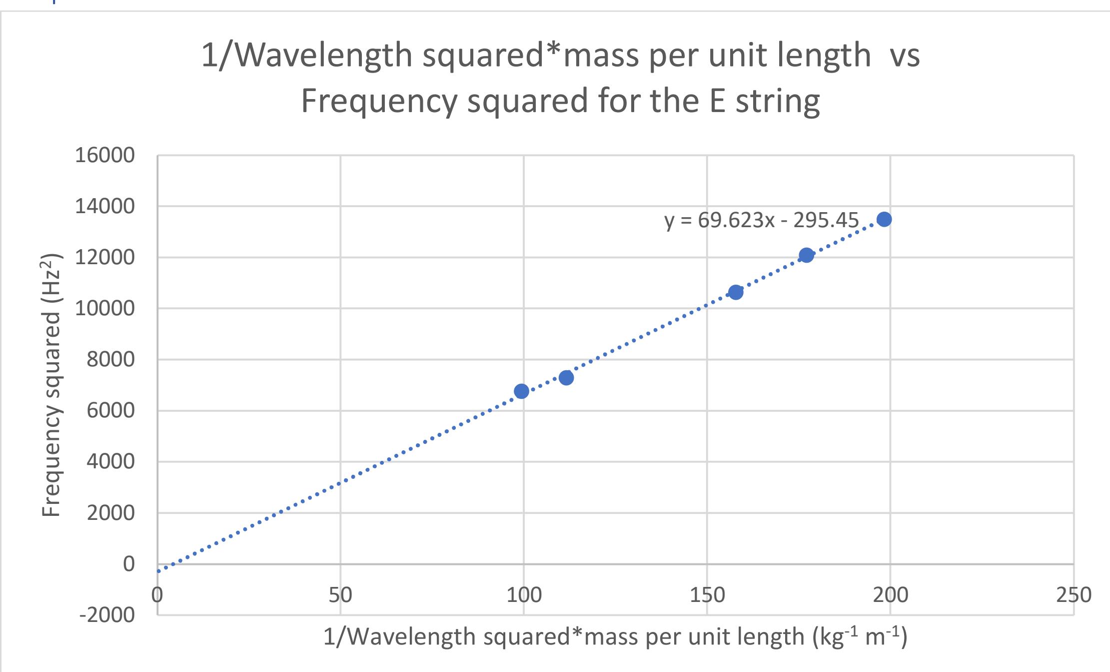
Sample uncertainty calculation (E string)

$$\Delta\mu = \left(\frac{0.000002}{0.003887} + \frac{0.001}{0.649}\right) = 0.21\%$$

Qualitative observations

One observation made was that as the string length was shortened, the sound was higher pitched. As well, as I moved down the strings, the pitch also increased. The B string had a slightly lighter feel to it, and had a bit more "give" compared to the other strings, who all had similar playing feels.

Graph



The error bars are too small to see, as the points themselves are larger than the error bars in both directions. There is a y intercept at (0, -295.45) and an x intercept at (4.243, 0).

Sample uncertainty calculation on point (99.33, 6773)

X axis	Y axis
$\Delta\mu = 0.21\%$ $\Delta\lambda = \frac{0.1}{64.9} = 0.15\%$ $\lambda * \lambda * \mu$ $0.15\% + 0.15\% + 0.21\% = 0.51\%$ $0.51\% * 99.33 = 0.506\ kg^{-1}m^{-1}$ $\Delta x = 0.506\ kg^{-1}m^{-1}$	$\Delta f_{mean} = 0.3$ $\frac{0.3}{82.4} = 0.36\%$ $f * f$ $0.36\% + 0.36\% = 0.72\%$ $0.72\% * 6773 = 48.76\ \text{Hz}^2$ $\Delta y = 48.76\ \text{Hz}^2$

Figure 4 Annotated uncertainty calculation (made with google drawings)

Comparison with literature

There was no officially published accepted value for the force of tension in my specific circumstance, as it would require my specific parameters to be met. The closest I could find online was a range of values (Achilles, 2000), saying that a normal guitar should have a tension of 50-80 N per string. In order to get a more definitive answer and a better measure of accuracy, I emailed Ernie Ball themselves to ask for the force of tension for my specific parameters. They replied with the following:

