Bilateral Control of Teleoperators with Time Delay

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Abstract—**When a robot is operated remotely by use of a teleoperator, it is desirable to communicate contact force information from the slave to the master, in order to kinesthetically couple the operator to the environment and increase the sense of telepresence. One problem, however, recognized as early as 1966, has remained unsolved until now: how to maintain stability in a force-reflecting bilateral teleoperator in the presence of substantial time delay. In this paper, we present a solution to this problem.**

I. Introduction

TELEOPERATION represents one of the first domains of robotics and one of the most challenging [1]. In teleoperation, a human operator conducts a task in a remote environment via master and slave manipulators. Providing contact force information to the human operator can improve task performance. Although this information can be obtained from visual displays, it is more useful when provided directly, by reflecting the measured force to motors on the master. When this is done, the contact force is said to be "reflected" to the human operator, and the teleoperator is said to be controlled bilaterally [2], [3]. When teleoperation is performed over a great distance, such as in undersea and outer space operations, a time delay is incurred in the transmission of information from one site to another. This time delay can destabilize a bilaterally controlled teleoperator.

In 1965 the first work [4] dealing with time delay in teleoperation appeared. Because force reflection was not used, however, instability was not a problem. In 1966, force reflection was used in the presence of time delay [5] and the instability was apparent. Delays on the order of a tenth of a second were shown to destabilize the teleoperator. Recently, Vertut and co-workers experimented with a force-reflecting system with time delay, and were able to achieve stability only when the bandwidth was severely reduced, allowing velocities of only 10 cm/s [6]. Since this pioneering work, little has appeared in the literature regarding the stability of bilateral teleoperators with time delay. One reason for this is the difficulty of the analysis. The time delay turns an otherwise finite-dimensional system into an infinite-dimensional system, and the nonlinearities in typical manipulators further complicate the analysis.

This paper presents a new control law for teleoperators which overcomes the instability caused by time delay. By using passivity and scattering theory, a criterion is developed which shows why existing bilateral control systems are unstable for certain environments, and why the proposed bilateral control law is stable for any environment and any time delay. The control law has been implemented on a single axis force reflecting hand controller, and preliminary results are shown.

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In order to keep the presentation clear we shall deal with a single degree-of-freedom (DOF) linear, time-invariant (LTI) teleoperator system. Nevertheless, our results can be extended, without loss of generality, to an n-DOF nonlinear teleoperation system for the following reasons. First the communication compensation presented remains linear, time-invariant, and uncoupled for any master and slave configuration. Second, the analysis for the master and slave is based on passivity concepts which are equally valid for the full nonlinear system.

II. Preliminaries

A teleoperator system can be represented by the block diagram of Fig. 1 and consists of five subsystems: the human operator, the master, the communication block, the slave, and the environment. The variables and parameters for the system are given in Table I. The operator commands a velocity forward, through the master, communication block, and slave, to the environment. Likewise, the force sensed at the environment is transmitted back through these blocks, to the human operator. Ideally, the slave is controlled so that in the steady state, the slave velocity v_s is equal to the operator velocity v_m and the backdriven force F_m is equal to the contact force F_e.

Since the teleoperator is controlled bilaterally, we can reverse the arrows shown in Fig. 1, and have the operator command forces forward to the environment, and have the environment's velocity sent back to the master. With this understanding, the teleoperator is acting as a hybrid control system, where the human determines the switching between the position and force control modes [7].

III. Network Representations, Passivity, and Scattering Theory

In this section we introduce some concepts from network theory which will be useful for our subsequent analysis. For simplicity, we restrict the discussion to linear, time-invariant (LTI) networks which can be either lumped or distributed.

In what follows $\boldsymbol{R}_+$ denotes the set of nonnegative real numbers, $\boldsymbol{R}^n$ denotes the usual n-dimensional Euclidean space over $\boldsymbol{R}$ with norm

$$\|x\| = \left(\sum_{i=1}^{n} x_i^2\right)^{1/2}; \qquad x=(x_1, \cdots, x_n)^T \tag{3.1}$$

$\boldsymbol{L}_2^n(\boldsymbol{R}_+)$ will denote the Hilbert space of Lebesque measurable functions $f:\boldsymbol{R}_+ \to \boldsymbol{R}^n$, which are square integrable, i.e.,

$$\|f\|_2 \triangleq \int_0^\infty \|f(t)\|^2\, dt < \infty. \tag{3.2}$$

If $T:\boldsymbol{L}_2^n(\boldsymbol{R}_+) \to \boldsymbol{L}_2^n(\boldsymbol{R}_+)$ is a (bounded) operator, the norm of T is defined by

$$\|T\| = \sup_{\substack{x\in \boldsymbol{L}_2^n(\boldsymbol{R}_+)\\ \|x\|\neq 0}} \frac{\|Tx\|_2}{\|x\|_2}. \tag{3.3}$$

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Fig. 1. Block diagram of the teleoperator system.

