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# A distributed controller approach for delay-independent stability of networked control systems ☆

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ABSTRACT

This article introduces a novel distributed controller approach for networked control systems (NCS) to achieve finite gain $\mathcal{L}_2$ stability independent of constant time delay. The proposed approach represents a generalization of the well-known scattering transformation which applies for passive systems only. The main results of this article are (a) a sufficient stability condition for general multi-input–multi-output (MIMO) input-feedforward–output-feedback-passive (IF–OFP) nonlinear systems and (b) a necessary and sufficient stability condition for linear time-invariant (LTI) single-input–single-output (SISO) systems. The performance advantages of the proposed approach are reduced sensitivity to time delay and improved steady state error compared to alternative known delay-independent small gain type approaches. Simulations validate the proposed approach.



## 1. Introduction

In networked control systems (NCS) plant and controller are connected through a communication network (see (Antsaklis & Baillieul, 2007; Hristu-Varsakelis & Levine, 2005; Tipsuwan & Chow, 2003) for excellent overviews of this field). The motivation for replacing the classical point-to-point control architecture by a NCS originates – among other things – from its flexible reconfiguration capabilities: Plant and controller nodes can be added or removed without additional wiring effort. The number of active nodes sharing the communication channel has an effect on the communication parameters such as the communication time delay and packet loss. In consequence, these communication parameters are, in general, not exactly known during the controller design stage. Depending on the communication protocol, the time delay might be constant or time-varying.

In this work the problem of an unknown, constant time delay is addressed. It is well-known that time delay in a control loop degrades performance and may cause instability. Time delay system approaches are classified into delay-dependent and delay-independent approaches (see Gu, Kharitonov, and Chen (2003); Richard (2003) and references therein for a concise overview and introduction to the rich literature of time delay systems). Input–output approaches, as considered in this work, are investigated in Bonnet and Partington (1999); Georgiou and Smith (1992) for known constant time delay and for uncertain, constant time delay in linear systems in Hale and Verduynn (1993); Miller and Davison (2005). The classical small gain result (Khalil, 1996) also provides delay-independent stability results for nonlinear systems, but is known to be rather conservative. For example, systems with an integrator in the open loop do not satisfy the small gain condition. In consequence, the steady state tracking performance to a reference input is poor in standard small gain designs. In order to overcome these performance issues in NCS we propose a novel distributed control approach.

The main contribution of this work is the analysis and design of a distributed control architecture to achieve input–output delay-independent stability. The proposed control design is in two steps: (i) the remote controller is designed without considering the time delay, (ii) a linear static input-feedforward–output-feedback is implemented locally at the plant side, and a similar modification at the remote controller side. The proposed approach can be applied to input-feedforward–output-feedback-passive (IF–OFP) nonlinear plants and controllers. The stability argument is based on a concept in line with the seminal works of (Zames, 1966a,b), where conditions for the open-loop behavior of feedback components are provided that guarantee input–output stability of the feedback interconnection. The main result is stated as follows: "If the open loop can be factored into two suitably proportioned,

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conic relations then the closed loop is bounded-input–bounded-output stable''. Here, by the IF–OFP assumption we require that the open loop system consisting of a plant and a controller but *without time delay* can be factored into such suitable conic sectors. The proposed modifications in step (ii) are designed such that the IF–OFP property of the plant is exactly preserved to the extended plant which additionally includes the modifications of step (ii) and the bidirectional communication line *with arbitrarily large constant time delay*. As a result every controller–plant pair, which is stable without the network and satisfies the IF–OFP assumption, is also stable with the network *with arbitrarily large constant time delay* and the proposed distributed controller. The approach has convincing performance advantages over standard small gain approaches as discussed in this article for linear time-invariant (LTI) single-input–single-output (SISO) systems. Among them are a low sensitivity to time delay resulting in a graceful performance degradation with increasing time delay and zero steady state error. Additionally, when the plant and pre-designed controller are known LTI systems a necessary and sufficient delay-independent stability condition is derived.

The additional control actions can also be interpreted as a linear transformation of the variables transmitted over the network: instead of the original plant and controller outputs a linear combination of the respective inputs and outputs is communicated. This is similar to the scattering transformation approach (Anderson & Spong, 1989; Niemeyer & Slotine, 1991), which stabilizes force feedback teleoperation systems in the presence of unknown constant time delay. For application of the scattering transformation in its original form, however, the subsystems are required to be passive. IF–OFP systems, as required here, represent a substantially larger class, with passive systems as a special case. In this sense, the proposed approach in this article is a generalization of the scattering transformation approach. In preliminary work of the same authors conditions are given under which the scattering transformation also applies to non-passive LTI NCS with arbitrary large constant time delay (Matiakis, Hirche, & Buss, 2005) and to NCS with IF–OFP subsystems in Matiakis, Hirche, and Buss (2006). A less conservative result is presented here using a general input–output transformation. Within the scattering transformation framework a number of modifications have been introduced that guarantee stability with time-varying delay (Lozano, Chopra, & Spong, 2002; Munir & Book, 2002), packet loss (Berestesky, Chopra, & Spong, 2004; Hirche & Buss, 2004; Secchi, Stramigioli, & Fantuzzi, 2003), and sampled-data systems (Stramigioli, 2001). It is important to note that all these extensions straightforwardly apply to the approach here.

The remainder of this article is organized as follows: In Section 2 the background on IF–OFP systems and finite gain $\mathcal{L}_2$ stability is presented, followed by the problem setting in Section 3, and the stability conditions in Section 4. Performance issues are discussed in Section 5 and validated through a numerical example in Section 6.

**Notation.** Let $\mathcal{L}_{2e}^m$ denote the extended $\mathcal{L}_2$ space of time functions of dimension $m$ with support on $[0, \infty)$. The notation $\|u\|$ stands for the $\mathcal{L}_2$ norm of a piecewise square-integrable function $u(\cdot) : \mathbb{R}_+ \to \mathbb{R}^m$ with $\mathbb{R}_+$ being the set of non-negative real numbers and $\mathbb{R}^m$ the Euclidean space of dimension $m$. The truncation of $u(\cdot)$ up to the time $t$ is denoted by $u_t(\cdot)$. The inner product of the truncated signals $u_t, y_t$ is denoted by $\langle u, y\rangle_t$, hence $\|u_t\|^2 = \langle u, u\rangle_t$. The $H_\infty$ norm of a transfer function $G(s)$ is denoted by $\|G\|_\infty$. $M > 0$ means that the matrix $M$ is positive definite; $I$ stands for the unit matrix.

## 2. Background

In this article the dynamical systems are considered from an input–output point of view as causal input–output mapping operators $h : \mathcal{U} \to \mathcal{Y}$ with $h(t \leq 0) = 0$ and $\mathcal{U} \subset \mathcal{L}_{2e}^m$ representing the admissible input space and $\mathcal{Y}$ accordingly the output space. The system is supposed to be well defined in the sense that each element in $\mathcal{U}$ is associated with an element in $\mathcal{Y}$. Input-feedforward–output-feedback-passive systems, as considered here, are a special class of dissipative systems. Recall that a dynamical system $h : \mathcal{U} \to \mathcal{Y}$ is called dissipative with respect to the supply rate $s(u, y)$ if for each admissible $u \in \mathcal{U}$ and each $t \geq 0$

$$\int_0^t s(u, y)\mathrm{d}\tau \geq 0, \tag{1}$$

holds (refer to Willems (1972a,b) and Hill and Moylan (1976) for more details). Often Willems (1972b) a quadratic supply rate $s(u, y) = z^{\mathrm{T}}Pz$ with $z^{\mathrm{T}} = [u\, y]$

$$P = \begin{bmatrix} Q & S \\ S^{\mathrm{T}} & R \end{bmatrix}, \tag{2}$$

is considered. With the special choice $Q = -\delta I$, $R = -\varepsilon I$, $S = \eta I$, $\delta, \varepsilon \in \mathbb{R}$, $\eta = \frac{1}{2}$ IF–OFP systems are characterized.