Super Slinky Nickel Wound (9-42)	STRING-1	STRING-2	STRING-3	STRING-4	STRING-5	STRING-6
PART NUMBER	P01009	P01011	P01016	P01124	P01132	P01142
GAUGE	9	11	16	24w	32	42
KEY	E4	B3	G3	D3	A2	E2
FREQUENCY	329.63	246.94	196	146.83	110	82.41
TENSION (lbs)	13.17	11.90	14.74	15.67	15.72	14.99

TOTAL TENSION (lbs)	**86.20**

Figure 5 Official string tension table

Note that in this case the strings are displayed in reverse order; the low e string is string 6 and the b string is string 2. The unit conversions from force-pounds to SI units are converting to kilograms and multiplying by the gravitational acceleration constant. A sample conversion is below.

$$14.99\ lbs * \frac{1kg}{2.205\ lbs} * 9.81\ m\ s^{-2} = 66.69\ N$$

The following is the tension table expressed in SI units with the percent errors for the force of tension values.

	B string	G string	D string	A string	E string
Accepted value	52.94 N	65.58 N	69.71 N	69.94 N	66.69 N
Actual value	48.15 N	65.68 N	73.24 N	70.01 N	69.62 N
Percent error	9.05%	0.15%	5.06%	0.10%	4.39%

Conclusion

The experimental results indicate that decreasing the wavelength of a plucked guitar string increases it's frequency by a factor of its tension.

Discussion

There is some random error involved, with a lot of variance in the level of error. This will be discussed further in "sources of error", but probably arose from inconsistent measurements. There appears to be a close relationship between the experimental values for the G and A string, however the other strings have a significantly higher error. Interestingly, the only actual value that was below the experimental one was the B string, which also had the largest error.

All of the points on every graph fit neatly into their trendlines with no major outliers, which would indicate that the percent error is solely based on a systematic error. Given that there are large errors in both directions, it would have to have been one of the variables that was different for each string. The only variable that was different was the mass per unit length. It is very possible that something went wrong in the cutting or the weighing of the strings, but it is difficult to tell exactly where due to the lack of reliable database information.

The largest uncertainty came from the tuner. Especially given that it was fluctuating, sometimes up to 0.3 Hz, I had to take every possibility into account. There is no real good way of minimizing this, as Snark clip on tuners are just about top of the line and they already avoid the uncertainties that arise in the propagation of sound through air. Perhaps in the future if a more accurate and precise tuner was developed then this experiment could be re-done with a higher level of accuracy.

Extensions

The obvious extension is to find the tension of the high E string. This could not be done in this experiment since the tuner available was unable to measure it, but with another tuner that can read those notes, the tension can easily be found using the same method as used here. Another extension would be to compare different string gauges and different brands. As the purpose of this was to determine the level of tension in order to be able to potentially change strings and guitars, it is only natural to determine which strings under which circumstances have similar tensions, in order to see which string combinations I could play with.