TABLE I
TELEOPERATOR VARIABLES AND PARAMETERS

	Variables		Parameters	
human operator	F_h	applied force		
master	F_m	force	M_m	inertia
	v_m	velocity	B_m	damping
	τ_m	motor torque		
	x_m	position		
commun. block	F_{md}	desired force	T	time delay
	v_{sd}	desired vel.	n	scale factor
slave	v_s	velocity	M_s	inertia
	F_s	force	B_{s1}	error damping
	x_s	position	B_{s2}	rate damping
	τ_s	motor torque	K_s	stiffness gain
environ-ment	F_e	contact force	Z_e	impedance
			α_f	force gain

If T is a convolution operator defining an LTI system with Laplace transform $T(s)$, then we have the Parseval identity

$$\|T\| = \sup_{\omega} \|T(j\omega)\| = \sup_{\omega} \lambda^{1/2}(T^*(j\omega)T(j\omega)) \tag{3.4}$$

where * denotes complex conjugate transpose and $\lambda^{1/2}(\cdot)$ denotes the square root of the maximum eigenvalue.

Using the analogy between mechanical and electrical systems [8], [9], we can represent a teleoperator as a network, shown in Fig. 2. Here the master, communication block, and slave are represented by two-ports, and the operator and environment are represented by one-ports [10].

An n-port is characterized by the relationship between effort F (force, voltage), and flow v (velocity, current). For an LTI one-port, this relationship is specified by its *impedance* $Z(s)$ according to

$$F(s) = Z(s)v(s) \tag{3.5}$$

where $F(s)$, $v(s)$ are the Laplace transforms of $F(t)$, $v(t)$, respectively. For an LTI two-port, this relationship is conveniently specified [11] by its *hybrid matrix*,[1] $H(s)$ according to

$$\begin{bmatrix} F_1(s) \\ -v_2(s) \end{bmatrix} = \begin{bmatrix} h_{11}(s) & h_{21}(s) \\ h_{12}(s) & h_{22}(s) \end{bmatrix} \begin{bmatrix} v_1(s) \\ F_2(s) \end{bmatrix} = H(s) \begin{bmatrix} v_1(s) \\ F_2(s) \end{bmatrix} \tag{3.6}$$

where F_1, F_2, v_1, and v_2 are defined as shown in Fig. 3(d).

Definition 3.1: An n-port is said to be *passive* if and only if for any independent set of n-port flows, v_i injected into the system, and efforts F_j applied across the system

$$\int_0^\infty F^T(t)v(t)\,dt \ge 0 \tag{3.7}$$

[1] The sign of v_2 is reversed here and in the scattering matrix, since v_2 is exiting rather than entering the two-port.

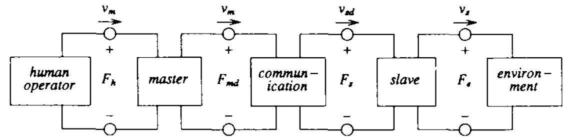


Fig. 2. Network representation of teleoperator.

where $F = [F_1, F_2, \cdots, F_n]^T \in L_2^n(\boldsymbol{R}_+)$ and $v(t) = [v_1, v_2, \cdots, v_n]^T \in L_2^n(\boldsymbol{R}_+)$.

It is assumed that the system is initially relaxed, i.e., no initial loading exists on any of the springs in the system, and none of the inertias have an initial velocity.

Condition (3.7) is simply a statement that a passive n-port may dissipate energy but cannot increase the total energy of a system in which it is an element. In addition, an n-port is said to be *lossless* if and only if

$$\int_0^\infty F^T(t)v(t)\,dt = 0. \tag{3.8}$$

Another useful tool for networks, especially for infinite-dimensional systems such as transmission lines and delay systems, is scattering theory [12].

Definition 3.2: The *scattering operator* $S: L_2^n(\boldsymbol{R}_+) \to L_2^n(\boldsymbol{R}_+)$ is defined by

$$F - v = S(F + v) \tag{3.9}$$

and maps effort plus flow into effort minus flow, where the flow is entering the system's ports, and the effort is measured across the system's ports.

For LTI systems, the scattering operator S can be expressed in the frequency domain as a scattering matrix $S(s)$, where

$$F(s) - v(s) = S(s)(F(s) + v(s)). \tag{3.10}$$

In the case of a two-port, this scattering matrix can be related to the hybrid matrix $H(s)$ as follows:

$$\begin{bmatrix} F_1(s) - v_1(s) \\ F_2(s) + v_2(s) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \left(\begin{bmatrix} F_1(s) \\ -v_2(s) \end{bmatrix} - \begin{bmatrix} v_1(s) \\ F_2(s) \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} (H(s) - I) \begin{bmatrix} v_1(s) \\ F_2(s) \end{bmatrix}. \tag{3.11}$$

Likewise,

$$\begin{bmatrix} F_1(s) + v_1(s) \\ F_2(s) - v_2(s) \end{bmatrix} = \begin{bmatrix} F_1(s) \\ -v_2(s) \end{bmatrix} + \begin{bmatrix} v_1(s) \\ F_2(s) \end{bmatrix} = (H(s) + I) \begin{bmatrix} v_1(s) \\ F_2(s) \end{bmatrix}. \tag{3.12}$$

Therefore,

$$S(s) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} (H(s) - I)(H(s) + I)^{-1}. \tag{3.13}$$

Theorem 3.1: A system is passive if and only if the norm of its scattering operator is less than or equal to one.