**Definition 1.** A dynamical system $h : \mathcal{U} \to \mathcal{Y}$ is called input-feedforward–output-feedback-passive (IF–OFP) if for each admissible $u \in \mathcal{U}$ and each $t \geq 0$

$$\langle u, y\rangle_t \geq \delta\|u_t\|^2 + \varepsilon\|y_t\|^2. \tag{3}$$

Note that the IF–OFP property represents a generalization of the passivity concept. If $\delta = \varepsilon = 0$ the system is called passive, if $\delta = 0$ and $\varepsilon > 0$ the system is called output-feedback strictly passive and if $\delta > 0$ and $\varepsilon = 0$ the system is called input-feedforward strictly passive. If one or both of the values $\delta, \varepsilon$ are negative there is a shortage of passivity.

Among the variety of stability notions we consider finite gain $\mathcal{L}_2$ stability in this article, which is another special case of quadratic dissipativity with $S = 0, R = I, Q = -\gamma^2 I, \gamma \in \mathbb{R}_+$.

**Definition 2** (*Khalil, 1996*). A dynamical system $h : \mathcal{U} \to \mathcal{Y}$ is called finite gain $\mathcal{L}_2$ stable if there exists a constant $\gamma \in \mathbb{R}_+$ such that for each admissible $u \in \mathcal{U}$ and each $t \in [0, \infty)$

$$\|y_t\| \leq \gamma\|u_t\|. \tag{4}$$

Finite gain $\mathcal{L}_2$ stability of a feedback interconnection can be concluded from the IF–OFP properties of its subsystems. Consider two IF–OFP systems $h_p$ and $h_c$ satisfying (3) with $\delta_i, \varepsilon_i, i \in \{p, c\}$.

**Proposition 1** (*Khalil, 1996*). *The negative feedback interconnection of $h_p$ and $h_c$ is finite gain $\mathcal{L}_2$ stable if*

$$\varepsilon_c + \delta_p > 0 \quad \text{and} \quad \varepsilon_p + \delta_c > 0. \tag{5}$$

**Remark 1.** Clearly, some of the $\delta_i$, $\varepsilon_i$ can be negative if it is compensated by appropriate positive values. Within the passivity formalism this can be interpreted as balancing shortage of passivity with excess of passivity between subsystems.

## 3. Problem setting

We consider a system comprising a plant $h_p : \mathcal{U}_p \to \mathcal{Y}_p$ and a controller $h_c : \mathcal{E} \to \mathcal{Y}_c$ as mappings from the plant input $u_p \in \mathcal{U}_p \subset \mathcal{L}_{2e}^m$ to the plant output $y_p \in \mathcal{Y}_p \subset \mathcal{L}_{2e}^m$ and from the control error $e \in \mathcal{E} \subset \mathcal{L}_{2e}^m$ to the controller output $y_c \in \mathcal{Y}_c \subset \mathcal{L}_{2e}^m$. The control error is defined as $e = w - u_c$, where $w \in \mathcal{W} \subset \mathcal{L}_{2e}^m$ is the reference input (see Fig. 1 for visualization). The blocks $M$ and its inverse $M^{-1}$ are introduced later. The network is modelled as a forward time delay operator $h_{T_1}$ (controller to plant channel) and backward time delay operator $h_{T_2}$ (plant to controller channel) with time delays $T_1$ and $T_2$, respectively. The input–output relations are given by $h_{T_1} : u_r(t) = u_l(t - T_1)$ and $h_{T_2} : v_l(t) = v_r(t - T_2)$. It is assumed that $u_l(t) = 0\ \forall t \in [-T_1, 0]$ and $v_r(t) = 0\ \forall t \in [-T_2, 0]$. The time delays $T_1, T_2 \in \mathbb{R}_+$ are assumed to be constant but unknown.

Without any further control measures a closed loop system with time delay can be unstable, as shown in Anderson and Spong (1989) for passive subsystems. In order to address this problem we propose a distributed controller architecture. Therefore a static output-feedback–input-feedforward controller is inserted at the plant side, which is represented by the block $M$ in Fig. 1. Additionally, the original remote controller $h_c$ is modified by $M^{-1}$. In line with the scattering transformation approach (Anderson & Spong, 1989) the additional control action $M$ and its inverse $M^{-1}$ is interpreted as a transformation of the input–output values of plant $h_p$ and controller $h_c$. Instead of the plant input–output vector $z_p^{\mathrm{T}} = [u_p^{\mathrm{T}}\ y_p^{\mathrm{T}}]$ a linear transformation $s_r^{\mathrm{T}} = [u_r^{\mathrm{T}}\ v_r^{\mathrm{T}}]$ is transmitted over the communication channel

$$s_r = Mz_p \tag{6}$$

with $M \in \mathbb{R}^{2m\times 2m}$ (see Fig. 1). Furthermore, the left-hand side transmitted values $s_l^{\mathrm{T}} = [u_l^{\mathrm{T}}\ v_l^{\mathrm{T}}]$ relate to the original controller input–output vector $z_c^{\mathrm{T}} = [y_c^{\mathrm{T}}\ u_c^{\mathrm{T}}]$ via

$$s_l = Mz_c. \tag{7}$$

Note that for $M = I$ the standard approach without transformation/local control at the plant side is recovered. For subsequent derivations the transformation matrix $M$ is decomposed into a block rotation matrix $R$ and scaling matrix $B$

$$M = RB, \quad R = \begin{bmatrix} \cos\theta\, I & \sin\theta\, I \\ -\sin\theta\, I & \cos\theta\, I \end{bmatrix}, \quad \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], \tag{8}$$

$$B = \begin{bmatrix} b_{11}I & b_{12}I \\ b_{21}I & b_{22}I \end{bmatrix},$$

with $b_{11}, b_{12}, b_{21}, b_{22} \in \mathbb{R}$ and $\det B \neq 0$. Throughout this article we assume that the closed loop system is well posed, i.e. for each input signal $w \in \mathcal{W}$ there exists a unique solution for the signals $e, u_c, y_c, u_l, v_l, u_r, v_r, u_p, y_p$ that causally depends on $w$. Note, that this requires the invertibility of the matrix $M$ as otherwise for a solution of $u_l, v_l, u_r, v_r$ there exists no unique solution for $u_p, y_p, u_c, y_c$. For further reference we define the *extended plant* $u_c = h_2(y_c)$, which consists of the original plant $h_p$, the right and left transformation $M, M^{-1}$, and the forward and backward time delay operators $h_{T_1}, h_{T_2}$ (see also Fig. 1).

The goal is now to find the transformation $M$, such that the overall system is finite gain $\mathcal{L}_2$ stable independently of the time delay.

## 4. Distributed architecture design for delay-independent stability of networked IF–OFP systems

The main focus of this section is on the design of the transformation matrix $M$, i.e. it is assumed that the original controller is already designed without considering time delay. Specifically, we make the following assumption throughout this section:

A1 Plant $h_p$ and controller $h_c$ are IF–OFP with $\delta_i$, $\varepsilon_i$ where $i \in \{p, c\}$ satisfying (5), i.e. the negative feedback interconnection of $h_p$ and $h_c$ *without time delay* is finite gain $\mathcal{L}_2$ stable (see Proposition 1).

