Overview: Basic concepts on Quantum Mechanics

Classical physics
Part II

Javier Orduz-Ducuara

[1] Baylor University

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Contents

Objectives

In this course [General objective] participants will. . .

General Objective (chap. 1)

Appraise historical facts and experiments that started the revolutionary change to Quantum Mechanics such as postulates, entanglement, superposition and others, which are used in Quantum Computing

How do we do?

In this lecture, participants will. . .

Particular Objectives

- Describe the relevance of Quantum Mechanics Postulates.
- Analyze concepts on Physics and Mathematics.

What is Quantum Mechanics?

Quantum Mechanics

Concept

is a fundamental theory in physics that provides a description of the physical properties of nature at the scale of atoms and subatomic particles.

Postulate One

State space

Associated to any isolated physical system is a complex vector space with inner product (that is, a Hilbert space) known as the state space of the system. The system is completely described by its state vector, which is a unit vector in the system's state space [2, pag. 80].

In other words...

It defines the arena where we work, or where the state vector of the system lives.

Postulate Two

Evolution

The evolution of a closed quantum system is described by a unitary transformation [2, pag. 81]. That is, the state $|\psi\rangle$ of the system at time t_1 is related to the state $|\psi'\rangle$ of the system at time t_2 by a unitary operator U which depends only on the times t_1 and t_2,

$$|\psi'\rangle = U|\psi\rangle. \tag{1}$$

In other words...

Quantum mechanics merely assures us that the evolution of any closed quantum system may be described through unitary operator.

Postulate Two (continue. . .)

QM requires the system being describing be closed:

Concept

A closed system is not interacting with the environment or other systems.

Postulate two

The time evolution of the state of a closed quantum system is described by the Shrödinger equation

$$i\hbar\frac{\partial|\psi\rangle}{\partial t} = H|\psi\rangle \qquad (2)$$

H is the Hamiltonian operator.

Postulate Three

Quantum measurement

For a given orthonormal basis $B = \{ |\varphi_i\rangle \}$ of a state space $\mathcal{H}_A$ for a system A, it is possible to perform a Von Neumann measurement on system $\mathcal{H}_A$ with respect to the basis B that, given a state

$$|\psi\rangle = \sum_i \alpha_i |\varphi_i\rangle$$

outputs a label i with probability $|\alpha_i|^2$ and leaves the system in state $|\varphi_i\rangle$.

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Postulate Three (continue. . .)

In other words. . .

QM tells us that if the state $\sum_i \alpha_i |i\rangle$ is provided as input to this apparatus, it will output label i with probability $|\alpha|^2$ and leave the system in state $|i\rangle$.

Besides. . .

Furthermore, given a state $|\psi\rangle = \sum_i \alpha_i |\varphi_i\rangle |\gamma_i\rangle$ from a bipartite state space $\mathcal{H}_A \otimes \mathcal{H}_B$, then performing a measurement on system A will yield outcome i with probability $|\alpha_i|^2$ and leave the bipartite system in state $|\varphi_i\rangle |\gamma_i\rangle$.

Postulate Three (continue...)

Quantum measurement

Quantum measurements are described by a collection $\{M_m\}$ of measurement operators. These are operators acting on the state space of the system being measured. If the state of the quantum system is $|\psi\rangle$ immediately before the measurement then the probability that result m occurs is given by $p(m) = \langle\psi|M_m^\dagger M_m|\psi\rangle$, and the state of the system after the measurement is $\frac{M_m|\psi\rangle}{\sqrt{\langle\psi|M_m^\dagger M_m|\psi\rangle}}$

The index m refers to the measurement outcomes that may occur in the experiment [1, fifth postulate, pag. 180].

Postulate Four

Composite systems

The state space of a composite physical system is the tensor product of the state spaces of the component physical systems. Moreover, if we have systems numbered 1 through n, and system number i is prepared in the state $|\psi_i\rangle$, then the joint state of the total system is

$$|\psi_1\rangle \otimes |\psi_2\rangle \otimes |\psi_3\rangle \otimes \cdots \otimes |\psi_n\rangle \,. \tag{3}$$

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Postulate Four (continue. . .)

In other words. . .

we will primarily be interested in composite systems composed of individual two-level systems. The state of each two-level system is described by a vector in a 2-dimensional Hilbert space. We can encode a qubit in such a two-level system. We would choose a basis for the corresponding 2-dimensional space.

If we want to study potentially useful quantum computations we will need to understand how quantum mechanics works for systems composed of several qubits interacting with each other.

Some questions to discuss

- What is a Hilbert space? and what is a qbit?
- what is an operator?
- Dirac notation?
- What is the Shrodinger equation?
- What does measure means in QM?
- What kind of probabilities we will use in QM?
- What tensorial product means?
- What does post-mesurement state mean?
- What tensorial product means?
- 2-dimensional Hilbert space?

and others. Check the following lectures

Conclusions

- We described the relevance of Quantum Mechanics Postulates.
- We analyzed concepts on Physics and Mathematics.

References

[1] M Loceff. A course in quantum computing (for the community college). foothill college, 2015.

[2] Michael A Nielsen and Isaac Chuang. Quantum computation and quantum information, 2002.