Proof: (Sufficiency)

If $\|S\| \le 1$, then $\|F - v\|_2 / \|F + v\|_2 \le 1$ for all $F, v \in L_2^n(\boldsymbol{R}_+)$. This implies that $\|F + v\|_2^2 - \|F - v\|_2^2 \ge 0$. Writing out the norms explicitly gives

$$\int_0^\infty (F+v)^T(F+v) - (F-v)^T(F-v)\,dt \ge 0. \tag{3.14}$$

But this is equivalent to

$$2\int_0^\infty (v^TF+F^Tv)\,dt=4\int_0^\infty F^Tv\,dt\geq 0. \tag{3.15}$$

Reversing the argument will show necessity. □

Corollary 3.1: A system is passive if and only if $\sup_\omega \lambda^{1/2}(S^T(j\omega)S(j\omega)) \leq 1$.

Proof: Follows directly from (3.4) and Theorem 3.1.

For a transmission line, the scattering operator relates the reflected wave $(F - v)$ to the incident wave $(F + v)$. Its norm can be interpreted as the square root of the maximum power gain for the element. Thus, it follows that for a passive element $\|S\| \leq 1$.

The type of network elements that we will use in our analysis, are the following.

1) An inertial element M, represented by an inductor, corresponding to the mass of the manipulator, which has input/output relationship $F(t) = M\dot{v}(t)$ [Fig. 3(a)].

2) A damping element B, represented by a resistor, corresponding either to velocity feedback or viscous friction, which has input/output relationship $F(t) = Bv(t)$ [Fig. 3(b)].

3) A stiffness element K, corresponding to position feedback, or environmental stiffness, which has input/output relationship $F(t) = K\int v(t)dt$ [Fig. 3(c)].

4) A generic one-port impedance element Z, corresponding to any parallel and/or series combination of the previous three elements, which has an input/output relationship in the frequency domain $F(s) = Z(s)v(s)$ [Fig. 3(b)].

5) A two-port ideal transformer n, corresponding to gear reduction or scaling, which has input/output relationship $F_1(t) = nF_2(t)$, $v_2(t) = nv_1(t)$ [Fig. 3(d)].

6) A linear two-port lossless transmission line element having an input/output relationship in the frequency domain of [14] [Fig. 3(e)].

$$F_1(s)=Z_0\tanh(sl/v_0)v_1(s)+\operatorname{sech}(sl/v_0)F_2(s) \tag{3.16}$$

$$-v_2(s)=-\operatorname{sech}(sl/v_0)v_1(s)+(\tanh(sl/v_0)/Z_0)F_2(s) \tag{3.17}$$

where $Z_0 = \sqrt{L/C}$, $v_0 = 1/\sqrt{LC}$, L is the characteristic inductance, and C is the capacitance for the transmission line.

Besides these passive elements, we also use active elements, namely independent and dependent effort (force) sources $F_d(t)$, and flow (velocity) sources $v_d(t)$. These are shown in Fig. 4.

It is well known that circuits made up of passive elements are themselves passive, and that linear, time-invariant causal systems which exhibit a passive input/output behavior can be realized solely by a circuit made up of passive elements. This does not imply, however, that the circuit must be constructed using passive elements. In fact, we will be using dependent sources, namely dc-motors, controlled in such a way as to imitate the input/output behavior of a passive system. For example, Fig. 5 shows two systems with identical input/output behavior. When representing the teleoperator, we will not distinguish between a truly passive system, such as shown in Fig. 5(a), and a system which is controlled passively, such as is shown in Fig. 5(b).

IV. Dynamics of the Teleoperator System

In order to keep the analysis as simple as possible we model the dynamics of the master and slave as

$$M_m\dot{v}_m=F_h+\tau_m, \tag{4.1}$$

$$M_s\dot{v}_s=-F_e+\tau_s \tag{4.2}$$

where v_m and v_s are the respective velocities for the master and slave, τ_m and τ_s are the respective motor torques, M_m and M_s are the respective inertias, F_h is the operator torque, and F_e is the environment torque.

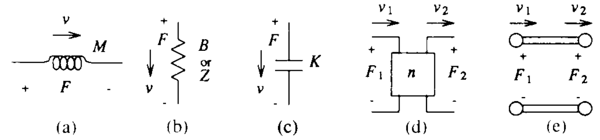


Fig. 3. Passive circuit elements: (a) inertia; (b) damping or impedance; (c) stiffness; (d) transformer; (e) transmission line.

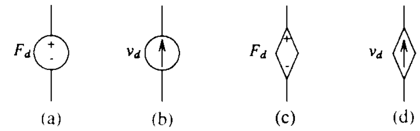


Fig. 4. Active circuit elements: (a) independent effort source; (b) independent flow source; (c) dependent flow source; (d) dependent effort source.

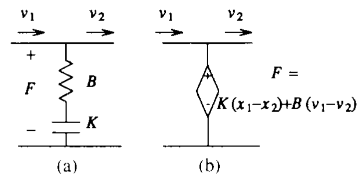


Fig. 5. (a) Passive system; (b) passively controlled system.

A simple control strategy for this system is then given by

$$\tau_m=-B_mv_m-F_{md} \tag{4.3}$$

$$\tau_s=-B_{s2}v_s+F_s-\alpha_fF_e \tag{4.4}$$

where F_s is called the coordinating torque and is given by

$$F_s=K_s\int(v_{sd}-v_s)\,dt+B_{s1}(v_{sd}-v_s). \tag{4.5}$$

The coordinating torque is a term used to describe a motor command which is based on the error between the master and slave variables. Its purpose is to cause the master and slave to track one another.