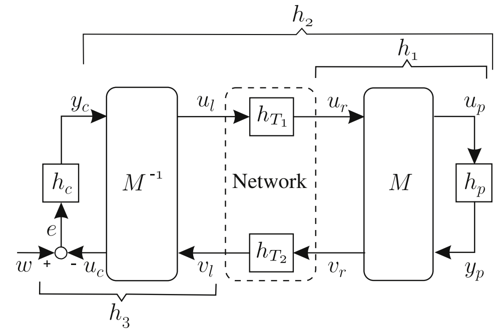


**Fig. 1.** Distributed control approach for NCS with input–output transformation $M$.

### 4.1. Delay-independent stability for IF–OFP systems

The main idea is to design the transformation $M$ such that the extended plant $h_2$ has *exactly* the same IF–OFP properties as the original plant $h_p$ *independently of the constant time delays* $T_1, T_2$. The subsequent theorem gives necessary and sufficient conditions for that. Based on Proposition 1 and Assumption A1 the finite gain $\mathcal{L}_2$ stability can then be concluded for the overall system, i.e. the feedback interconnection of the subsystems $h_c$ and $h_2$.

Before the theorem is stated some definitions and comments are given. We define the dissipativity matrix $P_p$ (2) with elements $\left(\delta_p, \varepsilon_p, \eta_p = \frac{1}{2}\right)$, and furthermore $\delta_B, \varepsilon_B, \eta_B$ as the elements of the matrix $P_B$

$$P_p = \begin{bmatrix} -\delta_p I & \eta_p I \\ \eta_p I & -\varepsilon_p I \end{bmatrix}; \qquad P_B = B^{-T}P_pB^{-1} = \begin{bmatrix} -\delta_B I & \eta_B I \\ \eta_B I & -\varepsilon_B I \end{bmatrix}. \tag{9}$$

Further note that without loss of generality we can assume that the parameters $\delta, \varepsilon, \eta$ of an IF–OFP system belong to the domain $\Omega = \Omega_1 \cup \Omega_2$ with $\Omega_1 = \{\delta, \varepsilon, \eta \in \mathbb{R} | \delta\varepsilon - \eta^2 < 0\}$ and $\Omega_2 = \{\delta, \varepsilon, \eta \in \mathbb{R} | \delta\varepsilon - \eta^2 = 0; \delta, \varepsilon > 0\}$, or equivalently that the dissipativity matrix $P$ (2) has eigenvalues either $m$ negative and $m$ positive, or $m$ negative and $m$ zero (see Lemma 1 in the Appendix). Where it is non-ambiguous, the time argument $t$ is dropped in the following for convenience.

**Theorem 1.** *Assume that the plant $h_p$ is IF–OFP with $\delta_p$, $\varepsilon_p$, $\eta_p = \frac{1}{2}$. Then the subsystem $h_2$ is IF–OFP with $\delta_p$, $\varepsilon_p$, $\eta_p = \frac{1}{2}$ independently of the constant delay if and only if for each $B$ the rotation matrix parameter $\theta$ (8) is chosen as the one of the two solutions of*

$$\cot 2\theta = \frac{\varepsilon_B - \delta_B}{2\eta_B}, \tag{10}$$

*which simultaneously satisfies*

$$\alpha(\theta) = 2\eta_B \sin(\theta)\cos(\theta) - \delta_B \cos^2(\theta) - \varepsilon_B \sin^2(\theta) \geq 0. \tag{11}$$

**Proof.** (*Sufficiency*). Rewriting (3) for the plant in matrix form, in terms of the transmitted variables $s_r$ yields

$$\int_0^t s_r^{\mathrm{T}} M^{-T} P_p M^{-1} s_r \mathrm{d}\tau \geq 0 \Leftrightarrow \int_0^t s_r^{\mathrm{T}} R^{-T} P_B R^{-1} s_r \mathrm{d}\tau \geq 0 \tag{12}$$

with $P_B$ given by (9) and

$$R^{-T} P_B R^{-1} = \begin{bmatrix} \alpha(\theta) I & \zeta(\theta) I \\ \zeta(\theta) I & -\beta(\theta) I \end{bmatrix}, \tag{13}$$

parameterized by $\theta, \delta_B, \varepsilon_B, \eta_B$ through $\alpha(\theta)$ (11),

$$\beta(\theta) = \alpha(\theta) + \delta_B + \varepsilon_B,$$

and

$$\zeta(\theta) = \eta_B \cos 2\theta - \frac{\varepsilon_B - \delta_B}{2} \sin 2\theta. \tag{14}$$

Choosing $\theta$ according to (10), it follows that $\zeta(\theta) = 0$ (14), and hence we can rewrite (12)

$$\alpha(\theta)\|u_{r,t}\|^2 - \beta(\theta)\|v_{r,t}\|^2 \geq 0.$$

According to Sylvester's law of inertia, congruence transformations do not change the inertia of the matrix, i.e. the number of positive, negative and zero eigenvalues. Thus $\left(\delta_p, \varepsilon_p, \eta_p = \frac{1}{2}\right) \in \Omega \Leftrightarrow (\delta_B, \varepsilon_B, \eta_B) \in \Omega$. For this domain of $(\delta_B, \varepsilon_B, \eta_B)$ we can always choose one of the two solutions to (10) in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, denoted by $\theta^+$ and $\theta^-$, respectively, so that $\alpha(\theta^+) \geq 0$ as required by (11) and furthermore $\beta(\theta^+) > 0$ (see Lemma 2 in the Appendix). For this choice of $\theta$ the subsystem $h_1$ is finite gain $\mathcal{L}_2$ stable with

$$\|v_{r,t}\| = \|h_1(u_{r,t})_t\| \leq \gamma_{h_1}\|u_{r,t}\| \qquad \forall t\ \gamma_{h_1}^2 = \frac{\alpha(\theta^+)}{\beta(\theta^+)}. \tag{15}$$

Considering further that the constant time delay operator has an $\mathcal{L}_2$ gain of one, and using the assumption that $u_l(t) = 0\ \forall t \in [-T_1, 0]$ and $v_r(t) = 0\ \forall t \in [-T_2, 0]$, we may state $\|u_{r,t}\|^2 \leq \|u_{l,t}\|^2$, $\|v_{l,t}\|^2 \leq \|v_{r,t}\|^2, \forall t > 0$. It follows that

$$\alpha(\theta^+)\|u_{l,t}\|^2 - \beta(\theta^+)\|v_{l,t}\|^2 \geq 0.$$

Analogously to (12) we may rewrite the latter equation as

$$\int_0^t s_l^{\mathrm{T}} M^{-T} P_p M^{-1} s_l \mathrm{d}\tau \geq 0 \tag{16}$$

which expressed in the variables $y_c, u_c$ becomes

$$\langle y_c, u_c\rangle_t \geq \delta_p\|y_{c,t}\|^2 + \varepsilon_p\|u_{c,t}\|^2.$$

Thus, the subsystem $h_2$ satisfies (1) with exactly the same dissipativity parameters $\delta_p, \varepsilon_p, \eta_p = \frac{1}{2}$ as the plant. For *necessity* it only has to be shown that without setting $\zeta(\theta) = 0$ the time delay alters the IF–OFP property of the subsystem $h_2$. This can be shown straightforwardly through the counter example $y_p(t) = k \cdot u_p(t)$. ■

Observe that $\theta^+$ exists for each $B$, i.e. $b_{11}, b_{12}, b_{21}, b_{22} \in \mathbb{R}$ can be chosen freely to meet performance requirements. From this result it is straightforward to conclude finite gain $\mathcal{L}_2$ stability.

