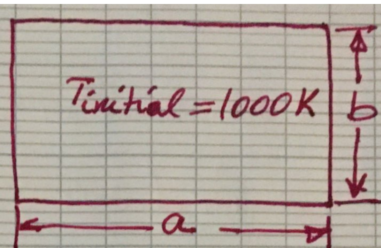
Assignment#3: (Two-dimensional unsteady conduction)

In this assignment, you are requested to extend the two-dimensional code you developed in assignments 1 and 2 to include the transient term. You are required to implement the first order transient backward Euler scheme and the second order crank-Nicholson scheme (use the two-step implementation approach). Use the developed general code to solve the problem shown below for which the analytical solution is given and can be used for comparison with the numerical solution. Use a time step of 1 second, solve for 60 seconds, and compare with analytical solutions at t=10, 30, and 60 seconds. Use a=0.1 m and b=0.2 m and a grid of size 20x40 elements.

$$\rho c_p \frac{\partial T}{\partial t} = k \nabla^2 T$$



$$T(x,y,0) = 1000$$

$$\frac{\partial T}{\partial x}(0,y,t) = \frac{\partial T}{\partial x}(a,y,t) = 0$$

$$T(x,0,t) = T(x,b,t) = 0$$

use $k = 15$ W/mK, $c_p = 500$ J/kg K, $\rho = 7800$ kg/m^3, $\alpha = \frac{k}{\rho c_p}$

$$T(x,y,t) = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} a_{mn}\, e^{-\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)\pi^2 \alpha t} \times \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

$$a_{0n} = \frac{2000}{n\pi}(1 - \cos n\pi)$$

$$a_{mn} = -\frac{4000}{mn\pi^2} \sin m\pi (\cos n\pi - 1)$$