The term F_{md} is the reflected force and v_{sd} is the velocity setpoint. These variables are determined by the two-port characteristics of the communication block, as shown in Fig. 6. The expressions for the latter terms will be discussed in the remaining sections of this paper.

It should be noticed that the coordinating torque term F_s and not the contact force F_e is available to the communication block for transmission to the master. This is necessary to ensure the passivity of the slave block. In cases where this has not been done, such as in [13], contact instabilities occurred even when there was no time delay.

The reason researchers have been reluctant to use F_s instead of F_e is that without local force feedback F_s is insensitive to changes in F_e. This occurs for a manipulator with a high-gear ratio since the environment impedance Z_e is small with respect to the slave's parameters B_{s2} and M_s. We can relate F_s to F_e by combining (4.2) to (4.4) and using the environments impedance relationship $F_e = Z_ev_s$ as follows:

$$F_s=\left(\frac{M_ss+B_{s2}}{(1+\alpha_f)Z_e}+1\right)(1+\alpha_f)F_e \tag{4.6}$$

and for suitably large α_f

$$F_s \cong (1+\alpha_f)F_e. \tag{4.7}$$

Therefore, when a local force loop is closed around the slave, as is done in (4.4), sensitivity to the environment is achieved, and the

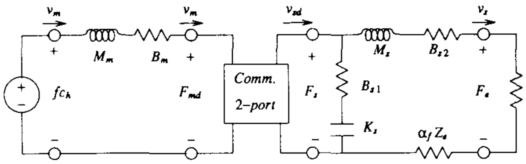


Fig. 6. Circuit representation of teleoperator.

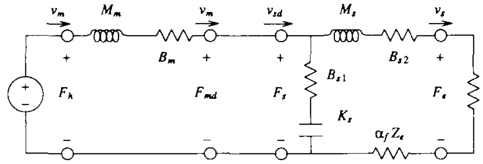


Fig. 7. Circuit representation of teleoperator without time delay.

coordinating torque signal becomes a valid signal for conveying contact force information.

V. Bilateral Teleoperation Without Time Delay

Referring to Fig. 6, we note that the desired behavior for the communication block is

$$F_{md} = F_s, \quad v_{sd} = v_m \tag{5.1}$$

which is the case if there is no time delay. Setting $F_{md} = F_s$ and $v_{sd} = v_m$ in (4.1)-(4.5) gives the following system:

$$M_m \dot{v}_m + B_m v_m = F_h - F_s \tag{5.2}$$

$$M_s \dot{v}_s + B_{s2} v_s = F_s - (1 + \alpha_f) F_e \tag{5.3}$$

where the coordinating torque term F_s is given by

$$F_s = K_s \int (v_m - v_s)\, dt + B_{s1}(v_m - v_s). \tag{5.4}$$

This system is represented in circuit form in Fig. 7.

The control law (4.3)-(4.5) and (5.1) has been implemented on a single DOF master slave system, developed at the Jet Propulsion Laboratory in Pasadena, CA. The master and slave are both dc-motor driven systems with position encoders and PWM drivers. Contact force information is acquired via a strain gauge bridge connected to the base of a beam which protrudes from the slave subsystem. The controller runs at a 500 Hz interrupt rate, and calculations are done using double precision floating point numbers. A preliminary version of this system is described further in [13]. Fig. 8 shows a diagram of the setup.

In order to analyze the fidelity of the teleoperator system, two different tasks were attempted. The first involves hard contact. The slave is commanded forward until a hard surface is contacted. The force is twice ramped up and back down until contact is lost in a period of about 20 s. Ideally, the slave position x_s tracks the master position x_m until contact. Once contact is made, the master motor torque τ_m should track the contact force F_e. Fig. 9 shows the response for the system without time delay.

The second task involves backdrivability. The master is allowed to swing freely, while the slave is hammered twice and then moved sinusoidally. Ideally, the master position should track the slave position without going unstable. Fig. 10 shows the results.

These two figures represent the response of a system without time delay. It is our goal to achieve equivalent responses from a system with time delay.

VI. Teleoperation with Time Delay

When a transmission delay exists, the communication block equation (5.1) becomes

$$F_{md}(t) = F_s(t - T), \tag{6.1}$$

$$v_{sd}(t) = v_m(t - T). \tag{6.2}$$

The backdrivability test was performed using this system with a time delay of only 40 ms. The results are given in Fig. 11.

The system, as is apparent from the plots, is easily destabilized

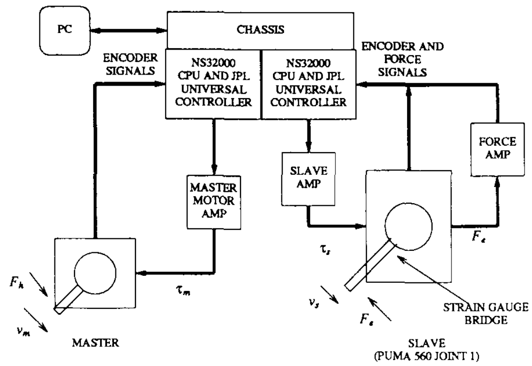


Fig. 8. Teleoperator demonstration setup.