**Proposition 2.** *Consider the distributed feedback control architecture consisting of the plant $h_p$, the remote controller $h_c$, satisfying Assumption A1, the bidirectional communication channel with time delays $T_1, T_2$ and the input–output transformation (8). If the transformation matrix satisfies Theorem 1, then the overall system is delay-independent finite gain $\mathcal{L}_2$ stable.*

**Proof.** We have to show that a bounded input $w \in \mathcal{L}_{2e}$ implies a bounded output $y_p \in \mathcal{L}_{2e}$. By applying Proposition 1 to the closed loop system decomposed into subsystems $h_2$ and $h_c$ it is straightforward that also the signals $u_c, y_c, e \in \mathcal{L}_{2e}$. Since $u_l, v_l$ are linear combinations of $u_c, y_c$ we have $u_c, y_c \in \mathcal{L}_{2e} \Rightarrow u_l, v_l \in \mathcal{L}_{2e}$. The forward constant time delay operator is finite gain $\mathcal{L}_2$ stable so $u_l \in \mathcal{L}_{2e} \Rightarrow u_r \in \mathcal{L}_{2e}$. Furthermore $h_1$ is finite gain $\mathcal{L}_2$ stable thus $u_r \in \mathcal{L}_{2e} \Rightarrow v_r \in \mathcal{L}_{2e}$. Since again $u_p, y_p$ are a linear transformation of $u_r, v_r$, we have that $u_r, v_r \in \mathcal{L}_{2e} \Rightarrow u_p, y_p \in \mathcal{L}_{2e}$, i.e. there exists a $\gamma < \infty$ such that $\|y_{p,t}\| \leq \gamma\|w_t\|$ holds $\forall t$. Assuming the plant output to be unbounded, i.e. $y_p \notin \mathcal{L}_{2e}$, results with the same arguments as above in a contradiction to the assumption $w \in \mathcal{L}_{2e}$. ■

**Remark 2.** In case of unstable plants the right-hand input–output transformation pre-stabilizes locally. This becomes clear from (15), where every IF–OFP plant $h_p$ results in a finite gain $\mathcal{L}_2$ stable system $h_1$.

**Remark 3.** For passive plants, i.e. with $\delta_p = \varepsilon_p = 0$, and the choice $b_{11} = \sqrt{b}, b_{22} = \frac{1}{\sqrt{b}}, b > 0$ and $b_{12}, b_{21} = 0$, the rotation parameter is computed to $\theta = \theta^+ = \frac{\pi}{4}$. The resulting transformation is equivalent to the well-known scattering transformation (Anderson & Spong, 1989; Niemeyer & Slotine, 1991).

### 4.2. Small gain interpretation

An interesting viewpoint gives the interpretation of Theorem 1 from a small gain perspective. For the analysis, the closed loop system is decomposed into the subsystems $v_r = h_1(u_r)$, $u_l = h_3(v_l, w)$, $h_{T_1}$, and $h_{T_2}$, where the *transmitted* signals $u_l, u_r, v_r, v_l$ act as inputs and outputs, and the open loop system

$$h_{OL} = h_3 \circ h_{T_1} \circ h_1 \circ h_{T_2} \tag{17}$$

is considered (see Fig. 1). In the following it is shown than $h_{OL}$ is finite gain $\mathcal{L}_2$ stable

$$\|h_{OL}(v_{l,t})_t\| \leq \gamma_{OL}\|v_{l,t}\| \quad \forall t, \tag{18}$$

with $\gamma_{OL} < 1$, i.e. the system satisfies the small gain condition in the transformed variables.

**Corollary 1.** *The open loop system $h_{OL}$ has an $\mathcal{L}_2$ gain $\gamma_{OL} < 1$.*

**Proof.** For the subsystem $h_{OL}$ it is straightforward to show that $\|h_{OL}(v_{l,t})_t\| \leq \gamma_{OL}\|v_{l,t}\|$ with $\gamma_{h_{OL}} \leq \gamma_{h_3}\gamma_{T_1}\gamma_{h_1}\gamma_{T_2} = \gamma_{h_3}\gamma_{h_1}$ since for the time delay operators $\gamma_{T_1} = \gamma_{T_2} = 1$ holds. It remains to show that $\gamma_{h_3}\gamma_{h_1} < 1$. From (15) the finite $\mathcal{L}_2$ gain stability of $h_1$ is certified with gain $\gamma_{h_1}^2$. For $h_3$, consider the dissipativity inequality of the controller expressed in the variables $s_l$

$$\int_0^t s_l^{\mathrm{T}} M^{-T} P_c M^{-1} s_l \mathrm{d}\tau \geq 0, \quad \text{with } P_c = \begin{bmatrix} -\varepsilon_c I & -\frac{1}{2}I \\ -\frac{1}{2}I & -\delta_c I \end{bmatrix}, \tag{19}$$

where the negative signs in the off-diagonals of $P_c$ result from the *negative* feedback interconnection. Setting $k = \min[(\varepsilon_p + \delta_c), (\varepsilon_c + \delta_p)] > 0$, where positivity comes from Assumption A1, it is straightforward to show that

$$P_c \leq -(P_p + kI). \tag{20}$$

Thus, by substituting (20) in (19) it follows that

$$-\int_0^t s_l^{\mathrm{T}} M^{-T} P_p M^{-1} s_l + k s_l^{\mathrm{T}} M^{-T} M^{-1} s_l \mathrm{d}\tau \geq 0 \Rightarrow$$

$$-\int_0^t s_l^{\mathrm{T}} M^{-T} P_p M^{-1} s_l + k\lambda_{\min} s_l^{\mathrm{T}} s_l \mathrm{d}\tau \geq 0$$

with $\lambda_{\min} > 0$ the minimum eigenvalue of $M^{-T}M^{-1} > 0$. Following the derivations of the proof of Theorem 1 using (12), (13) and choosing $\theta^+$, the quadratic term above involving $P_p$ is simplified and the inequality can be rewritten as

$$\|u_{l,t}\| = \|h_3(v_{l,t})_t\| \leq \gamma_{h_3}\|v_{l,t}\| \quad \forall t\ \gamma_{h_3}^2 = \frac{\alpha(\theta^+) - k\lambda_{\min}}{\beta(\theta^+) + k\lambda_{\min}}.$$

Therewith, the subsystem $h_3$ is certified to be finite $\mathcal{L}_2$ gain stable with gain $\gamma_{h_3}$. Accordingly, with (15)

$$\gamma_{h_3}^2\gamma_{h_1}^2 \leq \frac{\alpha(\theta^+)}{\beta(\theta^+)}\frac{\beta(\theta^+) - k\lambda_{\min}}{\alpha(\theta^+) + k\lambda_{\min}} < 1,$$

hence $\gamma_{h_3}\gamma_{h_1} < 1$, and thus $\gamma_{h_{OL}} < 1$. ■

Proposition 2 gives only a sufficient condition for finite gain $\mathcal{L}_2$ stability as it relies on the sufficient stability condition from Proposition 1. Furthermore, observe that the $\mathcal{L}_2$ gains of the subsystems $h_1$ and $h_3$ depend on the IF–OFP properties of plant and controller $\gamma_{h_1} = \gamma_{h_1}(\delta_p, \varepsilon_p)$ and $\gamma_{h_3} = \gamma_{h_3}(\delta_c, \varepsilon_c)$. More conservative, i.e. higher, values of $\delta_p$, $\varepsilon_p$ and $\delta_c$, $\varepsilon_c$ in Proposition 1 result in a smaller open loop gain, hence in a higher stability reserve.

