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– module TLABarrier –
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EXTENDS Integers
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CONSTANT threads, num\_threads

VARIABLE TState, n, BState

$$\begin{array}{ll} Init \; \stackrel{\triangle}{=} \; \; \wedge \; n = num\_threads \\ \; \; \wedge \; TState = [t \in threads \mapsto \text{``pre''}] \\ \; \; \wedge \; BState = \text{``init''} \end{array}$$

$$First(r) \triangleq \land BState = \text{``init''} \\ \land TState \in [threads \rightarrow \{\text{``pre''}\}] \\ \land BState' = \text{``stop''} \\ \land TState' = [TState \text{ EXCEPT } ![r] = \text{``at''}] \\ \land \text{UNCHANGED } \langle n \rangle$$

$$AdvanceTo(r) \triangleq \land BState = "stop" \\ \land TState[r] \neq "at" \\ \land (\exists \ t \in threads : TState[t] \neq "at") \\ \land TState' = [TState \ \texttt{EXCEPT} \ ![r] = "at"] \\ \land \texttt{UNCHANGED} \ \langle n, \ BState \rangle$$

$$Advance To Last(r) \triangleq \land BState = "stop" \\ \land TState[r] \neq "at" \\ \land (\forall t \in threads : \lor t = r \\ \lor TState[t] = "at") \\ \land TState' = [TState \ \texttt{EXCEPT} \ ![r] = "at"] \\ \land BState' = "pass" \\ \land \texttt{UNCHANGED} \ \langle n \rangle$$

$$\begin{array}{ll} AdvancePast(r) \triangleq & \land BState = \text{``pass''} \\ & \land TState' = [TState \text{ except } ![r] = \text{``thru''}] \\ & \land \text{UNCHANGED } \langle n, BState \rangle \end{array}$$

$$\begin{array}{ll} Done \; \stackrel{\Delta}{=} \; \; \wedge \; TState \in [threads \rightarrow \{\, \text{``thru''} \, \}] \\ & \; \wedge \; \text{UNCHANGED} \; \langle \, TState, \; BState, \; n \rangle \end{array}$$

$$TNext(r) \triangleq \forall First(r) \\ \lor AdvanceTo(r) \\ \lor AdvanceToLast(r) \\ \lor AdvancePast(r)$$

$$\begin{array}{ccc} TypeOK & \triangleq & \land \ TState \in [threads \rightarrow \{ \text{``pre''}, \text{``at''}, \text{``thru''} \}] \\ & \land \ BState \in \{ \text{``init''}, \text{``stop''}, \text{``pass''} \} \end{array}$$

$$BarrierNext \triangleq \forall Done \\ \forall (\exists t \in threads : TNext(t))$$

 $Spec \ \triangleq \ Init \land \Box [BarrierNext]_{\langle TState, \, n, \, BState \rangle}$