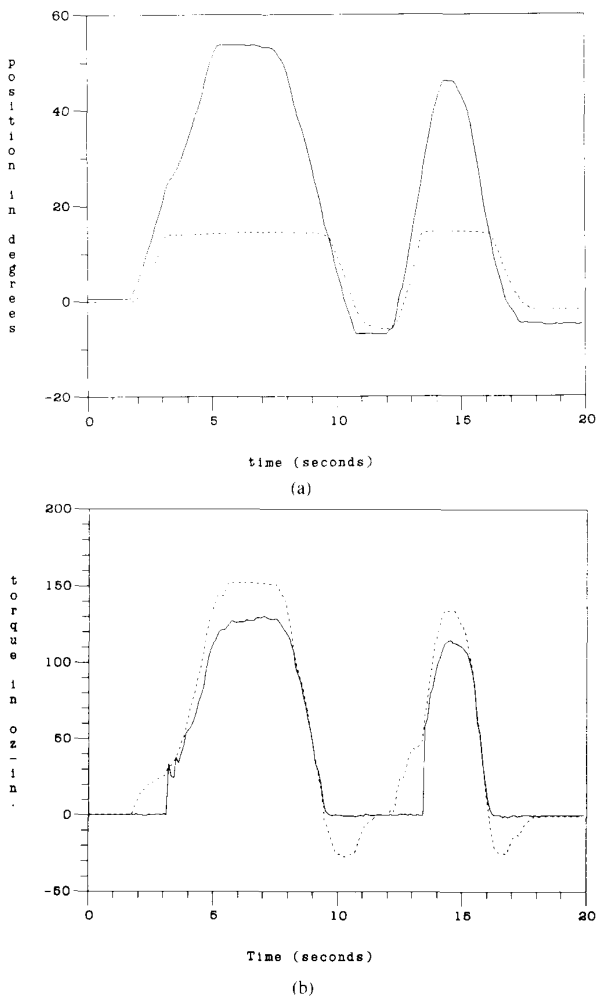


Fig. 9. Hard contact test for system without time delay: (a) x_m (solid) and x_s (dashed) versus time; (b) F_m (solid) and τ_m (dashed) versus time.

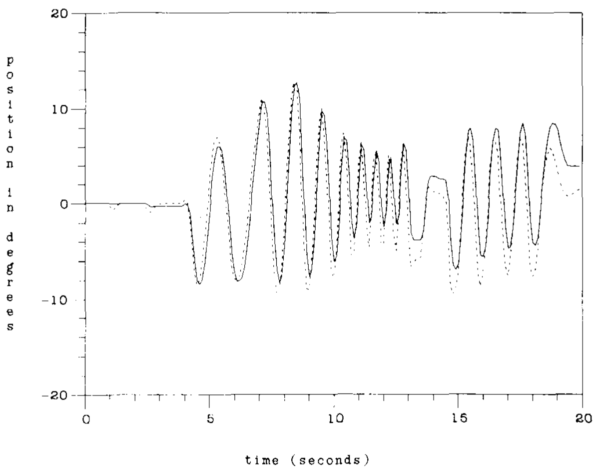


Fig. 10. Backdrivability test for system without time delay: x_m (solid) and x_s (dashed) versus time.

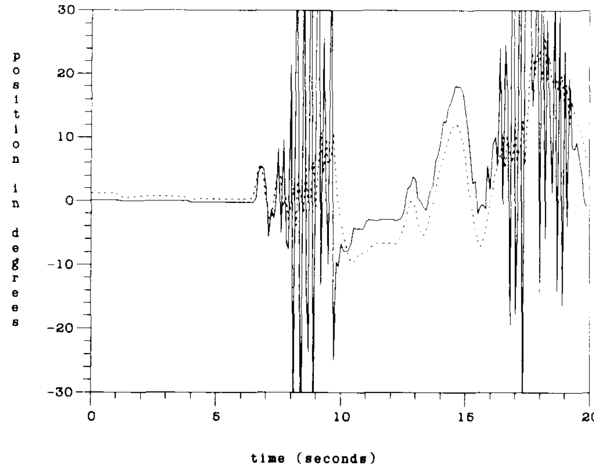


Fig. 11. Backdrivability test for the system with 40 ms time delay: x_m (solid) and x_s (dashed) versus time.

by a backdriven signal. Fig. 12 shows the equivalent circuit diagram of the system described by (4.1)–(4.5) and (6.1), (6.2). The two-port communication circuit for this system is nonpassive, and is the cause of the instability.

To show why the two-port communication circuit is nonpassive, let us examine the two-port given by (6.1) and (6.2). The associated hybrid matrix is derived directly as

$$H(s)=\begin{bmatrix} 0 & e^{-sT} \\ -e^{-sT} & 0 \end{bmatrix}. \tag{6.3}$$

Using (3.13) we can calculate the scattering matrix for the system.

$$\begin{aligned} S&=\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}(H(s)-I)(I+H(s))^{-1} \\ &=\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -1 & e^{-sT} \\ -e^{-sT} & -1 \end{bmatrix}\begin{bmatrix} 1 & e^{-sT} \\ -e^{-sT} & 1 \end{bmatrix}^{-1} \\ &=\begin{bmatrix} -\tanh(sT) & \operatorname{sech}(sT) \\ \operatorname{sech}(sT) & \tanh(sT) \end{bmatrix}. \end{aligned} \tag{6.4}$$