**Remark 4.** Using other known results the proposed approach can be easily extended to communication channels with time-varying delay and packet loss. Observe therefore that for the stability guarantee only the finite $\mathcal{L}_2$ gain $\gamma = 1$ property of the time delay operator is important. Accordingly, stability is also guaranteed for any other norm bounded uncertainty $h_*$ in the loop of the transformed variables, replacing the time delay operators $h_{T_1}$, $h_{T_2}$, or being in cascade with them, as long as $\gamma_{h_*} \leq 1$. Many scattering based approaches addressing time-varying delay (Lozano et al., 2002; Munir & Book, 2002), packet loss (Berestesky, Chopra, & Spong, 2004; Hirche & Buss, 2004; Secchi et al., 2003), and sampled-data systems (Stramigioli, 2001) are based on the same argument: They introduced control actions to keep the $\mathcal{L}_2$ gain of the corresponding input–output operator $\gamma \leq 1$. These approaches are straightforward to combine with the proposed approach.

### 4.3. Conic sectors interpretation

Conic sectors in the input–output space give a nice geometrical interpretation of IF–OFP system behavior (see e.g. Zames (1966a,b)). Following these lines, the input–output transformation can be interpreted as a rotation of conic sectors. For simplicity a memory-less, SISO IF–OFP system is considered as plant, even though stability related notions are futile in this case. With the choice $B = I$ the IF–OFP inequality (3) holds instantaneously

$$u_p y_p \geq \delta_p u_p^2 + \varepsilon_p y_p^2, \quad \forall t, \ \left(\delta_p, \varepsilon_p, \eta_p = \frac{1}{2}\right) \in \Omega. \tag{21}$$

Geometrically, this equation describes a conic sector in the $u_p$–$y_p$-plane which is described by its center-line angle $\theta_z$ and its apex angle $2\theta_{k,p}$. At each time instant $t$ the input and output lie within this conic sector $[\theta_z - \theta_{k,p}, \theta_z + \theta_{k,p}]$ or its mirrored counterpart (see Fig. 2(a) for a visualization). The center-line angle is straightforwardly derived by parameterizing the plant input and output in polar coordinates $u_p(t) = r_p(t)\cos\theta_p(t)$, $y_p(t) = r_p(t)\sin\theta_p(t)$ in (21), and is implicitly given as the solution of

$$\cot 2\theta_z = \varepsilon_p - \delta_p, \tag{22}$$

in the interval $[0, \frac{\pi}{2}]$. Observe that this describes the solution $\theta^+$ from Theorem 1 for $B = I$. Similarly, the apex angle $2\theta_{k,p}$ is given by the solution of

$$\cos 2\theta_{k,p} = \frac{\varepsilon_p + \delta_p}{\sqrt{(1 - 4\delta_p\varepsilon_p) + (\varepsilon_p + \delta_p)^2}},$$

with $\theta_{k,p} \in [0, \frac{\pi}{2})$. It is straightforward to compute that the output of any controller satisfying Proposition 1 lies within the conic sector $\theta_c(t) \in (\theta_z - \theta_{k,c}, \theta_z + \theta_{k,c})$ where $\theta_{k,c} = \frac{\pi}{2} - \theta_{k,p}$ (see Fig. 2(b)). The larger the sector of the plant is, the smaller the allowable sector for the controller.

By the proposed transformation $M$ the conic sector of the plant is rotated by $\theta^+$ (see Theorem 1) such that the sector of the subsystems $h_1$ has a center-line angle $\theta_z = 0$ (see Fig. 3(a)). This is exactly the conic sector representation for finite gain $\mathcal{L}_2$ stability, i.e. for the plant side in the transformed coordinates $\|v_{r,t}\| \leq \gamma_{h_1}\|u_{r,t}\|$. The apex angles $2\theta_{k,p}$, $2\theta_{k,c}$ of the plant and of the also

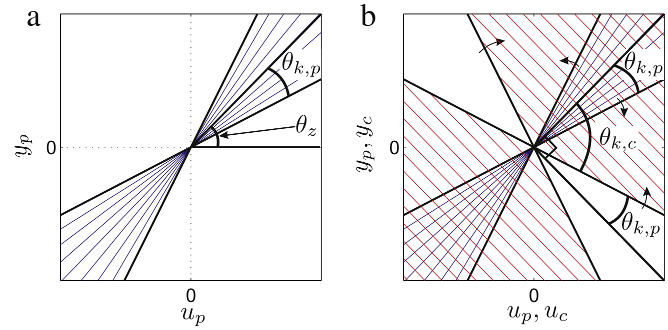


**Fig. 2.** (a) The conic sector of an IF–OFP plant. (b) The conic sector of the same plant and the corresponding controller satisfying Proposition 1.

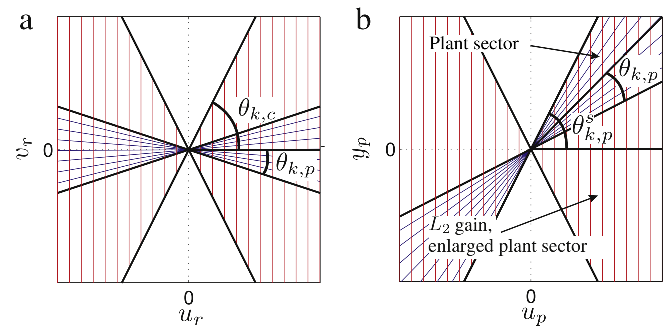


**Fig. 3.** (a) Finite gain $\mathcal{L}_2$ stable system after applying input–output transformation to the plant and controller from Fig. 2(b). (b) Equivalent $\mathcal{L}_2$ gain sector of an IF–OFP system.

rotated allowable controller sector are invariant to the rotation and are related to the $\mathcal{L}_2$ gain by $\tan\theta_{k,p} = \gamma_{h_1}$ and $\tan\theta_{k,c} = 1/\gamma_{h_1}$.

For comparison, the classical small gain approach without input–output transformation is discussed. The classical small gain approach can be applied only if the plant is initially finite gain $\mathcal{L}_2$ stable. This means that the plant's sector lies in the first and fourth quadrants. Clearly, in this case the IF–OFP plant sector from Fig. 2(a) can also be represented by an *enlarged* conic sector symmetric to the $u_p$ axis, as shown in Fig. 3(b). For the open loop gain $\gamma_p^s\gamma_c^s = \tan(\theta_{k,p}^s)\tan(\theta_{k,c}^s) < 1$ has to hold, where $|2\theta_{k,p}^s| \geq |2\theta_{k,p}|$ is the apex angle of the enlarged conic sector of the plant. Accordingly, the stability allowable controller sector with apex angle $|2\theta_{k,c}^s| \leq |2\theta_{k,c}|$ is smaller than with the transformation approach, i.e. it is more conservative. The same is true if the scattering transformation is applied to non-passive systems: The rotation by $\pi/4$ does not immediately lead to a small gain system, i.e. an enlarged sector must be considered, and conservatism is introduced.

With the intuition of conic sectors, the main idea of the proposed approach can be summarized into rotating the plant and controller conic sectors to achieve a non-conservative $\mathcal{L}_2$ gain representation in the communicated signals compared to the classical small gain approach.

### 4.4. Stronger stability condition for known LTI, SISO systems

So far the distributed control design has been derived for general nonlinear IF–OFP plants and pre-designed controllers based on a sufficient stability condition. From now on plant and controller are assumed to be LTI, SISO systems with *known* transfer functions and without requiring the IF–OFP property. This allows a less conservative design of the transformation matrix $M$ based on a necessary and sufficient condition for delay-independent stability.