Sources of error

Proposed set up

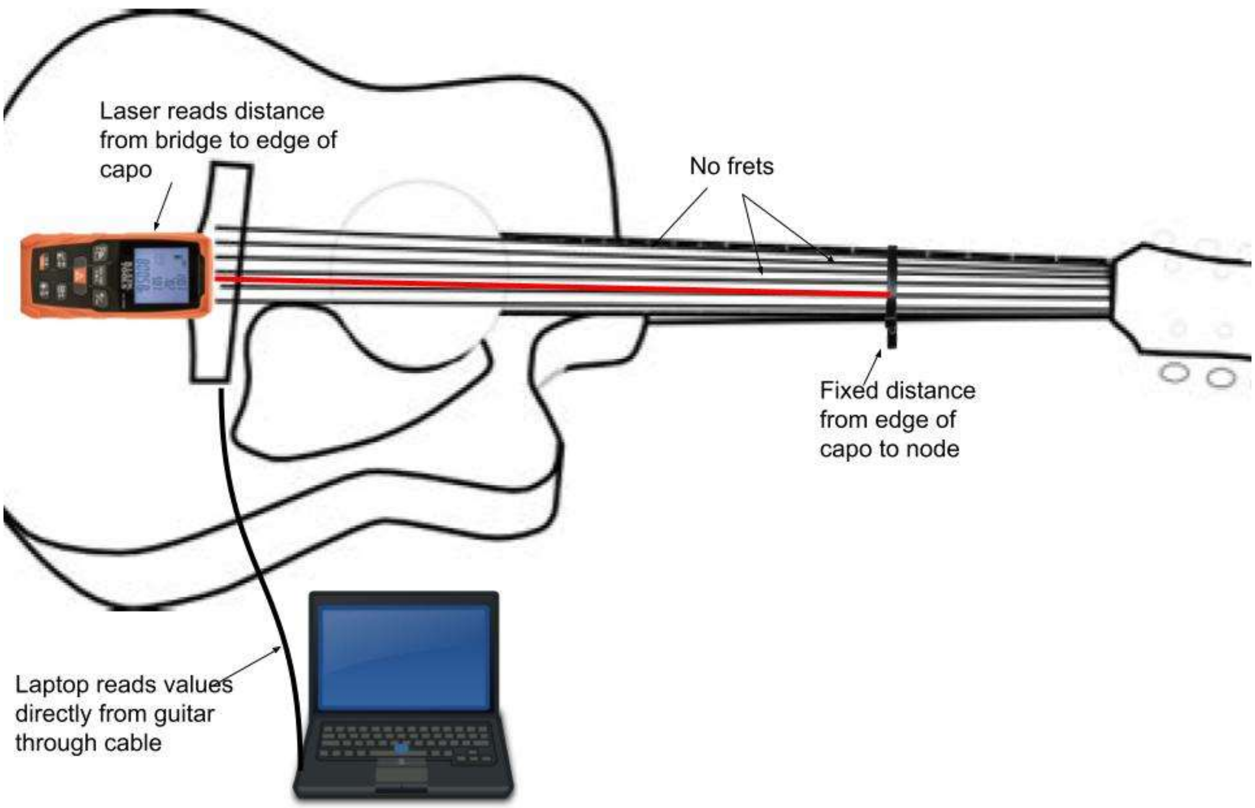


Figure 6 Proposed set up (made with photopea.com) (animations from https://www.guitarplayerbox.com/capo/using/, computer-laptop.png (PNG Image, 2400 × 2400 pixels) — Scaled (30%), and http://clipartlook.com/img-100912.html)

Source of error	Significance	Possible improvements
The capo may not be placed exactly on fret as the capo is significantly wider than the fret, making it hard to see. (Random)	The further away the capo from the fret, the more force it exerts on the string, and the sharper the note.	Use a fretless guitar, and then directly measure the distance from the bridge to where the capo was pressing down. I did not have a fretless guitar and I could not remove the frets of my guitar.
The string stretches slightly every time it is played, thus slightly	Highly insignificant, given that over years of experience, the guitar only needs to be slightly tuned up (0.5-1 Hz) every hour or so of constant playing,	Use fixed tuners. It is possible to retune to reach the exact notes that the frets are supposed to be tuned to, however this changes

reducing the tension. (Random)	and the cumulative playing time in this experiment was about 30 seconds.	the tension slightly back to an uncertain value and it also slightly changes the length.
Difficult to tell where exactly the bridge and the frets touched the string (Random)	Somewhat significant as this would have resulted in inaccurate wavelength measurements.	Use a locked in laser distance reader to measure the distance for more precise readings.