Computing the norm (3.4) gives

$$\begin{aligned} \|S\|&=\sup_\omega \lambda^{1/2}\left(\begin{bmatrix} j\tan(\omega T) & \sec(\omega T) \\ \sec(\omega T) & -j\tan(\omega T) \end{bmatrix}\begin{bmatrix} -j\tan(\omega T) & \sec(\omega T) \\ \sec(\omega T) & j\tan(\omega T) \end{bmatrix}\right) \\ &=\sup_\omega \lambda^{1/2}\left(\begin{bmatrix} \tan^2(\omega T)+\sec^2(\omega T) & 2j\tan(\omega T)\sec(\omega T) \\ -2j\tan(\omega T)\sec(\omega T) & \tan^2(\omega T)+\sec^2(\omega T) \end{bmatrix}\right) \end{aligned} \tag{6.5}$$

and thus $\|S\| = \sup_\omega(|\tan(\omega T)| + |\sec(\omega T)|) = \infty$. Therefore, the scattering operator for this system is unbounded, and hence the system is not passive. In practice, the signals entering the communication block are band-limited. However, since $|\tan(\omega T)| + |\sec(\omega T)| > 1$ for any ω, $T > 0$ the communication block is never passive for any range of frequencies. Furthermore, we would expect the instability problems to be most severe when $\omega T \to \pi/2$. Using the control laws (6.1) and (6.2), only a system with extremely band-limited behavior will be able to remain stable.

VII. Time Delay Compensation for Teleoperators

We have shown that the standard method of communicating forces and velocities between the master and slave in teleoperator systems leads to a nonpassive system for any time delay. This time delay is destabilizing unless the bandwidths of signals entering the communication block are severely restricted.

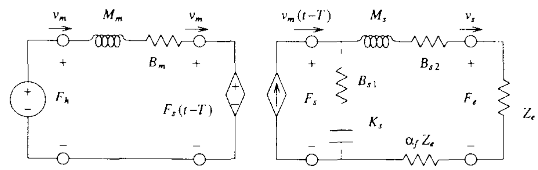


Fig. 12. Circuit diagram for system with time delay.

In this section we present our main result, which is a redefinition of the communication block which guarantees passivity independent of the amount of time delay present in the block. The basic idea is to choose the control law so that the two-port characteristics of the communcation block are identical to a two-port lossless transmission line.

Let us use the transmission line equations (3.16) and (3.17) to represent the communication block. Setting $Z_0 = 1$ and $v_0 = l/T$ gives

$$F_{md}(s)=\tanh(sT)v_m(s)+\operatorname{sech}(sT)F_s(s) \tag{7.1}$$

$$-v_{sd}(s)=-\operatorname{sech}(sT)v_m(s)+\tanh(sT)F_s(s). \tag{7.2}$$

After a little algebraic manipulation [applying (3.13)], we can represent (7.1) and (7.2) as

$$\begin{bmatrix} F_{md}(s)-v_m(s) \\ F_s(s)+v_{sd}(s) \end{bmatrix}=\begin{bmatrix} 0 & e^{-sT} \\ e^{-sT} & 0 \end{bmatrix}\begin{bmatrix} F_{md}(s)+v_m(s) \\ F_s(s)-v_{sd}(s) \end{bmatrix} \tag{7.3}$$

and thus the scattering matrix for the communication block S is given by

$$S=\begin{bmatrix} 0 & e^{-sT} \\ e^{-sT} & 0 \end{bmatrix}. \tag{7.4}$$

The norm (3.4) of the scattering matrix (7.4) is

$$\|S\|=\sup_{\omega} \lambda^{1/2}(S^*(j\omega)S(j\omega))=\sup_{\omega} \lambda^{1/2}\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)=1. \tag{7.5}$$

Therefore, by Theorem 3.1, the system (7.1) and (7.2) is passive.

In the time domain, (7.3) is equivalent to

$$\begin{bmatrix} F_{md}(t)-v_m(t) \\ F_s(t)+v_{sd}(t) \end{bmatrix}=\begin{bmatrix} F_s(t-T)-v_{sd}(t-T) \\ F_{md}(t-T)+v_m(t-T) \end{bmatrix}. \tag{7.6}$$

By regrouping terms in (7.6) we derive our control law for the communication circuit

$$F_{md}(t)=F_s(t-T)-v_{sd}(t-T)+v_m(t). \tag{7.7}$$

$$v_{sd}(t)=v_m(t-T)-F_s(t)+F_{md}(t-T) \tag{7.8}$$

where v_m is measured and F_s is computed from (5.4). It is clear that all information communicated between ends of the network has been delayed by T seconds. Furthermore, by setting $s = 0$ in (7.1) and (7.2) it is clear that in the steady state $F_{md} = F_s$ and $v_{sd} = v_m$.

The equations (7.7) and (7.8) represent a new control law for the communication circuit of a teleoperator with time delay. Because it is derived from the scattering matrix of a passive system, it is a passive control law, and therefore the communication circuit, no matter how large the time delay, cannot destabilize the teleoperator.