Consider the LTI plant and controller described by the transfer functions $G_p(s) = \frac{Y_p(s)}{U_p(s)}$ and $G_c(s) = \frac{Y_c(s)}{E(s)}$, respectively, where $Y_p(s)$ and $U_p(s)$ represent the Laplace transformations of the plant output $y_p(t)$ and input $u_p(t)$, and $Y_c(s)$ and $E(s)$ the Laplace transformations of the controller output $y_c(t)$ and input $e(t)$. Where it is unambiguous the Laplace variable $s$ is dropped for convenience of notation. Define the transfer function

$$G_{13} = G_1G_3 = \frac{m_{21}+m_{22}G_p}{m_{11}+m_{12}G_p}\frac{m_{12}-m_{11}G_c}{m_{22}-m_{21}G_c} \tag{23}$$

with $G_1$ and $G_3$ being the transfer functions of $h_1$ and $h_3$ respectively, and $\{m_{ij} \in \mathbb{R},\ \text{with } i, j \in \{1, 2\}\}$ the elements of $M \in \mathbb{R}^{2\times 2}$. The following proposition gives a necessary and sufficient condition for delay-independent stability:

**Proposition 3.** *Consider the distributed feedback control architecture consisting of the LTI, SISO plant $G_p$, the remote LTI, SISO controller $G_c$, the bidirectional communication channel with time delays $T_1, T_2$ and the input–output transformation (8). The overall closed loop system is delay-independently stable if and only if the transformation matrix $M \in \mathbb{R}^{2\times 2}$ is chosen such that the following two conditions are true*

(i) $G_1, G_3$ *are stable, and*
(ii) $|G_{13}| < 1\ \forall\omega > 0$.

**Proof.** Condition (i) is required to ensure stability for $T = T_1 + T_2 \to \infty$. The open loop transfer function in the transformed variables is given by $G_{13}\mathrm{e}^{-j\omega T}$. For stability $|G_{13}\mathrm{e}^{-j\omega T}| < 1$ must hold, when $\arg\{G_{13}\mathrm{e}^{-j\omega T}\} \leq -180°$. For arbitrary $T$ and $\omega \neq 0$, $\mathrm{e}^{-j\omega T}$ defines an arbitrary phase shift. Thus, for all $\omega > 0$, $|G_{13}| < 1$ must hold. ■

Observe that Theorem 1 leads to the more conservative stability requirement $\gamma_{h_1}\gamma_{h_3} = \|G_1\|_\infty\|G_3\|_\infty < 1$. The conservatism comes from the fact that in general $\max_{\omega>0}|G_1G_3| \leq \|G_1G_3\|_\infty \leq \|G_1\|_\infty\|G_3\|_\infty$ holds with strict inequality. Equality is given only if the maximum magnitude of $G_1$ and $G_3$ appears at the same frequency $\omega_{\max} = \arg\sup_\omega|G_1| = \arg\sup_\omega|G_3|$, which is not equal to zero.

## 5. Performance aspects for LTI, SISO systems

In the following the sensitivity of the proposed architecture to time delay and the steady state behavior are discussed for the proposed distributed controller architecture with LTI systems as plant and controller. Therefore the closed loop transfer function $G(s) = \frac{Y_p(s)}{W(s)}$, from the reference input $W$ to the plant output $Y_p$, is computed using (6) and (7) to be

$$G(s) = G_0(s)G_{tr}(s)\mathrm{e}^{-sT_1}, \quad G_{tr}(s) = \frac{1-G_{13}(s)}{1-G_{13}(s)\mathrm{e}^{-sT}}, \tag{24}$$

with $G_0 = (G_pG_c)(1 + G_pG_c)^{-1}$ and $G_{O*}$ given from (23). Obviously, the overall closed loop system can be decomposed into two components: (i) the standard closed loop system $G_0$ without time delay and without input–output transformation, and (ii) the component $G_{tr}$ which describes the influence of the time delay and the input–output transformation. Obviously, if $G_{tr}$ is far from identity, the behavior of the closed loop system with time delay and transformation largely differs from the behavior of the closed loop system without time delay and without transformation.

### 5.1. Sensitivity to time delay

Sensitivity to time delay is an interesting aspect of performance, especially in NCS where the time delay is not exactly known in advance. Low sensitivity to time delay means that a similar input–output behavior is achieved in a large range of time delay values. The sensitivity function with respect to the round trip time delay $T = T_1 + T_2$ is given by the infinite dimensional transfer function

$$S_T^{G^*} = \frac{T}{G^*}\frac{\mathrm{d}G^*}{\mathrm{d}T} = sT\mathrm{e}^{-sT}\frac{G_{13}}{1-G_{13}\mathrm{e}^{-sT}},$$

where $G^*(s) = G_0(s)G_{tr}(s)$ is the transfer function (24) without the purely time shifting part $\mathrm{e}^{-sT_1}$. For the norm of $S_T^{G^*}$ a frequency-dependent maximum can be computed as stated in the next theorem.

**Theorem 2.** *Assume that Proposition 3 is satisfied for the distributed control architecture, i.e. $\|G_{13}\|_\infty < 1$ holds. Then the norm of the time delay sensitivity function is for each frequency $\omega_0$ bounded from above by*

$$|S_T^{G^*}(j\omega_0)| \leq \frac{\omega_0T\|G_{13}\|_\infty}{1-\|G_{13}\|_\infty}. \tag{25}$$

**Proof.** Straightforward computation of the norm of the sensitivity function yields

$$|S_T^{G^*}(j\omega_0)| = \frac{\omega_0T|G_{13}|}{|1-G_{13}\mathrm{e}^{-j\omega_0T}|} \leq \frac{\omega_0T|G_{13}|}{1-|G_{13}|} \leq \frac{\omega_0T\|G_{13}\|_\infty}{1-\|G_{13}\|_\infty},$$

where the dependence on $j\omega_0$ in $|G_{13}(j\omega_0)|$ is suppressed for convenience of notation. ■

Interestingly, the performance requirement for low sensitivity to time delay is compatible with the demand for large stability reserve; in both cases $\|G_{13}\|_\infty$ is required to be small, and of course below one. This can be seen by taking the derivative of (25) with respect to $\|G_{13}\|_\infty$ and showing that the righthand part of (25) is a strictly increasing function of $\|G_{13}\|_\infty$ when $\|G_{13}\|_\infty < 1$. Thus, minimizing $\|G_{13}\|_\infty$ jointly achieves stability and sensitivity goals.

**Remark 5.** The design of the transformation $M$ that simultaneously guarantees delay-independent stability and low time delay sensitivity for known LTI plant and controller can be formulated as an optimization problem $\min_M\|G_{13}\|_\infty$ under bilinear matrix inequality constraints, as shown in Matiakis, Hirche, and Buss (2008).

The special case of $S_T^{G^*} = 0$ can be achieved by using a proportional controller $G_c(s) = \frac{m_{12}}{m_{11}}$, independently of the plant. This follows straightforwardly from substituting $G_c$ in (23) resulting in $G_{13} = 0 \Rightarrow S_T^{G^*} = 0 \Rightarrow G_{tr}(s) = 1$. The closed loop transfer function (24) reduces to $G(s) = G_0(s)\mathrm{e}^{-sT_1}$ with the time shifting part having no effect on the transient response. This fact reflects the intuition that if a static controller $G_c$ is used in the proposed setup, then it can be implemented at the plant side and no remote control action is required. However, a proportional controller does not usually meet the performance requirements and a compromise should be made between performance and sensitivity to time delay.

As the time delay reduces to zero, i.e. $T_1 = T_2 = T = 0$, the system reduces to that without input–output transformation, i.e. $G(s) = G_0(s)$ as straightforwardly computable from (24). The statement holds as well for the general nonlinear case, since $s_l = s_r$ when $T_1 = T_2 = 0$. This is interesting as the controller can be rather aggressively designed, compared to the standard small gain approach, without considering time delay. For zero time delay "nominal" performance is recovered. Together with low sensitivity this means that good performance is achieved in a large range of time delay values.