Works cited

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Appendices

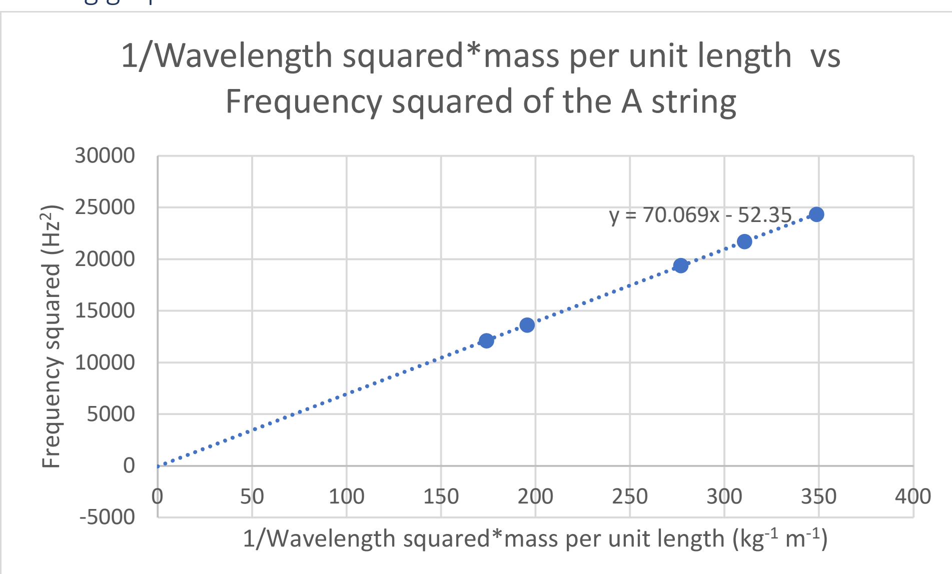
Appendix A

A string table

(All measured frequency values in Hz +/- 0.1 Hz)

Fret	Fret to bridge distance (cm +/- 0.2 cm)	f_1 (low)	f_1 (high)	f_2 (low)	f_2 (high)	f_3 (low)	f_3 (high)	f_{mean}	Δf_{mean}
Nut	64.7	110.1	110.2	110	110.2	109.9	110.1	110.1	0.3
1	61.0	116.8	116.9	116.8	116.9	116.7	116.9	116.9	0.3
4	51.3	139.2	139.3	139.4	139.5	139.2	139.4	139.3	0.3
5	48.4	147.3	147.4	147.3	147.4	147.4	147.4	147.4	0.2
6	45.7	156.0	156.1	155.9	156.1	156.1	156.1	156.1	0.3

A string graph



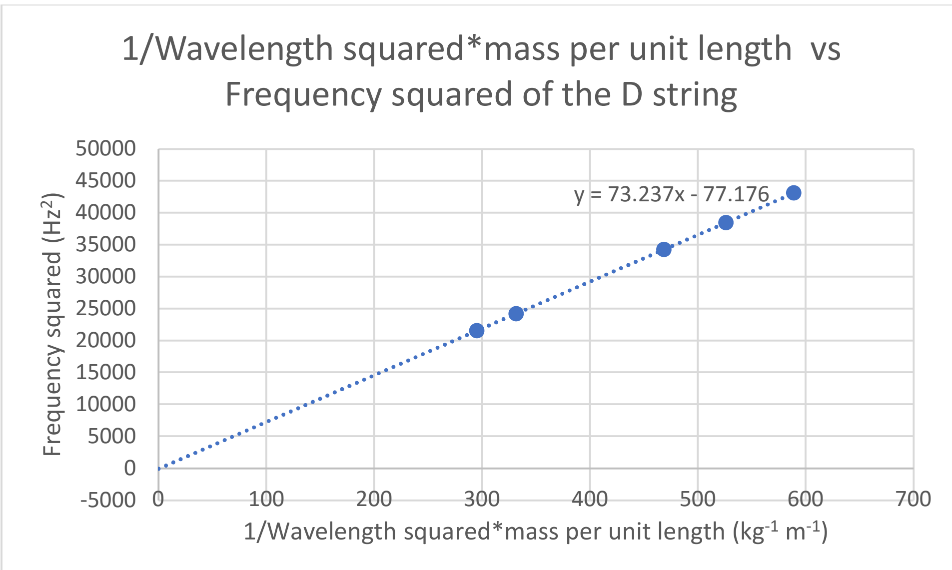
Appendix B

D string table

Fret	Fret to bridge distance	f_1 (low)	f_1 (high)	f_2 (low)	f_2 (high)	f_3 (low)	f_3 (high)	f_{mean}	Δf_{mean}