Because the force and the velocity signals may differ by orders of magnitude, the control law given by (7.7) and (7.8) may have implementation problems. In order to overcome this, some scaling is necessary. This is done by adding the equivalent of transformers, with scaling factors of n and $1/n$, respectively, to both ends of the transmission line. With this scaling factor the control law becomes

$$F_{md}(t)=F_s(t-T)+n^2(v_m(t)-v_{sd}(t-T)) \tag{7.9}$$

$$v_{sd}(t)=v_m(t-T)+\frac{1}{n^2}(F_{md}(t-T)-F_s(t)). \tag{7.10}$$

Because the resulting system is nothing more than the cascade connection of three passive two-ports, namely a transformer, a transmission line, and another transformer, the combination is still a passive two-port. The overall control law is now given by combining (7.9) and (7.10) with (4.3)–(4.5). In practice, this control law will be implemented digitally. Although this ensures that the time delayed signals can be reconstructed faithfully without additional dynamics, it also requires that a sampled control law be used. Achieving the best digital implementation of the continuous-time control law (4.3)–(4.5), (7.9) and (7.10) is still an open problem.

A circuit diagram of the system is given in Fig. 13, and a block diagram of the entire system is shown in Fig. 14.

The hard contact and backdrivability tests were conducted using the system with time delay compensation. The results are shown in the following plots. Fig. 15 shows the backdrivability test response of the system with 40 ms of time delay. Unlike the system without time-delay compensation, this system could not be destabilized.

Fig. 16 shows the hard contact test responses of the system with 40 ms of time delay. The desired position and force commands are tracked without noticeable degradation.

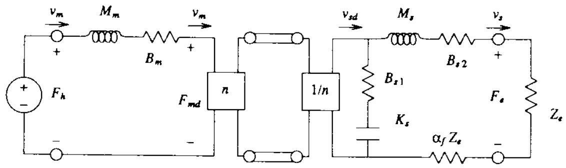


Fig. 13. Circuit diagram for teleoperator.

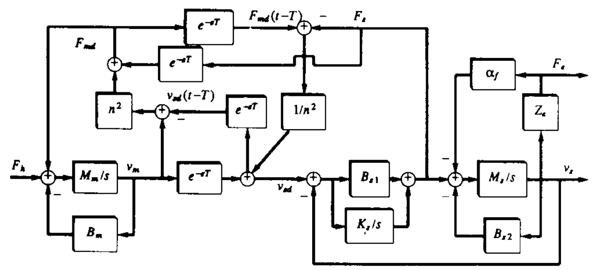


Fig. 14. Block diagram of implementation.

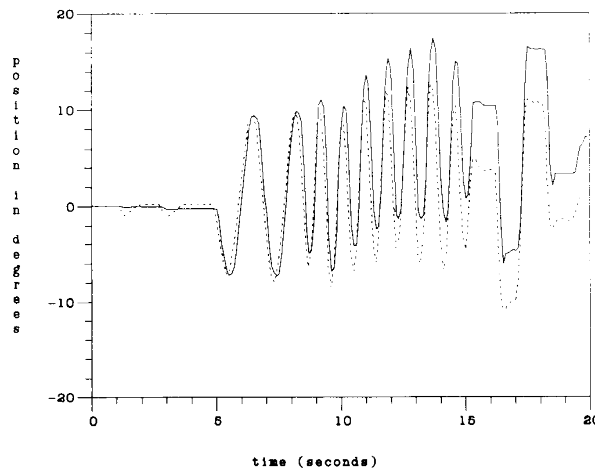


Fig. 15. Backdrivability test for system with 40 ms delay and compensation: x_m (solid) and x_s (dashed) versus time.

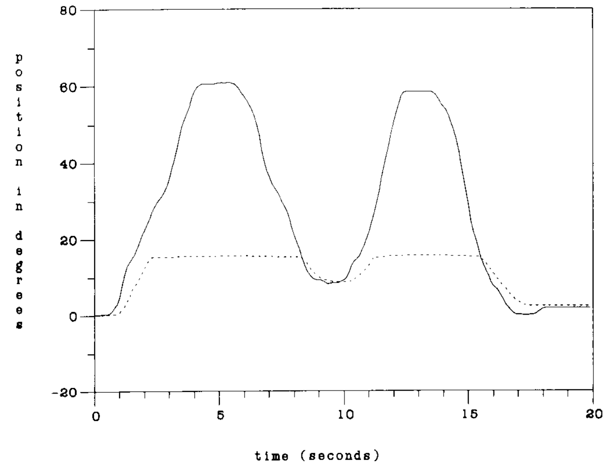
Fig. 17 shows backdrivability test responses of the system with 200 ms of time delay. Still, the system could not be destabilized. What appears to be an increasing signal magnitude is due only to an increase in the applied force F_e and not to instability.

Fig. 18 shows the hard contact test responses of the system with 200 ms of time delay. The position and force commands are still transmitted with high fidelity.

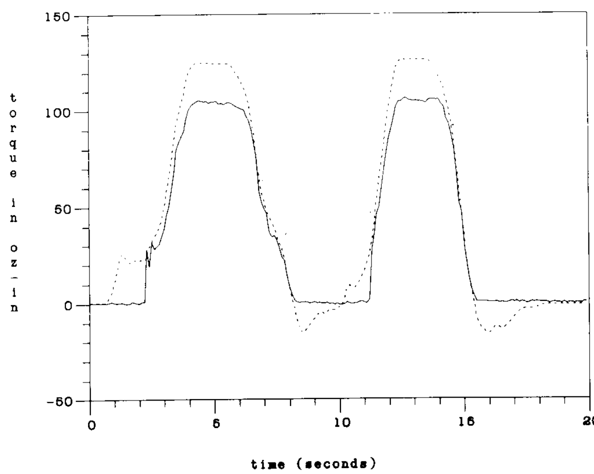
Time delays of up to 2 s were tested. In all cases the system could not be destabilized. Unfortunately, the force and position tracking begins to degrade for the higher time delays. Since this is not predicted in the continuous-time theory, and is not the case in simulations, it is believed that this degradation is due to the particular discrete implementation used on the system. Work is underway to see if a better implementation can be developed.