### 5.2. Steady state behavior

The steady state behavior of the system with the input–output transformation and time delay is equivalent to the steady state behavior of the system without the input–output transformation and without time delay as easily derivable by setting $s = 0$ in (24), resulting in $G(0) = G_0(0)$. For the nonlinear case this can be observed from the steady state condition $s_l = s_r$, hence $z_c = M^{-1}s_l = M^{-1}s_r = M^{-1}Mz_p = z_p$.

In terms of steady state error the proposed approach clearly outperforms the standard small gain approach which requires $|G_c(j\omega)G_p(j\omega)| < 1, \omega > 0$, i.e. free integrators in the open loop are not allowed, thus leading to a non-zero steady state error. In the proposed approach free integrators in plant or controller do not necessarily appear as free integrators in $G_{13}$ (23). As a result delay-independent stability based on Proposition 3 and steady state error zero can be simultaneously guaranteed. This is demonstrated in the following example.

**Example.** Consider the plant $G_p(s) = \frac{1}{s+1}$ and the controller $G_c(s) = \frac{s+1}{s(s+10)}$. The input–output transformation minimizing $\|G_{13}\|_\infty$ in numerical optimization is given by $m_{11} = m_{22} = 0.866$, $m_{12} = 0.5$, and $m_{21} = -0.5$. The open loop transfer function $G_cG_p$ violates the small gain condition. With transformation, i.e. the distributed control approach, zero steady state error is achieved.

In summary, the proposed distributed control approach indicates significant advantages over the standard small gain approach. In fact, even delay-dependent input–output approaches are outperformed in simulation and experiments as shown in Matiakis and Hirche (2006). Here we demonstrate its efficacy by a numerical example.

## 6. Numerical example

As plant we consider the NN8 example, extracted from the publicly available benchmark collection COMPl$_e$ib (Leibfritz, 2004)), regarding only its first input and output, resulting in a SISO system. The state space matrices are

$$A_p = \begin{bmatrix} -0.25 & 0.1 & 1 \\ -0.05 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \qquad \begin{aligned} B_p &= [0\ 0\ 1]^{\mathrm{T}} \\ C_p &= [1\ 0\ 0] \\ D_p &= 0. \end{aligned}$$

Three different controllers are compared. A linear quadratic regulator (LQR), with and without the transformation, and a small gain based controller with state feedback. The exact design procedure is described in the following.

### 6.1. Linear quadratic regulator

The controller $G_c$ is a LQR without considering the time delay minimizing the cost function

$$J = \int_0^\infty y^2(\tau) + 0.01u^2(\tau)\mathrm{d}\tau. \tag{26}$$

A full state observer is computed with its poles placed at the real axis to $[-2\ -3\ -4]$. The overall observer based controller is given by

$$A_c = \begin{bmatrix} -8 & 0.1 & 1 \\ -240 & 0 & 0 \\ -3.122 & -0.339 & -4.387 \end{bmatrix}, \qquad B_c = \begin{bmatrix} -7.8 \\ -239.95 \\ 6 \end{bmatrix},$$

$$C_c = [-9.122\ -0.339\ -3.387], \qquad D_c = 0.$$

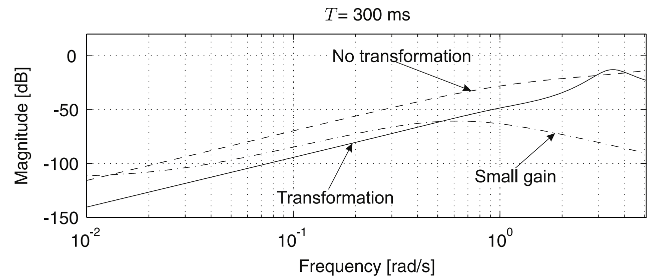


**Fig. 4.** Norm of the time delay sensitivity function $|S_T^{G^*}|$ of the systems with an LQR with and without the input–output transformation, and the small gain based controller.

### 6.2. Distributed control approach

The LQR described in Section 6.1 is used as the pre-designed controller. The transformation $M$ is designed by numerical optimization solving $\min_M \|G_{13}\|_\infty$. The optimization is performed using `fminsearch` of the Matlab optimization toolbox. Note that the optimization problem is not convex. Therefore the optimization algorithm is executed starting from different random initial conditions $M_0$, and the best achieved (locally optimal) solution after several trials is applied. The computation of $\|G_{13}\|_\infty$ is done by expressing $\|G_{13}\|_\infty$ as an optimization problem with linear matrix inequality constraints, and using the YALMIP Matlab toolbox (Löfberg, 2004) with the SDPT3 solver (Tütüncü, Toh, & Todd, 2003). The best achieved solution is $\|G_{13}\|_\infty = 0.5533$ for the transformation

$$M = \begin{bmatrix} 0.7778 & 5.2414 \\ 0.0474 & -11.9826 \end{bmatrix}. \tag{27}$$

### 6.3. Small gain based controller

For the small gain based controller the LQR state feedback problem is solved, formulated in LMIs (Boyd, Ghaoui, Feron, & Balakrishnan, 1994), with a additional small gain constraint of the open loop transfer function, which ensures delay-independent stability. The problem is described as

$$\textbf{minimize } x_0\mathbf{K_1}x_0 \quad \textbf{subject to}$$

$$\begin{bmatrix} 0.01I & B_p^{\mathrm{T}}\mathbf{K_1} \\ \mathbf{K_1}B_p & A_p^{\mathrm{T}}\mathbf{K_1} + \mathbf{K_1}A_p + C_p^{\mathrm{T}}C_p \end{bmatrix} < 0,$$

$$\begin{bmatrix} A_p^{\mathrm{T}}\mathbf{K_2} + \mathbf{K_2}A_p + \mathbf{K_1}B_p10^4B_p^{\mathrm{T}}\mathbf{K_1} & \mathbf{K_2}B_p \\ B_p^{\mathrm{T}}\mathbf{K_2} & -I \end{bmatrix} > 0,$$

$$\mathbf{K_1} > 0, \qquad \mathbf{K_2} > 0,$$

where with bold letters the optimization parameters are denoted, and $x_0^{\mathrm{T}} = [1\ 1\ 1]$ is the initial condition. For the solution the YALMIP Matlab toolbox (Löfberg, 2004) with the local solver PENBMI (Kočara & Stingl, 2003) is used for different initial conditions. The obtained state feedback is $F = [183.05\ 92.631\ 5.914]10^{-3}$.

### 6.4. Simulation results

The norm of the sensitivity function with respect to time delay $|S_T^{G^*}|$ is shown in Fig. 4 for the three different approaches for round trip time delay $T = 300$ ms, plotted until the maximum cutoff frequency of the three closed loop systems. The sensitivity of the proposed distributed control approach is lower than for an LQR without the transformation, in almost all the considered frequencies, except for a small range $\omega \in [10^{0.2}\ 10^{0.3}]$ rad/s. The state feedback small gain based controller shows lower sensitivity in the higher frequencies; it is, however, very conservative, as

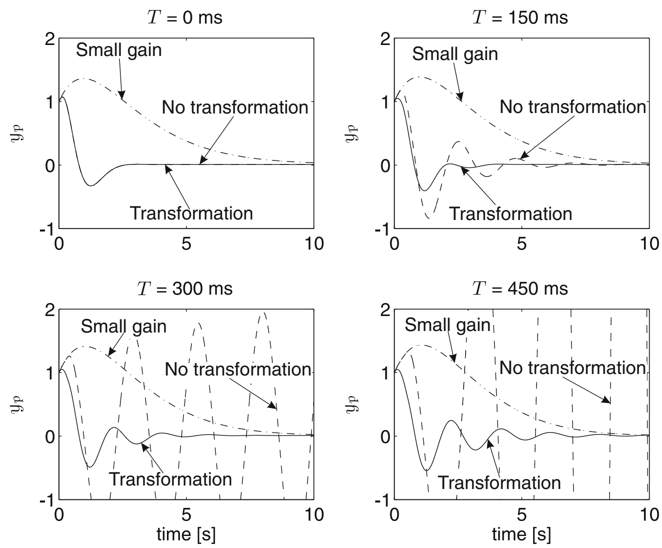


**Fig. 5.** Impulse response of the systems with an LQR with and without the input–output transformation, and the small gain based controller, for various time delay values.