	(cm +/- 0.1 cm)								
Nut	64.5	146.8	146.9	146.7	146.8	146.8	146.8	146.8	0.2
1	60.8	155.4	155.7	155.5	155.6	155.5	155.6	155.6	0.3
4	51.1	185.0	185.1	184.9	185.1	185.1	185.1	185.1	0.3
5	48.2	195.9	196.0	196.1	196.2	196.1	196.1	196.1	0.3
6	45.5	207.6	207.7	207.4	207.6	207.5	207.6	207.6	0.3

D string graph

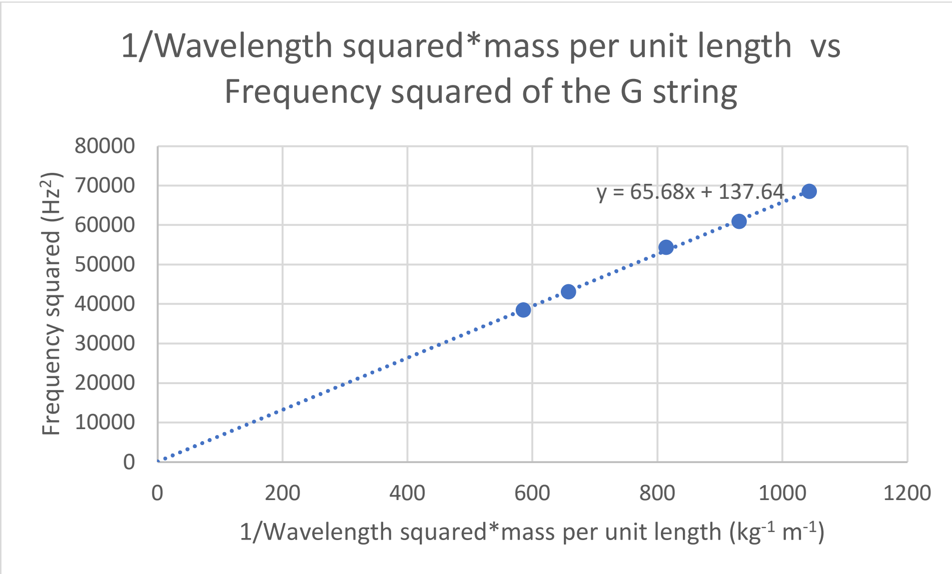


Appendix C

G string table

Fret	Fret to bridge distance (cm +/- 0.1 cm)	f_1 (low)	f_1 (high)	f_2 (low)	f_2 (high)	f_3 (low)	f_3 (high)	f_{mean}	Δf_{mean}
Nut	64.7	196.2	196.3	195.9	196.0	196.1	196.3	196.1	0.3
1	61.0	207.6	207.6	207.6	207.7	207.5	207.7	207.6	0.2
3	54.8	233.0	233.1	233.1	233.2	233.1	233.2	233.1	0.2
4	51.3	246.9	247.0	246.8	247.0	246.7	246.9	246.9	0.3
5	48.4	261.7	261.9	261.7	261.9	261.6	261.8	261.8	0.3

G string graph



Appendix D

B string table

Fret	Fret to bridge distance (cm +/- 0.1 cm)	f_1 (low)	f_1 (high)	f_2 (low)	f_2 (high)	f_3 (low)	f_3 (high)	f_{mean}	Δf_{mean}
Nut	64.5	246.9	247.1	246.8	246.9	247.0	247.1	247.0	0.3
1	60.8	261.5	261.7	261.6	261.7	261.5	261.6	261.6	0.2
4	51.1	311.0	311.1	311.2	311.3	310.9	311.1	311.1	0.3
5	48.2	329.6	329.7	329.6	329.7	329.5	329.7	329.6	0.3
6	45.5	349.0	349.2	349.1	349.2	349.1	349.3	349.2	0.3

B string graph