VIII. Discussion and Conclusions

This paper has introduced a new control law for controlling a teleoperator with time delay. The teleoperation system was divided into five blocks: the operator, master, communication block, slave, and environment. By using a mechanical/electrical analogy it was shown that the instability which occurs in bilateral teleoperation in the presence of time delay is due to a nonpassive communication block.

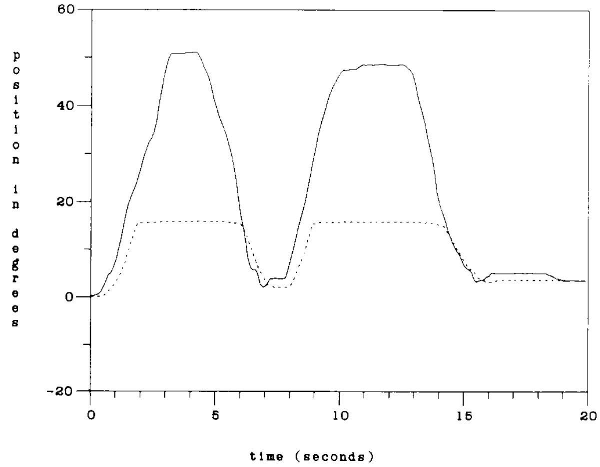


(a)

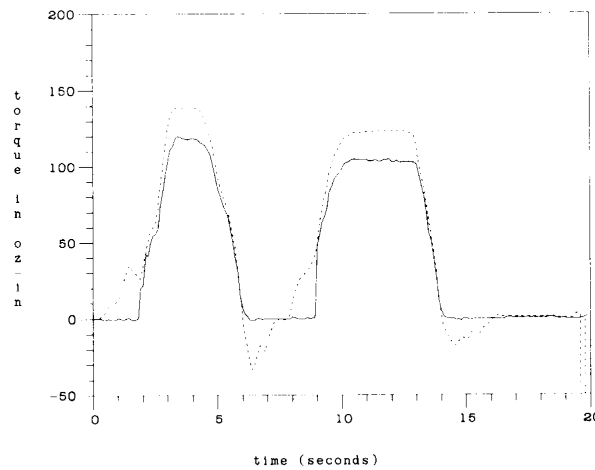


(b)

Fig. 16. Hard contact test for system with 40 ms of time delay and compensation: (a) x_m (solid) and x_s (dashed) versus time; (b) F_m (solid) and τ_m (dashed) versus time.



(a)



(b)

Fig. 18. Hard contact test for system with 200 ms of time delay and compensation: (a) x_m (solid) and x_s (dashed) versus time; (b) F_m (solid) and τ_m (dashed) versus time.

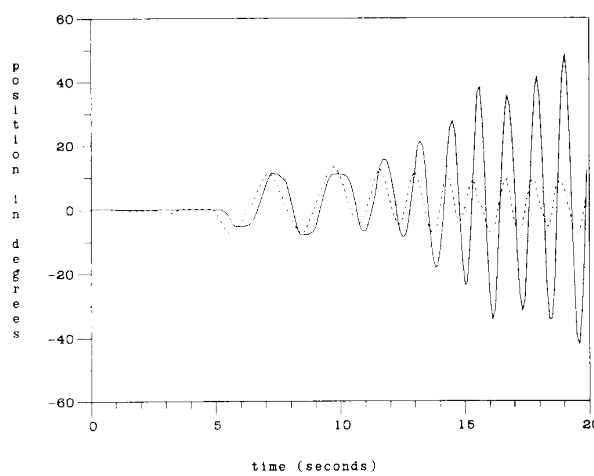


Fig. 17. Backdrivability test for system with 200 ms delay and compensation: x_m (solid) and x_s (dashed) versus time.

By active control we have achieved a communication block which mimics a lossless transmission line and is therefore guaranteed to be a passive element independent of time delay. Thus, stability for the teleoperator has been achieved without restricting the bandwidth of signals.

Insofar as the proposed control law maintains passivity for the master and slave subsystems, the closed-loop system is stable for a wide variety of perturbations. For instance, the communication delay can be unknown and time varying and the environment and human operator dynamics can vary within the entire class of passive system. In addition, the damping and stiffness gains can be arbitrarily assigned. Given this robust stability, the need to model the human and environment and to tune the damping and stiffness gains is dictated solely by performance, rather than stability issues.

Of course, when actuator dynamics are included, or when force feedback is applied around a flexible link or joint, the master and slave subsystems are no longer passive. Typically, this loss of passivity is negligible, affecting the system only at high frequencies where the system gains have rolled off substantially. In any case, this problem restricts the bandwidths of all robot systems and is not unique to bilateral teleoperators.

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