**Table 1**
Value of cost function $J$ (26) for time horizon of 10 s.

| Time delay (ms) | 0 | 150 | 300 | 450 |
|---|---|---|---|---|
| LQR + transformation | 0.273 | 0.289 | 0.316 | 0.353 |
| LQR w/o transformation | 0.273 | 0.615 | Unst. | Unst. |
| Standard small gain | 2.611 | 2.698 | 2.788 | 2.879 |

observable from the transient behavior. The response for the three approaches with initial condition $x_0^T = [1\ 1\ 1]$ and roundtrip time delay values $T = 0, 150, 300, 450$ ms equally divided in the forward and backward channel are presented in Fig. 5. The system with the input–output transformation remains stable in all cases, and its response is only slightly affected by the time delay value. On the contrary, the system without the transformation is sensitive to the time delay, and becomes unstable for $T = 288$ ms. The response of the system with the state feedback small gain based controller is also only slightly affected by the time delay value, but it is very conservative. This can be seen from the value of the cost function (26) for the simulation time horizon of 10 s, which is presented in Table 1. The results certify that the proposed approach shows superior performance for all time delay values.

In short, compared to an LQR without the transformation, the proposed approach shows significantly lower sensitivity, and significantly better performance compared to the state feedback small based controller.

## 7. Conclusions

This article presents a novel distributed controller approach for delay-independent stability of NCS. Instead of directly communicating the plant and controller inputs and outputs, a linear combination of them is transmitted. A design for this linear transformation is given. In the case of non-linear IF–OFP systems, delay-independent finite gain $\mathcal{L}_2$ stability is guaranteed for every plant–controller pair which satisfies a well-known stability condition without time delay. In the case of LTI systems with known transfer functions, a necessary and sufficient stability condition is given. The proposed approach allows non-conservative controller design without considering time delay in the loop, resulting in a superior tracking performance. Due to the low sensitivity to time delay the performance remains good even for high time delay values. Simulations validate the proposed approach in a comparison with an LQR without the input–output transformation, and with a small gain based state feedback controller. Extensions guaranteeing stability for time-varying delay and packet loss are subject of forthcoming papers to be published soon.

## Acknowledgement

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## Appendix

The following lemma imposes restrictions on the eigenvalues of the dissipativity matrix $P$.

**Lemma 1.** *The dissipativity parameters $\delta, \varepsilon, \eta$ of all dissipative systems belong to the domain $\Omega = \Omega_1 \cup \Omega_2$ with $\Omega_1 = \{\delta, \varepsilon, \eta \in \mathbb{R} | \delta\varepsilon - \eta^2 < 0\}$ and $\Omega_2 = \{\delta, \varepsilon, \eta \in \mathbb{R} | \delta\varepsilon - \eta^2 = 0; \delta, \varepsilon > 0\}$.*

**Proof.** For convenient notation the proof is given for the SISO case. In the case of MIMO system the proof is exactly the same, only the multiplicity of the eigenvalues changes accordingly. For $(\delta, \varepsilon, \eta) \in \bar{\Omega} = \Omega_3 \cup \Omega_4$ with $\Omega_3 = \{\delta, \varepsilon, \eta \in \mathbb{R} | \delta\varepsilon - \eta^2 > 0\}$, and $\Omega_4 = \{\delta, \varepsilon \in \mathbb{R} | \delta\varepsilon - \eta^2 = 0; \varepsilon, \delta < 0\}$ degenerate cases occur. The condition $(\delta, \varepsilon, \eta) \in \Omega_3$ is equivalent to positive or negative definiteness of matrix $P$, i.e. $\det P = \lambda_1\lambda_2 = \delta\varepsilon - \eta^2 > 0$ where $\lambda_1, \lambda_2$ are the two eigenvalues of $P$. Hence, $\lambda_1, \lambda_2 > 0 \Leftrightarrow P > 0$ or $\lambda_1, \lambda_2 < 0 \Leftrightarrow P < 0$. For $P > 0$ (1) is satisfied for any pair $u(\tau), y(\tau)$ imposing no restriction to the system input–output behavior. Analogously, for $P < 0$ (1) cannot be satisfied for any pair $u(\tau), y(\tau)$. When $(\delta, \varepsilon, \eta) \in \Omega_4$ we get $\lambda_1 = 0, \lambda_2 = -\delta - \varepsilon > 0$. Thus, $P$ is positive semidefinite and (1) is again satisfied for any pair $u(\tau), y(\tau)$. ∎

Lemma 1 implies that without loss of generality we can restrict $P$ to have either $m$ positive and $m$ negative, or $m$ zero and $m$ negative eigenvalues. For $\Omega$ the next lemma holds.

**Lemma 2.** *Consider the expressions*

$$\alpha(\theta) = 2\eta \sin(\theta)\cos(\theta) - \delta\cos^2(\theta) - \varepsilon\sin^2(\theta)$$
$$\beta(\theta) = \alpha(\theta) + \delta + \varepsilon$$

*where $\theta = \theta^+$ and $\theta = \theta^-$ are the two solutions of*

$$\cot(2\theta) = \frac{\varepsilon - \delta}{2\eta}$$

*in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$, and $(\delta, \varepsilon, \eta) \in \Omega$. The following statements are true:*

- $(\delta, \varepsilon, \eta) \in \Omega_1$
  $\Rightarrow \alpha(\theta^+) > 0, \beta(\theta^+) > 0$, *and* $\alpha(\theta^-) < 0, \beta(\theta^-) < 0$
- $(\delta, \varepsilon, \eta) \in \Omega_2$
  $\Rightarrow \alpha(\theta^+) = 0, \beta(\theta^+) > 0$, *and* $\beta(\theta^-) = 0, \alpha(\theta^-) < 0$.

**Proof.** For the two angles $\theta = \theta^+$ and $\theta = \theta^-$ it can be shown that $\alpha(\theta)\beta(\theta) = \eta^2 - \delta\varepsilon$. Thus for $(\delta, \varepsilon, \eta) \in \Omega_1, \alpha(\theta)\beta(\theta) > 0$, meaning that $\alpha, \beta$ always have the same sign for each angle. Furthermore $\alpha(\theta^-) = -\beta(\theta^+), \beta(\theta^-) = -\alpha(\theta^+)$ meaning that $\alpha(\theta), \beta(\theta)$ always have different signs for the two angle solutions $\theta = \theta^+$ and $\theta = \theta^-$. Combining the above, the first part of the lemma is proved. For $(\delta, \varepsilon, \eta) \in \Omega_2$ we get $\alpha(\theta)\beta(\theta) = \eta^2 - \delta\varepsilon = 0$ meaning that $\alpha(\theta)$ and/or $\beta(\theta)$ are zero. Furthermore, $\beta(\theta) = \alpha(\theta) + \delta + \varepsilon \Rightarrow \beta(\theta) > \alpha(\theta)$. If $\alpha(\theta^+) = -\beta(\theta^-) = 0$ then $\beta(\theta^+) > 0, \alpha(\theta^-) < 0$; analogously for the other case $\alpha(\theta^-) = -\beta(\theta^+) = 0$. ∎

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